FIITJEE
ALL INDIA TEST SERIES
JEE (Advanced)-2024
FULL TEST – I
PAPER –1
TEST DATE: 28-12-2023

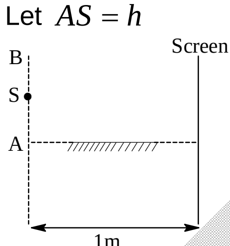
ANSWERS, HINTS & SOLUTIONS

Physics **PART – I**

SECTION – A

1. BC

Sol. Let $AS = h$



$$\frac{1}{4} = \frac{\lambda(1)}{2h} ....(i)$$

$$\frac{1}{6} = \frac{\lambda(1)}{2(h+0.6)} ....(ii)$$

$$\frac{\frac{1}{4}}{\frac{1}{6}} = \frac{h+0.6}{h}$$

$$\frac{3}{2} = \frac{h+0.6}{h} \Rightarrow h = 1.2mm$$

$$\frac{1}{4}mm = \frac{\lambda(1m)}{2\times(1.2)mm} \Rightarrow \lambda = 6000\mathring{A}$$

2. AD

Sol. $2(C_V)_{monoatomic} > (C_V)_{diatomic}$

Maximum change $= \dfrac{(2)\left(\frac{3R}{2}\right)-(1)\left(\frac{5R}{2}\right)}{\frac{5R}{2}} = \dfrac{1}{5}$

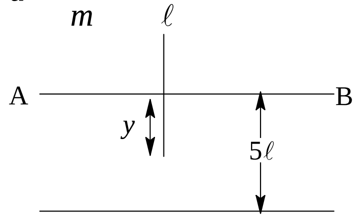
$\%\text{change} = 20\%$

3. ABC

Sol. Positive power lens (convex lens in rare medium) can from enlarged (or of same size) real, inverted image.
Positive power lens (convex lens in rare medium) can form virtual, erect, diminished image.

4. C

Sol. $a = \dfrac{\frac{m}{\ell}yg}{m} = \dfrac{gy}{\ell}$



$$\int_0^u v\,dv = \frac{g}{\ell}\int_0^{\ell} y\ dy$$

$u = \sqrt{g\ell}$........(1)

$v^2 = u^2 + 2g(4\ell)$

$u = 3\sqrt{gl}$

5. B

Sol. Work function $= \dfrac{hC}{\lambda} - \dfrac{hC}{2\lambda} = \dfrac{hC}{2\lambda}$

6. D

Sol. $v_1 + 2v_2 = 10$

$\underline{v_1 - v_2 = -e(10)}$

$3v_2 = 10(1+e)$

$3\sqrt{5(10)\left(\frac{1}{2}\right)} = 10(1+e) \Rightarrow e = \dfrac{1}{2}$

7. A

8. A

9. B

10. C

11. D

SECTION – B

12. 2

Sol. $U.\rho.=\text{Constant}$

$\frac{T}{V}=\text{constant}$

$P=\text{constant}$

$W=nR\Delta T \quad ......(i)$

$n\left(C_V\right)\Delta T=3$

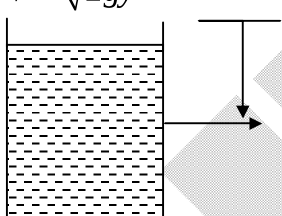
$n\left(\frac{3R}{2}\right)\Delta T = 3 \Rightarrow W = 2\,joule$

13. 4

14. 9

15. 5

Sol. $V=\sqrt{2gy}$



$$\Rightarrow dF = \left(1\times10^{-3}\right)\left(\sqrt{2gy}\ dy\sqrt{2gy}\times\rho\right)$$

$$dF = 1\times10^{-3}\times2\times10\times y\times10^{3}\ dy = 40\ ydy$$

$$\Rightarrow F = \frac{20}{2}\int ydy = \frac{40}{2}\left(0.75^2-0.25^2\right) = 20\times0.5 = 5$$

16. 5

17. 30

Chemistry

PART – II

SECTION – A

18. BC

Sol. $Z = \frac{V_{m,real}}{V_{m,ideal}}$

at high temperature and low pressure the gas follows PV=nRT

19. A

Sol. The volume of H_2SO_4 used at equivalence point is 15mL.

$15 \times 0.2 \times 2 = 20 \times conc.$

Conc. = 0.3

20. C

Sol. $NH_3 \uparrow \rightleftharpoons \frac{1}{2}N_2 \uparrow + \frac{3}{2}H_2 \uparrow$

$n \quad 0 \quad 0$

$n(1-\alpha) \quad \frac{n\alpha}{2} \quad \frac{3n\alpha}{2}$

Partial pressure at equation $\left(\frac{1-\alpha}{1+\alpha}\right)P; \frac{\alpha}{2(1+\alpha)}P; \frac{3}{2}\left(\frac{\alpha}{1+\alpha}\right)P$

$$K_p = \frac{\left(P_{N_2}\right)^{1/2}\left(P_{H_2}\right)^{3/2}}{P_{NH_3}} = \frac{3\sqrt{3}P}{4}\left[\frac{\alpha^2}{1-\alpha^2}\right]$$

$$\therefore \alpha = \left[1 + \frac{3\sqrt{3}}{4}\frac{P}{K_p}\right]^{-1/2}$$

21. D

Sol. Possible binary solutions are, Benzene + Toluene; Benzene + Xylene; Toluene + Xylene and ternary solution is Benzene + Toluene + Xylene.

In equimolar solutions, mole fractions are 1/2 in binary solution and 1/3rd in ternary solution for each component.

$\therefore$ For Benzene + Toluene, $P = \frac{1}{2}(75) + \frac{1}{2}(22) = \frac{97}{2} = 48.5 \, or \, 48\frac{1}{2}$

For Benzene + Xylene $P = \frac{1}{2}(75) + \frac{1}{2}(10) = \frac{85}{2} = 42.5$

For Xylene + Toluene $P = \frac{1}{2}(22+10) = 16$

For Ternary solution $P = \frac{1}{3}(75+22+10) = \frac{107}{3} = 35\frac{2}{3}$

Hence, option (D) is not possible

22. B

Sol. The chromyl chloride test is shown by ionic chlorides. $K_2\left[PtCl_6\right]$ and $Cl-Hg-Hg-Cl$ have covalently bonded chlorine atoms.

23. C

24. B

Sol. $Na\left[\overset{+I}{A}g\left(CN\right)_2\right]$

25. A

26. C

27. D

28. B

SECTION – B

29. 2

Sol. $\mu=\frac{m_{e^-}\times m_{e^+}}{m_{e^-}+m_{e^+}}=\frac{m_e}{2}$

$$IE=\frac{\mu\times 13.6}{m_e}\text{eV}=\frac{\frac{m_e}{2}\times 13.6}{m_e}=6.8\text{eV}$$

$A=6.8$

30. 6

Sol. $PO_4^{3-}+2H^+\rightarrow H_2PO_4^-$

5	10	0
0	0	5

$$\therefore pH=\frac{pK_{a1}+pK_{a2}}{2}=6$$

31. 34

32. 7

33. 5

34. 5

Mathematics

PART – III

SECTION – A

35. B, C

Sol. Given that $f_6(f_m(x)) = f_4(x)$

$$\frac{f_m(x)-1}{f_m(x)} = \frac{1}{1-x}$$

$$f_m(x) = \frac{x-1}{x} = f_6(x)$$

So, m = 6, and

$f_n\,[f_4(x)] = f_3(x)$

$$f_n\left[\frac{1}{1-x}\right] = \frac{1}{x}$$

Re Place $\frac{1}{1-x} \to x$ and x by $\left(1-\frac{1}{x}\right)$

$$f_n(x) = \frac{1}{1-\left(\frac{1}{x}\right)}$$

$$= \frac{x}{x-1}$$

$f_n(x) = f_5(x)$

n = 5

36. A, C, D

Sol. $x = \sqrt{5+\sqrt{5+\sqrt{5...}}} \quad \Rightarrow \quad x = \sqrt{5+x}$

$\Rightarrow\ x^2 - x - 5 = 0$

$\therefore\ P(x) = x^2 - x - 5$

$$\therefore\ \frac{1}{P(r)+5} = \frac{1}{r^2-r} = \frac{1}{r-1} - \frac{1}{r}$$

$$100\sum_{r=2}^{100}\left(\frac{1}{r-1}-\frac{1}{r}\right) = 100\left(1-\frac{1}{100}\right) = 99$$

$$\frac{1}{P(r)+3} = \frac{1}{r^2-r-2} = \frac{1}{(r-2)} - \frac{1}{r+1}$$

$$\therefore\ \sum_{r=3}^{n}\left(\frac{1}{r-2}-\frac{1}{r+1}\right) = \left(\frac{1}{1}-\frac{1}{4}\right)+\left(\frac{1}{2}-\frac{1}{5}\right)+...+\left(\frac{1}{n-2}-\frac{1}{n+1}\right)$$

$$\therefore\ \lim_{x\to\infty}\sum_{r=3}^{n}\left(\frac{1}{r-2}+\frac{1}{r+1}\right) = 1+\frac{1}{2}+\frac{1}{3} = \frac{11}{6}$$

And $\int_0^1 \frac{dx}{P(x)+x+6}dx = \int_0^1 \frac{1}{x^2+1}dx = \left(\tan^{-1}x\right)_0^1 = \frac{\pi}{4}$

And $\int_1^2 \frac{1}{\sqrt{P(x)+x+4}}dx = \int_1^2 \frac{1}{\sqrt{x^2-1}}dx = \left(\ln\left(x+\sqrt{x^2-1}\right)\right)_1^2 = \ln\left(2+\sqrt{3}\right)$

37. B, C

Sol. $R_n = \sum_{r=1}^{2n} x^{4r-3} = x + x^5 + x^9 ....$

$S_n = \sum_{r=1}^{2n} x^{4r-2} = x^2 + x^6 + x^{10} ....$

$T_n = \sum_{r=1}^{2n} x^{4r-1} = x^3 + x^7 + x^{11} ....$

$U_n = \sum_{r=1}^{2n} x^{4\times r} = x^4 + x^8 + x^{12} ....$

Clearly $R_n > S_n > T_n > U_n$ as $x \in \left(0, \frac{\pi}{4}\right)$

And $\lim_{n\to\infty}(R_n + S_n + T_n + U_n) = x + x^2 + ... = \frac{x}{1-x}$

$\because$ $R_n + S_n = T_n + U_n$

$\Rightarrow \frac{x\left(1-x^{8n}\right)}{1-x^4} + \frac{x^2\left(1-x^{8n}\right)}{1-x^4} = \frac{x^3\left(1-x^{8n}\right)}{1-x^4} + \frac{x^4\left(1-x^{8n}\right)}{1-x^4}$

$\Rightarrow x + x^2 = x^3 + x^4$ $\Rightarrow$ $(x + x^2)(x^2 - 1) = 0$ $\Rightarrow$ no value of x.

38. D

Sol. Now Locus of M is a circle having PQ as diameter Hence

E_1:- $(x - 2)(x + 2) + (y - 7)(y + 5) = 0$

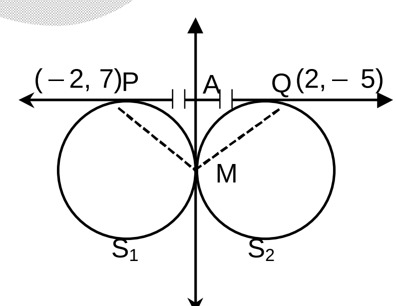
$\Rightarrow x^2 + y^2 - 2y - 39 = 0$ Locus of mid point of chord passing through (1, 1) is E_2

$BR^2 + BC^2 = CR^2$

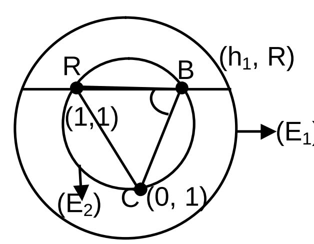
$(h - 1)^2 + (k - 1)^2 + h^2 + (k - 1)^2 = 1$

$\Rightarrow h^2 + k^2 - h - 2k + 1 = 0$

$x^2 + y^2 - x - 2y + 1 = 0$



AP = AQ = AM



39. A

Sol. $I = \int_{\pi/8}^{3\pi/8} \frac{11 + 4\cos 2x + \cos 4x}{1 - \cos 4x} dx$ …(1)

Using King

$I = \int_{\pi/8}^{3\pi/8} \frac{11 - 4\cos 2x + \cos 4x}{1 - \cos 4x} dx$ …(2)

(1) + (2)

$$2I = 2\int_{\pi/8}^{3\pi/8} \frac{11+\cos 4x}{1-\cos 4x}\,dx$$

$$\Rightarrow\ I = I = \int_{\pi/8}^{3\pi/8} \frac{12-(1-\cos 4x)}{1-\cos 4x} = 12\int_{\pi/8}^{3\pi/8} \frac{1}{2\sin^2 2x}dx - \int_{\pi/8}^{3\pi/8} dx = 6\int_{\pi/8}^{3\pi/8} \operatorname{cosec}^2 2x\,dx - \frac{\pi}{4}$$

$$= \left[-\frac{6}{2}\cot 2x\right]_{\frac{3\pi}{8}}^{\frac{\pi}{8}} - \frac{\pi}{4} = -3[(1)+(1)] - \frac{\pi}{4} = -6 - \frac{\pi}{4}$$

40. D

Sol. $V = \begin{vmatrix} a_1 & a_2 & a_3 \\ a_2 & a_3 & a_1 \\ a_3 & a_1 & a_2 \end{vmatrix} = \frac{1}{2}(a_1+a_2+a_3)\left[(a_2-a_3)^2+(a_1-a_2)^2+(a_3-a_1)^2\right]$

$= \frac{1}{2}(3a_2)(6d^2)$ (d is common difference)

$= 9d^2a_2 = 54 \Rightarrow d^2a_2 = 6$

$\Rightarrow\ d^2 = 1$ and $a_2 = 6$

$\Rightarrow\ d = 1$

41. B

Sol. $P = \begin{bmatrix} 1 & 0 & 0 \\ 4 & 1 & 0 \\ 16 & 4 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix} + \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$

Let $A = \begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix} \Rightarrow A^2 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 16 & 0 & 0 \end{bmatrix}$ and $A^3 = \begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix}$

So $A^n = 0\ \forall n \geq 3$ $= I + 50A + \frac{50 \times 49}{2}A^2$

$Q + I = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} + 50\begin{bmatrix} 0 & 0 & 0 \\ 4 & 0 & 0 \\ 16 & 4 & 0 \end{bmatrix} + 25 \times 49\begin{bmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 16 & 0 & 0 \end{bmatrix}$

So $\frac{q_{31}+q_{32}}{q_{21}} = \frac{16(50+25 \times 49)+50\times 4}{50 \times 4} = 103$

Sum of digits = 4

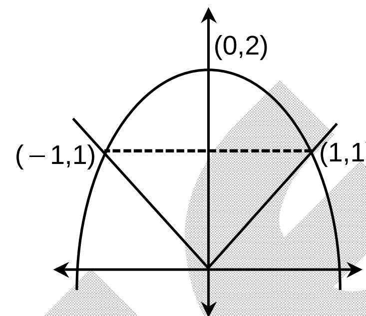
42. B

Sol. (P) CS_1 and CS_2 divided the triangle into three of equal area so that S_1 and S_2 are points of trisection of AB. Thus $S_1 = (0, 0)$, $S_2 = (2, -3)$. Equation of CS_1 is $y - x = 0$. Slope of $CS_2 = -4$. The line through (0, 0) drawn parallel to CS_2 is Required equation is $y^2 + 3xy - 4x^2 = 0$

Required equation $y^2 + 3xy - 4x^2 = 0$

$\Rightarrow \lambda + \mu = 3 + 4 = 7$

(Q) The required area is bounded by the lines $y = \pm x$ and the parabola $x^2 = -(y - 2)$ having the vertex at (0,2) and passing through $(\pm 1, 1)$. By summary about the y-axis,



$$\text{Area} = 2\left[\frac{1}{2} + \int_1^2 \sqrt{2-y}\,dy\right] = \frac{7}{3} = \frac{p}{q}$$

$\Rightarrow p - q = 7 - 3 = 4$

(R) With $y = xv' = \Rightarrow \frac{2v\,dv}{1-v^2} = \frac{dx}{x}$

Whose solution is $x(1 - v^2) = c$. When $x = 2$, $v = \frac{1}{2}$ so that $c = \frac{3}{2}$

The equation of the curve is $x^2 - y^2 = \frac{3x}{2}$ $\Rightarrow \left(x - \frac{3}{4}\right)^2 - y^2 = \frac{9}{16}$

This is a rectangular hyperbola with eccentricity $\sqrt{2}$.

$\therefore\ e^6 = 8$

(S) $P = \lim\limits_{x\to\infty} \frac{\left[\left(n^2+1\right)\left(n^2+2^2\right)...\left(n^2+n^2\right)\right]^{m/n}}{n^{2m}}$

$\ln P = \lim\limits_{x\to\infty}\left[\sum_{r=1}^{n} \ln\left(n^2+r^2\right) - 2m\ \ln\ n\right] \times \frac{m}{n}$

$\ln P = \lim\limits_{x\to\infty}\left[\frac{\sum_{r=1}^{n} \ln\left(n^2+r^2\right) - 2n\ \ln\ n}{n}\right] \times m$

$\ln P = \left[\ln 2 + \frac{\pi}{2} - 2\right] \times m$

$P = e^{m\left[\ln 2 + \frac{\pi}{2} - 2\right]} = e^{m\ln 2}.e^{-2m}.e^{\frac{m\pi}{2}} = \frac{2^m e^{\frac{m\pi}{2}}}{e^{2m}} = \frac{\left(2e^{\pi/2}\right)^m}{e^{2m}}$

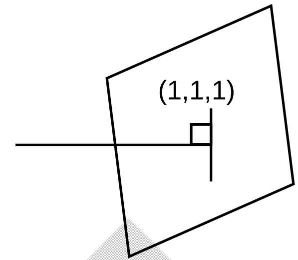
$= \left(\frac{2\sqrt{e^{\pi}}}{e^2}\right)^m \Rightarrow a = 2$

43. C

Sol. (P): Equation of the plane A(x – 1) + B(y – 1) + C(z – 1) = 0
Since the lines is perpendicular to the plane (1)
$\therefore$ 3(x – 1) + 0(y – 1) + 4(z – 1) = 0
3x + 0y + 4z – 7 = 0
Distance from (0, 0, 0)

$$d = \left|\frac{-7}{5}\right| = \frac{7}{5} = \frac{p}{q}$$

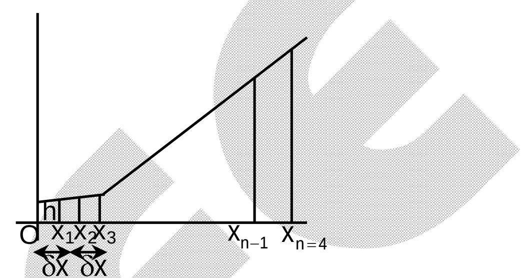
$\Rightarrow$ p – q = 2



(Q) $L = \lim\limits_{\delta x \to 0} \delta x\left(x_1 + x_2 + x_3 + ... + x_n\right)$

$$= \lim_{n\to\infty}\left[\frac{4}{n} + \frac{8}{n} + \frac{12}{n} + ... + 4.\frac{n}{n}\right]\left(\delta x = \frac{4}{n}\right)$$

$$= \lim_{n\to\infty}\frac{16}{n^2}\left(1+2+3+..+n\right) = \lim_{n\to\infty}\frac{16}{n^2}.\frac{n(n+1)}{2} = 8$$



(R) If x is replaced by – x in the given equation, then
– x f(– x) + (1 + x) f(x) = x^2 – x + 1
subtracting the two equations we get
f(– x) = f(x) + 2x,
subtracting the value of f(– x) in the given equation we get
xf(x) + (1 – x) (f(x) + 2x) = x^2 + x + 1 and thus

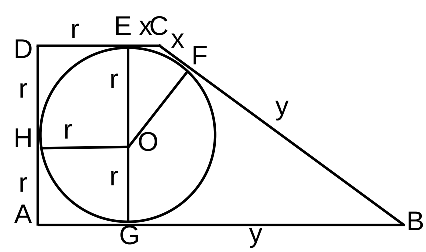
$$f(x) = x^2 + x + 1 - 2x(1 - x) = 3x^2 - x + 1 = 3\left(x - \frac{1}{6}\right)^2 + \frac{11}{12}$$

and hence f(x) $\geq \frac{11}{12}$. If x = $\frac{1}{6}$ than we get $f\left(\frac{1}{6}\right) = \frac{11}{12} = \frac{p}{q}$

$\Rightarrow$ (q – p) = 1

(S) Using the notation from the above diagram and the conditions from the problem one obtains:
3(r + x) = r + y $\Leftrightarrow y = 3x + 2r$
and $(x + y)^2 = 4r + \{(r + y) - (r + x)^2\} \Leftrightarrow xy = r^2$
Hence $x(3x + 2r) = r^2 \Leftrightarrow x = \frac{r}{3}$
And consequently y = 3r. On the other hand the area of the trapezoid ABCD is 4, thus (r + x + r + y)r = 4.
Substituting for x = $\frac{r}{3}$ and y = 3r,

we get r = $r = \frac{\sqrt{3}}{2} \Rightarrow 4r^2 = 3$



44. A

Sol. (P) x + 2x +… + 12x = 78x

$\therefore$ {x} = x. {2x} = 2x… {12x} = 12x

$\therefore$ $0 \le 12x < 1 \Rightarrow 0 \le x < \frac{1}{12}$

Since $x \in \left[\frac{1}{15}, \frac{1}{10}\right]$

$\therefore$ $x \in \left[\frac{1}{15}, \frac{1}{12}\right) \Rightarrow \frac{1}{15}, \frac{1}{14}, \frac{1}{13}$

$\therefore$ Number of values of x = 3

(Q) $(2x^2 + 3x + 4)^{10} = \sum_{r=0}^{20} a_r x^r$ replace x by $\frac{2}{x}$ we get

$$\left(\frac{8}{x^2} + \frac{6}{x} + 4\right)^{10} = \sum_{r=0}^{20} a_r \frac{2^r}{x^r}$$

$$\Rightarrow 2^{10}(4 + 3x + 2x^2)^{10} = \sum_{r=0}^{20} a_r 2^r x^{20-r}$$

$$\Rightarrow 2^{10}\sum_{r=0}^{20} a_r x^r = \sum_{r=0}^{20} a_r 2^r x^{20-r}$$

Comparing coefficient of x^9 both sides, we get

$$2^{10} . a_9 = 2^{11} a_{11} \Rightarrow \frac{a_9}{a_{11}} = 2$$

(R) Let P be origin and position vectors of point A, B, C be $\vec{\alpha}, \vec{\beta}, \vec{\gamma}$

$$\left|\vec{\alpha}+\vec{\beta}+\vec{\gamma}\right|^2 + \left|\vec{\alpha}-\vec{\beta}\right|^2 + \left|\vec{\beta}-\vec{\gamma}\right|^2 + \left|\vec{\gamma}-\vec{\alpha}\right|^2 = 3\left(\left|\vec{\alpha}\right| + \left|\vec{\beta}\right| + \left|\vec{\gamma}\right|^2\right)$$

$\Rightarrow$ $9PG^2 + AB^2 + BC^2 + CA^2 = 3(PA^2 + PB^2 + PC^2)$

$\Rightarrow$ $9 \times 1 + a^2 + b^2 + c^2 = 3(1 + 4 + 1)$

$\Rightarrow$ $a^2 + b^2 + c^2 = 9$

(S) f(xy) = f(x) + f(y)

$\Rightarrow$ f(16) = f(4) + f(4) = 3 $\Rightarrow$ f(4) = $\frac{3}{2}$

$\Rightarrow$ f(2) + f(2) = $\frac{3}{2}$ $\Rightarrow$ f(2) = $\frac{3}{4}$

$$\frac{1}{f(2)} + \frac{1}{f(4)} = \frac{4}{3} + \frac{2}{3} = 2$$

45. D

Sol. (P) The equation of tangents to hyperbola having slope m are

$y = mx \pm\sqrt{9m^2 - 49}$

Now, $\frac{2\sqrt{9m^2-49}}{\sqrt{1+m^2}} = 2(\text{Given})$

$\Rightarrow$ $m = \pm\frac{5}{2}$ $\Rightarrow \frac{2}{5}|m| = 1$

(Q) As, $\frac{e}{2}$ and $\frac{e'}{2}$ are eccentricities of a hyperbola and its conjugate hyperbola, so

$\frac{4}{e^2}+\frac{4}{e'^2}=1 \Rightarrow 4=\frac{e^2e'^2}{e^2+e'^2}...$ (1)

The equation of the lines e'x + ey − ee' = 0

It is tangent to the circle $x^2 + y^2 = r^2$

$\therefore \quad \frac{ee'}{\sqrt{e^2+e'2}}=r=2$ [usin(1)]

(R) The equation of common tangents is x – 2x + 4 = 0

(a + b) = 5

(S) The equation of chord to ellipse whose midpoint is (0, 3) is (T = S_1)

$\frac{x^2}{16}+\frac{y^2}{25}=1 \quad \Rightarrow \frac{x^2}{16}=1-\frac{9}{25} \quad \Rightarrow x=\pm\frac{16}{5}$

Length of chord = $\frac{32}{5} \Rightarrow p=4$

SECTION – B

46. 4

Sol. Give integral= $\frac{\pi}{2}\int_0^a \frac{1}{\tan^{-1}\alpha+\tan^{-1}x}.\frac{dx}{x^2+1}=\frac{\pi}{2}\left(\ln\left(\tan^{-1}\alpha+\tan^{-1}x\right)\right)_0^a=\frac{\pi}{2}\ln 2$

47. 5

Sol. $a^3 + b^3 + c^3 = p$,

$a^3b^3 + b^3c^3 + c^3a^3 = q$ and $a^3b^3c^3 = r$

$\because \quad \sin^3A + \sin^3B + \sin^3C - 3\sin A \sin B \sin C = 8\Delta^3$

$\Rightarrow \quad \frac{a^3+b^3-3abc}{8R^3}=8\Delta^3$

$\Rightarrow \quad \frac{\left(p-3r^{1/3}\right).64\Delta^3}{8(abc)^3}=8\Delta^3$

$\Rightarrow \quad p-3r^{1/3} = a^3b^3c^3 = r \quad \Rightarrow p = r + 3r^{1/3}$

$\because \quad p, q, r \in N$

$\therefore$ For minimum value r = 1 $\therefore$ p = 4

$\therefore$ p + r = 5

48. 4

Sol. Given that $N=\left(\prod_{K=1}^{14} P_k\right)\left(\prod_{k=1}^{14}\log_{(k+1)}(k+2)\right)$

$= 1. \log_2 16 = 4$

Now $(\tan x + \cot x)^2 = 4\sin^2 x$

$\Rightarrow \quad x = \frac{\pi}{4},\frac{3\pi}{4},\frac{5\pi}{4},\frac{7\pi}{4}$

Number of solution = 4.

49. 2

Sol. Replacing t by z^2 we get

$\int_0^4 f(t)dt = \int_0^2 2zf(z^2)dz$

From L.M.V. T $\dfrac{\int_0^2 2z f(z^2)dz - \int_0^0 2z f(z^2)dz}{2-0} = 2\gamma f(\gamma^2)$

Where $\gamma \in (0,2)$

for 0 < α. < β < 2, using mean value theorem

$$\int_0^2 2z f(z^2)dz - 2\left[2\gamma f\left(\gamma^2\right)\right] = 2\left[\frac{2\alpha f(\alpha)^2 + 2\beta f\left(\beta^2\right)}{2}\right]$$

$$\Rightarrow \int_0^4 f(t)\,dt = 2\left[\alpha f\left(\alpha^2\right) + f\left(\beta^2\right)\right] \text{ for } 0 < \alpha,\ \beta < 2$$

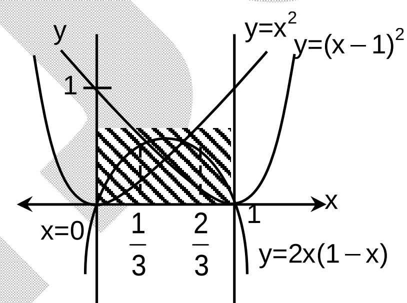
Hence k = 2

50. 44

Sol. Form fig it clear that

$$f(x) = \begin{cases} (1-x)^2 & 0 \le x \le \frac{1}{3} \\ 2x(1-x) & \frac{1}{3} < x \le \frac{2}{3} \\ x^2 & \frac{2}{3} < x \le 1 \end{cases}$$

The required Area



$$A = \int_0^1 f(x)$$

$$= \int_0^{\frac{1}{3}} (1-x)^2 dx + \int_{\frac{1}{3}}^{\frac{2}{3}} 2x(1-x)dx + \int_{\frac{2}{3}}^{1} x^2 dx$$

$$= \left[-\frac{1}{3}(1-x)^3\right]_0^{\frac{1}{3}} + \left[\left(x^2 - \frac{2x^3}{3}\right)\right]_{\frac{1}{3}}^{\frac{2}{3}} + \left[\frac{x^3}{3}\right]_{\frac{2}{3}}^{1}$$

$$= \frac{17}{27}$$

So, $\dfrac{p}{q} = \dfrac{17}{27}$ Hence p + q = 17 + 27 = 44

51. 72

Sol. LCM of $\alpha, \beta, \gamma = p^2 q^2 r$ & HCF = pqr

$\therefore$ $\alpha = p^{m_1} q^{m_1} r$

$\beta = p^{m_2} q^{n_2} r$

$\gamma = p^{m_3} q^{n_3} r$

Minimum of (m_1, m_2, m_3) = 1& maximum of (m_1, m_2, m_3) = 3

$\therefore$ Number of possibilities for m_1, m_2, m_3 = 12

and minimum of n_1, n_2, n_3 = 1 and maximum (n_1, n_2, n_3) = 2

$\therefore$ Number of possibilities = 6

$\therefore$ Total Number of ordered triplets = 12 x 6 = 72