FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
OPEN TEST – II
PAPER –2
TEST DATE: 21-04-2024

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. B

Sol. Using conservation of angular momentum of the system about the axis of rotation.

$$mv\frac{a}{2} = \frac{ma^2\omega}{3} - \frac{mv_1a}{2}$$

$$\frac{ma}{2}(v + v_1) = \frac{ma^2\omega}{3}$$

$$v + v_1 = \frac{2\omega a}{3} \quad \text{...(i)}$$

$$e = \frac{\left(\frac{\omega a}{2} + v_1\right)}{v}$$

$$\frac{\omega a}{2} + v_1 = v \quad \text{(Since, e = 1)}$$

$$v - v_1 = \frac{\omega a}{2} \quad \text{...(ii)}$$

Solving equation (i) and (ii), we get

$$2v = \frac{7\omega a}{6}$$

$$\Rightarrow \omega = \frac{12v}{7a} = \frac{12 \times 14}{7 \times 1} = 24 \text{ rad/s}$$

ω = 24 rad/s

2. A

Sol. In the process AB, $T = \alpha V^2$

$PV = \alpha RV^2$

PV^{-1} = constant

This is a polytropic process with a polytropic constant, $x = -1$

Molar heat capacity, $C = C_V + \frac{R}{(1-x)}$

$C = \frac{3R}{2} + \frac{R}{2}$

C = 2R

$\Delta Q_{AB} = nC\Delta T = 1\times 2R(1200-300) = 1800R$

$\Delta Q_{BC} = nC_V\Delta T = 1\times\frac{3R}{2}(600-1200) = -900\ R$

$\Delta Q_{CA} = nC_P\Delta T = 1\times\frac{5R}{2}(300-600) = -750\ R$

$\Delta W_{cycle} = \Delta Q_{cycle} = \Delta Q_{AB} + \Delta Q_{BC} + \Delta Q_{CA}$

$\Delta W_{cycle} = 1800R - 900R - 750R$

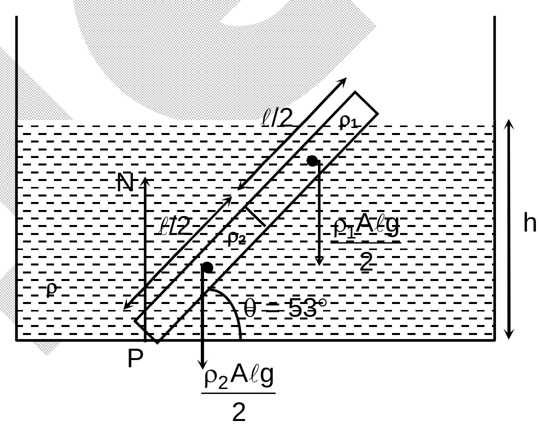
$\Delta W_{cycle} = 150\ R$

Efficiency of the cycle $\eta = \frac{\Delta W_{cycle}}{\Delta Q_{supplied}}\times 100 = \frac{150R}{1800R}\times 100 = 8.33\%$

3. D

Sol. Let the length of rod submerged in the water be x.

Since the rod is in equilibrium,



$\tau_{P} = 0$

$\rho_1\frac{A\ell g}{2}\frac{3\ell\cos\theta}{4} + \rho_2\frac{A\ell g}{2}\frac{\ell\cos\theta}{4} = \rho Axg\frac{x}{2}\cos\theta$

$\frac{\ell^2}{4}(3\rho_1+\rho_2) = \rho x^2$

$x = \frac{\ell}{2}\sqrt{\left(\frac{3\rho_1+\rho_2}{\rho}\right)} = \frac{100}{2}\sqrt{\frac{1.50+2.50}{1}}$

$\Rightarrow$ x = 100 cm

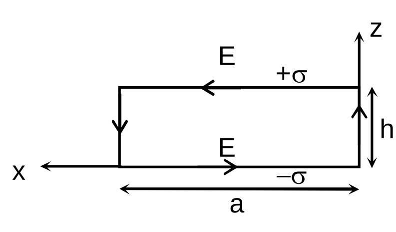
Hence the depth of water in the tub,

h = x sin θ = 100 sin 53°

h = $100\times\frac{4}{5} = 80$ cm

h = 80 cm

4. C



Sol. 2E (a + h) = $-ah\frac{dB}{dt}$

2Ea = $-ah\frac{dB}{dt}$ (∴ h <<a)

$E = -\frac{h}{2}\frac{dB}{dt}$ …(i)

Now, $mv = \int 2\sigma abEdt$

$mv = \sigma ab\int 2Edt$

$$\Rightarrow mv = -\sigma abh\int_{B_0}^{0} dB$$

$mv = \sigma abhB_0$

$$v = \frac{\sigma abhB_0}{m}$$

Hence the velocity acquired by the insulating box,

$$\vec{v} = \left(\frac{\sigma abhB_0}{m}\right)\hat{i}$$

5. A, C

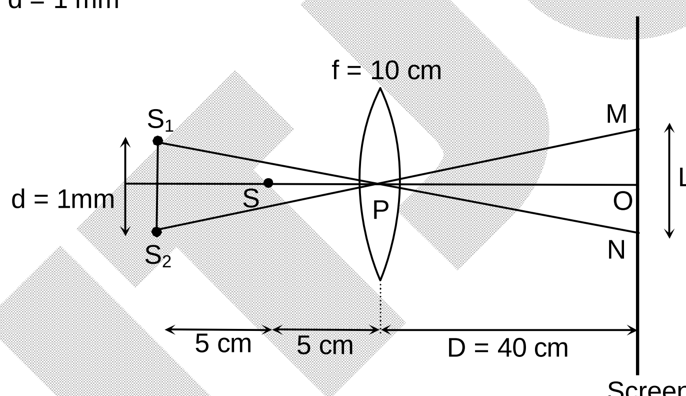
Sol. $\frac{1}{v} - \frac{1}{u} = \frac{1}{f}$

$$\frac{1}{v} - \frac{1}{-5} = \frac{1}{10}$$

$v = -10$ cm

$$m = \frac{v}{u} = \frac{-10}{-5} = +2$$

The two images S_1 and S_2 of the point source S will be formed due to the two halves of the lens at a separation d = 1 mm



Fringe width, $\omega = \frac{\lambda(D+10)}{d} = \frac{4\times10^{-7}\times0.50}{1\times10^{-3}} = 2\times10^{-4}\text{m} = 0.20\text{ mm}$

Now, $\frac{L}{d} = \frac{D}{10} \Rightarrow L = \frac{Dd}{10} = \frac{40}{10}\times1 = 4\text{ mm} \Rightarrow L = 4$ mm

The number of interference fringes obtained on the screen $n = \frac{L}{\omega} = \frac{4}{0.2} = 20$

$\Rightarrow n = 20$

6. B, C

Sol. Since the block stops on the plank before reversing its direction of motion relative to the plank.

$kx = \mu mg$

$$x = \frac{\mu mg}{k} \quad \ldots(i)$$

Now, using work energy theorem for the system relative to the centre of mass frame

$$\frac{1}{2}\left(\frac{mM}{M+m}\right)v_0^2 = \mu mgx + \frac{1}{2}kx^2$$

$$\frac{1}{2}\left(\frac{mM}{M+m}\right)v_0^2 = \frac{\mu^2 m^2 g^2}{k} + \frac{\mu^2 m^2 g^2}{2k}$$

$$\frac{1}{2}\left(\frac{mM}{M+m}\right)v_0^2 = \frac{3\mu^2 m^2 g^2}{2k}$$

$$\Rightarrow v_0 = \mu g\sqrt{\frac{3m(M+m)}{Mk}} = 0.5\times10\sqrt{\frac{3\times1\times4}{3\times100}} = 1 \text{ m/s}$$

Total work done by the friction on the system,

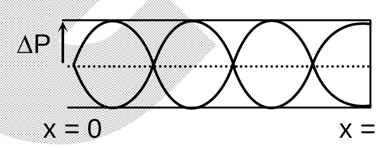
$$\Delta W_{fr} = -\mu mgx = -\frac{\mu^2 m^2 g^2}{k}$$

$$\Rightarrow \quad \Delta W_{fr} = \frac{-(0.5\times1\times10)^2}{100} = -0.25 \text{ J}$$

7. A, B, D

Sol. Since the air column in the pipe vibrates in third overtone,

$$f = \frac{7v}{4\ell} \quad \ldots(i)$$



$$\ell = \frac{7v}{4f} = \frac{7\times330}{4\times660} = \frac{7}{8} \text{ m}$$

$$\ell = \frac{7}{8} \text{ m}$$

Now amplitude of pressure variation,

a = $|\Delta P_0 \sin kx|$

$$a = \left|\Delta P_0 \sin\left(\frac{7\pi x}{2\ell}\right)\right| \quad \ldots(ii)$$

$$\left(\because \quad \frac{7\lambda}{4} = \ell \Rightarrow k = \frac{2\pi}{\lambda} = \frac{7\pi}{2\ell}\right)$$

At x = $\ell/3$, the amplitude of pressure variation is

$$a = \left|\Delta P_0 \sin\left(\frac{7\pi}{2\ell}\times\frac{\ell}{3}\right)\right| = \left|\Delta P_0 \sin\left(\frac{7\pi}{6}\right)\right| = \frac{\Delta P_0}{2}$$

The maximum pressure at x = $\ell/3$ will be $\left(P_0 + \frac{\Delta P_0}{2}\right)$

At $x = \frac{\ell}{2}$, the amplitude of pressure variation is

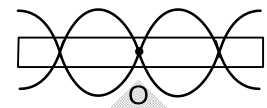
$$a = \left|\Delta P_0 \sin\left(\frac{7\pi}{2\ell}\times\frac{\ell}{2}\right)\right| = \left|\Delta P_0 \sin\left(\frac{7\pi}{4}\right)\right| = \frac{\Delta P_0}{\sqrt{2}}$$

The minimum pressure at x = $\ell/2$ will be $\left(P_0 - \frac{\Delta P_0}{\sqrt{2}}\right)$

SECTION – B

8. 450

Sol. Velocity of longitudinal wave in the solid rod,



$$v_r = \sqrt{\frac{Y_{rod}}{\rho_{rod}}} = \sqrt{\frac{2\times10^{11}}{8\times10^3}} = 5000 \text{ m/s}$$

$$\frac{3\lambda_r}{2} = \ell \Rightarrow \lambda_r = \frac{2\ell}{3} \quad \text{...(i)}$$

Also, $\frac{\lambda_g}{2} = \Delta\ell \Rightarrow \lambda_g = 2\Delta\ell$ ...(ii)

Now, $\frac{v_g}{v_r} = \frac{\lambda_g}{\lambda_r} \Rightarrow \frac{\lambda_g}{\lambda_r} = \frac{2\Delta\ell}{2\ell/3}$

$$\Rightarrow v_g = \frac{3\Delta\ell}{\ell} v_r$$

$$\sqrt{\frac{\gamma RT}{M}} = \frac{3\times0.1\times5\times10^3}{1.2}$$

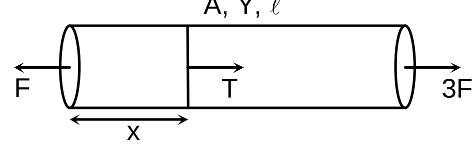
$$\sqrt{\frac{5\times25T}{3\times3\times4\times10^{-3}}} = \frac{5\times10^3}{4}$$

$$\frac{5\times25T}{9\times4\times10^{-3}} = \frac{25\times10^6}{4\times4}$$

$$T = \frac{9000}{20} = 450\text{ K}$$

9. 19

Sol. The acceleration of the rod



$$a = \frac{3F - F}{m} = \frac{2F}{m} \quad \text{...(i)}$$

Now, tension developed in the rod at a distance x from left end,

$$T - F = \left(\frac{mx}{\ell}\right)\left(\frac{2F}{m}\right)$$

$$\Rightarrow T = F\left(1 + \frac{2x}{\ell}\right) \quad \text{...(ii)}$$

The ratio of elastic potential energy stored in the right half to that in the left half of the rod,

$$\eta = \frac{\frac{1}{2Y}\int_{\ell/2}^{\ell}\left(\frac{T}{A}\right)^2 Adx}{\frac{1}{2Y}\int_{0}^{\ell/2}\left(\frac{T}{A}\right)^2 Adx} = \frac{\frac{F^2}{2AY}\int_{\ell/2}^{\ell}\left(1+\frac{2x}{\ell}\right)^2 dx}{\frac{F^2}{2AY}\int_{0}^{\ell/2}\left(1+\frac{2x}{\ell}\right)^2 dx}$$

$$\eta = \frac{\left[x + \frac{2x^2}{\ell} + \frac{4x^3}{3\ell^2}\right]_{\ell/2}^{\ell}}{\left[x + \frac{2x^2}{\ell} + \frac{4x^3}{3\ell^2}\right]_{0}^{\ell/2}}$$

$$\eta = \frac{19\frac{\ell}{6}}{7\frac{\ell}{6}} = \frac{19}{7}$$

Hence, K = 19

10. 15

Sol. Since initially the pistion is in equilibirum,

$$\left(\frac{nRT_0}{A\frac{3\ell}{4}} - \frac{nRT_0}{A\frac{5\ell}{4}}\right)A = \frac{k\ell}{4}$$

$$\frac{8nRT_0}{15\ell} = \frac{k\ell}{4}$$

$$k = \frac{32nRT_0}{15\ell^2} \qquad \ldots(i)$$

Now, when the new equilibrium is established, the total spring energy is utilized in increasing the internal energy of the gas,

$$\frac{1}{2}k\Delta\ell^2 = 2nC_V\Delta T$$

$$\frac{1}{2}k\left(\frac{\ell}{4}\right)^2 = 2n\frac{3R}{2}\Delta T$$

$$\frac{k\ell^2}{32} = 3nR\Delta T$$

$$\frac{nRT_0}{15} = 3nR\Delta T$$

$$\Delta T = \frac{T_0}{45} \Rightarrow \Delta T = \frac{675}{45}$$

$$\Rightarrow \Delta T = 15\text{ k}$$

11. 16

Sol. $$f = \left(\frac{D^2 - x^2}{4D}\right)$$

$$x^2 = D^2 - 4DF$$

$$x = \sqrt{D(D-4f)}$$

$$x = \sqrt{150(150-96)}$$

$$x = \sqrt{150 \times 54}$$

x = 90 cm

The ratio of the sizes of two images formed on the screen,

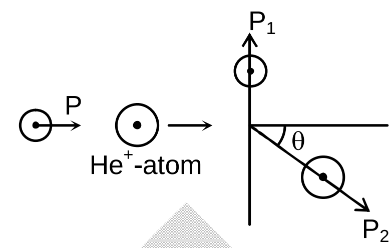
$$\beta = \left(\frac{D+x}{D-x}\right)^2$$

$$\beta = \left(\frac{150+90}{150-90}\right)^2$$

$$\beta = \left(\frac{240}{60}\right)^2 = (4)^2 = 16$$

12. 25

Sol. Using conservation of momentum



$P_2 \cos\theta = P$ …(i)

$P_2 \sin\theta = P_1$ …(ii)

From equation (i) and (ii), we get

$P_2^2 = P^2 + P_1^2$

$2m_2K_2 = 2mK + 2mK_1$

$4K_2 - K_1 = K$

$4K_2 - K_1 = 88$ …(iii)

Now,

$$E_n = -13.6\left(\frac{z^2}{n^2}\right)$$

For He^+ atom

$E_1 = -54.4$ eV

$E_4 = -3.4$ eV

$\Delta E = E_4 - E_1 = -3.4 + 54.4 = 51$ eV

$\Delta E = 51$ eV

Now, $K_1 + K_2 = K - \Delta E$

$K_1 + K_2 = 88 - 51$

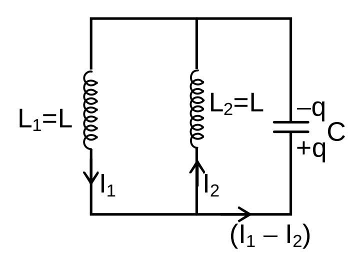
$K_1 + K_2 = 37$ …(iv)

Solving equation (iii) and (iv), we get

$5K_2 = 125 \Rightarrow K_2 = 25$ eV and $K_1 = 12$ eV

13. 120

Sol. The current through the inductor L_1 in the steady state when the switch 'S' is getting in the position-1



$$I_0 = \frac{\varepsilon}{r} = \frac{6}{1} = 6\,A \quad \text{…(i)}$$

After the switch 'S' is shifted to position-2

$$L_1\frac{dI_1}{dt} + L_2\frac{dI_2}{dt} = 0$$

$$L_2\int_0^{I_2} dI_2 = -L_1\int_{I_0}^{I_1} dI_1$$

$L_2I_2 = L_1(I_0 - I_1)$

$L_1I_1 + L_2I_2 = L_1I_0$

When the charge on the capacitor is maximum,

$I_1 = I_2 = I$

$$(L_1 + L_2)I = L_1I_0 \Rightarrow I = \frac{L_1I_0}{L_1 + L_2}$$

$$\Rightarrow I = \frac{I_0}{2} \quad \text{…(ii)}$$

Now, using conservation of energy

$$\frac{1}{2}L_1I_0^2 = \frac{1}{2}(L_1 + L_2)I^2 + \frac{q_{max}^2}{2C}$$

$$\Rightarrow LI_0^2 = 2L \times \frac{I_0^2}{4} + \frac{q_{max}^2}{C}$$

$$\frac{q_{max}^2}{C} = \frac{LI_0^2}{2}$$

$$q_{max} = I_0\sqrt{\frac{LC}{2}} = 6\sqrt{\frac{4\times10^{-4}\times2\times10^{-6}}{2}}$$

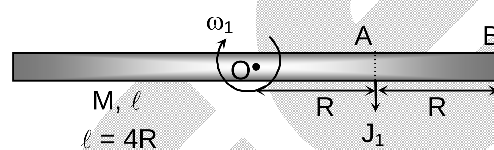
$$q_{max} = 6\times2\times10^{-5} = 120\ \mu\mathbf{C}$$

SECTION – C

14. 20.00

15. 10.00

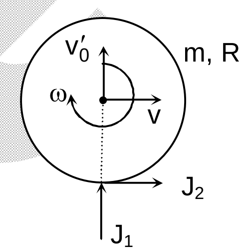
Sol. (Q.14-15): $J_1 = m(v_0' + v_0)$ …(i)



$$J_1R = \frac{M\ell^2}{12}\omega_1$$

$$m(v_0' + v_0)R = \frac{M16R^2}{12}\omega_1$$

$v_0 + v_0' = 4\omega_1R$ …(ii)



$\frac{v_0' + \omega_1R}{v_0} = e = 1$ (for elastic collision)

$v_0 - v_0' = \omega_1R$ …(iii)

Solving equations (ii) and (iii), we get

$$\omega_1 = \frac{2v_0}{5R} = \frac{2\times10}{5\times0.2} = 20\ \text{rad/s}$$

From equation (iii),

$$v_0' = v_0 - \omega_1R = 10 - 20\times0.2 = 6\ \text{m/s}$$

$$J_1 = m(v_0' + v_0) = 1(6+10) = 16\ \text{N-s}$$

$$J_{max} = \mu\int Ndt = \mu J_1 = 0.5\times16 = 8\ \text{N-s}$$

Let the ball starts pure rolling just after collison,

$v = \omega R$

Now, using conservation of angular momentum of the ball about an axis passing through contact point and fixed to the ground

$$\frac{2}{5}mR^2\omega_0 = \frac{2}{5}mR^2\omega + mvR$$

$$\Rightarrow \frac{2}{5}mR^2\omega_0 = \frac{7}{5}mR^2\omega$$

$$\Rightarrow \omega = \frac{2\omega_0}{7} = \frac{2\times35}{7} = 10\ \text{rad/s}$$

Now, $J_2 = mv = m\omega R = 1\times10\times0.2 = 2$ N-s

Hence $J_2 < J_{max}$

16. 1.80

17. 6.25

Sol. (Q.16-17): When $0 \le t < \frac{2R}{v}$

$$\frac{Kq_1}{r_1} + \frac{Kq_2}{r_2} + \frac{KQ}{R} = 0$$

$$Q = -R\left(\frac{q_1}{r_1} + \frac{q_2}{r_2}\right)$$

$$I = \frac{dQ}{dt} = -R\left[\frac{-q_1}{r_1^2}\frac{dr_1}{dt} + \frac{-q_2}{r_2^2}\frac{dr_2}{dt}\right]$$

$$I = R\left[\frac{q_1}{r_1^2}\frac{dr_1}{dt} + \frac{q_2}{r_2^2}\frac{dr_2}{dt}\right] \qquad \left[\text{when, } 0 \le t < \frac{2R}{v}\right]$$

At $t = \frac{R}{v}$

$$I = R\left[\frac{(-2q)(-v)}{4R^2} + \frac{(-q)(-v)}{16R^2}\right]$$

$$I = \frac{9qv}{16R} = \alpha\left(\frac{5qv}{16R}\right)$$

Hence, $\alpha = 1.80$

When, $\frac{2R}{v} \le t < \frac{4R}{v}$

$$\frac{kq_2}{r_2} + \frac{kQ}{R} = 0$$

$$\Rightarrow \quad Q = -R\left(\frac{q_2}{r_2}\right)$$

$$I = \frac{dQ}{dt} = \frac{Rq_2}{r_2^2}\frac{dr_2}{dt} = \frac{Rqv}{r_2^2}$$

At $t = \frac{5R}{2v}$, $I = \frac{4Rqv}{25R^2} = \frac{4qv}{25R}$

n = 6.25

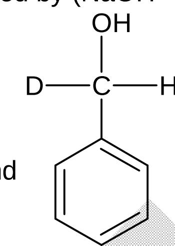
Chemistry

PART – II

SECTION – A

18. D

Sol. Borche's reagent 2,4-DNP which is also known as Brady's reagent.
Sodium hypoiodite solution is NaOI which is prepared by (NaOH + I_2).

Compound (IV) also gives optically active compound 

19. A

Sol.
$$2Y_2O_3 + 8BaCO_3 + 12CuO + O_2 \xrightarrow[\Delta]{850^oC} \underset{(A)}{4YBa_2Cu_3O_7} + 8CO_2 \uparrow$$

Common oxidation state of Y $\rightarrow$ + 3
Common oxidation state of Ba $\rightarrow$ + 2
Oxidation state of Cu $= \left(\frac{+7}{3}\right)$

20. B

Sol.
$$C_2O_4^{2-}(aq) + BaF_2(s) \rightleftharpoons BaC_2O_4(s) + 2F^-(aq)$$
$$0.1\,M \qquad\qquad\qquad\qquad\qquad 0$$
$$0.1 - x \qquad\qquad\qquad\qquad\qquad 2x$$
$$K_{eq} = \frac{[F^-]^2}{[C_2O_4^{2-}]} = \frac{K_{sp}\text{ of }BaF_2}{K_{sp}\text{ of }BaC_2O_4} = \frac{10^{-8}}{10^{-9}}$$
$$\frac{4x^2}{0.1 - x} = 10$$
$$x = 0.096$$
$$[C_2O_4^{2-}] = 4.0 \times 10^{-3}\,M$$
$$[Ba^{+2}] = \frac{10^{-9}}{4.0 \times 10^{-3}} = \frac{10^{-6}}{4.0} = 2.5 \times 10^{-7}\,M$$

21. A

Sol. With the help of data given following thermochemical equations:

$2C(s) + 3H_2(g) \longrightarrow C_2H_6(g)$ $\quad \Delta H_1 = ?$ … (a)

$3C(s) + 4H_2(g) \longrightarrow C_3H_8(g)$ $\quad \Delta H_2 = ?$ … (b)

(i) $H_2(g) + \frac{1}{2}O_2(g) \longrightarrow H_2O(\ell)$ $\quad \Delta H = -68.0\,kcal$

(ii) $C(s) + O_2(g) \longrightarrow CO_2(g)$ $\quad \Delta H = -94.0\,kcal$

(iii) $C_2H_6(g) + \frac{7}{2}O_2(g) \longrightarrow 2CO_2(g) + 3H_2O(\ell)$ $\Delta H = -372$ kcal

(iv) $C_3H_8(g) + 5O_2(g) \longrightarrow 3CO_2(g) + 4H_2O(\ell)$ $\Delta H = -530.0$ kcal

2(ii) + 3(i) – (iii) = $\Delta H_1 = -20$ Kcal

3(ii) + 4(i) – (iv) = $\Delta H_2 = -24$ Kcal

For bond energy calculated $\Delta H = \sum(BE)_R - \sum(BE)_P$

$\Delta H_1 = \left[2 \times \Delta H_f C(g) + 6 \times \Delta H_f H(g)\right] - \left[BE_{C-C} + 6 \times BE_{C-H}\right]$

$x + 6y = 676$ …. (v)

Similarly,

$\Delta H_2 = \left[3 \times \Delta H_f C(g) + 8 \times \Delta H_f H(g)\right] - \left[2x + 8y\right]$

$2x + 8y = 956$ … (vi)

$x = 82$ Kcal / mol. $y = 99$ Kcal / mol.

$\frac{x+y}{4} = \frac{181}{4} = 45.25$

22. A, C, D

Sol. Heat capacity is extensive property.

23. A, C, D

Sol. $\frac{d[P]}{dt} = k[A]^{\alpha}[B]^{\beta}[C]^{\gamma}$

On solving from data given $\alpha = 2, \beta = 0, \gamma = 0.5$

Over all order = 2.5.

$\frac{d[P]}{dt} = k[A]^2[C]^{\frac{1}{2}}$

$0.002 = k[0.01]^2[0.01]^{\frac{1}{2}}$

$k = 200\ M^{-3/2}\ h^{-1}$

There is large excess of B and C with respect to [A] and thus the rate equation simplifies and become

$\text{rate} = k[A]^2[B]^o[C]^{\frac{1}{2}}$

$= k'[A]^2$ where $k' = k.[2.0]^{\frac{1}{2}}$.

Pseudo second order Kinetics

$t_{1/2} = \frac{1}{[A]_o.k'} = \frac{1}{0.01 \times 200 \times (2)^{\frac{1}{2}}} \times 60$ Min

$= \frac{60}{2\sqrt{2}}\text{Min} = \frac{30}{\sqrt{2}} = \frac{30\sqrt{2}}{2} = 15\sqrt{2}$

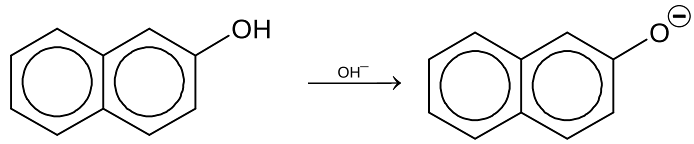
$= 15 \times 1.414$

$= 21.210$ Min

24. A, D

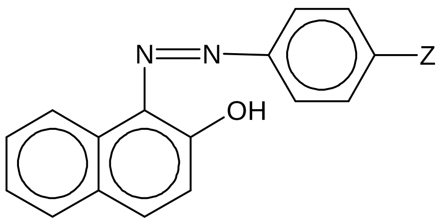
Sol.

X →



(9 canonical structure are possible)

Y →



Electron withdrawing group in diazonium salt increases the rate of coupling reaction.

SECTION – B

25. 15

Sol. x = 5, y = 4, z = 6

26. 110

Sol. $CH_3-CH_2-CH_2-C\equiv C-CH_2-CHO$

or

$Pr-C\equiv C-CH_2-CHO$

(A)

27. 5

Sol. All orders are correct except (V) and (VI)

(3 > 2 = 1)

C_6H_{14} total structural isomers are 5.

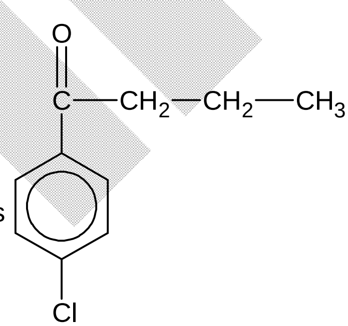
C_4H_8 total structural isomers are 5.

C_4H_6 total structural isomers are 9.

28. 73

Sol.

Compound 'X' is



a = 182.5

b = 5

$$\frac{2a}{b}=\frac{2\times 182.5}{5}=73$$

29. 76

Sol.
$$\underset{(X)}{FeSO_4.7H_2O} \xrightarrow{\Delta} \underset{(Y)}{FeSO_4} \xrightarrow{\Delta} \underset{(A)}{Fe_2O_3} + \underset{(B)}{SO_2} + \underset{(C)}{SO_3}$$

$$\underset{(Y)}{FeSO_4} + H_2O + NO \longrightarrow \underset{(D)}{\left[Fe(H_2O)_5 NO\right]SO_4}$$

Number of unpaired electron in (D) are 3.
Molar mass of compound (A) is 160.
Molar mass of compound (B and C) is 144.
x = 3, a = 160, b = 144.

30. 20

Sol. Correct statements are – III, V, VI, VII and VIII.
Incorrect statements are – I, II, IV.

SECTION – C

31. 4.93
[4.92 – 4.95]

Sol.
$$P = \frac{RT}{V-\beta} - \frac{\alpha}{TV^2}$$

$$\frac{PV}{RT} = \left(1-\frac{\beta}{V}\right)^{-1} - \frac{\alpha}{RT^2V} = \left(1+\frac{\beta}{V}+\frac{P^2}{V^2}+....\right) - \frac{\alpha}{RT^2V} = 1+\left(\beta-\frac{\alpha}{RT^2}\right)\frac{1}{V}+\frac{\beta^2}{V^2}+.....$$

32. 0.72

Sol.
$$\left(P+\frac{a}{V_m^2}\right)(V_m) = RT$$

$$PV_m^2 - RTV_m + a = 0$$

$$PV_m^2 - 2.4V_m + a = 0$$

For real gas this equation must have only one root.

D = 0, $b^2 - 4ac = 0$, $(2.4)^2 - 8P = 0$

$\Rightarrow (P = 0.72 \text{ atm})$

33. 0.20

34. 0.56

Sol. (Q.No. 33 and 34):
Ore is Chromite $FeO.Cr_2O_3$

$$\underset{(A)}{4FeO.Cr_2O_3} + 8Na_2CO_3 + 7O_2 \xrightarrow[\Delta]{CaO} \underset{(B)}{2Fe_2O_3} + \underset{(C)}{8Na_2CrO_4}$$

0.4 mole 0.2 mole 0.8 mole

$$Fe_2O_3 + 6HCl \longrightarrow 2FeCl_3 + 3H_2O$$

$$4FeCl_3 + 3K_4Fe(CN)_6 \longrightarrow \underset{(D)}{Fe_4\left[Fe(CN)_6\right]_3} + 12KCl$$

Prussian blue

$$2Na_2CrO_4 + H_2SO_4 \xrightarrow[\text{yield}]{70\%} Na_2Cr_2O_4 + Na_2SO_4 + H_2O$$

(C) (E)

0.8 mole 0.28 mole

$$Na_2Cr_2O_7 + 2KCl \longrightarrow K_2Cr_2O_7 + 2NaCl$$

(E)

$$K_2Cr_2O_7 + 4H_2SO_4 + 3H_2C_2O_4 \longrightarrow K_2SO_4 + Cr_2(SO_4)_3 + 6CO_2 + 7H_2O$$

$$Cr_2(SO_4)_3 + 6K_2C_2O_4 \longrightarrow 2K_3\left[Cr(C_2O_4)_3\right] + 3K_2SO_4$$

0.28 mole 0.56 mole (G)

Mathematics

PART – III

SECTION – A

35. C

Sol. $f\left(\frac{1}{x}\right)=\tan^{-1}x^{-2}+\tan^{-1}x^{-4}+\tan^{-1}x^{-6}+.....+\tan^{-1}x^{-2n}$

$=\frac{\pi}{2}-\tan^{-1}x^{2}+\frac{\pi}{2}-\tan^{-1}x^{4}+\frac{\pi}{2}-\tan^{-1}x^{6}+.....+\frac{\pi}{2}-\tan^{-1}x^{2n}$

$=\frac{n\pi}{2}-f(x)$

$\Rightarrow \int_a^{a+1} f\left(\frac{1}{x}\right)dx=\int_a^{a+1}\left(\frac{n\pi}{2}-f(x)\right)dx=\frac{n\pi}{2}-\int_a^{a+1} f(x)dx$

Now, $\int_a^{a+1} f(x)dx=\lim_{x\to\infty}\ln\left(\sum_{n=0}^{x}\int_0^{x/e}\frac{e^{n-1}\cdot u^{n-1}\cdot e^{-u}}{((n-1)!)^2}du\right)$

$=\lim_{x\to\infty}\ln\left(\sum_{n=0}^{x}\frac{e^{n-1}}{((n-1)!)^2}\int_0^{x/e}u^{n-1}\cdot e^{-u}\,du\right)$ (By putting $\frac{y}{e}=u$)

$=\lim_{x\to\infty}\ln\left(\sum_{n=0}^{x}\frac{e^{n-1}}{((n-1)!)^2}\cdot(n-1)!\right)$; $\left[\lim_{x\to\infty}e^{-\frac{x}{e}}=0\right]$

$=\lim_{x\to\infty}\ln\sum_{n=0}^{x}\frac{e^n}{n!}=\ln e^e=e$

So, $\int_a^{a+1} f(x)dx=\frac{n\pi}{2}-e$

36. A

Sol. Tangent to xy = 4 at $B\left(\frac{2}{3},6\right)$ is 9x + y = 12

37. B

Sol. $4^x+16^x+32^x+144^x=1-\sqrt{\frac{1}{4}-x^2}$

LHS is increasing within domain (–∞, ∞)

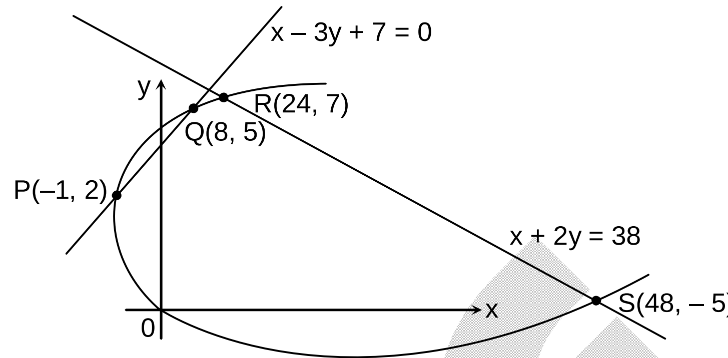
RHS is decreasing in $\left[-\frac{1}{2},0\right)$ and increasing in $\left[0,\frac{1}{2}\right)$

So, LHS > RHS no solution $x\in\left[0,\frac{1}{2}\right)$

LHS = RHS has one solution $x=-\frac{1}{2}$

38. C

Sol. $(a^2 - 1, a + 2)$ is on the parabola
$(x + 1) = (y - 2)^2$
So, $a + 2 \in (-5, 2) \cup (5, 7)$
$\Rightarrow a \in (-7, 0) \cup (3, 5)$



39. A, B, C, D

Sol. (A) $\frac{2n - \sin 2n}{4n} + \int_0^1 (f(\sin x))^2 dx \ge 2\int_0^1 \sin nx \cdot f(\sin x) dx$

Consider $\int_0^1 \left(\frac{1 - \cos 2nx}{2}\right) dx + \int_0^1 (f(\sin x))^2 dx - 2\int_0^1 \sin nx \cdot f(\sin x) dx$

$= \int_0^1 \sin^2(nx) dx + \int_0^1 (f(\sin x))^2 dx - 2\int_0^1 \sin nx \cdot f(\sin x) dx$

$= \int_0^1 (\sin x - f(\sin x))^2 dx \ge 0$ so TRUE

(B) $|z_2 + iz_1| = |z_1| + |z_2| \Rightarrow z_2, iz_1, 0$ are collinear

$z_3 = \left(\frac{z_2 - iz_1}{1 - i}\right) \Rightarrow \text{Arg}\left(\frac{z_3 - z_2}{z_3 - z_1}\right) = \frac{\pi}{2}$ and $|z_3 - z_2| = |z_3 - z_1|$

So, AC = BC, $AB^2 = AC^2 + BC^2$

So, area = $\frac{25}{4}$ square units

(C) Let $f \cdot g : R \to R$ be two bijective functions $h(x) = f(x) + g(x)$

Hence, $\frac{17}{4} - f(x) = -\frac{17}{4} + g(x) \Rightarrow f(x) + g(x) = \frac{17}{2}$

(D) $\left(\left(x + \frac{1}{x}\right)^2 - 2\right) + k\left(x + \frac{1}{x}\right) + 1 = 0$

$\Rightarrow t^2 + kt - 1 = 0$ where $t = x + \frac{1}{x}$, $x \ne -1$

$t < 0 \Rightarrow t < -2$

So, $f(-2) < 0 \Rightarrow 4 - 2k - 1 < 0 \Rightarrow k > \frac{3}{2}$

40. A, B, C, D

Sol. $f(m) = \lim_{x \to 0^+} \text{cosec}\, x \cdot \cos^{-1}\left(\frac{m^2 \text{cosec}^2 x - 1}{m^2 \text{cosec}^2 x + 1}\right)$

Let $\frac{m^2 - \sin^2 x}{m^2 + \sin^2 x} = y \Rightarrow \text{cosec}\, x = \frac{1}{m}\sqrt{\frac{1+y}{1-y}}$

So, $\lim_{y \to 1} \frac{1}{m}\sqrt{\frac{1+y}{1-y}} \cos^{-1}(y) = \frac{2}{m}$

(A) $\lim\limits_{y\to 1}\left(1+\frac{1}{2}+.....+\frac{1}{2^{n-1}}\right)=2$

(B) $\left[\frac{1}{1^{\frac{2}{3}}}+\frac{1}{2^{\frac{2}{3}}}+.....+\frac{1}{1000^{\frac{2}{3}}}\right]=27$

(C) $f(r)=\frac{2}{r} \Rightarrow f(2r)=\frac{1}{r}$

$$\text{So, } \prod_{r=1}^{n}\frac{\left((f(2r))^2+1\right)^2}{(f(r))^4+1}=\prod_{r=1}^{n}\left(\frac{\left(\frac{1}{r^2}+1\right)^2}{\frac{4}{r^4}+1}\right)=\prod_{r=1}^{n}\frac{\left(1+r^2\right)^2}{\left(4+r^4\right)}=2$$

41. A, B, D

Sol. $f^{-1}(x)=x^{2n+1}+x$ which is an odd function

SECTION – B

42. 2

Sol. 2n $T_r(A)$ + det(A) = 8 ⇒ det(A) = 8
det(A^2 + det(A)·A) + 0 = det(A)·(det(A + 8I)) = 8(64 + 8) = 8.72 = $2^6 \cdot 3^2$
Hence, 2 ways

43. 512

Sol. $x^4+3x^3+2x^2+x=1 \Rightarrow (x^2+2x-1)(x^2+x+1)=0$

$x=\sqrt{2}-1, -\sqrt{2}-1$

$x=\tan\frac{\pi}{8}, \tan\frac{5\pi}{8}$

$x=\frac{z_{n+1}-z_n}{1+z_n z_{n+1}}$ let $z_n = \tan\theta_n$

$x=\tan(\theta_{n+1}-\theta_n)=\tan\frac{\pi}{8} \Rightarrow \theta_{n+1}-\theta_n=\frac{\pi}{8}$

$\theta_2-\theta_1=\frac{\pi}{8}$

$\theta_3-\theta_2=\frac{\pi}{8}$

$\vdots$

$\theta_n-\theta_{n-1}=\frac{\pi}{8}$

$\theta_n=\theta_1+(n-1)\frac{\pi}{8}$

For $1\le p\le 8$, $r\ge 0$, $z_{8r+p}=\tan\left(\theta_1+(8r+p-1)\frac{\pi}{8}\right)=z_p$ (let)

$$\sum_{r=1}^{4096}=\sum_{j=1}^{511}\left(\sum_{k=1}^{r}z_{8r+r}\right)=\sum_{j=1}^{511}\left(\sum_{k=1}^{r}z_k\right) \Rightarrow \frac{\sum\limits_{r=1}^{4096}}{\sum\limits_{k=1}^{8}z_k}=512$$

44. 4

Sol. Equation of the circle passing through the point of intersection $x^2 + y^2 = 8\sqrt{2}$

So, radius of director circle = $4\left(2^{1/4}\right) = R$

$\therefore$ [R] = 4

45. 54

Sol. Possible cases 2B5W, 3B4W, 4B3W, 5B2W, 6B1W favourable case 4B3W

So, required probability = $\frac{9}{35}$

So, xyz = $3^2 \cdot 5 \cdot 7$

46. 1

Sol. By rewriting f(x), we have $f(x) = \frac{\sqrt{2}\sin\left(\pi x - \frac{\pi}{4}\right) + 2}{\sqrt{x}}$ for $\frac{1}{4} \le x \le \frac{5}{4}$

Define $g(x) = \sqrt{2}\sin\left(\pi x - \frac{\pi}{4}\right)$, where $\frac{1}{4} \le x \le \frac{5}{4}$

Then g(x) $\ge$ 0, g(x) is monotone increasing on $\left[\frac{1}{4}, \frac{3}{4}\right]$, and monotone decreasing on $\left[\frac{3}{4}, \frac{5}{4}\right]$

Further, the graph of y = g(x) is symmetric about $x = \frac{3}{4}$, i.e. for any $x_1 \in \left[\frac{1}{4}, \frac{3}{4}\right]$ there exists

$x_2 \in \left[\frac{3}{4}, \frac{5}{4}\right]$ such that $g(x_2) = g(x_1)$

Then, $f(x_1) = \frac{g(x_1)+2}{\sqrt{x_1}} = \frac{g(x_2)+2}{\sqrt{x_1}} \ge \frac{g(x_2)+2}{\sqrt{x_2}} = f(x_2)$

On the other hand, f(x) is monotone decreasing on $\left[\frac{3}{4}, \frac{5}{4}\right]$

Therefore $f(x) \ge f\left(\frac{5}{4}\right) = \frac{4\sqrt{5}}{5}$. That means the minimum value of f(x) on $\left[\frac{1}{4}, \frac{5}{4}\right]$ is $\frac{4\sqrt{5}}{5}$

47. 199

Sol. Let P = $(t^2, 2t)$, and we obtain the equation of l as $y = x + 2t - t^2$. Plug it into the equation for C_2, $(x-4)^2 + (x + 2t - t^2)^2 = 8$, and simplify, to get $2x^2 - 2(t^2 - 2t + 4)x + (t^2 - 2t)^2 + 8 = 0$ ..... (1)

Since, Q, R are distinct intersections, the discriminant of quadratic equation (1) is positive

So, $\frac{\Delta}{4} = (t^2 - 2t + 4)^2 - 2\left[(t^2 - 2t)^2 + 8\right] = (t^2 - 2t)^2 - 8(t^2 - 2t) + 16 - 2(t^2 - 2t)^2 - 16$

$= -(t^2 - 2t)^2 + 8(t^2 - 2t) = -(t^2 - 2t)(t^2 - 2t - 8) = -t(t-2)(t+2)(t-4) > 0$

and we derive $t \in (-2, 0) \cup (2, 4)$ ..... (2)

Let x_1, x_2 be the x-coordinates of Q, R respectively. By equation (1)

$x_1 + x_2 = t^2 - 2t + 4,\ x_1x_2 = \frac{1}{2}\left[(t^2 - 2t)^2 + 8\right]$

Furthermore, since l is parallel to y = x, $|PQ| \cdot |PR| = \sqrt{2}\left(x_1 - t^2\right) \cdot \sqrt{2}\left(x_2 - t^2\right)$

$= 2x_1x_2 - 2t^2(x_1 + x_2) + 2t^4 = (t^2 - 2t)^2 + 8 - 2t^2(t^2 - 2t + 4) + 2t^4$

$= (t^2 - 2)^2 + 4$ ..... (3)

By equation (2), $t^2 - 2 \in (-2, 2) \cup (2, 14)$, $(t^2 - 2)^2 \in [0, 4) \cup (4, 196)$

So, we conclude from (3), that $|PQ| \cdot |PR| = (t^2 - 2)^2 + 4 \in [4, 8) \cup (8, 200)$

SECTION – C

48. 1.00

Sol. We have f(x) = $\sin^2 x + a\sin x + a - \frac{3}{a}$. Let t = sin x ; ($-1 \le t \le 1$).

Then, $g(t) = t^2 + at + a - \frac{3}{a}$

The sufficient and necessary condition for $f(x) \le 0,\ \forall x \in R$ is $\begin{cases} g(-1) = 1 - \frac{3}{a} \le 0 \\ g(1) = 1 + 2a - \frac{3}{a} \le 0 \end{cases}$

Therefore, we obtain the range a of a is (0, 1]

49. 1.50

Sol. As $a \ge 2$, then $-\frac{a}{2} \le -1$, we have $g(t)_{min} = g(-1) = 1 - \frac{3}{a}$

Then $f(x)_{min} = 1 - \frac{3}{a}$. Therefore, the sufficient and necessary condition for $f(x) \le 0,\ \exists x \in R$ is

$1 - \frac{3}{a} \le 0$ or $0 < a \le 3$

Finally, we obtain that the range of a is [2, 3]

50. 6.00

Sol. We have, $f(x) = \lim_{\alpha \to 0} \frac{1}{\alpha^4} \int_0^{\alpha} \frac{(e^{x+t} - e^x)\ln^2(1+t)}{2t^3 + 3} dt$ = $f(x) = \lim_{\alpha \to 0} \frac{e^x \int_0^{\alpha} \frac{(e^t - 1)\ln^2(1+t)}{(2t^3+3)} dt}{\alpha^4}$

$= \lim_{\alpha \to 0} e^x \frac{(e^{\alpha} - 1)\ln^2(1+\alpha)}{(2\alpha^3 + 3)4\alpha^3}$ = $e^x \lim_{\alpha \to 0} \frac{(e^{\alpha} - 1)}{\alpha} \cdot \frac{\ln^2(1+\alpha)}{\alpha^2} \cdot \frac{1}{4(2\alpha^3 + 3)} = \frac{e^x}{12}$

Clearly, $f(\ln 2) = \frac{e^{\ln 2}}{12} = \frac{2}{12} = \frac{1}{6}$

51. 4.50

Sol. We have, $\int_0^x g(t)dt = 3x + \int_x^0 \cos^2 t \cdot g(t) dt$. Now, on differentiating both the sides with respect to x,

we get g(x) = 3 – $\cos^2 x$ g(x) $\Rightarrow g(x) = \frac{3}{1 + \cos^2 x}$

Clearly, $\frac{3}{2} \le \frac{3}{1 + \cos^2 x} \le 3$. Hence, range of $g(x) = \left[\frac{3}{2}, 3\right]$