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Units, Dimension, Measurements and Practical Physics

Fundamental or base quantities

The quantities which do not depend upon other quantities for their complete definition are known as *fundamental or base quantities*.

e.g. : length, mass, time, etc.

Derived quantities

The quantities which can be expressed in terms of the fundamental quantities are known as *derived quantities*.

e.g. Speed (=distance/time), volume, acceleration, force, pressure, etc.

Units of physical quantities

The chosen reference standard of measurement in multiples of which, a physical quantity is expressed is called the *unit* of that quantity.

e.g. Physical Quantity = Numerical Value × Unit

Systems of Units

	MKS	CGS	FPS	MKSQ	MKSA
(i)	Length (m)	Length (cm)	Length (ft)	Length (m)	Length (m)
(ii)	Mass (kg)	Mass (g)	Mass (pound)	Mass (kg)	Mass (kg)
(iii)	Time (s)	Time (s)	Time (s)	Time (s)	Time (s)
(iv)	–	–	–	Charge (Q)	Current (A)

Fundamental Quantities in S.I. System and their units

S.N.	Physical Qty.	Name of Unit	Symbol
1	Mass	kilogram	kg
2	Length	meter	m
3	Time	second	s
4	Temperature	kelvin	K
5	Luminous intensity	candela	Cd
6	Electric current	ampere	A
7	Amount of substance	mole	mol

SI Base Quantities and Units

Base Quantity	SI Units		
	Name	Symbol	Definition
Length	meter	m	The meter is the length of the path traveled by light in vacuum during a time interval of 1/(299, 792, 458) of a second (1983)
Mass	kilogram	kg	The kilogram is equal to the mass of the international prototype of the kilogram (a platinum-iridium alloy cylinder) kept at International Bureau of Weights and Measures, at Sevres, near Paris, France. (1889)
Time	second	s	The second is the duration of 9, 192, 631, 770 periods of the radiation corresponding to the transition between the two hyperfine levels of the ground state of the cesium-133 atom (1967)
Electric Current	ampere	A	The ampere is that constant current which, if maintained in two straight parallel conductors of infinite length, of negligible circular cross-section, and placed 1 metre apart in vacuum, would produce between these conductors a force equal to 2 x 10^{-7} Newton per metre of length. (1948)
Thermodynamic Temperature	kelvin	K	The kelvin, is the fraction 1/273.16 of the thermodynamic temperature of the triple point of water. (1967)
Amount of Substance	mole	mol	The mole is the amount of substance of a system, which contains as many elementary entities as there are atoms in 0.012 kilogram of carbon-12. (1971)
Luminous Intensity	candela	Cd	The candela is the luminous intensity, in a given direction, of a source that emits monochromatic radiation of frequency 540 x 10^{12} hertz and that has a radiant intensity in that direction of 1/683 watt per steradian (1979).

Supplementary Units

- Radian (rad) - for measurement of plane angle
- Steradian (sr) - for measurement of solid angle

Dimensional Formula

Relation which express physical quantities in terms of appropriate powers of fundamental units.

Use of dimensional analysis

- To check the dimensional correctness of a given physical relation
- To derive relationship between different physical quantities
- To convert units of a physical quantity from one system to another

$$n_1u_1 = n_2u_2 \Rightarrow n_2 = n_1\left(\frac{M_1}{M_2}\right)^a\left(\frac{L_1}{L_2}\right)^b\left(\frac{T_1}{T_2}\right)^c$$

where $u = M^aL^bT^c$

Limitations of dimensional analysis

- In Mechanics the formula for a physical quantity depending on more than three other physical quantities cannot be derived. It can only be checked.
- This method can be used only if the dependency is of multiplication type. The formulae containing exponential, trigonometrical and logarithmic functions can't be derived using this method. Formulae containing more than one term which are added or subtracted like $s = ut + \frac{1}{2}at^2$ also can't be derived.
- The relation derived from this method gives no information about the dimensionless constants.
- If dimensions are given, physical quantity may not be unique as many physical quantities have the same dimensions.
- It gives no information whether a physical quantity is a scalar or a vector.

SI PREFIXES

The magnitudes of physical quantities vary over a wide range. The CGPM recommended standard prefixes for magnitude too large or too small to be expressed more compactly for certain powers of 10.

PREFIXES USED FOR DIFFERENT POWERS OF 10

Power of 10	Prefix	Symbol	Power of 10	Prefix	Symbol
10^{18}	exa	E	10^{-1}	deci	d
10^{15}	peta	P	10^{-2}	centi	c
10^{12}	tera	T	10^{-3}	milli	m
10^{9}	giga	G	10^{-6}	micro	μ
10^{6}	mega	M	10^{-9}	nano	n
10^{3}	kilo	k	10^{-12}	pico	p
10^{2}	hecto	h	10^{-15}	femto	f
10^{1}	deca	da	10^{-18}	atto	a

UNITS OF IMPORTANT PHYSICAL QUANTITIES

Physical quantity	Unit	Physical quantity	Unit
Angular acceleration	rad s^{-2}	Frequency	hertz
Moment of inertia	kg – m^2	Resistance	kg m^2 A^{-2} s^{-3}
Self inductance	henry	Surface tension	newton/m
Magnetic flux	weber	Universal gas constant	joule K^{-1} mol^{-1}
Pole strength	A–m	Dipole moment	coulomb–meter
Viscosity	poise	Stefan constant	watt m^{-2} K^{-4}
Reactance	ohm	Permittivity of free space (ε_0)	$coulomb^2/N–m^2$
Specific heat	J/kg°C	Permeability of free space (μ_0)	weber/A-m
Strength of magnetic field	newton A^{-1} m^{-1}	Planck's constant	joule–sec
Astronomical distance	Parsec	Entropy	J/K

DIMENSIONS OF IMPORTANT PHYSICAL QUANTITIES

Physical quantity	Dimensions	Physical quantity	Dimensions
Momentum	$M^1 L^1 T^{-1}$	Capacitance	$M^{-1} L^{-2} T^4 A^2$
Calorie	$M^1 L^2 T^{-2}$	Modulus of rigidity	$M^1 L^{-1} T^{-2}$
Latent heat capacity	$M^0 L^2 T^{-2}$	Magnetic permeability	$M^1 L^1 T^{-2} A^{-2}$
Self inductance	$M^1 L^2 T^{-2} A^{-2}$	Pressure	$M^1 L^{-1} T^{-2}$
Coefficient of thermal conductivity	$M^1 L^1 T^{-3} K^{-1}$	Planck's constant	$M^1 L^2 T^{-1}$
Power	$M^1 L^2 T^{-3}$	Solar constant	$M^1 L^0 T^{-3}$
Impulse	$M^1 L^1 T^{-1}$	Magnetic flux	$M^1 L^2 T^{-2} A^{-1}$
Hole mobility in a semi conductor	$M^{-1} L^0 T^2 A^1$	Current density	$M^0 L^{-2} T^0 A^1$
Bulk modulus of elasticity	$M^1 L^{-1} T^{-2}$	Young modulus	$M^1 L^{-1} T^{-2}$
Potential energy	$M^1 L^2 T^{-2}$	Magnetic field intensity	$M^0 L^{-1} T^0 A^1$
Gravitational constant	$M^{-1} L^3 T^{-2}$	Magnetic Induction	$M^1 T^{-2} A^{-1}$
Light year	$M^0 L^1 T^0$	Electric Permittivity	$M^{-1} L^{-3} T^4 A^2$
Thermal resistance	$M^{-1} L^{-2} T^3 K$	Electric Field	$M^1 L^1 T^{-3} A^{-1}$
Coefficient of viscosity	$M^1 L^{-1} T^{-1}$	Resistance	$ML^2 T^{-3} A^{-2}$

SETS OF QUANTITIES HAVING SAME DIMENSIONS

S.N.	Quantities	Dimensions
1.	Strain, refractive index, relative density, angle, solid angle, phase, distance gradient, relative permeability, relative permittivity, angle of contact, Reynolds number, coefficient of friction, mechanical equivalent of heat, electric susceptibility, etc.	$[M^0 L^0 T^0]$
2.	Mass or inertial mass	$[M^1 L^0 T^0]$
3.	Momentum and impulse.	$[M^1 L^1 T^{-1}]$
4.	Thrust, force, weight, tension, energy gradient.	$[M^1 L^1 T^{-2}]$
5.	Pressure, stress, Young's modulus, bulk modulus, shear modulus, modulus of rigidity, energy density.	$[M^1 L^{-1} T^{-2}]$
6.	Angular momentum and Planck's constant (h).	$[M^1 L^2 T^{-1}]$
7.	Acceleration, g and gravitational field intensity.	$[M^0 L^1 T^{-2}]$
8.	Surface tension, free surface energy (energy per unit area), force gradient, spring constant.	$[M^1 L^0 T^{-2}]$
9.	Latent heat capacity and gravitational potential.	$[M^0 L^2 T^{-2}]$
10.	Thermal capacity, Boltzmann constant, entropy.	$[ML^2 T^{-2} K^{-1}]$
11.	Work, torque, internal energy, potential energy, kinetic energy, moment of force, (q^2/C), (LI^2), (qV), (V^2C), (I^2Rt), $\frac{V^2}{R}t$, (VIt), (PV), (RT), (mL), $(mc\,\Delta T)$	$[M^1 L^2 T^{-2}]$
12.	Frequency, angular frequency, angular velocity, velocity gradient, radioactivity $\frac{R}{L}, \frac{1}{RC}, \frac{1}{\sqrt{LC}}$	$[M^0 L^0 T^{-1}]$
13.	$\left(\frac{\ell}{g}\right)^{1/2}, \left(\frac{m}{k}\right)^{1/2}, \left(\frac{L}{R}\right), (RC), (\sqrt{LC})$, time	$[M^0 L^0 T^1]$
14.	(VI), (I^2R), (V^2/R), Power	$[M L^2 T^{-3}]$

SOME FUNDAMENTAL CONSTANTS

Gravitational constant (G)	6.67×10^{-11} N m^2 kg^{-2}
Speed of light in vacuum (c)	3×10^8 ms^{-1}
Permeability of vacuum (μ_0)	$4\pi \times 10^{-7}$ H m^{-1}
Permittivity of vacuum (ε_0)	8.85×10^{-12} F m^{-1}
Planck constant (h)	6.63×10^{-34} Js
Atomic mass unit (amu)	1.66×10^{-27} kg
Energy equivalent of 1 amu	931.5 MeV
Electron rest mass (m_e)	9.1×10^{-31} kg $\equiv$ 0.511 MeV
Avogadro constant (N_A)	6.02×10^{23} mol^{-1}
Faraday constant (F)	9.648×10^4 C mol^{-1}
Stefan–Boltzmann constant (σ)	5.67×10^{-8} W m^{-2} K^{-4}
Wien constant (b)	2.89×10^{-3} mK
Rydberg constant (R_∞)	1.097×10^7 m^{-1}
Triple point for water	273.16 K (0.01°C)
Molar volume of ideal gas (NTP)	22.4 L = 22.4×10^{-3} m^3 mol^{-1}

KEY POINTS

- Trigonometric functions sinθ, cosθ, tanθ etc and their arrangements θ are dimensionless.
- Dimensions of differential coefficients $\left[\frac{d^n y}{dx^n}\right] = \left[\frac{y}{x^n}\right]$
- Dimensions of integrals $\left[\int y\,dx\right] = [yx]$
- We can't add or subtract two physical quantities of different dimensions.
- Independent quantities may be taken as fundamental quantities in a new system of units.

PRACTICAL PHYSICS

Rules for Counting Significant Figures

For a number greater than 1

- All non-zero digits are significant.
- All zeros between two non-zero digits are significant. Location of decimal does not matter.
- If the numbe is without decimal part, then the terminal or trailing zeros are not significant.
- Trailing zeros in the decimal part are significant.

For a Number Less than 1

Any zero to the right of a non-zero digit is significant. All zeros between decimal point and first non-zero digit are not significant.

Significant Figures

All accurately known digits in measurement plus the first uncertain digit together form significant figure.

Ex. 0.108 → 3SF, 40.000 → 5SF, 1.23×10^{-19} → 3SF, 0.0018 → 2SF

Significant Digits

The product or quotient will be reported as having as many significant digits as the number involved in the operation with the least number of significant digits.

For example : 0.000170 × 100.40 = 0.017068

Another example : $2.000 \times 10^4 / 6.0 \times 10^{-3} = 0.33 \times 10^7$

For example : $3.0 \times 800.0 = 2.4 \times 10^3$

The sum or difference can be no more precise than the least precise number involved in the mathematical operation. Precision has to do with the number of positions to the RIGHT of the decimal. The more position to the right of the decimal, the more precise the number. So a sum or difference can have no more indicated positions to the right of the decimal as the number involved in the operation with the LEAST indicated positions to the right of its decimal.

For example : 160.45 + 6.732 = 167.18 (after rounding off)

Another example : 45.621 + 4.3 – 6.41 = 43.5 (after rounding off)

Rules for rounding off digits :

1. If the digit to the right of the last reported digit is less than 5 round it and all digits to its right off.
2. If the digit to the right of the last reported digit is greater than 5 round it and all digits to its right off and increased the last reported digit by one.
3. If the digit to the right of the last reported digit is a 5 followed by either no other digits or all zeros, round it and all digits to its right off and if the last reported digit is odd round up to the next even digit. If the last reported digit is even then leave it as is.

For example if we wish to round off the following number to 3 significant digits : 18.3682

The answer is : 18.4. Another example : Round off 4.565 to three significant digits.

The answer would be 4.56.

Rounding off

$6.87 \to 6.9$, $6.84 \to 6.8$, $6.85 \to 6.8$,

$6.75 \to 6.8$, $6.65 \to 6.6$, $6.95 \to 7.0$

Order of magnitude

Power of 10 required to represent a quantity

$49 = 4.9 \times 10^1 \approx 10^1 \Rightarrow$ order of magnitude $=1$

$51 = 5.1 \times 10^1 \approx 10^2 \Rightarrow$ order of magnitude $= 2$

$0.051 = 5.1 \times 10^{-2} \approx 10^{-1}$ order of magnitude $= -1$

Errors

Whenever an experiment is performed, two kinds of errors can appear in the measured quantity.

(1) random and (2) systematic errors.

1. Random errors appear randomly because of operator, fluctuations in external conditions and variability of measuring instruments. The effect of random error can be some what reduced by taking the average of measured values. Random errors have no fixed sign or size.
2. Systematic errors occur due to error in the procedure, or miscalibration of the instrument etc. Such errors have same size and sign for all the measurements. Such errors can be determined.

A measurement with relatively small random error is said to have high precision. A measurement with small random error and small systematic error is said to have high accuracy.

Least Count Error :– If the instrument has known least count, the absolute error is taken to be equal to the least count unless otherwise stated.

$$\text{Relative error} = \frac{\text{absolute error in a measurement}}{\text{size of the measurement}}$$

A. ***Systematic errors*:**

They have a known sign. The systematic error is removed before beginning calculations. Bench error and zero error are examples of systematic error.

Propagation of combination of errors

Error in Summation and Difference :

$x = a + b$ then $\Delta x = \pm (\Delta a + \Delta b)$

Error in Product and Division

A physical quantity X depend upon Y & Z as $X = Y^a Z^b$ then maximum possible fractional error in X.

$$\frac{\Delta X}{X} = |a|\frac{\Delta Y}{Y} + |b|\frac{\Delta Z}{Z}$$

Error in Power of a Quantity

$$x = \frac{a^m}{b^n} \text{ then } \frac{\Delta x}{x} = \pm\left[m\left(\frac{\Delta a}{a}\right) + n\left(\frac{\Delta b}{b}\right)\right]$$

The quotient rule is not applicable if the numerator and denominator are dependent on each other.

e.g if $R = \frac{XY}{X+Y}$. We cannot apply quotient rule to find the error in R. Instead we write the equation as follows $\frac{1}{R} = \frac{1}{X} + \frac{1}{Y}$. Differentiating both the sides, we get $-\frac{dR}{R^2} = -\frac{dX}{X^2} - \frac{dY}{Y^2}$.

Thus $\frac{r}{R^2} = \frac{x}{X^2} + \frac{y}{Y^2}$

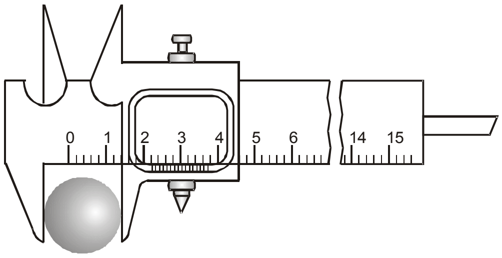
Least count

The smallest value of a physical quantity which can be measured accurately with an instrument is called the least count of the measuring instrument.

Vernier Callipers

Least count = 1MSD – 1 VSD

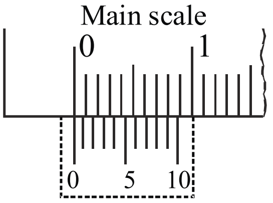
(MSD → main scale division, VSD → Vernier scale division)



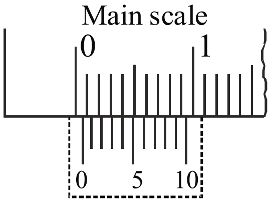
Ex. A vernier scale has 10 parts, which are equal to 9 parts of main scale having each path equal to 1 mm then least count $= 1\text{ mm} - \frac{9}{10}\text{ mm} = 0.1\text{ mm}$

$[\because 9 \text{ MSD} = 10 \text{ VSD}]$

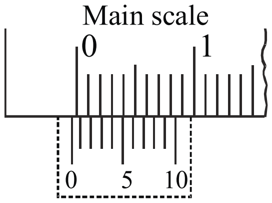
Zero Error



Vernier scale
without zero error



Vernier scale
with positive zero error
(i)

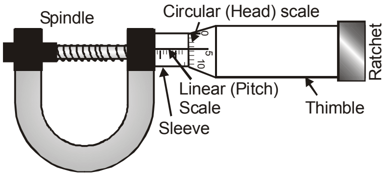


Vernier scale
with negative zero error
(ii)

The zero error is always subtracted from the reading to get the corrected value.
If the zero error is positive, its value is calculated as we take any normal reading.
Negative zero error = – [Total no. of vsd – vsd coinciding] ×L.C.

Screw Gauge

$$\text{Least count} = \frac{\text{pitch}}{\text{total no. of divisions on circular scale}}$$



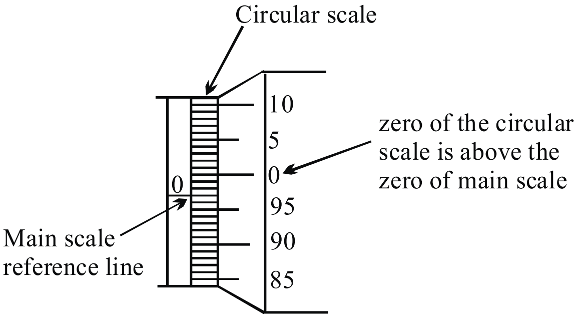
Ex. The distance moved by spindle of a screw gauge for each turn of head is 1mm. The edge of the humble is provided with a angular scale carrying 100 equal divisions. The least count = $\frac{1\text{mm}}{100}$ = 0.01 mm

Zero Error

If there is no object between the jaws (i.e. jaws are in contact), the screwgauge should give zero reading. But due to extra material on jaws, even if there is no object, it gives some excess reading. This excess reading is called Zero error.

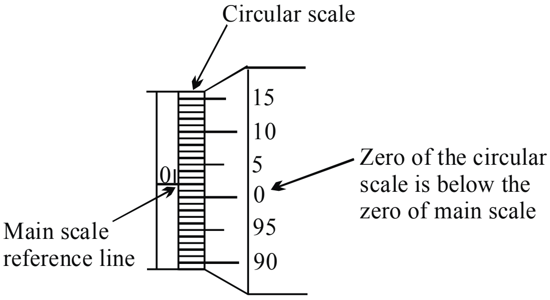
Negative Zero Error

(3 division error) i.e., –0.003 cm


Circular scale
10
5
0
95
90
85
0
zero of the circular scale is above the zero of main scale
Main scale reference line

Positive Zero Error

(2 division error) i.e., +0.002 cm


Circular scale
15
10
5
0
95
90
0
Zero of the circular scale is below the zero of main scale
Main scale reference line

CHAPTER 2

Basic Mathematics used in Physics

Quadratic Equation

Roots of $ax^2 + bx + c=0$ are $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Sum of roots $x_1 + x_2 = -\dfrac{b}{a}$;

Product of roots $x_1x_2 = \dfrac{c}{a}$

For real roots, $b^2 - 4ac \geq 0$

For imaginary roots, $b^2 - 4ac < 0$

Logarithm

$\log_{10}N = x \Rightarrow 10^x = N$

$\log_b N = \log_b a * \log_a N$

$\log_b 1 = 0$, $\log_a a = 1$

$\log mn = \log m + \log n$ $\quad$ $\log \dfrac{m}{n} = \log m - \log n$

$\log m^n = n \log m$ $\quad$ $\log_e m = 2.303 \log_{10} m$

$\log 2 = 0.3010$ $\quad$ $\log 3 = 0.4771$

Arithmetic progression-AP

$a, a+d, a+2d, a+3d, \ldots\ldots a+(n-1)d$

here d = common difference

Sum of n terms $S_n = \dfrac{n}{2}[2a+(n-1)d]$

n^{th} term, $a_n = a + (n-1)d$

Note: (i) $1+2+3+4+5\ldots.+ n = \dfrac{n(n+1)}{2}$

(ii) $1^2+2^2+3^2+\ldots+ n^2 = \dfrac{n(n+1)(2n+1)}{6}$

(iii) $1^3+2^2+3^2+\ldots+ n^3 = \left[\dfrac{n(n+1)}{2}\right]^2$

Binomial Theorem

$(1+x)^n = 1 + nx + \dfrac{n(n-1)}{2}x^2 + \dfrac{n(n-1)(n-2)}{6}x^3 + \ldots.$

$(1-x)^n = 1 - nx + \dfrac{n(n-1)}{2}x^2 - \dfrac{n(n-1)(n-2)}{6}x^3 + \ldots..$

If $x<<1$ then $(1+x)^n \approx 1 + nx$ & $(1-x)^n \approx 1-nx$

Componendo and dividendo theorem

If $\dfrac{p}{q} = \dfrac{a}{b}$ then $\dfrac{p+q}{p-q} = \dfrac{a+b}{a-b}$

Geometrical progression-GP

$a, ar, ar^2, ar^3, \ldots\ldots$ here, r = common ratio

n^{th} term, $a_n = a.r^{n-1}$

Sum of n terms $S_n = \dfrac{a(1-r^n)}{1-r}$

Sum of ∞ terms $S_\infty = \dfrac{a}{1-r}$ [where $|r| < 1$]

TRIGONOMETRY

2π radian = 360° ⇒ 1 rad = 57.3°

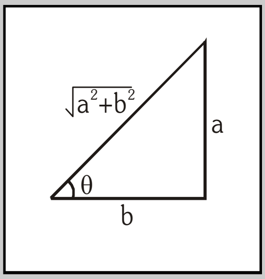
$\sin\theta = \dfrac{\text{perpendicular}}{\text{hypotenuse}}$ $\cos\theta = \dfrac{\text{base}}{\text{hypotenuse}}$ $\tan\theta = \dfrac{\text{perpendicular}}{\text{base}}$

$\cot\theta = \dfrac{\text{base}}{\text{perpendicular}}$ $\sec\theta = \dfrac{\text{hypotenuse}}{\text{base}}$ $\text{cosec}\,\theta = \dfrac{\text{hypotenuse}}{\text{perpendicular}}$

$\sin\theta = \dfrac{a}{\sqrt{a^2+b^2}}$ $\cos\theta = \dfrac{b}{\sqrt{a^2+b^2}}$ $\tan\theta = \dfrac{a}{b}$

$\text{cosec}\theta = \dfrac{1}{\sin\theta}$ $\sec\theta = \dfrac{1}{\cos\theta}$ $\cot\theta = \dfrac{1}{\tan\theta}$

$\sin^2\theta + \cos^2\theta = 1$ $1 + \tan^2\theta = \sec^2\theta$ $1 + \cot^2\theta = \text{cosec}^2\theta$



$\sin(A\pm B) = \sin A\cos B \pm \cos A\sin B$ $\cos(A\pm B) = \cos A\cos B \mp \sin A\sin B$

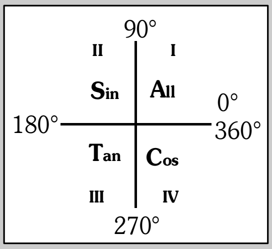
$\tan(A\pm B) = \dfrac{\tan A \pm \tan B}{1 \mp \tan A\tan B}$ $\sin 2A = 2\sin A\cos A$

$\cos 2A = \cos^2 A - \sin^2 A = 1 - 2\sin^2 A = 2\cos^2 A - 1$

$\tan 2A = \dfrac{2\tan A}{1-\tan^2 A}$ $\sin 3\alpha = 3\sin\alpha - 4\sin^3\alpha$

$\cos 3\alpha = 4\cos^3\alpha - 3\cos\alpha$ $2\sin A\sin B = \cos(A-B) - \cos(A+B)$

$2\cos A\cos B = \cos(A-B) + \cos(A+B)$ $2\sin A\cos B = \sin(A+B) + \sin(A-B)$



θ	0° (0)	30° ($\pi/6$)	*45°* ($\pi/4$)	*60°* ($\pi/3$)	*90°* ($\pi/2$)	*120°* ($2\pi/3$)	*135°* ($3\pi/4$)	*150°* ($5\pi/6$)	*180°* (π)	*270°* ($3\pi/2$)	*360°* (2π)
sinθ	0	$\frac{1}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{\sqrt{3}}{2}$	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	–1	**0**
cosθ	1	$\frac{\sqrt{3}}{2}$	$\frac{1}{\sqrt{2}}$	$\frac{1}{2}$	0	$-\frac{1}{2}$	$-\frac{1}{\sqrt{2}}$	$-\frac{\sqrt{3}}{2}$	–1	0	**1**
tanθ	**0**	$\frac{1}{\sqrt{3}}$	**1**	$\sqrt{3}$	∞	$-\sqrt{3}$	**–1**	$-\frac{1}{\sqrt{3}}$	**0**	∞	**0**

$\sin(90°+\theta) = \cos\theta$
$\cos(90°+\theta) = -\sin\theta$
$\tan(90°+\theta) = -\cot\theta$

$\sin(180°+\theta) = -\sin\theta$
$\cos(180°+\theta) = -\cos\theta$
$\tan(180°+\theta) = \tan\theta$

$\sin(180°-\theta) = \sin\theta$
$\cos(180°-\theta) = -\cos\theta$
$\tan(180°-\theta) = -\tan\theta$

$\sin(270°-\theta) = -\cos\theta$
$\cos(270°-\theta) = -\sin\theta$
$\tan(270°-\theta) = \cot\theta$

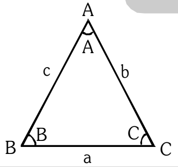
$\sin(-\theta) = -\sin\theta$
$\cos(-\theta) = \cos\theta$
$\tan(-\theta) = -\tan\theta$

$\sin(270°+\theta) = -\cos\theta$
$\cos(270°+\theta) = \sin\theta$
$\tan(270°+\theta) = -\cot\theta$

$\sin(90°-\theta) = \cos\theta$
$\cos(90°-\theta) = \sin\theta$
$\tan(90°-\theta) = \cot\theta$

$\sin(360°-\theta) = -\sin\theta$
$\cos(360°-\theta) = \cos\theta$
$\tan(360°-\theta) = -\tan\theta$

sine law



$$\frac{\sin A}{a} = \frac{\sin B}{b} = \frac{\sin C}{c}$$

For small θ

$\sin\theta \approx \theta$ $\cos\theta \approx 1$ $\tan\theta \approx \theta$ $\sin\theta \approx \tan\theta$

cosine law

$$\cos A = \frac{b^2+c^2-a^2}{2bc},\ \cos B = \frac{c^2+a^2-b^2}{2ca},\ \cos C = \frac{a^2+b^2-c^2}{2ab}$$

Differentiation

- $y = x^n \rightarrow \frac{dy}{dx} = nx^{n-1}$ • $y = \ell nx \rightarrow \frac{dy}{dx} = \frac{1}{x}$
- $y = \sin x \rightarrow \frac{dy}{dx} = \cos x$ • $y = \cos x \rightarrow \frac{dy}{dx} = -\sin x$
- $y = e^{\alpha x+\beta} \rightarrow \frac{dy}{dx} = \alpha e^{\alpha x+\beta}$ • $y = uv \rightarrow \frac{dy}{dx} = u\frac{dv}{dx} + v\frac{du}{dx}$
- $y = f(g(x)) \Rightarrow \frac{dy}{dx} = \frac{df(g(x))}{dg(x)} \times \frac{d(g(x))}{dx}$
- $y=k(\text{constant}) \Rightarrow \frac{dy}{dx} = 0$
- $y = \frac{u}{v} \Rightarrow \frac{dy}{dx} = \frac{v\frac{du}{dx} - u\frac{dv}{dx}}{v^2}$

Integration

C = Arbitrary constant, k = constant

- $\int f(x)dx = g(x) + C$
- $\frac{d}{dx}(g(x)) = f(x)$
- $\int kf(x)dx = k\int f(x)dx$
- $\int (u+v+w)dx = \int udx + \int vdx + \int wdx$
- $\int e^x dx = e^x + C$
- $\int x^n dx = \frac{x^{n+1}}{n+1} + C, n \neq -1$
- $\int \frac{1}{x}dx = \ell nx + C$
- $\int \sin x dx = -\cos x + C$
- $\int \cos x dx = \sin x + C$
- $\int e^{\alpha x+\beta}dx = \frac{1}{\alpha}e^{\alpha x+\beta} + C$
- $\int (\alpha x+\beta)^n dx = \frac{(\alpha x+\beta)^{n+1}}{\alpha(n+1)} + C$

Definite integration

$$\int_a^b f(x)dx = |g(x)|_a^b = g(b) - g(a)$$

Area under the curve y = f(x) from x =a to a = b is

$$A = \int_a^b f(x)dx$$

Maxima & Minima of a function y=f(x)

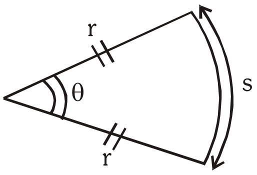
- For maximum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = -ve$
- For minimum value $\frac{dy}{dx} = 0$ & $\frac{d^2y}{dx^2} = +ve$

Average of a varying quantity

If y = f(x) then $< y >= \bar{y} = \frac{\int_{x_1}^{x_2} ydx}{\int_{x_1}^{x_2} dx} = \frac{\int_{x_1}^{x_2} ydx}{x_2 - x_1}$

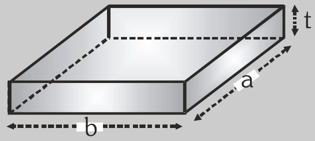
FORMULAE FOR DETERMINATION OF AREA

- Area of a square = $(\text{side})^2$
- Area of rectangle = length × breadth
- Area of a triangle = $\frac{1}{2}$ × base × height
- Area of a trapezoid
  $= \frac{1}{2} \times$ (distance between parallel sides) × (sum of parallel sides)
- Area enclosed by a circle = πr^2 (r = radius)
- Surface area of a sphere = $4\pi r^2$ (r = radius)
- Area of a parallelogram = base × height
- Area of curved surface of cylinder = $2\pi r\ell$
  where r = radius and ℓ = length
- Area of whole surface of cylinder = $2\pi r(r + \ell)$ where ℓ = length
- Area of ellipse = πab
  (a & b are semi major and semi minor axis respectively)
- Surface area of a cube = $6(\text{side})^2$
- Total surface area of a cone = $\pi r^2 + \pi r\ell$
  where $\pi r\ell = \pi r\sqrt{r^2 + h^2}$ = lateral area
- Arc length $s = r.\theta$
- Area of sector = $\frac{r^2\theta}{2}$



- Plane angle, $\theta = \frac{s}{r}$ radian
- Solid angle, $\Omega = \frac{A}{r^2}$ steradian

FORMULAE FOR DETERMINATION OF VOLUME



- Volume of a rectangular slab
  = length × breadth × height
  = abt
- Volume of a cube = $(\text{side})^3$
- Volume of a sphere = $\frac{4}{3}\pi r^3$
  (r = radius)
- Volume of a cylinder = $\pi r^2\ell$
  (r = radius and ℓ = length)
- Volume of a cone = $\frac{1}{3}\pi r^2 h$
  (r = radius and h = height)

KEY POINTS

- To convert an angle from degree to radian, we have to multiply it by $\frac{\pi}{180°}$ and to convert an angle from radian to degree, we have to multiply it by $\frac{180°}{\pi}$.
- By help of differentiation, if y is given, we can find $\frac{dy}{dx}$ and by help of integration, if $\frac{dy}{dx}$ is given, we can find y.
- The maximum and minimum values of function

$$A\cos\theta + B\sin\theta \text{ are } \sqrt{A^2 + B^2} \text{ and } -\sqrt{A^2 + B^2} \text{ respectively.}$$

- $(a+b)^2 = a^2 + b^2 + 2ab$ $(a-b)^2 = a^2 + b^2 - 2ab$
  $(a+b)(a-b) = a^2 - b^2$ $(a+b)^3 = a^3 + b^3 + 3ab(a+b)$
  $(a-b)^3 = a^3 - b^3 - 3ab(a-b)$

IMPORTANT NOTES

VECTORS

Vector Quantities

A physical quantity which requires magnitude and a particular direction, when it is expressed.
Parallel vectors – two vectors having same direction.

antiparallel vectors – vectors in opposite direction.

Equal vectors – Vectors which have equal magnitude and same direction
Negative or opposite vectors – Vectors having equal magnitude but opposite direction.

Null vector or Zero vector

A vector having zero magnitude. The direction of a zero vector is indeterminate.

$$\vec{A} + (-\vec{A}) = \vec{0}$$

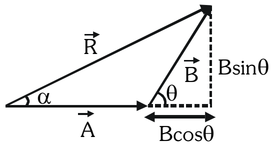
Unit vector

A vector having unit magnitude. It is used to speciafy direction.

Unit vector in direction of $\vec{A}$, $\hat{A} = \dfrac{\vec{A}}{|\vec{A}|}$

Triangle law of Vector addition

$\boxed{\vec{R} = \vec{A} + \vec{B}}$



$R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

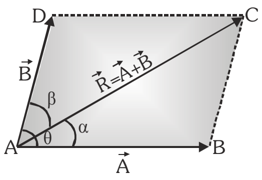
$\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$

If A = B then $R = 2A\cos\dfrac{\theta}{2}$ & $\alpha = \dfrac{\theta}{2}$

$R_{max} = A+B$ for $\theta = 0°$; $R_{min} = A \sim B$ for $\theta = 180°$

Parallelogram Law of Addition of Two Vectors

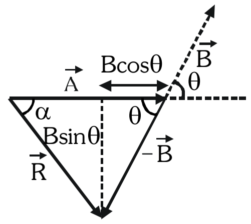
If two vectors are represented by two adjacent sides of a parallelogram which are directed away from their common point then their sum (i.e. resultant vector) is given by the diagonal of the parallelogram passing away through that common point.



$\overrightarrow{AB} + \overrightarrow{AD} = \overrightarrow{AC} = \vec{R}$ or $\vec{A} + \vec{B} = \vec{R} \Rightarrow R = \sqrt{A^2 + B^2 + 2AB\cos\theta}$

$\tan\alpha = \dfrac{B\sin\theta}{A + B\cos\theta}$ and $\tan\beta = \dfrac{A\sin\theta}{B + A\cos\theta}$

Vector subtraction



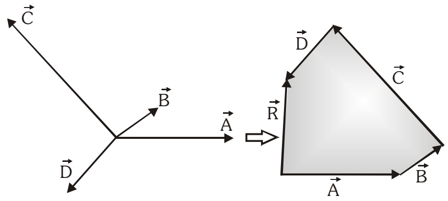
$\boxed{\vec{R} = \vec{A} - \vec{B}} \Rightarrow \vec{R} = \vec{A} + (-\vec{B})$

$R = \sqrt{A^2 + B^2 - 2AB\cos\theta}$, $\tan\alpha = \dfrac{B\sin\theta}{A - B\cos\theta}$

If A = B then $R = 2A\sin\dfrac{\theta}{2}$

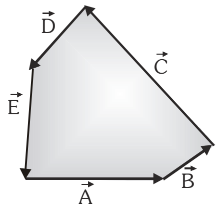
Addition of More than Two Vectors (Law of Polygon)

If some vectors are represented by sides of a polygon in same order, then their resultant vector is represented by the closing side of polygon in the opposite order.



$$\vec{R} = \vec{A} + \vec{B} + \vec{C} + \vec{D}$$

In a polygon if head of the last vector coincide with the tail of the first vectors, in other words vectors are forming closed polygon, then their resultant is null vector.



$$\vec{A} + \vec{B} + \vec{C} + \vec{D} + \vec{E} = 0$$

Rectangular component of a 3–D vector

- $\vec{A} = A_x\hat{i} + A_y\hat{j} + A_z\hat{k}$

  Angle made with x-axis

  $$\cos\alpha = \frac{A_x}{A} = \frac{A_x}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = \ell$$

  Angle made with y-axis

  $$\cos\beta = \frac{A_y}{A} = \frac{A_y}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = m$$

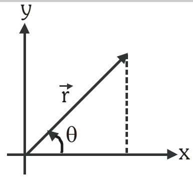
Angle made with z-axis

$$\cos\gamma = \frac{A_z}{A} = \frac{A_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}} = n$$

- ❐ ℓ, m, n are called direction cosines
$\ell^2+m^2+n^2=\cos^2\alpha+\cos^2\beta+\cos^2\gamma$

$$= \frac{A_x^2 + A_y^2 + A_z^2}{\left(\sqrt{A_x^2 + A_y^2 + A_z^2}\right)^2} = 1 \text{ or } \sin^2\alpha + \sin^2\beta + \sin^2\gamma = 2$$

General Vector in x-y plane



$$\vec r = x\hat i + y\hat j = r\left(\cos\theta\hat i + \sin\theta\hat j\right)$$

EXAMPLES :

1. Construct a vector of magnitude 6 units making an angle of 60° with x-axis.

Sol. $\vec r = r(\cos 60\hat i + \sin 60\hat j) = 6\left(\frac{1}{2}\hat i + \frac{\sqrt3}{2}\hat j\right) = 3\hat i + 3\sqrt3\hat j$

2. Construct an unit vector making an angle of 135° with x axis.

Sol. $\hat r = 1(\cos 135°\hat i + \sin 135°\hat j) = \frac{1}{\sqrt2}(-\hat i + \hat j)$

Multiplication of a vector by a number

If $\vec b = k_x\vec a$ then magnitude of $\vec b$ is k times $|\vec a|$, and direction of $\vec b$ is same as $\vec a$

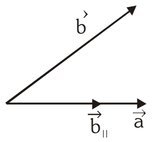
Scalar product (Dot Product)

- ❐ $\vec A.\vec B = AB\cos\theta \Rightarrow$ $\boxed{\text{Angle between two vectors}}$ $\boxed{\theta = \cos^{-1}\left(\frac{\vec A\cdot\vec B}{AB}\right)}$
- ❐ If $\vec A = A_x\hat i + A_y\hat j + A_z\hat k$ & $\vec B = B_x\hat i + B_y\hat j + B_z\hat k$ then $\vec A.\vec B = A_xB_x + A_yB_y + A_zB_z$ and angle between $\vec A$ & $\vec B$ is given by

$$\cos\theta = \frac{\vec A.\vec B}{AB} = \frac{A_xB_x + A_yB_y + A_zB_z}{\sqrt{A_x^2 + A_y^2 + A_z^2}\sqrt{B_x^2 + B_y^2 + B_z^2}}$$

- ❐ $\hat i.\hat i = 1$, $\hat j.\hat j = 1$, $\hat k.\hat k = 1$, $\hat i.\hat j = 0$, $\hat i.\hat k = 0$, $\hat j.\hat k = 0$
- ❐ Component of vector $\vec b$ along vector $\vec a$,

$\vec b_{\parallel} = \left(\vec b\,.\,\hat a\right)\hat a$

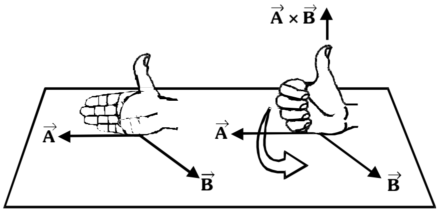


- ❐ Component of $\vec b$ perpendicular to $\vec a$,

$\vec b_{\perp} = \vec b - \vec b_{\parallel} = \vec b - \left(\vec b\cdot\hat a\right)\hat a$

Cross Product (Vector product)

- ❐ $\vec A \times \vec B = AB\sin\theta\,\hat n$ where $\hat n$ is a vector perpendicular to $\vec A$ & $\vec B$ or their plane and its direction given by right hand thumb rule.



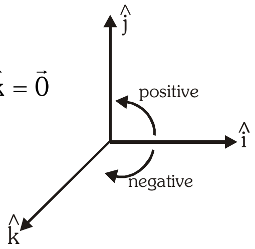
- ❐ $\vec A \times \vec B = \begin{vmatrix} \hat i & \hat j & \hat k \\ A_x & A_y & A_z \\ B_x & B_y & B_z \end{vmatrix}$

$$= \hat i\left(A_yB_z - A_zB_y\right) - \hat j\,(A_xB_z - B_xA_z) + \hat k\,(A_xB_y - B_xA_y)$$

- ❐ $\vec A \times \vec B = -\vec B \times \vec A$
- ❐ $(\vec A \times \vec B).\vec A = (\vec A \times \vec B).\vec B = 0$
- ❐ $\hat i \times \hat i = \vec 0$, $\hat j \times \hat j = \vec 0$, $\hat k \times \hat k = \vec 0$
- ❐ $\hat i \times \hat j = \hat k$; $\hat j \times \hat k = \hat i$,

$\hat k \times \hat i = \hat j$; $\hat j \times \hat i = -\hat k$

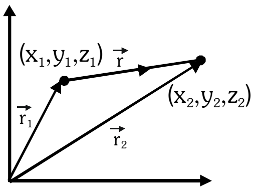
$\hat k \times \hat j = -\hat i$, $\hat i \times \hat k = -\hat j$



Differentiation

- ❐ $\frac{d}{dt}(\vec A.\vec B) = \frac{d\vec A}{dt}.\vec B + \vec A.\frac{d\vec B}{dt}$
- ❐ $\frac{d}{dt}(\vec A \times \vec B) = \frac{d\vec A}{dt} \times \vec B + \vec A \times \frac{d\vec B}{dt}$

When a particle moved from (x_1, y_1, z_1) to (x_2, y_2, z_2) then its displacement vector

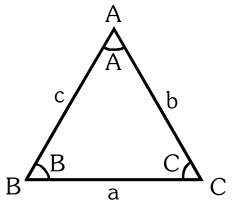


$$\vec r = \vec r_2 - \vec r_1 = (x_2\hat i + y_2\hat j + z_2\hat k) - (x_1\hat i + y_1\hat j + z_1\hat k)$$

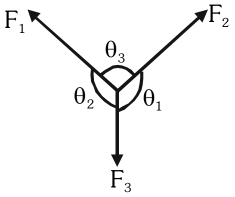
$$= (x_2 - x_1)\hat i + (y_2 - y_1)\hat j + (z_2 - z_1)\hat k$$

Magnitude: $r = |\vec r| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + (z_2 - z_1)^2}$

Lami's theorem



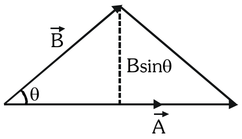
$$\frac{\sin A}{a}=\frac{\sin B}{b}=\frac{\sin C}{c}$$



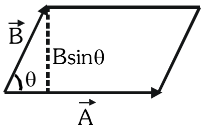
$$\frac{F_1}{\sin\theta_1}=\frac{F_2}{\sin\theta_2}=\frac{F_3}{\sin\theta_3}$$

Area of triangle

$$\text{Area}=\frac{|\vec{A}\times\vec{B}|}{2}=\frac{1}{2}AB\sin\theta$$



Area of parallelogram



$$\text{Area}=|\vec{A}\times\vec{B}|=AB\sin\theta$$

For parallel vectors

$$\vec{A}\times\vec{B}=\vec{0}$$

For perpendicular vectors

$$\vec{A}.\vec{B}=0$$

For coplanar vectors

$$\vec{A}.(\vec{B}\times\vec{C})=0$$

If A,B,C points are collinear

$$\overrightarrow{AB}=\lambda\overrightarrow{BC}$$

Examples of dot products

- Work, $W = \vec{F}.\vec{d} = Fd\cos\theta$ where $F \rightarrow$ force, $d \rightarrow$ displacement
- Power, $P = \vec{F}.\vec{v} = Fv\cos\theta$ where $F \rightarrow$ force, $v \rightarrow$ velocity
- Electric flux, $\phi_E = \vec{E}.\vec{A} = EA\cos\theta$ where $E \rightarrow$ electric field, $A \rightarrow$ Area
- Magnetic flux, $\phi_B = \vec{B}.\vec{A} = BA\cos\theta$ where $B \rightarrow$ magnetic field, $A \rightarrow$ Area
- Potential energy of dipole in where $p \rightarrow$ dipole moment,

  uniform field, $U = -\vec{p}.\vec{E}$ where $E \rightarrow$ Electric field

Examples of cross products

- Torque $\vec{\tau} = \vec{r} \times \vec{F}$ where $r \rightarrow$ position vector, $F \rightarrow$ force
- Angular momentum $\vec{J} = \vec{r} \times \vec{p}$ where $r \rightarrow$ position vector, $p \rightarrow$ linear momentum
- Linear velocity $\vec{v} = \vec{\omega} \times \vec{r}$ where $r \rightarrow$ position vector, $\omega \rightarrow$ angular velocity
- Torque on dipole placed in electric field $\vec{\tau} = \vec{p} \times \vec{E}$

  where $p \rightarrow$ dipole moment, $E \rightarrow$ electric field

KEY POINTS

- Tensor : A quantity that has different values in different directions is called tensor.

  Example : Moment of Inertia

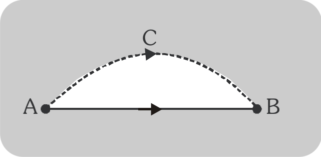
  In fact tensors are merely a generalisation of scalars and vectors; a scalar is a zero rank tensor, and a vector is a first rank tensor.
- Electric current is not a vector as it does not obey the law of vector addition.
- A unit vector has no unit.
- To a vector only a vector of same type can be added and the resultant is a vector of the same type.
- A scalar or a vector can never be divided by a vector.

IMPORTANT NOTES

KINEMATICS

- **Distance and Displacement**

  Total length of path (ACB) covered by the particle, in definite time interval is called distance. Displacement vector or displacement is the minimum distance (AB) and directed from initial position to final position.

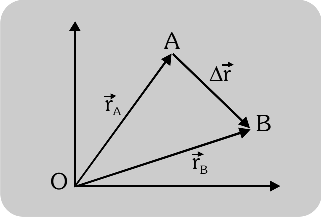


- **Displacement in terms of position vector**

From ΔOAB $\Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k} \text{ and } \vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$$

$$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k}$$



- **Average velocity** $\dfrac{\text{Displacement}}{\text{Time interval}} = \vec{v}_{av} = \dfrac{\Delta\vec{r}}{\Delta t}$
- **Average speed** $\dfrac{\text{Distance travelled}}{\text{Time interval}}$
- **For uniform motion** Average speed = | average velocity | = | instataneous velocity |
- **Velocity** $\vec{v} = \dfrac{d\vec{r}}{dt} = \dfrac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \dfrac{dx}{dt}\hat{i} + \dfrac{dy}{dt}\hat{j} + \dfrac{dz}{dt}\hat{k} = v_x\hat{i} + v_y\hat{j} + v_z\hat{k}$
- **Average Acceleration** $\dfrac{\text{change in velocity}}{\text{total time taken}} = \vec{a}_{av} = \dfrac{\Delta\vec{v}}{\Delta t}$
- **Acceleration** $\vec{a} = \dfrac{d\vec{v}}{dt} = \dfrac{d}{dt}\left(v_x\hat{i} + v_y\hat{j} + v_z\hat{k}\right) = \dfrac{dv_x}{dt}\hat{i} + \dfrac{dv_y}{dt}\hat{j} + \dfrac{dv_z}{dt}\hat{k} = a_x\hat{i} + a_y\hat{j} + a_z\hat{k}$

Important points about 1D motion

- Distance ≥ | displacement | and Average speed ≥ | average velocity |
- If distance > | displacement | this implies

  (a) atleast at one point in path, velocity is zero.

  (b) The body must have retarded during the motion
- Speed increase if acceleration and velocity both are positive or negative

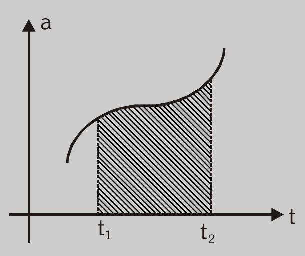
  (i.e. both have same sign)

- **In 1-D motion** $a = \dfrac{dv}{dt} = v\dfrac{dv}{dx}$
- **Graphical integration in Motion analysis**

If a = f(t)

$$a = \frac{dv}{dt} \Rightarrow \int_{v_1}^{v_2} dv = \int_{t_1}^{t_2} a\,dt \Rightarrow v_2 - v_1 = \int_{t_1}^{t_2} a\,dt$$

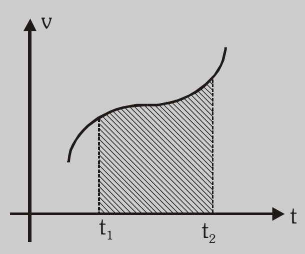
⇒ Change in velocity = Area between acceleration curve and time axis, from t_1 to t_2



shaded area = change in velocity

If v = f(t)

$$v = \frac{dx}{dt} \Rightarrow \int_{x_1}^{x_2} dx = \int_{t_1}^{t_2} v\,dt \Rightarrow x_2 - x_1 = \int_{t_1}^{t_2} v\,dt$$



shaded area = displacement

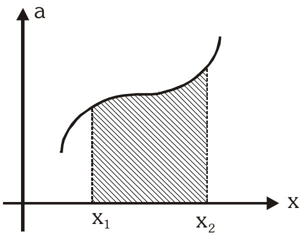
$\Rightarrow$ Change in position = displacement

= area between velocity curve and time axis, from t_1 to t_2.

If $a = f(x)$

$$a = \frac{vdv}{dx} \Rightarrow \int_{v_i}^{v_f} vdv = \int_{x_i}^{x_f} adx$$

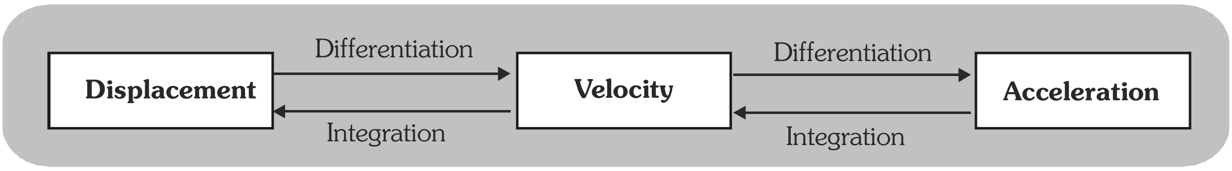
$$\frac{v_f^2 - v_i^2}{2} = \text{Area under a-x curve}$$

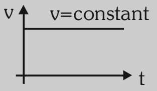
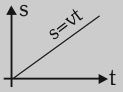
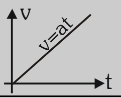
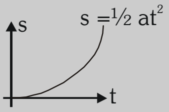
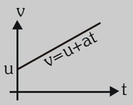
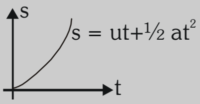
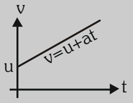
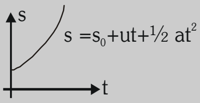
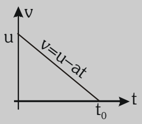
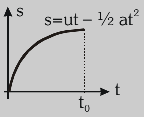
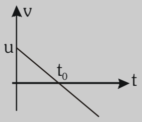
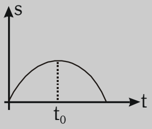


shaded area = $\frac{v_2^2 - v_1^2}{2}$

Important point about graphical analysis of motion

- Instantaneous velocity is the slope of position time curve $\left(v = \frac{dx}{dt}\right)$
- Slope of velocity-time curve = instantaneous acceleration $\left(a = \frac{dv}{dt}\right)$
- v-t curve area gives displacement. $\left[\Delta x = \int vdt\right]$
- a-t curve area gives change in velocity. $\left[\Delta v = \int adt\right]$



Different Cases	v–t graph	s–t graph
1. Uniform motion		
2. Uniformly accelerated motion with $u = 0$ at $t = 0$		
3. Uniformly accelerated with $u \neq 0$ at $t = 0$		
4. Uniformly accelerated motion with $u \neq 0$ and $s = s_0$ at $t = 0$		
5. Uniformly retarded motion till velocity becomes zero		
6. Uniformly retarded then accelerated in opposite direction		

Motion with constant acceleration : Equations of motion

- *In vector form* **:** $\vec{v} = \vec{u} + \vec{a}t$ $\quad \Delta\vec{r} = \vec{r}_2 - \vec{r}_1 = \vec{s} = \left(\frac{\vec{u}+\vec{v}}{2}\right)t = \vec{u}t + \frac{1}{2}\vec{a}t^2 = \vec{v}t - \frac{1}{2}\vec{a}t^2$

  $v^2 = u^2 + 2\vec{a}.\vec{s}$ $\quad \vec{s}_{n^{th}} = \vec{u} + \frac{\vec{a}}{2}(2n-1)$

  [$S_{n^{th}}$ → displacement in n^{th} second]

- *In scalar form* $v = u + at$ $\quad s = \left(\frac{u+v}{2}\right)t = ut + \frac{1}{2}at^2 = vt - \frac{1}{2}at^2$

  (for one dimensional motion) : $v^2 = u^2+2as$ $\quad s_{n^{th}} = u + \frac{a}{2}(2n-1)$

RELATIVE MOTION

There is no meaning of motion without reference or observer. If reference is not mentioned then we take the ground as a reference of motion. Generally velocity or displacement of the particle w.r.t. ground is called actual velocity or actual displacement of the body. If we describe the motion of a particle w.r.t. and object which is also moving w.r.t. ground then velocity of particle w.r.t. ground is its actual velocity $\left(\vec{v}_{act}\right)$ and velocity of particle w.r.t. moving object is its relative velocity $\left(\vec{v}_{rel.}\right)$ and the velocity of moving object (w.r.t. ground) is the reference velocity $\left(\vec{v}_{ref.}\right)$ then $\vec{v}_{rel} = \vec{v}_{act} - \vec{v}_{ref}$

$$\boxed{\vec{v}_{actual} = \vec{v}_{relative} + \vec{v}_{reference}} \xrightarrow{\text{Differentiation}} \boxed{\vec{a}_{actual} = \vec{a}_{relative} + \vec{a}_{reference}}$$

If $\vec{a}_{rel} = 0$

$\Rightarrow \vec{v}_{rel} = \text{constant}$

then $\Rightarrow \vec{S}_{rel} = \vec{v}_{rel} \times \text{time}$

If $\vec{a}_{rel} = \text{constant}$

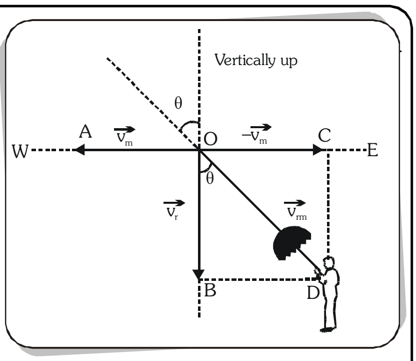
then we can we equation of metion in relative form

$$\vec{v}_{rel} = \vec{u}_{rel} + \vec{a}_{rel}t \qquad \text{... (i)}$$

$$\vec{s}_{rel} = \vec{u}_{rel}t + \frac{1}{2}\vec{a}_{rel}t^2 \qquad \text{... (ii)}$$

$$\vec{v}_{rel} \cdot \vec{v}_{rel} = \vec{u}_{rel} \cdot \vec{u}_{rel} + 2\left(\vec{a}_{rel} \cdot \vec{s}_{rel}\right)$$

Relative velocity of Rain w.r.t. the Moving Man :



A man walking west with velocity $\vec{v}_m$, represented by $\overrightarrow{OA}$. Let the rain be falling vertically downwards with velocity $\vec{v}_r$, represented by $\overrightarrow{OB}$ as shown in figure.

The relative velocity of rain w.r.t. man $\vec{v}_{rm} = \vec{v}_r - \vec{v}_m$ will be represented by diagonal $\overrightarrow{OD}$ of rectangle OBDC.

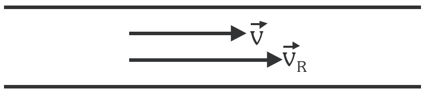
$$\therefore v_{rm} = \sqrt{v_r^2 + v_m^2 + 2v_r v_m \cos 90^\circ} = \sqrt{v_r^2 + v_m^2}$$

If θ is the angle which $\vec{v}_{rm}$ makes with the vertical direction then $\tan\theta = \frac{BD}{OB} = \frac{v_m}{v_r} \Rightarrow \theta = \tan^{-1}\left(\frac{v_m}{v_r}\right)$

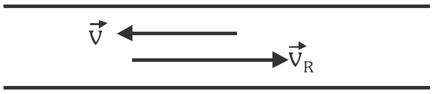
Swimming into the River :

A man can swim with velocity $\vec{v}$, i.e. it is the velocity of man w.r.t. still water. If water is also flowing with velocity $\vec{v}_R$ then velocity of man relative to ground $\vec{v}_m = \vec{v} + \vec{v}_R$

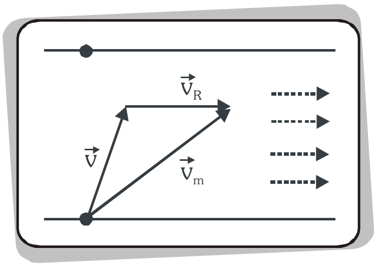
- If the swimming is in the direction of flow of water or along the downstream then $v_m = v + v_R$



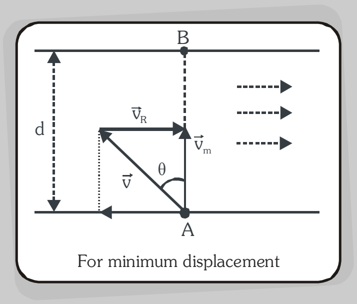
- If the swimming is in the direction opposite to the flow of water or along the upstream then $v_m = v - v_R$



- If man is crossing the river as shown in the figure i.e. $\vec{v}$ and $\vec{v}_R$ not collinear then use the vector algebra $\vec{v}_m = \vec{v} + \vec{v}_R$ **(assuming v > v_R)**



For shortest path

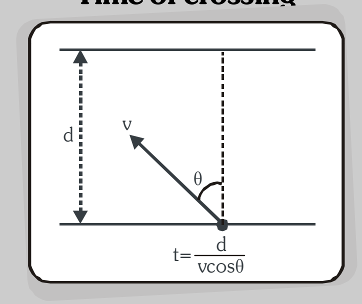


For minimum displacement

To reach at B:

$v\sin\theta = v_R \Rightarrow \sin\theta = \frac{v_R}{v}$

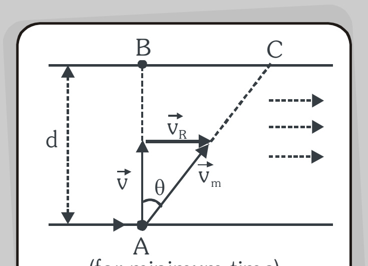
Time of crossing



$t = \frac{d}{v\cos\theta}$

Note : If $v_R > v$ then for minimum drifting $\sin\theta = \frac{v}{v_R}$

For minimum time



(for minimum time)

then $\boxed{t_{min} = \frac{d}{v}}$

MOTION UNDER GRAVITY

If a body is thrown vertically up with a velocity u in the uniform gravitational field (neglecting air resistance) then

(i) Maximum height attained $H = \frac{u^2}{2g}$

(ii) Time of ascent = time of descent = $\frac{u}{g}$

(iii) Total time of flight = $\frac{2u}{g}$

(iv) Velocity of fall at the point of projection = u (downwards)

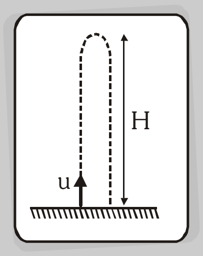
(v) **Gallileo's law of odd numbers** : For a freely falling body ratio of successive distance covered in equal time internval 't'

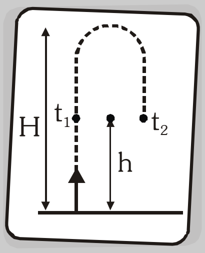
$S_1 : S_2 : S_3 : ....S_n = 1 : 3 : 5 : .... : 2n-1$

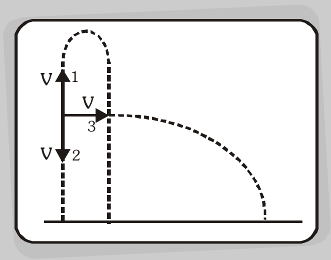
At any point on its path the body will have same speed for upward journey and downward journey. If a body thrown upwards crosses a point in time t_1 & t_2 respectively then height of point $h = ½\, g t_1 t_2$

Maximum height $H = \frac{1}{8} g(t_1 + t_2)^2$

A body is thrown upward, downward & horizontally with same speed takes time t_1, t_2 & t_3 respectively to reach the ground then $t_3 = \sqrt{t_1 t_2}$ & height from where the particle was throw is $H = \frac{1}{2} g t_1 t_2$



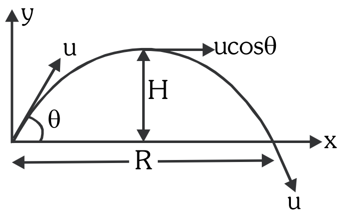




PROJECTILE MOTION

Horizontal Motion

$u\cos\theta = u_x$; $a_x = 0$; $x = u_x t = (u\cos\theta)t$



Vertical Motion

$v_y = u_y - gt$ where $u_y = u\sin\theta$;

$y = u_y t - \frac{1}{2}gt^2 = u\sin\theta t - \frac{1}{2}gt^2$

Net acceleration = $\vec{a} = a_x\hat{i} + a_y\hat{j} = -g\hat{j}$

At any instant :

$v_x = u\cos\theta$, $v_y = u\sin\theta - gt$

Velocity of particle at time t :

$$\vec{v} = v_x\hat{i} + v_y\hat{j} = u_x\hat{i} + (u_y - gt)\hat{j} = u\cos\theta\hat{i} + (u\sin\theta - gt)\hat{j}$$

If angle of velocity $\vec{v}$ from horizontal is α, then

$$\tan\alpha = \frac{v_y}{v_x} = \frac{u_y - gt}{u_x} = \frac{u\sin\theta - gt}{u\cos\theta} = \tan\theta - \frac{gt}{u\cos\theta}$$

- ☐ At highest point : $v_y = 0$, $v_x = u\cos\theta$
- ☐ Time of flight : $T = \frac{2u_y}{g} = \frac{2u\sin\theta}{g}$
- ☐ Horizontal range :

$$R = (u\cos\theta)T = \frac{2u^2\sin\theta\cos\theta}{g} = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

It is same for θ and (90°–θ) and maximum for θ = 45°

- ☐ Maximum height : $H = \frac{u_y^2}{2g} = \frac{u^2\sin^2\theta}{2g} = \frac{1}{8}gT^2$
- ☐ $\frac{H}{R} = \frac{1}{4}\tan\theta$
- ☐ Equation of trajectory

$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta} = x\tan\theta\left(1 - \frac{x}{R}\right)$$

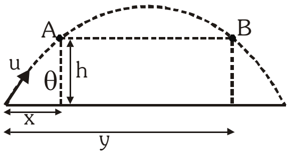
For projectile motion :

A body crosses two points at same height in time t_1 and t_2 the points are at distance x and y from starting point then

(a) $x + y = R$ (b) $t_1 + t_2 = T$

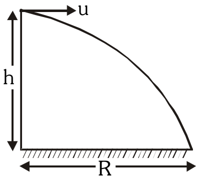
(c) $h = \frac{1}{2} g t_1 t_2$

(d) Average velocity from A to B is $u\cos\theta$



Note :- If a person can throw a ball to a maximum distance 'x' then the maximum height to which he can throw the ball will be (x/2)

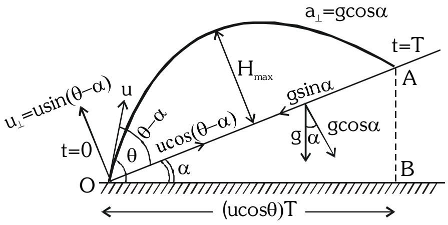
Horizontal projection from some height



- Time of flight $T = \sqrt{\frac{2h}{g}}$
- Horizontal range $R = uT = u\sqrt{\frac{2h}{g}}$
- Angle of velocity at any instant with horizontal

$$\theta = \tan^{-1}\left(\frac{gt}{u}\right)$$

Projectile motion on inclined plane- up motion

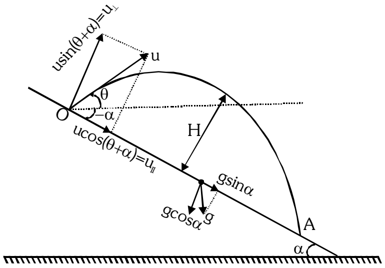


- Time of flight: $T = \frac{2u_\perp}{g_\perp} = \frac{2u\sin(\theta-\alpha)}{g\cos\alpha}$
- Maximum height : $H_{max} = \frac{u_\perp^2}{2g_\perp} = \frac{u^2\sin^2(\theta-\alpha)}{2g\cos\alpha}$
- Range on inclined plane :

$$R = OA = \frac{2u^2\sin(\theta-\alpha)\cos\theta}{g\cos^2\alpha}$$

- Max. range : $R_{max} = \frac{u^2}{g(1+\sin\alpha)}$ at angle $\theta = \frac{\pi}{4} + \frac{\alpha}{2}$

Projectile motion on inclined plane – down motion (put α = –α in above)



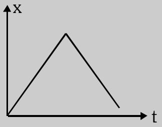
- Time of flight : $T = 2t_H = \frac{2u_\perp}{a_\perp} = \frac{2u\sin(\theta+\alpha)}{g\cos\alpha}$
- Maximum height : $H = \frac{u_\perp^2}{2a_\perp} = \frac{u^2\sin^2(\theta+\alpha)}{2g\cos\alpha}$
- Range on inclined plane :

$$R = OA = \frac{2u^2\cos\theta\sin(\theta+\alpha)}{g\cos^2\alpha}$$

- Max. range: $R_{max} = \frac{u^2}{g(1-\sin\alpha)}$ at angle $\theta = \frac{\pi}{4} - \frac{\alpha}{2}$

KEY POINTS :

- A positive acceleration can be associated with a "slowing down" of the body because the origin and the positive direction of motion are a matter of choice.
- The x-t graph for a particle undergoing rectilinear motion, cannot be as shown in figure because infinitesimal changes in velocity are physically possible only in infinitesimal time.



- In oblique projection of a projectile the speed gradually decreases up to the highest point and then increases because the tangential acceleration opposes the motion till the particle reaches the highest point, and then it favours the motion of the particle.
- In free fall, the initial velocity of a body may not be zero.
- A body can have acceleration even if its velocity is zero at an instant.
- Average velocity of a body may be equal to its instantaneous velocity.
- The trajectory of an object moving under constant acceleration can be straight line or parabola.
- The path of one projectile as seen from another projectile is a straight line as relative acceleration of one projectile w.r.t. another projectile is zero.

IMPORTANT NOTES

LAWS OF MOTION & FRICTION

FORCE

A push or pull that one object exerts on another.

Forces in nature

There are four fundamental forces in nature :

- Gravitational force
- Electromagnetic force
- Strong nuclear force
- Weak force

Types of forces on macroscopic objects

(a) Field Forces or Range Forces :

These are the forces in which contact between two objects is not necessary.

Ex. (i) Gravitational force between two bodies.
(ii) Electrostatic force between two charges.

(b) Contact Forces :

Contact forces exist only as long as the objects are touching each other.

Ex. (i) Normal force. (ii) Frictional force

(c) Attachment to Another Body :

Tension (T) in a string and spring force (F = kx) comes in this group.

NEWTON'S FIRST LAW OF MOTION (or Galileo's law of Inertia)

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external unbalanced force to change that state.

Inertia : Inertia is the property of the body due to which body opposes the change of it's state. Inertia of a body is measured by mass of the body.

$$\boxed{\text{inertia} \propto \text{mass}}$$

Newton's second law

$$\vec{F} = \frac{d\vec{p}}{dt} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$$

(Linear momentum $\vec{p} = m\vec{v}$)

For constant mass system $\vec{F} = m\vec{a}$

Momentum

It is the product of the mass and velocity of a body

i.e. momentum $\vec{p} = m\vec{v}$

- **SI Unit** : kg m s^{-1}
- **Dimensions** : [M L T^{-1}]

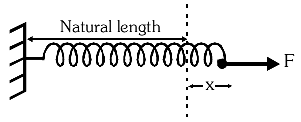
Newton's third law of motion :

Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

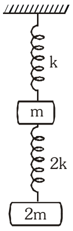
i.e. $\vec{F}_{A/B} = -\vec{F}_{B/A}$

Spring Force (According to Hooke's law)

In equilibrium F=kx (k is spring constant)



Note : Spring force is non impulsive in nature.



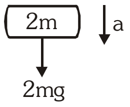
Ex. If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring.

Sol. Initial stretches $x_{upper} = \frac{3mg}{k}$

and $x_{lower} = \frac{mg}{k}$

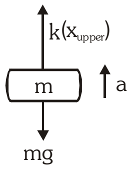
On cutting the lower spring, by virtue of non–impulsive nature of spring the stretch in upper spring remains same immediately after cutting the spring. Thus,

Lower block :



$2mg = 2ma \Rightarrow a = g$

Upper block :



$k\left(\frac{3mg}{k}\right) - mg$

$= ma \Rightarrow a = 2g$

Motion of bodies in contact

When two bodies of masses m_1 and m_2 are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called force of contact. These two bodies will move with same acceleration a.

(i) When the force F acts on the body with mass m_1 as shown in figure (i) : $F = (m_1 + m_2)a$

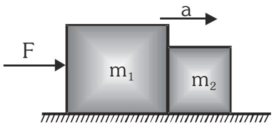


Fig.(1) : When the force F acts on mass m_1

If the force exerted by m_2 on m_1 is f_1 (force of contact) then for body m_1: $(F - f_1) = m_1 a$

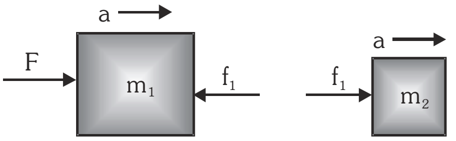


Fig. 1(a) : F.B.D. representation of action and reaction forces.

For body m_2 **:**

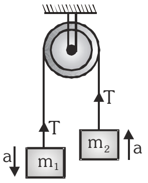
$f_1 = m_2 a \Rightarrow$ action of m_1 on m_2: $f_1 = \dfrac{m_2 F}{m_1 + m_2}$

Pulley system

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

SOME CASES OF PULLEY

Case – I :



Let $m_1 > m_2$ now for mass m_1, $m_1 g - T = m_1 a$

for mass m_2, $T - m_2 g = m_2 a$

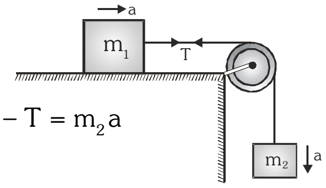
$$\text{Acceleration} = a = \frac{(m_1 - m_2)}{(m_1 + m_2)} g = \frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

$$\text{Tension} = T = \frac{2 m_1 m_2}{(m_1 + m_2)} g = \frac{2 \times \text{Product of masses}}{\text{Sum of two masses}} g$$

Reaction at the suspension of pulley :

$$R = 2T = \frac{4 m_1 m_2 g}{(m_1 + m_2)}$$

Case – II



For mass m_1 :

$T = m_1 a$

For mass m_2 : $m_2 g - T = m_2 a$

Acceleration:

$$a = \frac{m_2 g}{(m_1 + m_2)} \text{ and } T = \frac{m_1 m_2}{(m_1 + m_2)} g$$

FRAME OF REFERENCE

- **Inertial frames of reference :** A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

  All the fundamental laws of physics have been formulated in respect of inertial frame of reference.
- **Non–inertial frame of reference :** An accelerating frame of reference is called a non–inertial frame of reference. Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

Pseudo force:

The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where $\vec{a}_0$ is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.

When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass). But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.

$\sum \vec{F}_{real} + \vec{F}_{pseudo} = m\vec{a}$ (where $\vec{a}$ is acceleration of object in non inertial reference frame) & $\vec{F}_{pseudo} = -m\vec{a}_a$

(where $\vec{a}_0$ is acceleration of non inertial reference frame).

Man in a Lift

(a) If the lift moving with constant velocity v upwards or downwards. In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift.

So apparent weight $W' = Mg$ = Actual weight.

(b) If the lift is accelerated upward with constant acceleration a. Then forces acting on the man w.r.t. observed inside the lift are

(i) Weight $W = Mg$ downward

(ii) Fictitious force $F_0 = Ma$ downward.

So apparent weight $W' = W + F_0 = Mg + Ma = M(g+a)$

(c) If the lift is accelerated downward with acceleration $a < g$.

Then w.r.t. observer inside the lift fictitious force $F_0 = Ma$ acts upward while weight of man $W = Mg$ always acts downward.

So apparent weight

$W' = W - F_0 = Mg - Ma = M(g-a)$

Special Case :

If $a = g$ then $W' = 0$ (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

(d) If lift accelerates downward with acceleration $a > g$. Then as in Case (c).

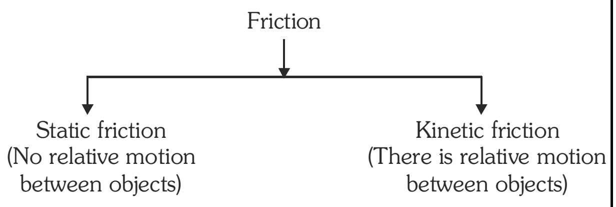
Apparent weight $W' = M(g-a)$ is negative, i.e., the man will be accelerated upward and will stay at the ceiling of the lift.

FRICTION

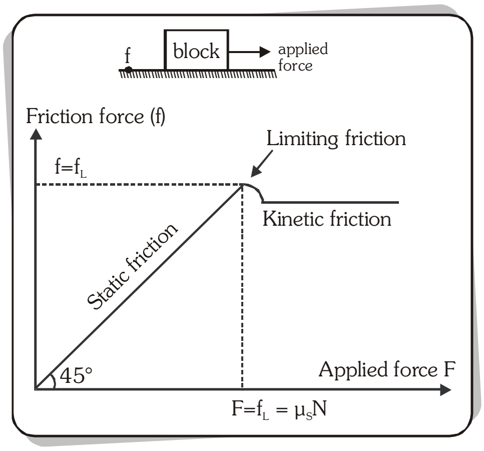
Friction is the force of two surfaces in contact, or the force of a medium acting on a moving object. (i.e. air on aircraft.)

Frictional forces arise due to molecular interactions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.

- **Cause of Friction:** Friction arises on account of strong atomic or molecular forces of attraction between the two surfaces at the point of actual contact.
- **Types of friction**


Friction
Static friction
(No relative motion between objects)
Kinetic friction
(There is relative motion between objects)

Graph between applied force and force of friction



- **Static friction coefficient**

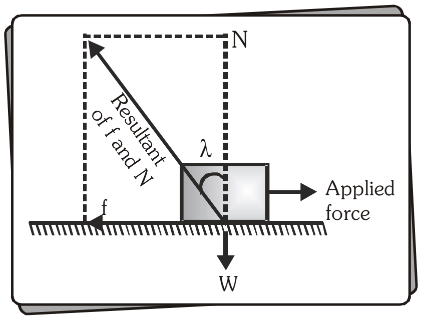
$$\mu_s = \frac{(f_s)_{max}}{N},\ 0 \le f_s \le \mu_s N,\ \vec{f}_s = -\vec{F}_{applied}$$

$$(f_s)_{max} = \mu_s N = \text{ limiting friction}$$

- **Kinetic friction coefficient**

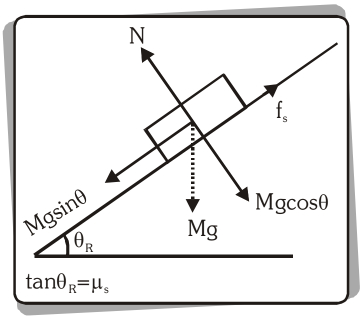
$$\mu_k = \frac{f_k}{N},\ \vec{f}_k = -(\mu_k N)\hat{v}_{rel}$$

- **Angle of Friction (λ)**



$$\tan\lambda = \frac{f_S}{N} = \frac{\mu_s N}{N} = \mu_S$$

- **Angle of repose** : The maximum angle of an inclined plane for which a block remains stationary on the plane.



Dependent Motion of Connected Bodies

Method I : Method of constraint equations

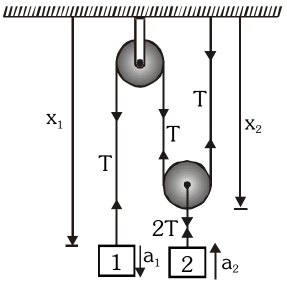
$$\boxed{\sum x_i = \text{constant} \Rightarrow \sum \frac{dx_i}{dt} = 0 \Rightarrow \sum \frac{d^2x_i}{dt^2} = 0}$$

- For n moving bodies we have $x_1, x_2, \ldots x_n$
- No. of constraint equations = no. of strings

Method II : Method of virtual work :

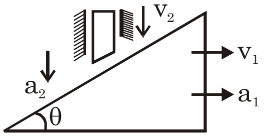
The sum of scalar products of tension forces applied by connecting links of constant length and displacement of corresponding contact points equal to zero.

$$\sum \vec{T} \cdot \vec{x} = 0 \Rightarrow \sum \vec{T} \cdot \vec{v} = 0 \Rightarrow \sum \vec{T} \cdot \vec{a} = 0$$



Here $2a_2 = a_1$

Normal constraint : displacements, velocities & accelerations of both objects should be same along C.N.



e.g. $a_2 = a_1 \tan\theta$ & $v_2 = v_1 \tan\theta$

KEY POINTS

- Aeroplanes always fly at low altitudes because according to Newton's III law of motion as aeroplane displaces air & at low altitude density of air is high.
- Rockets move by pushing the exhaust gases out so they can fly at low & high altitude.
- Pulling (figure I) is easier than pushing (figure II) on a rough horizontal surface because normal reaction is less in pulling than in pushing.

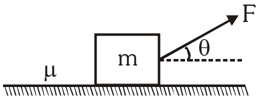


Fig. I

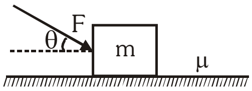


Fig. II

- While walking on ice, one should take small steps to avoid slipping. This is because smaller step increases the normal reaction and that ensure smaller friction.

IMPORTANT NOTES

CIRCULAR MOTION

Definition of Circular Motion

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point. That fixed point is called centre and the distance is called radius of circular path.

Radius Vector :

The vector joining the centre of the circle and the center of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction. It is directed outwards.

$\vec{k}\cdot\vec{v}=0$ & $\vec{r}$ & $\vec{v}$ always in same plane.

Frequency (n) :

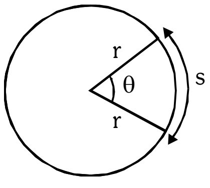
No. of revolutions described by particle per sec. is its frequency. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.)

Time Period (T) :

It is time taken by particle to complete one revolution.

$$T=\frac{1}{n}$$

- Angle $\theta=\frac{\text{arc length}}{\text{radius}}=\frac{s}{r}$



- Average angular velocity $\omega=\frac{\Delta\theta}{\Delta t}$ (a scalar quantity)
- Instantaneous angular velocity

$$\omega=\frac{d\theta}{dt}\text{ (a vector quantity)}$$

- For uniform angular velocity $\omega=\frac{2\pi}{T}=2\pi f$ or $2\pi n$
- Angular displacement $\theta=\omega t$

$\omega\to$ Angular frequency n or f = frequency

- Relation between ω and v $\omega=\frac{v}{r}$

In vector form velocity $\vec{v}=\vec{\omega}\times\vec{r}$

- Acceleration $\vec{a}=\frac{d\vec{v}}{dt}=\frac{d}{dt}(\vec{\omega}\times\vec{r})=\frac{d\vec{\omega}}{dt}\times\vec{r}+\vec{\omega}\times\frac{d\vec{r}}{dt}$

$$=\vec{\alpha}\times\vec{r}+\vec{\omega}\times\vec{v}=\vec{a}_t+\vec{a}_C$$

- Tangential acceleration: $a_t=\frac{dv}{dt}=\alpha r$

$$\left[\vec{a}_t=\text{component of }\vec{a}\text{ along }\vec{v}=(\vec{a}\cdot\hat{v})\hat{v}=\left(\frac{dv}{dt}\right)\hat{v}\right]$$

- Centripetal acceleration :

$$a_C=\omega v=\frac{v^2}{r}=\omega^2 r\text{ or }\vec{a}_C=\omega^2 r(-\hat{r})$$

$$\vec{a}_C\cdot\vec{v}=0$$

- Magnitude of net acceleration :

$$a=\sqrt{a_C^2+a_t^2}=\sqrt{\left(\frac{v^2}{r}\right)^2+\left(\frac{dv}{dt}\right)^2}$$

For uniform circular motion $\frac{d|\vec{v}|}{dt}=0=a_t$

- If a is contant, then following equations hold

(i) $\Delta\theta=\theta_f-\theta_i$

(ii) $\omega_f=\omega_i+\alpha t$

(iii) $\theta=\omega_i t+\frac{1}{2}\alpha t^2$

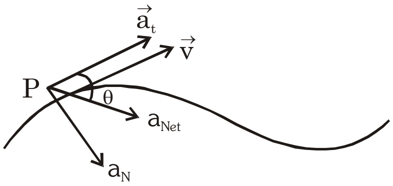
(iv) $\omega_f^2=\omega_i^2+2\alpha\theta$

(v) $\theta=\omega_f t-\frac{1}{2}\alpha t^2$

(vi) $\theta=\frac{(\omega_i+\omega_f)t}{2}$

(vii) $\alpha=\left(\frac{\omega_f-\omega_i}{t}\right)$

Curvilinear Motion :



$\vec{a}_{Net} = \frac{d\vec{v}}{dt}$;

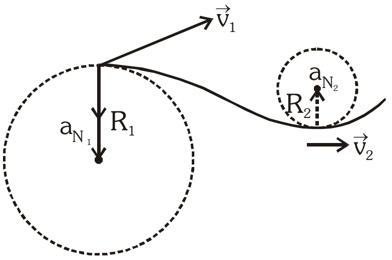
components of $\vec{a}_{Net}$ along $\vec{v} = \vec{a}_t$

components of $\vec{a}_{Net}$ perpendicular to $\vec{v} = \vec{a}_N$

$\vec{a}_T = \frac{d|\vec{v}|}{dt}$; a_N is responsible for change of direction

Radius of Curvature :

$a_N = \frac{v^2}{R} \Rightarrow R = \frac{v^2}{a_N}$



$R_1 > R_2$; Radius of curvature doesn't remains constant

R is a property of curves, not of the particle

(If a bee follows this path instead of the particle then its radius of curvature will be the same)

Maximum speed of in circular motion :

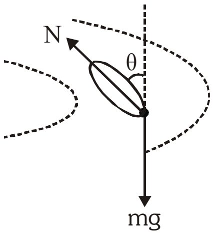
- On unbanked road : $v_{max} = \sqrt{\mu_s Rg}$
- On banked road :

$$v_{max} = \sqrt{\left(\frac{\mu_s + \tan\theta}{1 - \mu_s \tan\theta}\right)Rg} = \sqrt{\tan(\theta + \phi)Rg}$$

$$v_{min} = \sqrt{Rg\tan(\theta - \phi)} \; ; \; v_{min} \le v_{car} \le v_{max}$$

where ϕ = angle of friction = $\tan^{-1}\mu_s$; θ = angle of banking

◆ Bending of cyclist : $\tan\theta = \frac{v^2}{rg}$



Circular motion in vertical plane

A. Condition to complete vertical circle $u \geq \sqrt{5gR}$

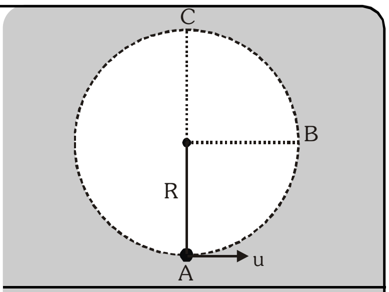
If $u = \sqrt{5gR}$ then Tension at C is equal to 0 and tension at A is equal to 6mg

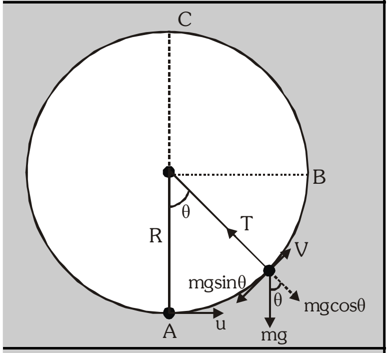
Velocity at B: $v_B = \sqrt{3gR}$

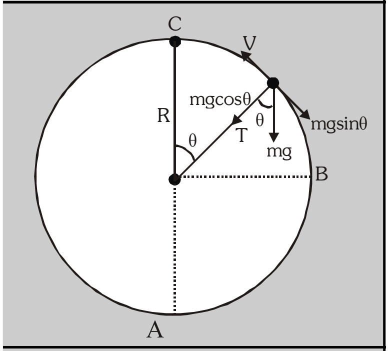
Velocity at C: $v_C = \sqrt{gR}$

From A to B : $T = mg\cos\theta + \dfrac{mv^2}{R}$

From B to C : $T = \dfrac{mv^2}{R} - mg\cos\theta$







B. Condition for pendulum motion (oscillating condition)

$u \leq \sqrt{2gR}$ (in between A to B)

Velocity can be zero but T never be zero between A & B.

Because T is given by $T = mg\cos\theta + \dfrac{mv^2}{R}$

C. Condition for leaving path : $\sqrt{2gR} < u < \sqrt{5gR}$

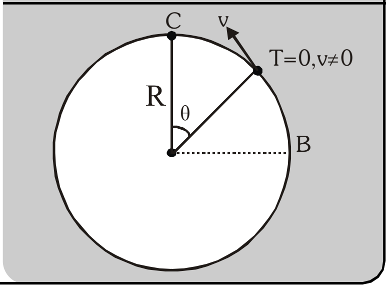
Particle crosses the point B but not complete the vertical circle.

Tension will be zero in between B to C & the angle where T = 0

$\cos\theta = \dfrac{u^2 - 2gR}{3gR}$; θ is from vertical line

Note : After leaving the circle, the particle will follow a parabolic path.

* T is maximum at the bottom & minimum at the top.



KEY POINTS

- Average angular velocity is a scalar physical quanity whereas instantaneous angular velocity is a vector physical quanity.
- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity.

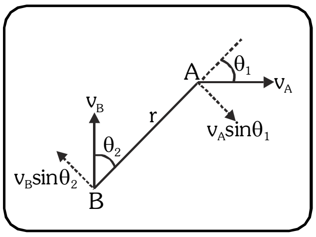
$$d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1$$

But $\vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$

- **Relative Angular Velocity**

  Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B.

  That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant



$$\omega_{AB} = \frac{(v_{AB})_\perp}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{seperation between A and B}}$$

here $(v_{AB})_\perp = v_A \sin\theta_1 + v_B \sin\theta_2 \therefore \omega_{AB} = \dfrac{v_A \sin\theta_1 + v_B \sin\theta_2}{r}$

WORK, ENERGY & POWER

WORK DONE

$$W = \int dW = \int \vec{F}.d\vec{r} = \int F dr \cos\theta$$

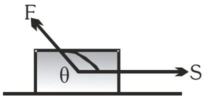
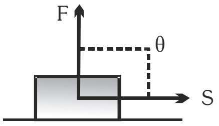
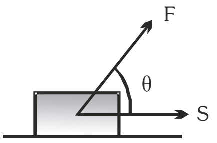
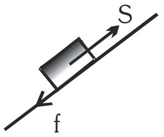
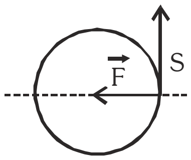
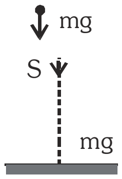
[where θ is the angle between $\vec{F}$ & $\vec{dr}$]

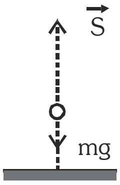
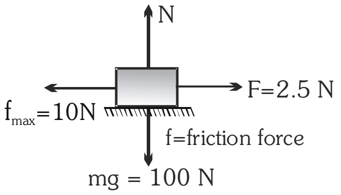
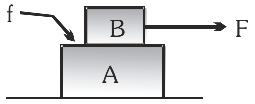
- For constant force $W = \vec{F}.\vec{d} = Fd\cos\theta$
- For Unidirectional force

  $W = \int dW = \int F dx =$ Area between F–x curve and x-axis.

NATURE OF WORK DONE

Although work done is a scalar quantity, yet its value may be positive, negative or even zero

Negative work	Zero work	Positive work
 (θ>90°)	 (θ = 90°)	 (θ< 90°)
 Work done by friction force (θ=180°)	 Motion of particle on circular path (uniform) (θ=90°)	 Motion under gravity (θ=0°)

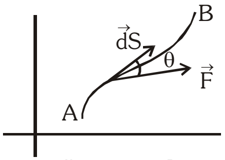

Work done by gravity (θ=180°)
	 N
	F=2.5 N
	f_{max}=10N
	f=friction force
	mg = 100 N
	As f = F, hence S = 0
		
		Work done by friction force on block A (θ=0°)

WORK DONE BY VARIABLE FORCE

A force varying with position or time is known as the variable force

$\vec{F} = F_x\hat{i} + F_y\hat{j} + F_z\hat{k}$

$d\vec{S} = dx\hat{i} + dy\hat{j} + dz\hat{k}$



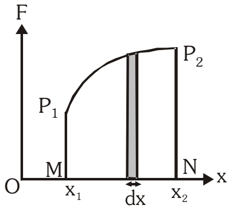
$$W_{AB} = \int_A^B \vec{F}\cdot d\vec{S} = \int_{x_A}^{x_B} F_x dx + \int_{y_A}^{y_B} F_y dy + \int_{z_A}^{z_B} F_z dz$$

Calculation of work done from force–displacement graph :

Total work done,

$$W = \sum_{x_1}^{x_2} F dx$$

$= \text{Area of } P_1P_2NM$



Kinetic energy

- The energy possessed by a body by virtue of its motion is called kinetic energy.

$$K = \frac{1}{2}mv^2 = \frac{1}{2}m(\vec{v}.\vec{v})$$

- Kinetic energy is a frame dependent quantity.

Work energy theorem $(W = \Delta KE)$

Change in kinetic energy = work done by all force

For conservative force $F(x) = -\frac{dU}{dx}$

Change in potential energy $\Delta U = -\int F(x)dx$

Conservative Forces

- Work done does not depend upon path.
- Work done in a round trip is zero.
- Central forces, spring forces etc. are conservative forces
- When only a conservative force acts within a system, the kinetic energy and potential energy can change into each other. However, their sum, the mechanical energy of the system, doesn't change.
- Work done is completely recoverable.
- If $\vec{F}$ is a conservative force then $\vec{\nabla} \times \vec{F} = \vec{0}$ (i.e. curl of $\vec{F}$ is zero)

Non–conservative Forces

- Work done depends upon path.
- Work done in a round trip is not zero.
- Forces are velocity–dependent & retarding in nature e.g. friction, viscous force etc.
- Work done against a non–conservative force may be dissipated as heat energy.
- Work done is not recoverable.

POTENTIAL ENERGY

- The energy possessed by a body by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservative force field is defined as the external work done against the action of conservative force in order to shift it from a certain reference point (PE = 0) to the present position.
- Potential energy of a body in a conservative force field is equal to the work done by the conservative force in moving the body from its present position to reference position.
- At a certain reference position, the potential energy of the body is assumed to be zero or the body is assumed to have lost the capacity of doing work.
- Relationship between conservative force field and potential energy :

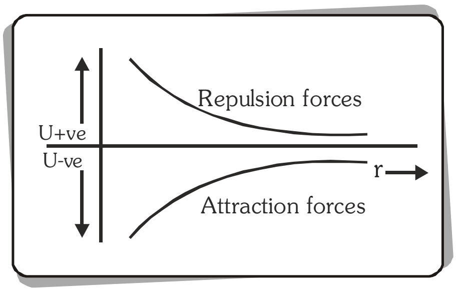
$$\vec{F} = -\nabla U = -\text{grad}(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

- If force varies with only one dimension (say along x-axis) then

$$F = -\frac{dU}{dx} \Rightarrow dU = -Fdx \Rightarrow \int_{U_1}^{U_2} dU = -\int_{x_1}^{x_2} Fdx$$

$$\Rightarrow \boxed{\Delta U = -W_C}$$

- Potential energy may be positive or negative or even zero



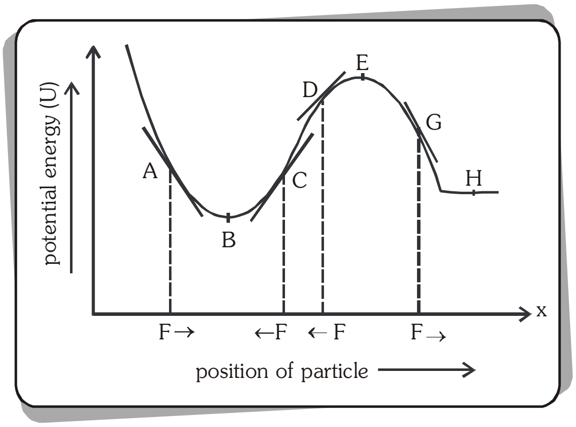
i) Potential energy is positive, if force field is repulsive in nature

ii) Potential energy is negative, if force field is attractive in nature

- If r ↑ (separation between body and force centre), U ↑, force field is attractive or vice–versa.
- If r ↑, U ↓, force field is repulsive in nature.

Potential energy curve and equilibrium

It is a curve which shows the change in potential energy with the position of a particle.



Stable Equilibrium :

After a particle is slightly displaced from its equilibrium position if it tends to come back towards equilibrium
then it is said to be in stable equilibrium.

At point **A :** slope $\frac{dU}{dx}$ is negative so F is positive

At point **C :** slope $\frac{dU}{dx}$ is positive. so F is negative

$$\boxed{\text{At equilibrium}\quad F = -\frac{dU}{dx} = 0}$$

At point **B :** it is the point of stable equilibrium.

At point **B :** $U = U_{min}$, $\frac{dU}{dx} = 0$ & $\frac{d^2U}{dx^2}$ = positive

Unstable equilibrium :

After a particle is slightly displaced from its equilibrium position, if it tends to move away from equilibrium position then it is said to be in unstable equilibrium.

At point **D** : slope $\frac{dU}{dx}$ is positive so F is negative ; At point **G** : slope $\frac{dU}{dx}$ is negative so F is positive

At point **E** : it is the point of unstable equilibrium;

At point **E** $U=U_{max}$, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2}$ = negative

Neutral equilibrium

After a particle is slightly displaced from its equilibrium position if no force acts on it then the equilibrium is said to be neutral equilibrium.

Point **H** corresponds to neutral equilibrium ⇒ U = constant ; $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$.

Law of conservation of Mechanical energy

Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles or the work done by all other forces is zero. From work energy theorem $W = \Delta KE$

Proof : For internal conservative forces $W_{int} = -\Delta U$

So $W = W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = \Delta KE$
$\Rightarrow \Delta(KE+U) = 0 \Rightarrow KE+U = \text{constant}$

- Spring force $F=-kx$, Elastic potential energy stored in spring $U(x) = \frac{1}{2}kx^2$
- Mass and energy are equivalent and are related by $E = mc^2$

Power

- Power is a scalar quantity with dimension $M^1L^2T^{-3}$
- SI unit of power is J/s or watt
- 1 horsepower = 746 watt = 550 ft–lb/sec.

Average power : $P_{av} = W/t$

Instantaneous power : $P = \frac{dW}{dt} = \frac{\vec{F}.d\vec{r}}{dt} = \vec{F}.\vec{v}$

fig.(a)

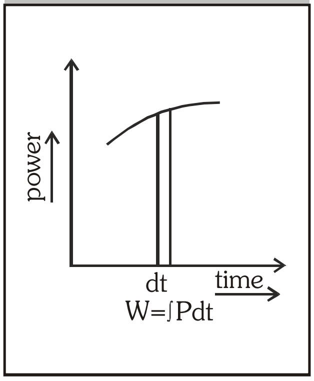


fig.(b)

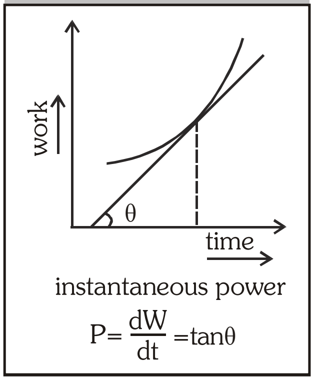
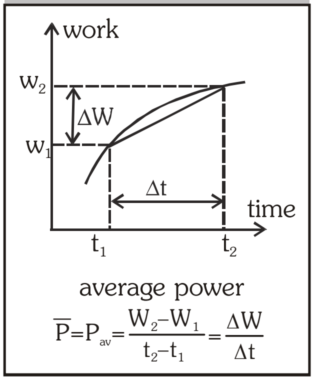


fig.(c)



- For a system of varying mass $\vec{F} = \frac{d}{dt}(m\vec{v}) = m\frac{d\vec{v}}{dt} + \vec{v}\frac{dm}{dt}$
- If $\vec{v}$ = constant then $\vec{F} = \vec{v}\frac{dm}{dt}$ then $P = \vec{F}.\vec{v} = v^2\frac{dm}{dt}$
- In rotatory motion : $P = \tau\frac{d\theta}{dt} = \tau\omega$
- Efficiency $\eta = \frac{\text{Output Energy}}{\text{Input Energy}}$

KEY POINTS

- A body may gain kinetic energy and potential energy simultaneously because principle of conservation of mechanical energy may not be valid every time.
- Comets move around the sun in elliptical orbits. The gravitational force on the comet due to sun is not normal to the comet's velocity but the work done by the gravitational force is zero in complete round trip because gravitational force is a conservative force.
- Work done by static friction may be positive because static friction may acts along the direction of motion of an object.

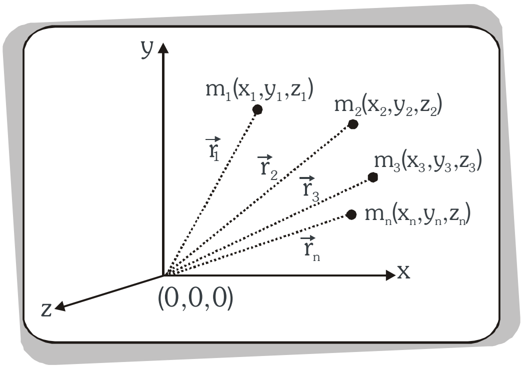
CHAPTER 7

COLLISIONS & CENTRE OF MASS

Centre of mass :

Centre of mass of system is the point associated with the system which have same acceleration as the acceleration of point mass (of same mass as that of system) would have under the application of same external force.

Centre of mass of system of discrete particles



Total mass of the body :

$M = m_1 + m_2 + ... + m_n$ then

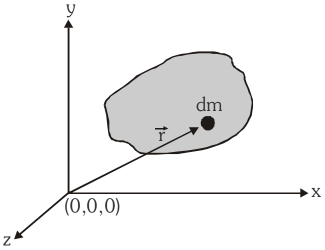
$$\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...} = \frac{1}{M}\Sigma m_i \vec{r}_i$$

co-ordinates of centre of mass :

$$x_{cm} = \frac{1}{M}\Sigma m_i x_i\ ,\ y_{cm} = \frac{1}{M}\Sigma m_i y_i\ \&\ z_{cm} = \frac{1}{M}\Sigma m_i z_i$$

Centre of mass of continuous distribution of particles

$$\vec{R}_{CM} = \frac{1}{M}\int \vec{r}dm$$



$$x_{cm} = \frac{1}{M}\int x\ dm\ ,\ y_{cm} = \frac{1}{M}\int y\ dm \text{ and } z_{cm} = \frac{1}{M}\int z\ dm$$

x, y, z are the co-ordinate of the COM of the dm mass.

The centre of mass after removal of a part of a body

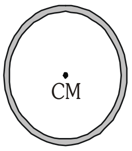
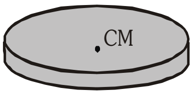
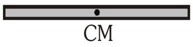
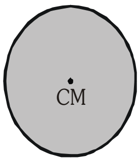
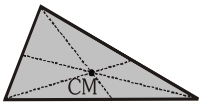
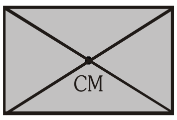
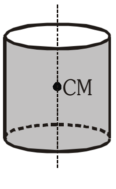
Original mass (M) – mass of the removed part (m)

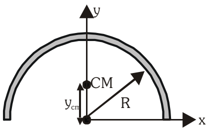
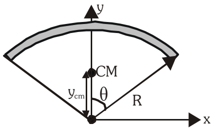
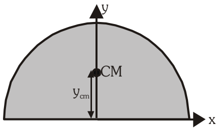
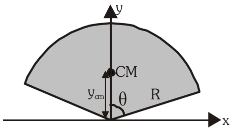
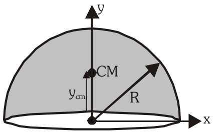
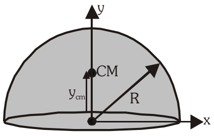
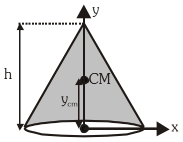
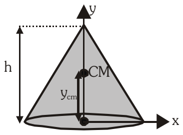
= {original mass (M)}+{–mass of the removed part (m)}

The formula changes to:

$$x_{CM} = \frac{Mx - mx'}{M - m}\ ;\ y_{CM} = \frac{My - my'}{M - m}\ ;\ y_{CM} = \frac{Mz - mz'}{M - m}$$

CENTRE OF MASS OF SOME COMMON OBJECTS

Body	Shape of body	Position of centre of mass
Uniform Ring		Centre of ring
Uniform Disc		Centre of disc
Uniform Rod		Centre of rod
Solid sphere/ hollow sphere		Centre of sphere
Triangular plane lamina		Point of intersection of the medians of the triangle i.e. centroid
Plane lamina in the form of a square or rectangle or parallelogram		Point of intersection of diagonals
Hollow/solid cylinder		Middle point of the axis of cylinder

Body	Shape of body	Position of centre of mass
Half ring		$y_{cm} = \frac{2R}{\pi}$
Segement of a ring		$y_{cm} = \frac{R\sin\theta}{\theta}$
Half disc (plate)		$y_{cm} = \frac{4R}{3\pi}$
Sector of a disc (plate)		$y_{cm} = \frac{2R\sin\theta}{3\theta}$
Hollow hemisphere		$y_{cm} = \frac{R}{2}$
Solid hemisphere		$y_{cm} = \frac{3R}{8}$
Hollow cone		$y_{cm} = \frac{h}{3}$
Solid cone		$y_{cm} = \frac{h}{4}$

MOTION OF CENTRE OF MASS

For a system of particles,
velocity of centre of mass

$$\vec{v}_{CM} = \frac{d\vec{R}_{CM}}{dt} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2 + ...}{m_1 + m_2 + ...}$$

Similarly acceleration

$$\vec{a}_{CM} = \frac{d}{dt}\left(\vec{v}_{CM}\right) = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$$

Law of conservation of linear momentum

Linear momentum of a system of particles is equal to the product of mass of the system with velocity of its centre of mass.

From Newton's second law $\vec{F}_{ext.} = \frac{d\left(M\vec{v}_{CM}\right)}{dt}$

If $\vec{F}_{ext.} = \vec{0}$ then $M\vec{v}_{CM} = \text{constant}$

If no external force acts on a system the velocity of its centre of mass remains constant, i.e., velocity of centre of mass is unaffected by internal forces.

Impulse – Momentum theorem

Impulse of a force is equal to the change of momentum

$$\boxed{\int_{t_1}^{t_2} \vec{F}dt = \Delta\vec{p}}$$

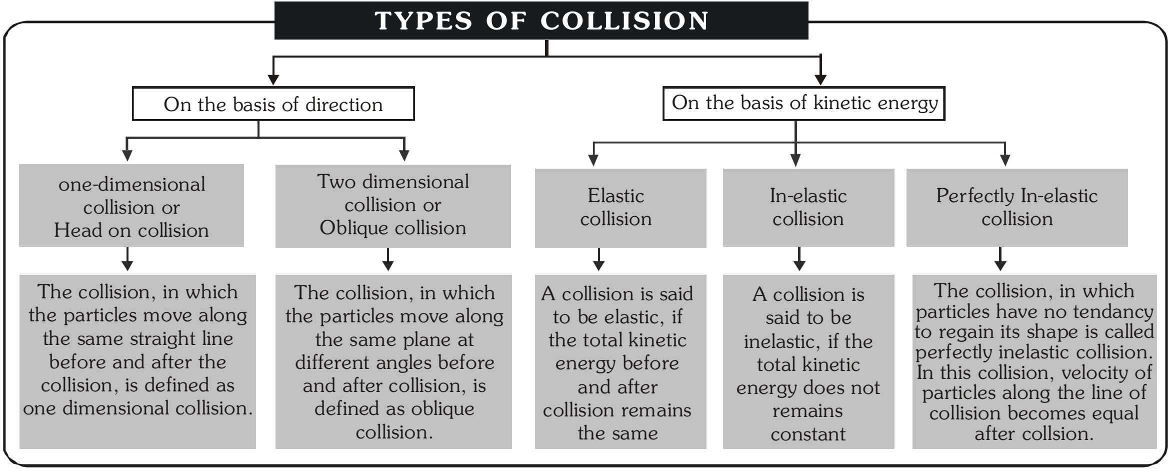
Force time graph area gives change in momentum.

Collision of bodies

The event or the process, in which two bodies either coming in contact with each other or due to mutual interaction at distance apart, affect each others motion (velocity, momentum, energy or direction of motion) is defined as a collision.

In collision

- The particles come closer before collision and after collision they either stick together or move away from each other.
- The particles need not come in contact with each other for a collision.
- The law of conservation of linear momentum is necessarily applicable in a collision, whereas the law of conservation of mechanical energy is not.



TYPES OF COLLISION

On the basis of direction

- one-dimensional collision or Head on collision: The collision, in which the particles move along the same straight line before and after the collision, is defined as one dimensional collision.
- Two dimensional collision or Oblique collision: The collision, in which the particles move along the same plane at different angles before and after collision, is defined as oblique collision.

On the basis of kinetic energy

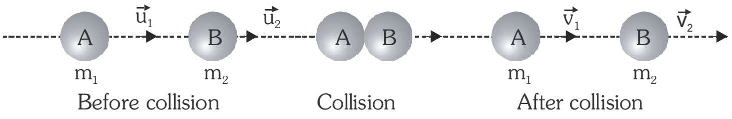
- Elastic collision: A collision is said to be elastic, if the total kinetic energy before and after collision remains the same
- In-elastic collision: A collision is said to be inelastic, if the total kinetic energy does not remains constant
- Perfectly In-elastic collision: The collision, in which particles have no tendancy to regain its shape is called perfectly inelastic collision. In this collision, velocity of particles along the line of collision becomes equal after collsion.

Coefficient of restitution (Newton's law)

$$e = -\frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}} = \frac{v_2 - v_1}{u_1 - u_2}$$

Value of e is 1 for elastic collision, 0 for perfectly inelastic collision and $0 < e < 1$ for inelastic collision.

Head on collision



Head on elastic collision

(i) Linear momentum is conserved

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$$

(ii) KE is not conserved but initial KE is equal to final KE

$$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2$$

(iii) Rate of separation = Rate of approach

i.e. $e = 1$

$\Rightarrow u_1 - u_2 = v_2 - v_1$

Head on inelastic collision of two particles

Let the coefficient of restitution for collision is e

(i) Momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i) (ii) Kinetic energy is not conserved.

(iii) According to Newton's law $e = \frac{v_2 - v_1}{u_1 - u_2}$ ...(ii)

By solving eq. (i) and (ii) :

$$v_1 = \left(\frac{m_1 - em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2 = \frac{m_1u_1 + m_2u_2 - m_2e(u_1 - u_2)}{m_1 + m_2}$$

$$v_2 = \left(\frac{m_2 - em_1}{m_1 + m_2}\right)u_2 + \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1 = \frac{m_1u_1 + m_2u_2 - m_1e(u_2 - u_1)}{m_1 + m_2}$$

Elastic Collision (e=1)

- **If the two bodies are of equal masses :** $m_1 = m_2 = m$, $v_1 = u_2$ and $v_2 = u_1$

  Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.

- **If the mass of a body is negligible as compared to other.** If $m_1 >> m_2$ and $u_2 = 0$ then $v_1 = u_1$, $v_2 = 2u_1$

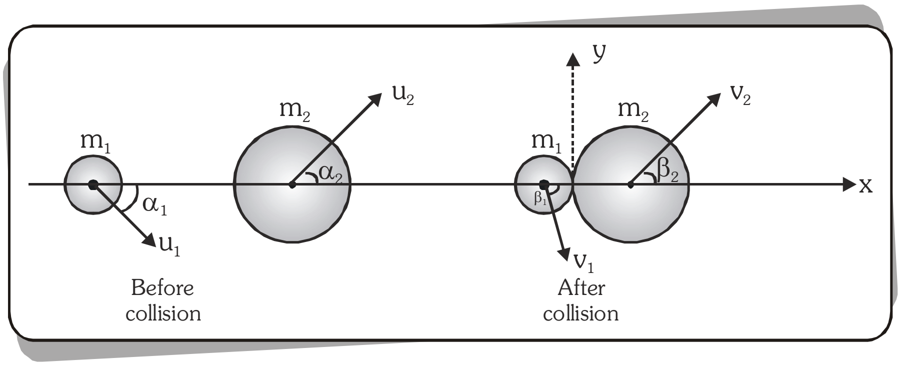
  when a heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will move with velocity double that of A. If $m_2 >> m_1$ and $u_2 = 0$ then $v_2 = 0$, $v_1 = -u_1$

  When light body A collides against a heavy body B at rest, the body A should start moving with same speed just in opposite direction while the body B should practically remains at rest.

◆ **Loss in kinetic energy in inelastic collision** $\Delta K = \frac{m_1 m_2}{2(m_1+m_2)}(1-e^2)\,|u_1-u_2|^2$

Oblique Collision

Conserving the momentum of system in directions along normal (x axis in our case) and tangential (y axis in our case) $m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$



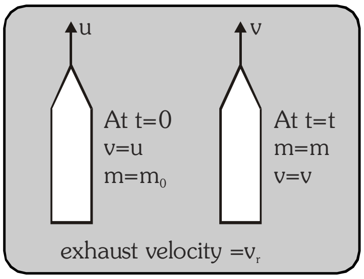
Since no force is acting on m_1 and m_2 along the tangent (i.e. y–axis) the individual momentum of m_1 and m_2 remains conserved. $m_1u_1\sin\alpha_1 = m_1v_1\sin\beta_1$ & $m_2u_2\sin\alpha_2 = m_2v_2\sin\beta_2$

By using Newton's experimental law along the line of impact $e = \frac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_1\cos\alpha_1 - u_2\cos\alpha_2}$

Rocket propulsion :

Thrust force on the rocket= $v_r\left(-\frac{dm}{dt}\right)$

Velocity of rocket at any instant $v = u - gt + v_r \ln\left(\frac{m_0}{m}\right)$



KEY POINTS

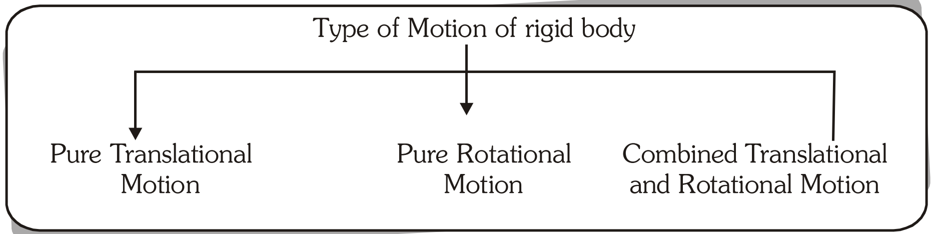
- Sum of mass moments about centre of mass is zero. i.e. $\sum m_i \vec{r}_{i/cm} = \vec{0}$
- A quick collision between two bodies is more violent then slow collision, even when initial and final velocities are equal because the rate of change of momentum determines that the impulsive force small or large.
- Heavy water is used as moderator in nuclear reactors as energy transfer is maximum if $m_1 \simeq m_2$
- Impulse-momentum theorem is equivalent to Newton's second law of motion.
- For a system, conservation of linear momentum is equivalent to Newton's third law of motion.

ROTATIONAL MOTION

RIGID BODY

Rigid body is defined as a system of particles in which distance between each pair of particles remains constant (with respect to time) that means the shape and size do not change, during the motion.

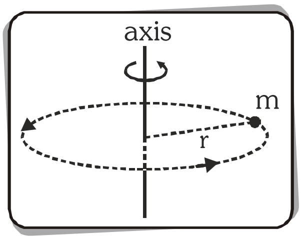
Eg : Fan, Pen, Table, stone and so on.


Type of Motion of rigid body
Pure Translational Motion
Pure Rotational Motion
Combined Translational and Rotational Motion

ROTATIONAL MOTION

Moment of Inertia

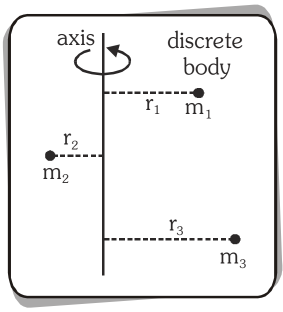
The virtue by which a body revolving about an axis opposes the change in rotational motion is known as moment of inertia.

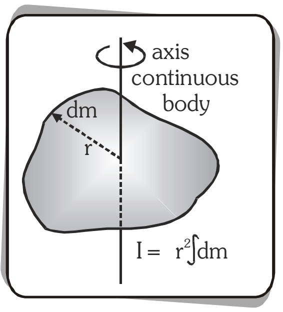


- The moment of inertia of a particles with respect to an axis of rotation is equal to the product of mass of the particle and square of distance from rotational axis. $I = mr^2$

  r = perpendicular distance from axis of rotation

- Moment of inertia of system of particle





$$I = m_1r_1^2 + m_2r_2^2 + m_3r_3^2 + ...$$

For Rigid Bodies :

Moment of inertia of a rigid body about any axis of rotation. $I = \int dm\, r^2$

Radius of Gyration (K)

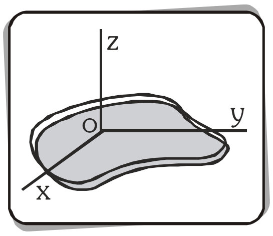
K has no meaning without axis of rotation.

$I = MK^2$ K is a scalar quantity

Radius of gyaration : $K = \sqrt{\frac{I}{M}}$

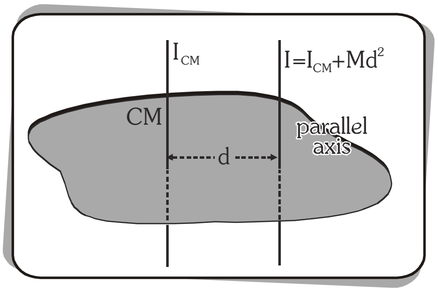
Perpendicular axis Theorems : $I_z = I_x + I_y$

(body lies on the x-y plane)



(Valid only for 2-dimensional body)

Parallel axis Theorem : $I = I_{CM} + Md^2$

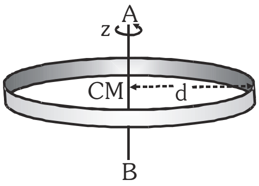


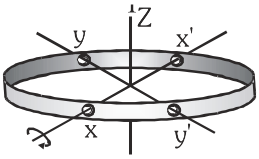
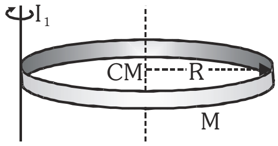
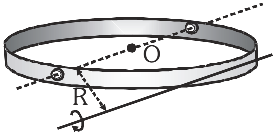
(for all type of bodies)

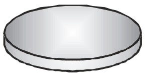
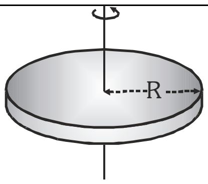
I_{CM} = moment of inertia about the axis Passing through the centre of mass

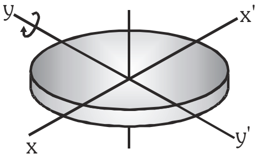
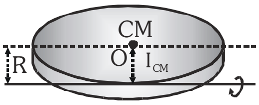
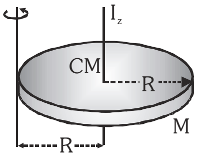
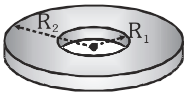
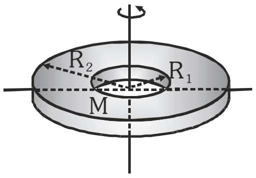
MOMENT OF INERTIA OF SOME REGULAR BODIES

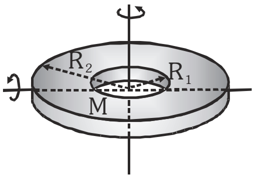
Shape of the body	Position of the axis of rotation	Figure	Moment of Inertia (I)	Radius of gyration (K)

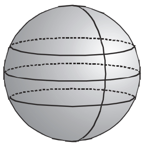
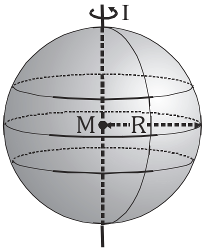
(1) Circular Ring
 	
Mass = M
Radius = R
	(a) About an axis perpendicular to the plane and passes through the centre		MR^2	R

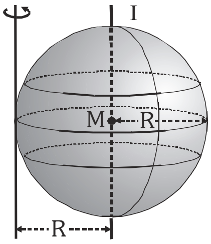
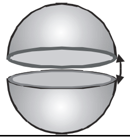
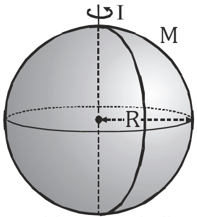
	(b) About the diametric axis		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$
	(c) About an axis tangential to the rim and perpendicular to the plane of the ring		$2MR^2$	$\sqrt{2}\,R$
	(d) About an axis tangential to the rim and lying in the plane of ring		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}\,R$

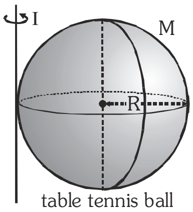
(2) Circular Disc

M = Mass
R = Radius
	(a) About an axis passing through the centre and perpendicular to the plane of disc		$\frac{1}{2}MR^2$	$\frac{R}{\sqrt{2}}$

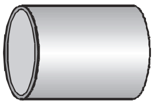
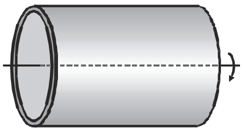
	(b) About a diametric axis		$\frac{MR^2}{4}$	$\frac{R}{2}$
	(c) About an axis tangential to the rim and lying in the plane of the disc		$\frac{5}{4}MR^2$	$\frac{\sqrt{5}}{2}R$
	(d) About an axis tangential to the rim & perpendicular to the plane of disc		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
(3) Annular disc  M = Mass R_1 = Internal Radius R_2 = Outer Radius	(a) About an axis passing through the centre and perpendicular to the plane of disc		$\frac{M}{2}\left[R_1^2+R_2^2\right]$	$\sqrt{\frac{R_1^2+R_2^2}{2}}$

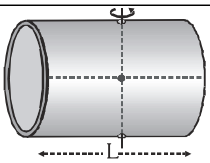
Shape of the body	Position of the axis of rotation	Figure	Moment of Inertia (I)	Radius of gyration (K)
	(b) About a diameteric axis		$\frac{M}{4}\left[R_1^2+R_2^2\right]$	$\frac{\sqrt{R_1^2+R_2^2}}{2}$

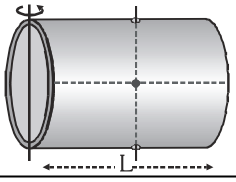
(4) Solid Sphere

M = Mass
R = Radius
	(a) About its diametric axis which passes through its centre of mass		$\frac{2}{5}MR^2$	$\sqrt{\frac{2}{5}}R$

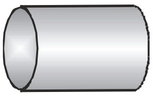
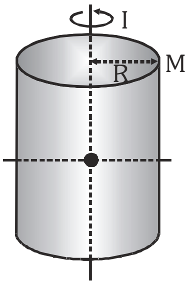
	(b) About a tangent to the Sphere		$\frac{7}{5}MR^2$	$\sqrt{\frac{7}{5}}R$
(5) Hollow Sphere (Thin spherical Shell) 	(a) About diametric axis passing through centre of mass	 table tennis ball	$\frac{2}{3}MR^2$	$\sqrt{\frac{2}{3}}R$

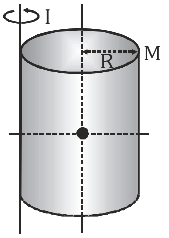
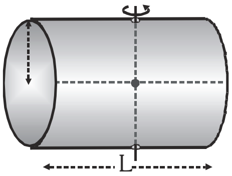
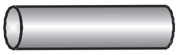
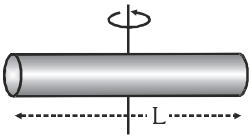
M = Mass
R = Radius
Thickness negligible
	(b) About a tangent to the surface	
		table tennis ball
			$\frac{5}{3}MR^2$	$\sqrt{\frac{5}{3}}R$

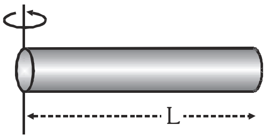
(6) Hollow Cylinder 	(a) About its geometrical axis which is parallel to its length		MR^2	R

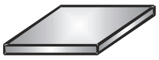
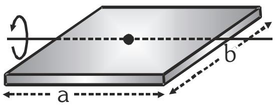
M = Mass
R = Radius
L = Length
	(b) About an axis which is perpendicular to its length and passes through its centre of mass		$\frac{MR^2}{2}+\frac{ML^2}{12}$	$\sqrt{\frac{R^2}{2}+\frac{L^2}{12}}$

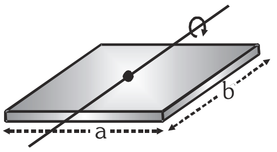
Shape of the body	Position of the axis of rotation	Figure	Moment of Inertia (I)	Radius of gyration (K)
	(c) About an axis perpendicular to its length and passing through one end of the cylinder		$\frac{MR^2}{2}+\frac{ML^2}{3}$	$\sqrt{\frac{R^2}{2}+\frac{L^2}{3}}$

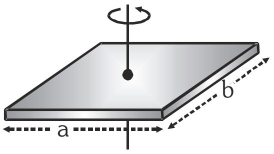
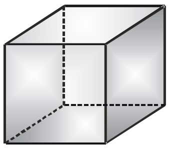
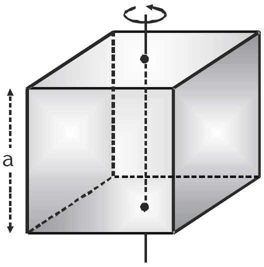
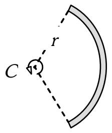
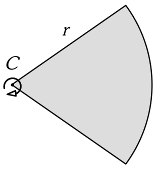
(7) Solid Cylinder
M = Mass
R = Radius
L = Length

	(a) About its geometrical axis, which is along its length		$\frac{MR^2}{2}$	$\frac{R}{\sqrt{2}}$

	(b) About an axis tangential to the cylinderical surface and parallel to its geometrical axis		$\frac{3}{2}MR^2$	$\sqrt{\frac{3}{2}}R$
	(c) About an axis passing through the centre of mass and perpendicular to its length		$\frac{ML^2}{12}+\frac{MR^2}{4}$	$\sqrt{\frac{L^2}{12}+\frac{R^2}{4}}$
(8) Thin Rod  Thickness is negligible w.r.t. length	(a) About an axis passing through centre of mass and perpendicular to its length		$\frac{ML^2}{12}$	$\frac{L}{\sqrt{12}}$

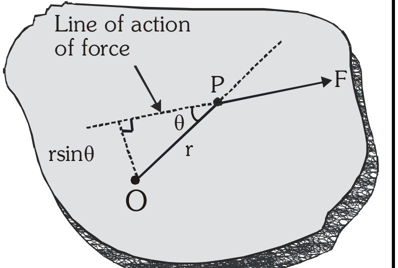
Mass = M
Length = L
	(b) About an axis passing through one end and perpendicular to length of the rod		$\frac{ML^2}{3}$	$\frac{L}{\sqrt{3}}$

(9) Rectangular Plate 	(a) About an axis passing through centre of mass and perpendicular to side b in its plane		$\frac{Mb^2}{12}$	$\frac{b}{2\sqrt{3}}$

M = Mass
a = Length
b = Breadth
	(b) About an axis passing through centre of mass and perpendicular to side a in its plane.		$\frac{Ma^2}{12}$	$\frac{a}{2\sqrt{3}}$

Shape of the body	Position of the axis of rotation	Figure	Moment of Inertia (I)	Radius of gyration (K)
	(c) About an axis passing throught centre of mass and perpendicular to plane		$\frac{M\left(a^2+b^2\right)}{12}$	$\sqrt{\frac{a^2+b^2}{12}}$
(10) Cube  Mass = M Side a	About an axis passes through centre of mass and perpendicular to face		$\frac{Ma^2}{6}$	$\frac{a}{\sqrt{6}}$
(11) Uniform thin rod bent into shape of an arc of mass *m*	About an axis Passing through center and perpendicular to the plane containing the arc		$I_C = mr^2$	r
(12) Sector of a uniform disk of mass *m*	About an axis Passing through center and perpendicular to the plane containing the sector.		$I_C = \frac{mr^2}{2}$	$r/\sqrt{2}$

TORQUE



Torque about point : $\vec{\tau}=\vec{r}\times\vec{F}$

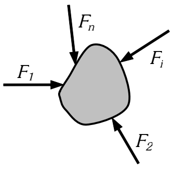
Magnitude of torque = Force × perpendicular distance of line of action of force from the axis of rotation.

$$t = rF\sin\theta$$

Direction of torque can be determined by using right hand thumb rule.

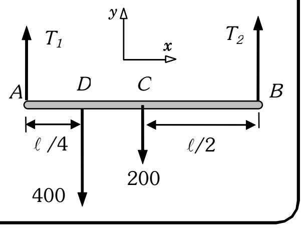
ROTATIONAL EQUILIBRIUM

If a rigid body is in rotational equilibrium under the action of several coplanar forces, the resultant torque of all the forces about any axis perpendicular to the plane containing the forces must be zero.



In the figure a body is shown under the action of several external coplanar forces $F_1, F_2, \ldots F_i$, and F_n.

$$\sum \vec{\tau}_P = 0$$



Here *P* is a point in the plane of the forces about which we calculate torque of all the external forces acting on the body. The flexibility available in selection of the point *P* provides us with advantages that we can select such a point about which torques of several unknown forces will become zero or we can make as many number of equations as desired by selecting several different points. The first situation yields to a simpler equation to be solved and second situation though does not give independent equation, which can be used to determine additional unknowns yet may be used to check the solution.

The above condition reveals that a body cannot be in rotational equilibrium under the action of a single force unless the line of action passes through the mass center of the body.

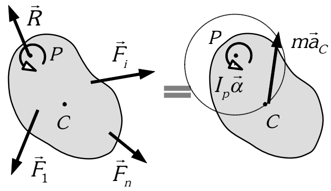
A case of particular interest arises where only three coplanar forces are involved and the body is in rotational equilibrium. It can be shown that *if a body is in rotational equilibrium under the action of three forces, the lines of action of the three forces must be either concurrent or parallel.* This condition provides us with a graphical technique to analyze rotational equilibrium.

Equilibrium of Rigid Bodies

A rigid body is said to be in equilibrium, if it is in translational as well as rotational equilibrium both. To analyze such problems conditions for both the equilibriums must be applied.

Rotation about fixed axis not passing through mass center

In this kind of rotation the axis of rotation remains fixed and does not passes through the mass center. Rotation of door is a common example of this category. Doors are hinged about their edges; therefore their axis of rotation does not pass through the mass center. In this kind of rotation motion the mass center executes circular motion about the axis of rotation.



In the figure, free body diagram and kinetic diagram of a body rotating about a fixed axis through point *P* is shown. It is easy to conceive that as the body rotates its mass center moves on a circular path of radius $\vec{r}_{P/C}$. The mass center of the body is in translation motion with acceleration $\vec{a}_C$ on circular path of radius $r_{P/C}$. To deal with this kind of motion, we have to make use of both the force and the torque equations.

Translation of mass center $\quad \Sigma\vec{F}_i = M\vec{a}_C = M\vec{\alpha}\times\vec{r}_{C/P} - M\omega^2\vec{r}_{C/P}$

Centroidal Rotation $\quad \Sigma\vec{\tau}_C = I_C\vec{\alpha}$

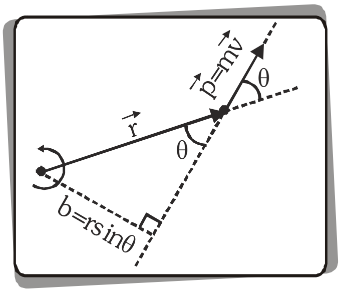
Making use of parallel axis theorem $\quad \left(I_P = Mr_{P/C}^2 + I_C\right)$ and $\vec{a}_{C/P} = \vec{\alpha}\times\vec{r}_{C/P} - \omega^2\vec{r}_{C/P}$

we can write the following equation also.

Pure Rotation about *P* $\quad \Sigma\vec{\tau}_P = I_P\vec{\alpha}$

ANGULAR MOMENTUM (MOMENT OF LINEAR MOMENTUM)

Angular momentum of a particle about a given axis is the product of its linear momentum and perpendicular distance of line of action of linear momentum vector from the axis of rotation, $\vec{L} = \vec{r}\times\vec{p}$



Magnitude of Angular momentum

= Linear momentum × Perpendicular distance of line of action of momentum from the axis of rotation

$$L = mv \times r\ \sin\theta$$

Direction of angular momentum can be used by using right hand thumb rule.

- According to Newtons Second Law's for rotatory motion $\vec{\tau} = \frac{d\vec{L}}{dt} = I\vec{\alpha}$.
- Angular Impulse = Change in angular momentum.
- If a large torque acts on a body for a small time then, angular impulse= $\vec{\tau}dt$

Conservation of Angular Momentum

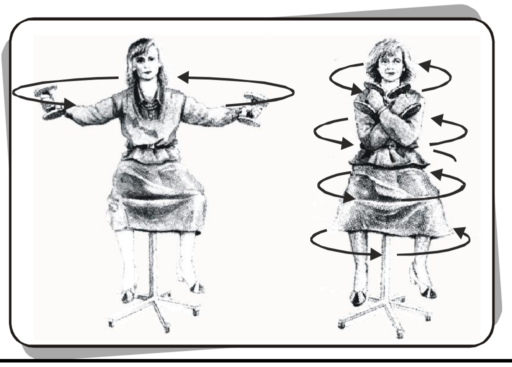
Angular momentum of a particle or a system remains constant if $\tau_{ext} = 0$ about that point or axis of rotation.

If $\tau = 0$ then $\frac{\Delta L}{\Delta t} = 0 \Rightarrow L =$ constant

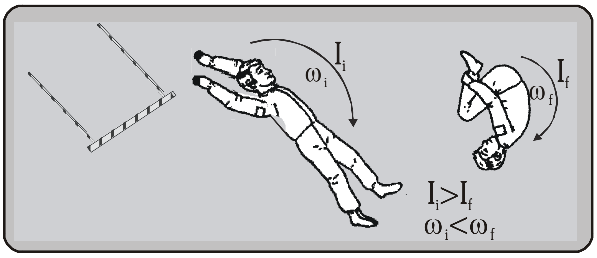
$$\Rightarrow L_f = L_i \quad \text{or } I_1\omega_1 = I_2\omega_2$$

Examples of Conservation of Angular Momentum

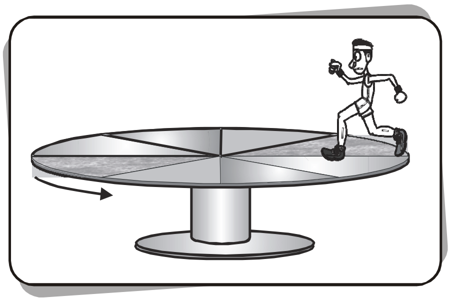
- If a person skating on ice folds his arms then his M.I. decreases and 'ω' increases.



- A diver jumping from a height folds his arms and legs (I decrease) in order to increase no. of rotation in air by increasing 'ω'.



- If a person moves towards the centre of rotating platform then 'I' decrease and 'ω' increase.



ROTATIONAL KINETIC ENERGY

Kinetic Energy of Rotation $KE_R = \frac{1}{2}I\omega^2$

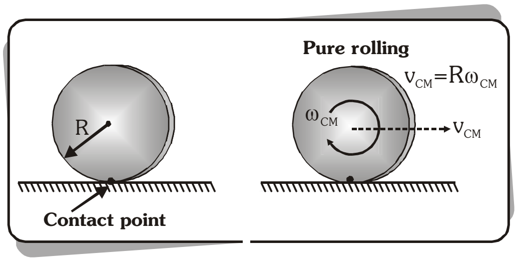
- Other forms $K = \frac{1}{2}I\omega^2 = \frac{L^2}{2I} = \frac{1}{2}L\omega$
- If external torque acting on a body is equal to zero ($\tau = 0$), L=constant $K \propto \frac{1}{I}$, $K \propto \omega$
- Rotational Work : $W_r = \tau\theta$ (If torque is constant) $W_r = \int_{\theta_1}^{\theta_2} \tau d\theta$ (If torque is variable)
- The work done by torque = Change in kinetic energy of rotation. $W = \frac{1}{2}I\omega_2^2 - \frac{1}{2}I\omega_1^2 = \frac{1}{2}I(\omega_2^2 - \omega_1^2)$
- Instantaneous power $= \frac{dW}{dt} = \tau\frac{d\theta}{dt} = \tau\omega$ Average power $P_{av} = \frac{\Delta W}{\Delta t}$

COMBINED TRANSLATIONAL AND ROTATIONAL MOTION OF A RIGID BODY

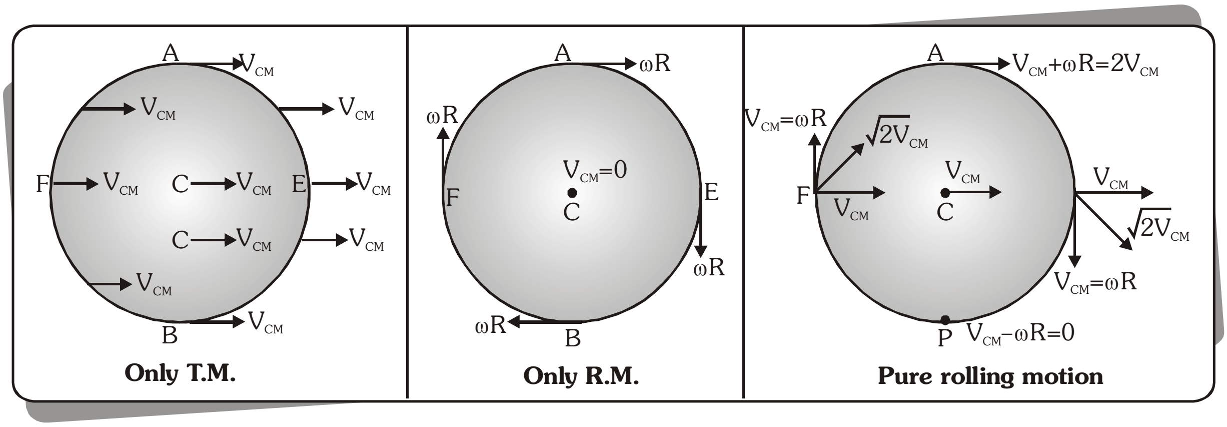
When a body perform translatory motion as well as rotatory motion then it is known as rolling.

In Pure Rolling

(i) If the velocity of point of contact with respect to the surface is zero then it is known as pure rolling.



(ii) If a body is performing rolling then the velocity of any point of the body with respect to the surface is given by $\vec{v} = \vec{v}_{CM} + \vec{\omega}_{CM} \times \vec{R}$



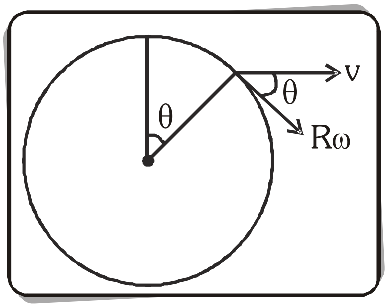
Only T.M. **Only R.M.** **Pure rolling motion**

Only Translatory motion + Only Rotatory Motion = Rolling motion. For pure rolling above body

$V_A = 2V_{CM}$ $V_E = \sqrt{2}\ V_{CM}$ $V_F = \sqrt{2}\ V_{CM}$ $V_B = 0$

Velocity at a point on rim of sphere

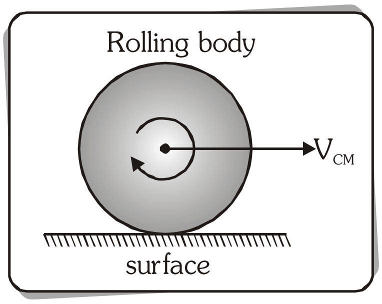
$$v_{net} = \sqrt{v^2 + R^2\omega^2 + 2vR\omega\cos\theta}$$



For pure rolling v = Rω

$$\boxed{v_{net} = 2v\cos\frac{\theta}{2}}$$

Rolling Kinetic Energy under pure rolling



Rolling Kinetic Energy

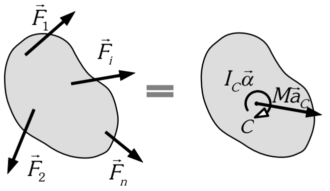
$$E = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2 = \frac{1}{2}mv^2 + \frac{1}{2}mK^2\left(\frac{v^2}{R^2}\right)$$

Rolling Kinetic Energy $E = \frac{1}{2}mv^2\left(1 + \frac{K^2}{R^2}\right)$

$$E_{translation} : E_{rotation} : E_{Total} = 1 : \frac{K^2}{R^2} : 1 + \frac{K^2}{R^2}$$

Body	$\frac{K^2}{R^2}$	$\frac{E_{trans}}{E_{rotation}} = \frac{1}{\frac{K^2}{R^2}}$	$\frac{E_{trans}}{E_{total}} = \frac{1}{1+\frac{K^2}{R^2}}$	$\frac{E_{rotation}}{E_{total}} = \frac{\frac{K^2}{R^2}}{1+\frac{K^2}{R^2}}$
Ring	1	1	1/2	1/2
Disc	1/2	2	2/3	1/3
Solid sphere	2/5	5/2	5/7	2/7
Spherical shell	2/3	3/2	3/5	2/5
Solid cylinder	1/2	2	2/3	1/3
Hollow cylinder	1	1	1/2	1/2

General Plane Motion: Rotation about axis in translation motion



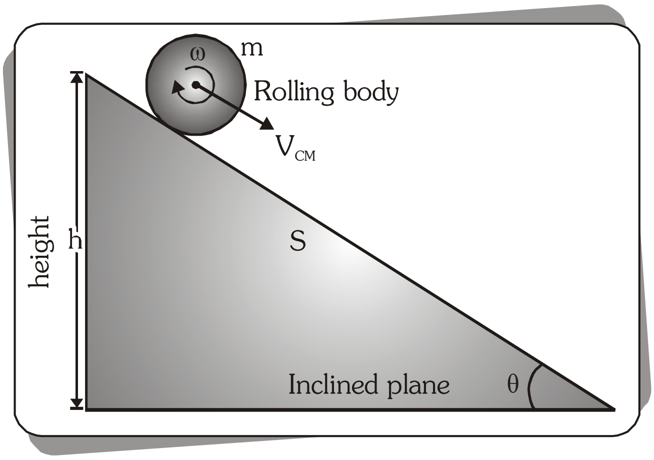
Rotation of bodies about an axis in translation motion can be dealt with either as superposition of translation of mass center and centroidal rotation or assuming pure rotation about the instantaneous axis of rotation. In the figure is shown the free body diagram and kinetic diagram of a body in general plane motion.

Translation of mass center $\sum_{i=0}^{n}\vec{F}_i = M\vec{a}_C$

Centroidal Rotation $\sum_{i-1}^{n}\vec{\tau}_C = I_C\vec{\alpha}$

This kind of situation can also be dealt with considering it rotation about IAR. It gives sometimes quick solutions, especially when IAR is known and forces if acting at the IAR are not required to be found.

Rolling Motion on an inclined plane



Applying Conservation of energy

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}I\omega^2$$

$$mgh = \frac{1}{2}mv^2 + \frac{1}{2}mK^2\left(\frac{v^2}{R^2}\right)$$

$$mgh = \frac{1}{2}mv^2\left(1+\frac{K^2}{R^2}\right) \quad \text{...(1)}$$

$$h = s\sin\theta \quad \text{...(2)}$$

from (1) & (2)

$$\boxed{V_{\text{Rolling}} = \sqrt{\frac{2gh}{1+\frac{K^2}{R^2}}} = \sqrt{\frac{2gs\sin\theta}{1+\frac{K^2}{R^2}}}}$$

- Linear accleration on reaching the lowest point a

$$= \frac{g\sin\theta}{1+K^2/R^2}$$

- Time taken to reach the lowest point of the plane is

$$t = \sqrt{\frac{2s(1+K^2/R^2)}{g\sin\theta}}$$

- $\frac{K^2}{R^2}$ Least, will reach first

  $\frac{K^2}{R^2}$ Maximum, will reach last

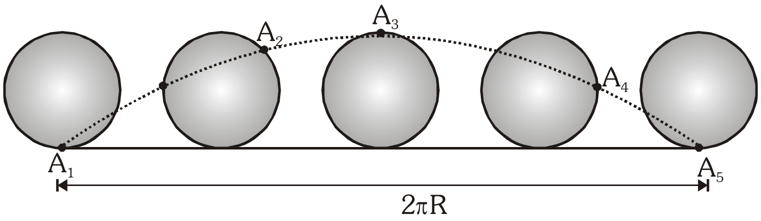
  $\frac{K^2}{R^2}$ equal, will reach together

- When ring, disc, hollows sphere, solid sphere rolls on same inclined plane then

  $v_S > v_D > v_H > v_R$ $a_S > a_D > a_H > a_R$

  $t_S < t_D < t_H < t_R$

For a pure rolling body after one full rotation

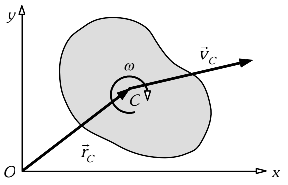


displacement of lowermost point =2πR

distance = 8R

Angular Momentum in general plane motion

Angular momentum of a body in plane motion can also be written similar to torque equation or kinetic energy as sum of angular momentum about the axis due to translation of mass center and angular momentum of centroidal rotation about centroidal axis parallel to the original axis.



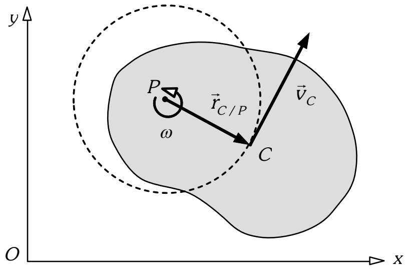
Consider a rigid body of mass M in plane motion. At the instant shown its mass center has velocity $\vec{v}$ and it is rotating with angular velocity $\vec{\omega}$ about an axis perpendicular to the plane of the figure. It angular momentum $\vec{L}_o$ about an axis passing though the origin and parallel to the original is expressed by the following equation.

$$\vec{L}_o = \vec{r}_C \times \left(M\vec{v}_C\right) + I_C\vec{\omega}$$

The first term of the above equation represent angular momentum due to translation of the mass center and the second term represents angular momentum in centroidal rotation.

Angular momentum in rotation about fixed axis

Consider a body of mass M rotating with angular velocity ω about a fixed axis perpendicular to plane of the figure passing through point P. Making use of the parallel axis theorem $I_P = Mr_{C/P}^2 + I_C$ and equation $\vec{v}_C = \vec{\omega} \times \vec{r}_{C/P}$ we can express the angular momentum $\vec{L}_P$ of the body about the fixed rotational axis.

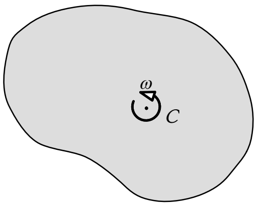


$$\vec{L}_P = I_P\vec{\omega}$$

The above equation reveals that the angular momentum of a rigid body in plane motion can also be expressed in a single term due to rotation about the instantaneous axis of rotation.

Angular momentum in pure centroidal rotation

In pure centroidal rotation, mass center remains at rest, therefore angular momentum due to translation of the mass center vanishes.

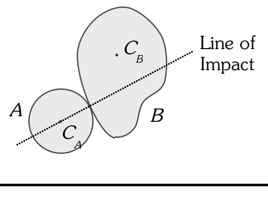


$$\vec{L}_C = I_C\vec{\omega}$$

Eccentric Impact

In eccentric impact the line of impact which is the common normal drawn at the point of impact does not passes through mass center of at least one of the colliding bodies. It involves change in state of rotation motion of either or both the bodies.

Consider impact of two A and B such that the mass center C_B of B does not lie on the line of impact as shown in figure. If we assume bodies to be frictionless their mutual forces must act along the line of impact. The reaction force of A on B does not passes through the mass center of B as a result state of rotation motion of B changes during the impact.



Problems of Eccentric Impact

Problems of eccentric impact can be divided into two categories. In one category both the bodies under going eccentric impact are free to move. No external force act on either of them. There mutual forces are responsible for change in their momentum and angular momentum. In another category either or both of the bodies are hinged.

Eccentric Impact of bodies free to move

Since no external force acts on the two body system, we can use principle of conservation of linear momentum, principle of conservation of angular momentum about any point and concept of coefficient of restitution.

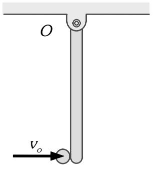
The coefficient of restitution is defined for components of velocities of points of contacts of the bodies along the line of impact.

While applying principle of conservation of angular momentum care must be taken in selecting the point about which we write the equation. The point about which we write angular momentum must be at rest relative to the selected inertial reference frame and as far as possible its location should be selected on line of velocity of the mass center in order to make zero the first term involving moment of momentum of mass center.

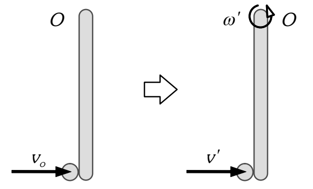
Eccentric Impact of hinged bodies

When either or both of the bodies are hinged the reaction of the hinge during the impact act as external force on the two body system, therefore linear momentum no longer remain conserved and we cannot apply principle of conservation of linear momentum. When both the bodies are hinged we cannot also apply conservation of angular momentum, and we have to use impulse momentum principle on both the bodies separately in addition to making use of coefficient of restitution. But when one of the bodies is hinged and other one is free to move, we can apply conservation of angular momentum about the hinge.

Ex. A uniform rod of mass m and length ℓ is suspended from a fixed support and can rotate freely in the vertical plane. A small ball of mass m moving horizontally with velocity v_o strikes elastically the lower end of the rod as shown in the figure. Find the angular velocity of the rod and velocity of the ball immediately after the impact.



Sol. The rod is hinged and the ball is free to move. External forces acting on the rod ball system are their weights and reaction from the hinge. Weight of the ball as well as the rod are finite and contribute negligible impulse during the impact, but impulse of reaction of the hinge during impact is considerable and cannot be neglected. Obviously linear momentum of the system is not conserved. The angular impulse of the reaction of hinge about the hinge is zero. Therefore angular momentum of the system about the hinge is conserved. Let velocity of the ball after the impact becomes v'_B and angular velocity of the rod becomes ω'.



Before the impact

Immediately after the impact

We denote angular momentum of the ball and the rod about the hinge before the impact by L_{B1} and L_{R1} and after the impact by L_{B2} and L_{R2}.

Applying conservation of angular momentum about the hinge, we have

$$\vec{L}_{B1}+\vec{L}_{R1}=\vec{L}_{B2}+\vec{L}_{R2}\rightarrow \quad mv_o\ell+0=mv'_B\ell+I_o\omega'$$

Substituting $\frac{1}{3}M\ell^2$ for I_o, we have

$$3mv'_B+M\ell\omega'=3mv_o \qquad (1)$$

The velocity of the lower end of the rod before the impact was zero and immediately after the impact it becomes $\ell\omega'$ towards right. Employing these facts we can express the coefficient of restitution according to eq.

$$e=\frac{v'_{Qn}-v'_{Pn}}{v_{pn}-v_{Qn}}\rightarrow \qquad \ell\omega'-v'_B=ev_o \qquad (2)$$

From eq. (1) and (2), we have

Velocity of the ball immediately after the impact $\quad v'_B=\dfrac{(3m-eM)v_o}{3m+M}$ **Ans.**

Angular velocity of the rod immediately after the impact $\quad \omega'=\dfrac{3(1+e)mv_o}{(3m+M)\ell}$ **Ans.**

IMPORTANT NOTES

CHAPTER

9

SIMPLE HARMONIC MOTION

Periodic Motion

Any motion which repeats itself after regular interval of time (i.e. time period) is called periodic motion or harmonic motion.

Example:

(i) Motion of planets around the sun.

(ii) Motion of the pendulum of wall clock.

Oscillatory Motion

The motion of body is said to be oscillatory or vibratory motion if it moves back and forth (to and fro) about a fixed point after regular interval of time.

The fixed point about which the body oscillates is called mean position or equilibrium position.

Example:

(i) Vibration of the wire of 'Sitar'.

(ii) Oscillation of the mass suspended from spring.

Simple Harmonic Motion (SHM)

Simple harmonic motion is the simplest form of vibratory or oscillatory motion. In which restoring force is directly proporiontal to distance from mean.

Some Basic Terms in SHM

- **Mean Position**

  The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.

- **Restoring Force**

  The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.

  Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.

- **Amplitude**

  The maximum (positive or negative) value of displacement of particle from mean position is defined as amplitude.

- **Time period (T)**

  The minimum time after which the particle keeps on repeating its motion is known as time period.

  The smallest time taken to complete one oscillation or vibration is also defined as time period.

  It is given by $T = \frac{2\pi}{\omega} = \frac{1}{n}$ where ω is angular frequency and n is frequency.

- **Frequency (n or f)**

  The number of oscillations per second is defined as frequency. It is given by $n = \frac{1}{T} = \frac{\omega}{2\pi}$

- **Phase**

  Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.

  In the equation $x = A \sin(\omega t + \phi)$, $(\omega t + \phi)$ is the phase of the particle.

  The phase angle at time t = 0 is known as initial phase or epoch.

  The difference of total phase angles of two particles executing SHM with respect to the mean position is known as phase difference.

  Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π, i.e. $\Delta\phi = 2n\pi$ where n = 0, 1, 2, 3,....

  Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π i.e., $\Delta\phi = (2n + 1)\pi$ where n = 0, 1, 2, 3,....

- **Angular frequency (ω)** :The rate of change of phase angle of a particle with respect to time is defined as its angular frequency.

$$\omega = \sqrt{\frac{k}{m}}$$

◆ **For linear SHM**

$$(F \propto -x) : F = m\frac{d^2x}{dt^2} = -kx = -m\omega^2 x \text{ where } \omega = \sqrt{\frac{k}{m}}$$

◆ **For angular SHM** $(\tau \propto -\theta)$:

$$\tau = I\frac{d^2\theta}{dt^2} = I\alpha = -k\theta = -m\omega^2\theta \text{ where } \omega = \sqrt{\frac{k}{m}}$$

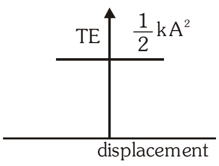
◆ **Displacement** $x = A\sin(\omega t + \phi)$,

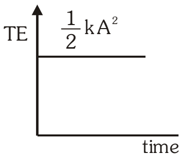
◆ **Angular displacement** $\theta = \theta_0 \sin(\omega t + \phi)$

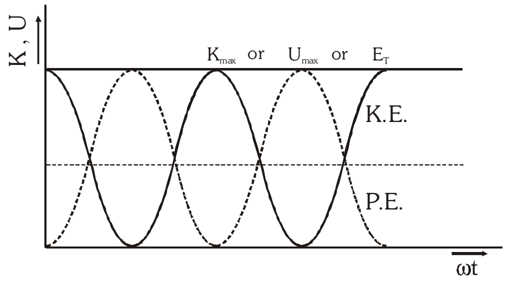
- **Velocity** $v = \frac{dx}{dt} = A\omega\cos(\omega t + \phi) = \omega\sqrt{A^2 - x^2}$
- **Angular velocity** $\frac{d\theta}{dt} = \theta_0\omega\cos(\omega t+\phi)$
- **Acceleration** $a = \frac{d^2x}{dt^2} = -A\omega^2\sin(\omega t + \phi) = -\omega^2 x$
- **Angular acceleration**

$$\frac{d^2\theta}{dt^2} = -\theta_0\omega^2\sin(\omega t+\phi) = -\omega^2\theta$$

- **Kinetic energy** $K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2A^2\cos^2(\omega t+\phi)$
- **Potential energy** $U = \frac{1}{2}kx^2 = \frac{1}{2}m\omega^2A^2\sin^2(\omega t+\phi)$
- **Total energy** $E = K+U = \frac{1}{2}m\omega^2A^2 =$ constant







Note :

(i) Total energy of a particle in S.H.M. is same at all instant and at all displacement.

(ii) Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

Average energy in SHM

(i) The time average of P.E. and K.E. over one cycle is

(a) $\langle K\rangle_t = \frac{1}{4}kA^2$ (b) $\langle PE\rangle_t = \frac{1}{4}kA^2$

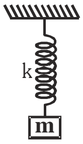
(c) $\langle TE\rangle_t = \frac{1}{2}kA^2 + U_0$

(ii) The position average of P.E. and K.E. between x = – A to x=A

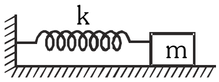
(a) $\langle K\rangle_x = \frac{1}{3}kA^2$ (b) $\langle PE\rangle_x = U_0 + \frac{1}{6}kA^2$

(c) $\langle TE\rangle_x = \frac{1}{2}kA^2 + U_0$

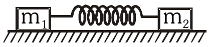
- **Spring block system**



$T = 2\pi\sqrt{\frac{m}{k}}$



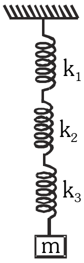
$T = 2\pi\sqrt{\frac{m}{k}}$

- 

$T = 2\pi\sqrt{\frac{\mu}{k}}$

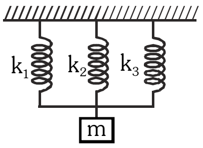
where μ=reduced mass $= \frac{m_1m_2}{m_1+m_2}$

- **Series combination of springs**



$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$ where $\frac{1}{k_{eff}} = \frac{1}{k_1} + \frac{1}{k_2} + \frac{1}{k_3}$

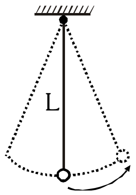
- **Parallel combination of springs**



$T = 2\pi\sqrt{\frac{m}{k_{eff}}}$

where $k_{eff} = k_1 + k_2 + k_3$

- **Time period of simple pendulum**

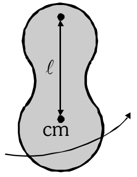


Time period $T = 2\pi\sqrt{\frac{L}{g}}$

- **Second pendulum**

Time period = 2 seconds, Length ≈ 1 meter (on earth's surface)

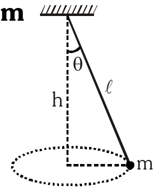
- **Time period of Physical pendulum**



$$T = 2\pi\sqrt{\frac{I}{mg\ell}} = 2\pi\sqrt{\frac{\frac{k^2}{\ell}+\ell}{g}}$$

where $I_{cm} = mk^2$

- **Time period of Conical pendulum**



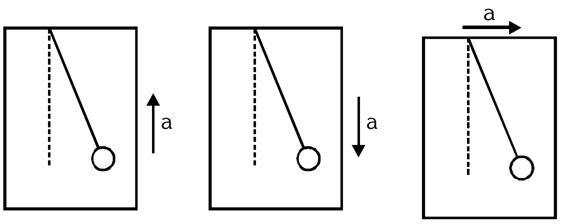
$$T = 2\pi\sqrt{\frac{\ell\cos\theta}{g}} = 2\pi\sqrt{\frac{h}{g}}$$

- **Time period of Torsional pendulum**

$T=2\pi\sqrt{\frac{I}{k}}$ where k = torsional constant of the wire

I=moment of inertia of the body about the vertical axis

In accelerating cage

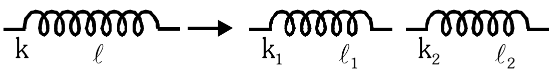


$g_{eff} = g + a$ $g_{eff} = g-a$ $g_{eff} = \sqrt{g^2+a^2}$

$T = 2\pi\sqrt{\frac{\ell}{g+a}}$ $T = 2\pi\sqrt{\frac{\ell}{g-a}}$ $T = 2\pi\sqrt{\frac{\ell}{(g^2+a^2)^{1/2}}}$

KEY POINTS

- SHM is the projection of uniform circular motion along one of the diameters of the circle.
- The periodic time of a hard spring is less as compared to that of a soft spring because the spring constant is large for hard spring.
- For a system executing SHM, the mechanical energy remains constant.
- Maximum kinetic energy of a particle in SHM may be greater than mechanical energy as potential energy of a system may be negative.
- The frequency of oscillation of potential energy and kinetic energy is twice as that of displacement or velocity or acceleration of a particle executing S.H.M.
- **Spring cut into two parts :**



Here $\frac{\ell_1}{\ell_2} = \frac{m}{n}$ $\ell_1 = \left(\frac{m}{m+n}\right)\ell$, $\ell_2 = \left(\frac{n}{m+n}\right)\ell$ But $k\ell = k_1\ell_1 = k_2\ell_2 \Rightarrow k_1 = \frac{(m+n)}{m}k$; $k_2 = \frac{(m+n)}{n}k$

IMPORTANT NOTES

IMPORTANT NOTES

CHAPTER 10

WAVE MOTION & DOPPLER'S EFFECT

A wave is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

CLASSIFICATION OF WAVES

- Medium Necessity
  - Mechanical (Elastic) waves
  - Non-mechanical waves (EM waves)
- Propagation of energy
  - Progressive waves
  - Stationary (standing) waves
- Vibration of medium particle
  - Transverse waves
  - Longitudinal waves
- Dimension
  - 1-D (Waves on strings)
  - 2-D (Surface waves or ripples on water)
  - 3-D (Sound or light waves)

- If particle vibrates in pependiuclar direction to the wave motion then its called transverse wave.
- In strings, mechanical waves are always transverse.
- In gases and liquids, mechanical waves are always longitudinal because fluids cannot sustain shear.
- Partially transverse waves are possible on a liquid surface because surface tension provide some rigidity on a liquid surface. These waves are called as ripples as they are combination of transverse & longitudinal.
- In solids mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation.
- In longitudinal wave motion, oscillatory motion of the medium particles produce regions of compression (high pressure) and rarefaction (low pressure).

PLANE PROGRESSIVE WAVES

- Wave equation : $y = A\sin(\omega t - kx)$ where $k = \frac{2\pi}{\lambda}$ = wave propagation constant

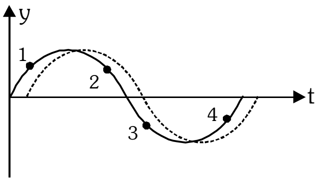
  (Wave is moving along x-axis and paticle is moving along y axis)
- If coefficient of x & t are of opposite sign than wave move in +ve x-direction

  Ex. $y = A\sin(\omega t - kx)$ wave is moving in +x direction and particle is moving in y direction

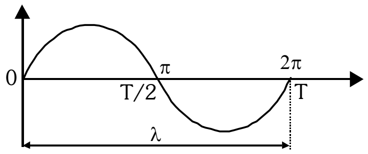
  Similarly Ex. $z = A\sin(\omega t - ky)$ here wave move in +y direction and particle move in z direction
- Differential equation : $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$

  Wave velocity (phase velocity) $\boxed{v_w = \frac{dx}{dt} = \frac{\omega}{k}}$ $\because \omega t - kx = \text{constant} \Rightarrow \frac{dx}{dt} = \frac{\omega}{k}$
- Particle velocity $v_p = \frac{dy}{dt} = A\omega\cos(\omega t - kx)$ $\boxed{v_p = -v_w \times \text{slope} = -v_w\left(\frac{dy}{dx}\right)}$
- Particle acceleration : $a_p = \frac{\partial^2 y}{\partial t^2} = -\omega^2 A\sin(\omega t - kx) = -\omega^2 y$

  For particle 1 : $v_p \downarrow$ and $a_p \downarrow$
  For particle 2 : $v_p \uparrow$ and $a_p \downarrow$
  For particle 3 : $v_p \uparrow$ and $a_p \uparrow$
  For particle 4 : $v_p \downarrow$ and $a_p \uparrow$



- Relation between phase difference, path difference & time difference



$$\boxed{\frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T}}$$

ENERGY IN WAVE MOTION

- $\dfrac{KE}{\text{volume}} = \dfrac{1}{2}\left(\dfrac{\Delta m}{\text{volume}}\right)v_p^2$

  $= \dfrac{1}{2}\rho v_p^2 = \dfrac{1}{2}\rho\omega^2 A^2\cos^2(\omega t - kx)$

- $\dfrac{PE}{\text{volume}} = \dfrac{1}{2}\rho v^2\left(\dfrac{dy}{dx}\right)^2 = \dfrac{1}{2}\rho\omega^2A^2\cos^2(\omega t - kx)$

- $\dfrac{TE}{\text{volume}} = \rho\omega^2A^2\cos^2(\omega t - kx)$

- Pressure energy density $\boxed{u = \dfrac{1}{2}\rho\omega^2A^2}$

  [i.e. Average total energy / volume]

- **Power :** P = (energy density) (volume/ time)

  $$P = \left(\frac{1}{2}\rho\omega^2A^2\right)(Sv)$$

  [where S = Area of cross-section]

- **Intensity :** $I = \dfrac{\text{Power}}{\text{area of cross-section}} = \dfrac{1}{2}\rho\omega^2A^2v$

Speed of transverse wave on string :

$v = \sqrt{\dfrac{T}{\mu}}$ where μ = mass/length and

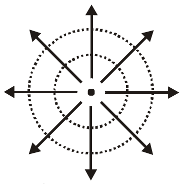
T = tension in the string.

KEY POINTS

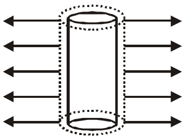
- A wave can be represented by function $y=f(kx \pm \omega t)$ because it satisfy the differential equation $\dfrac{\partial^2 y}{\partial x^2} = \dfrac{1}{v^2}\left(\dfrac{\partial^2 y}{\partial t^2}\right)$ where $v = \dfrac{\omega}{k}$.
- A pulse whose wave function is given by $y=4 / [(2x+5t)^2+2]$ propagates in –x direction as this wave function is of the form $y=f(kx + \omega t)$ which represent a wave travelling in –x direction.
- Longitudinal waves can be produced in solids, liquids and gases because bulk modulus of elasticity is present in all three.

WAVE FRONT

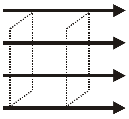
- Spherical wave front (source →point source)



- Cylindrical wave front (source →linear source)



- Plane wave front (source → point / linear source at very large distance)



INTENSITY OF WAVE

- Due to point source $I \propto \dfrac{1}{r^2}$

  $$y(r,t) = \frac{A}{r}\sin(\omega t - \vec{k}\cdot\vec{r})$$

- Due to cylindrical source $I \propto \dfrac{1}{r}$

  $$y(r,t) = \frac{A}{\sqrt{r}}\sin(\omega t - \vec{k}\cdot\vec{r})$$

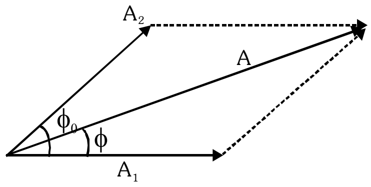
- Due to plane source I = constant

  $$y(r,t) = A\sin(\omega t - \vec{k}\cdot\vec{r})$$

INTERFERENCE OF WAVES

$y_1 = A_1\sin(\omega t - kx)$, $y_2 = A_2\sin(\omega t - kx + \phi_0)$

$y = y_1 + y_2 = A\sin(\omega t - kx + \phi)$



where $A = \sqrt{A_1^2 + A_2^2 + 2A_1A_2\cos\phi_0}$

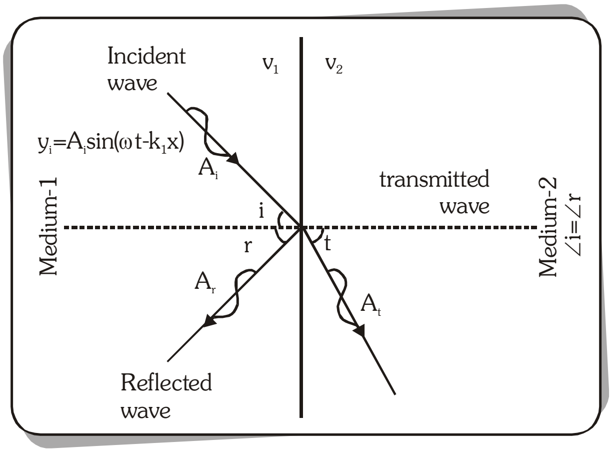
and $\tan\phi = \dfrac{A_2\sin\phi_0}{A_1 + A_2\cos\phi_0}$

As $I \propto A^2$

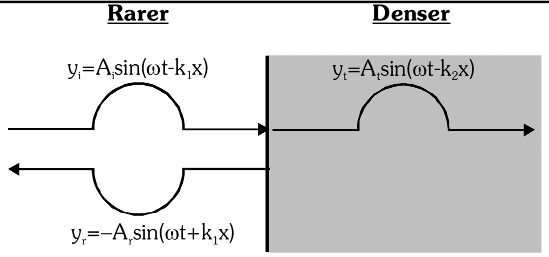
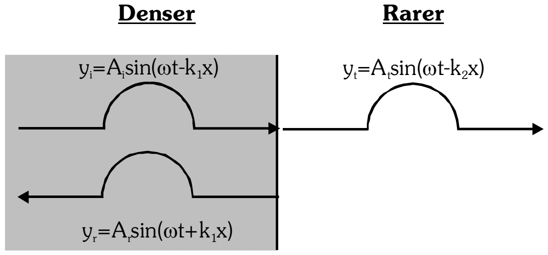
So $\boxed{I = I_1 + I_2 + 2\sqrt{I_1I_2}\cos\phi_0}$

- For constructive interference [Maximum intensity] $\phi_0 = 2n\pi$ or path difference = $n\lambda$ where n = 0, 1, 2, 3, ... $\boxed{I_{max} = \left(\sqrt{I_1} + \sqrt{I_2}\right)^2}$
- For destructive interference [Minimum Intensity] $\phi_0 = (2n+1)\pi$ or path difference = $(2n+1)\frac{\lambda}{2}$ where n = 0, 1, 2, 3, ... $\boxed{I_{min} = \left(\sqrt{I_1} - \sqrt{I_2}\right)^2}$
- Degree of hearing= $\frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100$

REFLECTION AND REFRACTION (TRANSMISSION) OF WAVES



- The frequency of the wave remain unchanged.
- Amplitude of reflected wave→ $A_r = \left(\frac{v_2 - v_1}{v_1 + v_2}\right)A_i$
- Amplitude of transmitted wave → $A_t = \left(\frac{2v_2}{v_1 + v_2}\right)A_i$
- If $v_2 > v_1$ i.e. medium-2 is rarer $A_r > 0 \Rightarrow$ no phase change in reflected wave
- If $v_2 < v_1$ i.e. medium-1 is rarer $A_r < 0 \Rightarrow$ There is a phase change of π in reflected wave
- As A_t is always positive whatever be v_1 & v_2 the phase of transmitted wave always remains unchanged.
- In case of reflection from a denser medium or rigid support or fixed end, there is inversion of reflected wave i.e. phase difference of π between reflected and incident wave.
- The transmitted wave is never inverted.

- 
Rarer
Denser
$y_i=A_i\sin(\omega t-k_1x)$
$y_t=A_t\sin(\omega t-k_2x)$
$y_r=-A_r\sin(\omega t+k_1x)$
- 
Denser
Rarer
$y_i=A_i\sin(\omega t-k_1x)$
$y_t=A_t\sin(\omega t-k_2x)$
$y_r=A_r\sin(\omega t+k_1x)$

BEATS

When two sound waves of nearly equal (but not exactly equal) frequencies travel in same direction, at a given point due to their super position, intensity alternatively increases and decreases periodically. This periodic waxing and waning of sound at a given position is called beats.
Beat frequency = difference of frequencies of two interfering waves
Beat frequency = $|f_1 - f_2|$

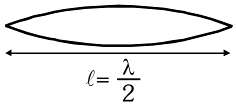
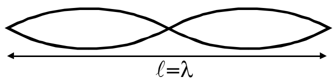
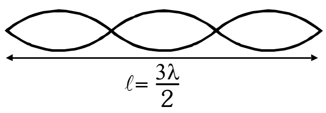
STATIONARY WAVES OR STANDING WAVES

When two waves of same frequency and amplitude travel in opposite direction at same speed, their superstition gives rise to a new type of wave, called stationary waves or standing waves. Formation of standing wave is possible only in bounded medium.

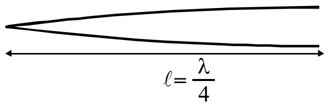
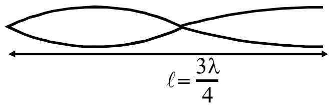
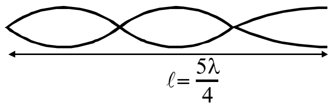
- Let two waves are $y_1=A\sin(\omega t-kx)$; $y_2=A\sin(\omega t+kx)$ by principle of superposition $y=y_1+y_2=2A\cos kx\sin\omega t$ ← Equation of stationary wave
- As this equation satisfies the wave equation $\frac{\partial^2 y}{\partial x^2} = \frac{1}{v^2}\frac{\partial^2 y}{\partial t^2}$, it represent a wave.
- Its amplitude is not constant but varies periodically with position.
- **Nodes**→amplitude is minimum : $\cos kx = 0 \Rightarrow x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}, ...$
- **Antinodes** → amplitude is maximum : $\cos kx = 1 \Rightarrow x = 0, \frac{\lambda}{2}, \lambda, \frac{3\lambda}{2}, ...$
- The nodes divide the medium into segments (loops). All the particles in a segment vibrate in same phase but in opposite phase with the particles in the adjacent segment.
- As nodes are permanently at rest, so no energy can be transmitted across them, i.e. energy of one region (segment) is confined in that region.

Transverse stationary waves in stretched string

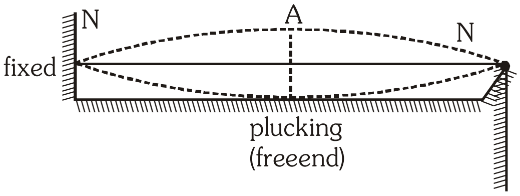
[Fixed at both ends] [fixed end →Node & free end→Antinode]

 $\ell=\frac{\lambda}{2}$	Fundamental or first harmonic or zero overtone	$f=\frac{v}{2\ell}$
 $\ell=\lambda$	second harmonic first overtone	$f=\frac{2v}{2\ell}$
 $\ell=\frac{3\lambda}{2}$	third harmonic second overtone	$f=\frac{3v}{2\ell}$
	n^{th} harmonic $(n-1)^{th}$ overtone	$f_\ell=\frac{nv}{2\ell}$

Fixed at one end

 $\ell=\frac{\lambda}{4}$	Fundamental	$f=\frac{v}{4\ell}$
 $\ell=\frac{3\lambda}{4}$	third harmonic first overtone	$f=\frac{3v}{4\ell}$
 $\ell=\frac{5\lambda}{4}$	fifth harmonic second overtone	$f=\frac{5v}{4\ell}$
	$(2n+1)^{th}$ harmonic n^{th} overtone	$f=\frac{(2n+1)v}{4\ell}$

Sonometer

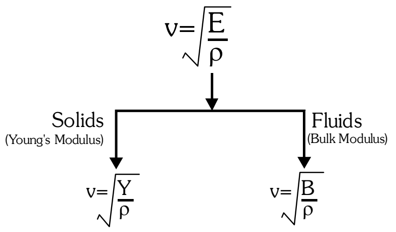


$$f_n=\frac{p}{2\ell}\sqrt{\frac{T}{\mu}}$$

[p : number of loops]

Sound Waves

Velocity of sound in a medium of elasticity E and density ρ is


$v=\sqrt{\frac{E}{\rho}}$
Solids
(Young's Modulus)
Fluids
(Bulk Modulus)
$v=\sqrt{\frac{Y}{\rho}}$
$v=\sqrt{\frac{B}{\rho}}$

- **Newton's formula** : Sound propagation is isothermal $B=P\Rightarrow v=\sqrt{\frac{P}{\rho}}$
- **Laplace correction** : Sound propagation is adiabatic $B=\gamma P\Rightarrow v=\sqrt{\frac{\gamma P}{\rho}}$

KEY POINTS

- With rise in temperature, velocity of sound in a gas increases as $v=\sqrt{\frac{\gamma RT}{M_W}}$
- With rise in humidity velocity of sound increases due to presence of water in air.
- Pressure has no effect on velocity of sound in a gas as long as temperature remains constant.

Displacement and pressure wave

A sound wave can be described either in terms of the longitudinal displacement suffered by the particles of the medium (called displacement wave) or in terms of the excess pressure generated due to compression and rarefaction (called pressure wave).

Displacement wave $y=A\sin(\omega t-kx)$

Pressure wave $p=p_0\cos(\omega t-kx)$

where $p_0=ABk=\rho A v\omega$

Note : As sound-sensors (e.g., ear or mike) detect pressure changes, description of sound as pressure wave is preferred over displacement wave.

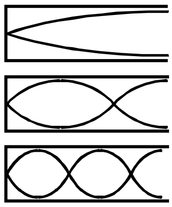
KEYPOINTS

- The pressure wave is 90° out of phase w.r.t. displacement wave, i.e. displacement will be maximum when pressure is minimum and vice-versa.
- Intensity in terms of pressure amplitude $I=\frac{p_0^2}{2\rho v}$

Vibrations of organ pipes

Stationary longitudinal waves closed end → displacement node, open end→ displacement antinode

- **Closed end organ pipe**



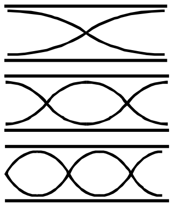
$$\ell = \frac{\lambda}{4} \Rightarrow f = \frac{v}{4\ell}$$

$$\ell = \frac{3\lambda}{4} \Rightarrow f = \frac{3v}{4\ell}$$

$$\ell = \frac{5\lambda}{4} \Rightarrow f = \frac{5v}{4\ell}$$

  - Only odd harmonics are present
  - Maximum possible wavelength = 4ℓ
  - Frequency of mth overtone $= (2m+1)\frac{v}{4\ell}$

- **Open end organ pipe**



$$\ell = \frac{\lambda}{2} \Rightarrow f = \frac{v}{2\ell}$$

$$\ell = \lambda \Rightarrow f = \frac{2v}{2\ell}$$

$$\ell = \frac{3\lambda}{2} \Rightarrow f = \frac{3v}{2\ell}$$

  - All harmonics are present
  - Maximum possible wavelength is 2ℓ.
  - Frequency of mth overtone $= (m+1)\frac{v}{2\ell}$

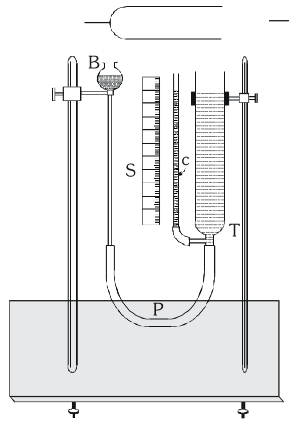
- **End correction** :
  Due to finite momentum of air molecules in organ pipes reflection takes place not exactly at open end but some what above it, so antinode is not formed exactly at free end but slightly above it.

  In closed organ pipe $f_1 = \frac{v}{4(\ell+e)}$

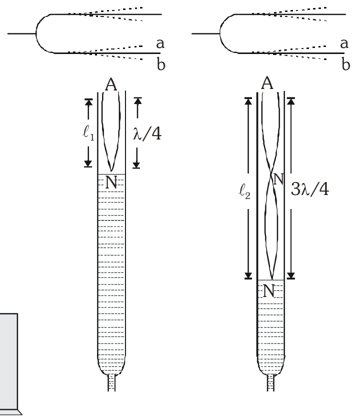
  where e = 0.6 R (R=radius of pipe)

  In open organ pipe $f_1 = \frac{v}{2(\ell+2e)}$

- **Resonance Tube**



Wavelength $\lambda = 2(\ell_2 - \ell_1)$



End correction $e = \frac{\ell_2 - 3\ell_1}{2}$

Intensity of sound in decibels

Sound level, $SL = 10\log_{10}\left(\frac{I}{I_0}\right)$

Where I_0 = threshold of human ear = 10^{-12} W/m^2

Characteristics of sound

- Loudness → Sensation received by the ear due to intensity of sound.
- Pitch → Sensation received by the ear due to frequency of sound.
- Quality (or Timbre)→ Sensation received by the ear due to waveform of sound.

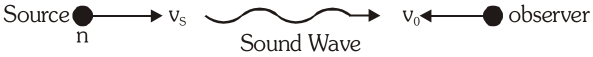
Doppler's effect in sound :

A stationary source emits wave fronts that propagate with constant velocity with constant separation between them and a stationary observer encounters them at regular constant intervals at which they were emitted by the source.

A moving observer will encounter more or lesser number of wavefronts depending on whether he is approaching or receding the source.

A source in motion will emit different wave front at different places and therefore alter wavelength i.e. separation between the wavefronts.

The apparent change in frequency or pitch due to relative motion of source and observer along the line of sight is called Doppler Effect.



Observed frequency

$$n' = \frac{\text{speed of sound wave w.r.t. observer}}{\text{observed wavelength}}$$

$$n' = \frac{v+v_0}{\left(\frac{v-v_s}{n}\right)} = \left(\frac{v+v_0}{v-v_s}\right)n$$

If v_0, v_s <<<v then $n' \approx \left(1+\frac{v_0+v_s}{v}\right)n$

- Mach Number=$\frac{\text{speed of source}}{\text{speed of sound}}$

Doppler's effect in light :

Case I : **Observer** ● O **Light Source** ← v S

$$\left.\begin{aligned} &\text{Frequency} \quad \nu' = \left(\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\right)\nu \approx \left(1+\frac{v}{c}\right)\nu \\ &\text{Wavelength} \quad \lambda' = \left(\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\right)\lambda \approx \left(1-\frac{v}{c}\right)\lambda \end{aligned}\right\}\text{Violet Shift}$$

Case II : **Observer** ● O **Light Source** → v S

$$\left.\begin{aligned} &\text{Frequency} \quad \nu' = \left(\sqrt{\frac{1-\frac{v}{c}}{1+\frac{v}{c}}}\right)\nu \approx \left(1-\frac{v}{c}\right)\nu \\ &\text{Wavelength} \quad \lambda' = \left(\sqrt{\frac{1+\frac{v}{c}}{1-\frac{v}{c}}}\right)\lambda \approx \left(1+\frac{v}{c}\right)\lambda \end{aligned}\right\}\text{Red Shift}$$

IMPORTANT NOTES

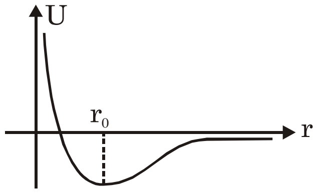
CHAPTER 11

PROPERTIES OF MATTER AND FLUID MECHANICS

(A) ELASTICITY

* **It is property of material to resist the deformation so steel is more elastic than rubber.**

*



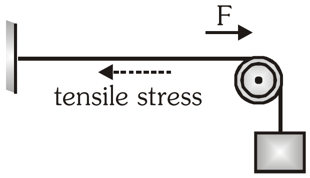
U = Potential energy,
r = inter atomic distance

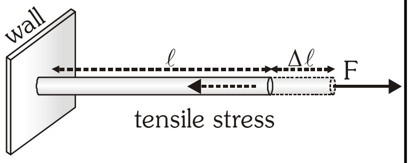
* **Stress** $= \dfrac{\text{Internal restoring force}}{\text{Area of cross-section}} = \dfrac{F_{Res}}{A}$.

There are three types of stress :-

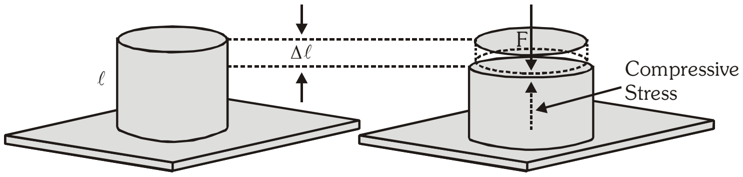
Longitudinal Stress

(a) Tensile Stress:

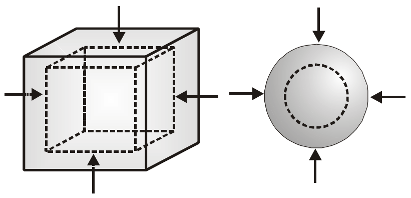




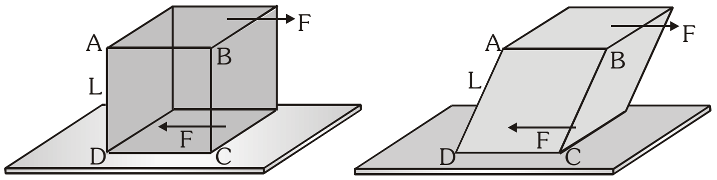
(b) Compressive Stress :



Volume Stress



Tangential Stress or Shear Stress



Strain $= \dfrac{\text{Change in size of the body}}{\text{Original size of the body}}$

Longitudinal strain

$$= \frac{\text{change in length of the body}}{\text{initial length of the body}} = \frac{\Delta L}{L}$$

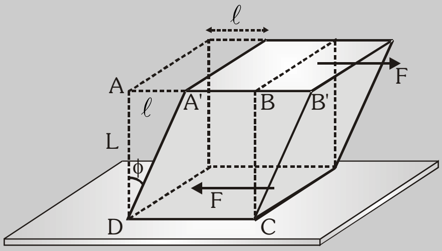
Volume strain

$$= \frac{\text{change in volume of the body}}{\text{original volume of the body}} = \frac{\Delta V}{V}$$

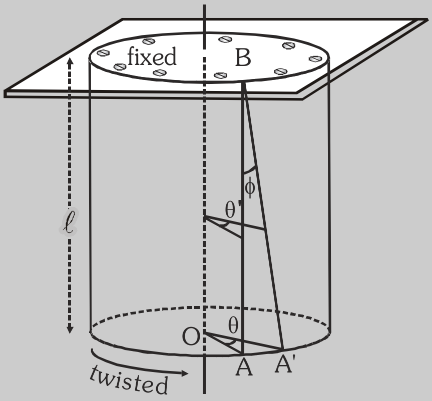
Shear strain

$$\tan\phi = \frac{\ell}{L} \text{ or}$$

$$\phi = \frac{\ell}{L} = \frac{\text{displacement of upper face}}{\text{distance between two faces}}$$



Relation between angle of twist (θ) & angle of shear (φ)

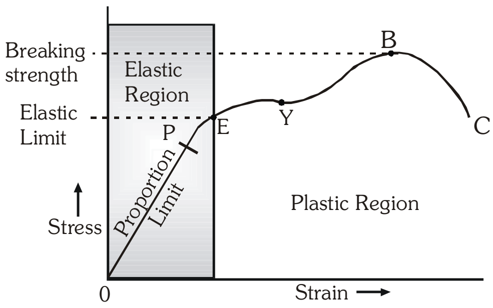


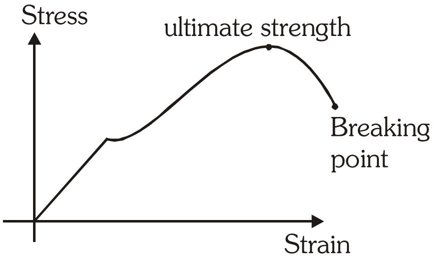
AA' = rθ and Arc AA' = ℓφ

So $r\theta = \ell\phi \Rightarrow \phi = \dfrac{r\theta}{\ell}$

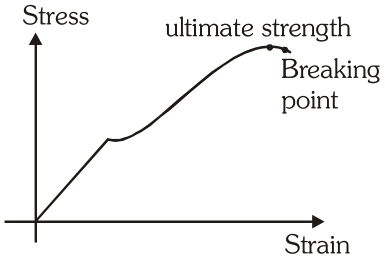
where θ = angle of twist, φ = angle of shear

Stress – Strain Graph





These type of materials are called ductile



These type of materials are called brittle materials

Hooke's Law

within elastic limit $\boxed{\text{Stress} \propto \text{strain}}$

- **Young's modulus of elasticity**

$$Y = \frac{\text{Longitudinal stress}}{\text{Longitudinal strain}} = \frac{F\ell}{A\,\Delta\ell}$$

- If L is the length of wire, r is radius and ℓ is the increase in length of the wire by suspending a weight Mg at its one end then Young's modulus of elasticity of the material of wire

$$Y = \frac{\left(Mg/\pi r^2\right)}{\left(\ell/L\right)} = \frac{MgL}{\pi r^2 \ell}$$

- **Increment in length due to own weight**

$$\Delta\ell = \frac{MgL}{2AY} = \frac{\rho g L^2}{2Y}$$

- **Bulk modulus of elasticity**

$$K = \frac{\text{Volume stress}}{\text{Volume strain}} = \frac{F/A}{\left(\frac{-\Delta V}{V}\right)} = \frac{P}{\left(\frac{-\Delta V}{V}\right)}$$

- **Bulk modulus of an ideal gas is process dependent.**
  - For isothermal process PV = constant

    $$\Rightarrow PdV + VdP = 0 \Rightarrow P = \frac{-dP}{dV/V}$$

    So bulk modulus = P
  - For adiabatic process PV^{γ} = constant

    $$\Rightarrow \gamma P V^{\gamma-1} dV + V^{\gamma} dP = 0$$

    $$\Rightarrow \gamma P dV + VdP = 0 \Rightarrow \gamma P = \frac{-dP}{dV/V};$$

    So bulk modulus = γP
  - For any polytropic process PV^n= constant

    $$\Rightarrow nPV^{n-1}dV + V^n dP = 0 \Rightarrow PdV + VdP = 0$$

    $$\Rightarrow nP = \frac{-dP}{dV/V}$$

    So bulk modulus = nP

Compressibility $C = \frac{1}{\text{Bulk modulus}} = \frac{1}{K}$

Modulus of rigidity

$$\eta = \frac{\text{shearing stress}}{\text{shearing strain}} = \frac{\left(F_{\text{tangential}}\right)/A}{\phi}$$

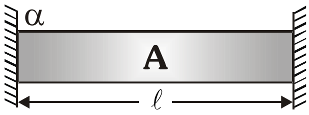
Poisson's ratio $(\sigma) = \frac{\text{lateral strain}}{\text{Longitudinal strain}}$

Work done in stretching wire

$$W = \frac{1}{2}\times\text{stress}\times\text{strain}\times\text{volume}:$$

$$W = \frac{1}{2}\times\frac{F}{A}\times\frac{\Delta\ell}{\ell}\times A\times\ell = \frac{1}{2}F\times\Delta\ell$$

Rod is rigidly fixed between walls



- Thermal Strain = $\alpha\Delta\theta$
- Thermal stress = $Y\alpha\Delta\theta$
- Thermal tension = $Y\alpha A\Delta\theta$

Effect of Temperature on elasticity

When temperature is increased then due to weakness of inter molecular force the elastic properties in general decreases i.e. elastic constant decreases. Plasticity increases with temperature. For example, at ordinary room temperature, carbon is elastic but at high temperature, carbon becomes plastic. Lead is not much elastic at room temperature but when cooled in liquid nitrogen exhibit highly elastic behaviour.

For a special kind of steel, elastic constants do not vary appreciably temperature. This steel is called 'INVAR steel'.

Effect of Impurity on elasticity

Y may increase or decrease depends upon type impurity.

(B) HYDROSTATICS

- **Density** $=\dfrac{\text{mass}}{\text{volume}}$

- **Specific weight** $=\dfrac{\text{weight}}{\text{volume}}\;\dfrac{\text{weight}}{\text{volume}}=\rho g$

- **Relative density** $=\dfrac{\text{density of given liquid}}{\text{density of pure water at }4°\text{C}}$

- **Density of a Mixture of substance in the proportion of mass**

the density of the mixture is $\rho=\dfrac{M_1+M_2+M_3....}{\dfrac{M_1}{\rho_1}+\dfrac{M_2}{\rho_2}+\dfrac{M_3}{\rho_3}+....}$

- **Density of a mixture of substance in the proportion of volume**

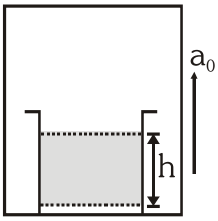
the density of the mixture is $\rho=\dfrac{\rho_1V_1+\rho_2V_2+\rho_3V_3}{V_1+V_2+V_3+....}$

- **Pressure** $=\dfrac{\text{normal force}}{\text{area}}$

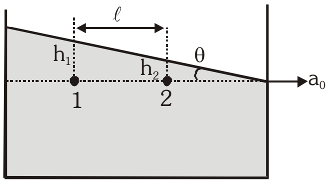
- **Variation of pressure with depth**
  Pressure is same at two points in the same horizontal level $P_1=P_2$
  The difference of pressure between two points separated by a depth h : $(P_2-P_1)=h\rho g$

Pressure in case of accelerating fluid

(i) Liquid placed in elevator: When elevator accelerates upward with acceleration a_0 then pressure in the fluid, at depth h may be given by, $P=h\rho[g+a_0]+P_0$



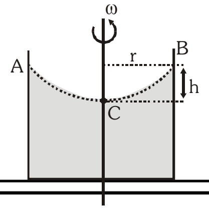
(ii) Free surface of liquid in case of horizontal acceleration:



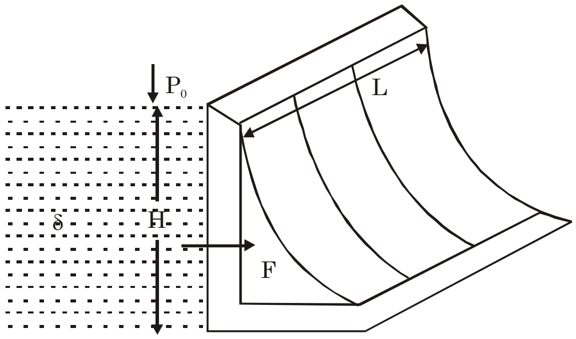
$$\tan\theta=\frac{ma_0}{mg}=\frac{a_0}{g}$$

If P_1 and P_2 are pressures at point 1 & 2 then $P_1-P_2=\rho g\,(h_1-h_2)=\rho g\ell\tan\theta\;=\rho\ell a_0$

(iii) Free surface of liquid in case of rotating cylinder



$$h = \frac{v^2}{2g} = \frac{\omega^2 r^2}{2g}$$



Force on the wall of dam = (Pressure of the centre) (contact area)
$(P_0 + \rho gH/2)$ (HL)

Pascal's Law

- The pressure in a fluid at rest is same at all the points if gravity is ignored.
- A liquid exerts equal pressures in all directions.
- If the pressure in an enclosed fluid is changed at a particular point, the change is transmitted to every point of the fluid and to the walls of the container without being diminished in magnitude. *[for ideal fluids]*

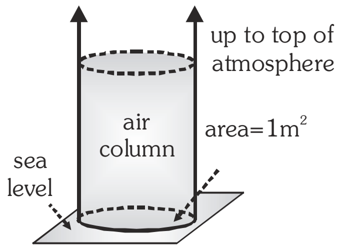
Types of Pressure :

Pressure is of three types

(i) Atmospheric pressure (P_0)
(ii) Gauge pressure (P_{gauge})
(iii) Absolute pressure ($P_{abs.}$)

◆ **Atmospheric pressure :**

Force exerted by air column on unit cross–section area of sea level called atmospheric pressure (P_o)



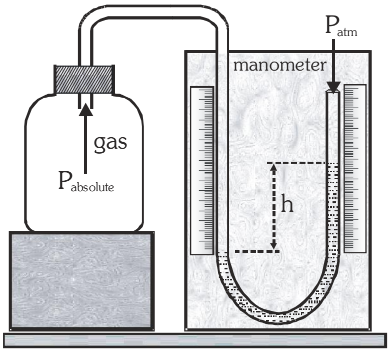
$$P_o = \frac{F}{A} = 101.3 \text{ kN/m}^2 = 1.013 \times 10^5 \text{ N/m}^2$$

Barometer is used to measure atmospheric pressure.
Which was discovered by **Torricelli**.
Atmospheric pressure varies from place to place and at a particular place from time to time.

- **Gauge Pressure :**

  Excess Pressure ($P - P_{atm}$) measured with the help of pressure measuring instrument called Gauge pressure.

$$P_{gauge} = h\rho g \text{ or } P_{gauge} \propto h$$



Gauge pressure is always measured with help of **"manometer"**

- **Absolute Pressure :**

  Sum of atmospheric and Gauge pressure is called absolute pressure.

$$P_{abs} = P_{atm} + P_{gauge} \Rightarrow P_{abs} = P_o + h\rho g$$

The pressure which we measure in our automobile tyres is gauge pressure.

◆ **Buoyant force**

Weight of displaced fluid = $V\rho g$

◆ **Apparent weight**

Weight – Upthrust

◆ **Rotatory – Equilibrium in Floatation :**

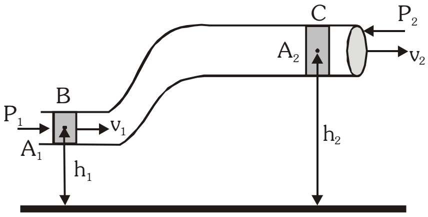
For rotational equilibrium of floating body the meta–centre must always be higher than the centre of gravity of the body.

◆ **Relative density of body** $= \dfrac{\text{Density of body}}{\text{Density of water}}$

(C) HYDRODYNAMICS

- **Steady and Unsteady Flow :** *Steady flow* is defined as that type of flow in which the fluid characteristics like velocity, pressure and density at a point do not change with time.
- **Streamline Flow :** In steady flow all the particles passing through a given point follow the same path and hence a unique line of flow. This line or path is called a *streamline*.
- **Laminar and Turbulent Flow :** *Laminar flow* is the flow in which the fluid particles move along well–defined streamlines which are straight and parallel.
- **Compressible and Incompressible Flow :** In *compressible flow* the density of fluid varies from point to point i.e. the density is not constant for the fluid whereas in *incompressible flow* the density of the fluid remains constant through out.
- **Rotational and Irrotational Flow :** *Rotational flow* is the flow in which the fluid particles while flowing along path–lines also rotate about their own axis. In *irrotational flow* particles do not rotate about their axis.
- **Equation of continuity** $\rho A_1v_1 = \rho A_2v_2 \Rightarrow A_1v_1 = A_2v_2$(if ρ = constant) Based on conservation of mass
- **Bernoulli's theorem** :

$\boxed{P+\frac{1}{2}\rho v^2+\rho gh = \text{constant}}$ Based on energy conservation

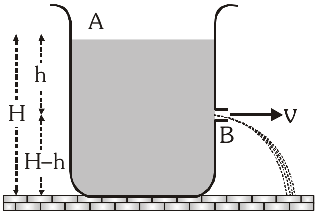


- **Kinetic Energy :**

Kinetic energy per unit volume $= \frac{\text{Kinetic Energy}}{\text{volume}} = \frac{1}{2}\frac{m}{V}v^2 = \frac{1}{2}\rho v^2$

- **Potential Energy :** Potential energy per unit volume $= \frac{\text{Potential Energy}}{\text{volume}} = \frac{m}{V}gh = \rho gh$
- **Pressure Energy :** Pressure energy per unit volume $= \frac{\text{Pressure energy}}{\text{volume}} = P$
- **For horizontal flow in venturimeter** : $P_1+\frac{1}{2}\rho v_1^2 = P_2+\frac{1}{2}\rho v^2 \Rightarrow v_1 = A_2\sqrt{\frac{2gh}{A_1^2-A_2^2}}$
- **Rate of flow :**

Volume of water flowing per second $: Q = A_1v_1 = A_1A_2\sqrt{\frac{2gh}{A_1^2-A_2^2}}$



- **Velocity of efflux** : $v = \sqrt{2gh}$
- **Horizontal range** : $R = 2\sqrt{h(H-h)}$

(D) SURFACE TENSION

Surface tension is basically a property of liquid. The liquid surface behaves like a stretched elastic membrane which has a natural tendency to contract and tends to have a minimum surface area. This property of liquid is called *surface tension*.

Intermolecular forces

(a) **Cohesive force** : The force acting between the molecules of one type of molecules of same substance is called cohesive force.

(b) **Adhesive force :** The force acting between different types of molecules or molecules of different substance is called adhesive force.

- ❐ Intermolecular forces are different from the gravitational forces and do not obey the inverse–square law
- ❐ The distance upto which these forces effective, is called molecular range. This distance is nearly 10^{-9} m. Within this limit this increases very rapidly as the distance decreases.
- ❐ Molecular range depends on the nature of the substance

Properties of surface tension

- Surface tension is a scalar quantity.
- It acts tangential to liquid surface.
- Surface tension is always produced due to cohesive force.
- More is the cohesive force, more is the surface tension.
- When surface area of liquid is increased, molecules from the interior of the liquid rise to the surface. For this, work is done against the downward cohesive force.

Dependency of Surface Tension

- **On Cohesive Force :** Those factors which increase the cohesive force between molecules increase the surface tension and those which decrease the cohesive force between molecules decrease the surface tension.
- **On Impurities :** If the impurity is completely soluble then on mixing it in the liquid, its surface tension increases. e.g., on dissolving ionic salts in small quantities in a liquid, its surface tension increases. If the impurity is partially soluble in a liquid then its surface tension decreases because adhesive force between insoluble impurity molecules and liquid molecules decreases cohesive force effectively, e.g.

  (a) On mixing detergent in water its surface tension decreases.

  (b) Surface tension of water is more than (alcohol + water) mixture.
- **On Temperature** : On increasing temperature surface tension decreases. At critical temperature and boiling point it becomes zero. **Note :** Surface tension of water is maximum at 4°C
- **On Contamination** : The dust particles or lubricating materials on the liquid surface decreases its surface tension.
- **On Electrification** :The surface tension of a liquid decreases due to electrification because a force starts acting due to it in the outward direction normal to the free surface of liquid.

Definition of surface tension

The force acting per unit length of an imaginary line drawn on the free liquid surface at right angles to the line and in the plane of liquid surface, is defined as surface tension.

❐ For floating needle $2T\ell \sin\theta = mg$

◆ **Required excess force for lift**

- ❐ Wire $F_{ex} = 2T\ell$
- ❐ Hollow disc $F_{ex} = 2\pi T(r_1 + r_2)$
- ❐ For ring $F_{ex} = 4\pi rT$
- ❐ Circular disc $F_{ex} = 2\pi rT$
- ❐ Square frame $F_{ex} = 8aT$
- ❐ Square plate $F_{ex} = 4aT$

◆ **Work** = surface energy = $T\Delta A$

- ❐ Liquid drop $W = 4\pi r^2 T$
- ❐ Soap bubble $W = 8\pi r^2 T$

◆ **Splitting of bigger drop into smaller droples** $R = n^{1/3} r$

Work done= Change in surface energy

$$= 4\pi R^3 T\left(\frac{1}{r} - \frac{1}{R}\right) = 4\pi R^2 T\,(n^{1/3} - 1)$$

◆ **Excess pressure** $P_{ex} = P_{in} - P_{out}$

- ❐ In liquid drop : $P_{ex} = \dfrac{2T}{R}$
- ❐ In soap bubble : $P_{ex} = \dfrac{4T}{R}$

ANGLE OF CONTACT (θ_C)

The angle enclosed between the tangent plane at the liquid surface and the tangent plane at the solid surface at the point of contact inside the liquid is defined as the *angle of contact.*

The angle of contact depends the nature of the solid and liquid in contact.

- **Angle of contact** $\theta < 90° \Rightarrow$ concave shape, Liquid rise up
  Angle of contact $\theta > 90° \Rightarrow$ convex shape, Liquid falls
  Angle of contact $\theta = 90° \Rightarrow$ plane shape, Liquid neither rise nor falls
- **Effect of Temperature on angle of contact**
  On increasing temperature surface tension decreases, thus $\cos\theta_c$ increases $\left[\because \cos\theta_c \propto \frac{1}{T}\right]$ and θ_c decrease. So on increasing temperature, θ_c decreases.
- **Effect of Impurities on angle of contact**

(a) Solute impurities increase surface tension, so $\cos\theta_c$ decreases and angle of contact θ_c increases.

(b) Partially solute impurities decrease surface tension, so angle of contact θ_c decreases.

- **Effect of Water Proofing Agent**
  Angle of contact increases due to water proofing agent. It gets converted acute to obtuse angle.

- **Capillary rise** $h = \dfrac{2T\cos\theta}{r\rho g}$
  - Zurin's law $h \propto \dfrac{1}{r}$
  - Jeager's method $T = \dfrac{rg}{2}(H\rho - hd)$
  - The height 'h' is measured from the bottom of the meniscus. However, there exist some liquid above this line also. If correction of this is applied then the formula will be

$$T = \frac{r\rho g\left[h + \frac{1}{3}r\right]}{2\cos\theta}$$

  - If height of capillary is injufficicnt, then liquid does not come out of the tube. And radius of curvature of maniscus is given by :
    hR = h'R'
    h = calculated height
    h' = actual height
    R = Calculated radius of curvature of mainscus
    R' = actual radius of curvature of mainscus

- When two soap bubbles are in contact then $r = \dfrac{r_1 r_2}{r_1 - r_2}$ $(r_1 > r_2)$ radius of curvature of the common surface
- When two soap bubbles are combining to form a new bubble then radius of new bubble $r = \sqrt{r_1^2 + r_2^2}$
- Force required to separate two plates $F = \dfrac{2AT}{d}$

(E) VISCOSITY

Newton's law of viscosity:

$$F = \eta A \frac{\Delta v_x}{\Delta y}$$

- **SI UNITS :** $\dfrac{N \times s}{m^2}$ or deca poise
- **CGS UNITS :** dyne–s/cm^2 or poise (1 decapoise = 10 poise)

Dependency of viscosity of fluids

On Temperature of Fluid

(a) Since cohesive forces decrease with increase in temperature as increase in K.E.. Therefore with the rise in temperature, the viscosity of liquids decreases.

(b) The viscosity of gases is the result of diffusion of gas molecules from one moving layer to other moving layer. Now with increase in temperature, the rate of diffusion increases. So, the viscosity also increases. Thus, the viscosity of gases increases with the rise of temperature.

On Pressure of Fluid

(a) The viscosity of liquids increases with the increase of pressure.

(b) The viscosity of gases is practically independent of pressure.

On Nature of Fluid

- **Poiseuillel's formula**

$$Q = \frac{dV}{dt} = \frac{\pi p r^4}{8\eta L}$$

- **Viscous force on spherical body (stokes' law)** $F_v = 6\pi\eta r v$
- **Terminal velocity**

$$v_T = \frac{2}{9}\frac{r^2(\rho - \sigma)g}{\eta} \Rightarrow v_T \propto r^2$$

- **Reynolds number** $R_e = \dfrac{\rho v d}{\eta}$

  $R_e < 1000$ laminar flow,

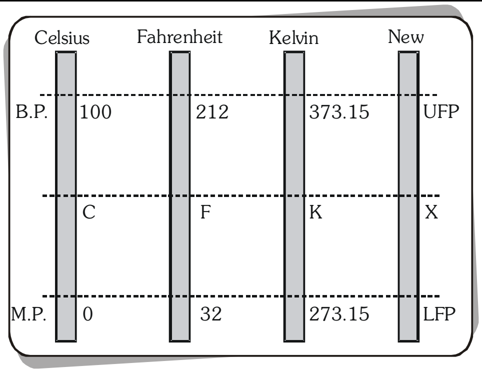
  $1000 < R_e < 2000$ may be steady or may be turbulant

  $R_e > 3000$ turbulant flow.

THERMAL PHYSICS

TEMPERATURE SCALES AND THERMAL EXPANSION

Name of the scale	Symbol for each degree	Lower fixed point (LFP)	Upper fixed point (UFP)	Number of divisions on the scale
Celsius	°C	0°C	100°C	100
Fahrenheit	°F	32°F	212°F	180
Kelvin	K	273.15 K	373.15 K	100

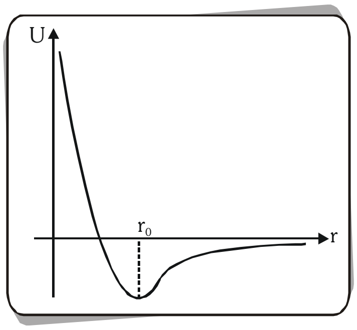


$$\frac{C-0}{100-0}=\frac{F-32}{212-32}=\frac{K-273.15}{373.15-273.15}=\frac{X-LFP}{UFP-LFP}\Rightarrow\frac{\Delta C}{100}=\frac{\Delta F}{180}=\frac{\Delta K}{100}=\frac{\Delta X}{UFP-LFP}$$

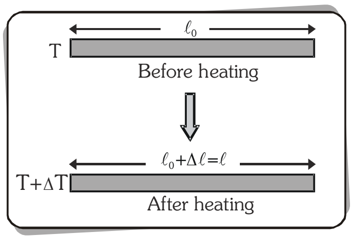
- **Old thermometry** : $\frac{\theta-0}{100-0}=\frac{X-X_0}{X_{100}-X_0}$ [two fixed points –ice & steam points] where X is thermometric property i.e. length, resistance etc.
- **Modern thermometry** : $\frac{T-0}{273.16-0}=\frac{X}{X_{tr}}$ [Only one reference point–triple point of water is chosen]

THERMAL EXPANSION

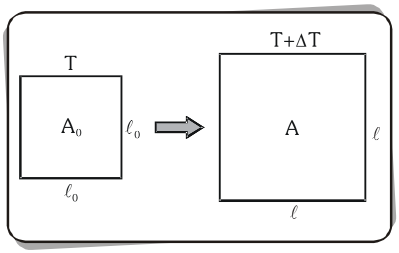
It is due to asymmetry in potential energy curve.



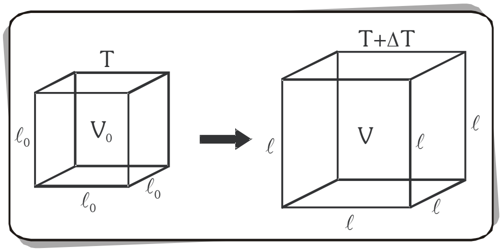
In solids → Linear expansion $\ell=\ell_0(1+\alpha\Delta T)$



In solids → Areal expansion $A=A_0(1+\beta\Delta T)$



In solids, liquids and gases → Volume expansion $V=V_0(1+\gamma\Delta T)$



[For isotropic solids : $\alpha : \beta : \gamma = 1 : 2 : 3$]

Thermal expansion of an isotropic object may be imagined as a photographic enlargement.

For anisotropic materials $\beta_{xy}=\alpha_x+\alpha_y$ and $\gamma=\alpha_x+\alpha_y+\alpha_z$

If α is variable : $\Delta\ell=\int_{T_1}^{T_2}\ell_0\alpha dT$

Measurement of length by metallic scale

Measured value at temp θ_2 °C,

$MV=\ell_a\{1+(\alpha_0-\alpha_s)(\theta_2-\theta_1)\}$

where,

ℓ_a = actual length of object at θ_1 °C

α_0 = linear expansion coefficient of object.

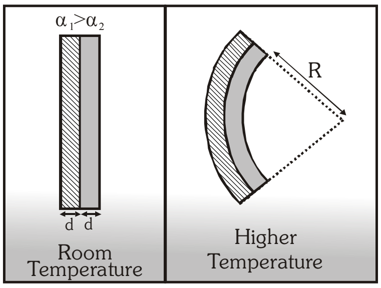
α_s = linear expansion coefficient of scale.

(i) if $\alpha_0>\alpha_s$, then MV is more than ℓ_a

(ii) if $\alpha_0<\alpha_S$, then MV is less than ℓ_a

Application of Thermal expansion in solids

I. Bi-metallic strip *(used as thermostat or auto-cut in electric heating circuits)* $R = \dfrac{d}{(\alpha_1 - \alpha_2)\Delta T}$



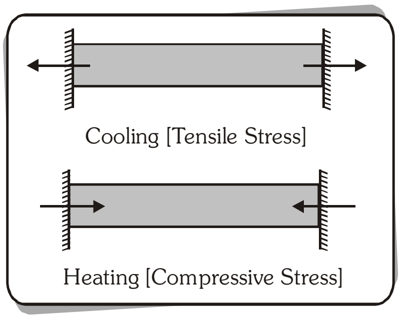
II. Simple pendulum :

$$T = 2\pi\sqrt{\frac{\ell}{g}} \Rightarrow T \propto \ell^{1/2} \Rightarrow \frac{\Delta T}{T} = \frac{1}{2}\frac{\Delta \ell}{\ell}$$

Fractional change in time period $= \dfrac{\Delta T}{T} = \dfrac{1}{2}\alpha\Delta\theta$

III. Scale reading : Due to linear expansion / contraction, scale reading will be lesser / more than actual value.
If temperature ↑ then actual value = scale reading $(1+\alpha\Delta\theta)$

IV. Thermal Stress



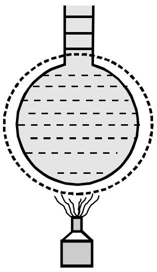
Thermal strain $= \dfrac{\Delta \ell}{\ell} = \alpha\Delta\theta$

As Young's modulus $Y = \dfrac{F/A}{\Delta\ell/\ell}$;

So thermal stress = $YA\alpha\Delta\theta = \dfrac{YA\alpha\Delta\theta}{(1+\alpha\Delta\theta)}$

Thermal expansion in liquids

(Only volume expansion)



$$\gamma_a = \frac{\text{Apparent increase in volume}}{\text{Initial volume} \times \text{Temperature rise}}$$

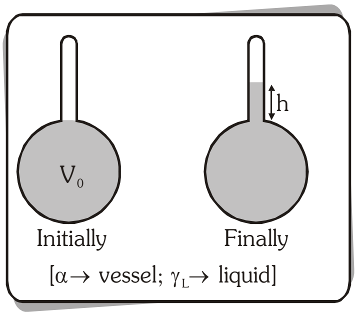
$$\gamma_r = \frac{\text{real increse in volume}}{\text{initial volume} \times \text{temperature rise}}$$

$$\gamma_r = \gamma_a + \gamma_{\text{vessel}}$$

Change in volume of liquid w.r.t. vessel

$$\Delta V = V_0(\gamma_r - 3\alpha)\Delta T$$

Expansion in enclosed volume


V_0
h
Initially
Finally
[$\alpha \rightarrow$ vessel; $\gamma_L \rightarrow$ liquid]

Increase in height of liquid level in tube when bulb was initially completely filled.

$$h = \frac{\begin{array}{c}\text{apparent change}\\ \text{in volume of liquid}\end{array}}{\text{area of tube}} = \frac{V_0(\gamma_L - 3\alpha)\Delta T}{A_0(1+2\alpha)\Delta T}$$

Anomalous expansion of water :
In the range 0°C to 4 °C water contract on heating and expands on cooling. At 4°C → density is maximum.
Aquatic life is able to survive in very cold countries as the lake bottom remains unfrozen at the temperature around 4°C.

Thermal expansion of gases :

- Coefficient of volume expansion : $\gamma_V = \dfrac{\Delta V}{V_0 \Delta T} = \dfrac{1}{T}$

  [$PV = nRT$ at constant pressure $V \propto T \Rightarrow \dfrac{\Delta V}{V} = \dfrac{\Delta T}{T}$]

- Coefficient of pressure expansion $\gamma_P = \dfrac{\Delta P}{P_0 \Delta T} = \dfrac{1}{T}$

KEY POINTS :

- Liquids usually expand more than solids because the intermolecular forces in liquids are weaker than in solids.
- Rubber contract on heating because in rubber as temperature increases, the amplitude of transverse vibrations increases more than the amplitude of longitudinal vibrations.
- Water expands both when heated or cooled from 4°C because volume of water at 4°C is minimum.
- In cold countries, water pipes sometimes burst, because water expands on freezing.

CALORIMETRY

$1 \text{ cal} = 4.186 \text{ J} \simeq 4.2 \text{ J}$

- **Thermal capacity of a body** $= \dfrac{Q}{\Delta T}$

  Amount of heat required to raise the temperature of a given body by 1°C (or 1K).

- **Specific heat capacity** $= \dfrac{Q}{m\Delta T}$ (m = mass)

  Amount of heat required to raise the temperature of unit mass of a body through 1°C (or 1K)

- **Molar heat capacity**$= \dfrac{Q}{n\Delta T}$ (n=number of moles)

- **Water equivalent :** If thermal capacity of a body is expressed in terms of mass of water, it is called water equivalent. Water equivalent of a body is the mass of water which when given same amount of heat as to the body, changes the temperature of water through same range as that of the body. Therefore water equivalent of a body is the quantity of water, whose heat capacity is the same as the heat capacity of the body.

  Water equivalent of the body,

  $$W = \text{mass of body} \times \left(\frac{\text{specific heat of body}}{\text{specific heat of water}}\right)$$

  Unit of water equivalent is g or kg.

- **Latent Heat (Hidden heat) :** The amount of heat that has to supplied to (or removed from) a body for its complete change of state (from solid to liquid, liquid to gas etc) is called latent heat of the body. Remember that phase transformation is an isothermal (i.e. temperature = constant) change.

- **Principle of calorimetry** :

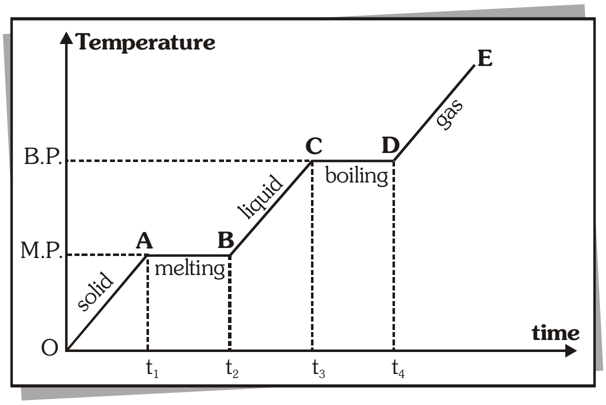
  Heat lost = heat gained

  For temperature change $Q = ms\Delta T$,

  For phase change $Q = mL$

- **Heating curve :**

  If to a given mass (m) of a solid, heat is supplied at constant rate (Q) and a graph is plotted between temperature and time, the graph is called heating curve.



Specific heat $\propto \dfrac{1}{\text{slope of curve}}$

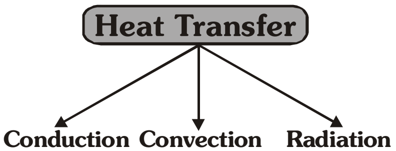
(or thermal capacity)

Latent heat ∝ length of horizontal line.

KEY POINTS

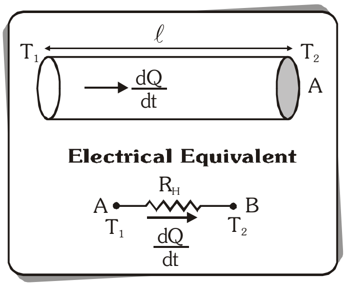
- Specific heat of a body may be greater than its thermal capacity as mass of the body may be less than unity.
- The steam at 100°C causes more severe burn to human body than the water at 100°C because steam has greater internal energy than water due to latent heat of vaporization.
- Heat is energy in transit which is transferred from hot body to cold body.
- One calorie is the amount of heat required to raise the temperature of one gram of water through 1°C (more precisely from 14.5°C to 15.5°C).
- Clausins & clapeyron equation (effect of pressure on boiling point of liquids & melting point of solids related with latent heat) $\dfrac{dP}{dT} = \dfrac{L}{T(V_2 - V_1)}$

THERMAL CONDUCTION



In conduction, heat is transferred from one point to another without the actual motion of heated particles. In the process of convection, the heated particles of matter actually move. In radiation, intervening medium is not affected and heat is transferred without any material medium.

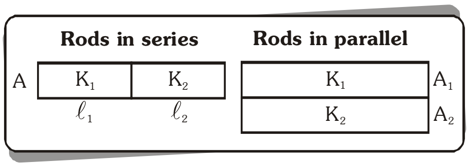
Conduction	***Convection***	***Radiation***
Heat Transfer due to Temperature difference	Heat transfer due to density difference	Heat transfer with out any medium
Due to free electron or vibration motion of molecules	Actual motion of particles	Electromagnetic radiation
Heat transfer in solid body (in mercury also)	Heat transfer in fluids (Liquid+gas)	All
Slow process	Slow process	Fast process (3×10^8 m/sec)
Irregular path	Irregular path	Straight line (like light)



Rate of heat flow

$$\frac{dQ}{dt} = -KA\frac{dT}{dx} \quad \text{or} \quad \frac{Q}{t} = \frac{KA(T_1 - T_2)}{\ell}$$

Thermal resistance $R_H = \dfrac{\ell}{KA}$

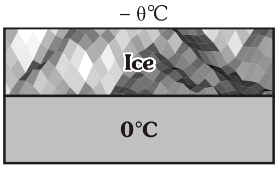


$$K_{eq} = \frac{\Sigma\ell}{\Sigma\ell/K} \quad ; \quad K_{eq} = \frac{\Sigma KA}{\Sigma A}$$

Growth of Ice on Ponds

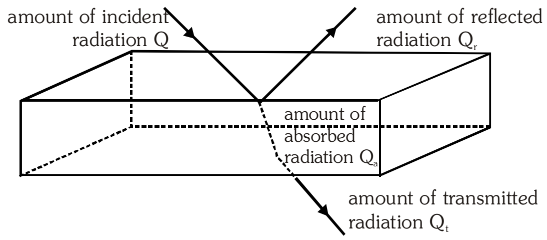
Time taken by ice to grow a thickness from x_1 to x_2: $t = \dfrac{\rho L}{2K\theta}\left(x_2^2 - x_1^2\right)$

[K=thermal conductivity of ice, ρ=density of ice]



RADIATION

- **Spectral, emissive, absorptive and transmittive power of a given body surface:**
  Due to incident radiations on the surface of a body following phenomena occur by which the radiation is divided into three parts.
  (a) Reflection (b) Absorption (c) Transmission


amount of incident radiation Q
amount of reflected radiation Q_r
amount of absorbed radiation Q_a
amount of transmitted radiation Q_t

From energy conservation

$$Q = Q_r + Q_a + Q_t \Rightarrow \frac{Q_r}{Q} + \frac{Q_a}{Q} + \frac{Q_t}{Q} = 1$$
$$\Rightarrow r + a + t = 1$$

- Reflective Coefficient : $r = \dfrac{Q_r}{Q}$
- Absorptive Coefficient : $a = \dfrac{Q_a}{Q}$
- Transmittive Coefficient : $t = \dfrac{Q_t}{Q}$

  r = 1 and a = 0 , t = 0 ⇒ Perfect reflector
  a = 1 and r = 0, t = 0 ⇒ Ideal absorber (ideal black body)
  t = 1 and a = 0, r = 0 ⇒ Perfect transmitter (daithermanous)

  Reflection power (r) $= \left[\dfrac{Q_r}{Q} \times 100\right]\%$

  Absorption power (a) $= \left[\dfrac{Q_a}{Q} \times 100\right]\%$

  Transmission power (t) $= \left[\dfrac{Q_t}{Q} \times 100\right]\%$

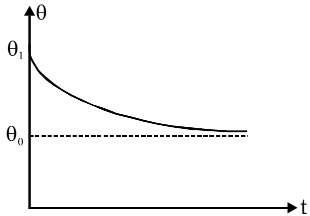
- **Stefan's Boltzmann law :**
  Radiated energy emitted by a perfect black body per unit area/sec $E = \sigma T^4$
  For a general body $E = \sigma e_r T^4$ [where $0 \le e_r \le 1$]
- **Prevost's theory of heat exchange :**
  A body is simultaneously emitting radiations to its surrounding and absorbing radiations from the surroundings. If surrounding has temperature T_0 then $E_{net} = e_r\sigma(T^4 - T_0^4)$

- **Kirchhoff's law :**
  The ratio of emissive power to absorptive power is same for all surfaces at the same temperature and is equal to the emissive power of a perfectly black body at that temperature.

$$\frac{e}{a}=\frac{E}{A}=\frac{E}{1}\Rightarrow\frac{e}{a}=E\Rightarrow e\propto a$$

Therefore a good absorber is a good emitter.

- **Perfectly Black Body :**
  A body which absorbs all the radiations incident on it is called a perfectly black body.
- **Absorptive Power (a) :**
  Absorptive power of a surface is defined as the ratio of the radiant energy absorbed by it in a given time to the total radiant energy incident on it in the same time.
  For ideal black body, absorptive power =1
- **Emissive power(e) :**
  For a given surface it is defined as the radiant energy emitted per second per unit area of the surface.
- **Newton's law of cooling:**



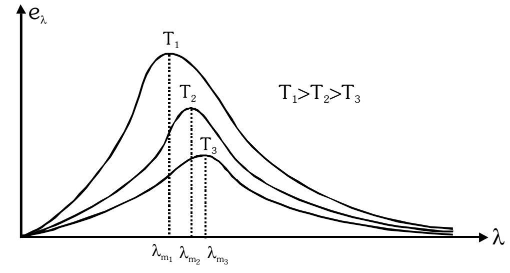
If temperature difference is small
Rate of cooling

$$\frac{d\theta}{dt}\propto\left(\theta-\theta_0\right)\Rightarrow\theta=\theta_0+\left(\theta_1-\theta_0\right)e^{-kt}$$

[where k = constant]
when a body cools from θ_1 to θ_2 in time 't' in a surrounding of temperature θ_0 then

$$\frac{\theta_1-\theta_2}{t}=k\left[\frac{\theta_1+\theta_2}{2}-\theta_0\right]\text{ [where k= constant]}$$

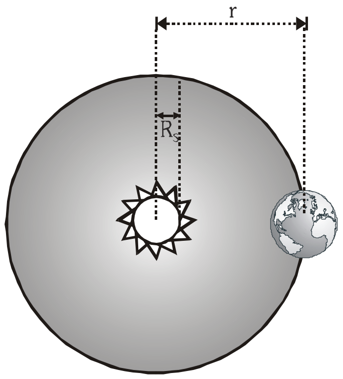
- **Wien's Displacement Law :**
  Product of the wavelength λ_m of most intense radiation emitted by a black body and absolute temperature of the black body is a constant
  $\lambda_m T=b = 2.89 \times 10^{-3}$ mK = Wein's constant



Area under $e_\lambda-\lambda$ graph$=\int_0^\infty e_\lambda d\lambda=e=\sigma T^4$

Solar constant

The Sun emits radiant energy continuously in space of which an insignificant part reaches the Earth. The solar radiant energy received per unit area per unit time by a black surface held at right angles to the Sun's rays and placed at the mean distance of the Earth (in the absence of atmosphere) is called solar constant.



$$S=\frac{P}{4\pi r^2}=\frac{4\pi R_S^2\sigma T^4}{4\pi r^2}=\sigma\left(\frac{R_S}{r}\right)^2T^4$$

where R_S = radius of sun
r = average distance between sun and earth.

Note :- S = 2 cal $cm^{-2}min$=1.4 kWm^{-2}
T = temperature of sun ≈ 5800 K

KEY POINTS

- Stainless steel cooking pans are preferred with extra copper bottom because thermal conductivity of copper is more than steel.
- Two layers of cloth of same thickness provide warmer covering than a single layer of cloth of double the thickness because air (which is better insulator of heat) is trapped between them.
- Animals curl into a ball when they feel very cold to reduce the surface area of the body.
- Water cannot be boiled inside a satellite by convection because in weightlessness conditions, natural movement of heated fluid is not possible.
- Metals have high thermal conductivity because metals have free electrons.

KINETIC THEORY OF GASES

It related the macroscopic properties of gases to the microscopic properties of gas molecules.

Basic postulates of Kinetic theory of gases

- Every gas consists of extremely small particles known as molecules. The molecules of a given gas are all identical but are different than those another gas.
- The molecules of a gas are identical, spherical, rigid and perfectly elastic point masses.
- The size is negligible in comparision to inter molecular distance (10^{-9} m)

Assumptions regarding motion :

- Molecules of a gas keep on moving randomly in all possible direction with all possible velocities.
- The speed of gas molecules lie between zero and infinity (very high speed).
- The number of molecules moving with most probable speed is maximum.

Assumptions regarding collision:

- The gas molecules keep colliding among themselves as well as with the walls of containing vessel. These collision are perfectly elastic. (ie., the total energy before collision = total energy after the collisions.)

Assumptions regarding force:

- No attractive or repulsive force acts between gas molecules.
- Gravitational attraction among the molecules is ineffective due to extremely small masses and very high speed of molecules.

Assumptions regarding pressure:

- Molecules constantly collide with the walls of container due to which their momentum changes. This change in momentum is transferred to the walls of the container. Consequently pressure is exerted by gas molecules on the walls of container.

Assumptions regarding density:

- The density of gas is constant at all points of the container.

Kinetic interpretation of pressure :

$$PV = \frac{1}{3} mNv_{rms}^2$$

[m = mass of a molecule, N = no. of molecules]

Ideal gas equation

$$PV = \mu RT \Rightarrow P = \frac{\mu RT}{V} = \frac{\mu N_A kT}{V} = \left(\frac{N}{V}\right) kT = nkT$$

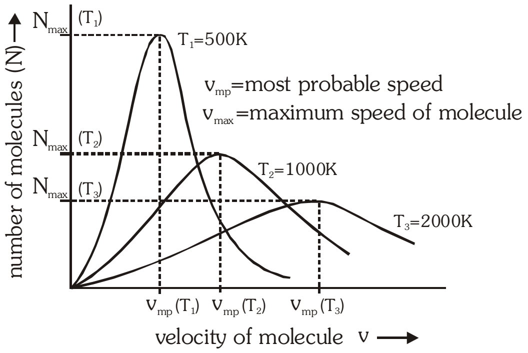
Gas laws

- ☐ **Boyle's law** :For a given mass at constant temperature. $V \propto \frac{1}{P}$
- ☐ **Charles' law** : For a given mass at constant pressure $V \propto T$
- ☐ **Gay–Lussac's law** For a given mass at constant volume $P \propto T$
- ☐ **Avogadro's law**:If P,V & T are same then no. of molecules $N_1 = N_2$

Different speeds of molecules

$$v_{rms} = \sqrt{\frac{3RT}{M_W}} = \sqrt{\frac{3kT}{m}} \ ; \ v_{mp} = \sqrt{\frac{2RT}{M_W}} = \sqrt{\frac{2kT}{m}}$$

$$v_{av} = \sqrt{\frac{8RT}{\pi M_W}} = \sqrt{\frac{8kT}{\pi m}}$$



Kinetic Interpretation of Temperature :

Temperature of an ideal gas is proportional to the average KE of molecules,

$$PV = \frac{1}{3} mNV_{rms}^2 \ \& \ PV = \mu RT \Rightarrow \frac{1}{2} mv_{rms}^2 = \frac{3}{2} kT$$

Degree of Freedom (F) :

Number of minimum coordinates required to specify the dynamical state of a system.

At higher temperature, diatomic molecules have two more degree of freedom due to vibrational motion (one for KE + one for PE)

At higher temperature diatomic gas has f = 7

Maxwell's Law of equipartition of energy:

Kinetic energy associated with each degree of freedom of particles of an ideal gas is equal to $\frac{1}{2}kT$

- Average KE of a particle having f degree of freedom= $\frac{f}{2}kT$
- Translational KE of a molecule = $\frac{3}{2}kT$
- Translational KE of a mole = $\frac{3}{2}RT$
- Internal energy of an ideal gas: $U = \frac{f}{2}\mu RT$

Specific heats (C_P and C_V) :

- Molar specific heat of a gas $C = \frac{dQ}{\mu dT}$
- $C_V = \left(\frac{dQ}{\mu dT}\right)_{V=\text{constant}} = \frac{dU}{\mu dT}$
- $C_P = \left(\frac{dQ}{\mu dT}\right)_{dP=0} = C_V + R$ ← Mayer's equation

Atomicity	**Translational**	**Rotational**	**Total (f)**	$\gamma = \frac{C_P}{C_V}$	$C_V = \frac{f}{2}R$	$C_P = C_V + R$
Monoatomic [He, Ar, Ne…]	3	0	3	$\frac{5}{3} = 1.67$	$\frac{3}{2}R$	$\frac{5}{2}R$
Diatomic [H_2, N_2..]	3	2	5	$\frac{7}{5} = 1.4$	$\frac{5}{2}R$	$\frac{7}{2}R$
Triatomic (Linear CO_2)	3	2	5	$\frac{7}{5} = 1.4$	$\frac{5}{2}R$	$\frac{7}{2}R$
Triatomic Non-linear-NH_3) & Polyatomic	3	3	6	$\frac{4}{3} = 1.33$	3R	4R

For mixture of non-reacting gases

Molecular weight : $M_{W_{mix}} = \frac{\mu_1 M_{W_1} + \mu_2 M_{W_2} + \ldots.}{\mu_1 + \mu_2 + \ldots.}$

Specific heat at constant V: $C_{V_{mix}} = \frac{\mu_1 C_{V_1} + \mu_2 C_{V_2} + \ldots..}{\mu_1 + \mu_2 + \ldots.}$

Specific heat at constant P: $C_{P_{mix}} = \frac{\mu_1 C_{P_1} + \mu_2 C_{P_2} + \ldots\ldots}{\mu_1 + \mu_2 + \ldots\ldots}$

$$\gamma_{mix} = \frac{C_{P_{mix}}}{C_{V_{mix}}} = \frac{\mu_1 C_{P_1} + \mu_2 C_{P_2} + \ldots\ldots}{\mu_1 C_{V_1} + \mu_2 C_{V_2} + \ldots..}$$

KEY POINTS

- Kinetic energy per unit volume

$$E_V = \frac{1}{2}\left(\frac{mN}{V}\right)v_{rms}^2 = \frac{3}{2}P$$

- At absolute zero, the motion of all molecules of the gas stops.
- At higher temperature and low pressure or at higher temperature and low density, a real gas behaves as an ideal gas.
- **For any general process**

(a) Internal energy change $\Delta U = nC_V dT$

(b) Heat supplied to a gas $\Delta Q = nCdT$

where C for any polytropic process

PV^x = constant is $C = C_V + \frac{R}{1-x}$

(c) Work done for any process $\Delta W = P\Delta V$

It can be calculated as area under P-V curve

(d) Work done = $\Delta Q - \Delta U = \frac{nR}{1-x}dT$

For any polytropic process PV^x = constant

THERMODYNAMICS

- **Zeroth law of thermodynamics :** If two systems are each in thermal equilibrium with a third, they are also in thermal equilibrium with each other.
- **First law of thermodynamics :** Heat supplied (ΔQ) to a system is equal to algebraic sum of change in internal energy (ΔU) of the system and mechanical work (W) done by the system

  $$\Delta Q = W + \Delta U \quad [\text{Here } W = \int P dV \ ; \ \Delta U = nC_V \Delta T]$$

  For differential change $$\boxed{\underbrace{\delta Q = \delta W}_{\text{path dependent}} + \underbrace{dU}_{\text{path independent}}}$$

  Area between P-V curve & V-axis gives work done by gas from one state to another state.
- **Sign Convention**
  Heat absorbed by the system $\rightarrow$positive
  Heat rejected by the system $\rightarrow$ negative
  Increase in internal energy
  (i.e. rise in temperature)$\rightarrow$positive
  Decrease in internal energy
  (i.e. fall in temperature)$\rightarrow$ negative
  Work done by the system $\rightarrow$positive
  Work done on the system $\rightarrow$negative

- **For cyclic process** $\Delta U = 0 \Rightarrow \Delta Q = W$
- **For isochoric process**
  V = constant $\Rightarrow P \propto T$ & $W = 0$
  $\Delta Q = \Delta U = \mu C_V \Delta T$
  Isochoric $\Delta V = 0$ B = not defined.
  Bulc modulus
  Volume expansion coefficient = 0
- **For isobaric process**
  P = constant $\Rightarrow V \propto T$
  $\Delta Q = \mu C_P \Delta T$, $\Delta U = \mu C_V \Delta T$
  $W = P(V_2 - V_1) = \mu R \Delta T$
  Isobaric $\Delta P = 0$ Bulk modulus (B) = 0

  Volume Expansion coefficient = $\dfrac{1}{T}$

- **For adiabatic process** PV^{γ} = constant
  or $T^{\gamma} P^{1-\gamma}$ = constant
  or $TV^{\gamma-1}$ = constant
  In this process $\Delta Q = 0$ and

  $$W = -\Delta U = \mu C_V (T_1 - T_2) = \frac{P_1V_1 - P_2V_2}{\gamma - 1}$$

  adiabatic Bulk modulus $B = -v\left(\dfrac{\partial P}{\partial v}\right)$

  $$B = \gamma P$$

  Volume Expansion coefficient = $\dfrac{1}{(1-\gamma)T}$

◆ **For Isothermal Process**
  T = constant or $\Delta T = 0 \Rightarrow PV$ = constant
  In this process $\Delta U = \mu C_V \Delta T = 0$

  So, $\Delta Q = W = \mu RT \,\ell n\left(\dfrac{V_2}{V_1}\right) = \mu RT \ell n\left(\dfrac{P_1}{P_2}\right)$

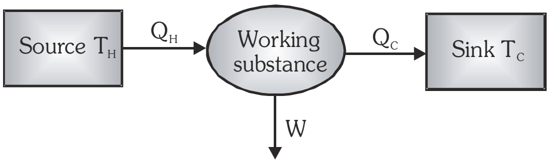
  Isothermal Bulk modulus $B = -v\left(\dfrac{\partial P}{\partial v}\right)_{T=\text{const}}$

  $$B = -v\left(\frac{-P}{v}\right) = P$$

  $\Delta T = 0$
  Volume expansion coefficient not defined.
- **For any general polytropic process**
  PV^x = constant
  - Molar heat capacity $C = C_V + \dfrac{R}{1-x}$
  - Work done by gas
    $$W = \frac{nR(T_1 - T_2)}{x-1} = \frac{(P_1V_1 - P_2V_2)}{x-1}$$
  - Slope of P-V diagram (also known as indicator diagram at any point $\dfrac{dP}{dV} = -x\dfrac{P}{V}$)

  Polytropic Bulk modulus $B = xP$

Efficiency of a cycle



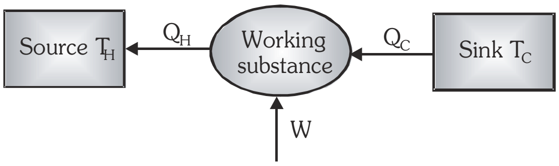
$$\eta = \frac{\text{Work done by working substance}}{\text{Heat supplied}}$$

$$= \frac{W}{Q_H} = \frac{Q_H - Q_C}{Q_H} = 1 - \frac{Q_C}{Q_H}$$

For carnot cycle

$$\frac{Q_C}{Q_H} = \frac{T_C}{T_H} \text{ so } \eta = 1 - \frac{Q_C}{Q_H} = 1 - \frac{T_C}{T_H}$$

For refrigerator

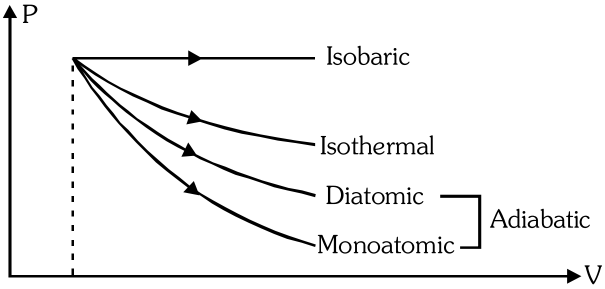


Coefficient of performance

$$\beta = \frac{Q_C}{W} = \frac{Q_C}{Q_H - Q_C} = \frac{T_C}{T_H - T_C}$$

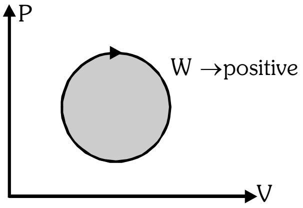
KEY POINTS

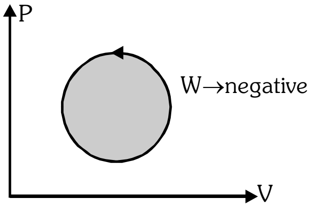
Work done is least for monoatomic gas (adiabatic process) in shown expansion.

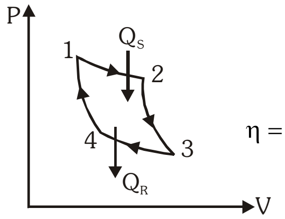


At a particular pressure and volume, magnitude of slope of P-V curve is greater for adiabatic $\left(\gamma\frac{P}{V}\right)$ then isothermal $\left(\frac{P}{V}\right)$

Air quickly leaking out of a balloon becomes cooler as the leaking air undergoes adiabatic expansion.

- First law of thermodynamics does not forbid flow of heat from lower temperature to higher temperature.
- First law of thermodynamics allows many processes which actually don't happen.
- 





$\eta = 1 - \frac{T_{3-4}}{T_{1-2}}$
- **CARNOT ENGINE**

It is a hypothetical engine with maximum possible efficiency

Process 1→2 & 3→4 are isothermal

Process 2→3 & 4→1 are adiabatic.

Second Law :- First law is quantitative analysis while second law is qualitative analysis of thermodynamics processes. Second law tells us in what conditions best can be converted into useful work.

$\boxed{ds = \frac{dQ}{T}}$ ds → change in entropy, dθ → best exchanged.

IMPORTANT NOTES

ELECTROSTATICS

ELECTRIC CHARGE

Charge of a material body is that property due to which it interacts with other charges. There are two kinds of charges- positive and negative.

S.I. unit → Coulomb (C)

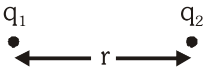
Properties of charge :-

(a) Charge is a scalar quantity
(b) Charge is quantised
(c) Charge is conserved.
(d) Charge is independent of frame of reference.

Methods of charging :-

(a) Friction (b) Induction (c) Conduction

COULOMB'S LAW



Force between two charges $\vec{F} = \frac{1}{4\pi \epsilon_0} \frac{q_1 q_2}{r^2} \hat{r}$

where, $\frac{1}{4\pi \epsilon_0} = 9 \times 10^9 \frac{Nm^2}{C^2}$

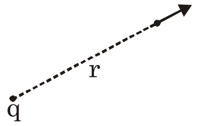
If medium is present then $\vec{F} = \frac{1}{4\pi \epsilon_0 \epsilon_r} \frac{q_1 q_2}{r^2} \hat{r}$

Note : The Law is applicable only for static and point charges. Moving charges may result in magnetic interaction. And if charges are spread on bodies then induction may change the charge distribution.

ELECTRIC FIELD OR ELECTRIC INTENSITY OR ELECTRIC FIELD STRENGTH

Electric field intensity is defined as force on unit test charge.

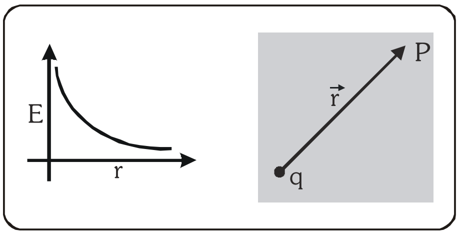
$$\vec{E} = \frac{kq}{r^2} \hat{r} = \frac{kq}{r^3} \vec{r}$$



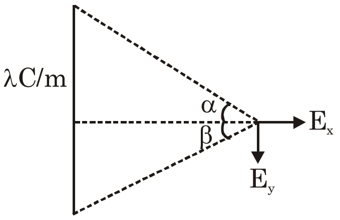
SI unit : Newton/coulomb (N/C)

ELECTRIC FIELD DUE TO SPECIAL CHARGE DISTRIBUTION

(a) Due to point charge $\vec{E} = \frac{kq}{r^2} \hat{r}$



(b) Due to linear change distribution :-

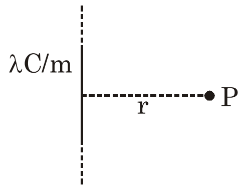


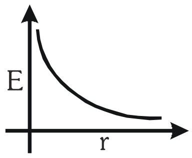
$$E_x = \frac{\lambda}{4\pi \epsilon_0 r} (\sin\alpha + \sin\beta)$$

$$E_y = \frac{\lambda}{4\pi \epsilon_0 r} (\cos\beta - \cos\alpha)$$

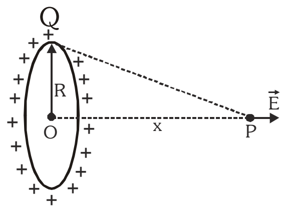
(c) Due to infinite line of charge

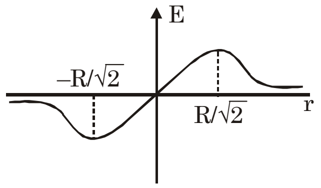
$$\vec{E}_P = \frac{\lambda}{2\pi \epsilon_0 r}\hat{r}$$





(d) Electric field due to uniformly changed ring



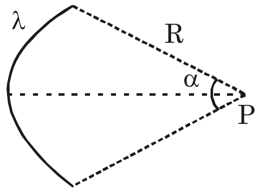


$$E_P = \frac{kQx}{\left(R^2 + x^2\right)^{3/2}}$$

(i) At centre of the ring, x = 0. So E = 0

(ii) Electric field is maximum at $x = \pm \frac{R}{\sqrt{2}}$

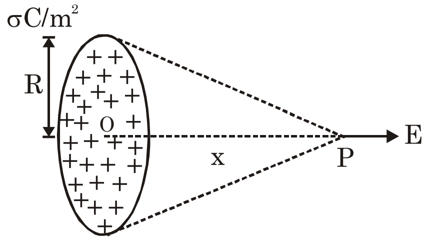
(e) Due to segment of ring



$$E_P = \frac{2k\lambda}{R}\sin\left(\frac{\alpha}{2}\right)$$

Direction of electric field is along the direction of angle bisector of the arc.

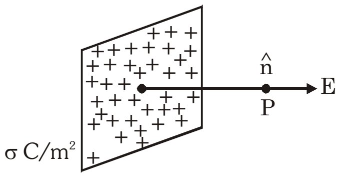
(f) Due to charged disk

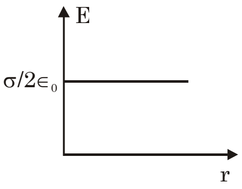


$$E_P = \frac{\sigma}{2\epsilon_0}\left(1 - \frac{x}{\sqrt{R^2 + x^2}}\right)$$

(g) Due to infinite plane sheet of charge

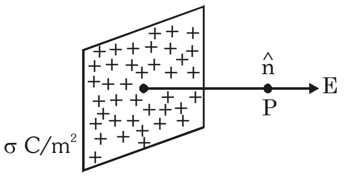
$$\vec{E}_P = \frac{\sigma}{2\epsilon_0}\hat{n}$$

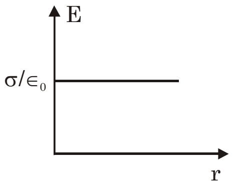




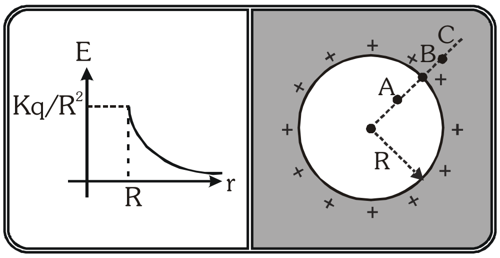
(h) Due to infinite charged conducting plate

$$\vec{E}_P = \frac{\sigma}{\epsilon_0}\hat{n}$$





(i) Due to hollow non-conducting sphere

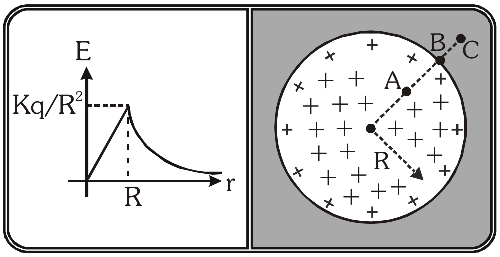


(a) For point inside the sphere (r < R) : $E_A = 0$

(b) For point on the surface (r = R) : $E_B = \frac{kQ}{R^2}$

(c) For point outside the sphere : $E_C = \frac{kQ}{r^2}$

(j) Due to uniformly charged non-conducting sphere



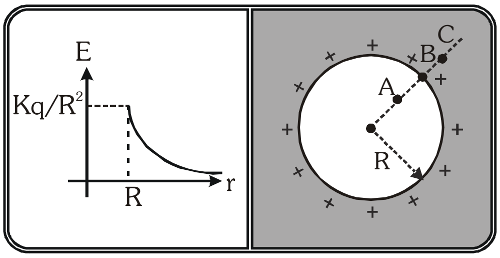
(a) For point inside the sphere (r < R)

$$E_A = \frac{kQr}{R^3} = \frac{\rho r}{3\in_0}$$

(b) For point on the surface (r = R) : $E_B = \dfrac{kQ}{R^2}$

(c) For point outside the sphere (r > R) : $E_C = \dfrac{kQ}{r^2}$

(k) Due to solid or Hollow conducting sphere

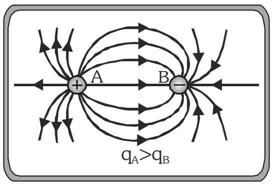


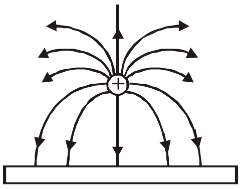
(a) For point inside the sphere (r < R) : $E_A = 0$

(b) For point on the surface (r = R): $E_B = \dfrac{kQ}{R^2}$

(c) For point outside the sphere (r > R) : $E_C = \dfrac{kQ}{r^2}$

ELECTRIC FIELD LINES





Fixed point charge near infinite metal plate

Electric field lines have the following properties :-

(a) Imaginary curves
(b) Never intersect each other
(c) Never form closed loops
(d) Start from (+ve) charge and ends on (–ve) charge.
(e) If there is no electric field then there will no field lines
(f) Number of electric field lines per unit area normal to the area at a point represents magnitude of electric field intensity. Crowded lines represent strong field while distant lines weak field.
(g) Number of lines originating from or terminating on a charge is proportional to magnitude of charge.
(h) Field lines start or end normally at the surface of a conductor.
(i) Tangent to the lines of force at a point in an electric field gives direction of intensity of electric field.

ELECTRIC FLUX

$$\phi = \int \vec{E}.\vec{dA}$$

(a) Scalar quantity
(b) SI unit :- Nm²/C or V–m

(i) For uniform electric field $\phi = \vec{E}.\vec{A} = EA\cos\theta$

where, θ = Angle between $\vec{E}$ and area vector ($\vec{A}$).

(ii) For non-uniform field $\phi = \int \vec{E}.\vec{dA}$

Gauss's Law

For a closed surface, total flux $\phi = \oint \vec{E}.\vec{dA} = \dfrac{q_{in}}{\in_0}$

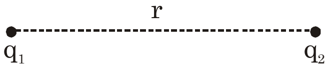
where q_{in} = net charge enclosed by the closed surface.

(i) Flux through Gaussian surface is independent of its shape.
(ii) Flux depends only on charges present inside the closed surface.
(iii) Flux through a closed surface is independent of position of charges inside it.
(iv) Electric field intensity at the Gaussian surface is due to all charges present (inside as well as outside).

ELECTROSTATIC POTENTIAL ENERGY

It is the amount of energy required to bring any charge from ∞ to any particular point without any charge in K.E.

Interection energy of a system of two charged particles



$$U = \frac{kq_1q_2}{r}$$

{ Assuming potential energy at ∞ to be zero}

ELECTRIC POTENTIAL

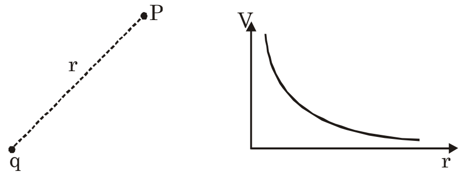
It is the work done against the field to take a unit positive charge from infinity (reference point) to the given point P without gaining any kinetic energy.

$$V_P = \frac{(W_{\infty-p})_{ext}}{q} = \frac{U}{q}$$

(i) Electric potential is a scalar quantity
(ii) SI unit :- Volt (V) or J/C
(iii) In presence of dielectric medium, potential decreases and becomes $\dfrac{1}{\varepsilon_r}$ times of its free space value.

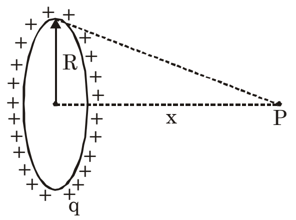
ELECTRIC POTENTIAL DUE TO SPECIAL CHARGE DISTRIBUTION :-

(a) Due to a point charge :-



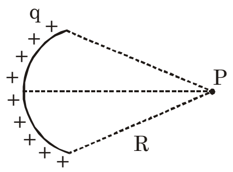
$$V_P = \frac{kq}{r} = \frac{q}{4\pi\varepsilon_0 r}$$

(b) Due to a charged ring :-



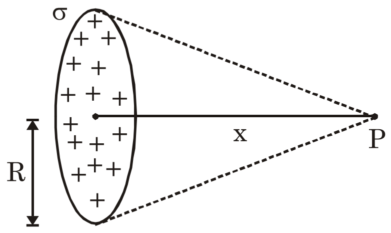
$$V_P = \frac{kq}{\sqrt{R^2 + x^2}}$$

(c) Due to segment of ring :-



$$V_P = \frac{kQ}{R}$$

(d) due to charged disk :-

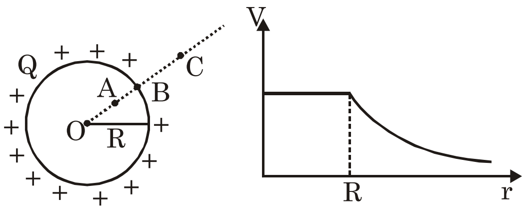


$$V_P = \frac{\sigma}{2\epsilon_0}\left(\sqrt{x^2 + R^2} - x\right)$$

(e) Due to non-conducting spherical shell :-

(a) For point inside the sphere (r < R) :-

$$V_A = \frac{kQ}{R}$$



(b) For point on the surface (r = R) :-

$$V_B = \frac{kQ}{R}$$

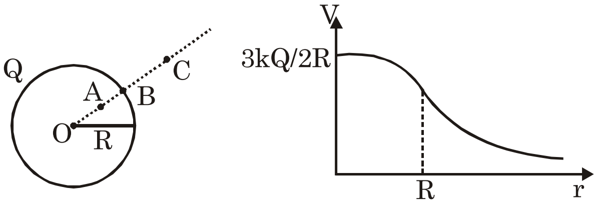
(c) For point outside the sphere (r > R) :-

$$V_C = \frac{kQ}{r}$$

(f) Due to solid non-conducting sphere :-

(a) For point inside the sphere (r < R) :-

$$V_A = \frac{kQ}{2R^3}\left(3R^2 - r^2\right)$$



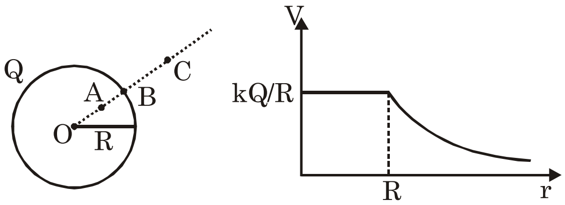
(b) For point on the surface (r = R)

$$V_B = \frac{kQ}{R}$$

(c) For point outside the sphere (r > R) :-

$$V_C = \frac{kQ}{r}$$

(g) Due to conducting sphere or shell :-



(a) For point inside the sphere (r < R) :-

$$V_A = \frac{kQ}{R}$$

(b) For point on the surface (r = R) :-

$$V_B = \frac{kQ}{R}$$

(c) For point outside the surface (r > R) :-

$$V_C = \frac{kQ}{r}$$

POTENTIAL DIFFERENCE

The potential difference between two points A & B is work done by external agent against electric field in taking a unit positive charge from B to A keeping kinetic energy constant :

$$V_A - V_B = \frac{(W_{BA})_{ext}}{q}$$

Relation between electric field & electric potential :-

$$\vec{E} = -\vec{\Delta}V = -\text{grad}V = -\frac{dV}{dr}\hat{r}$$

$$\vec{E} = \frac{-\partial v}{\partial x}\hat{i} - \frac{\partial v}{\partial y}\hat{j} - \frac{\partial v}{\partial z}\hat{k}\ ;\ V = -\int \vec{E}.\vec{dr}$$

(a) Direction of $\vec{E}$ is from high potential to low potential.

(b) If V = constant over a region, then E = 0 (in that region)

EQUIPOTENTIAL SURFACE.

The locus of all points having same potential is called

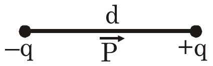
(a) Equipotential surfaces can never cross each other.

(b) Equipotential surfaces are always perpendicular to the direction of electric field.

(c) No work is done in moving a charge from one point to other over an equipotential surface.

ELECTRIC DIPOLE

A system of two equal and opposite charges separated by a small distance is called electric dipole :



Dipole Moment :- $\vec{p} = q\vec{d}$

Direction of dipole moment is from negative to positive charge :

DIPOLE PLACED IN UNIFORM ELECTRIC FIELD

(a) Torque $\vec{\tau} = \vec{p} \times \vec{E}$

(b) Net force = 0

(c) Work done in rotation of dipole from θ_1 to θ_2 angle in external electric field $W = pE(\cos\theta_1 - \cos\theta_2)$

(d) Electrostatic potential energy = $-\vec{p}.\vec{E} = -pE\cos\theta$

(e) In non-uniform electric field, force on electric dipole

$$\vec{F} = -\vec{p}.\frac{d\vec{E}}{dr}$$

ELECTRIC FIELD DUE TO DIPOLE

(a) At an axial point :-

$$\vec{E} = \frac{2k\vec{p}}{r^3}$$

(b) On the equitorial line :-

$$\vec{E} = \frac{-k\vec{p}}{r^3}$$

(c) At any general point :-

$$\vec{E} = \frac{k\vec{p}}{r^3}\sqrt{1 + 3\cos^2\theta}$$

ELECTRIC POTENTIAL DUE TO DIPOLE

(a) At an axial point :-

$$V = \frac{kp}{r^2}$$

(b) At equitorial point :-

V = 0

(c) At a general point :-

$$V = \frac{kp\cos\theta}{r^2}$$

CONDUCTORS AND ITS PROPERTIES

(a) Conductors are always equipotential surfaces.

(b) Charge always reside on the outer surface of a conductor.

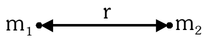
(c) Electric field is always perpendicular to conducting surface.

(d) Electric field lines never exist within conducting materials.

(e) When a conductor is grounded, its potential becomes zero.

GRAVITATION

Newton's law of gravitation



Force of attraction between two point masses

$$F = \frac{Gm_1m_2}{r^2}$$

Directed along the line joining of point masses.

- It is a conservative force field ⇒ mechanical energy is conserved.
- It is a central force field ⇒ angular momentum is conserved.

$$G = 6.67\times 10^{-11}\frac{m^3}{kgs^2}$$

Gravitational field due to point mass at distance x

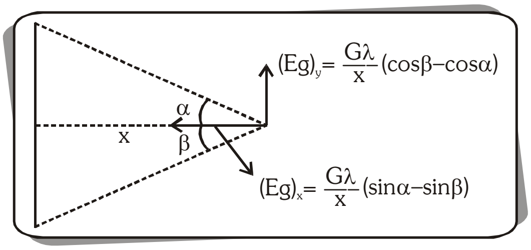
$$Eg = \frac{Gm}{r^2} \text{ [Radially inwards]}$$

Gravitational field on the axis of uniform thin ring at distance x

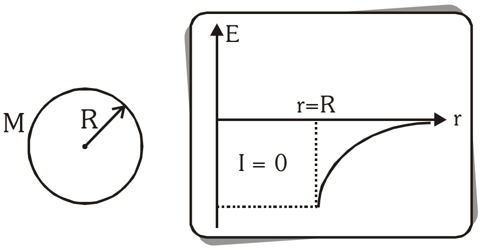
$$Eg = \frac{Gm}{(R^2+x^2)^{3/2}}x \text{ [Directed towards centre]}$$

Eg is max at $x = \pm\frac{R}{\sqrt{2}}$

Uniform linear mass (mass density λ)


$(Eg)_y = \frac{G\lambda}{x}(\cos\beta-\cos\alpha)$
α
x
β
$(Eg)_x = \frac{G\lambda}{x}(\sin\alpha-\sin\beta)$

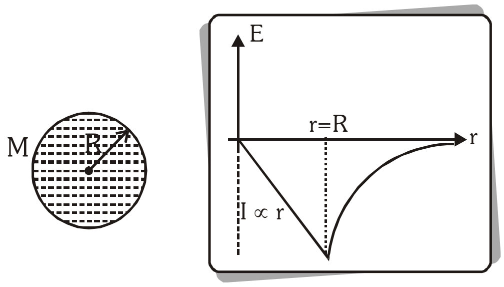
Gravitational field due to spherical shell



- ☐ Outside the shell $E_g = \frac{GM}{r^2}$, where r > R
- ☐ On the surface $E_g = \frac{GM}{r^2}$, where r=R
- ☐ Inside the shell $E_g = 0$, where r<R

[Note:Direction always towards the centre of the sphere]

Gravitational field due to solid sphere



- ☐ Outside the sphere $E_g = \frac{GM}{r^2}$, where r > R
- ☐ On the surface $E_g = \frac{GM}{r^2}$, where r=R
- ☐ Inside the sphere $E_g = \frac{GMr}{R^3}$, where r< R

Acceleration due to gravity $g = \frac{GM}{R^2}$

- ☐ At height h : $g_h = \frac{GM}{(R+h)^2}$

  If h << R : $g_h \approx g_s\left(1-\frac{2h}{R}\right)$

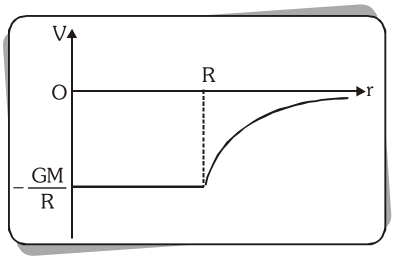
- ☐ At depth d : $g_d = \frac{GM(R-d)}{R^3} = g_s\left(1-\frac{d}{R}\right)$
- ☐ Effect of rotation on g : $g' = g-\omega^2R\cos^2\lambda$

  where λ is angle of latitude.

Gravitational potential

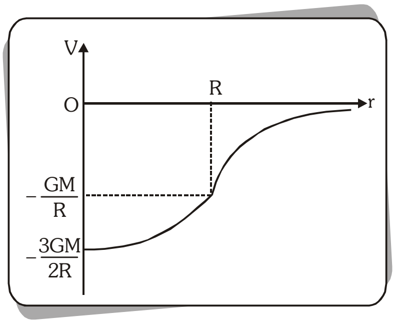
Due to a point mass at a distance $V = -\frac{GM}{r}$

Gravitational potential due to spherical shell



- Outside the shell $V = -\frac{GM}{r}$, r>R
- Inside/on the surface the shell $V = -\frac{GM}{R}$, $r \le R$

Potential due to solid sphere



- Outside the sphere $V = -\frac{GM}{r}$, r > R
- On the surface $V = -\frac{GM}{R}$, r = R
- Inside the sphere $V = -\frac{GM\left(3R^2 - r^2\right)}{2R^3}$, r < R

Potential on the axis of a thin ring at a distance x

$$V = -\frac{GM}{\sqrt{R^2 + x^2}}$$

Electrostatic self-energy

- For two point masses $U = -\frac{Gm_1m_2}{r}$
- Uniform thin spherical shell $U = -\frac{GM^2}{2R}$
- Uniform solid sphere $U = \frac{3}{2}\frac{GM^2}{R}$

Escape velocity from the surface a planet of mass M & radius R

$$v_e = \sqrt{\frac{2GM}{R}}$$

Orbital velocity of satellite

$$v_0 = \sqrt{\frac{GM}{r}} = \sqrt{\frac{GM}{(R+h)}}$$

- For nearby satellite $v_0 = \sqrt{\frac{GM}{R}} = \frac{v_e}{\sqrt{2}}$

  Here v_e = escape velocity on earth surface.

Time period of satellite

$$T = \frac{2\pi r}{v} = \frac{2\pi r^{3/2}}{\sqrt{GM}}$$

Energies of a satellite

- Potential energy $U = -\frac{GMm}{r}$
- Kinetic energy $K = \frac{1}{2}mv^2 = \frac{GMm}{2r}$
- Mechanical energy $E = U + K = -\frac{GMm}{2r}$
- Binding energy $BE = -E = \frac{GMm}{2r}$

Kepler's laws

- **I**

IMPORTANT NOTES

CURRENT ELECTRICTY

Some Definations

ELECTRIC CURRENT

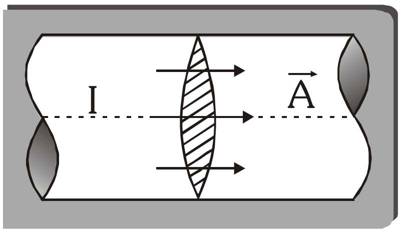
The rate of flow of electric charge across any cross-section is called electric current.

(a) Instantaneous electric current $I = \frac{dq}{dt}$

(b) Average electric current $I_{av} = \frac{\Delta q}{\Delta t}$

Current Density

Current flowing per unit area through any cross-section is called current density.



$J = \frac{I}{A}$

$$I = \vec{J}.\vec{A} = JA\cos\theta$$

Drift Velocity

Average velocity with which electrons drift from low potential end to high potential end of the conductor (v_d). Drift velocity is given by

$\vec{v}_d = -\frac{e\tau}{m}\vec{E}$ (in terms of applied electric field)

$v_d = \frac{I}{neA}$ (in terms of current through the conductor) From second relation

$I = neAv_d$ where A is the area of cross-section and "Av_d" represents the rate of flow.

The term $\frac{v_d}{E}$ is called mobility of charge carriers, represented by $\mu = \frac{v_d}{E} = \frac{e\tau}{m}$.

(here $\tau \rightarrow$ mean relaxation time depends on temperature $\tau \propto \frac{1}{\sqrt{T}}$, $T \rightarrow$ absolute temperature of the conductor)

OHM'S LAW

$I = \frac{V}{R}$ where $R = \frac{1}{\sigma}\frac{\ell}{A} = \frac{\rho\ell}{A}$ where ρ (resistivity) $= \frac{1}{\sigma}$

Hence according to Ohm's law when R is constant $I \propto V \Rightarrow I \sim V$ curve is a straight line (at constant temperature)

- Resistance of a conductor is given by

$$R = \frac{\rho\ell}{A} = \frac{m\ell}{ne^2\tau A}$$

where ρ is resistivity. Its units is Ω m.

- Resistivity of a conductor, $\rho = \frac{m}{ne^2\tau}$ (where m is mass of electron, n is number density of free electrons, τ is average relaxation time).

Variatio in resistance (R)

Variation with length: $R = \rho\frac{\ell}{A}$

(a) If a wire is cut to alter its length, then area remains same. $\therefore R \propto \ell$

(b) If a wire is stretched or drawn out or folded, area varies but volume remains constant. $\Rightarrow R \propto \ell^2$

For small percentage changes (< 5%) in length by stretching or folding, then, $\frac{\Delta R}{R} = \frac{2\Delta\ell}{\ell}$

Variation with area of cross-section or thickness

(a) If area is increased / decreased but length is kept same.

$\therefore R \propto \frac{1}{A}$ or $R \propto \frac{1}{r^2}$ (r = radius / thickness)

(b) If area is increased/decreased but volume remains same.

$$R \propto \frac{1}{A^2} \text{ or } R \propto \frac{1}{r^4}$$

For Conductors :

$\rho_t = \rho_0(1+\alpha t)$, where '$\alpha$' is temperature.

Coefficient of resistivity.

As $R \propto \rho \Rightarrow R = R_0(1 + \alpha t)$

(R_0 is the resistance at reference temperature)

At temperature t_1, $R_1 = R_0(1 + \alpha t_1)$

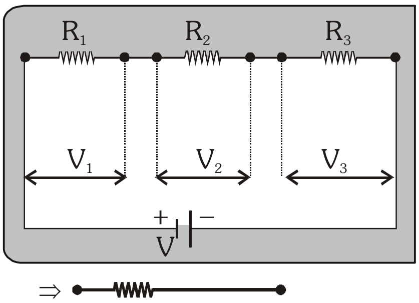
At temperature t_2, $R_2 = R_0(1 + \alpha t_2)$

$$\Rightarrow \alpha = \frac{R_2 - R_1}{R_0(t_2 - t_1)}, R_0 = \frac{R_2 - R_1}{\alpha(t_2 - t_1)}$$

COMBINATION OF RESISTANCES

Resistance In Series (Same Current)

$R = R_1 + R_2 + R_3 + \ldots\ldots\ldots\ldots\ldots + R_n$ and
$V = V_1 + V_2 + V_3 + \ldots\ldots\ldots\ldots\ldots + V_n$.



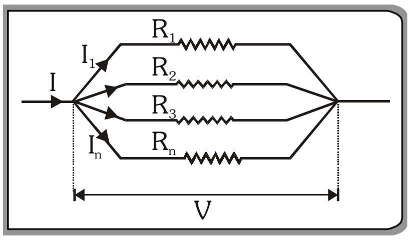
$R_{net} = R_1 + R_2 + R_3$

⇒ $V_1 = IR_1,\ V_2 = IR_2,\ V_3 = IR_3$

Resistance In Parallel (Same Potential difference)

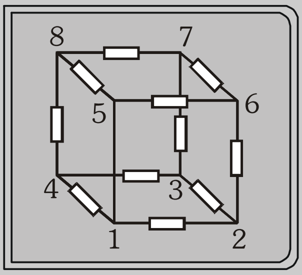
Effective resistance (R) then

$$\frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3} + \ldots\ldots\ldots + \frac{1}{R_n}$$



For two resistance $R = \dfrac{R_1R_2}{R_1 + R_2}$

Equivalent Resistance In Cube (Symmatry)



(a) Resistance between two nearer corners

$R_{12} = \dfrac{7}{12}r \qquad C_{12} = \dfrac{12C}{7}$

(b) Resistance across face diagonal

$R_{13} = \dfrac{3}{4}r \qquad C_{13} = \dfrac{4C}{3}$

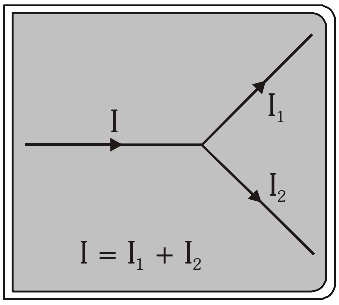
(c) Resistance across main diagonal

$R_{17} = \dfrac{5}{6}r \qquad C_{17} = \dfrac{6C}{5}$

KIRCHHOFF'S LAWS

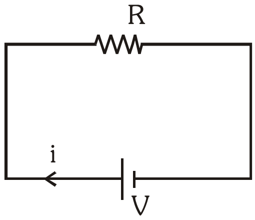
1. Junction Rule (K.C.L.)

It is based on conservation of charge.



2. Loop Rule (K.V.L.)

For any closed loop, total rise in potential + total fall in potential = 0.
It is based on conservation of energy.



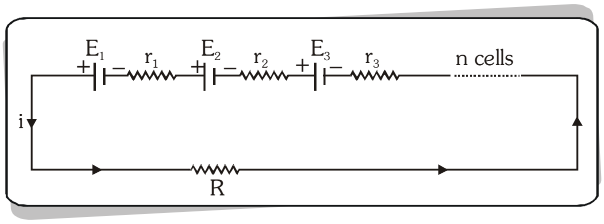
$+\ v - iR = 0$

Terminal Voltage V = E − iR discharging,
E + iR charging

Cell

- **EMF (E)** : The potential difference across the terminals of a practical cell when no current is being drawn from it.
- **Internal Resistance (r)** : The opposition of flow of current inside the cell. It depends on
  (i) Distance between electrodes : ↑r↑
  (ii) Area of electrodes : ↑r↓
  (iii) Concentration of electrolyte : ↑r↑
  (iv) Temperature : ↑r↓

Series Compbination of Cells :



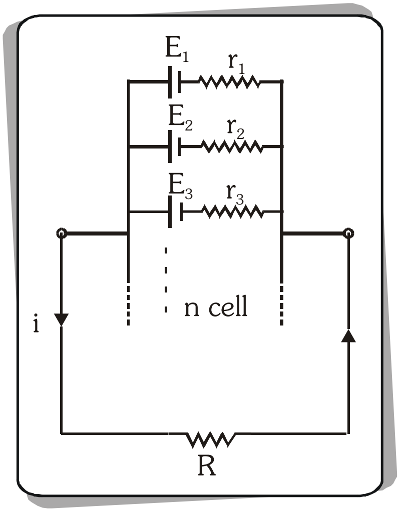
(a) $E_{equivalent} = E_1 + E_2 + E_3 + \ldots\ldots. E_n$
(b) $r_{equivalent} = r_1 + r_2 + r_3 + \ldots\ldots r_n$
(c) Current $i = \dfrac{\sum E_i}{\sum r_i + R}$
(d) If all cells have equal emf E and equal internal resistance r then $i = \dfrac{nE}{nr + R}$

Cases :

(i) If nr >> R $\Rightarrow i = \dfrac{E}{r}$

(ii) If nr << R $\Rightarrow i = \dfrac{nE}{R}$

Parallel Combination of cells :



(a) $E_{equivalent} = \dfrac{\dfrac{E_1}{r_1}+\dfrac{E_2}{r_2}+\dfrac{E_3}{r_3}+......}{\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}+........}$

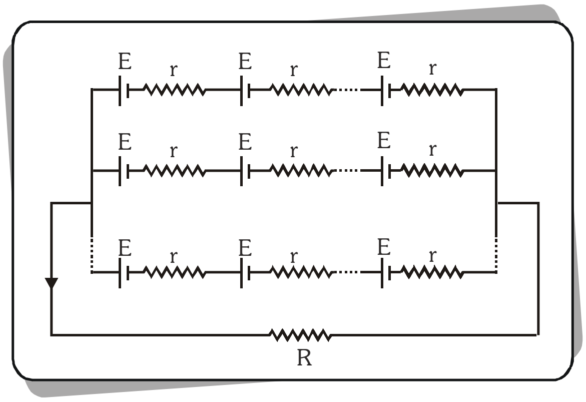
(b) $r_{equivalent} = \dfrac{1}{\dfrac{1}{r_1}+\dfrac{1}{r_2}+\dfrac{1}{r_3}+......}$

(c) If all cells have equal emf. E and internal resistance r then $E_{equivalent} = E$

$r_{equivalent} = \dfrac{r}{n} \Rightarrow$ current $i = \dfrac{E}{\frac{r}{n}+R}$

Mixed combination

Total number of identical cell in this circuit is nm. If n cells connected in a series and there are m such branches in the circuit than the internal resistance of the cells connected in a row = nr



Total internal resistance of the circuit $\dfrac{1}{r_{net}} = \dfrac{1}{nr}+\dfrac{1}{nr}+$

....upto m turns

($\because$ There are such m rows) $r_{net} = \dfrac{nr}{m}$

Total e.m.f. of the circuit = total e.m.f. of the cells connected in a row $E_T = nE$

$$I = \frac{E_{net}}{R+r_{net}} = \frac{nE}{R+\dfrac{nr}{m}}$$

Current in the circuit is maximum when external resistance in the circuit is equal to the total internal resistance of the cells $R = \dfrac{nr}{m}$

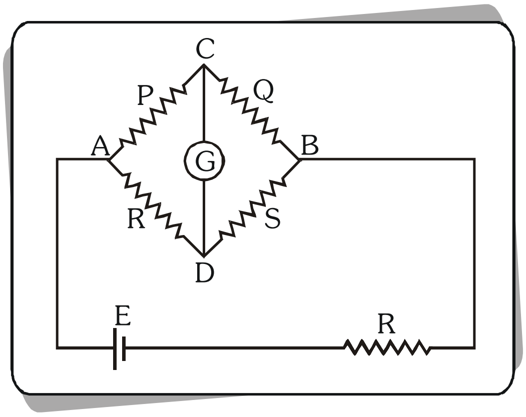
WHEAT STONE BRIDGE

When current through the galvanometer is zero (null point or zero defluction) $\dfrac{P}{Q} = \dfrac{R}{S}$.

When $PS > QR$, $V_C < V_D$ & $PS < QR$, $V_C > V_D$ or

$PS = QR \Rightarrow$ products of opposite arms are equal.

Potential difference between C & D at null point is zero . The null point is not affected by resistance of G & E. It is not affected even if the positions of G & E are interchanged.

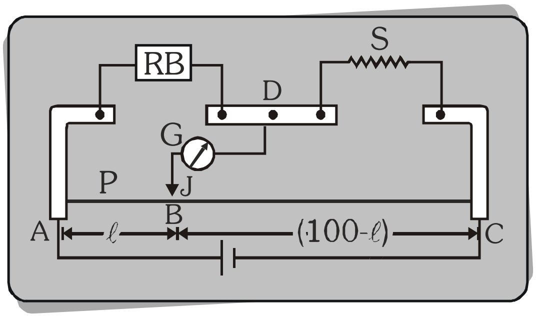


Electrical Instruments

Metre Bridge : Works on principle of wheet stone bridge

At balance condition :

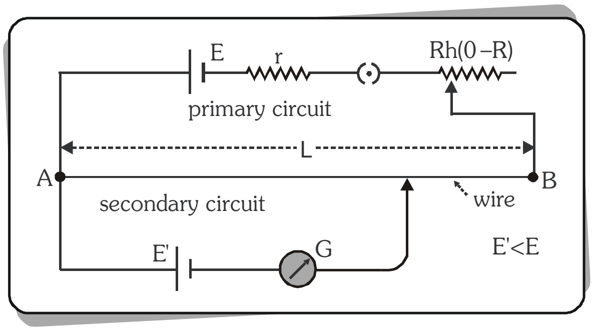
$$\frac{P}{Q}=\frac{R}{S}\Rightarrow\frac{\ell}{(100-\ell)}=\frac{R}{S}\Rightarrow S=\frac{(100-\ell)}{\ell}R$$



Potentiometer :

A potentiometer is a linear conductor of uniform cross-section with a steady current set up in it. This maintains a uniform potential gradient along the length of the wire . Any potential difference which is less than the potential difference maintained across the potentiometer wire can be measured using this .

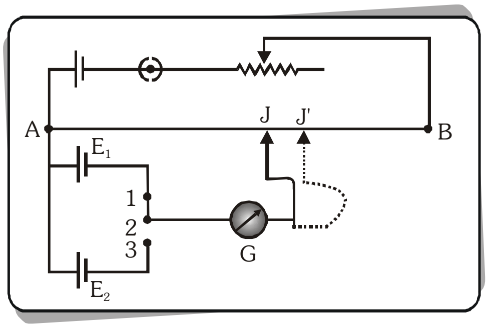
Circuits of potentiometer :



$$x=\frac{V}{L}=\frac{\text{current}\ \times\ \text{resistance of potentiometer wire}}{\text{length of potentiometer wire}}=I\left(\frac{R}{L}\right)$$

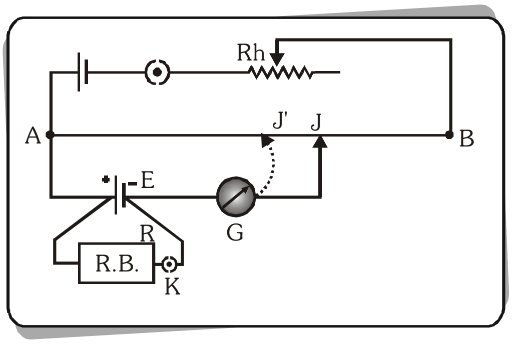
Applications of potentiometer :

(i) Comparision of emfs of two cells $\frac{E_1}{E_2}=\frac{\ell_1}{\ell_2}$



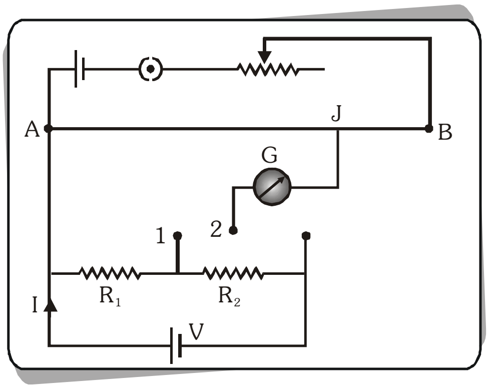
(ii) Internal Resistance of a given primary cell

$$\boxed{r=\left(\frac{\ell_1-\ell_2}{\ell_2}\right)R}$$



(iii) Comparision of two resistances

$$\frac{R_1}{R_1+R_2}=\frac{\ell_1}{\ell_2}$$



Galvanometer :

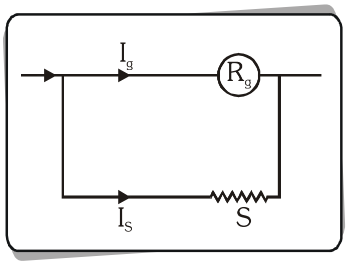
An instrument used to measure shtrength of current by measuring the diflection of me coil due to torque produced by a magnetic field.

$$T \propto i \propto \theta$$

A galvanometer can be converted into ammeter & voltmeter of varied scale as below.

Ammeter :

It is a modified form of suspended coil galvanometer, it is used to measure current. A shunt (small resistance) is connected in parallel with galvanometer to convert into ammeter .



$$S = \frac{I_g R_g}{I - I_g}$$

where

R_g = galvanometer resistance

I_g = Maximum current that can flow through the galvanometer .

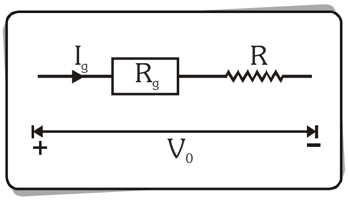
I = Maximum current that can be measured using the given ammeter .

An ideal ammeter has zero resistance.

Voltmeter :

A high resistance is put in series with galvanometer. It is used to measure potential difference.

$$I_g = \frac{V_o}{R_g + R}\ ;\ R \to \infty\ ,\ \text{Ideal voltmeter}$$



Electrical Power :

The energy liberated per second in a device is called its power. The electrical power P delivered by an electrical device is given by P = VI, where V = potential difference across device & I = current. If the current enters the higher potential point of the device then power is consumed by it (i.e. acts as load) . If the current enters the lower potential point then the device supplies power (i.e. acts as source). Power consumed by a resistor

$$P = I^2R = VI = \frac{V^2}{R}$$

Heating Effect Of Electric Current :

When a current is passed through a resistor energy is wasted in over coming the resistances of the wire. This energy is converted into heat

$$W = VIt = I^2Rt = \frac{V^2}{R}t$$

Joules Law Of Electrical Heating :

The heat generated (in joules) when a current of I ampere flows through a resistance of R ohm for T second is given by :

$$H = I^2RT \text{ joule} = \frac{I^2RT}{4.2} \text{ calories.}$$

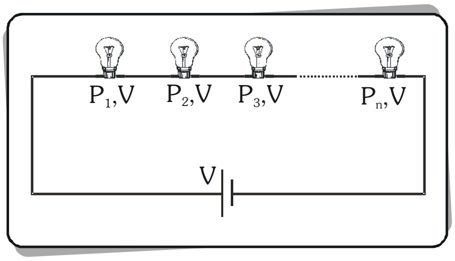
If current is variable passing through the conductor then we use for heat produced in resistance in time 0 to T is: $H = \int_0^T I^2R\,dt$

Unit Of Electrical Energy Consumption :

1 unit of electrical energy
= kilowatt hour
= 1 kWh = 3.6×10^6 joules.

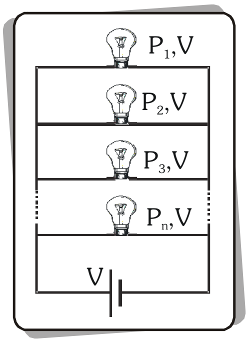
- **Series combination of Bulbs**

$$\frac{1}{P_{total}} = \frac{1}{P_1} + \frac{1}{P_2} + \frac{1}{P_3} + ...$$



- **Parallel combination of Bulbs**

$$P_{total} = P_1 + P_2 + P_3 + ...$$



KEY POINTS

- A current flows through a conductor only when there is an electric field within the conductor because the drift velocity of electrons is directly proportional to the applied electric field.
- Electric field outside the conducting wire which carries a constant current is zero because net charge on a current carrying conductor is zero.
- A metal has a resistance and gets often heated by flow of current because when free electrons drift through a metal, they make occasional collisions with the lattice. These collisions are inelastic and transfer energy to the lattice as internal energy.
- Ohm's law holds only for small current in metallic wire, not for high currents because resistance increased with increase in temperature.
- Potentiometer is an ideal instrument to measure the potential difference because potential gradient along the potentiometer wire can be made very small.
- An ammeter is always connected in series whereas a voltmeter is connected in parallel because an ammeter is a low-resistance galvanometer while a voltmeter is a high-resistance galvanometer.
- Current is passed through a metallic wires, heating it red, when cold water is poured over half of the portion, rest of the portion becomes more hot because resistance decreases due to decrease in temperature so current through wire increases.

IMPORTANT NOTES

CAPACITANCE

CONCEPT OF CAPACITANCE

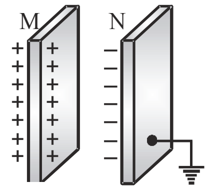
When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor. $Q \propto V \Rightarrow Q = CV$

The constant C is known as the capacity of the conductor.

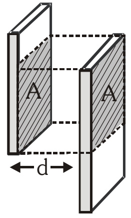
Capacitance is a scalar quantity with dimension $C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1}L^{-2}T^4A^2$

Unit :- farad, coulomb/volt

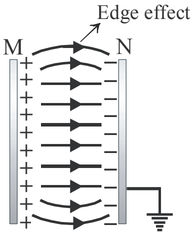
PARALLEL PLATE CAPACITOR



$$C = \frac{q}{V} = \frac{\varepsilon_r \varepsilon_0 A}{d}$$



- If one of the plates of parallel plate capacitor slides relatively than C decrease (As overlapping area decreases).
- If both the plates of parallel plate capacitor are touched each other resultant charge and potential became zero.
- Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



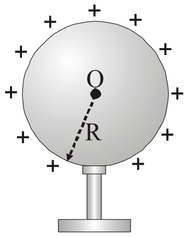
$\vec{E}$ = uniform in the centre

$\vec{E}$ = non-uniform at the edges

THE CAPACITANCE OF A SPHERICAL CONDUCTOR

When a charge Q is given to a isolated spherical conductor then its potential rises.

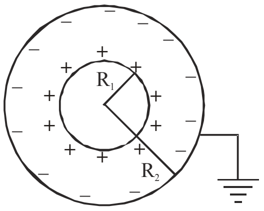
$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$



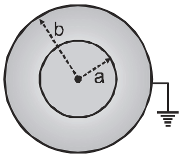
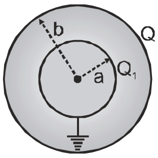
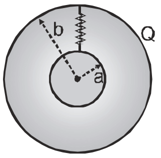
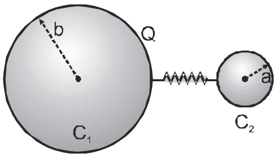
If conductor is placed in a medium then

$$C_{medium} = 4\pi\varepsilon R = 4\pi\varepsilon_0\varepsilon_r R$$

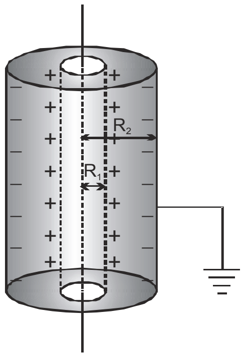
SPHERICAL CAPACITOR OUTER SPHERE IS EARTHED



$$C=\frac{Q}{V}=4\pi\varepsilon_0\frac{R_1R_2}{R_2-R_1}\text{ (in air or vacuum)}$$

Spherical capacitor outer is earthed	Inner is earthed and outer is given a charge	Connected and outer is given a charge	Connected spheres
			
$C=\frac{4\pi\varepsilon_0 ab}{b-a}$ (b > a)	$C=\frac{4\pi\varepsilon_0 b^2}{b-a}$ (b > a)	$C=4\pi\varepsilon_0 b$	$C=C_1+C_2$ $C=4\pi\varepsilon_0(a+b)$

CYLINDRICAL CAPACITOR



Electrical field between cylinders $E=\frac{\lambda}{2\pi\varepsilon_0 r}=\frac{Q/\ell}{2\pi\varepsilon_0 r}$

Potential difference between plates $V=\int_{R_1}^{R_2}\frac{Q}{2\pi\varepsilon_0 r\ell}dr=\frac{Q}{2\pi\varepsilon_0\ell}\ell n\left(\frac{R_2}{R_1}\right)$

Capacitance $C=\frac{Q}{V}=\frac{2\pi\varepsilon_0\ell}{\log_e(R_2/R_1)}$

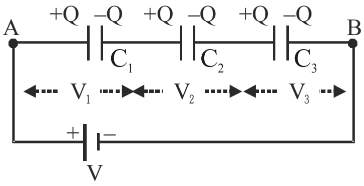
(ii) Force between the plates

$$F=-QE=-\frac{Q^2}{2\varepsilon_0 A}$$

Magnitude of force $F=\frac{Q^2}{2\varepsilon_0 A}=\frac{1}{2}\varepsilon_0 AE^2$

Force per unit area or energy density or electrostatic pressure $=\frac{F}{A}=u=p=\frac{1}{2}\epsilon_0 E^2$

COMBINATION OF CAPACITOR



- **Capacitor in series**:
  In this arrangement of capacitors the charge has no alternative path(s) to flow.

(i) The charges on each capacitor are equal
i.e. $Q=C_1V_1=C_2V_2=C_3V_3$

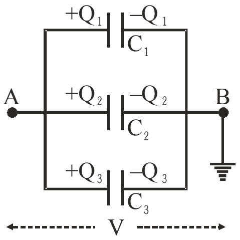
(ii) The total potential difference across AB is shared by the capacitors in the inverse ratio of the capacitances $V = V_1 + V_2 + V_3$

If C_S is the net capacitance of the series combination, then

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- **Capacitors in parallel**

In such arrangement of capacitors the charge has an alternative path(s) to flow.



(i) The potential difference across each capacitor is same and equal the total potential applied. i.e. $V = V_1 = V_2 = V_3 \Rightarrow V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$

(ii) The total charge Q is shared by each capacitor in the direct ratio of the capacitances. $Q = Q_1 + Q_2 + Q_3$

If C_P is the net capacitance for the parallel combination of capacitors :

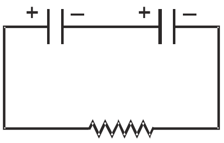
$$C_PV = C_1V + C_2V + C_3V \quad \Rightarrow C_P = C_1 + C_2 + C_3$$

- For a given voltage to store maximum energy capacitors should be connected in parallel.

- If N identical capacitors each having breakdown voltage V are joined in

(i) series then the break down voltage of the combination is equal to NV

(ii) parallel then the breakdown voltage of the combination is equal to V.



- Two capacitors are connected in series with a battery. Now battery is removed and loose wires connected together then final charge on each capacitor is zero.

- If N identical capacitors are connected then $C_{\text{series}} = \frac{C}{N}$, $C_{\text{parallel}} = NC$

- In DC capacitor's offers infinite resistance in steady state, so there will be no current flows through capacitor branch.

ENERGY STORED IN A CHARGED CONDUCTOR/CAPACITOR

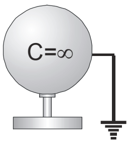
Let C is capacitance of a conductor. On being connected to a battery. It charges to a potential V from zero potential. If q is charge on the conductor at that time then q = CV. Let battery supplies small amount of charge dq to the conductor at constant potential V. Then small amount of work done by the battery against the force exerted by exsiting charge is

$$dW = Vdq = \frac{q}{C}dq \Rightarrow W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q \Rightarrow W = \frac{Q^2}{2C}$$

where Q is the final charge acquired by the conductor. This work done is stored as potential energy, so

$$U = \frac{Q^2}{2C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV \quad \therefore \quad U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- As the potential of the Earth is assumed to be zero, capacity of earth or a conductor connceted to earth will be infinite $C = \frac{q}{V} = \frac{q}{0} = \infty$



- Actual capacity of the Earth $C = 4\pi\varepsilon_0 R = \frac{1}{9\times10^9}\times 64\times10^5 = 711\ \mu F$
- Work done by battery W_b = (charge given by battery) × (emf) = QV but

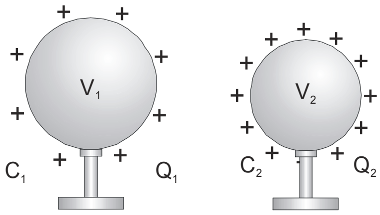
  Energy stored in conductor $= \frac{1}{2}QV$

  so 50% energy supplied by the battery is lost in form of heat.

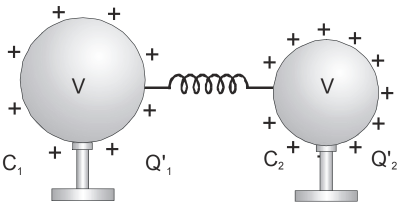
REDISTRIBUTION OF CHARGES AND LOSS OF ENERGY

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of two conductors became equal.

Let the amounts of charges after the conductors are connected are Q_1' and Q_2' respectively and potential is V then



(Before connection)



(After connection)

- **Common potential**

  According to law of Conservation of charge $Q_{\text{before connection}} = Q_{\text{after connection}}$

  $\Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$

  Common potential after connection $\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$

- **Charges after connection**

  $Q_1' = C_1V = C_1\left(\frac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\frac{C_1}{C_1 + C_2}\right)Q$ (Q : Total charge on system)

  $Q_2' = C_2V = C_2\left(\frac{Q_1 + Q_2}{C_1 + C_2}\right) = \left(\frac{C_2}{C_1 + C_2}\right)Q$

  Ratio of the charges after redistribution $\boxed{\frac{Q_1'}{Q_2'} = \frac{C_1V}{C_2V} = \frac{R_1}{R_2}\ \text{(in case of spherical conductors)}}$

- **Loss of energy in redistribution**

  When charge flows through the conducting wire then **energy is lost mainly on account of Joule effect**, electrical energy is converted into heat energy, so change in energy of this system,

  $$\Delta U = U_f - U_i \Rightarrow \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right) - \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) \Rightarrow \Delta U = -\frac{1}{2}\left(\frac{C_1C_2}{C_1 + C_2}\right)(V_1 - V_2)^2$$

  Here negative sign indicates that energy of the system decreases in the process.

EFFECT OF DIELECTRIC

- The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as **dielectrics**.
- Dielectrics are non conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called **dielectric strength** they lose their insulating property and begin to conduct.
- **Dielectric strength** is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre. Dimensions $M^1L^1T^{-3}A^{-1}$

Polar dielectrics

- In absence of external field the centres of positive and negative charge do not coincide-due to asymmetric shape of molecules.
- Each molecule has permanent dipole moment.
- The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.
- In presence of external field dipoles tends to align in direction of field.

  Ex. Water, Alcohol, CO_2, $HC\ell$, NH_3

Non polar dielectrics

- In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In presence of external field they acquire induced dipole moment.

  Ex. Nitrogen, Oxygen, Benzene, Methane

Polarisation :

The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

- **Polarisation vector** $\vec{p}$

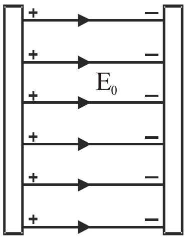
  This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

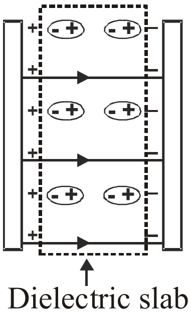
  $\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

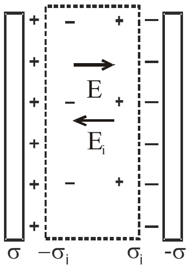
  where n is number of atoms per unit volume of dielectric and $\vec{p}$ is dipole moment of an atom or molecule.

  $\vec{P} = n\vec{p} = \frac{q_i d}{Ad} = \left(\frac{q_i}{A}\right) = \sigma_i$ = induced surface charge density.

  Unit of $\vec{P}$ is C/m^2 Dimension is $L^{-2}T^1A^1$







Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric. Let E, V, C are electric field, potential difference and capacitance in presence of dielectric respectively.

Electric field in absence of dielectric $E_0 = \frac{V_0}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric $E = E_0 - E_i = \frac{\sigma - \sigma_i}{\varepsilon_0} = \frac{Q - Q_i}{\varepsilon_0} = \frac{V}{d}$

Capacitance in absence of dielectric $C_0 = \frac{Q}{V_0}$

Capacitance in presence of dielectric $C = \frac{Q - Q_i}{V}$

The dielectric constant or relative permittivity K

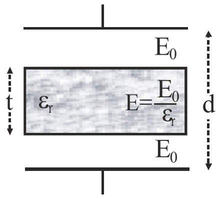
or $\varepsilon_r = \frac{E_0}{E} = \frac{V_0}{V} = \frac{C}{C_0} = \frac{Q}{Q - Q_i} = \frac{\sigma}{\sigma - \sigma_i} = \frac{\varepsilon}{\varepsilon_0}$

From $K = \frac{Q}{Q - Q_i} \Rightarrow Q_i = Q\left(1 - \frac{1}{K}\right)$ and $K = \frac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma_i = \sigma\left(1 - \frac{1}{K}\right)$

If capacitor is partially filled with dielectric

$V = E_0(d - t) + Et$

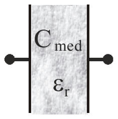
$\Rightarrow \quad V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right] \quad \because \frac{E_0}{E} = \varepsilon_r$ = Dielectric constant



$\Rightarrow V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] \Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)}$ ...(i)

If medium is fully present between the space.

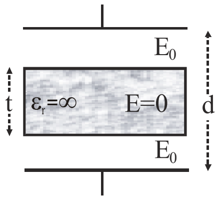
$\because \quad t = d$



Now from equation (i) $\quad C_{medium} = \frac{\varepsilon_0 \varepsilon_r A}{d}$

If capacitor is partialy filled by a conducting slab of thickness (t< d).

$\because \quad \varepsilon_r = \infty$ for conductor $C = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\infty}\right)} = \frac{\varepsilon_0 A}{(d - t)}$

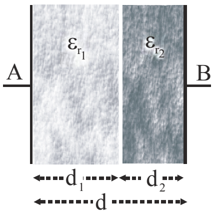


DISTANCE AND AREA DIVISION BY DIELECTRIC

- **Distance Division**

(i) Distance is divided and area remains same.

(ii) Capacitors are in series.



(iii) Individual capacitances are $C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A}{d_1}$, $C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A}{d_2}$

These two are in series $\frac{1}{C} = \frac{1}{C_1} + \frac{1}{C_2} \Rightarrow \frac{1}{C} = \frac{d_1}{\varepsilon_0 \varepsilon_{r_1} A} + \frac{d_2}{\varepsilon_0 \varepsilon_{r_2} A}$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 A}\left[\frac{d_1\varepsilon_{r_2} + d_2\varepsilon_{r_1}}{\varepsilon_{r_1}\varepsilon_{r_2}}\right] \Rightarrow C = \varepsilon_0 A\left[\frac{\varepsilon_{r_1}\varepsilon_{r_2}}{d_1\varepsilon_{r_2} + d_2\varepsilon_{r_1}}\right]$$

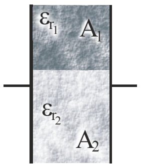
Special case : If $d_1 = d_2 = \frac{d}{2}$ $\Rightarrow C = \frac{\varepsilon_0 A}{d}\left[\frac{2\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right]$

- **Area Division**

(i) Area is divided and distance remains same.

(ii) Capacitors are in parallel.

(iii) Individual capacitances are $C_1 = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d}$ $C_2 = \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d}$



These two are in parallel so $C = C_1 + C_2 = \frac{\varepsilon_0 \varepsilon_{r_1} A_1}{d} + \frac{\varepsilon_0 \varepsilon_{r_2} A_2}{d} = \frac{\varepsilon_0}{d}(\varepsilon_{r_1} A_1 + \varepsilon_{r2} A_2)$

Special case : If $A_1 = A_2 = \frac{A}{2}$ Then $C = \frac{\varepsilon_0 A}{d}\left(\frac{\varepsilon_{r_1} + \varepsilon_{r2}}{2}\right)$

- **Variable Dielectric Constant :**

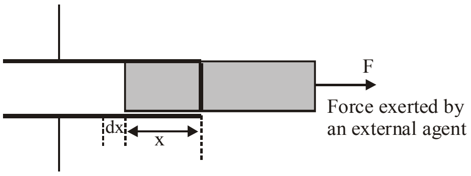
If the dielectric constant is variable, then equivalent capacitance can be obtained by selecting an element as per the given condition and then integrating.

(i) If different elements are in parallel, then $C = \int dC$, where dC = capacitance of selected differential element.

(ii) If different element are in series, then $\frac{1}{C} = \int d\left(\frac{1}{C}\right)$ is solved to get equivalent capacitance C.

FORCE ON A DIELECTRIC IN A CAPACITOR

Consider a differential displacement dx of the dielectric as shown in figure always keeping the net force on it zero so that the dielectric moves slowly without acceleration. Then, $W_{\text{Electrostatic}} + W_F = 0$, where W_F denotes the work done by external agent in displacement dx



$W_F = -W_{\text{Electrostatic}}$ $W_F = \Delta U$

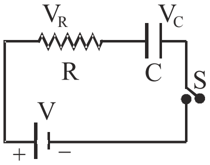
$$\Rightarrow -F.dx = \frac{Q^2}{2} d\left[\frac{1}{C}\right] \left[U = \frac{Q^2}{2C}\right] \Rightarrow -F.dx = \frac{-Q^2}{2C^2} dC \Rightarrow F = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

This is also true for the force between the plates of the capacitor. If the capacitor has battery connected to it, then as the p.d. across the plates is maintained constant. $V = \frac{Q}{C} \Rightarrow F = \frac{1}{2}V^2\frac{dC}{dx}$.

CHARGING & DISCHARGING OF A CAPACITOR

Charging

- When a capacitor, resistance, battery, and key is conected in series and key is closed, then



- **Charge at any instant**

$$V = V_C + V_R = \frac{Q}{C} + IR = \frac{Q}{C} + \frac{dQ}{dt}R$$

$$Q = CV\left[1 - e^{-t/RC}\right] = Q_0\left[1 - e^{-t/RC}\right]$$

At $t = \tau = RC$ = time constant

$Q = Q_0[1 - e^{-1}] = 0.632\,Q_0$

So, in charging, charge increases to 63.2% of charge in the time equal to τ.

- **Current at any instant**

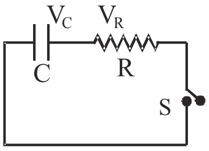
$i = dQ/dt = i_0 e^{-t/RC}$ $\{i_0 = Q_0/RC\}$

- **Potential at any instant**

$V = V_0(1 - e^{-t/RC})$

Discharging

- When a charged capacitor, resistance and keys is conected in series and key is closed. Then energy stored in capacitor is used to circulate current in the circuit.



- **Charge at any instant**

$V_C + V_R = 0$

$Q = Q_0 e^{-t/RC}$

At $t = \tau = RC$ = time constant

$Q = Q_0 e^{-1} = 0.368\,Q_0$

So, in discharging, charge decreases to 36.8% of the initial charge in the time equal to τ.

- **Current at any instant**

$i = dQ/dt = -i_0 e^{-t/RC}$ $\{i_0 = Q_0/RC\}$

- **Potential at any instant**

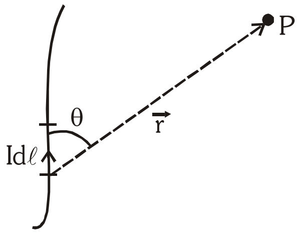
$V = V_0\ e^{-t/RC}$

Magnetic Effect of Current & Magnetism

Magnetic effect of current discovered by : Orested

BIOT-SAVART'S LAW :-

→ The magnetic field dB at a point due to current element $I\vec{d\ell}$ is given by ,

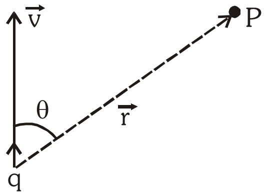


$I\vec{d\ell}$ is given by

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{(Id\vec{\ell} \times \vec{r})}{r^3}$$

$$\Rightarrow dB = \frac{\mu_0}{4\pi}\frac{Id\ell\sin\theta}{r^2}$$

→ Magnetic field at P due to moving charge is given by,

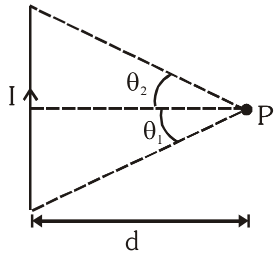


$$\vec{B} = \frac{\mu_0}{4\pi}\frac{q(\vec{v} \times \vec{r})}{r^3}$$

$$\Rightarrow \vec{B} = \frac{\mu_0}{4\pi}\frac{qv\sin\theta}{r^2}$$

1. Magnetic field due to finite current carrying wire at point P,

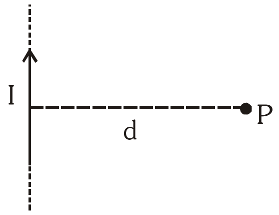
$$B_P = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$$



(a) For infinite wire,

$\theta_1 = \theta_2 = 90°$

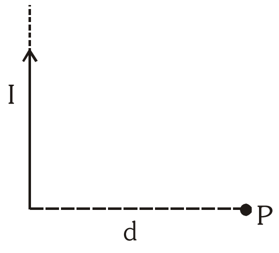
$$B_P = \frac{\mu_0 I}{2\pi d}$$



(b) For semi-infinite wire,

$\theta_1 = 0°, \theta_2 = 90°$

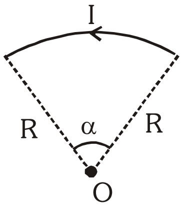
$$B_P = \frac{\mu_0 I}{4\pi d}$$



Note : For points along the length of the wire (but not on it), the field is always zero.

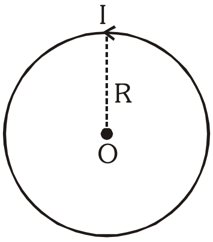
2. Magnetic field at the centre of current carrying circular arc.

$$B_0 = \frac{\mu_0 I}{4\pi R}(\alpha)$$



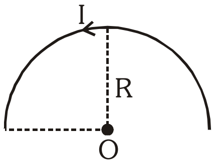
(a) At the centre of current carrying circular loop,

$$B_0 = \frac{\mu_0 I}{2R}$$



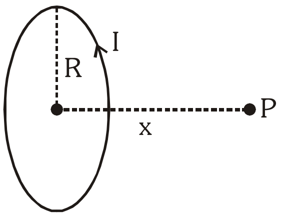
(b) At the centre of semi-circular arc

$$B_0 = \frac{\mu_0 I}{4R}$$



3. Magnetic-field at an axial point of current carrying circular loop,

$$B_P = \frac{\mu_0 I R^2}{2\left(R^2 + x^2\right)^{\frac{3}{2}}}$$



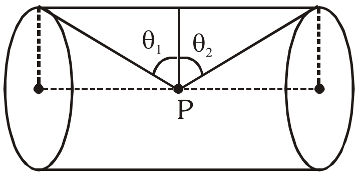
4. Magnetic field at the axis of solenoid :

(a) Finite length :

$$B_P = \frac{\mu_0 n I}{2}\left[\sin\theta_1 + \sin\theta_2\right]$$

(b) Infinite length :

$$B_P = \mu_0 n I$$



n → number of turns per unit length.

Note : Magnetic field outside solenoid is zero.

AMPERE'S LAW :

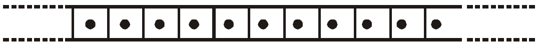
The line integral of magnetic field over the closed path $\left(\oint \vec{B}\cdot\vec{d\ell}\right)$ is equal to μ_0 times the net current crossing the area inclosed by the path.

$$\oint \vec{B}\cdot\vec{d\ell} = \mu_0 I_{\text{enclosed}}$$

Here $\vec{B}$ is net magnetic field.

(i) Magnetic field due to infinite current sheet.

$$B = \frac{\mu_0 k}{2}$$



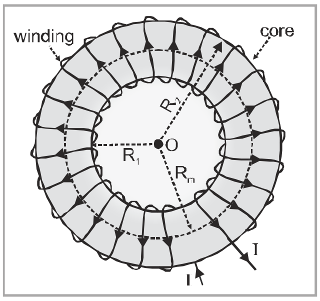
Here k is linear current density

(ii) Magnetic field inside toroid :

Field inside toroid :-

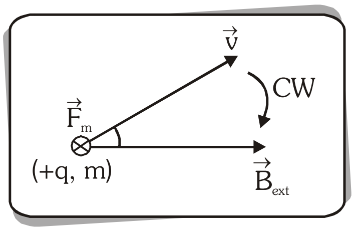
$B = \mu_0 nI$, where $n = N/2\pi R_m$, turn density

mean radius $R_m = \dfrac{R_1 + R_2}{2}$



Magnetic force on moving charge in magnetic field

Vector from $\vec{F}_m = q(\vec{v}\times\vec{B}_{ext})$ Always $\begin{bmatrix}\vec{F}_m \perp \vec{v}\\ \vec{F}_m \perp \vec{B}_{ext}\end{bmatrix}$

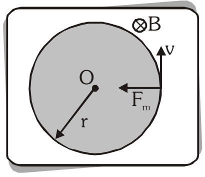


Magnitude form :

$$F_m = qvB\sin\theta \begin{cases}\theta=90^\circ\ (\vec{v}\perp\vec{B}) \Rightarrow \boxed{F_m = qvB}_{(max)}\\ \theta=0^\circ \text{ or } 180^\circ \Rightarrow \boxed{F_m = 0}_{(min)}\end{cases}$$

Motion of charge in uniform field

$(\vec{v}\perp\vec{B},\ \theta=90^\circ)$ $\boxed{qvB = \dfrac{mv^2}{r}}$



(a) Radius of circular path :

$$r = \frac{mv}{qB},\quad \text{where } P = mv = \sqrt{2mE_K} = \sqrt{2mqV_{acc}}$$

(b) Time period : $T = \dfrac{2\pi m}{qB}$

(c) Kinetic energy of charge : $E_K = \dfrac{(qBr)^2}{2m}$

Motion of charge in uniform field at any angle except 0° or 180° or 90°

(a) Radius of helical path : $r = \dfrac{mv\sin\theta}{qB}$

(b) Time period : $T = \dfrac{2\pi m}{qB}$

(c) Pitch of helix : $P = (v\cos\theta)\,T$, where $T = \dfrac{2\pi m}{qB}$

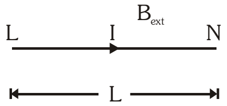
Combined effect of $\vec{E}$ & $\vec{B}$ on moving charge

Electromegnetic or Lorentz force

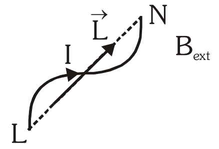
$$\vec{F}_L = \vec{F}_e + \vec{F}_m \quad \boxed{\vec{F}_L = q\vec{E} + q(\vec{v}\times\vec{B})}$$

Magnetic force on current carrying wire (or conductor)

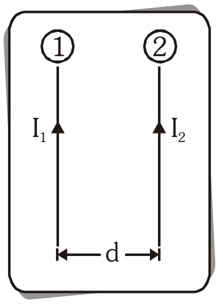
(a) Straight wire :- $\vec{F}_m = I(\vec{L}\times\vec{B}_{ext/uniform})$



(b) Arbitrary wire :- $\vec{F}_m = I(\vec{L}\times\vec{B}_{ext/uniform})$



Magnetic force b/w two long parallel wires



$$f = \frac{\mu_0 I_1 I_2}{2\pi d}\ N/m$$

$\begin{bmatrix}\text{parallel currents} \Rightarrow \text{Attraction}\\ \text{antiparallel currents} \Rightarrow \text{Repulsion}\end{bmatrix}$

MAGNETIC TORQUE ON A CLOSED CURRENT CIRCUIT

When a plane closed current circuit is placed in uniform magnetic field , it experience a zero net force, but experience a torque given by $\vec{\tau} = NI\vec{A}\times\vec{B} = \vec{M}\times\vec{B} = BINA\sin\theta$ where $\vec{A}$ = area vector outward from the face of the circuit where the current is anticlockwise, $\vec{B}$ = magnetic induction of the uniform magnetic field. $\vec{M}$ = magnetic moment of the current circuit = $IN\vec{A}$

Note : This expression can be used only if $\vec{B}$ is uniform.

Moving Coil Galvanometer

It consists of a plane coil of many turns suspended in a radial magnetic field. When a current is passed in the coil it experiences a torque which produces a twist in the suspension.

This deflection is directly proportional to the torque

$\therefore NIAB = K\theta$;

$$I = \left(\frac{K}{NAB}\right)\theta;$$

K=elastic torsional constant of the suspension

$$I = C\theta;\ C = \frac{K}{NAB}$$

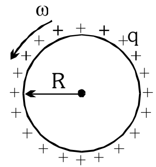
= Galvanometer constant

Magnetic dipole

- ❒ Magnetic moment $M = m \times 2\ell$, where m is pole strength of the magnet
- ❒ Magnetic field at axial point (or End-on position) of dipole $\vec{B} = \frac{\mu_0}{4\pi}\frac{2\vec{M}}{r^3}$
- ❒ Magnetic field at equitorial position (Broad-side on position) of dipole $\vec{B} = \frac{\mu_0}{4\pi}\frac{(-\vec{M})}{r^3}$
- ❒ Magnetic field at a point which is at a distance r from dipole midpoint and making angle θ with dipole axis. $$B = \frac{\mu_0}{4\pi}\frac{M\sqrt{1+3\cos^2\theta}}{r^3}$$
- ❒ Torque on dipole placed in uniform magnetic field $\vec{\tau} = \vec{M}\times\vec{B}$
- ❒ Potential energy of dipole placed in an uniform field $U = -\vec{M}\cdot\vec{B}$

Magnetic moment of a rotating charge

If a charge q is rotating at an angular velocity ω, its equivalent current is given as $I = \frac{q\omega}{2\pi}$ & its magnetic moment is $M = I\pi R^2 = \frac{1}{2}q\omega R^2$.



NOTE: The ratio of magnetic moment to angular momentum called gyromagnetic ratio of a uniform rotating object which is charged uniformly is always a constant and equal to half of specific charge. Irrespective of the shape of conductor $\boxed{M/L = q/2m}$

GILBERT'S MAGNETISM (EARTH'S MAGNETIC FIELD)

(a) Imaginary vertical plane passing through the magnetic North - South poles at that place. This plane is called the **MAGNETIC MERIDIAN**. The Earth's Magnetic poles are opposite to the geometric poles i.e. at earth's north pole, its geomagnetic south pole is situated and vice versa.

(b) On the magnetic meridian, the magnetic induction vector of the earth at any point, generally inclined to the horizontal at an angle called the **MAGNETIC DIP** at that place , such that $\vec{B}$ = total magnetic induction of the earth at that point.

$\vec{B}_v$ = the vertical component of $\vec{B}$ in the magnetic meridian plane = B sin θ

$\vec{B}_H$ = the horizontal component of $\vec{B}$ in the magnetic meridian plane=B cos θ. $\frac{B_v}{B_H} = \tan\theta$

(c) At a given place on the surface of the earth , the magnetic meridian and the geographic meridian may not coincide . The angle between them is called "**DECLINATION AT THAT PLACE**"

- ◆ **Intensity of magnetisation** $I = M/V$
- ◆ **Magnetic induction** $B = \mu H = \mu_0(H + I)$
- ◆ **Magnetic permeability** $\mu = \frac{B}{H}$
- ◆ **Magnetic susceptibility** $\chi_m = \frac{I}{H} = \mu_r - 1$
- ◆ **Curie law**
  - ❒ For paramagnetic materials $\chi_m \propto \frac{1}{T}$
- ◆ **Curie Wiess law**
  - ❒ For Ferromagnetic materials $\chi_m \propto \frac{1}{T - T_C}$

    Where T_C = curie temperature

KEY POINTS

- A charged particle moves perpendicular to magnetic field. Its kinetic energy will remain constant but momentum changes because magnetic force acts perpendicular to velocity of particle.
- If a unit north pole rotates around a current carrying wire then work has to be done because magnetic field produced by current is always non-conservative in nature.
- In a conductor, free electrons keep on moving but no magnetic force acts on a conductor in a magnetic field because in a conductor, the average thermal velocity of electrons is zero.
- Magnetic force between two charges is generally much smaller than the electric force between them because speeds of charges are much smaller than the free space speed of light.

Note : $\frac{F_{\text{magnetic}}}{F_{\text{electric}}} = \frac{v^2}{c^2}$

IMPORTANT NOTES

IMPORTANT NOTES

Electromagnetic Induction

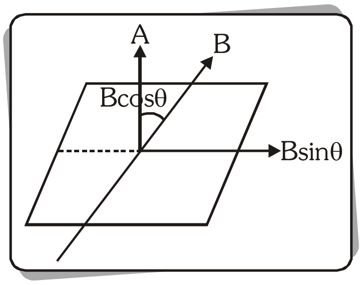
FARADAY'S LAW

Rate of change of magnetic flux is propotional to incuced emf $e = \frac{d\phi}{dt}$

MAGNETIC FLUX (ϕ)

Number of magnetic line of forces passing through a area perpendicular is known as magnetic flux.

Magnetic flux (ϕ) = $N\vec{B}.\vec{A} = NBA\cos\theta$



$$\text{emf} = \frac{-d\phi}{dt} = \frac{-d(\vec{B}.\vec{A})}{dt}$$

B changes	A changes	θ changes
$\text{emf} = -A\frac{dB}{dt}$	$\text{emf} = -B\frac{dA}{dt}$	$\text{emf} = -NBA\omega\sin\omega t$ (where $\theta = \omega t$)
Avg emf	Avg emf	Avg emf
$= \frac{A(B_i - B_f)}{t}$	$= \frac{B(A_i - A_f)}{t}$	$= \frac{NBA(\cos\theta_1 - \cos\theta_2)}{t}$

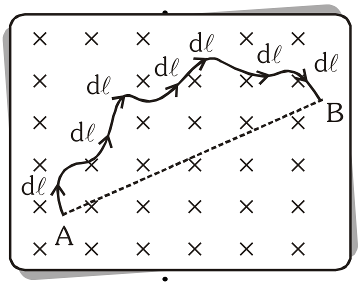
Lenz Law

Direction of induced current is such that it always try to oppose the course of change.

$$e = -\frac{d\phi}{dt}$$

Lanz Law Faraday Law

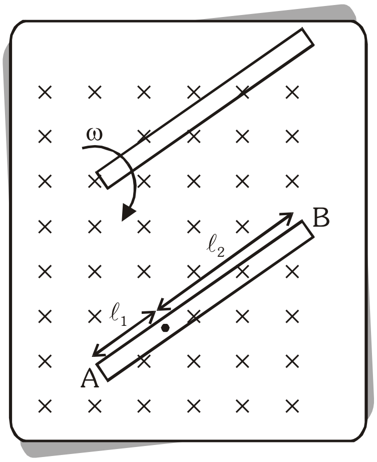
Motional EMF



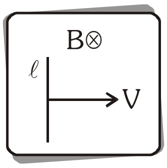
$e = \vec{B}\int(d\vec{\ell}\times\vec{v})$, $e = \vec{B}.(\vec{L}_{BA}\times\vec{v})$

Rod is rotating with angular velocity

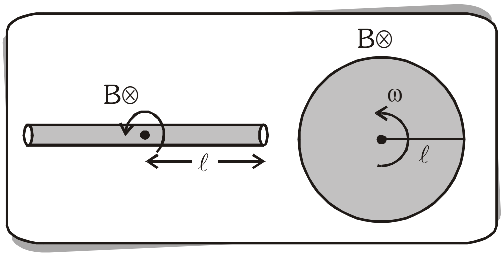
$$e = \frac{Bw\ell^2}{2} \qquad v_B - v_A = \frac{Bw(\ell_2^2 - \ell_1^2)}{2}$$



- When a rod moves perpendicular to its length and perpendicular to magnetic field then induces emf in rod $= B\ell v$ $(\vec{v}\perp\vec{\ell}\perp\vec{B})$



- When a conducting disc or conducting rod is rotated about its axis ⊥ to magnetic field then emf induced between its centre and periphery is given by emf $= \frac{B\omega\ell^2}{2}$



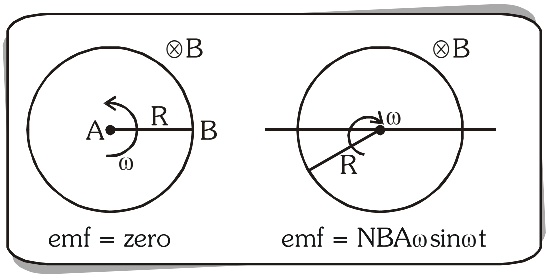
When a loop of area A is rotated about its diameter in uniform magnetic field B then maximum induced emf = NBAω

Case-II : If angle between area vector and magnetic field changes

$$\phi = \vec{B}.\vec{A}\cos\theta$$

$$e = \frac{d\phi}{dt} = BA\omega\sin\omega t$$

Note :



Self induction

Phenomena of inducing emf due to change in its own current $\phi = Li$ emf $= -\frac{Ldi}{dt}$

Self inductance (L) for solenoid

$$L = \frac{\mu_0 N^2 A}{\ell} = \mu_0 n^2 A\ell$$

N = number of turns; n = number of turns/length

Combination of inductors

Series $L = L_1 + L_2$ Parallel $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$

Energy stored in inductor :

$$U = \frac{1}{2}Li^2 = \frac{\phi^2}{2L} = \frac{\phi i}{2}$$

Mutual induction :

Phenomena of including emf in a coil due to change in current in another coil is known as mutual induction.

$\phi = Mi$ $\qquad$ $\text{emf} = -M\frac{di}{dt}$

Mutual inductance between 2 solenoids

$$M = \frac{\mu_0 N_1 N_2 A}{\ell}$$

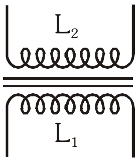
Relation between self inductance and mutal inductance.

Real

$M = k\sqrt{L_1L_2}$

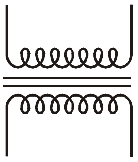
k = coupling factor

Ideal

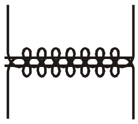


$M = \sqrt{L_1L_2}$

(k = 1)

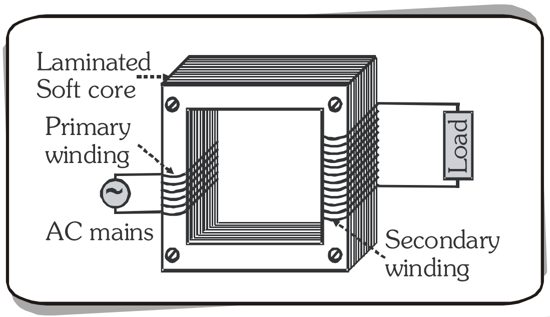


$0 < k < 1$



k = 1

TRANSFORMER



Works only for AC

Principle : Mutual induction

For ideal transformer

(1) Power loss = 0 ⇒ efficiency = 100%

(2) Flux loss = 0

But practicaly $P_{out} < P_{in}$ ∴ efficiency < 100%

$$\boxed{\frac{V_S}{V_P} = \frac{N_S}{N_P} = \frac{i_P}{i_S} = \text{turn ratio}} \qquad \eta(\text{efficiency}) = \frac{i_s^2 R}{V_P i_P} \times 100$$

Types of transformer

- Step up
  - $V_S > V_P$
  - $N_S > N_P$
  - $i_P > i_S$
- Step down
  - $V_P > V_S$
  - $N_P > N_S$
  - $i_S > i_P$

Transmissions are done at high voltage and low current by using step up transformer.

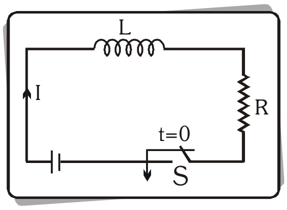
Losses in tranformers

- Coil (Cu losses)
  - Heat loss: Can be minimize by using thick wires
  - Flux loss: Can zero by making coupling factor1
- Core (Iron losses)
  - Hysteresis: Can be controlled by using substance of high μ_r (soft iron)
  - Eddy current: Can be controlled by laminating the core

Induced electric field :

Produced due to change in magnetic field and is non-conservative in nature $\boxed{\oint \vec{E}.d\vec{\ell} = -\frac{d\phi_B}{dt}}$

Growth of a Current in an L – R Circuit



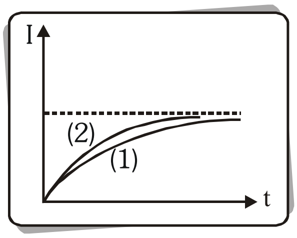
$I = \frac{E}{R}(1 - e^{-Rt/L})$. [If initial current = 0]

$I = I_0(1 - e^{-t/\tau})$

$\tau = \frac{L}{R}$ = time constant of the circuit.

$I_0 = \frac{E}{R}$.

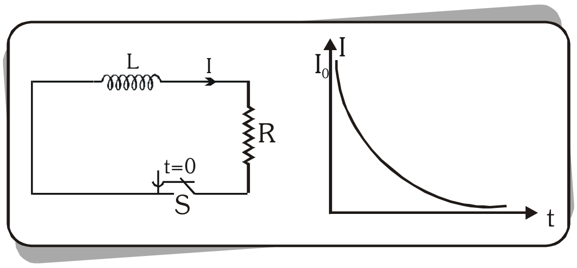
(i) L behaves as open circuit at $t = 0$ [as if $I = 0$]
(ii) L behaves as short circuit at $t = \infty$ always.



Curve (1) $\longrightarrow \frac{L}{R}$ Large Curve (2) $\longrightarrow \frac{L}{R}$ Small

Decay of Current

Initial current through the inductor = I_0 ; Current at any instant $i = I_0e^{-Rt/L}$



KEY POINTS

- An emf is induced in a closed loop where magnetic flux is varied. The induced electric field is not conservative field because for induced electric field, the line integral $\oint \vec{E}.d\vec{\ell}$ around a closed path is non-zero.
- Acceleration of a magnet falling through a long solenoid decrease because the induced current produced in a circuit always flows in such direction that it opposes the change or the cause that produces it.
- The mutual inductance of two coils is doubled if the self inductance of the primary and secondary coil is doubled because mutual inductance $M \propto \sqrt{L_1L_2}$.

IMPORTANT NOTES

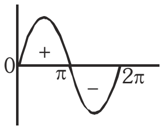
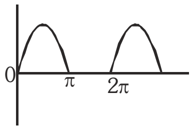
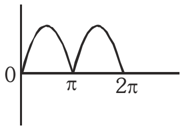
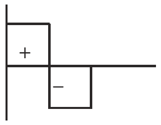
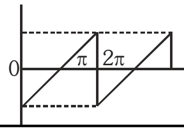
Alternating Current & EM Waves

Voltage or current is said to be alternating if periodically it changes its dir. and magnitude.

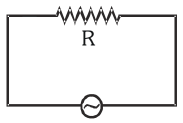
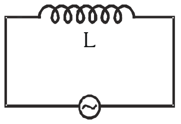
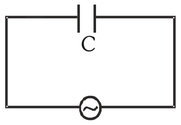
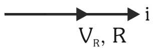
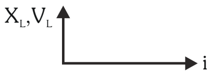
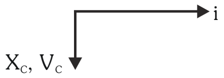
$i = I_0 \sin \omega t \qquad v = V_0 \sin(\omega t + \phi)$

$$\text{Average current} = \frac{\int_0^t i\,dt}{\int_0^t dt} \qquad i_{rms} = \sqrt{\frac{\int_0^t i^2 dt}{\int_0^t dt}}$$

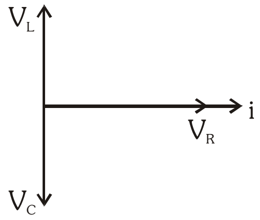
AC ammeter and voltmeter reads RMS value of current and voltages respectively $\Rightarrow i_0 > i_{rms} > i_{AV}$

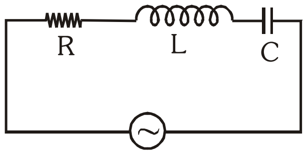
Nature of wave form	Wave–form Value for half cycle	RMS Value	Average or mean
Sinusoidal		$\frac{I_0}{\sqrt{2}} = 0.707\ I_0$	$\frac{2I_0}{\pi} = 0.637\ I_0$(half cycle)
Half wave rectifier		$\frac{I_0}{2} = 0.5\ I_0$	$\frac{I_0}{\pi} = 0.318\ I_0$ (full cycle)
Full wave rectifier		$\frac{I_0}{\sqrt{2}} = 0.707\ I_0$	$\frac{2I_0}{\pi} = 0.637\ I_0$
Square or Rectangular		I_0	I_0
Saw Tooth wave		$\frac{I_0}{\sqrt{3}}$	$\frac{I_0}{2}$

AC CIRCUITS

R	L	C
		
$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
$i = \frac{V_0}{R} \sin \omega t$	$i = \frac{V_0}{\omega L}(-\cos \omega t)$	$i = \frac{V_0}{(1/\omega C)} \cos \omega t$
Resistance = R	Reactance $X_L = \omega L$	Reactance $X_C = \frac{1}{\omega C}$
		
$V_R = iR$	$V_L = iX_L$	$V_C = iX_C$

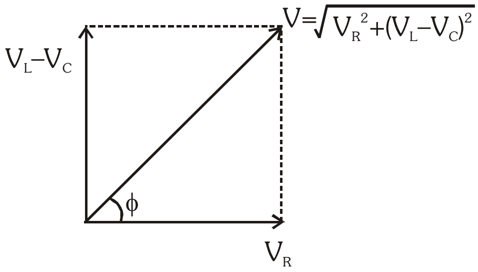
AC THROUGH LCR CIRCUIT

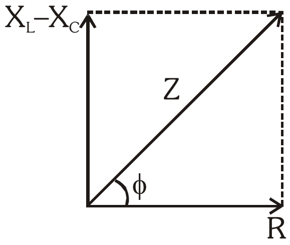




$v=V_0\sin\omega t$

If $V_L > V_C \Rightarrow X_L > X_C$ then

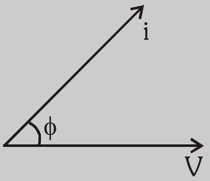




Impedence= $Z=\sqrt{R^2+(X_L-X_C)^2}$ and **admittance** = $\frac{1}{Z}$ and $\left(i=\frac{V}{Z}\right)$ and $\left(\cos\phi=\frac{R}{Z}\right)$

Power in AC Circuit :

$V = V_0\sin\omega t$
$i = I_0\sin(\omega t+\phi)$
Power = $V_{rms}\, i_{rms}\cos\phi = i^2R$
Wattfull current = $i_{rms}\cos\phi$
Wattless current = $i_{rms}\sin\phi$
Wattless power = $v_{rms}\, i_{rms}\sin\phi$
Where $\cos\phi$ = Power factor

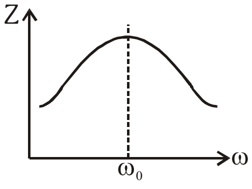


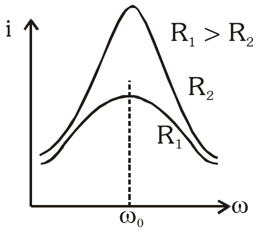
RESONANCE IN SERIES LCR CIRCUIT

At resonance

- $X_L = X_C$ or $V_L = V_C$
- $Z = R = \min \Rightarrow i=\frac{V}{R}=\max$
- Power factor $\left(\cos\phi=\frac{R}{Z}=1\right)$
- Angle (or phase defference) Between v and i = 0°
- $V_R = V_{Source}$

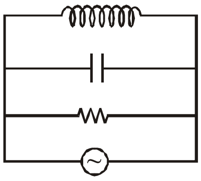
Resonating frequency $\boxed{\omega_0=\frac{1}{\sqrt{LC}}}$





Sharpness∝ quality factor = $\frac{X_L}{R}=\frac{1}{R}\sqrt{\frac{L}{C}}=\frac{f_0}{\text{Band width}}$

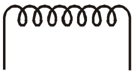
L-C-R PARALLEL COMBINATION



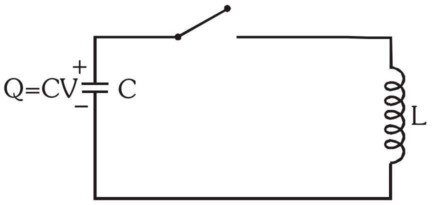
$$\frac{1}{Z}=\sqrt{\frac{1}{R^2}+\left(\frac{1}{x_L}-\frac{1}{x_C}\right)^2}$$

CHOKE COIL

It is used to control alternating current without any power loss. It is an inductor and low resistance.

high L, low R  $Z \approx X_L \Rightarrow$ Power = 0

LC - OSCILLATION

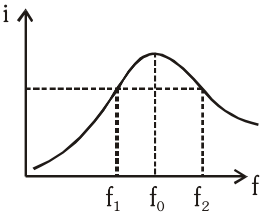


$V_C + V_L = 0$ $\boxed{L\dfrac{d^2Q}{dt^2} + \dfrac{Q}{C} = 0}$

$Q = Q_0 \cos \omega t \Rightarrow i = -i_0 \sin \omega t$ where $i_0 = Q_0\omega$

where $\boxed{\omega = \dfrac{1}{\sqrt{LC}}}$ frequency of oscillation

- Q value $= \dfrac{\text{Resonance frequency}}{\text{Band width}}$



E.M.W

Maxwell's equations

1. $\oint \vec{E}\cdot\vec{dA} = \dfrac{Q}{\varepsilon_0}$ (Gauss's law of electricity)
2. $\oint \vec{B}\cdot\vec{dA} = 0$ (Gauss's law of magnetism)
3. $\oint \vec{E}\cdot\vec{d\ell} = -\dfrac{d\phi_E}{dt}$ (Faraday is law)
4. $\oint \vec{B}\cdot\vec{d\ell} = \mu_0\left[i_c + \varepsilon_0\dfrac{d\phi_E}{dt}\right]$ (Empere – maxwell law)

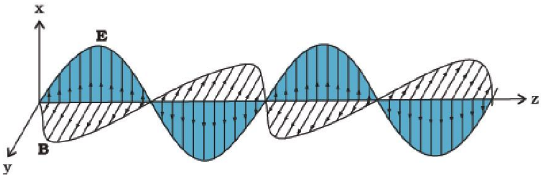
DISPLACEMENT CURRENT

$$\phi = EA = \frac{Q}{\varepsilon_0}$$

$$\frac{d\phi}{dt} = \frac{1}{\varepsilon_0}\frac{dQ}{dt} \Rightarrow i_d = \varepsilon_0\frac{d\phi_E}{dt}$$

Electromagnetic wave :
$E_x = E_0 \sin(kz - wt)$
$B_y = B_0 \sin(kz - wt)$

$$C = \frac{1}{\sqrt{\mu_0\varepsilon_0}} = \frac{E_0}{B_0}$$



Direction of propagation of light $\hat{E}\times\hat{B}$ ·

Poynting vector $\vec{S} = \dfrac{1}{\mu_0}(\vec{E}\times\vec{B})$

Electric field energy density $= \dfrac{1}{2}\varepsilon_0 E^2$;

Magnetic field energy density $= \dfrac{B^2}{2\mu_0}$

Total energy density $= \dfrac{1}{2}\varepsilon_0 E^2 + \dfrac{B^2}{2\mu_0} = \dfrac{1}{2}\varepsilon_0 E_0^2 = \dfrac{B_0^2}{2\mu_0}$

If total energy transferred to a surface in time t is U, total momentum delivered to this surface is p = U/c.

Type	Wavelength range	Production	Detection
Radio	> 0.1 m	Rapid acceleration and decelerations of electrons in aerials	Receiver's aerials
Microwave	0.1m to 1 mm	Klystron valve or magnetron valve	Point contact diodes
Infra-red	1mm to 700 nm	Vibration of atoms and molecules	Thermopiles Bolometer, Infrared photographic film
Light	700 nm to 400 nm	Electrons in atoms emit light when they move from one energy level to a lower energy level	The eye Photocells Photographic film
Ultraviolet	400 nm to 1nm	Inner shell electrons in atoms moving from one energy level to a lower level	Photocells Photographic film
X-rays	1nm to 10^{-3} nm	X-ray tubes or inner shell electrons	Photographic film Geiger tubes Ionisation chamber
Gamma rays	$<10^{-3}$ nm	Radioactive decay of the nucleus	-do-

KEY POINTS

- An alternating current of frequency 50 Hz becomes zero, 100 times in one second because alternating current changes direction and becomes zero twice in a cycle.
- An alternating current cannot be used to conduct electrolysis because the ions due to their inertia, cannot follow the changing electric field.
- Average value of AC is always defined over half cycle because average value of AC over a complete cycle is always zero.
- AC current flows on the periphery of wire instead of flowing through total volume of wire. This known as skin effect.

CHAPTER 21

Modern Physics

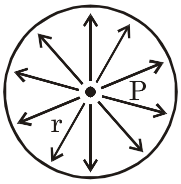
PHOTON

- Max plank said light is mode up of discrete packets of energy called 'Photon' according to his quantum theory.
- Energy of photon, E = hν, where h = Plank's constant = 6.63×10^{-34} Fs and ν = frequency of light used.
- Effective mode of photon, $m = \frac{E}{C^2}$; $m \propto \frac{1}{\lambda}$. So effective mass of violet light is more then effective mass of red light photon.
- Rest mass of a photon is zero.
- Linear momentum of photon

$$P = \frac{E}{C} = \frac{h\nu}{c} = \frac{h}{\lambda}$$

INTENSITY OF LIGHT

$$\text{Intensity} = \frac{\text{Energy}}{\text{time} \times \text{Area}}$$



Point source

$I = P\ 4\pi r^2$

where P = power of source

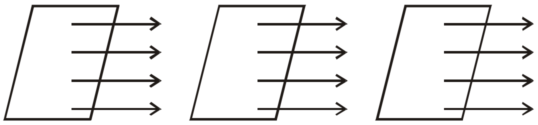
Infinite linear source

$$I = \frac{P}{2\pi r \ell}$$

Infinite planar source

I = constant (independent of distance)

$I \propto P$

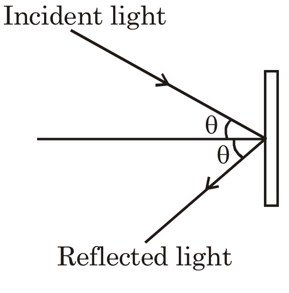


$$I = \frac{Nh\nu}{At},\ n = \frac{N}{t} = \frac{I \times A}{h\nu} =$$

No of photons incident per second

Force exerted by light on a surface

$F = \frac{IA}{C}(1 + r)\cos\theta$, where r = reflectivity



Radiation Pressure (P)

$$P = \frac{F}{A} = \frac{I}{C}(1 + r)\cos^2\theta$$

Photo Electric Effect (PEE)

This experiment shows particle nature of light.

Electrons are emitted a metal surface when light is incident upon it only when $\nu \geq \nu_0$ (Threshold frequency) and P.E.E. is independant of intensity of light. This is shown as ther is no time rag in emission of electron.

Einsten's Photoelectric Equation

$h\nu = \phi t\ KE_{max}$.

where ϕ = work function which depends on metal

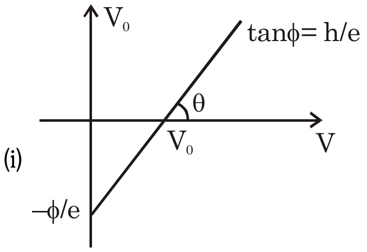
KE_{max} = maximum KE that an e^- can have after emission.

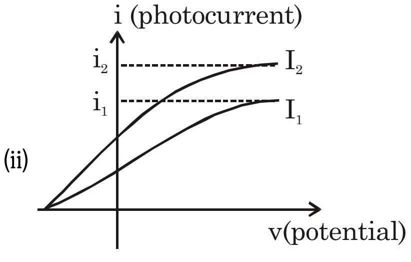
$KE_{max} = eV_0$, where V_0 = stopping potential or cut of potential

So, $h\nu = \phi + eV_0$

or $V_0 = \frac{h\nu}{e} - \frac{\phi}{e}$

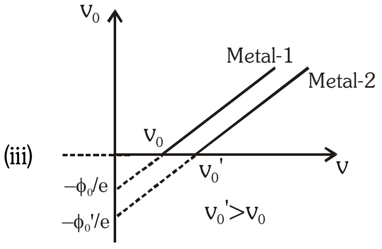
Graphs of Photo Electric Effect



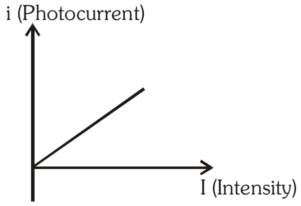


i_1, i_2 = Saturation currents

$I_2 > I_1$ (ν = same)



(iv)



Quantum efficiency

Quantum efficiency =

$$\frac{\text{No. of electron emitted per second}}{\text{Total no. of photon incident per second}}$$

$i = n\frac{IA\lambda}{hc}$ (Saturation current)

Refer graph (iv)

Photocell : Works on PEE, Photocell light energy is converted into electrical energy.

Matter waves theory

Light has dual nature

Experiments like reflection, refraction, interference deffraction are explained only ont the basis of wave theory of light.

Experiments the PEE and crompton effect, pair production and positon axilation are explained on the basis of particle nature of light.

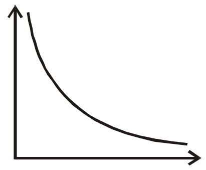
Atomic Structure

Various Models for structure of Atom

(i) Dalton's Theory

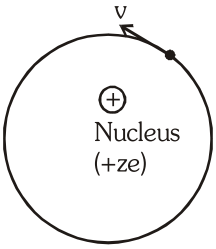
(a) Thomson Model

(b) Rutherford model



(Number of α-particles scattered at an angle θ)

Bohr atomic model



(i) Electron revolves circular orbit around nucleus

(ii) $mvr = \frac{nh}{2\pi}$

So electron has discrete angular momentum and is allowed to be present in certain fixed orbits only (called as stationary energy levels or shells.)

(iii) Electrons denot radiate energy when in shells but energy is radiated or aborbed when an electrons jumps to lower or higher orbit respectively.

Mathematical Analysis of Bohr's Theory

(i) $v = 2.2 \times 10^6 \ 2/n \ \text{m/s}$

(ii) $r = 0.53 \ \frac{n^2}{z}\text{Å}$

(iii) Total energy of electron in nth orbit,

$$E_n = -13.6\frac{z^2}{n^2}\text{eV}$$

(iv) $T = \left(\frac{n^3}{z^2}\right)1.51 \times 10^{-16}\text{s}$

(v) $f = \left(\frac{z^2}{n^3}\right)6.6 \times 10^{15}\text{Hz}$

(vi) $\frac{1}{\lambda} = Rz^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$,

R_H = Rydberg constant = $\frac{1.097 \times 10^{-3}}{\text{Å}}$

(For stationary nucleus)

$R' = \frac{R_H}{1 + m/M}$ (If nuceus is not stationary)

i.e. mass of nucleus and revolving particle are comparable)

where m = mass of revolving particle

M = mass of nucleus

S.No.	Series Observed	Value of n_1	Values of n_2	Position in the Spectrum
1.	**Lyman Series**	**1**	**2, 3, 4 ..... ∞**	**Ultra Violet**
2.	**Balmer Series**	**2**	**3, 4, 5 ..... ∞**	**Visible**
3.	**Paschen Series**	**3**	**4, 5, 6 ..... ∞**	**Infra-red**
4.	**Brackett Series**	**4**	**5, 6, 7 ..... ∞**	**Infra-red**
5.	**Pfund Series**	**5**	**6, 7, 8 ..... ∞**	**Infra-red**

De Broglie Hypothesis

He said there wave nature of very particle just the light has dual nature.

$$\lambda_D = \frac{h}{p} = \frac{h}{\sqrt{2mK\sqrt{2}}},$$

where λ_D = De-Broglie wvaelength of any particle.

$$\lambda_D = \frac{h}{\sqrt{2mqV}}$$; if particle has charger and is accelerated by V

$$\lambda_{\text{electron}} = \frac{12.27}{\sqrt{V}}$$

$$\lambda_{\text{Proton}} = \frac{0.286}{\sqrt{V}}\text{Å}$$

$$\lambda_{\text{Deutron}} = \frac{0.202}{\sqrt{V}}\text{Å}$$

$$\lambda_{\alpha-\text{Particle}} = \frac{0.101}{\sqrt{V}}\text{Å}$$

Bohr's quantigation

$$mvr = \frac{nh}{2\pi}$$

Total emission spectral lines

From n_1 = n to n_2 = 1 state $= \frac{n(n-1)}{2}$

From n_1 = n to n_2 = m state is $= \left(\frac{(n-m)(n-m+1)}{2}\right)$

$= {}^{n-m}C_2$

Excitation potential of atom

$$n_1 \to n_2 = \frac{E_{n2} - E_{n1}}{\text{electron charge}}$$

Ionization energy of hydrogen atom

It is the energy required to remove an electron from an atom

ex I.E.of Hydrogen = 0 – (– 13.6) = 13.6 eV.

Ionization Potential

It is the potential required correspongin to ionization energy in order to remove the electron fro the atom

$$= \frac{-E_n}{\text{electronic charge}}$$

X - RAYS

Produced by bombording high speed electrons on a target of high atomic weight and high melting point. The we basically highly magnetic photons.

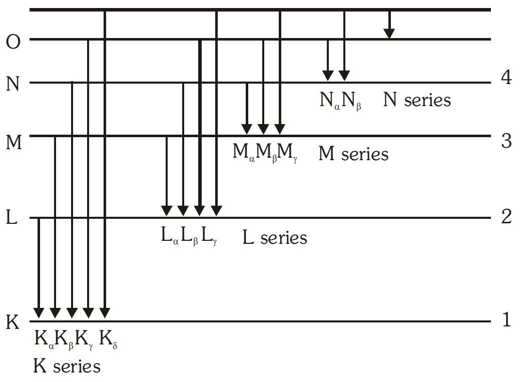
	Soft X–ray	Hard X–ray
Wavelength	10 Å to 100 Å	0.1 Å – 10 Å
Energy	$\frac{12400}{\lambda}$ eV–Å	$\frac{12400}{\lambda}$ eV–Å
Penetrating power	Less	More
UseRadio photography	Radio therapy	

- Intensity of X-ray × current flowing through filament
- Rentetrating power × Applied Potential difference.

- Continous X-ray – Produce when electron while they hit the target
- Cutt-off wavelength – Mininum wavelength of continuous X-rays which appears when as electron loosses all its KE. in its first collision only. Hence producing photon of maximum energy and of minimum wavelength.

  $\lambda_{min} = \frac{12400}{V}\text{Å}$, where V is applied potential difference
- Characteristic X-ray – Produce when electron hitting the metal target ejects on electron from shell and that vacant space when occupied by an electron from upper shell, produces a photon.

From Bohr Model


O
N
M
L
K
4
3
2
1
$N_\alpha N_\beta$ N series
$M_\alpha M_\beta M_\gamma$ M series
$L_\alpha L_\beta L_\gamma$ L series
$K_\alpha K_\beta K_\gamma K_\delta$
K series

$n_1 = 1$, $n_2 = 2, 3, 4$.......K series

$n_1 = 2$, $n_2 = 3, 4, 5$.......L series

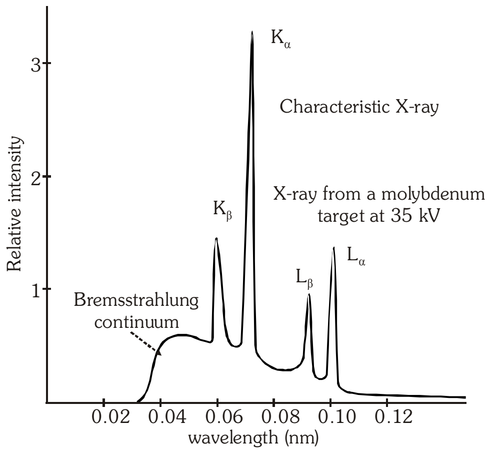
$n_1 = 3$, $n_2 = 4, 5, 6$.......M series

First line of series = α

Second line of series = β

Third line of series = γ

Transition	Wave–length	Energy	Energy difference	Wavelength
$L \to K$	$\lambda_{K\alpha}$	$h\nu_{K\alpha}$	$-(E_K - E_L)$	$\lambda_{K\alpha} = \frac{hc}{(E_K - E_L)}$
$(2 \to 1)$			$= h\nu_{K\alpha}$	$= \frac{12400}{(E_K - E_L)}\text{eVÅ}$
$M \to K$	$\lambda_{K\beta}$	$h\nu_{K\beta}$	$-(E_K - E_M)$	$\lambda_{K\beta} = \frac{hc}{(E_K - E_M)}$
$(3 \to 1)$			$= h\nu_{K\beta}$	$= \frac{12400}{(E_K - E_M)}\text{eVÅ}$
$M \to L$	$\lambda_{L\alpha}$	$h\nu_{L\alpha}$	$-(E_L - E_M)$	$\lambda_{L\alpha} = \frac{hc}{(E_L - E_M)}$
$(3 \to 2)$			$= h\nu_{L\alpha}$	$= \frac{12400}{(E_L - E_M)}\text{eVÅ}$



MOSELEY'S LAW

$\sqrt{\nu} \propto (Z - b)$ where ν = frequancy of characteristic x-ray

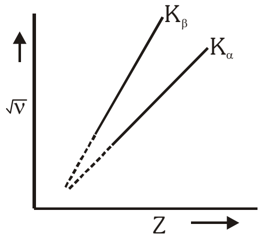
Z = atomic number of target

ν = frequency of characteristic spectrum

b = screening constant (for K– series b=1, L series b=7.4)

a = proportionality constant

$$\nu = RcZ^2\left[\frac{1}{n_1^2} - \frac{1}{n_2^2}\right]$$



	Bohr model		Moseley's correction
1.	For single electron species	1.	For many electron species
2.	$\Delta E = 13.6Z^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$ eV	2.	$\Delta E = 13.6\,(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$ eV
3.	$\nu = RcZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$	3.	$\nu = Rc(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$
4.	$\frac{1}{\lambda} = RZ^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$	4.	$\frac{1}{\lambda} = R\,(Z-1)^2\left[\frac{1}{n_1^2}-\frac{1}{n_2^2}\right]$

- For X–ray production, Moseley formulae are used because heavy metal are used.

When target is same $\lambda \propto \dfrac{1}{\frac{1}{n_1^2}-\frac{1}{n_2^2}}$ **When transition is same** $\lambda \propto \dfrac{1}{(Z-b)^2}$

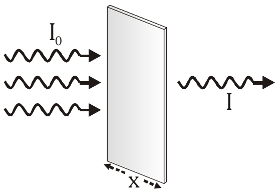
ABSORPTION OF X–RAY

$I = I_0e^{-\mu x}$

where I_0 = Intensity of incident X–ray

I = Intensity of X–ray after passing through x distance

μ = absorption coefficient of material



- Intensity of X–ray decrease exponentially.
- Maximum absorption of X–ray → Lead
- Minimum absorption of X–ray → Air

Half thickness ($x_{1/2}$)

It is the distance travelled by X–ray when intensity become half the original value $x_{1/2} = \frac{\ell n_2}{\mu}$

DIFFRACTION OF X–RAY

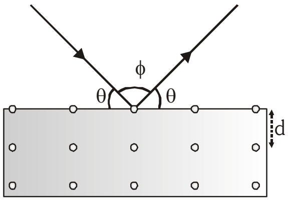
$2d\sin\theta = n\lambda$

where, d = spacing of crystal plane or lattice constant or

distance between adjacent atomic plane

θ = Bragg's angle or glancing angle

ϕ = Diffracting angle n = 1, 2, 3 ....



For Maximum Wavelength

$\sin\theta = 1, n = 1 \Rightarrow \lambda_{max} = 2d$

so if $\lambda > 2d$ diffraction is not possible i.e. solution of Bragg's equation is not possible.

NUCLEAR PHYSICS

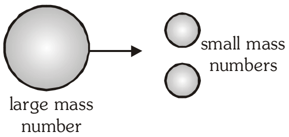
BINDING ENERGY

- Binding energy of a nucleus = Δmc^2
  where Δm = mass defect = $[Zm_p + (A-Z)m_n] - m_{nuc}$
  = Totall mass of nucleus mass of nucleus.
- Binding energy per nucleus = $\frac{\text{B.E.}}{\text{mass number}} = \frac{\text{B.E.}}{A}$

  Stability $\propto \frac{\text{B.E.}}{A}$
- $\frac{\text{B.E.}}{A}$ is maximum for A =62 (Ni), It is 8.79460 ± 0.00003 MeV/nucleon

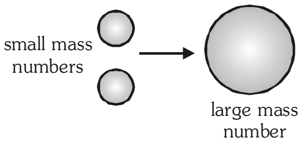
NUCLEAR FISSION

When heavy nuclei achieve stability by braking into two smaller nuclei, then it is called as nuclear fission reaction.



NUCLEAR FUSSION

When lighter nuclei achieve stability by combining and resulting into heavy nucleus and this reaction is called fusion reaction.



Q VALUE

It is teh theory released after trhe nuclei avhieve stability through nuclear fissiion or fussion reactions

It is always positive when the reaction occurs towards achieving more stability.

It Q value of a reaction is negative, it means taht reaction cannot occur as the products in that case will be more constable than the reaction.

NUCLEAR FISSION OF U^{235}

$U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3{}_0n^1 + 200\text{ MeV}$ or $U^{235} + {}_0n^1 \rightarrow Xe + Sr + 2{}_0n^1 + 200\text{ MeV}$

and many other reactions are possible.

- The average number of secondary neutrons is 2.5.
- Nuclear fission can be explained by using "liquid drop model" also.
- Δm is about 0.1% of mass of fissioned nucleus
- About 93% of released energy (Q) is appear in the form of kinetic energies of products and about 7% part in the form of γ – rays.

NATURAL URANIUM :

It is mixture of U^{235} (0.7%) and U^{238} (99.3%).

U^{235} is easily fissionable, by slow neutron (or thermal neutrons) having K.E. of the order of 0.03 eV. But U^{238} is fissionable with fast neutrons.

ENRICHED URANIUM

It contains 97% U^{238} and 3% U^{235}.

CRITICAL SIZE (OR MASS) :

It is minimum mass of uranium required to sustain a chain reacter.

REPRODUCTION FACTOR :

$$(K) = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

(i) If K = 1; chain reaction is steady or sustained (As in nuclear reaction)

(ii) If K > 1; chain reaction will accelerate resulting in a explosion (As in atom bomb)

(iii) If K < 1; chain reaction will retard and will stop.

NUCLEAR REACTOR (K = 1) :

- **Nuclear Fuel :** U^{235} , Pu^{239} .Pu^{239} is the **best**. Its critical size is less than that of U^{235}. But Pu^{239} is not naturally available. So U^{235} is used in most of the reactors.
- **Moderator :** They are used to slow down the fast secondary neutrons D_2O, Graphite etc.
- **Control rods :** They are used absorbs slow neutrons e.g. Boron and Cadmium.
- **Coolant :** It is used to absorb heat and transfers it to water for further use.

FAST BREADER REACTORS

It is the atomic reactor in which fresh fissionable fuel (Pu^{239}) is produced along with energy. The amount of produced fuel (Pu^{239}) is more than consumed fuel (U^{235})

- **Fuel :** Used in this reactor Natural uranium.
- **Process :**

$${}_{92}U^{238} + {}_0n^1 \longrightarrow {}_{92}U^{239} \xrightarrow{2\beta^-} {}_{94}Pu^{239} \text{ (best fuel of fission)}$$

- **Moderator :** Are not used in these reactors.
- **Coolant :** Liquid sodium

REQUIRED CONDITION FOR NUCLEAR FUSION

- **High temperature**
- **High Pressure (or density)**

HYDROGEN BOMB

Based on nuclear fusion and produces more energy than an atom bomb (based on nuclear fission).

PAIR PRODUCTION

e^- and e^+ pair is produced when in γ-photon having energy >, 1.02 MeV strikes a nucleus.

PAIR ANMIHILATION

When electron and positron combines, 2 γ-photon are formed, reach photon having energy >, 0.5 MeV.

RADIOACTIVITY

Radioactive Decays

- **α decay :** Occurs in nucleus having A>210
- **β decay :**
  - **A type**

    $(N/Z)_A > (N/Z)$ stable

    To achieve stability, it increases Z by conversion of neutron into proton

    $${}_0n^1 \to {}_1p^1 + e^{-1} + \bar{\nu},\ {}_ZX^A \to {}_{Z+1}Y^A + \underset{(\beta\ \text{particle})}{e^{-1}} + \bar{\nu}$$

    This decay is called β^{-1} decay.

    Kinetic energy available for e^{-1} and $\bar{\nu}$ is, $Q = K_\beta + K_{\bar{\nu}}$

- **B type**

  $(N/Z)_B < (N/Z)$ stable

  To achieve stability it decreases Z by the conversion of a proton into neutron. That is,

  $$p \to n + \underset{(\text{positron})}{e^+} + \underset{(\text{neutrino})}{\nu}\ ,\ {}_Z X^A \to {}_{Z-1} Y^A + \underset{(\beta^+ \text{particle})}{e^+} + \nu$$

- **γ decay :** When daughter nucleus in higher energy state comes to ground state. by emitting a photon or photons. a r photon is released.

e.g. $${}^{67}_{27}Co \to \underset{(\text{higher energy state})}{{}^{67}_{28}Ni^*} + \beta^- + \bar{\nu}\ ,\ {}^{67}_{28}Ni^* \to {}^{67}_{28}Ni + \gamma \text{ photon}$$

Properties of α, β and γ rays

Features	**α–particles**	**β–particles**	**γ–rays**
Identity	Helium nucleus or doubly ionised helium atom (${}_2He^4$)	Fast moving electrons $({}_{-}\beta^0$ or $\beta^-)$	Electromagnetic wave (photons)
Charge	Twice of proton (+2e) ≈ $4m_p$	Electronic (– e)	Neutral
Mass	(rest mass of β) m_p–mass of proton	rest mass = 0 = (rest mass of electron)	
Speed	1.4×10^7 m/s. to 2.2×10^7 m/s. (Only certain value between this range). Their speed depends on nature of the nucleus. So that it is a characteristic speed.	1% of c to 99% of c (All possible values between this range) β–particles come out with different speeds from the same type of nucleus. So that it can not be a characteristic speed.	Only $c = 3 \times 10^8$ m/s γ–photons come out with same speed from all types of nucleus. So, can not be a characteristic speed.
K.E.	≈ MeV	≈ MeV	≈ MeV
Energy spectrum	Line and discrete (or linear)	Continuous (or linear)	Line and discrete
Ionization	10,000 times	100 times of γ–rays	
power (α>β>γ)	of γ–rays	(or $\frac{1}{100}$ times of α)	1 (or $\frac{1}{100}$ times of β)
Penetration	$\frac{1}{10000}$ times of γ–rays	$\frac{1}{100}$ times of γ–rays	1(100 times of β)
power (γ>β>α)		(100 times of α)	
Effect of electric or magnetic field	Deflection	Deflection (More than α)	No deflection
Explanation of emission	By Tunnel effect (or quantum mechanics)	By weak nuclear interactions	With the help of energy levels in nucleus

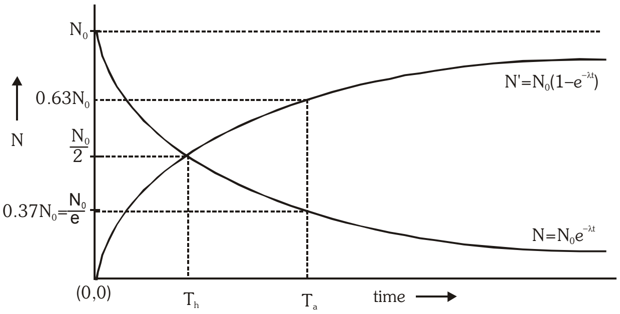
Laws of Radioactive Decay

Rate of decay $\propto$ number of nuclei.

$$\frac{dN}{dt} = -\lambda N$$

where λ is called the decay constant. This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$.

$$N = N_0 e^{-\lambda t}$$



- **Half life (T_h)** : Time during which number of active nuclei reduce to half of initial value.

$$T_h = \frac{\ell n(2)}{\lambda}$$

- **Mean or Average Life (T_a)** : It is the average of age of all active nuclei i.e.

$$T_a = \frac{\text{sum of times of existance of all nuclei in a sample}}{\text{initial number of active nuclei in that sample}} = \frac{1}{\lambda}$$

ACTIVITY OF A SAMPLE (A OR R) (OR DECAY RATE)

$$R = -\frac{dN}{dt} = N\lambda \text{ or } R = R_0 e^{-\lambda t}$$

SI UNIT of R : 1 becquerel (1 Bq)= 1 decay/sec

Other Unit is curie : 1 Ci = 3.70×10^{10} decays/sec

1 Rutherford : (1 Rd) = 10^6 decays/s

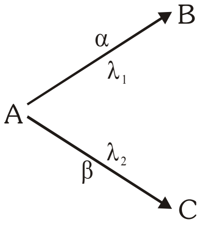
Specific activity : Activity of 1 gm sample of radioactive substance. Its unit is Ci/gm e.g. specific activity of radium (226) is 1 Ci/gm.

- **Parallel radioactive disintegration**

$$\Rightarrow \frac{dN_A}{dt} = -(\lambda_1 + \lambda_2)N_A$$

$$\Rightarrow \lambda_{eff} = \lambda_1 + \lambda_2$$

$$\Rightarrow t_{eff} = \frac{t_1 t_2}{t_1 + t_2}$$



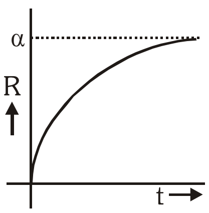
Radioactive Disintegration with Successive Production

$$\xrightarrow[(\alpha=\text{rate of production})]{\alpha} A \xrightarrow{\lambda} B$$

$$\frac{dN_A}{dt} = \alpha = \lambda N_A \text{ ....(i)}$$

when N_A in maximum $\frac{dN_A}{dt} = 0 = \alpha - \lambda N_A = 0$, N_A or max $= \frac{\alpha}{\lambda}$

$$\int_0^t \frac{dN_A}{\alpha - \lambda N_A} = \int_0^t dt, \text{ Number of nuclei is } N_A = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$$

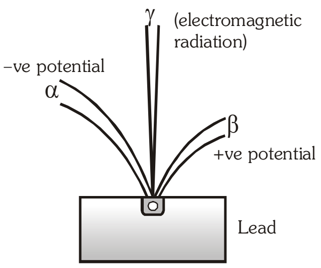


Soddy and Fajan's Group Displacement Laws :

(i) **α–decay :** After emission of one α–particle reduces the mass number by 4 units and atomic number by 2 units. ${}_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}(\alpha)$

(ii) **β–decay :** Mass number remains same and atomic bumber increases by 1. ${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + \beta + \overline{\nu}$

(iii) **γ–decay :** Both mass number and atomic number remains same, only energy is released in the form of γ–photons.



Ray Optics & Optical Instruments

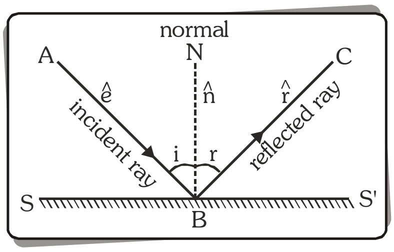
REFLECTION

LAWS OF REFLECTION

The incident ray (AB), the reflected ray (BC) and normal (NB) to the surface (SS') of reflection at the point of incidence (B) lie in the same plane. This plane is called the plane of incidence (also plane of reflection).

The angle of incidence (the angle between normal and the incident ray) and the angle of reflection (the angle between the reflected ray and the normal) are equal

$$\boxed{\angle i = \angle r}$$



In vector form $\boxed{\hat{\mathbf{r}} = \hat{\mathbf{e}} - 2(\hat{\mathbf{e}}.\hat{\mathbf{n}})\hat{\mathbf{n}}}$

OBJECT :

- **Real :** Point from which rays actually diverge.
- **Virtual:** Point towards which rays appear to converge

IMAGE :

- Image is decided by reflected or refracted rays only. The point image for a mirror is that point towards which the rays reflected from the mirror, actually converge (real image).

OR

- From which the reflected rays appear to diverge (virtual image).

CHARACTERISTICS OF REFLECTION BY A PLANE MIRROR :

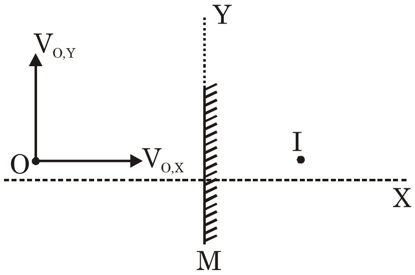
- The size of the image is the same as that of the object.
- For a real object the image is virtual and for a virtual object the image is real.
- For a fixed incident light ray, if the mirror be rotated through an angle θ the reflected ray turns through an angle 2θ in the same sense.

Number of images (n) in inclined mirror

Find $\frac{360}{\theta} = m$

- ☐ If m even, then n = m – 1, for all positions of object.
- ☐ If m odd , then n = m, If object not on bisector and n = m – 1, If object at bisector

VELOCITY OF IMAGE OF MOVING OBJECT (PLANE MIRROR)



(i) Velocity component along X-axis

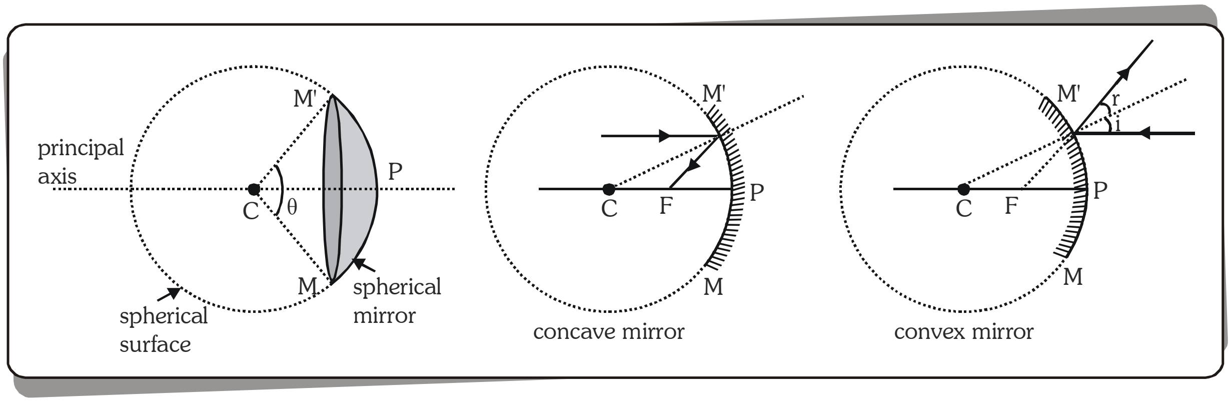
$$\vec{V}_{I,M} = -\vec{V}_{O,M}$$

$$\Rightarrow \vec{V}_{I,X} = 2\vec{V}_{M,X} - \vec{V}_{O,X}$$

(ii) Along Y-axis

$$\vec{V}_{I,Y} = \vec{V}_{O,Y}$$

SPHERICAL MIRRORS



PARAXIAL RAYS :

Rays which forms very small angle with principl axis are called paraxial rays. All formulae are valid for paraxial ray only.

SIGN CONVENTION :

- We follow cartesian co-ordinate system convention according to which the pole of the mirror is the origin.
- The direction of the incident rays is considered as positive x-axis. Vertically up is positive y-axis.
- All distance are measured from pole.

Note : According to above convention radius of curvature and focus of concave mirror is negative and of convex mirror is positive.

MIRROR FORMULA :

$$\frac{1}{f}=\frac{1}{v}+\frac{1}{u}.$$

f = x-coordinate of focus

u = x-coordinate of object

v = x-coordinate of image

Note : Valid only for paraxial rays.

TRANSVERSE MAGNIFICATION :

$$m=\frac{h_2}{h_1}=-\frac{v}{u}$$

h_2 = y co-ordinate of image

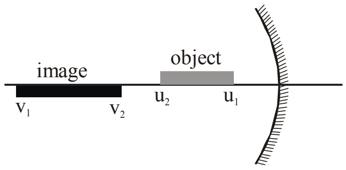
h_1 = y co-ordinate of the object

(both perpendicular to the principal axis of mirror)

Magnification	**Image**
$\lvert m\rvert > 1$	enlarged
$\lvert m\rvert < 1$	diminished
$m < 0$	inverted
$m > 0$	erect

LONGITUDINAL MAGNIFICATION

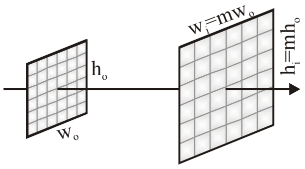
$$m_L=\frac{\text{length of image}}{\text{length of object}}=\frac{|v_2-v_1|}{|u_2-u_1|}$$



For small objects only : $m_L=-\frac{dv}{du}=m^2$

SUPERFICIAL MAGNIFICATION

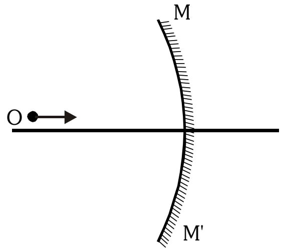
Linear magnification $m=\frac{h_i}{h_o}=\frac{w_i}{w_o}$



$$m_s=\frac{\text{area of image}}{\text{area of object}}=\frac{(ma)\times(mb)}{(a\times b)}=m^2$$

VELOCITY OF IMAGE OF MOVING OBJECT (SPHERICAL MIRROR)

Velocity component along axis (Longitudinal velocity)



When an object is coming from infinite towards the focus of concave mirror

$$\because \frac{1}{v}+\frac{1}{u}=\frac{1}{f} \qquad \therefore -\frac{1}{v^2}\frac{dv}{dt}-\frac{1}{u^2}\frac{du}{dt}=0$$

$$\Rightarrow \vec{v}_{IM}=-\frac{v^2}{u^2}\vec{v}_{ox}=-m^2\,\vec{v}_{OM}$$

- $v_{IM}=\frac{dv}{dt}$ = velocity of image with respect to mirror
- $v_{OM}=\frac{du}{dt}$ = velocity of object with respect to mirror.

(b) **Velocity component perpendicular to axis (Transverse velocity)**

$$m=\frac{h_I}{h_0}=-\frac{v}{u}=\frac{f}{f-u}\Rightarrow h_I=\left(\frac{f}{f-u}\right)h_0$$

$$\frac{dh_I}{dt}=\left(\frac{f}{f-u}\right)\frac{dh_0}{dt}+\frac{f\,h_0}{(f-u)^2}\frac{du}{dt}\;;$$

$$\vec{v}_{iy}=\left[m\vec{v}_{oy}+\frac{m^2h_0}{f}\vec{v}_{ox}\right]\hat{j}$$

$$\left[\begin{array}{l}\frac{dh_I}{dt}=\text{velocity of image}\perp^r\text{ to principal-axis}\\ \frac{dh_o}{dt}=\text{velocity of object }\perp^r\text{ to principal-axis}\end{array}\right]$$

Note : Here principal axis has been taken to be along x–axis.

NEWTON'S FORMULA :

Applicable to a pair of real object and real image position only. They are called conjugate positions or foci, X_1, X_2 are the distance along the principal axis of the real object and real image respectively from the principal focus

$$\boxed{X_1X_2=f^2}$$

OPTICAL POWER :

Optical power of a mirror (in Diopters) $=-\frac{1}{f}$

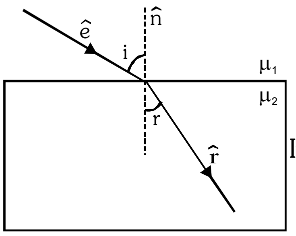
where f = focal length (in meters) with sign.

REFRACTION - PLANE SURFACE

LAWS OF REFRACTION (at any refracting surface)

Laws of Refraction

(i) Incident ray, refracted ray and normal always lie in the same plane.



In vector form $\boxed{(\hat{e}\times\hat{n}).\hat{r}=0}$

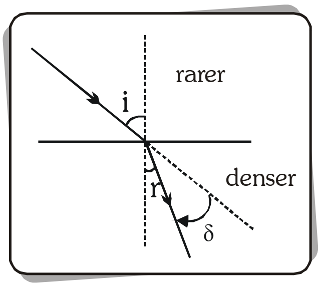
(ii) The product of refractive index and sine of angle of incidence at a point in a medium is constant. $\mu_1\sin i=\mu_2\sin r$ (Snell's law)

Snell's law : $\boxed{\frac{\text{Sin } i}{\text{Sin } r}={}_1\mu_2=\frac{\mu_2}{\mu_1}=\frac{v_1}{v_2}=\frac{\lambda_1}{\lambda_2}}$

In vector form $\boxed{\mu_1\,|\hat{\mathbf{e}}\times\hat{\mathbf{n}}|=\mu_2\,|\hat{\mathbf{r}}\times\hat{\mathbf{n}}|}$

Note : Frequency of light does not change during refraction.

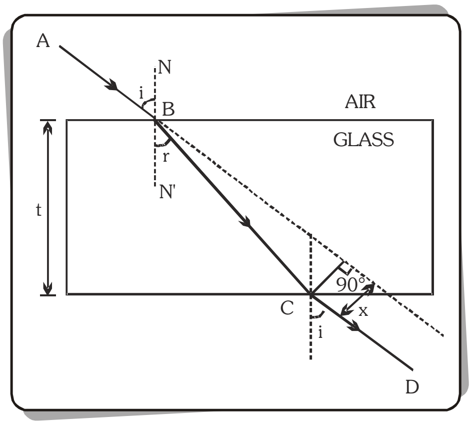
DEVIATION OF A RAY DUE TO REFRACTION



$\boxed{\text{angle of deviation, } \delta=i-r}$

REFRACTION THROUGH A PARALLEL SLAB

Emergent ray is parallel to the incident ray, if medium is same on both sides.

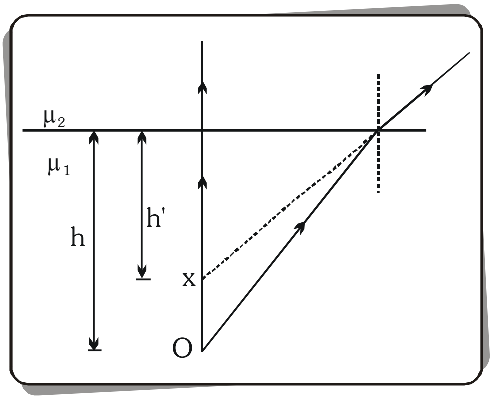


Lateral shift $x=\frac{t\sin(i-r)}{\cos r}$; t = thickness of slab

Note : Emergent ray will not be parallel to the incident ray if the medium on both the sides are different.

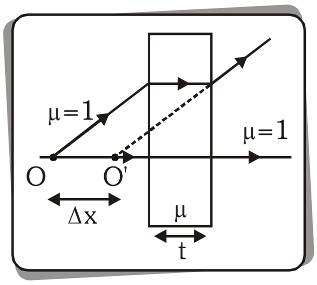
APPARENT DEPTH OF SUBMERGED OBJECT

$(h' < h)\ \mu_1 > \mu_2$



For near normal incidence $h' = \frac{\mu_2}{\mu_1} h$

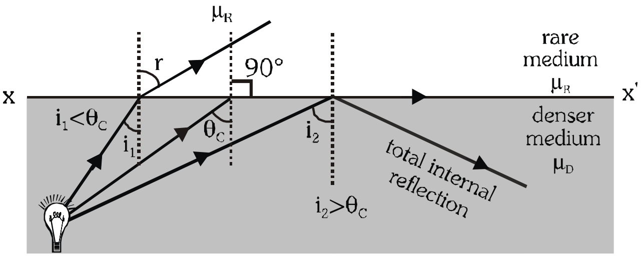
Note : h and h' are always measured from surface.



Δx = Apparent normal shift = $t\left(1 - \frac{1}{\mu}\right)$

Note : Shift is always in direction of incidence ray.

CRITICAL ANGLE & TOTAL INTERNAL REFLECTION (TIR)



CONDITIONS

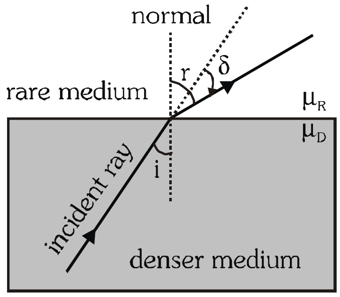
- Angle of incident > critical angle $[i > \theta_c]$
- Light should travel from denser to rare medium
  $\Rightarrow$ Glass to air, water to air, Glass to water
  Snell's Law at boundary xx', $\mu_D \sin\theta_C = \mu_R \sin 90°$

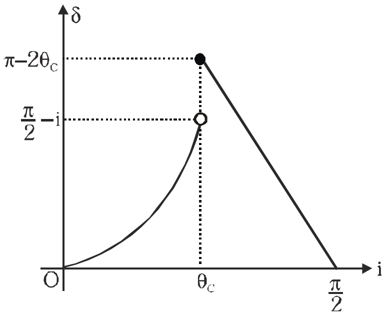
$$\Rightarrow\ \sin\theta_C = \frac{\mu_R}{\mu_D}$$

Graph between angle of deviation (δ) and angle of incidence (i) as rays goes from denser to rare medium

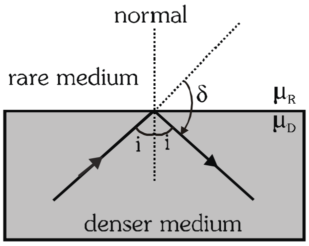
- If $i < \theta_c$ $\mu_D \sin i = \mu_R \sin r$; $r = \sin^{-1}\left(\frac{\mu_D}{\mu_R}\sin i\right)$ so

$$\delta = r - i = \sin^{-1}\left(\frac{\mu_D}{\mu_R}\sin i\right) - i$$





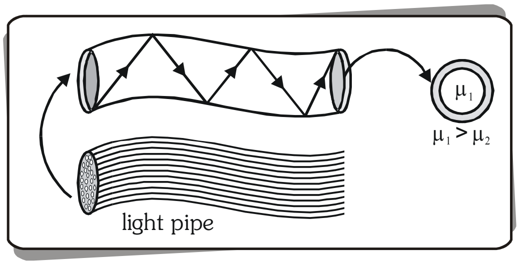
- If $i > \theta_c; \delta = \pi - 2i$



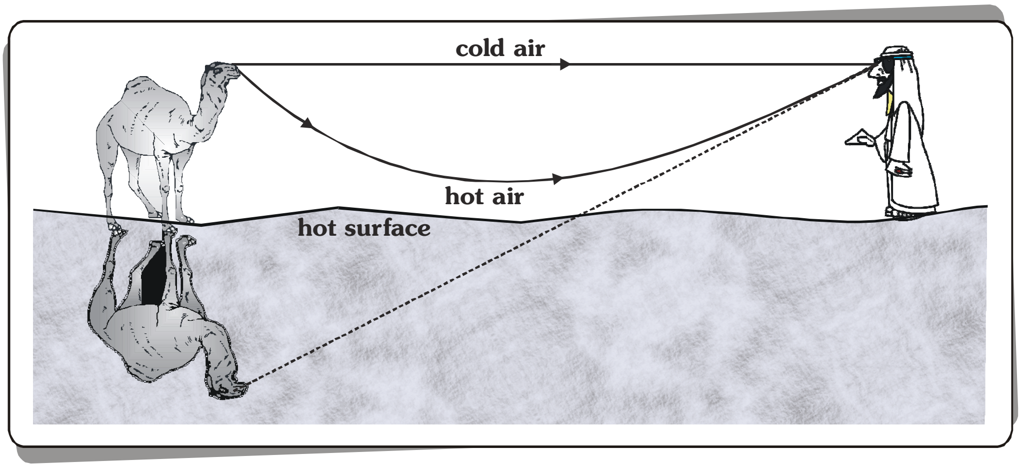
SOME ILLUSTRATIONS OF TOTAL INTERNAL REFLECTION

- **Sparkling of diamond :** The sparkling of diamond is due to total internal reflection inside it. As refractive index for diamond is 2.5 so C= 24°. Now the cutting of diamond are such that i > C. So TIR will take place again and again inside it. The light which beams out from a few places in some specific directions makes it sparkle.

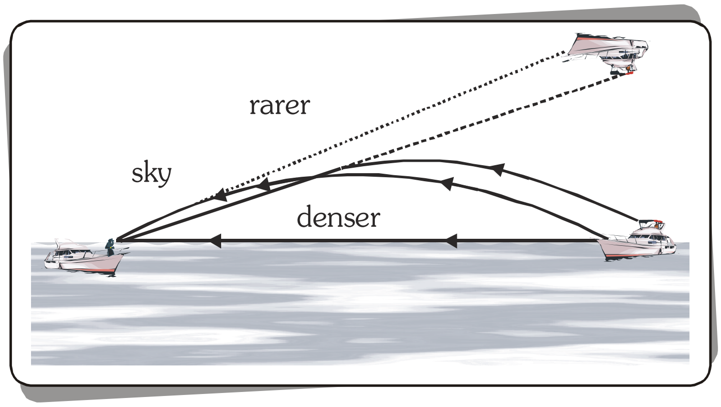
- **Optical Fibre :** In it light through multiple total internal reflections is propagated along the axis of a glass fibre of radius of few microns in which index of refraction of core is greater than that of surroundings (cladding)



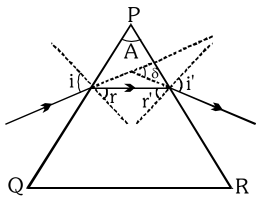
- **Mirage and looming** : Mirage is caused by total internal reflection in deserts where due to heating of the earth, refractive index of air near the surface of earth becomes lesser than above it. Light from distant objects reaches the surface of earth with $i > \theta_C$ so that TIR will take place and we see the image of an object along with the object as shown in figure.

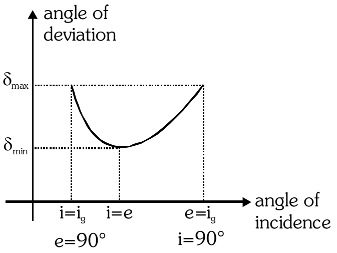


Similar to 'mirage' in deserts, in polar regions 'looming' takes place due to TIR. Here μ decreases with height and so the image of an object is formed in air if $(i>\theta_C)$ as shown in figure.



REFRACTION THROUGH PRISM





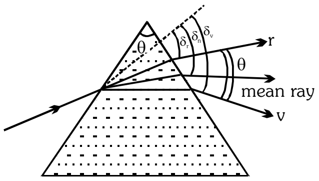
- $\delta = (i + i') - (r + r')$
- $r + r' = A$
- There is one and only one angle of incidence for which the angle of deviation is minimum.

  When $\delta = \delta_m$ then $i = i'$ & $r = r'$, the ray passes symetrically about the prism, & then $n = \dfrac{\sin\left[\frac{A+\delta m}{2}\right]}{\sin\left[\frac{A}{2}\right]}$, where n = w.r.t. surroundings R.I. of glass.
- For a thin prism ($A \le 10°$) ; $\delta=(n-1)A$
- **Dispersion Of Light :** The angular splitting of a ray of white light into a number of components when it is refracted in a medium other than air is called ***Dispersion of Light.***
- **Angle of Dispersion :** Angle between the rays of the extreme colours in the refracted (dispersed) light is called Angle of Dispersion.

$$\theta = \delta_v - \delta_r$$

- Dispersive power (ω) of the medium of the material of prism.

$$\omega = \frac{\text{angular dispersion}}{\text{deviation of mean ray (yellow)}}$$



For small angled prism ($A \le 10°$);

$$\omega = \frac{\delta_v - \delta_R}{\delta_y} = \frac{n_v - n_R}{n-1} \;;\; n = \frac{n_v + n_R}{2}$$

n_v, n_R & n are R. I. of material for violet, red & yellow colours respectively .

COMBINATION OF TWO PRISMS

- **Achromatic Combination :**

  It is used for deviation without dispersion . Condition for this $(n_v - n_r) A + (n'_v - n'_r) A' = 0$
  $\omega\delta + \omega'\delta' = 0$ where ω, ω' are dispersive powers for the two prisms & δ , δ' are the mean deviation.
  Net mean deviation

$$= \left[\frac{n_v + n_R}{2} - 1\right]A + \left[\frac{n'_v + n'_R}{2} - 1\right]A'$$

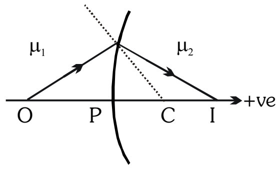
- **Direct Vision Combination :**
  It is used for producing disperion without deviation condition for this

$$\left[\frac{n_v + n_R}{2} - 1\right]A = -\left[\frac{n'_v + n'_R}{2} - 1\right]A'$$

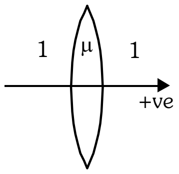
Net angle of dispersion = $(n_v - n_r) A + (n_v' - n_r') A'$.

- $\dfrac{\mu_2}{v} - \dfrac{\mu_1}{u} = \dfrac{\mu_2 - \mu_1}{R}$

  v, u & R are to be kept with sign as
  v = PI
  u = –PO
  R = PC
  (Note : Radius is with sign)



- $m = \dfrac{\mu_1 v}{\mu_2 u}$
- **Lens Formula :**



- $\dfrac{1}{v} - \dfrac{1}{u} = \dfrac{1}{f}$

- $\dfrac{1}{f} = (\mu - 1)\left(\dfrac{1}{R_1} - \dfrac{1}{R_2}\right)$
- $m = \dfrac{v}{u}$

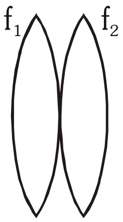
- **Power of Lenses**

  Reciprocal of focal length in meter is known as power of lens.
  - **SI unit** : dioptre (D)
  - **Power of lens :** $P = \dfrac{1}{f(m)} = \dfrac{100}{f(cm)}$ dioptre

Combination of Lenses

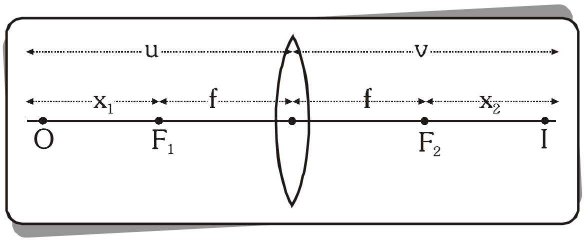
Two thin lens are placed in contact to each other

Power of combination. $P = P_1 + P_2 \Rightarrow \frac{1}{f} = \frac{1}{f_1} + \frac{1}{f_2}$



Use sign convention when solving numericals

Newton's Formula

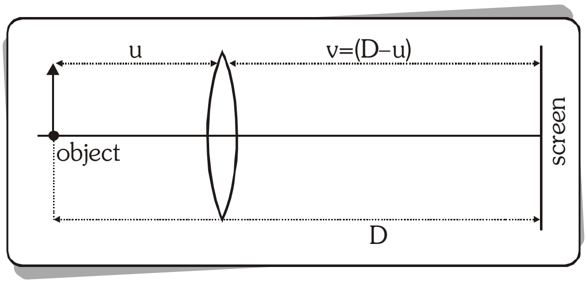


$$\boxed{f = \sqrt{x_1 x_2}}$$

x_1 = distance of object from focus
x_2 = distance of image from focus

Displacement Method

It is used for determination of focal length of convex lens in laboratory. A thin convex lens of focal length f is placed between an object and a screen fixed at a distance D apart.



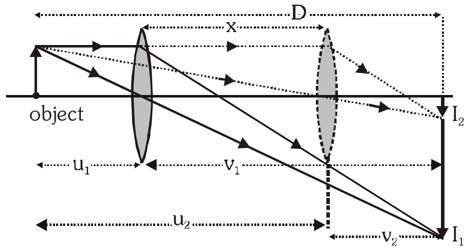
(i) For D<4f: u will be imaginary hence physically no position of lens is possible

(ii) For D = 4f : $u = \frac{D}{2} = 2f$ so only one position of lens is possible and since $v = D - u = 4f - 2f = u$

(iii) For D > 4f :

$$u_1 = \frac{D - \sqrt{D(D-4f)}}{2} \text{ and } u_2 = \frac{D + \sqrt{D(D-4f)}}{2}$$

So there are two positions of lens for which real image will be formed on the screen.(for two distances u_1 and u_2 of the object from lens)



If the distance between two positions of lens is x then $x = u_2 - u_1$

$$= \frac{D + \sqrt{D(D-4f)}}{2} - \frac{D - \sqrt{D(D-4f)}}{2} = \sqrt{D(D-4f)}$$

$$\Rightarrow x^2 = D^2 - 4Df \Rightarrow f = \frac{D^2 - x^2}{4D}$$

Distance of image corresponds to two positions of the lens :

$$v_1 = D - u_1 = D - \frac{1}{2}[D - \sqrt{D(D-4f)}]$$

$$= \frac{1}{2}[D + \sqrt{D(D-4f)}] = u_2 \Rightarrow v_1 = u_2$$

$$v_2 = D - u_2 = D - \frac{1}{2}[D + \sqrt{D(D-4f)}]$$

$$= \frac{1}{2}[D - \sqrt{D(D-4f)}] = u_1 \Rightarrow v_2 = u_1$$

Distances of object and image are interchangeable. for the two positions of the lens. Now

$x = u_2 - u_1$ and $D = v_1 + u_1 = u_2 + u_1$ $[\because v_1 = u_2]$

so $u_1 = v_2 = \frac{D - x}{2}$ and $u_2 = v_1 = \frac{D + x}{2}$;

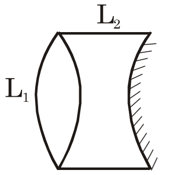
$$m_1 = \frac{I_1}{O} = \frac{v_1}{u_1} = \frac{D + x}{D - x} \text{ and } m_2 = \frac{I_2}{O} = \frac{v_2}{u_2} = \frac{D - x}{D + x}$$

Now $m_1 \times m_2 = \frac{D + x}{D - x} \times \frac{D - x}{D + x} \Rightarrow \frac{I_1 I_2}{O^2} = 1 \Rightarrow O = \sqrt{I_1 I_2}$

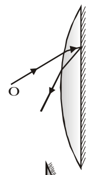
Silvering of one surface of lens

$$P_{eff} = 2P_{L1} + 2P_{L2} + P_M$$

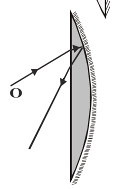
$$\frac{1}{f_{eff}} = \frac{2}{f_{L1}} + \frac{2}{f_{L2}} - \frac{1}{f_M}$$



When plane surface is silvered $f = \frac{R}{2(\mu - 1)}$



When convex surface is silvered $f = \frac{R}{2\mu}$



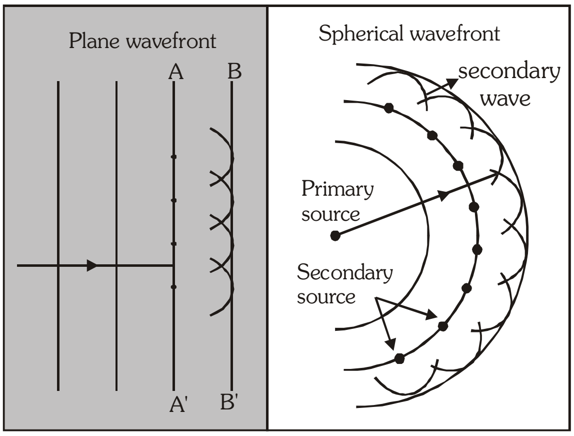
IMPORTANT NOTES

Wave Nature of Light & Wave Optics

HUYGEN'S WAVE THEORY

Huygen's in 1678 assumed that a body emits light in the form of waves.

- Each point source of light is a centre of disturbance from which waves spread in all directions. The locus of all the particles of the medium vibrating in the same phase at a given instant is called a wavefront.
- Each point on a wave front is a source of new disturbance, called secondary wavelets. These wavelets are spherical and travel with speed of light in that medium.
- The forward envelope of the secondary wavelets at any instant gives the new wavefront.
- In homogeneous medium, the wave front is always perpendicular to the direction of wave propagation.



COHERENT SOURCES :

Two sources will be coherent if and only if they produce waves of same frequency (and hence wavelength) and have a constant initial phase difference.

INCOHERENT SOURCES :

Two sources are said to be incoherent if they have different frequency and initial phase difference is not constant w.r.t. time.

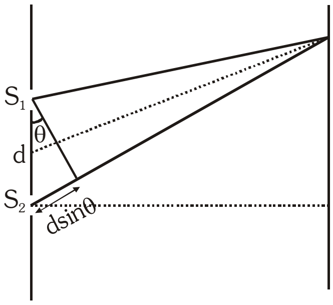
INTERFERENCE : YDSE

- Resultant intensity for coherent sources

  $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi_0$
- Resultant intensity for incoherent sources $I=I_1+I_2$
- Intensity ∝ width of slit ∝ (amplitude)^2

$$\Rightarrow \frac{I_1}{I_2} = \frac{W_1}{W_2} = \frac{a_1^2}{a_2^2} \Rightarrow \frac{I_{max}}{I_{min}} = \frac{\left(\sqrt{I_1} + \sqrt{I_2}\right)^2}{\left(\sqrt{I_1} - \sqrt{I_2}\right)^2} = \left(\frac{a_1 + a_2}{a_1 - a_2}\right)^2$$

- Distance of n^th^ bright fringe $x_n = \frac{n\lambda D}{d}$



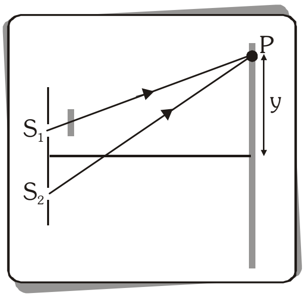
Path difference = nλ where n =0, 1, 2, 3, ...

- Distance of m^th^ dark fringe $x_m = \frac{(2m+1)\lambda D}{2d}$

Path difference=(2m+1)$\frac{\lambda}{2}$ where m= 0,1,2, 3,.....

- Fringe width $\beta = \frac{\lambda D}{d}$
- Angular fringe width $= \frac{\beta}{D} = \frac{\lambda}{d}$
- Fringe visibility $= \frac{I_{max} - I_{min}}{I_{max} + I_{min}} \times 100\ \%$
- If a transparent sheets of referactive index μ and thickness t is introduced in one of the paths of interfering waves, optial path will becomes 'μt' instead of 't'. Entire fringe pattern is displaced by $\frac{D\left[(\mu-1)t\right]}{d} = \frac{\beta}{\lambda}(\mu-1)t$ towards the side in which the thin sheet is introduced without any change in fringe width.

SHIFTING OF FRINGES



- Path difference produced by a slab $\Delta x = (\mu-1)t$
- Fringe shift, $\Delta x = \frac{\beta}{\lambda}(\mu-1)t = \frac{D}{d}(\mu-1)t$
- Number of fringes shift

$$= \frac{\text{shift}}{\text{fringe width}} = \frac{t(\mu\text{-}1)D/d}{\lambda D/d} = \frac{(\mu-1)t}{\lambda}$$

INTEREREFERENCE IN THE FILM

- For reflected Light :

  Maxima $\rightarrow 2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

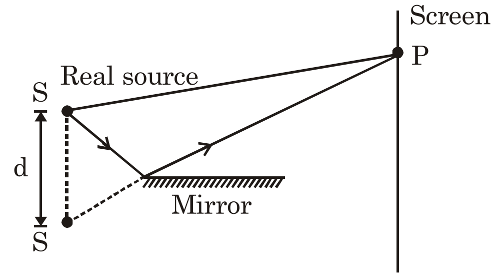
  Minima$\rightarrow 2\mu t \cos r = n\lambda$
- For transmitted light :

  Maxima $\rightarrow 2\mu t \cos r = n\lambda$

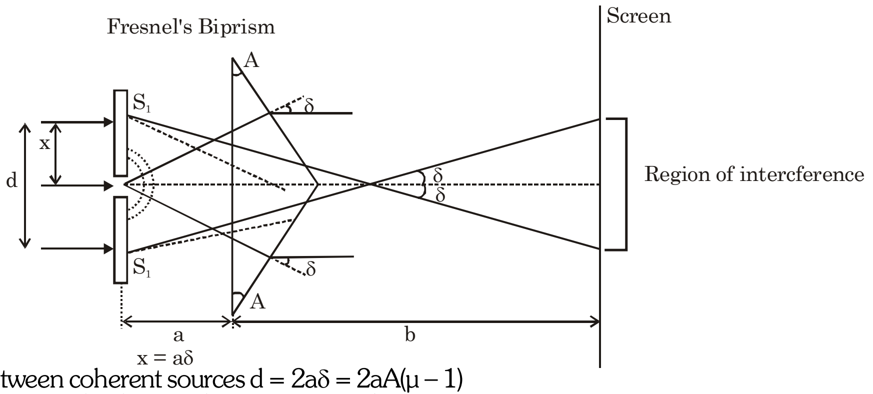
  Minima $\rightarrow 2\mu t \cos r = (2n+1)\frac{\lambda}{2}$

  (t =thickness of film, μ=R.I. of the film)

- **Newton's Ring** : When a lens of large Radius of curvature is placed on a plane glass plate, an air film is formed between lower surface of the lens and the upper surface of the plate. It this film is illuminated by sodium light, due to interference concentric bright and dark rings called Newton's Rings are seen.
- If the film is wedge shaped. The fringes will be straight lines parallel to the edge of apex with minimum at the apex and fringe width $\beta = \frac{\lambda}{2\mu\theta}$ where $\theta = \frac{t}{x}$.
- If case of Newton's rings the centre is dark spacing between rings goes on decreasing as we move away from centre and radius of dark rings is proportional to the square root of all natural number while bright rings 1.0 the square root of odd numbers.
- **Llayd's Mirror** :



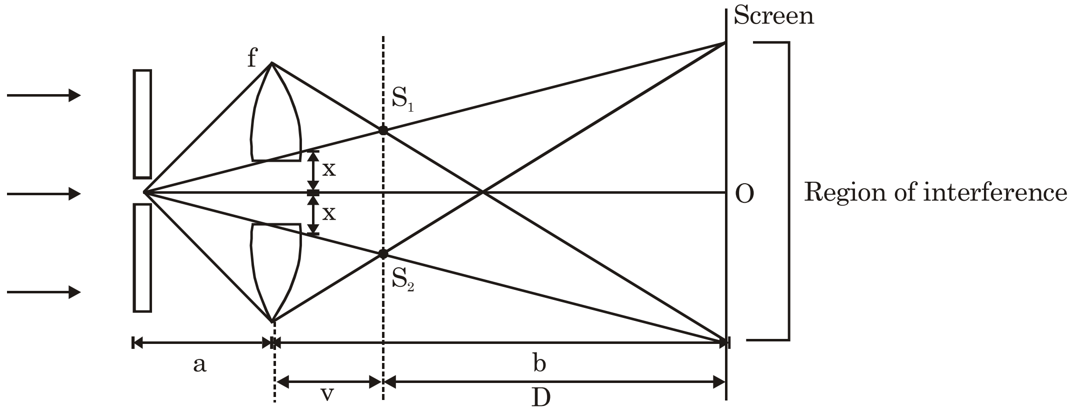
The position of dark and bright fringes are reversed relative to the pattern of two real sources because there is a 180^0 phase change produced by reflection.



Seperation between coherent sources $d = 2a\delta = 2aA(\mu - 1)$
Seperation between slit plane and screen $\Delta = a + b$

Frindge width on screen $\beta = \dfrac{AD}{d} = \dfrac{\lambda(a+b)}{2aA(\mu-1)}$

Frillet split lens as a limiting case of YDSE



from lens

formula $v = \dfrac{af}{a-f}$ $D = a + b - |V|$

$$d = 2x + 2\left|\frac{v}{u}\right|x$$

$$d = 2x\left(1 + \left|\frac{v}{u}\right|\right)$$

Frindge width $\beta = \dfrac{\lambda D}{d}$

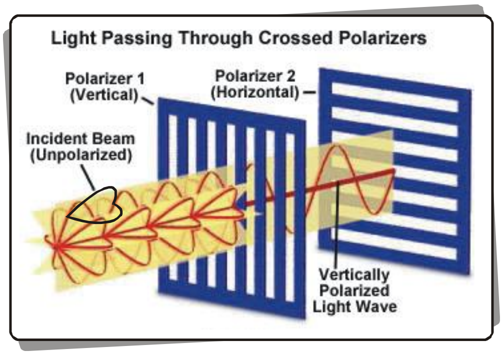
DIFFRACTION

- **Fresnel's diffraction** : In Fresnel's diffraction, the source and screen are placed close to the aperture or the obstacle and light after diffraction appears converging towards the screen and hence no lens is required to observed it. The incident wave fronts are either spherical or cylindrical.
- **Fraunhofer's diffraction :** The source and screen are placed at large distances from the aperture or the obstacle and converging lens is used to observed the diffraction pattern. The incident wavefront is planar one.
  - ❒ For minima : $a \sin\theta_n = n\lambda$
  - ❒ For maxima : $a \sin\theta_n = (2n + 1)\dfrac{\lambda}{2}$
  - ❒ Linear width of central maxima : $W_x = \dfrac{2\lambda D}{a}$
  - ❒ Angular width of central maxima $W_\theta = \dfrac{2\lambda}{a}$
  - ❒ Intensity of maxima
    where I_0 = Intensity of central maxima

$$I = I_0\left[\frac{\sin(\beta/2)}{\beta/2}\right]^2 \text{ and } \beta = \frac{2\pi}{\lambda} a \sin\theta$$

POLARISATION OF LIGHT

If the vibrations of a wave are present in just one direction in a plane perpendicular to the direction of propagation, the wave is said to be **polarised or plane polarised**. The phenomenon of restricting the oscillations of a wave to just one direction in the transverse plane is called polarisation of waves.


Light Passing Through Crossed Polarizers
Polarizer 1 (Vertical)
Polarizer 2 (Horizontal)
Incident Beam (Unpolarized)
Vertically Polarized Light Wave

MALUS LAW

The intensity of transmitted light passed through an analyser is $I=I_0\cos^2\theta$

(θ=angle between transmission directions of polariser and analyser)

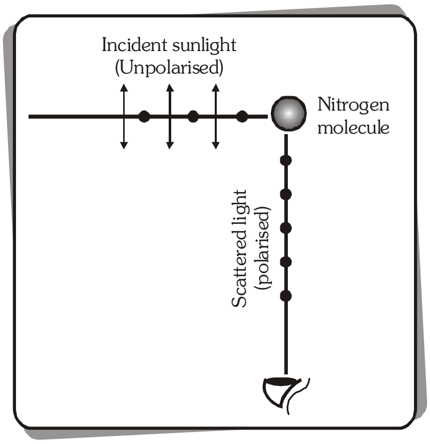
POLARISATION BY REFLECTION

Brewster's Law : The tangent of polarising angle of incidence at which reflected light becomes completely plane polarised is numerically equal to refractive index of the medium. $\mu=\tan i_p$;

i_p =Brewster's angle and $i_p+r_p=90°$

POLARISATION BY SCATTERING

If we look at the blue portion of the sky through a polaroid and rotate the polaroid, the transmitted light shows rise and fall of intensity.



The scattered light screen in a direction perpendicular to the direction of incidence is found to be plane polarised.

KEY POINTS

- The law of conservation of energy holds good in the phenomenon of interference.
- Fringes are neither image nor shadow of slit but locus of a point which moves such a way that its path difference from the two sources remains constant.
- In YDSE the interference fringes for two coherent point sources are hyperboloids with axis S_1S_2.
- If the interference experiment is repeated with bichromatic light, the fringes of two wavelengths will be coincident for the first time when

$$n(\beta)_{\text{longer}}=(n+1)(\beta)_{\text{shorter}}$$

- No interference pattern is detected when two coherent sources are infinitely close to each other, because $\beta \propto \frac{1}{d}$
- If maximum number of maximas or minimas are asked in the question, use the fact that value of $\sin\theta$ or $\cos\theta$ can't be greater than 1. $n_{\max}=\frac{d}{\lambda}$ Total maxima = $2n_{\max}+1$
- **Limit of resolution for microscope** $=\frac{1.22\lambda}{2a\sin\theta}=\frac{1}{\text{resolving power}}$
- **Limit of resolution for telescope** $=\frac{1.22\lambda}{a}=\frac{1}{\text{resolving power}}$

IMPORTANT NOTES

CHAPTER 24

Errors

Error

Random
(Cann't be removed
but minimized)
No fixed sign

Syotematic
(Can be removed)
Ex- zero error, Bench error
Fixed sign

Absolute Error : Expressed in absolute term ex-least count

Relative/ Fractional Error : $\dfrac{\text{Absolute error}}{\text{Size of measurement}}$

1. ***Addition and subtraction rule*:**
   The absolute random errors add.
   Thus if $R = A + B$, $r = a + b$ and if $R = A - B$, $r = a + b$
2. ***Product and quotient rule*:**
   The relative random errors add.
   Thus if $R = AB$, $\dfrac{r}{R} = \dfrac{a}{A} + \dfrac{b}{B}$
   and if $R = \dfrac{A}{B}$, then also $\dfrac{r}{R} = \dfrac{a}{A} + \dfrac{b}{B}$
3. ***Power rule*:**
   When a quantity Q is raised to a power P, the relative error in the result is P times the relative error in Q.
   This also holds for negative powers. If $R = Q^P$, $\dfrac{r}{R} = P \times \dfrac{q}{Q}$
4. The quotient rule is not applicable if the numerator and denominator are dependent on each other.
   e.g if $R = \dfrac{XY}{X+Y}$. We cannot apply quotient rule to find the error in R. Instead we write the equation as follows $\dfrac{1}{R} = \dfrac{1}{X} + \dfrac{1}{Y}$. Differentiating both the sides, we get $-\dfrac{dR}{R^2} = -\dfrac{dX}{X^2} - \dfrac{dY}{Y^2}$.
   Thus $\dfrac{r}{R^2} = \dfrac{x}{X^2} + \dfrac{y}{Y^2}$

Significant Digits

Rules for determining the number of significant digits in number with indicated decimals.

- All nonzero digits (1-9) are to be counted as significant.
- Zeros that have any nonzero digits anywhere to the LEFT of them are considered significant zeros.
- All other zeros not covered in rule (2) above are NOT be considered significant digits.

Determining the number of significant digits in number is not having an indicated decimals.

Express in eicientific notation

Rule for expressing the correct number of significant digits in an addition or substraction :

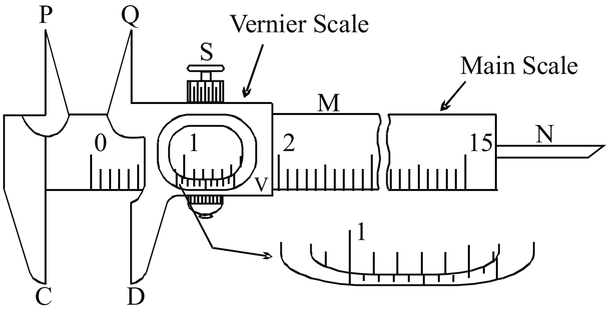
A sum or difference can have no more indicated positions to the right of the decimal as the number involved in the operation with the LEAST indicated positions to the right of its decimal.

Rules for rounding off digits

There are a set of conventional rules for rounding off.

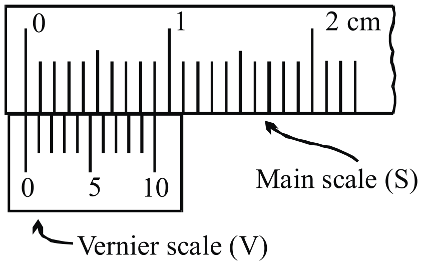
- Determine according to the rule what the last reported digit should be.
- Consider the digit to the right of the last reported digit.
- If the digit to the right of the last reported digit is less than 5 round it and all digits to its right off.
- If the digit to the right of the last reported digit is greater than 5 round it and all digits to its right off and increased the last reported digit by one.
- If the digit to the right of the last reported digit is a 5 followed by either no other digits or all zeros, round it and all digits to its right off and if the last reported digit is odd round up to the next even digit. If the last reported digit is even then leave it as is.

Vernier Callipers



Vernier Callipers

Principle of Vernier



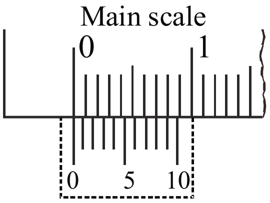
Least count of Vernier Callipers

The least count or Vernier constant (v. c) is the minimum value of correct estimation of length without eye estimation. If N division of vernier coincides with (N-1) division of main scale, then

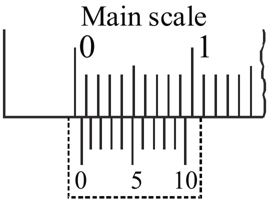
$$N\,(VS) = (N-1)\,ms \;\Rightarrow\; 1VS = \frac{N-1}{N}\,ms$$

Vernier constant = $1\ ms - 1\ vs = \left(1-\frac{N-1}{N}\right) ms = \frac{1ms}{N}$, which is equal to the value of the smallest division on the main scale divided by total number of divisions on the vernier scale.

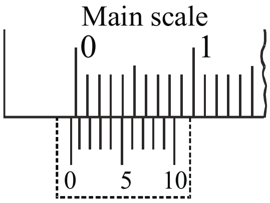
Zero error:



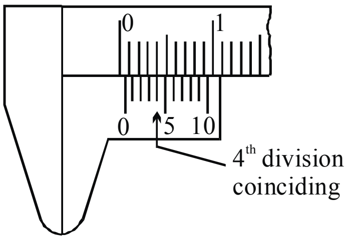
Vernier scale
without zero error



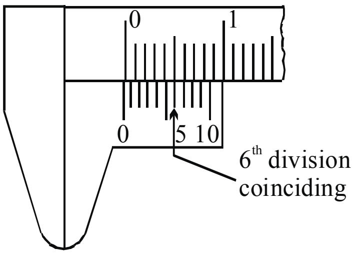
Vernier scale
with positive zero error
(i)



Vernier scale
with negative zero error
(ii)



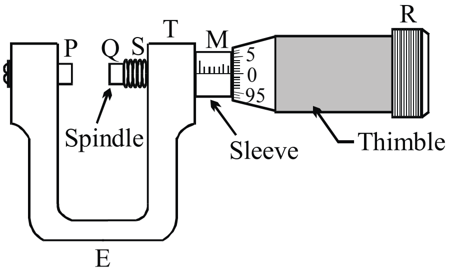
Positive zero error (+0.04 cm)
and its correction



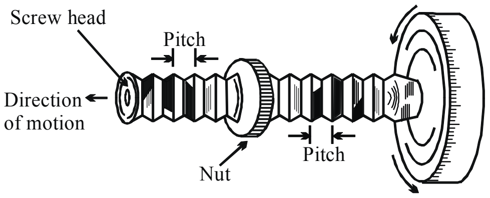
Negative zero error = (–0.04 cm)
and its correction

Negative zero error = – [Total no. of vsd – vsd coinciding] ×L.C.

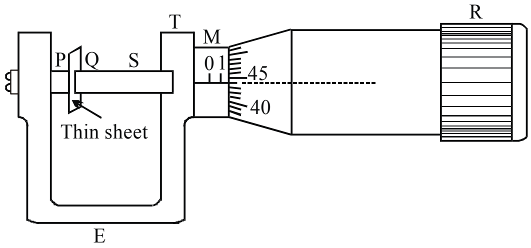
Screw Gauge

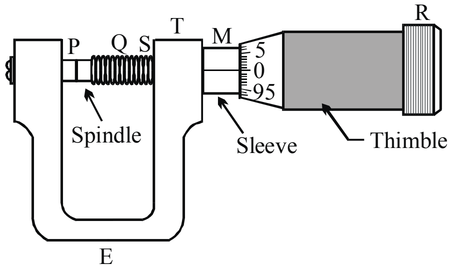


Screw Gauge

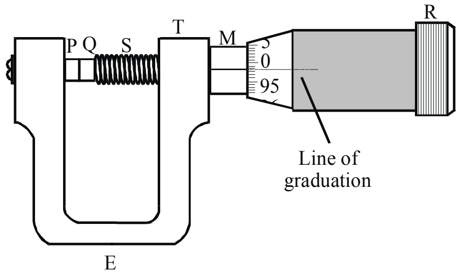


Principle of a micrometer

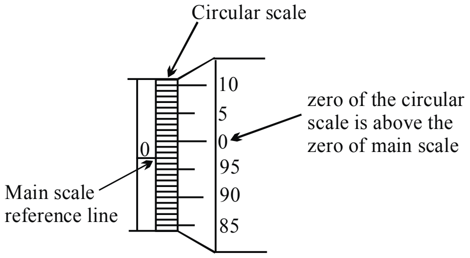


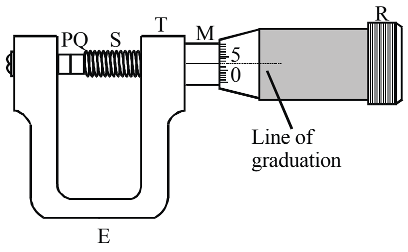


Screw gauge with no zero error

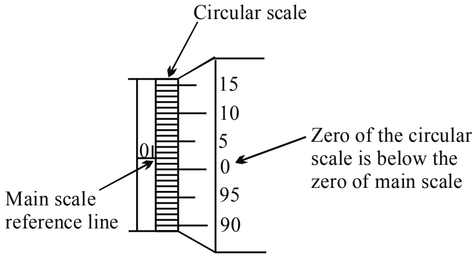


Negative zero error
(3 division error) i.e., - 0.003 cm


Circular scale
10
5
0
95
90
85
0
zero of the circular scale is above the zero of main scale
Main scale reference line



Positive zero error
(2 division error) i.e., + 0.002 cm

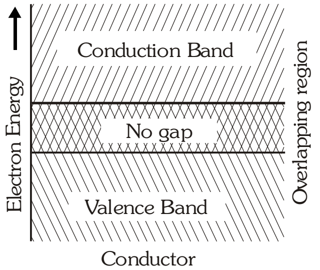
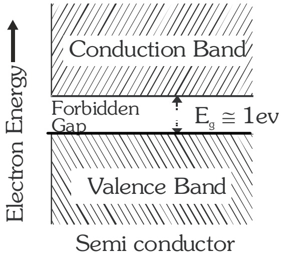
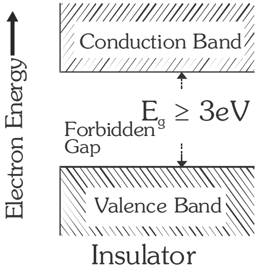

Circular scale
15
10
5
0
95
90
0
Zero of the circular scale is below the zero of main scale
Main scale reference line

Constants of the Screw Gauge

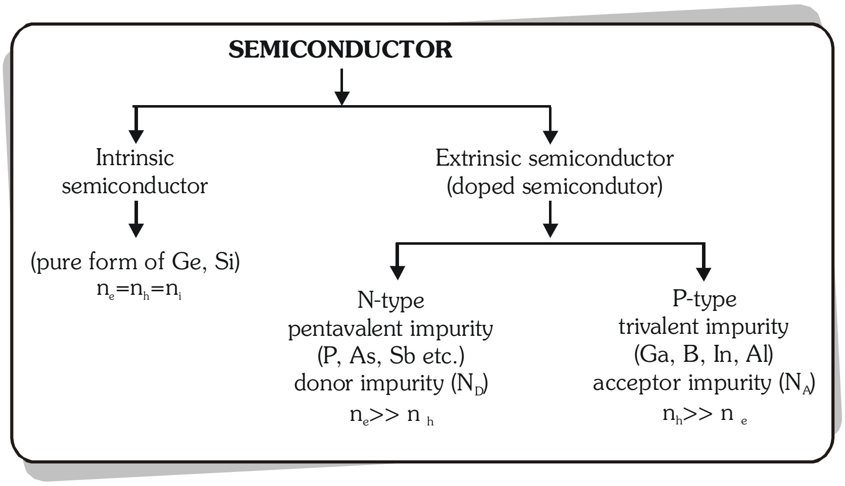
(a) Pitch
(b) Least count
(c) Measurement of length by screw gauage

Semiconductor & Digital Electronics

COMPARISON BETWEEN CONDUCTOR, SEMICONDUCTOR AND INSULATOR

Properties	**Conductor**	**Semiconductor**	**Insulator**
Resistivity	$10^{-2} - 10^{-8}\ \Omega m$	$10^{-5} - 10^{6}\ \Omega m$	$10^{11} - 10^{19}\ \Omega m$
Conductivity	$10^{2} - 10^{8}$ mho/m	$10^{-6} - 10^{5}$ mho/m	$10^{-19} - 10^{-11}$ mho/m
Temp. Coefficient of resistance (α)	Positive	Negative	Negative
Current	Due to free electrons	Due to electrons and holes	No current
Energy band diagram	 Conduction Band No gap Valence Band Electron Energy Overlapping region Conductor	 Conduction Band Forbidden Gap $E_g \cong 1ev$ Valence Band Electron Energy Semi conductor	 Conduction Band $E_g \geq 3eV$ Forbidden Gap Valence Band Electron Energy Insulator
Forbidden energy gap	$\cong 0eV$	$\cong 1eV$	$\geq 3eV$
Example	Pt, Al, Cu, Ag	Ge, Si, GaAs, GaF_2	Wood, plastic, Diamond, Mica

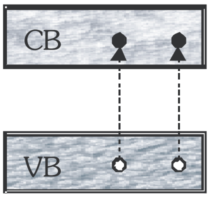
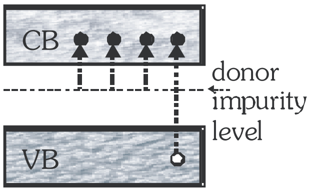
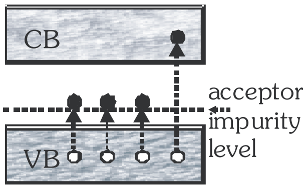
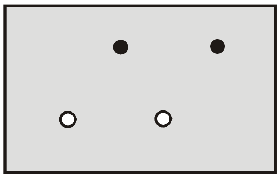
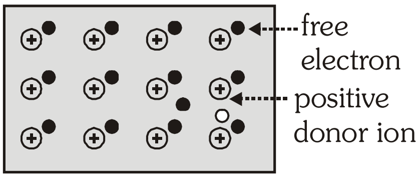
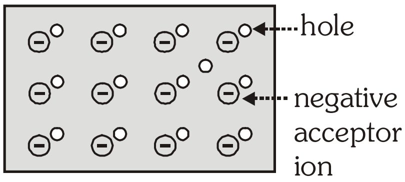
- **Number of electrons reaching from valence band to conduction band :** $n = AT^{3/2}e^{-\frac{\Delta Eg}{2kT}}$
- **CLASSIFICATION OF SEMICONDUCTORS :**



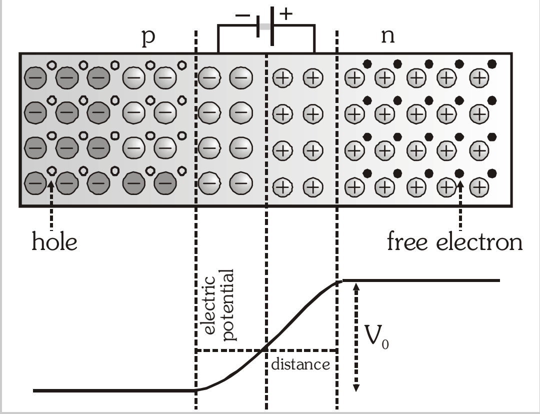
- **MASS-ACTION LAW** : $n_i^2 = n_e \times n_h$
  - ❐ For N-type semiconductor $n_e \simeq N_D$
  - ❐ For P-type semiconductor $n_h \simeq N_A$

CONDUCTION IN SEMICONDUCTOR

Intrinsic semiconductor	**P - type**	**N - type**
$n_e = n_h$ $J = ne\,[v_e + v_h]$ (Current density) $\sigma = \frac{1}{\rho} = en\,[\mu_e + \mu_h]$ (Conductivity)	$n_h >> n_e$ $J \cong e\,n_h\,v_h$ $\sigma = \frac{1}{\rho} \cong e\,n_h\,\mu_h$	$n_e >> n_h$ $J \cong e\,n_e\,v_e$ $\sigma = \frac{1}{\rho} \cong e\,n_e\,\mu_e$

Intrinsic Semiconductor	N-type (Pentavalent impurity)	P-type (Trivalent impurity)
		
		
Current due to electron and hole	Mainly due to electrons	Mainly due to holes
$n_e = n_h = n_i$	$n_h << n_e$ ($N_D \approx n_e$)	$n_h >> n_e$ ($N_A \approx n_h$)
$I = I_e + I_h$	$I \approx I_e$	$I \approx I_h$
Entirely neutral	Entirely neutral	Entirely neutral
Quantity of electrons and holes are equal	Majority Electrons Minority Holes	Majority Holes Minority - Electrons

P-N JUNCTION
(At equilibrium condition)

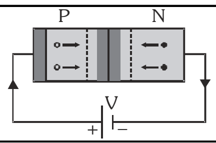
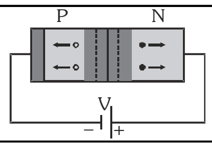
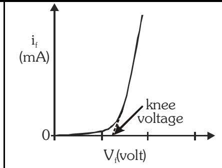
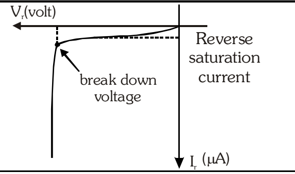


Direction of diffusion current : P to N side and drift current : N to P side

If there is no biasing then diffusion current = drift current. So total current is zero

In junction N side is at high potential relative to the P side. This potential difference tends to prevent the movement of electron from the N region into the P region. This potential difference called a **barrier potential**.

COMPARISON BETWEEN FORWARD BIAS AND REVERSE BIAS

	Forward Bias		Reverse Bias
			
1	Potential Barrier reduces	**1**	Potential Barrier increases.
2	Width of depletion layer decreases	**2**	Width of depletion layer increases.
3	P-N jn. provide very small resistance	**3**	P-N jn. provide high resistance
4	Forward current flows in the circuit	**4**	Very small current flows.
5	Order of forward current is milli ampere.	**5**	Order of current is micro ampere for Ge or Nano ampere for Si.
6	Current flows mainly due to majority carriers.	**6**	Current flows mainly due to minority carriers.
7	Forward characteristic curves.	**7**	Reverse characteristic curve
			
8	Forward Resistance : $R_f = \frac{\Delta V_f}{\Delta I_f} \cong 100\Omega$	**8**	Reverse Resistance : $R_r = \frac{\Delta V_r}{\Delta I_r} \cong 10^6\Omega$
9	Order of knee or cut in voltage Ge→ 0.3 V Si → 0.7 V Special point : Generally $\frac{R_r}{R_f} = 10^3 : 1$ for Ge	**9**	Breakdown voltage Ge → 25 V Si → 35 V $\frac{R_r}{R_f} = 10^4 : 1$ for Si

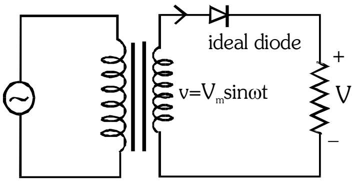
BREAKDOWN ARE OF TWO TYPES

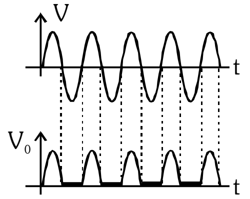
Zener Break down	**Avalanche Break down**
Where covalent bonds of depletion layer, itself break, due to high electric field	Here covalent bonds of depletion layers are broken by collision of "Minorities" which aquire high kinetic energy from high electric field
This phenomena take place in (i) P – N junction having "High doping" (ii) P – N junction having thin depletion layer Here P – N junction does not damage permanently "In D.C voltage stabilizer zener phenomena is used".	This phenomena takes place in (i) P – N junction having "Low doping" (ii) P – N junction having thick depletion layer Here P – N junction damages permanentaly due to abruptly increment of minorities during repeatative collisions.

APPLICATION OF DIODE

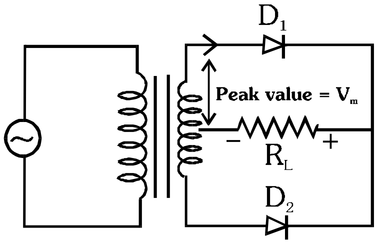
•	**Zener diode** :	It is highly doped p-n junction diode used as a voltage regulator.
•	**Photo diode** :	A p-n junction diode use to detect light signals operated in reverse bias.
•	**LED** :	A p-n junction device that emits optical radiation under forward bias conditions
•	**Solar cell** :	Generates emf of its own due to the effect of sun radiations.

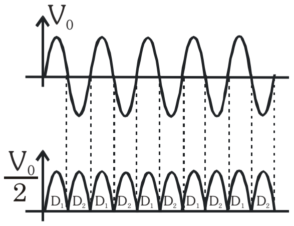
HALF WAVE RECTIFIER



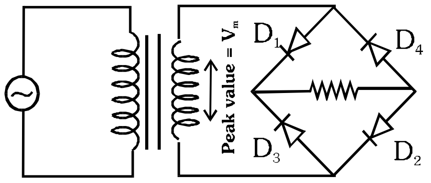


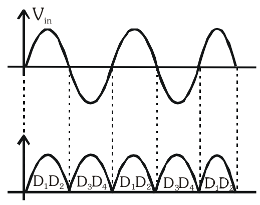
CENTRE – TAP FULL WAVE RECTIFIER





FULL WAVE BRIDGE REACTIFIER





RIPPLE FACTOR : $r = \frac{I_{ac}}{I_{dc}}$

❒ For HWR $r = 1.21$

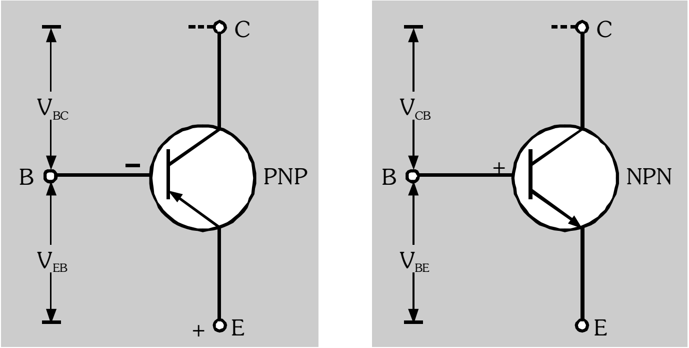
❒ For FWR $r = 0.48$

RECTIFIER EFFICIENCY: $\eta = \frac{P_{dc}}{P_{ac}} = \frac{I_{dc}^2 R_L}{I_{rms}^2 (R_F + R_L)}$

For HWR : $\eta\% = \frac{40.6}{1 + \frac{R_F}{R_L}}$ & FWR $\eta\% = \frac{81.2}{1 + \frac{R_F}{R_L}}$

FOR TRANSISTOR

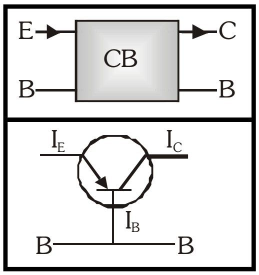
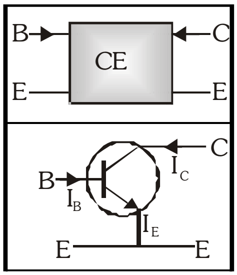
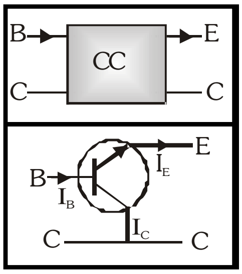
$I_E = I_B + I_C$



(a) (b)

COMPARATIVE STUDY OF TRANSISTOR CONFIGURATIONS

1. Common Base **(CB)** 2. Common Emitter **(CE)** 3. Common Collector **(CC)**

	CB	**CE**	**CC**
		 B C CE E E C I_C B I_B I_E E E	 B E CC C C E I_E B I_B I_C C C
Input Resistance	Low (100 Ω)	High (750 Ω)	Very High ≅ 750 kΩ
Output resistance	Very High	High	Low
Current Gain	(A_I or α)	(A_I or β)	(A_I or γ)
	$\alpha = \frac{I_C}{I_E} < 1$	$\beta = \frac{I_C}{I_B} > 1$	$\gamma = \frac{I_E}{I_B} > 1$
Voltage Gain	$A_V = \frac{V_o}{V_i} = \frac{I_C R_L}{I_E R_i}$	$A_V = \frac{V_o}{V_i} = \frac{I_C R_L}{I_B R_i}$	$A_V = \frac{V_o}{V_i} = \frac{I_E R_L}{I_B R_i}$
	$A_v = \alpha \frac{R_L}{R_i} \cong 150$	$A_v = \beta \frac{R_L}{R_i} \cong 500$	$A_v = \gamma \frac{R_L}{R_i} < 1$
Power Gain	$A_p = \frac{P_o}{P_i} = \alpha^2 \frac{R_L}{R_i}$	$A_p = \frac{P_o}{P_i} = \beta^2 \frac{R_L}{R_i}$	$A_p = \frac{P_o}{P_i} = \gamma^2 \frac{R_L}{R_i}$
Phase difference (between output and input)	same phase	opposite phase	same phase
Application	For High Frequency	For Audible frequency	For Impedance Matching

TRANSISTOR CHARACTERISTICS

❒ **Input resistance (r_1)** $\left(\frac{\Delta V_{BE}}{\Delta I_B}\right)_{V_{CE=\text{constant}}}$

❒ **Output resistance (r_0)** $\left(\frac{\Delta V_{CE}}{\Delta I_C}\right)_{I_{B=\text{constant}}}$

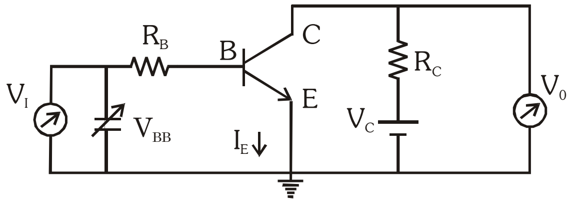
❒ **Relation between α and β :** $\beta = \frac{\alpha}{1-\alpha}$ **&** $\alpha = \frac{\beta}{1+\beta}$

❒ **Current amplification factor**

- **For CE(n-p-n)** $\beta_{ac} = \left(\frac{\Delta I_C}{\Delta I_B}\right)_{V_{CE=\text{constant}}}$
- **For CB (p-n-p)** $\alpha_{ac} = \left(\frac{\Delta I_C}{\Delta I_E}\right)_{V_{CB=\text{constant}}}$

TRANSISTOR IN CE CONFIGURATION

Appyling Kirchof's voltage Law



$V_i = V_{BB} = I_BR_B + V_{BE}$ ....(i)

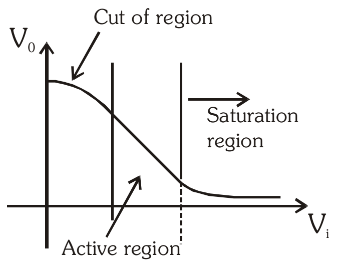
And $V_0 = V_{CE} = V_{CC} - I_CR_C$ ....(ii)

When V_i is less then Knee voltage, the trnasistor will be in cut off state and current I_C will be zero

So $\Delta V_i = R_B\Delta I_B + \Delta V_{BE}$ (ΔV_{BE} is negligible)

From equation (i) & (ii)

and $\Delta V_0 = -R_C\Delta I_C$ ($\because V_{CC}$ = constant)

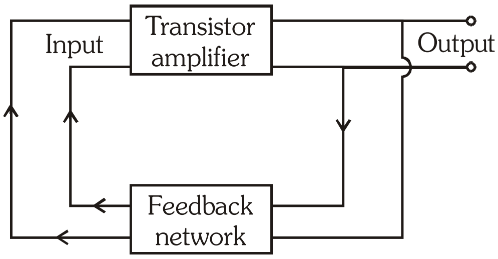


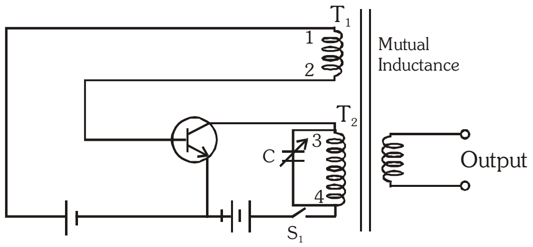
Voltage gain $A_V = \frac{\Delta V_0}{\Delta V_i}$

$\therefore$ Voltage gain $A_V = -\frac{R_C\Delta I_C}{R_B\Delta I_B}$

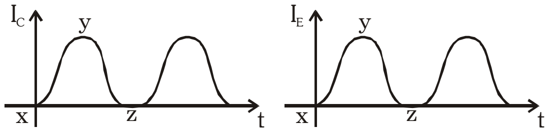
$A_V = -\beta_{ac}\frac{R_C}{R_B}$

Feeedback amplifier and transistor oscillator





Working



Initially surge of collector current (I_C) flows through T_2. This current increases from x to y. By innductive coupling between coils T_2 and T_1 causes a current in emitter. It is the feedback from input to output. As soon as field becomes static and there will be not feed back. Without feedback, the emitter current begins to fall. So collector current decreases and it will further, untill transistor reaches at cut-off region. The whole process repeat it self. The frequency of tuned circuit at which oscillator will oscillate is $f = \frac{1}{2\pi\sqrt{LC}}$

APPLICATIONS OF TRANSISTORS

There are three regions of transistor operation:

- **Cut off region * Active region * Saturation region**
- **Transistor as Voltage amplifier**

  * To operate it as an amplifer we need to fix its operating voltage somewhere in active region where it increases the strength of input ac signal and produces an amplified output signal.

  * Voltage gain $A_V = \frac{V_0}{V_i} = -\beta_{ac}\frac{R_{out}}{R_{in}}$

  * Power gain $A_P = A_V \times \beta_{ac}$

- **Transistor as a Switch**

  A transistor can be used as a switch if it is operated in its cutoff and saturation states only.

- **Transisot as an Oscillator**

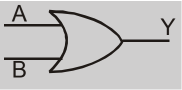
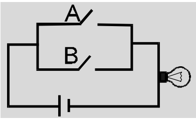
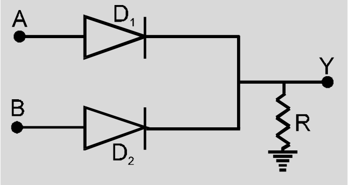
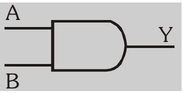
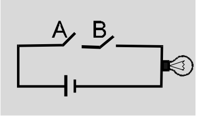
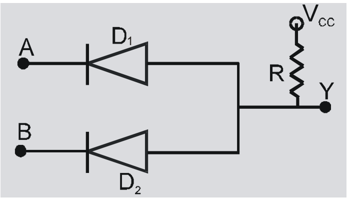
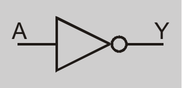
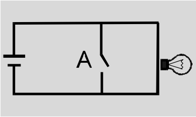
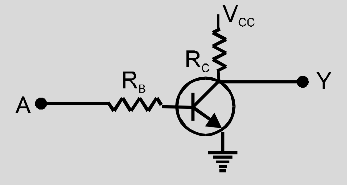
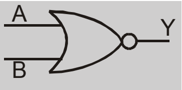
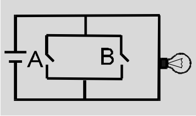
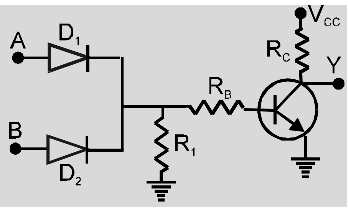
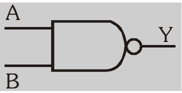
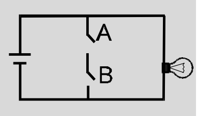
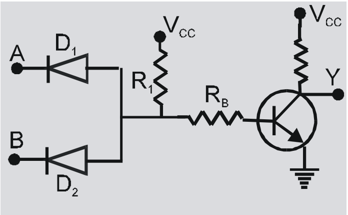
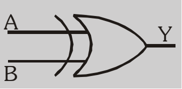
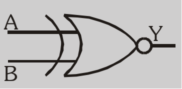
  An oscillator is a generator of an ac signal using positive feedback

  Frequency of oscillations if $f = \frac{1}{2\pi\sqrt{LC}}$

- **Relation between α, β and γ :**

$$\beta = \frac{\alpha}{1-\alpha},\ \gamma = 1+\beta,\ \gamma = \frac{1}{1-\alpha}$$

SUMMARY OF LOGIC GATES

Names	Symbol	Boolean Expression	Truth table	Electrical analogue	Circuit diagram (Practical Realisation)
OR		$Y = A + B$	A B Y 0 0 0 0 1 1 1 0 1 1 1 1		
AND		$Y = A.B$	A B Y 0 0 0 0 1 0 1 0 0 1 1 1		
NOT or Inverter		$Y = \overline{A}$	A Y 0 1 1 0		
NOR (OR +NOT)		$Y = \overline{A + B}$	A B Y 0 0 1 0 1 0 1 0 0 1 1 0		
NAND (AND+NOT)		$Y = \overline{A.B}$	A B Y 0 0 1 0 1 1 1 0 1 1 1 0		
XOR (Exclusive OR)		$Y = A \oplus B$ or $Y = \overline{A}.B + A\overline{B}$	A B Y 0 0 0 0 1 1 1 0 1 1 1 0		
XNOR (Exclusive NOR)		$Y = A \odot B$ or $Y = A.B + \overline{A}.\overline{B}$ or $Y = \overline{A \oplus B}$	A B Y 0 0 1 0 1 0 1 0 0 1 1 1		

DE MORGAN'S THEOREM

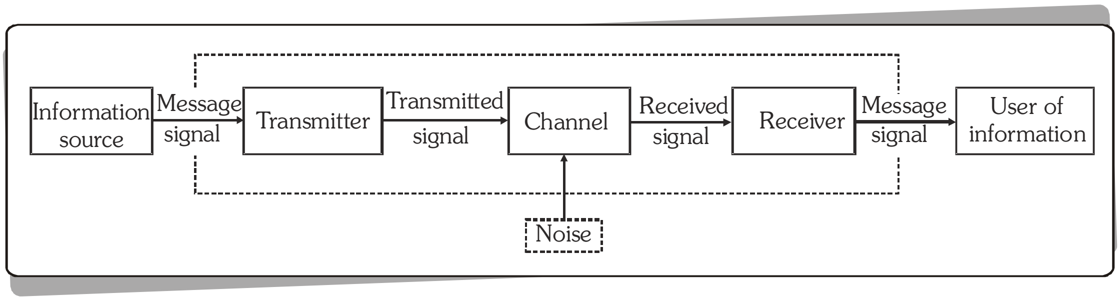
$\overline{A+B} = \overline{A}\bullet\overline{B}$, $\overline{A\bullet B} = \overline{A}+\overline{B}$

OR	AND	NOT
$A + 0 = A$	$A.\ 0 = 0$	$A + \overline{A} = 1$
$A + 1 = 1$	$A.\ 1 = A$	$A \cdot \overline{A} = 0$
$A + A = A$	$A\ .\ A = A$	$\overline{\overline{A}} \cdot A = A$

Communication System

Faithful transmission of information from one place to another place is called communication.

BASIC COMPONENTS OF A COMMUNICATION SYSTEM



- **Transmitter :** Transmitter converts the message signal produced by information source into a form (e.g. electrical signal) that is suitable for transmission through the channel to the receiver.
- **Communication channel** : Communication channel is a medium (transmission line, an optical fibre or free space etc) which connects a receiver and a transmitter. It carries the modulated wave from the transmitter to the receiver.
- **Receiver** : It receives and decode the signal into original form.

IMPORTANT TERMS USED IN COMMUNICATION

- **Transducer.** Transducer is the device that converts one form of energy into another. Microphone, photo detectors and piezoelectric sensors are types of transducer.
- **Signal** Signal is the information converted in electrical form. Signals can be analog or digital. Sound and picture signals in TV are analog.

  It is defined as a single–valued function of time which has a unique value at every instant of time.

  - **Analog Signal :–** A continuously varying signal (Voltage or Current) is called an analog signal. A decimal number with system base 10 is used to deal with analog signal.
  - **Digital Signal :**– A signal that can have only discrete stepwise values is called a digital signal. A binary number system with base 2 is used to deal with digital signals. (See Fig. 1)
- **Noise :** There are unwanted signals that tend to disturb the transmission and processing of message signals. The source of noise can be inside or outside the system.
- **Attenuation :** It is the loss of strength of a signals while propagating through a medium. It is like damping of oscillations.
- **Amplification :** It is the process of increasing the amplitude (and therefore the strength) of a signal using an electronic circuit called the amplifier. Amplification is absolutely necessary to compensate for the attenuation of the signal in communication systems.
- **Range :** It is the largest distance between the source and the destination upto which the signal is received with sufficient strength.
- **Repeater :** A repeater acts as a receiver and a transmitter. A repeater picks up the signal which is coming from the transmitter, amplifies and retransmits it with a change in carrier frequency. Repeaters are necessary to extend the range of a communication system as shown in figure A communication satellite is basically a repeater station in space. (See Fig. 2)

Fig.1

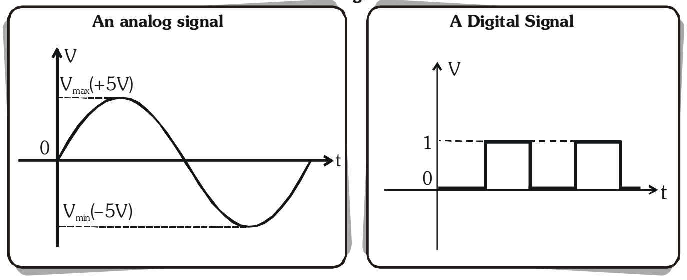
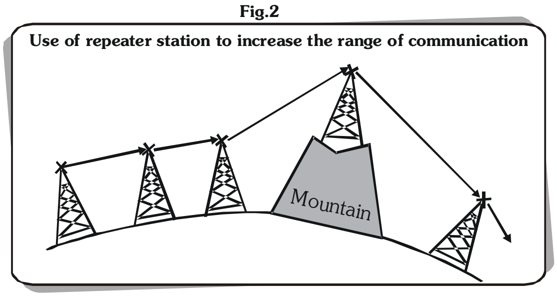


Fig.2


Use of repeater station to increase the range of communication
Mountain

BANDWIDTH

BANDWIDTH OF SIGNALS

Different signals used in a communication system such as voice, music, picture, computer data etc. all have different ranges of frequency. The difference of maximum and minimum frequency in the range of each signal is called bandwidth of that signal. Bandwidth can be of message signal as well as of transmission medium.

(i) Bandwidth for analog signals :

Bandwidth for some analog signals are listed below

Signal	Frequency range	Bandwidth required
Speech	300-3100 Hz	3100-300 =2800 Hz
Music	High frequencies produced by musical instrument audible range =20 Hz - 20 kHz	20 kHz
Picture	—	4.2 MHz
TV	Contains both voice and picture	6 MHz

(ii) Bandwidth for digital signal :

Basically digital signals are rectangular waves and these can be splited into a superposition of sinusoidal waves of frequencies ν_0, $2\nu_0$, $3\nu_0$, $4\nu_0$, $n\nu_0$, where n is an integer extending to infinity. This implies that the infinite band width is required to reproduce the rectangular waves. However, for practical purposes, higher harmonics are neglected for limiting the bandwidth

BANDWIDTH OF TRANSMISSION MEDIUM

Different types of transmission media offer different band width of which some are listed below

	Service	Frequency range	Remarks
1	Wire (most common : Coaxial Cable)	750 MHz (Bandwidth)	Normally operated below 18 GHz
2	Free space (radio waves)	540 kHz-4.2 GHz	
	(i) Standard AM	540 kHz to 30 MHz	
	(ii) FM	88-108 MHz	
	(iii) Television	54-72 MHz 76-88 MHz 174-216 MHz 420-890 MHz	VHF (Very high frequencies) TV UHF (Ultra hight frequency) TV
	(iv) Cellular mobile radio	896-901 MHz 840-935 MHz	Mobile to base Station Base station to mobile
	(v) Satellite Communication	5.925-6.425 GHz 3.7 - 4.2 GHz	Uplinking Downlinking
3	Optical communication using fibres	1THz-1000 THz (microwaves- ultra violet)	One single optical fibre offers bandwidth > 100 GHz

GROUND WAVE PROPAGATION

(a) The radio waves which travel through atmosphere following the surface of earth are known as ground waves or surface waves and their propagation is called ground wave propagation or surface wave propagation. These waves are vertically polarised in order to prevent short-circuiting of the electric component. The electrical field due to the wave induce charges in the earth's surface. As the wave travels, the induced charges in the earth also travel along it. This constitutes a current in the earth's surface. As the ground wave passes over the surface of the earth, it is weakened as a result of energy absorbed by the earth. Due to these losses the ground waves are not suited for very long range communication. Further these losses are higher for high frequency. Hence, ground wave propagation can be sustained only at low frequencies (500 kHz to 1500 kHz).

(b) The ground wave transmission becomes weaker with increase in frequency because more absorption of ground waves takes place at higher frequency during propagation through atmosphere.

(c) The ground wave propagation is suitable for low and medium frequency i.e. upto 2 MHz only.

(d) The ground wave propagation is generally used for local band broadcasting and is commonly called medium wave.

(e) The maximum range of ground or surface wave propagation depends on two factors :

(i) The frequency of the radio waves and (ii) Power of the transmitter

SKY WAVE PROPAGATION

(a) The sky waves are the radio waves of frequency between 2 MHz to 30 MHz.

(b) The ionospheric layer acts as a reflector for a certain range of frequencies (3 to 30 MHz). Therefore it is also called has ionospheric propagation or short wave propagation. Electromagnetic waves of frequencies higher than 30 MHz penetrate the ionosphere and escape.

(c) The highest frequency of radio waves which when sent straight (i.e. normally) towards the layer of ionosphere gets reflected from ionosphere and returns to the earth is called critical frequency. It is given by $\boxed{f_c = 9\sqrt{N_{max}}}$, where N is the number density of electron/m^3.

SPACE WAVE PROPAGATION

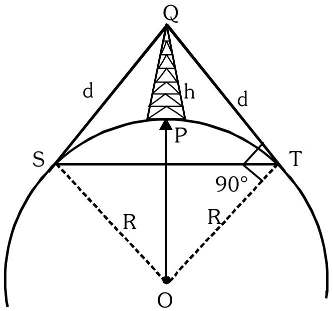
(a) The space waves are the radio waves of very high frequency (i.e. between 30 MHz. to 300 MHz or more).

(b) The space waves can travel through atmosphere from transmitter antenna to receiver antenna either directly or after reflection from ground in the earth's troposphere region. That is why the space wave propagation is also called as tropospherical propagation or line of sight propagation.

(c) The range of communication of space wave propagation can be increased by increasing the heights of transmitting and receiving antenna.

(d) **Height of transmitting Antenna :**

The transmitted waves, travelling in a straight line, directly reach the received end and are then picked up by the receiving antenna as shown in figure.



Due to finite curvature of the earth, such waves cannot be seen beyond the tangent points S and T.

$$(R+h)^2 = R^2 + d^2$$

As R>>h, So $h^2 + 2Rh = d^2 \Rightarrow d = \sqrt{2Rh}$

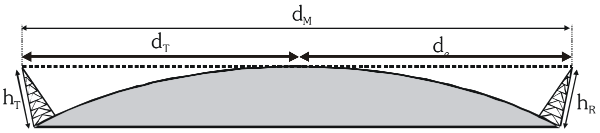
Area covered for TV transmission :

$$A = \pi d^2 = 2\pi Rh$$

Population covered=population density ×area covered

If height of receiving antenna is also given in the question then the maximum line of sight

$$d_M = \sqrt{2Rh_T} + \sqrt{2Rh_R}$$



Line of sight communication by space waves

where ;

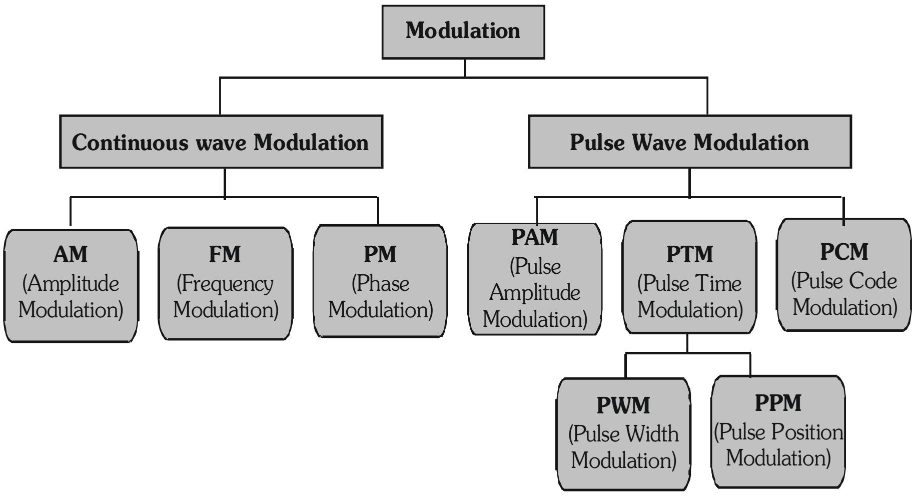
R=radius of earth (approximately 6400 km)

h_T = height of transmitting antenna

h_R = height of receiving antenna

MODULATION

The phenomenon of superposition of information signal over a high frequency carrier wave is called modulation. In this process, amplitude, frequency or phase angle of a high frequency carrier wave is modified in accordance with the instantaneous value of the low frequency information.


Modulation
Continuous wave Modulation
Pulse Wave Modulation
AM (Amplitude Modulation)
FM (Frequency Modulation)
PM (Phase Modulation)
PAM (Pulse Amplitude Modulation)
PTM (Pulse Time Modulation)
PCM (Pulse Code Modulation)
PWM (Pulse Width Modulation)
PPM (Pulse Position Modulation)

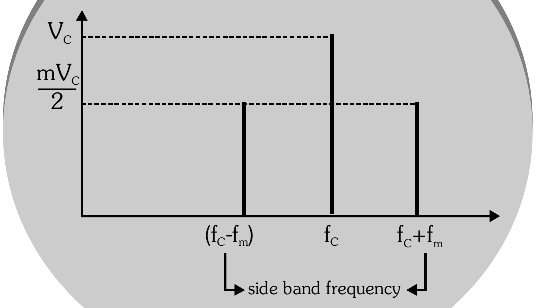
NEED FOR MODULATION

(i) **To avoid interference:** If many modulating signals travel directly through the same transmission channel, they will interfere with each other and result in distortion.

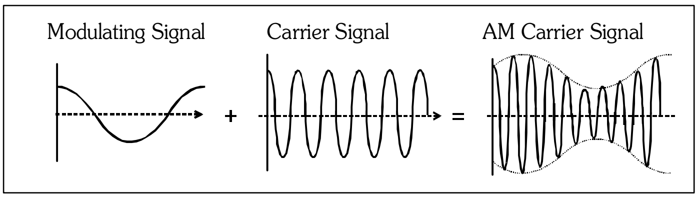
(ii) **To design antennas of practical size :** The minimum height of antenna (not of antenna tower) should be λ/4 where λ is wavelength of modulating signal. This minimum size becomes impracticale because the frequency of the modulating signal can be upto 5 kHz which corresponds to a wavelength of $3 \times 10^8/5 \times 10^3$ = 60 km. This will require an antenna of the minimum height of λ/4 = 15 km. This size of an antenna is not practical.

(iii) **Effective Power Radiated by an Antenna :** A theoretical study of radiation from a linear antenna (length ℓ) shows that the power radiated is proportional to $(\text{frequency})^2$ i.e. $(\ell/\lambda)^2$. For a good transmission, we need high powers and hence this also points out to the need of using high frequency transmission.

FREQUENCY SPECTRUM OF AM WAVE



AMPLITUDE MODULATION



Modulation factor, $m = \dfrac{\text{amplitude of modulating wave}}{\text{amplitude of normal carrier wave}}$

If $v_m = V_m \cos\omega_m t$ and $v_c = V_c \cos\omega_c t$ then $m = \dfrac{V_m}{V_c}$

- As amplitude of the carrier wave varies at signal frequency f_m so the amplitude of AM wave = $V_C + mV_C \cos\omega_m t$ & frequency of AM wave = $\dfrac{\omega_c}{2\pi}$

Therefore $v = \left[V_c(1+m)\cos\omega_m t\right]\cos\omega_c t$

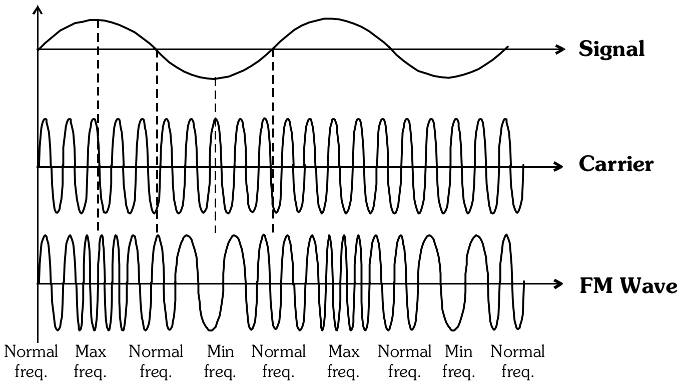
$$\Rightarrow v = V_C \cos\omega_c t + \frac{mV_C}{2}\cos(\omega_C + \omega_m)t + \frac{mV_C}{2}\cos(\omega_C - \omega_m)t$$

POWER IN AM WAVE

- Power of carrier wave : $P_C = \dfrac{V_C^2}{2R}$ where R = resistance of antenna in which power is dissipated.
- Total power of side bands : $P_{\text{sidebands}} = 2 \times \dfrac{1}{2R}\left(\dfrac{mV_C}{2}\right)^2 = \dfrac{m^2}{2}P_C$
- Total power of AM wave = $P_C\left(1 + \dfrac{m^2}{2}\right)$
- Fraction of total power carried by sidebands = $\dfrac{m^2}{2+m^2}$

FREQUENCY MODULATION (FM)

When the frequency of carries wave is changed in accordance with the instantaneous value of the modulating signal, it is called frequency modulation.



MODULATION FACTOR OR INDEX AND CARRIER SWING (CS)

- ***Modulation factor*:** $m = \dfrac{\text{max. frequency deviation}}{\text{Modulating frequency}} = \dfrac{\Delta f}{f_m}$

  $\Delta f = f_{max.} - f_c = f_c - f_{min.}$; $v_{FM} = V_C \cos[\omega_C t + m_f \cos\omega_m t]$

- ***Carrier Swing (CS)***

  The total variation in frequency from the lowest to the highest is called the carrier swing ⇒ $CS = 2x\Delta f$

- ***Side Bands***

  FM wave consists of an infinite number of side frequency components on each side of the carrier frequency f_C, $f_C \pm f_m$, $f_C \pm 2f_m$, $f_C \pm 3f_m$, & so on.

	Amplitude Modulation		Frequency Modulation
1	The amplitude of FM wave is constant, whatever be the modulation index.	1	The amplitude of AM signal varies depending on modulation index.
2	It require much wider channel (Band width) [7 to 15 times] as compared to AM.	2	Band width* is very small (One of the biggest advantage).
3	Transmitters are complex and hence expensive.	3	Relatively simple and cheap.
4	Area of reception is small since it is limited to line of sight. (This limits the FM mobile communication over a wide area)	4	Area of reception is Large.
5	Noise can be easily minimised amplitude variation can be eliminated by using limiter.	5	It is difficult to eliminate effect of noise.
6	Power contained in the FM wave is useful. Hence full transmitted power is useful.	6	Most of the power which contained in carrier is not useful. Therefore carrier power transmitted is a waste.
7	The average power is the same as the carrier wave.	7	The average power in modulated wave is greater than carrier power.
8	No restriction is placed on modulation index (m).	8	Maximum m = 1, otherwise over modulation (m > 1) would result in distortion.
9	It is possible to operate several independent transmitter on same frequency.	9	It is not possible to operate without interference.

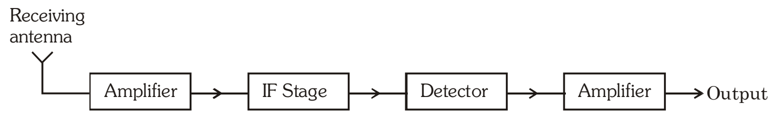
MODEM

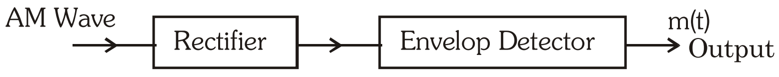
The name modem is a contraction of the terms Modulator and Demodulator. Modem is a device which can modulate as well as demodulate the signal.

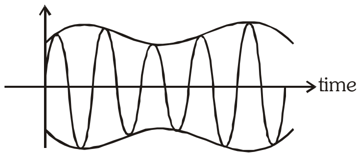
FAX (Facsimile Telegraphy)

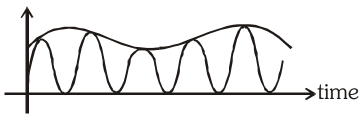
FAX is abbreviation for facsimile which means exact reproduction. The electronic reproduction of a document at a distance place is called Fax.

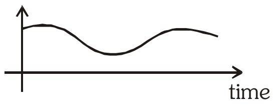
DETECTION OF AMPLITUDE MODULATION WAVE











IMPORTANT NOTES

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