FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
OPEN TEST – I
PAPER –1
TEST DATE: 04-02-2024

ANSWERS, HINTS & SOLUTIONS

Physics PART – I

SECTION – A

1. A, C, D

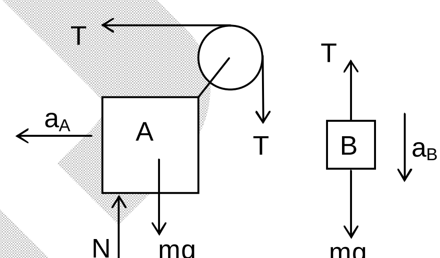
Sol. $T = ma_A$

$mg - T = ma_B$

$a_A = a_B$

By solving

$a_A = a_B = \frac{g}{2}$

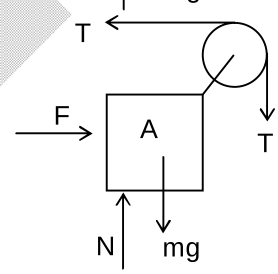


FBD of wedge + block

F = T

and T = mg

$\Rightarrow$ T = mg



$a_A = 0$

2. A, B, D

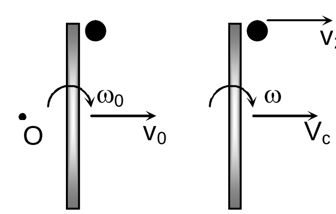
Sol. Conservation of momentum

$4v_0 + 0 = 4v_C + 1v_2$ …(i)

Conservation of angular momentum about point O

$$\frac{4\times 2^2}{12}\times 3 = \frac{4\times 2^2}{12}\times \omega + 1\times v_2\times 1$$

$$4 = \frac{4}{3}\omega + v_2 \quad \text{…(ii)}$$



By using Conservation of energy

$$\frac{1}{2}\times 4\times 1^2 + \frac{1}{2}\times\frac{4\times 2^2}{12}\times 3^2 = \frac{1}{2}\times 4\times v_c^2 + \frac{1}{2}\times 1\times v_2^2 + \frac{1}{2}\frac{4\times 2^2}{12}\omega^2$$

$$16 = 4\times v_c^2 + v_2^2 + \frac{4}{3}\omega^2 \qquad \ldots(iii)$$

By solving $v_C = \omega = 0$, $v_2 = 4$, $J = 4$

3. A, C

Sol. Since, $C \rightarrow D$, PT = constant

$\Rightarrow P_C = P_B = 3P_0$

$\therefore\ \Delta Q_{AB} = -nRT_0\ell n3$

$$\Delta Q_{BC} = nC_P\Delta T = n\frac{5}{2}R\left(\frac{4}{3}T_0 - T_0\right) = \frac{5nRT_0}{6}$$

$C \rightarrow D$, PT = constant $\Rightarrow PV^{1/2} = \text{constant}$

$$\therefore\ \Delta Q_{CD} = \frac{3}{2}nR\left(2T_0 - \frac{4}{3}T_0\right) + \frac{nR\left(2T_0 - \frac{4}{3}T_0\right)}{1-\frac{1}{2}} = \frac{7}{3}nRT_0$$

$$A \rightarrow B,\ \Delta Q = nRT_0\ell n\left(\frac{P_A}{P_B}\right)$$

$D \rightarrow A$

$\Rightarrow$ V = constant

$$\therefore\ \Delta Q_{D\rightarrow A} = nC_V\Delta T = n\frac{3}{2}R\left(T_0 - 2T_0\right) = -\frac{3}{2}nRT_0$$

$$\eta = 1 - \frac{\left(\frac{3}{2}nRT_0 + nRT_0\ell n3\right)}{\left(\frac{7}{3}+\frac{5}{6}\right)nRT_0} = 0.1793$$

4. A

Sol. For $N_1 = 0$ and $a = 0$ and $a_1 = 0$

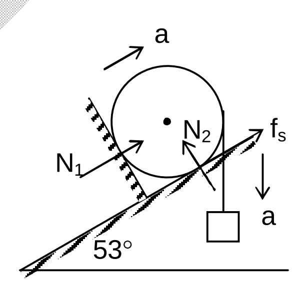
$T = 4g$, $f\times R = TR$

$\Rightarrow T = f_S$

$f_s \le \mu N_2$

$4g < \mu\left[1g + 4g\right]\cos 53°$

$$\mu \ge \frac{4}{3}$$

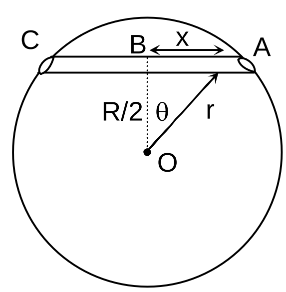


For $\left(\mu = \frac{3}{4}\right) < \frac{4}{3}$

$N_1 \neq 0$, friction decreases and cylinder will rotate and block will move down.

5. C

Sol. $$F = -\left(\frac{GM}{R^3}r\frac{x}{r} - \frac{\mu GM}{R^3}\frac{R}{2}\right)$$



$$a = -\frac{g}{R}x + \frac{\mu g}{2}$$

$$a = 0 \text{ at } x = \frac{\mu R}{2} = \frac{\sqrt{3}}{2}\frac{R}{2} = \frac{\sqrt{3}R}{4}$$

$$AB = \sqrt{R^2 - \frac{R^2}{4}} = \frac{R\sqrt{3}}{2}$$

$\therefore$ equilibrium position point 'P' is midway of AB

Apply work energ theorem for AP

$$v = \frac{\sqrt{3gR}}{4}$$

6. B

Sol. $mg = F_B$

$$\Rightarrow \pi r^2 h\rho g = \pi r^2 \frac{h}{2}\rho_w g$$

$$\Rightarrow \rho = \frac{\rho_w}{2}$$

For just dipping, if ℓ is pushed down

$$\pi r^2 \ell = \pi(4r^2 - r^2)\left(\frac{2h}{3} - \ell\right)$$

$\Rightarrow \ell = h/2$

Work done by gravity $W_g = mg\frac{h}{2}$

Change in potential energy of water

$$= (\pi r^2 \ell \rho_w g)\left[\frac{\ell}{2} + \frac{h}{2} + \frac{\frac{2h}{3} - \ell}{2}\right] = \frac{2}{3}mgh$$

For equilibrium, $kx_0 = mg - F_B = mg - \frac{2mg}{3} = \frac{mg}{3}$

$$x_0 = \frac{mg}{3k} = \frac{\sqrt{3}}{6}b$$

Change in potential energy of spring

$$U_s = \frac{1}{2}\left(\frac{mg}{\sqrt{3}\frac{h}{2}}\right)\left[\left(\frac{h}{2} + \frac{\sqrt{3}h}{6}\right)^2 - \left(\frac{\sqrt{3}h}{6}\right)^2\right] = \frac{mgh}{12\sqrt{3}}(3 + 2\sqrt{3})$$

$$W_F + W_g = \Delta U$$

$$W_F + mg\frac{h}{2} = \frac{mgh}{12\sqrt{3}}(3 + 2\sqrt{3}) + \frac{2}{3}mgh$$

$$W_F = mgh\left(\frac{\sqrt{3} + 4}{12}\right)$$

7. A

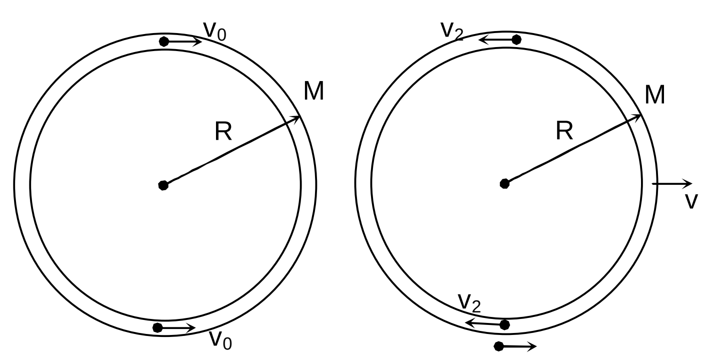
Sol. Conservaion of momentum

$2mv_0 = Mv - 2mv_2$

Conservation of energy

$$2 \times \frac{1}{2}mv_0^2 = \frac{1}{2}Mv^2 + 2 \times \frac{1}{2}mv_2^2$$

$$\Rightarrow v = \frac{4mv_0}{M + 2m} = \frac{2v_0}{3}$$



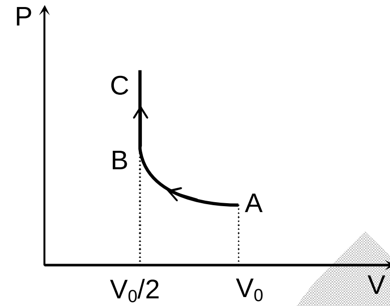
8. A

Sol. **(P)** $P_0V_0 = \frac{P_BV_0}{2} \Rightarrow \mathbf{P_B = 2P_0}$

B → C, $Q = \frac{3}{2}nR(T_C - T_B)$

$P_0V_0 = \frac{3}{2}\left(P_C\frac{V_0}{2} - 2P_0\frac{V_0}{2}\right)$

$P_C = \frac{10}{3}P_0$



(Q) $\frac{P_0V_0}{T_0} = \frac{P_0\frac{V_0}{2}}{T_B}$

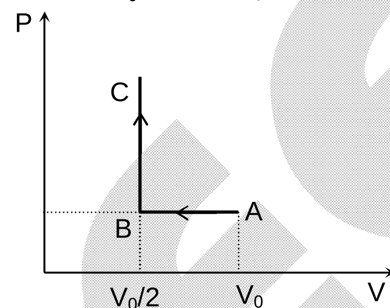
$\Rightarrow T_B = \frac{T_0}{2}$

$Q = \frac{3}{2}nR(T_C - T_B)$

$P_0V_0 = \frac{3}{2}\left(P_C\frac{V_0}{2} - P_0\frac{V_0}{2}\right)$

$P_0 = \frac{3}{2}\left(\frac{P_C}{2} - \frac{P_0}{2}\right)$

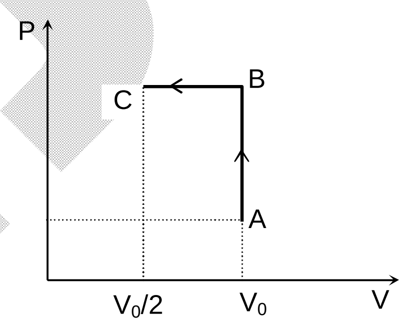
$P_C = \frac{7P_0}{3}$



(R) $Q = \frac{3}{2}nR(T_B - T_0)$

$P_0V_0 = \frac{3}{2}(P_BV_0 - P_0V_0)$

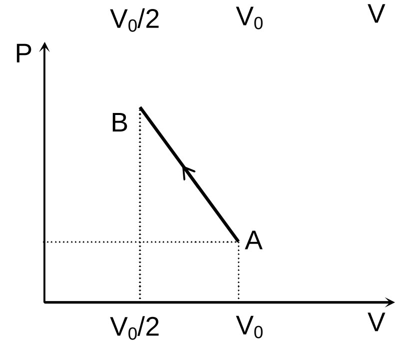
$P_B = \frac{5}{3}P_0$



(S) $Q + W_{ex} = \frac{3}{2}nR(T_B - T_A)$

$P_0V_0 + \frac{1}{2}\frac{V_0}{2}(P_B + P_0) = \frac{3}{2}\left(P_B\frac{V_0}{2} - P_0V_0\right)$

$P_B = \frac{11}{2}P_0$



9. C

Sol. (P) $q_1 = 4 \times \frac{8}{6} = \frac{16}{3}$

$u_i = \frac{1}{2} \times \frac{8}{6} \times 4^2 = \frac{32}{3}$

$q_2 = 8$

$u_2 = \frac{1}{2}\times 2\times 4^2 = 16$

$4\left(8-\frac{16}{3}\right) = \left(16-\frac{32}{3}\right)+H$

$\Rightarrow H = \frac{16}{3}$

(Q) $q_i = 4 \times 2 = 8$

$u_i = \frac{1}{2}\times 2\times 4^2 = 16$

$q_f = 4\times 4 = 16$

$u_f = \frac{1}{2}\times 4\times 4^2 = 32$

$4(16 - 8) = (32 - 16) + H$

$H = 16$

(R) $u_i = \frac{1}{2}\times 1\times 4^2 = 8$

$q_i = 1 \times 4 = 4$

$q_2 = 4 \times 2 = 8$

$u_2 = \frac{1}{2}\times 2\times 4^2 = 16$

$4(8 - 4) = 16 - 8 + H_2$

$H_2 = 16$

$\therefore H = H_1 + H_2 = 24$

(S) $q_1 = 4$

$u_1 = \frac{1}{2}\times 1\times 4^2 = 8$

$q_2 = 4\times\frac{8}{6} = \frac{16}{3}$, $u_2 = \frac{1}{2}\times\frac{8}{6}\times 4^2 = \frac{32}{3}$

$\left(\frac{16}{3}-4\right)\times 4 = \left(\frac{32}{3}-8\right)+H$

$H_1 = 8/3$

$q_3 = 4 \times 4 = 16$

$u_3 = \frac{1}{2}\times 4\times 4^2 = 32$

$4\left(16-\frac{16}{3}\right) = \left(32-\frac{32}{3}\right)+H_2$

$H_2 = 64/3$

$H = H_1 + H_2 = \frac{8}{3}+\frac{64}{3} = 24$

10. B

Sol. Using angular impulse momentum theorem

(P) $J\frac{7R}{5} = mV_C R + \frac{2}{5}mR^2\left(\frac{V_C}{R}\right)$

$V_C = J/M$

(Q) Using angular impulse momentum theorem

$\frac{3R}{5}J = \frac{7}{5}mV_C R$

$\Rightarrow V_C = \frac{3J}{7m}$

(R) Using angular Impulse momentum theorem

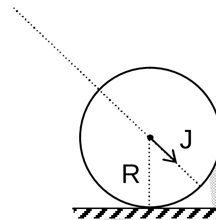
$J\frac{2R}{5} = \frac{7}{5}mV_C R$

$V_C = \frac{2J}{7M}$

(S) Using angular impulse momentum theorem

$JR\sin 53° = \frac{7}{5}mV_C R$

$V_C = \frac{4J}{7m}$



11. A

Sol. (P) $F = -kx - Ax\sigma g$

$a = -\frac{(k + A\sigma g)x}{A\rho h}$

$T = 2\pi\sqrt{\frac{\rho h}{2\sigma g}}$

(Q) $F = -k(x + x_0) + T,\ kx_0 = \rho A\frac{\ell}{2}g$

$-T + \rho A\frac{\ell}{2}g = \rho A\frac{\ell}{2}a$

$\therefore\ F = -kx - kx_0 + \rho A\frac{\ell}{2}(g - a)$

$a = \frac{-kx}{\rho A\ell}$

(R) $\frac{Mg}{a'} = Y\frac{h}{L}$

After extending it by y

$F = mg - \frac{a'Y}{L}(h + y)$

$ma = mg - \frac{a'Y}{L}h - \frac{a'Yy}{L}$

$\therefore\ a = -\frac{a'Y}{ML}y \quad \ldots(ii)$

$\Rightarrow \quad a = -\frac{g}{h}y$

$T = 2\pi\sqrt{\frac{h}{g}}$

(S) $Mg\frac{\ell}{2} = (kx_0)\ell$

$\tau_0 = \left(Mg\frac{\ell}{2}\right)\cos\theta - k(x_0 + \ell\theta)\ell$

$$\tau_0 = Mg\frac{\ell}{2} - kx_0\ell - k\ell^2\theta$$

$$\frac{M\ell^2}{3}\alpha = -k\ell^2\theta$$

$$\alpha = -\frac{3k}{M}\theta = \frac{-3A\sigma g}{A\ell\rho}\theta$$

$$\alpha = -\left(\frac{3g\sigma}{\ell\rho}\right)\theta$$

SECTION – B

12. 9

Sol. Using conservation of momentum of the system

$$9mv_1 = mv_2$$

$$v_2 = 9v_1 \qquad \ldots(i)$$

Using conservation of energy of the system

$$-\frac{G9m^2}{14R} = -\frac{G9m^2}{4R} + \frac{1}{2}\times 9mv_1^2 + \frac{1}{2}mv_2^2$$

$$\frac{45Gm^2}{28R} = 45mv_1^2$$

$$\Rightarrow v_1 = \sqrt{\frac{Gm}{28R}}$$

Hence, $v_2 = 9v_1 = 9\sqrt{\frac{Gm}{28R}}$

13. 2

Sol. $204 = 13.6z^2\left(\frac{1}{1^2} - \frac{1}{4^2}\right)$

$\Rightarrow$ Z = 4

$$\frac{hC}{\lambda} = 13.6\times 4^2\left(\frac{1}{2^2} - \frac{1}{4^2}\right) = 40.8 \text{ eV}$$

$$\phi = \frac{hC}{\lambda} - eV_0 = 40.8 - 32.22 = 8.58 \text{ eV}$$

$$13.6\times 4^2\left(\frac{1}{3^2} - \frac{1}{4^2}\right) = 10.58 \text{ eV}$$

$$eV = \frac{hC}{\lambda} - \phi$$

V = 10.58 – 8.58 = 2 volt

14. 4

Sol. $nh\nu = IA$

$$n = \frac{IA}{h\nu}$$

$$I_P = \frac{ne}{10^5}$$

$$1\times10^{-3} = \frac{e}{10^5}\frac{IA}{h\nu}$$

Also, $eV_0 = h\nu - \phi$

$$h\nu = \phi + eV_0 = (3.5+2.5)eV = 6\ eV$$

$$I = \frac{10^{-3}\times10^5\times6}{15\times10^{-4}} = 4\times10^5\ \text{W/m}^2$$

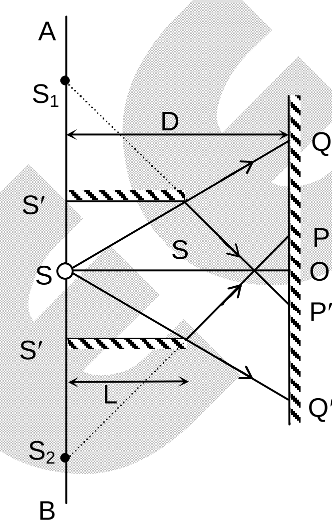
15. 4

Sol. Overlaping region where interference takes place is PQ + P′Q′ = 2PQ

By geometry PQ = 2d

$\therefore$ Distance = 4d

No. of fringes N = $\frac{4d}{\left(\frac{D\lambda}{d}\right)} = 4\frac{d^2}{D\lambda} = 4$



16. 4

Sol. One small division on main scale = 1 mm

One division vernier scale = $\frac{9}{10}$mm

4th line on vernier scale is in line with main scale.

X = 4 mm − $4\times\frac{9}{10}$mm $= \frac{4}{10}$mm

17. 2

Sol. Water surface behaves as mirror

y = A cos (ωt −kx)

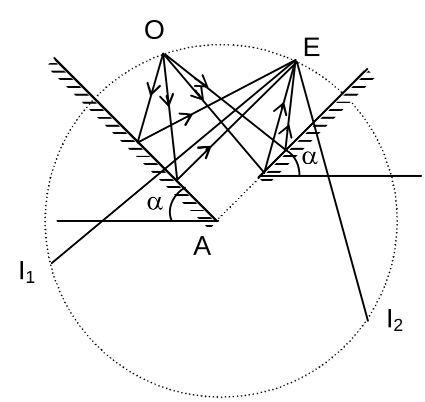
$$\tan\alpha = \frac{\partial y}{\partial x} = Ak\cos(\omega t - kx)$$

$$\tan(\alpha_{max}) = Ak$$

$$\alpha_{max} = \tan^{-1}\left(0.1\times\frac{10}{\sqrt{3}}\right) = 30^\circ$$

Angle between two reflecting surface is 2α. O, E, I and I_2 all lies on a circle

Angle between I_1E and I_2E is 2α i.e. 60°



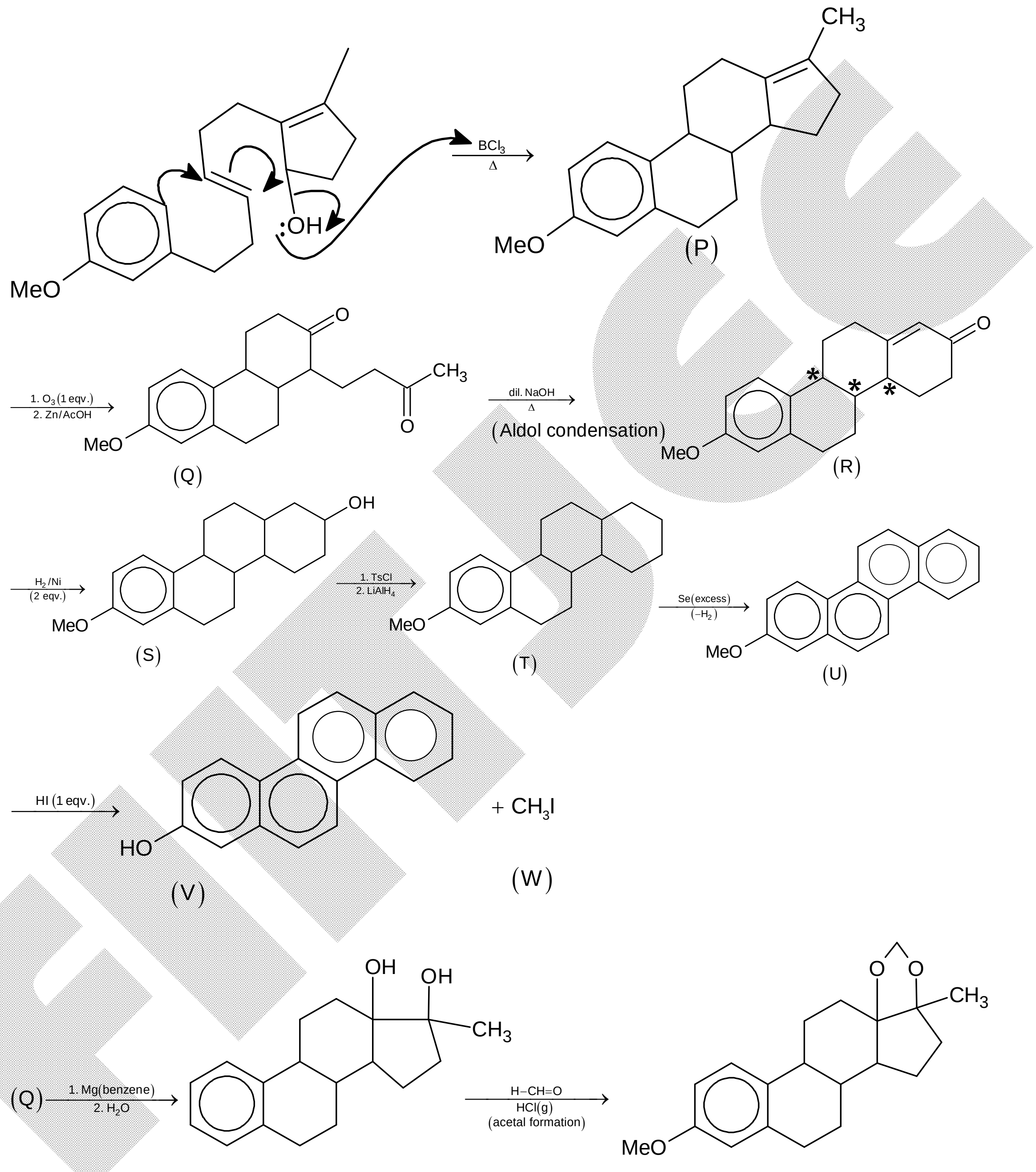
Chemistry

PART – II

SECTION – A

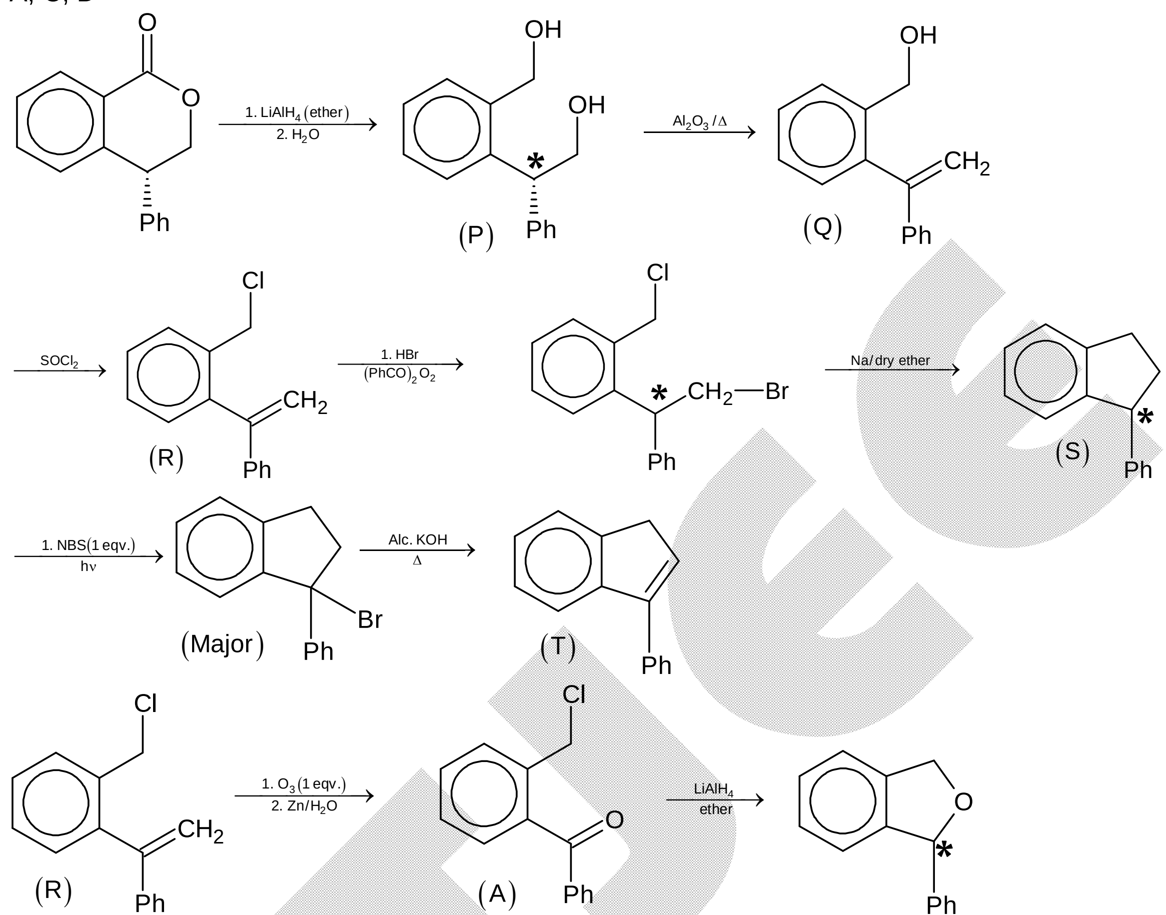
18. A, B, C

Sol.



19. A, C, D

Sol.



20. A, B, C

Sol. $x \rightarrow y$ is a reversible isothermal expansion. So,

$$\Delta S_{x\rightarrow y} = 2.303\ nR\log\frac{P_x}{P_y} = 2.303\times 5\times 8.314\times\log\frac{10}{1} = 95.74\ JK^{-1}$$

$x \rightarrow z$ is a reversible adiabatic expansion. So, $\Delta S_{x\rightarrow z} = 0$

Since, $x\rightarrow z\rightarrow y\rightarrow x$ is cyclic process. So,

$\Delta S_{x\rightarrow z} + \Delta S_{z\rightarrow y} + \Delta S_{y\rightarrow x} = 0$ (because 'S' is a state function).

$\therefore \Delta S_{x\rightarrow z} + \Delta S_{z\rightarrow y} = -\Delta S_{y\rightarrow x} = \Delta S_{x\rightarrow y}$

$\Delta S_{x\rightarrow z\rightarrow y} = \Delta S_{x\rightarrow z} + \Delta S_{z\rightarrow y} = \Delta S_{z\rightarrow y}$

Now, $\Delta S_{x\rightarrow z} + \Delta S_{z\rightarrow y} + \Delta S_{y\rightarrow x} = 0$

$0 + \Delta S_{z\rightarrow y} + (-95.74) = 0$

$\therefore \Delta S_{z\rightarrow y} = 95.74\ JK^{-1}$

21. A

Sol. $\left[Cu(CN)_4\right]^{3-} = 0.1\,M; \left[CN^-\right] = 0.2\,M$

$$\left[Cu^+\right] = \frac{K_{instability}\left[Cu(CN)_4^{3-}\right]}{\left[CN^-\right]^4} = \frac{6.4\times10^{-15}\times 0.1}{(0.2)^4} = 4\times10^{-13}\ M.$$

Now, $\left[S^{2-}\right] = \frac{K_{sp}}{\left[Cu^{+}\right]^2} = \frac{2.56\times10^{-27}}{\left(4\times10^{-13}\right)^2} = 1.6\times10^{-2}$ M

$\therefore \left[H^{+}\right] = \sqrt{\frac{K_a\left(H_2S\right)}{\left[S^{2-}\right]}} = \sqrt{\frac{1.6\times10^{-21}\times0.1}{1.6\times10^{-2}}} = 10^{-10}$

22. C

Sol. Pressure (P) of first container, when all stop-cocks are closed,

$P\times3 = 2\times RT$

$\therefore P = \frac{2RT}{3}$

After opening all stop-cocks',

Total volume (V_{total}) $= 3 + 6 + 9 + \ldots + 150$

$= 3\,[1 + 2 + \ldots + 50]$

$= 3\times\frac{50\left(50+1\right)}{2}$

$= 3 \times 25 \times 51$ litres

Total moles (n_{total}) $= 2 + 8 + 18 + \ldots 5000$

$= 2[1 + 4 + 9 + \ldots 2500]$

$= 2\left[1^2+2^2+3^2+\ldots50^2\right]$

$= 2\left[\frac{50\times\left(50+1\right)\times\left(2\times50+1\right)}{6}\right]$

$= \frac{1}{3}\times50\times51\times101$

$= 50\times17\times101$

So, let the total pressure be P_{total}.

$P_{total}\times\left(3\times25\times51\right) = \left(50\times17\times101\right)\times RT$

$P_{total} = \frac{50\times17\times101\times RT}{3\times25\times51} = 33.66\times\left(\frac{2}{3}\times RT\right) = 33.66$ P

23. C

Sol. (A) $NO_2^- + H^+ + I^- \longrightarrow \underset{(Paramagnetic)}{NO} + I_2$

(B) $O_3 + KOH \longrightarrow \underset{(Paramagnetic)}{KO_3} + H_2O + \underset{(Paramagnetic)}{O_2}$

(C) $\left(NH_4\right)_2Cr_2O_7\left(s\right) \longrightarrow \underbrace{N_2\left(g\right) + Cr_2O_3\left(s\right) + 4H_2O}_{\text{All are diamagnetic substances}}$

(D) $Pb\left(NO_3\right)_2 \xrightarrow{\Delta} PbO + \underbrace{NO_2 + O_2}_{\text{Paramagnetic}}$

24. B

Sol.

	n – butane $\rightleftharpoons$	isobutane
at t=0 (conc.)	1	0
at time 't'	$(1-x)$	x
at eqbm.	$(1-x_e)$	x_e

At eqbm, $1.2\times\left(1-x_e\right) = 0.8\times x_e$

$\therefore x_e = 0.6\ \text{mol L}^{-1}$

$K_c = \frac{K_f}{K_b} = \frac{1.2}{0.8} = 1.5$

So, $\Delta_r G^o = -2.303 RT \log 1.5 = -1.034 \text{ kJ mol}^{-1}$ (Option A)

Rate of conversion of n-butane to isobutane $\left(\frac{dx}{dt}\right)$ can be written as

$\frac{dx}{dt} = K_f(a-x) - K_b x$

So, $t = \frac{2.303}{(K_f + K_b)} \log \frac{K_f a}{K_f a - (K_f + K_b)x}$

$t_{1/2} = \frac{0.69}{(K_f + K_b)} = \frac{0.69}{2} \text{min}^{-1}$

At $t = t_{1/2}$, $x = \frac{x_e}{2} = \frac{0.6}{2} = 0.3 \text{ M}$

So, after 0.345 min, $\frac{dx}{dt} = 1.2 \times (1-0.3) - 0.8 \times 0.3 = 0.60 \text{ Mmin}^{-1}$

$= 0.01 \text{ M sec}^{-1}$

Option (D)

When $t = t_{1/4}$, $x = \frac{1}{4} \times x_e = \frac{1}{4} \times 0.6 = \frac{0.3}{2} = 0.15 \text{ M}$

So, $t_{1/4} = \frac{2.303}{2} \times \log \frac{1.2 \times 1}{1.2 \times 1 - 2 \times 0.15}$

$= \frac{2.303}{2} \log \frac{1.2}{1.2 - 0.30}$

$= \frac{2.303}{2} \times \log \frac{1.2}{0.9}$

$= \frac{2.303}{2} \times [2 \log 2 - \log 3]$

$= \frac{2.303}{2} [0.60 - 0.48]$

$= \frac{2.303}{2} \times 0.12$

$t_{1/4} = 0.138 \text{ min}$ (So, option (B) is wrong)

Similarly when $t = t_{3/4}$, $x = \frac{3}{4} \times x_e = \frac{3}{4} \times 0.6 = \frac{0.9}{2} = 0.45 \text{ M}$

So, $t_{3/4} = \frac{2 \times 0.69}{2} = 0.69 \text{ min}$. (option (C) is correct).

25. B

Sol. $\underline{P}_4 + O_2(\text{Excess}) \longrightarrow \underline{P}_4O_{10} \xrightarrow{6PCl_5} \underline{P}OCl_3$

$\underline{P}_4(\text{Excess}) + O_2 \longrightarrow P_4O_6 \xrightarrow{6PCl_5} \underline{P}OCl_3 + \underline{P}Cl_3$

$\underline{P}_4 + \text{Conc. } HNO_3 \longrightarrow H_3\underline{P}O_4 \xrightarrow{3PCl_5} \underline{P}OCl_3 + HCl$

$P_4(\text{White}) + \underline{S}O_2Cl_2 \longrightarrow \underline{S}O_2 + PCl_5$

$\underline{S}O_2 \xrightarrow{PCl_5} \underline{S}OCl_2 + POCl_3$

26. C

Sol. $CuSO_4 + 4NH_3(aq.) \longrightarrow \left[Cu(NH_3)_4\right]^{2+} \rightarrow$ Blue, sp^2d, $\mu = \sqrt{3}$ BM; paramagnetic

$CoCl_2 + KNO_2 + CH_3COOH \xrightarrow{pH>7} \left[Co(NO_2)_6\right]^{3-} \rightarrow$ yellow, diamagnetic, (d^2sp^3), $\mu = 0$

$CoCl_2 + NH_4SCN \longrightarrow \left[Co(SCN)_4\right]^{2-} \rightarrow$ blue, paramagnetic, $\mu = 3.87$ BM, sp^3

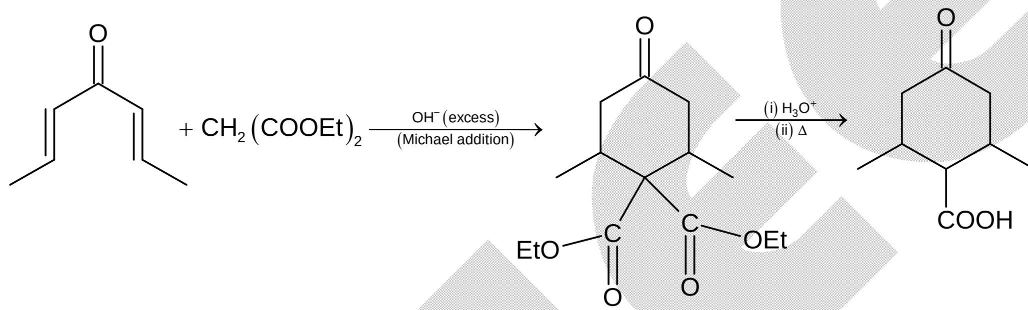
$$CuSO_4 + \underset{(excess)}{KCN} \longrightarrow \left[Cu(CN)_4\right]^{3-} + (CN)_2$$

$\downarrow$

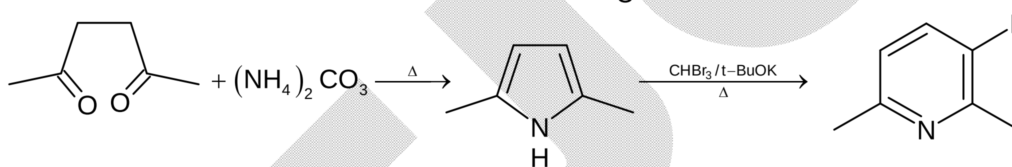
sp^3, colourless, $\mu = 0$, diamagnetic

27. A

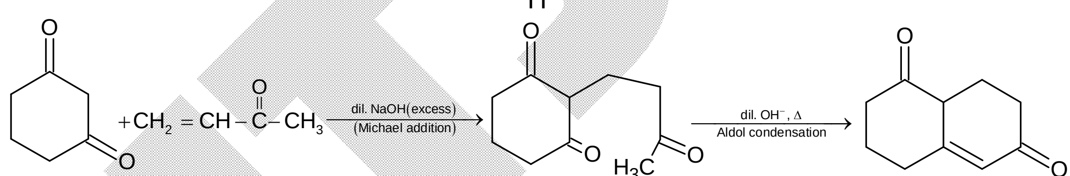
Sol. (P)



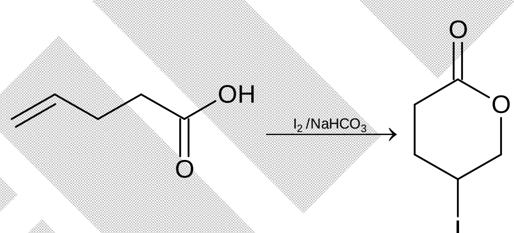
(Q)



(R)



(S)



28. B

Sol. $K_2Cr_2O_7 + 7H_2O_2 + 4KOH \longrightarrow \underset{(\text{red brown solid})}{2K_3CrO_8} + 9H_2O$

$K_2Cr_2O_7 + H_2O_2 + H_2SO_4 \longrightarrow \underset{(\text{blue})}{2CrO_5} + K_2SO_4 + 5H_2O$

$3K_4\left[Fe(CN)_6\right] + \underset{(\text{dil.})}{6H_2SO_4} \xrightarrow{\Delta} \underset{(\text{triatomioc gas})}{12HCN} \uparrow + Fe_2\left[Fe(CN)_6\right] + 6K_2SO_4$

$2KMnO_4 + \text{Conc. } 2H_2SO_4 (\text{cold}) \longrightarrow \underset{(\text{green oily})}{Mn_2O_7} + 2KHSO_4 + H_2O$

SECTION – B

29. 40

Sol. $'M' + NaNO_3 + NaOH \longrightarrow Na_2MO_2 + NH_3$

g-equivalents of NH_3 produced = $(100\times1 - 80\times0.25)\times10^{-3}$

$= 80\times10^{-3}$

$= 8\times10^{-2}$

g-equivalents of 'M' = g-equivalents of NH_3 = $8\times10^{-2} = \dfrac{2.016}{\left(\dfrac{M}{2}\right)}$

M = 65.4

So, 'M' is zinc (Zn)

$Zn + 2NaOH \longrightarrow Na_2ZnO_2 + H_2(g)$

1 mol　　　　　　　　1 mol

20 moles　　　　　　　20 moles

So, mass of H_2 gas produced $= 20\times2 = 40$ g.

30. 768

Sol.

	$2KI(aq)$	$+ HgI_2(s)$	$\longrightarrow K_2[HgI_4](aq)$
millimoles initially :	20	5	0
millimoles after reaction :	$20-10 = 10$	0	5

m moles of $KI = 100\times0.2 = 20$

m. moles of $HgI_2 = \dfrac{2.27\times10^3}{454} = 5$

$i_{KI} = 1 + (2-1)\times0.9 = 1.9$

$i_{K_2[HgI_4]} = 1 + (3-1)\times0.8 = 2.6$

So, molarity of resulting solution $(M_{mix}) = \dfrac{1.9\times10 + 2.6\times5}{100}$

$M_{mix} = 0.32$

Now, $\pi = M_{mix}RT$

$= 0.32\times0.08\times300 = 7.68$ atm $= x$

So, $100x = 768$

31. 36

Sol. From the graph, we can calculate that

$\Lambda^o_{Na_2SO_4} = 500$ Scm^2 mol^{-1};　　$\Lambda^o_{CH_3COONa} = 1500$ Scm^2 mol^{-1};

$\Lambda^o_{H_2SO_4} = 2500$ Scm^2 mol^{-1};　　$\Lambda^o_{HCOONa} = 4000$ Scm^2 mol^{-1};

From Kohlrausch law:

$\Lambda^o_{CH_3COOH} = \dfrac{1}{2}\times\left[2\times\Lambda^o_{CH_3COONa} + \Lambda^o_{H_2SO_4} - \Lambda^o_{Na_2SO_4}\right] = 2500$ Scm^2 mol^{-1};

Similarly

$\Lambda^o_{HCOOH} = 5000$ Scm^2 mol^{-1}

Conductivity (κ) of CH_3COOH solution $= \dfrac{1}{10}\times0.1 = 10^{-2}$ S / cm

Conductivity (κ) of HCOOH solution $= \frac{1}{20} \times 0.1 = 0.5 \times 10^{-2}$ S / cm

$$\Lambda_{CH_3COOH} = \frac{10^{-2} \times 1000}{10^{-1}} = 100 \text{ Scm}^2 \text{ mol}^{-1}$$

$$\Lambda_{HCOOH} = \frac{0.5 \times 10^{-2} \times 1000}{10^{-2}} = 500 \text{ Scm}^2 \text{ mol}^{-1}$$

$$\alpha_{CH_3COOH} = \frac{100}{2500} = 4 \times 10^{-2}$$

$$\alpha_{HCOOH} = \frac{500}{5000} = 10^{-1}$$

Now, $\left[H^+\right]_{CH_3COOH} = 10^{-1} \times 4 \times 10^{-2} \Rightarrow (pH)_{CH_3COOH} = 2.4$

Similarly, $(pH)_{HCOOH} = 3$

Now, $E_{cell} = 0 - \frac{0.06}{2} \log \frac{\left[H^+\right]_a^2}{\left[H^+\right]_c^2}$

$$E_{cell} = 0.06 \times \left[(pH)_{HCOOH} - (pH)_{CH_3COOH}\right]$$

$$= 0.06 \times [3 - 2.4]$$

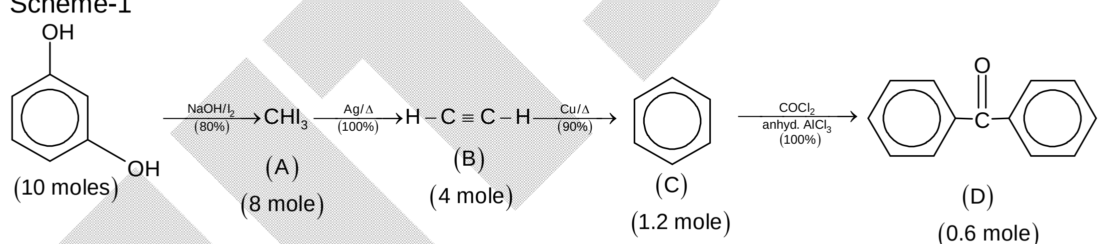
$$= 0.036 \text{ V} = 36 \text{ mV}.$$

32. 10

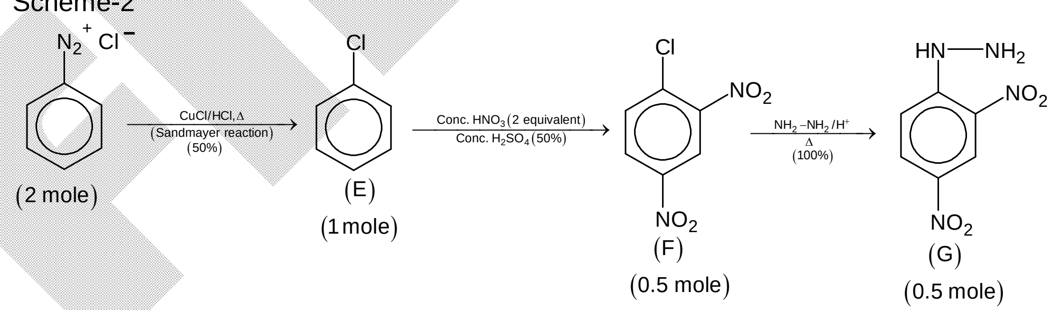
Sol. Reaction (i), (v), (vi), (vii), (viii), (ix), (xiii), (xvii), (xviii) and (xix)

33. 181

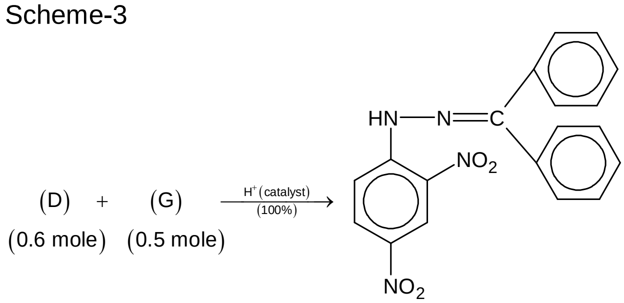
Sol. Scheme-1



Scheme-2



Scheme-3



(Z)

(0.5 mole)

Molar mass of Z = 362 g

So, mass of Z formed = 362 × 0.5 = 181 g.

34. 504

Sol. **Step 1:** Calculation of resonance energy of benzene (liq.):

$$\Delta_f H_{cal.}(\text{benzene}) = \left[6\times\Delta H_{C(s)\to C(g)} + 3\times BE_{H-H}\right] - \left[1\times\Delta H_{vap}(\text{benzene}) + 3\times 350 + 3\times 610 + 6\times 414\right]$$

$$\Delta_f H_{cal.} = \left[6\times 714 + 3\times 434\right] - \left[31 + 6\times 414 + 3\times 350 + 3\times 610\right]$$

$$\Delta_f H_{cal.} = \left[4284 + 1302\right] - \left[31 + 2484 + 1050 + 1830\right]$$

$$= 5586 - 5395$$

$$\Delta_f H_{cal.} = 191\ \text{kJ mol}^{-1}$$

Resonance energy of benzene $= \Delta_f H_{(obs)} - \Delta_f H_{(cal)} = 49 - 191 = -142\ \text{kJ mol}^{-1}$

Step 2: Calculation of ΔS :

$$H_2O(s, 273K) \xrightarrow[\Delta S_1]{q_1 = 15\times 6 = 90\text{kJ}} H_2O(\ell, 273\text{ K}) \xrightarrow[\Delta S_2]{q_2 = 142-90 = 52\text{ kJ}} H_2O(\ell, T_2)$$

$$10^3 \times 52 = 15\times(4\times 18)\times(T_2 - 273) \Rightarrow T_2 = 321.15\text{ K}$$

$$\Delta S_{syst} = \Delta S_1 + \Delta S_2$$

$$\Delta S_{syst} = \frac{15\times 6\times 10^3}{273} + 15\times 4\times 18\,\ell n\frac{321.15}{273}$$

$$= 329.67 + 1080\times 2.303\times \log(1.18)$$

$$= 329.67 + 174.10$$

$$= 503.77\ \text{JK}^{-1} \approx 504\ \text{JK}^{-1}$$

Mathematics

PART – III

SECTION – A

35. B, C, D

Sol. $x^2 + (x + 1)^2 = 3x + k,\ x \neq -1, 0$

$2x^2 - x + 1 - k = 0$

$x = 0,\ k = 1$

$x = -1,\ k = 2 \times (-1)^2 + 1 + 1 = 4$

$D = 0 \Rightarrow 1 - 4 \times 2(1 - k) = 0$

$1 - k = \frac{1}{8}\ ;\ k = 1 - \frac{1}{8} = \frac{7}{8}$

36. A, B, C

Sol. Let h(x) = $e^{\beta x} f(x)$ apply Rolle's on [a, b]

Let $h(x) = e^{g(nx)} f(x)$

Let $h(x) = \frac{f(x)}{g(x)}$

37. A, C

Sol. Circumcircle of ΔTPQ is circle with diameter RC where C is centre of C_3

If Q_1 and O_2 are centers of C_1 and C_2, then $\tan\alpha = \frac{3}{5}, \tan\beta = \frac{4}{5},\ \tan(\alpha + \beta) = \frac{r}{5}$

$\Rightarrow r = \frac{175}{13}$

38. B

Sol. $\sum_{r=0}^{14} {}^{30}C_{2r} \cdot {}^{30}C_{2r+2}$ = coefficient of x^{28} in $\frac{\left((1+x)^{30} + (1-x)^{30}\right)^2}{4} = \frac{{}^{60}C_{28} + {}^{30}C_{14}}{2}$

39. C

Sol. $|z_1| = \sqrt{25^2 - 15^2} = 5\sqrt{25 - 9} = 20$ as $|z_1 - z_2| = |z_1 - z_3|$

So, α be foot of perpendicular drawn from $A(z_1)$ on y = x

So, $|\alpha| = \sqrt{OA^2 - |z_1 - \alpha|^2} = \sqrt{|z_1|^2 - |z_1 - \alpha|^2} = \sqrt{20^2 - \left(\frac{20}{5\sqrt{2}}\right)^2} = 20 \times \frac{7}{5\sqrt{2}} = 14\sqrt{2}$

40. D

Sol. $P_n = \frac{3}{4}\prod_{r=2}^{n}\left(1 - \left(\frac{1}{4}\right)^r\right)$

41. A

Sol. $\sum_{k=1}^{3} \frac{1}{|z_k|^2} = 50 \times 3 - 14\left\{\cos\frac{\pi}{6} + \cos\left(\frac{3\pi}{6}\right) + \cos\frac{5\pi}{6}\right\} = 150$

42. A

Sol. Let S be the focus.

So, equation of AS is $y - 4 = \frac{3}{2}(x-4)$; $2y - 3x = -4$

BS, $y - 0 = -\frac{2}{3}(x-2)$; $3y + 2x = 4$

$\Rightarrow 5\left(\frac{20}{13}, \frac{4}{13}\right)$

43. B

Sol. (P) $\alpha_1 = \alpha_2 = \alpha_3 = 2$, AM of $\alpha_1, \alpha_2, \alpha_3$ is HM of $\alpha_1, \alpha_2, \alpha_3$

(Q) Range of $-x^2 + x + a$ $(-\infty, -1]$, $a = 1$

(R) $g(g(3)) = g(9) = 1 \Rightarrow g(g(g(3))) = g(1) = 2$

$g(g(0)) = 0$; $\frac{-2+0}{3\times 2 - 2\times 5 - 1} = \frac{2}{5}$

(S) $\hat{a}\times\left(\hat{b}\times\hat{c}\right) = \left(\frac{\hat{b}+\hat{c}}{\sqrt{2}}\right)$; $\left(\hat{a}\cdot\hat{c}\right)\hat{b} - \left(\hat{a}\cdot\hat{b}\right) = \frac{\hat{b}+\hat{c}}{\sqrt{2}}$

$\hat{a}\cdot\hat{c} = \frac{1}{\sqrt{2}}$ and $-\hat{a}\cdot\hat{b} = \frac{1}{\sqrt{2}}$

44. D

Sol. (P) $I = \int_0^{4\pi} \ln\left|14\sin(x+\alpha)\right| dx = 4\int_0^{\pi} \ln\left|\sin(x+\alpha)\right| dx + 4\pi\ln 14$

Put $\pi + \alpha$; $x + \alpha = t$

$= 4\int_{\alpha}^{\pi+\alpha} \ln\left|\sin t\right| dt + 4\pi\ln 14 = 4\times 2\times -\frac{\pi}{2}\ln 2 + 4\pi\ln 14 = 4\pi \ln 7$

(Q) Let $a = 2^{\alpha_1}\cdot 3^{\beta_1}\cdot 5^{\gamma_1}$; $b = 2^{\alpha_2}\cdot 3^{\beta_2}\cdot 5^{\gamma_2}$; $b = 2^{\alpha_3}\cdot 3^{\beta_3}\cdot 5^{\gamma_3}$

Given L.C.M. of a, b, c is $2^3\cdot 3^2\cdot 5^1$

H.C.F of a, b, c is $2\times 3\times 5$

So, for $\alpha_1, \alpha_2, \alpha_3$ there are 12 cases and $\beta_1, \beta_2, \beta_3$ there are 6 cases

So, total number of triplets is $12 \times 6 = 72$

(R) $\left\{\frac{x}{10}\right\}$ is discontinuous at 10, 20, 30, 40, 50

$\left[\frac{x}{2}\right]$ is discontinuous at 2, 4, 6, ..., 50

So, number of points of discontinuous 25 – 5 = 20

(S) $f(x) - g(x) = a(x-1)^2(x-2)^2$

Given $\int_0^1 \left(f(x) - g(x)\right)dx = a\int_0^1 \left((x-1)^2\left((x-1)-1\right)^2\right)dx$

$= a\int_0^1 \left((x-1)^4 + (x-1)^2 - 2(x-1)^3\right)dx = a\left|\frac{(x-1)^5}{5} + \frac{(x-1)^3}{3} - \frac{2(x-1)^4}{4}\right|_0^1$

$= a\left\{0 - \left(-\frac{1}{5} - \frac{1}{3} - \frac{2}{4}\right)\right\} = a\left\{\frac{31}{30}\right\}$

45. C

Sol. (P) $x = 1$

(Q) Area = $e + \frac{1}{e} - 3$

(R) $y^2 = e^{2x} - 2e^{2x} \ln y$

So, given expression is 4

(S) $I = \int_0^{\frac{\pi}{2}} (\cos x)^{2023} \sin(2025x)\,dx = \int_0^{\frac{\pi}{2}} (\cos x)^{2023} \{\sin 2024x \cos x + \text{cod}2024 \sin x\}\,dx$

$= \int_0^{\frac{\pi}{2}} \sin 2024x \cos^{2024} x + \cos 2024x \cos^{2025} x \sin x\,dx = \frac{-1}{2024}\int_0^{\frac{\pi}{2}} \frac{d}{dx}\left(\cos 2024x \cos^{2024} x\right) dx$

$= \frac{-1}{2024}\left|\cos 2024x (\cos x)^{2024}\right|_0^{\frac{\pi}{2}} = \frac{-1}{2024}\{0-1\} = \frac{1}{2024}$

SECTION – B

46. 8

Sol. $P + Q^T = \text{adj } Q$

$\Rightarrow P^T + Q = (\text{adj } Q)^T \Rightarrow |\text{adj } P| = |(\text{adj } Q)^T|$

$\Rightarrow |P|^2 = |Q|^2 \Rightarrow |P| = \pm|Q| \Rightarrow P = Q^T$, adj Q = 2P

$2PQ = Q \text{ adj } Q = |Q|I$

$PQ = 4I$ as $|Q| = 8$ and $QP = 4I$

47. 3

Sol. $R_1\ B_1\ R_2\ B_2\ R_3\ B_3\ R_4\ B_4\ R_5\ B_5\ R_6$

R_1 are to be filled with red balls and

B_1 are to be filled with blue balls

$B_1 + B_2 + B_3 + B_4 + B_5 = 10$ with $R_1 + R_2 + R_3 + R_4 + R_5 + R_6 = 6$

$R_1, R_6 \in W$, $R_2, R_3, R_4, R_5 \in N$

Total arrangements = ${}^{10-1}C_{5-1} \cdot {}^{2+6-1}C_{6-1} = {}^{9}C_4 \cdot {}^{7}C_5$

48. 2

Sol. $\text{sgn}\left(\left[\frac{x}{\tan^{-1} x}\right]\right) = 1$

49. 9

Sol. $\lim_{n\to\infty} \frac{\int_0^{\frac{1}{n}} x^{x+k-1}}{\left(\frac{1}{n}\right)^k} = \lim_{\alpha\to 0} \frac{\int_0^{\alpha} x^{x+k-1}dx}{\alpha^k} = \lim_{\alpha\to 0} \frac{\alpha^{(\alpha+k-1)}}{k\,\alpha^{k-1}} = \frac{1}{k}$

50. 1

Sol. Let $\vec{p} = 2\hat{i} + 3\hat{j} + 5\hat{k}$; $\vec{q} = \sin\alpha \sin\beta \hat{i} + \cos\beta \hat{j} + \cos\alpha \sin\beta \hat{k}$

$|\vec{q}| = \sqrt{\sin^2\alpha \sin^2\beta + \cos^2\beta + \cos^2\alpha \sin^2\beta} = 1$

$\Rightarrow \sin\alpha \sin\beta = 2\lambda$; $\cos\beta = 3\lambda$; $\cos\alpha \sin\beta = 5\lambda$

$1 = 38\lambda^2$; $\lambda = \frac{1}{\sqrt{38}}$

$$\det A = \left[\frac{\sin\alpha\sin\beta}{\cos\beta} + \frac{1}{3}\right] = 1$$

51. 8

Sol. Let equation of tangent be $\frac{x}{a}\cos\theta + \frac{7}{b}\sin\theta = 1$ as it passes through (–2, 0)

So, $-\frac{2}{a}\cos\theta = 1$; a –2 cos θ and $-\frac{b}{a}\frac{\cos\theta}{\sin\alpha} = 4$

$\tan\theta = -\frac{b}{4a}$; $\sec^2\theta = 1 + \tan^2\theta$

$\frac{4}{a^2} = 1 + \frac{b^2}{16a^2}$; $64 = 16a^2 + b^2$

$16a^2 + b^2 \geq 2\sqrt{16a^2b^2}$; $ab \leq \frac{64}{8} = 8$