FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
FULL TEST – V
PAPER –1
TEST DATE: 08-02-2024

ANSWERS, HINTS & SOLUTIONS

Physics

PART – I

SECTION – A

1. ACD

Sol. By Lenz law we can predict that East end will be at higher potential.

2. ABC

Sol. $\varepsilon = \frac{d\phi}{dt} = 4n\,t^{n-1}$

3. BC

Sol. $v_L = v_C = v_R$;

$\Rightarrow x_L = x_C = R$

when inductor is short circuited

$Z = \sqrt{R^2 + x_C^2} = \sqrt{2}\,R$

$\therefore I = \frac{30}{Z} = \frac{30}{\sqrt{2}R}$

$\therefore V_L = ix_L = \frac{30}{\sqrt{2}R} \times R = \frac{30}{\sqrt{2}}$

$\therefore$ (A) is incorrect and with similar calculations (B) will be correct.

Here f_0 is the resonance frequency as $v_L = v_C$

$\omega_0 = 2\pi f_0 = \frac{1}{\sqrt{LC}}$

$\frac{x_L}{x_C} = \frac{\omega L}{1/\omega C} = \omega^2 LC$

Given $f = 2f_0$

$\Rightarrow \quad \omega = 2\omega_0$

$\therefore \quad \frac{x_L}{x_C} = 4$

$\therefore$ (C) is also correct.

4. B

Sol. $E_p = \frac{\sigma}{4\varepsilon_0}(2-\sqrt{2})$, $V_p = \frac{\sigma r}{2\varepsilon_0}(2-\sqrt{2})$

5. A

Sol. $3G_1 = 2G_2$; $\frac{G_1}{G_2} = \frac{2}{3}$

6. B

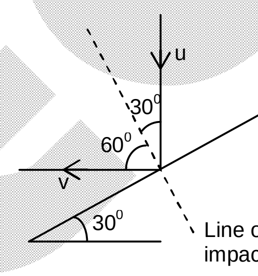
Sol. $W = B^2V^2\ell^2 t\left[\frac{1}{R}+\frac{t}{2L}\right] = 32$ J.

7. B

Sol. Let velocity of ball just before collision is u and just after collision is v.

u sin 30° = v sin 60°

$$e = \frac{v\cos 60^0}{u\cos 30^0} = \frac{1}{3}$$



8. B

Sol. In clockwise cyclic process $\theta_{cycle} > 0$ and $W_{cycle} > 0$.

In anticlockwise cyclic process $\theta_{cycle} < 0$ and $W_{cycle} < 0$.

9. A

Sol. Stress at a distance x from lower end = $\frac{F + \frac{mgx}{\ell}}{A}$

Speed of longitudinal wave in rod = $\sqrt{\frac{Y}{\rho}}$

Speed of transverse wave in rod at a distance x from lower end = $\sqrt{\frac{F+\frac{mgx}{\ell}}{\frac{m}{\ell}}}$

If elongation of rod is Δl then elastic pot. energy in the rod will be = $\frac{1}{2}Y\left(\frac{\Delta\ell}{\ell}\right)^2 A\ell$

10. C

Sol. Let the radius of circle in which particle moves is R. In this magnitude of region electric field is $E = \frac{R}{2}\left(\frac{dB}{dt}\right)$ as $qE = m\frac{dV}{dt}$

$$\Rightarrow \quad \frac{qR}{2}\left(\frac{dB}{dt}\right) = m\frac{dV}{dt}$$

also $R = \frac{mV}{Bq}$

$\Rightarrow \quad \frac{dV}{V} = \frac{1}{2}\frac{dB}{B}$

as $R = \frac{mV}{Bq}, \frac{dV}{V} = \frac{dq}{q} + \frac{dB}{B}, \frac{dq}{q} = -\frac{1}{2}\frac{dB}{B}$

11. D

Sol. P $\rightarrow$ 2, 4 from Newton's law of cooling $\frac{dT}{dt} \propto A$

Q $\rightarrow$ 3, 1 the potential at an outside point, $V = \frac{kQ}{r}$ as Q and V both are fixed.

R $\rightarrow$ 2, 4 $F = \rho a v^2$ with time v decreases

S $\rightarrow$ 1, 3 $\phi = BA \cos 90 = 0$.

SECTION – B

12. 8

Sol. $F.x - \mu m_1 g x - \frac{1}{2}kx^2 = 0$

$kx = \mu m_2 g$ for just shifting m_2

$F.x - \mu m_1 g x - \frac{1}{2}\mu m_2 g x = 0$

$F = \mu\left(m_1 + \frac{m_2}{2}\right)g = 0.4\left(1 + \frac{2}{2}\right)(10) = 8N$

13. 2

Sol. $\delta = x + y - z$

14. 5

Sol. Strain $= \frac{2 \times 10^{-3}}{4} = 5 \times 10^{-4}$

Stress = Y × strain = $2 \times 10^{11} \times 5 \times 10^{-4} = 10^8$.

$\therefore$ Energy density $= \frac{1}{2} \times \text{Stress} \times \text{Strain} = \frac{1}{2} \times 10^8 \times 5 \times 10^{-4} = \frac{5}{2} \times 10^4 \, J/m^3$.

15. 6

Sol. As source (horn of bus) is approaching stationary wall (say, listener), therefore, apparent frequency striking the wall is

$$\nu' = \frac{\upsilon \nu}{\upsilon - \upsilon_s} \quad \ldots(1)$$

Sound of this frequency will be reflected by the wall (now, source). The passenger is the listener moving towards source. Therefore, frequency heard by the listener

$$\nu'' = \frac{(\upsilon + \upsilon_L)\nu'}{\upsilon}$$

using (1) $v'' = \frac{\upsilon + \upsilon_L}{\upsilon} \times \frac{\upsilon v}{\upsilon - \upsilon_s} = \frac{(\upsilon + \upsilon_L)v}{\upsilon - \upsilon_s}$

$= \frac{(330+5)\times 200}{330-5} = \frac{335}{325} \times 200$

$v'' = 206$ Hz

$\therefore$ Beat frequency = $(v'' - v)$

$= 206 - 200 = 6$ Hz

16. 1

Sol. K = 50

$\frac{1}{2}KA^2 = \frac{1}{2}mv^2$

$\Rightarrow \quad \frac{1}{2} \times 50 \times A^2 = \frac{1}{2} \times \frac{1}{2} \times 10^2 \qquad \Rightarrow$ A = 1 m.

17. 6

Sol. In process BC, $(2P_o)(2V_o)^{7/5} = P_C V_C^{7/5}$

In process CA, $P_o V_o = P_C V_C$

Take ratio and solve.

Chemistry PART – II

SECTION – A

18. ABCD

Sol. $\Delta S = \frac{q}{T} = nC_V \ln\frac{T_2}{T_1} + nR\ln\frac{V_2}{V_1}$

Apply above formula for ΔS calculation. In reversible process $\Delta S_{total} = 0$

19. ABC

Sol. Options A, B & C are examples of E_{CB}^1 mechanism as E_{CB}^1 is favoured by resonance stabilized conjugate base, poor leaving group and strong base.

20. ABC

Sol. Non-aqueous ionizing solvents are characterized by facts in ABC.

21. D

Sol. $[B]_t + [C]_t = 2$ M

$$\frac{[B]_t}{[C]_t} = \frac{2k_1}{3k_2} = \frac{2\times 2\times 10^{-4}}{3\times 3\times 10^{-4}} = \frac{4}{9}$$

$[B]_t = \frac{4}{9}C_t$ putting it in

$$\frac{4}{9}C_t + C_t = 2M$$

$$C_t = \frac{18}{13}M$$

Hence $B_t = \frac{8}{13}M$

22. D

Sol. Mutarotation involve both acid as well as base in ring opening of cyclic structure of monosaccharides in rate determining steps.

23. B

Sol. $HA \rightleftharpoons H^+ + A^-$

$$K_a = \frac{C\alpha^2}{1-\alpha} \quad ----(i)$$

$$K_a = \frac{(C\alpha)\alpha}{1-\alpha} \quad \because H^+ = C\alpha$$

$$K_a = \frac{[H^+]\alpha}{1-\alpha}$$

$$[H^+] = K_a\frac{(1-\alpha)}{\alpha}$$

Learning log value calculated

$-\log[H^+] = \left[-\log K_a^+\right] - \log\frac{1-\alpha}{\alpha}$

$$pH = P^{K_a} - \log\frac{1-\alpha}{\alpha}$$

$$-\log\left(\frac{1-\alpha}{\alpha}\right) = pH - P^{K_a}$$

$$\log\frac{1-\alpha}{\alpha} = P^{K_a} - pH$$

$$\frac{1-\alpha}{\alpha} = 10^{P^{K_a} - pH}$$

$$\frac{1}{\alpha} - 1 = 10^{P_{K_a} - pH}$$

$$\frac{1}{\alpha} = 10^{P_{K_a} - pH} + 1$$

$$\alpha = \frac{1}{1+10^{P_{K_a} - pH}}$$

24. A

Sol. Apply MO concept in O_2, in ground state it is triplete and electronic transition convert into excited singlet state.

25. C

Sol. Consider the nature of groups +R/-R and special nature of X(halogen).

26. A

Sol. As we known cope elimination involve syn elimination.

27. A

Sol. Thermodynamic stability of substance depend on temperature.

28. A

Sol. CaC_2 + Ca_3P_2 mixture is known as holme's signal, by charge balancing we can identify nature of carbide.

SECTION – B

29. 11

Sol. If there is no hydrogen bonding in hydrazone, further oxidation of alcoholic group is possible.

30. 5

Sol. Apply the concept of Keq and put the values of respective species.

31. 6

Sol. Heteroatom are atoms other than C and H present in organic compound.

32. 7

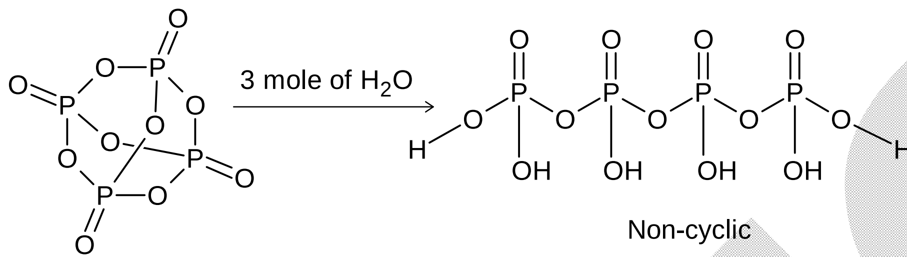
Sol. Apply Huckel's rule of aromaticity.

33. 5

Sol. Compounds having active or acidic 'H' atoms can show tautomerism.

34. 3

Sol.



Cyclic

Non-cyclic

Mathematics PART – III

SECTION – A

35. AB

Sol. $h(x) = [f(x)]^2 + [f'(x)]^2$

$h'(x) = 2f'(x)[f(x) + f''(x)] = 2f'(x)[-xg(x)f'(x)]$

$= -2x[f'(x)]^2 g(x)$

When $x > 0 \Rightarrow h'(x) \le 0$

So, $h(x)$ is decreasing.

$h(0) = (f(0))^2 + [f'(0)]^2 = 16 + 9 = 25$

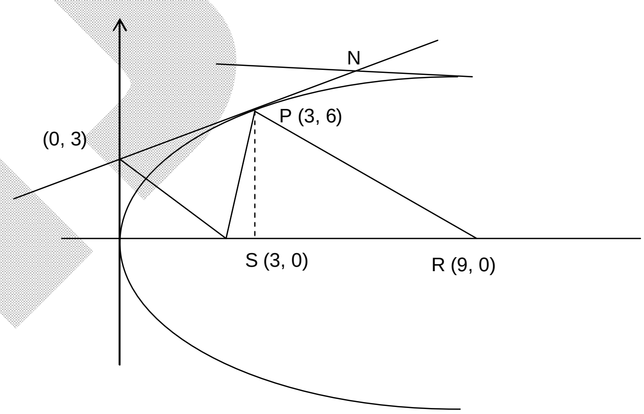
$h(x)$ is decreasing so $\begin{matrix} x > 0 \\ h(x) < h(0) \end{matrix}$

$[f(x)]^2 + [f'(x)]^2 \le 25$

$\Rightarrow [f(x)]^2 \le 25 \Rightarrow |f(x)| \le 5$

36. BC

Sol. Let $P = (3t^2, 6t)$



$PS = 3 + 3t^2 = 6 \Rightarrow t = \pm 1$ but $t > 0$

$\Rightarrow t = 1 \Rightarrow P(3, 6)$

Let $M = (3t_1^2, 6t_1)$

Tangent at

$M: t_1 y = x + 3t_1^2$ ....(1)

$P: 6y = \frac{12}{2}(x+3)$

$y = x + 3$ ....(2)

Intersection of (1) and (2)

$(t_1 - 1)y = 3t_1^2 - 3$

$y = 3(t_1 + 1)$

$x = 3t_1$

$\Rightarrow N = (3t_1, 3(t_1 + 1))$

$SN = 5 = \sqrt{(3t_1 - 3)^2 + 3^2(t_1 + 1)^2}$

$\Rightarrow 25 = 9[2t_1^2 + 2] \Rightarrow t_1^2 = \frac{7}{18}$

$t_1 = \pm\sqrt{\frac{7}{18}}$

$$\Rightarrow M = \left(3 \times \frac{7}{18}, \pm 6\sqrt{\frac{7}{18}}\right)$$

Sum of square of possible ordinates of M $= 36 \times \frac{7}{18} + 36 \times \frac{7}{18}$

= 28

Area of quadrilateral PRST

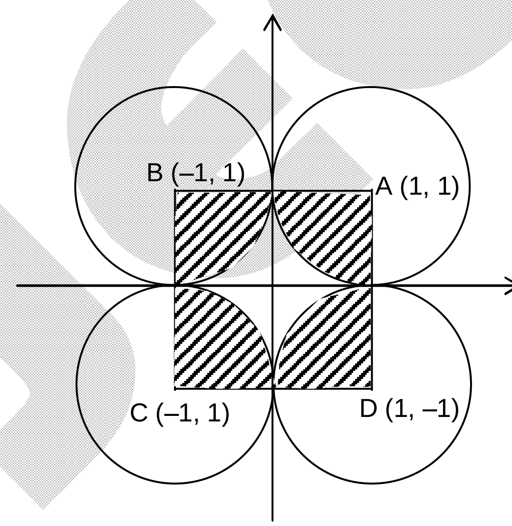
$$= \frac{1}{2}\left|\left[\begin{vmatrix} 3 & 9 \\ 6 & 0 \end{vmatrix} + \begin{vmatrix} 9 & 3 \\ 0 & 0 \end{vmatrix} + \begin{vmatrix} 3 & 0 \\ 0 & 3 \end{vmatrix} + \begin{vmatrix} 0 & 3 \\ 3 & 6 \end{vmatrix}\right]\right|$$

$$= \frac{1}{2}\left|-54 + 0 + 9 - 9\right| = 27$$

37. BC

Sol. Shaded region is required area

$= \pi$



38. A

Sol. $\left(\frac{-b}{2a}, -\frac{(b^2 - 4ac)}{4a}\right) = (4, 2)$

$$\Rightarrow \frac{-b}{2a} = 4 \Rightarrow b = -8a \quad \ldots(1)$$

$$-\left(\frac{b^2 - 4ac}{4a}\right) = 2$$

From (1) $-\left(\frac{64a^2 + 4ac}{4a}\right) = 2$

$\Rightarrow -16a + c = 2 \Rightarrow c = 16a + 2$

Now $abc = a(-8a)(16a + 2)$

$= -8\left[16a^3 + 2a^2\right]$

If $a \in [1, 3]$

$\Rightarrow -8\left[16a^3 + 2a^2\right]$ decreases

So $(abc)_{max}$ is at $a=1$

$= -8[16+2] = -144$

$(abc)_{min}$ is at $a=3$

$= -8[16\times 27+18]$

$= -s[432+18] = -8\times 450$

= –3600

39. B

Sol. Let A, B, be (a, 0) and (0, b)

Equation of circle

$(x-a)(x-0)+(y-0)(y-b)=0$

$x^2+y^2-ax-by=0$

Tangent at origin

$0+0-\frac{a}{2}(x+0)-\frac{b}{2}(y+0)=0$

$\Rightarrow ax+by=0$

$m=\frac{|a^2+0|}{\sqrt{a^2+b^2}},\ n=\left|\frac{0+b^2}{a^2+b^2}\right|$

$m+n=\sqrt{a^2+b^2}=\sqrt{(2r)^2}=2r$

40. A

Sol. $S=\sum_{r=0}^{n}\frac{3^{r+4}\ {}^{n}C_r\times 4!}{(r+4)(r+3)(r+2)(r+1)}+\frac{\sum_{r=0}^{3}{}^{n+4}C_r\,3^r}{{}^{n+4}C_4}$

Now consider $\frac{{}^{n}C_r\ (r+1)}{(r+4)(r+3)(r+2)(r+1)}=\frac{{}^{n+1}C_{r+1}}{(n+1)(r+4)(r+3)(r+2)}$

$=\frac{{}^{n+2}C_{r+2}}{(n+1)(n+2)(r+3)(r+4)}=\frac{{}^{n+4}C_{r+4}}{(n+1)(n+2)(n+3)(n+4)}$

So $S=\sum_{r=0}^{n}3^{r+4}\frac{{}^{n+4}C_{r+4}\times 4}{(n+1)(n+2)(n+3)(n+4)}+\sum_{r=0}^{3}\frac{{}^{n+4}C_r\,3^r\times 4!}{(n+4)(n+3)(n+2)(n+1)}$

$=\frac{4!}{(n+1)(n+2)(n+3)(n+4)}\left[\sum_{r=0}^{n}3^{r+4}\ {}^{n}C_{r+4}+\sum_{r=0}^{3}{}^{n+4}C_r\,3^r\right]$

$=\frac{4!(1+3)^{n+4}}{(n+1)(n+2)(n+3)(n+4)}$

$=\frac{4^{n+4}}{{}^{n+4}C_4}$

41. D

Sol. $\sum_{k=1}^{88}(-1)^{k+1}\frac{1}{\sin(k+2)\sin k}$

$=\frac{1}{\sin 2}\sum_{k=1}^{88}(-1)^{k+1}\frac{\sin\left(\overline{k+2}-\overline{k}\right)}{\sin(k+2)\sin k}=\frac{1}{\sin 2}\sum_{k=1}^{88}(-1)^{k+1}\left[\cot k-\cot(k+2)\right]$

$=\frac{1}{\sin 2}\left[(\cot 1-\cot 3)-(\cot 2-\cot 4)+(\cot 3-\cot 5)+......-(\cot 88-\cot 90)\right]$

$=\frac{1}{\sin 2}\left[\cot 1-\cot 2-\cot 89+\cot 90\right]=\frac{1}{\sin 2}\left[\cot 1-\cot 2-\cot 89\right]$

$=\frac{1}{\sin 2}\left[\cot 1-\tan 1-\cot 2\right]$

$=\frac{1}{\sin 2}\left[\frac{\cos 1}{\sin 1}-\frac{\sin 1}{\cos 1}-\cot 2\right]$

$=\frac{1}{\sin 2}\left[\frac{\cos^2 1-\sin^2 1}{\cos 1\times\sin 1}-\cot 2\right]$

$=\frac{1}{\sin 2}\left[\frac{2\cos 2}{\sin 2}-\cot 2\right]$

$=\frac{\cot 2}{\sin 2}$

42. B

Sol.

(P) Let $C(1,0,1), D(3,2,-1)$

$\vec{n}\perp$ to $\overrightarrow{CD}=2\hat{i}+2\hat{j}-2\hat{k}$

$\vec{n}=\begin{vmatrix}\hat{i} & \hat{j} & \hat{k}\\ 0 & 1 & 0\\ 2 & 2 & -2\end{vmatrix}=-2\hat{i}+0\hat{j}-2\hat{k}$

Equation of plane π: $-2x-2z=-2-2\Rightarrow x+z=2$

(Q) $\overrightarrow{AB}=2\hat{i}-2\hat{k}\perp\vec{n}$ hence $\overrightarrow{AB}$ is parallel to plane π and both A and B are on same side of π

Mirror image of A (4, 0, 0) about π.

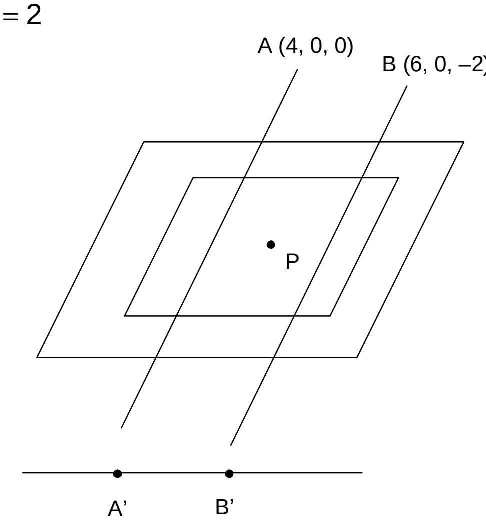
$\frac{x-4}{1}=\frac{y-0}{0}=\frac{z-0}{1}=-2\left(\frac{4+0-2}{1+1}\right)=-2$

$x=2,\ y=0,\ z=-2\Rightarrow A'=(2,0,-2)$

If PA + PB is minimum then P is intersection of plane π with BA'.

$BA':\frac{x-2}{4}=\frac{y-0}{0}=\frac{z+2}{0}=\alpha$ and π is

$x+z=2$



Let $P' = (4\alpha + 2, 0, -2)$ lies on $x + z = 2$

$$4\alpha + 2 - 2 = 2 \Rightarrow \alpha = \frac{1}{2}$$

So, $P' = (4, 0, -2) = (x_0, y_0, z_0)$

(R) $0 \le |PA - PB| < AB$

$0 \le |PA - PB| < \sqrt{4+4}$

$0 \le |PA - PB| < \sqrt{8}$

(S) Reflected line is parallel to AB i.e. $(2\hat{i} - 2\hat{k})$ and passes through $n'(2, 0, -2)$

So equation $\frac{x-2}{2} = \frac{y-0}{0} = \frac{z+2}{-2}$

$\Rightarrow \frac{x-2}{1} = \frac{y}{0} = \frac{z+2}{-1}$

So $\alpha = 0, \beta = 2$

43. D

Sol. Total numb of ways $= {}^{30}C_4$

(P) $P(\min(p, q, r, s) < 15) = 1 - P$ (when all are ≥ 15)

$= 1 - \frac{{}^{16}C_4}{{}^{30}C_4} = 1 - \frac{16 \times 15 \times 14 \times 13}{30 \times 29 \times 28 \times 27}$

$= 1 - \frac{52}{783} = \frac{731}{783}$

(Q) $P = \frac{{}^{19}C_1 \times {}^{10}C_2}{{}^{30}C_4} = \frac{19 \times 10 \times 10 \times 41}{2 \times 30 \times 29 \times 28 \times 27}$

$= \frac{19 \times 10 \times 9 \times 24}{2 \times 30 \times 29 \times 28 \times 27} = \frac{19}{609}$

(R) (i) all are odd

(ii) Two odd two even

(iii) All even

$P = \frac{{}^{15}C_4 + {}^{15}C_2 \times {}^{15}C_2 + {}^{15}C_4}{{}^{30}C_4}$

(S) $P = 1 - P$ (all odd)

$= 1 - \frac{{}^{15}C_4}{{}^{30}C_4} = 1 - \frac{15 \times 14 \times 13 \times 12}{30 \times 29 \times 28 \times 27}$

$= 1 - \frac{13}{261} = \frac{248}{261}$

44. C

Sol.

(P) $4\int_0^{\infty}\frac{\ln t}{x^2+t^2}dt=\pi\ln 2$

$t = x\tan\theta$

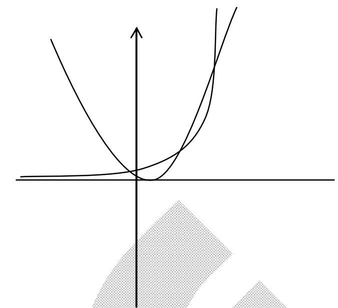
$4\int_0^{\pi/2}\frac{\ln(x\tan\theta)}{x^2\sec^2\theta}\times x\sec^2\theta d\theta=\pi\ln 2$

$4\int_0^{\pi/2}\frac{\ln x+\ln\tan\theta}{x}$

$4\int_0^{\pi/2}\frac{\ln x}{x}d\theta+4\int_0^{\pi/2}\frac{\ln\tan\theta}{x}d\theta=\pi\ln 2$

$4\frac{\ln x}{x}.\frac{\pi}{2}=\pi\ln 2\Rightarrow 2\ln x=2\ln 2$

$\Rightarrow x^2=2^x$



2 Solutions when $x>0$

(Q) $\int_1^e\frac{x^4\ln x+2}{x^3\ln x+x}dx=\int_1^e\frac{x^4\ln x+x^2-x^2+2}{x^2\ln x+x}dx$

$=\int_1^e x\,dx+\int_1^e\frac{2-x^2}{x^3\ln x+x}dx$

$=\frac{e^2-1}{2}+\int_1^e\frac{\frac{2}{x^3}-\frac{1}{x}}{\ln x+\frac{1}{x^2}}dx$

$=\frac{e^2-1}{2}+\left[-\ln\left(\ln x+\frac{1}{x^2}\right)\right]_1^e$

$=\frac{e^2-1}{2}+\left[-\ln\left(1+\frac{1}{e^2}\right)+0\right]=\frac{e^2-1}{2}-\ln\left(e^2+1\right)+2$

$=\frac{e^2+3}{2}-\ln\left(e^2+1\right)$

(R) $2\tan^{-1}\left(\sec^2\pi x\right)=\sin^{-1}\left(x^3-x^2+x+2\right)$

$\sec^2 2\pi x\ge 1$

So $2\tan^{-1}\left(\sec^2\pi x\right)\ge\frac{\pi}{2}$

$\sin^{-1}\left(x^3-x^2+x+2\right)\le\frac{\pi}{2}$

So only possibility is when both are $\frac{\pi}{2}$ but there is no common value of x hence no solution

(S) $f(x)=x^4-x^2+1$

$f(x)$ is symmetric about y axis

So $f(x)$ positive values of x i.e. $x>0$

$f(x) = x^2 + (1 - x^2)^2$

$\Rightarrow f(x) \ge x^2$

$\Rightarrow f\{f(x)\} \ge [f(x)]^2 \ge x^4$

$\Rightarrow f[f\{f(x)\}] \ge [f(x)]^4$

$\Rightarrow f[f\{f(x)\}] \ge x^8$

For $f[f\{f(x)\}] \le x^8$

$\Rightarrow (1 - x^2)^2 = 0$

$\Rightarrow x = +1$

Since f(x) is symmetric so x is also –1

Hence two values of x.

45. A

Sol. $b_1 = 1, b_2 = r, b_3 = r^2, b_4 = 2r^2 - r, b_5 = \frac{(2r^2 - r)^2}{r^2} = (2r - 1)^2$

$= 4r^2 - 4r + 1$

$b_6 = 2[4r^2 - 4r + 1] - (2r^2 - r) = (6r^2 - 7r + 2)$

$b_5 + b_6 = 4r^2 - 4r + 1 + 6r^2 - 7r + 2 = 198$

$\Rightarrow 10r^2 - 11r - 195 = 0$

$= 10r^2 - 50r + 39r - 195 = 0$

$\Rightarrow (10r + 39)(r - 5) - 0 \Rightarrow r = 5$

$b_1 = 1, b_2 = 5, b_3 = 25, b_4 = 45, b_5 = 81, b_6 = 117$

$b_7 = \frac{(117)^2}{81} = 13^2 = 169$

$b_8 = 2 \times 169 - 117 = 338 - 117 = 221$

$b_9 = \frac{(221)^2}{169} = \frac{(13 \times 17)^2}{169} = 289$

$b_{10} = 2 \times 289 - 221 = (578 - 221) = 357$

SECTION – B

46. 1488

Sol. Let A_i be number of ways in which couple A_i is seated next to each other

Hence required is $n(A_1^c \cap A_2^c \cap A_3^c \cap A_4^c) = \text{Total} - n(A_1 \cup A_2 \cup A_3 \cup A_4)$

$= 7! - \left[{}^4C_1 \times 2! \times 6! - {}^4C_2 \times 2!2!5 + {}^4C_3 2!^3 4! - {}^4C_4 (2!)^4 3!\right]$

= 1488

47. 6

Sol. For consistency $\begin{vmatrix} a & 1 & b \\ b & 1 & a \\ a & b & ab \end{vmatrix} = 0 = a\left[ab - ab\right] - 1\left(ab^2 - a^2\right) + b\left(b^2 - a\right) = 0$

$= -ab^2 + a^2 + b^3 - ab = 0$

$\Rightarrow b^2\left(b - a\right) - a\left(b - a\right) = 0$

$\Rightarrow \left(b^2 - a\right)\left(b - a\right) = 0$

$\Rightarrow$ either $b^2 = a$ as $a = b$

$a = b \Rightarrow 5$ ways

$a = b^2 \Rightarrow a = 4,\ b = 2$

$\Rightarrow$ One additional way

So total = 6

48. 2

Sol. $2\cos^2 x - 1 + 1 - \sqrt{3} = \left(2 - \sqrt{3}\right)\cos x$

$\Rightarrow 2\cos^2 x - 2\cos x + \sqrt{3}\cos x - \sqrt{3} = 0$

$\Rightarrow 2\cos x\left(\cos x - 1\right) + \sqrt{3}\left(\cos x - 1\right) = 0 \Rightarrow \cos x = 1$ or $\dfrac{-\sqrt{3}}{2}$

$\Rightarrow x = 0, 2\pi, 4\pi, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{7\pi}{6}, \dfrac{9\pi}{6}, \dfrac{29\pi}{6}$ ……..(1)

$\sin 3x = 2\sin x \Rightarrow 3\sin x - 4\sin^3 x = 2\sin x \to \sin x = 0, \pm\dfrac{1}{2}$

$x = 0, \pi, 2\pi, 3\pi, 4\pi, 5\pi, \dfrac{\pi}{6}, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{11\pi}{6}, \dfrac{13\pi}{6}, \dfrac{17\pi}{6}, \dfrac{19\pi}{6}, \dfrac{23\pi}{6}, \dfrac{25\pi}{6}, \dfrac{29\pi}{6}$ …(2)

Common of (1) and (2) are $0, 2\pi, 4\pi, \dfrac{5\pi}{6}, \dfrac{7\pi}{6}, \dfrac{17\pi}{6}, \dfrac{19\pi}{6}, \dfrac{29\pi}{6}$

Out of these 8 only $\dfrac{7\pi}{6}$ and $\dfrac{17\pi}{6}$ satisfy $\sqrt{3}\tan x - 1 \ge 0,$ so only two solutions.

49. 1

Sol. $a_{n+1} = a_n\left(1 + a_n\right)$

$\Rightarrow \dfrac{1}{a_{n+1}} = \dfrac{1}{a_n\left(a_n + 1\right)} = \dfrac{1}{a_n} - \dfrac{1}{a_{n+1}}$

$\Rightarrow \dfrac{1}{a_n + 1} = \dfrac{1}{a_n} - \dfrac{1}{a_{n+1}}$

$\Rightarrow \sum_{n=1}^{100}\dfrac{1}{a_n + 1} = \sum_{n=1}^{100}\left[\dfrac{1}{a_n} - \dfrac{1}{a_{n+1}}\right]$

$= \dfrac{1}{a_1} - \dfrac{1}{a_{101}} = 2 - \dfrac{1}{a_{101}}$

Since $a_{101} > 1$

$$1 < \sum_{n=1}^{100} \frac{1}{a_n + 1} < 2$$

So, $[s] = 1$

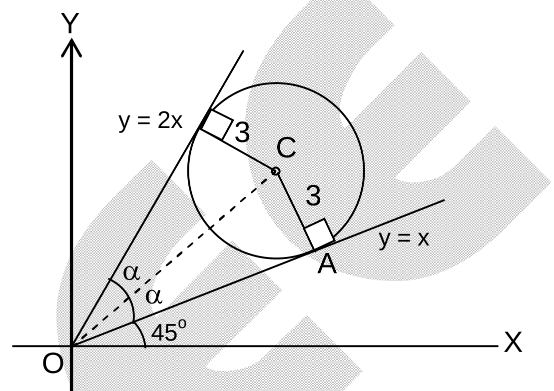
50. 1

Sol. $2x^2 + y^2 - 3xy = 0$

$\Rightarrow \quad (2x - y)(x - y) = 0$

$\Rightarrow \quad y = 2x,\ y = x$

are the equations of straight lines passing through origin. Now, let the angle between tangents is 2α, then $\tan(45^o + 2\alpha) = 2$



$\Rightarrow \quad \frac{\tan 45^o + \tan 2\alpha}{1 - \tan 45^o \tan 2\alpha} = 2$

$\Rightarrow \quad \frac{1 + \tan 2\alpha}{1 - \tan 2\alpha} = \frac{2}{1}$

$\Rightarrow \quad \frac{2\tan 2\alpha}{2} = \frac{1}{3}$

(By componendo and dividendo rule)

$\Rightarrow \quad \frac{2\tan\alpha}{1 - \tan^2\alpha} = \frac{1}{3}$

$\Rightarrow \quad \tan^2\alpha + 6\tan\alpha - 1 = 0$

$\therefore \quad \tan\alpha = \frac{-6 \pm \sqrt{(36 + 4)}}{2} = -3 \pm \sqrt{(10)}$

$= -3 + \sqrt{10} \qquad \left(\because 0 < \alpha < \frac{\pi}{4}\right)$

Now, in ΔOAC, $\tan\alpha = \frac{3}{OA} = \left(\sqrt{10} - 3\right)$

$\therefore\ OA = \frac{3\left(\sqrt{10} + 3\right)}{\left(\sqrt{10} - 3\right)\left(\sqrt{10} + 3\right)} = 3\left(3 + \sqrt{10}\right)$

$\Rightarrow \quad 9 + \sqrt{90} = \lambda + \sqrt{\mu}$

$\therefore \lambda = 9$ and $\mu = 90$, then $\lambda^2 + \mu = 81 + 90 = 171$

51. 7

Sol. Let E_1 : first ball drawn red from Urn A and 2nd ball drawn black from B.

E_2 : first ball drawn while from A and 2nd ball drawn black from B

E_3 : first ball drawn red from B and 2nd ball drawn black from B

E_4 : first ball drawn black from B and 2nd ball drawn black from B.

$$P = \frac{P(E_1) + P(E_3)}{P(E_1) + P(E_2) + P(E_3) + P(E_4)}$$

$$P(E_1) = \left(\frac{1}{2}\right)\left(\frac{2}{6}\right)\left(\frac{3}{6}\right)\left(\frac{1}{2}\right) = \frac{6}{144}$$

$$P(E_2) = \left(\frac{1}{2}\right)\left(\frac{4}{6}\right)\left(\frac{1}{2}\right)\left(\frac{3}{6}\right) = \frac{12}{144}$$

$$P(E_3) = \left(\frac{1}{2}\right)\left(\frac{3}{6}\right)\left(\frac{1}{2}\right)\left(\frac{3}{5}\right) = \frac{9}{120}$$

$$P(E_4) = \left(\frac{1}{2}\right)\left(\frac{3}{6}\right)\left(\frac{1}{2}\right)\left(\frac{2}{5}\right) = \frac{6}{120}$$

$$\Rightarrow P = \frac{7}{15}$$