ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
path to success

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PHYSICS

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

Chapter Contents 01

COMPENDIUM-1

Important Notes

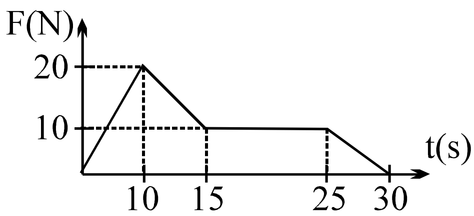
KINEMATICS

Advanced Objective (AO)

Linked Comprehension Type Question

Paragraph for Question Nos. 1 to 4

In figure shown, the graph shows the variation of a unidirectional force F acting on a body of mass 10kg (in gravity free space), with time t. The velocity of the body at t = 0 is zero. (Area under F-t curve gives change in momentum)



1. The velocity of the body at t = 30 s is

(A) 30 m/s (B) 20 m/s (C) 40 m/s (D) none

2. The power of the force at t = 12 s is (Power = force × velocity)

(A) 225.0 W (B) 217.6 W (C) 226.7 W (D) none

3. The average acceleration of the body from t = 0 to t = 15 s is :-

(A) 1.25 m/s^2 (B) 4/7 m/s^2 (C) 5/6 m/s^2 (D) 7/6 m/s^2

4. The change in momentum of the body between the time t = 10 s to 15 s is :-

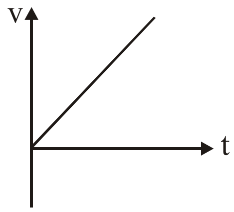
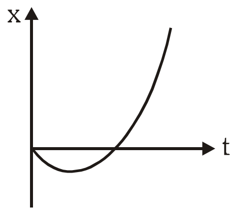
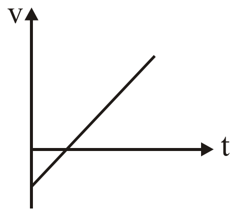
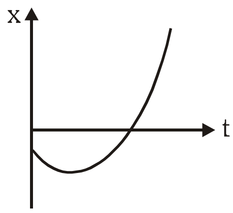
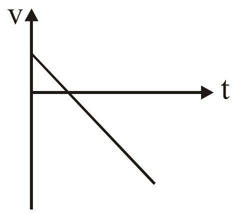
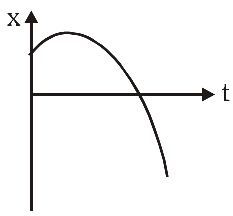
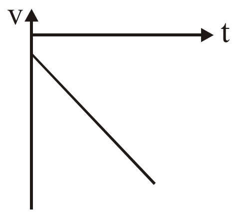
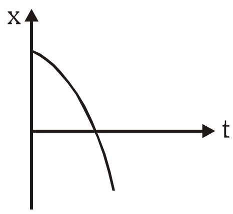
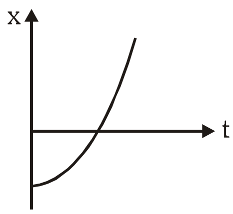
(A) 100 kg.m/s (B) 75 kg.m/s (C) 125 kg.m/s (D) none

Numerical Grid Type Question

5. A person decided to walk on an escalator which is moving at constant rate (speed). When he moves at the rate 1 step/sec, then he reaches top in 20 steps. Next day he goes 2 steps / sec. and reaches top in 32 steps. If speed of escalator is n steps / sec. Find the value of n.

Matrix Match Type Question

6. In the first column of the given table, some velocity-time (v-t) graphs and in the second column some position-time (x-t) graphs are shown. Suggest suitable match or matches.

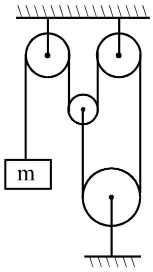
Column-I	**Column-II**
(A) 	(P) 
(B) 	(Q) 
(C) 	(R) 
(D) 	(S) 
	(T) 

NLM & FRICTION

Advanced Objective (AO)

Single Correct Answer Type Question

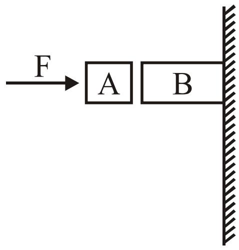
1. If the string & all the pulleys are ideal, acceleration of mass m is :-



(A) $\frac{g}{2}$ (B) 0 (C) g (D) dependent on m

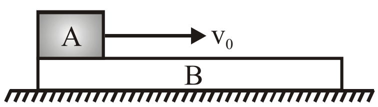
2. Given in the figure are two blocks A and B of weight 20 N and 100 N, respectively. These are being pressed against a wall by a force F as shown. If the coefficient of friction between the blocks is 0.1 and between block B and the wall is 0.15, the frictional force applied by the wall on block B is :-

[JEE-MAIN-2015]

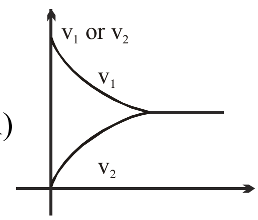


(A) 120 N (B) 150 N (C) 100 N (D) 80 N

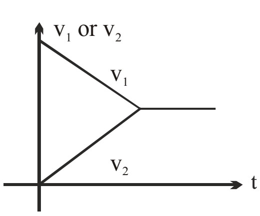
3. A block A is placed over a long rough plank B of same mass as shown in figure. The plank is placed over a smooth horizontal surface. At time t = 0, block A is given a velocity v_0 in horizontal direction. Let v_1 and v_2 be the velocities of A and B at time t. Then choose the correct graph between v_1 or v_2 and t.



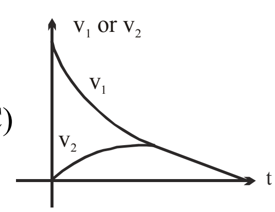
(A)



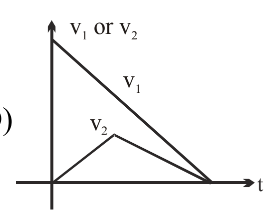
(B)



(C)

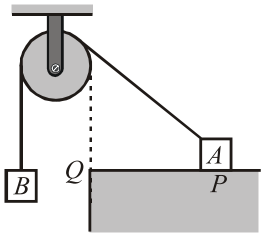


(D)



Multiple Correct Answer Type Question

4. Two small identical blocks are connected to the ends of a string passing over pulley as shown when the system is released from rest.



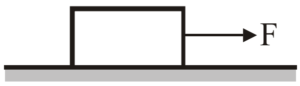
(A) block *A* and *B* do not move

(B) block *A* accelerates towards pulley along the string.

(C) block *A* does not leave contact with table till it reaches to the edge *Q* of the table

(D) Normal reaction of table on block A is less than weight of block *A* between *P* and Q and at Q it vanishes

5. A block is kept on a rough surface and applied with a horizontal force as shown which is gradually increasing from zero. The coefficient of static and kinetic friction are $1/\sqrt{3}$ then



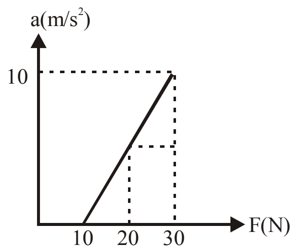
(A) When F is less than the limiting friction, angle made by net force on the block by the surface is less than 30° with vertical.

(B) When the block is just about to move, the angle made by net force by the surface on the block becomes equal to 30° with vertical.

(C) When the block starts to accelerate, the angle made by net force by the surface on the block becomes constant and equal to 30° vertical.

(D) The angle made by net force with vertical on the block by the surface, depends on the mass of the block.

6. A block placed on a rough horizontal surface is pushed with a force F acting horizontally on the block. The magnitude of F is increased and acceleration produced is plotted in the graph shown.



(A) Mass of the block is 2 kg.

(B) Coefficient of friction between block and surface is 0.5.

(C) Limiting friction between block and surface is 10 N.

(D) When $F = 8$ N, friction between block and surface is 10 N.

7. A block is released from rest from a point on a rough inclined place of inclination 37°. The coefficient of friction is 0.5.

(A) The time taken to slide down 9 m on the plane is 3 s.

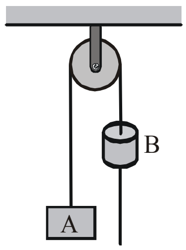
(B) The velocity of block after moving 4 m is 4 m/s.

(C) The block travels equal distances in equal intervals of time.

(D) The velocity of block increases linearly.

Advanced Subjective (AS)

1. A 2 kg block *A* is attached to one end of a light string that passes over an an ideal pulley and a 1 kg sleeve *B* slides down the other part of the string with an acceleration of 5 m/s^2 with respect to the string. Find the acceleration of the block, acceleration of sleeve and tension in the string. [g = 10 m/s^2]

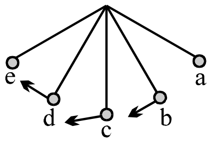


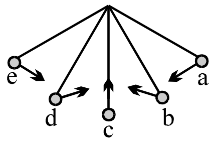
CIRCULAR MOTION

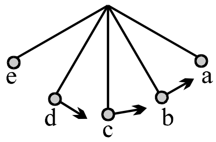
Advanced Objective (AO)

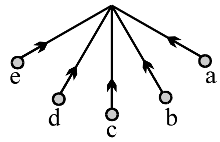
Single Correct Answer Type Question

1. Which of the following best illustrates the acceleration of a pendulum bob at points a through e?

(A) 

(B) 

(C) 

(D) 

2. A uniform rod of mass m and length ℓ rotates in a horizontal plane with an angular velocity ω about a vertical axis passing through one end. The tension in the rod at a distance x from the axis is :

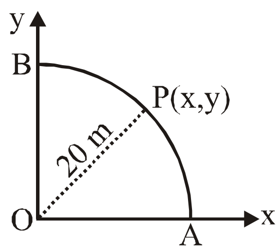
(A) $\frac{1}{2}m\omega^2 x$

(B) $\frac{1}{2}m\omega^2 \frac{x^2}{\ell}$

(C) $\frac{1}{2}m\omega^2 \ell \left(1-\frac{x}{\ell}\right)$

(D) $\frac{1}{2}\frac{m\omega^2}{\ell}[\ell^2 - x^2]$

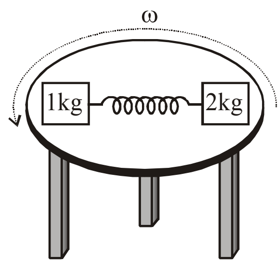
3. A point P moves in counter clockwise direction on a circular path as shown in the figure. The movement of 'P' is such that it sweeps out a length $s = t^3 + 5$, where s is in metres and t is in seconds. The radius of the path is 20 m. The acceleration of 'P' when t = 2s is nearly : **[AIEEE - 2010]**



(A) 14 m/s^2 (B) 13 m/s^2 (C) 12 m/s^2 (D) 7.2 m/s^2

Multiple Correct Answer Type Question

4. On a circular turn table rotating about its center horizontally with uniform angular velocity ω rad/s placed two blocks of mass 1 kg and 2 kg, on a diameter symmetrically about center. Their separation is 1m and friction is sufficient to avoid slipping. The spring between them as shown is stretched and applied force of 5N. If f_1 and f_2 are values of friction on 1 kg & 2kg block respectively:-



(A) For ω = 2 rad/s, f_1 = 3N & f_2 = 1N

(B) For ω = 3 rad/s, f_1 = 0.5 N & f_2 = 4N

(C) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 5N

(D) For ω = $\sqrt{10}$ rad/s, f_1 = 0 & f_2 = 0N

Advanced Subjective (AS)

1. Two blocks of mass m_1=10kg and m_2=5kg connected to each other by a massless inextensible string of length 0.3m are placed along a diameter of a turn table. The coefficient of friction between the table and m_1 is 0.5 while there is no friction between m_2 and the table. The table is rotating with an angular velocity of 10 rad/sec about a vertical axis passing through its centre. The masses are placed along the diameter of the table on either side of the centre O such that m_1 is at a distance of 0.124 m from O. The masses are observed to be at rest with respect to an observer on the turn table.

(i) Calculate the frictional force on m_1

(ii) What should be the minimum angular speed of the turn table so that the masses will slip from this position.

(iii) How should the masses be placed with the string remaining taut, so that there is no frictional force acting on the mass m_1.

WORK, POWER & ENERGY

Advanced Objective (AO)

Single Correct Answer Type Question

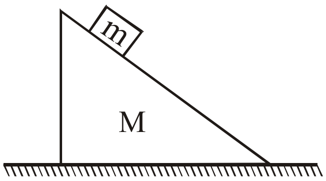
1. The potential energy function for the force between two atoms in a diatomic molecule is approximately given by $U(x) = \frac{a}{x^{12}} - \frac{b}{x^{6}}$, where a and b are constant and x is the distance between the atoms. if the dissociation energy of the molecule is $D = [U(x=\infty) - U_{\text{at equilibrium}}]$, D is : **[AIEEE - 2010]**

(A) $\frac{b^2}{6a}$ (B) $\frac{b^2}{2a}$

(C) $\frac{b^2}{12a}$ (D) $\frac{b^2}{4a}$

Matrix Match Type Question

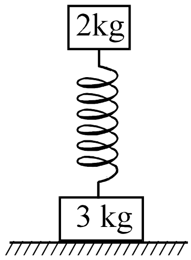
2. A block of mass m lies on wedge of mass M. The wedge in turn lies on smooth horizontal surface. Friction is absent everywhere. The wedge block system is released from rest. All situation given in column–I are to be estimated in duration the block undergoes a vertical displacement 'h' starting from rest (assume the block to be still on the wedge, g is acceleration due to gravity).



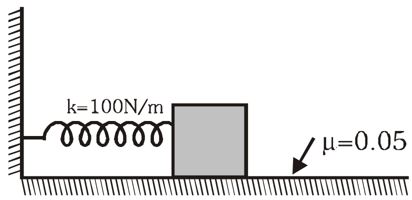
	Column I		Column II
(A)	Work done by normal reaction acting on the block is	(P)	Positive
(B)	Work done by normal reaction (exerted by block) acting on wedge is	(Q)	Negative
(C)	The sum of work done by normal reaction on block and work done by normal reaction (exerted by block) on wedge is	(R)	Zero
(D)	Net work done by all forces on block is	(S)	Less than mgh in magnitude

Advanced Subjective (AS)

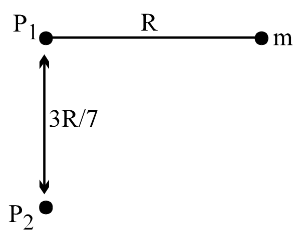
1. The ends of spring are attached to blocks of mass 3kg and 2kg. The 3kg block rests on a horizontal surface and the 2kg block which is vertically above it is in equilibrium producing a compression of 1cm of the spring. The 2kg mass must be compressed further by at least _______, so that when it is released, the 3 kg block may be lifted off the ground.



2. A block of mass 1 kg is attached to a spring with a force constant 100 N/m and rests on a rough horizontal ground as shown in the figure. Initial displacement of block from natural length is 50 cm. The total distance covered by the block if coefficient of friction between block & ground is 0.05. [$g = 10 m/s^2$]



3. A simple pendulum consists of a bob of mass m and a string of length R suspended from a peg P_1 on the wall. A second peg P_2 is fixed vertically below the first one at a distance $3R/7$ from it. The pendulum is drawn aside such that the string is horizontal and released. Calculate the maximum height (with respect to the lowest point) to which it rises

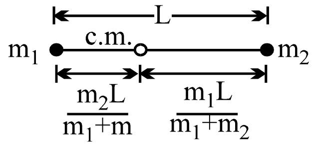


CENTER OF MASS & COLLISION

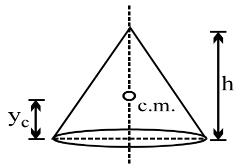
THEORY

Centre of Mass of Some Common Systems :

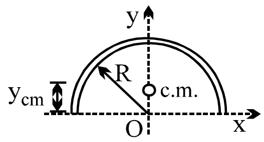
(i) A system of two point masses.
The centre of mass lie closer to the heavier mass.



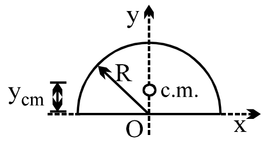
(ii) A circular cone $y_c = \dfrac{h}{4}$



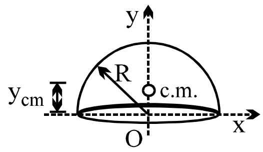
(iii) A semi-circular ring $y_c = \dfrac{2R}{\pi}$; $x_c = 0$



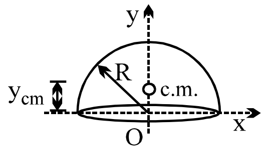
(iv) A semi-circular disc $y_c = \dfrac{4R}{3\pi}$; $x_c = 0$



(v) A hemispherical shell $y_c = \dfrac{R}{2}$; $x_c = 0$



(vi) A solid hemisphere $y_c = \dfrac{3R}{8}$; $x_c = 0$



Advanced Objective (AO)

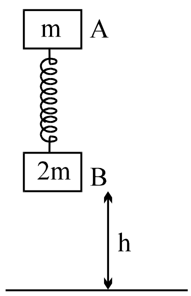
Single Correct Answer Type Question

1. A mass 'm' moves with a velocity 'v' and collides inelastically with another identical mass at rest. After collision the 1st mass moves with velocity $\frac{v}{\sqrt{3}}$ in a direction perpendicular to the initial direction of motion. Find the speed of the 2nd mass after collision :-

(A) $\frac{2v}{\sqrt{3}}$ (B) $\frac{v}{\sqrt{3}}$

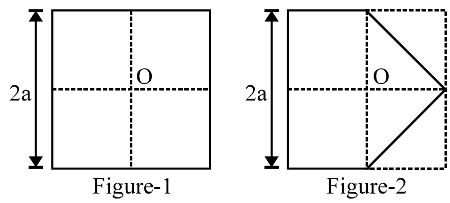
(C) $v\sqrt{\frac{2}{3}}$ (D) the situation of the problem is not possible without external impulse

2. From what minimum height h must the system be released when spring is unstretched so that after perfectly inelastic collision (e = 0) with ground, B may be lifted off the ground (Spring constant = k).



(A) mg/(4k) (B) 4mg/k (C) mg/(2k) (D) none

3. A piece of paper (shown in figure-1) is in form of a square. Two corners of this square are folded to make it appear like figure-2. Both corners are put together at centre of square 'O'. If O is taken to be (0, 0), the centre of mass of new system will be at



(A) $\left(\frac{-a}{8}, 0\right)$ (B) $\left(\frac{-a}{6}, 0\right)$ (C) $\left(\frac{a}{12}, 0\right)$ (D) $\left(\frac{-a}{12}, 0\right)$

4. A man weighing 80 kg is standing at the centre of a flat boat and he is 20 m from the shore. He walks 8 m on the boat towards the shore and then halts. The boat weight 200 kg. How far is he from the shore at the end of this time ?

(A) 11.2 m (B) 13.8 m (C) 14.3 m (D) 15.4 m

5. **Statement-1 :** Two particles moving in the same direction do not lose all their energy in a completely inelastic collision.

Statement-2 : Principle of conservation of momentum holds true for all kinds of collisions.

(A) Statement–1 is true, Statement–2 is false

(B) Statement–1 is true, Statement–2 is true; Statement–2 is the correct explanation of Statement–1

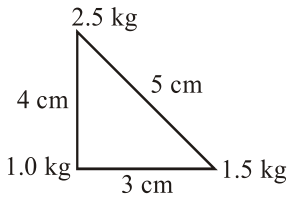
(C) Statement–1 is true, Statement–2 is true; Statement–2 is not the correct explanation of Statement–1

(D) Statement–1 is false, Statement–2 is true

6. A particle of mass 'm' is moving with speed '2v' and collides with a mass '2m' moving with speed 'v' in the same direction. After collision, the first mass is stopped completely while the second one splits into two particles each of mass 'm', which move at angle 45° with respect to the origianl direction. The speed of each of the moving particle will be :-

(A) $v/\left(2\sqrt{2}\right)$ (B) $2\sqrt{2}v$ (C) $\sqrt{2}v$ (D) $v/\sqrt{2}$

7. Three point particles of masses 1.0 kg, 1.5 kg and 2.5 kg are placed at three corners of a right angle triangle of sides 4.0 cm, 3.0 cm and 5.0 cm as shown in the figure. The center of mass of the system is at a point:



(A) 1.5 cm right and 1.2 cm above 1 kg mass

(B) 0.9 cm right and 2.0 cm above 1 kg mass

(C) 0.6 cm right and 2.0 cm above 1 kg mass

(D) 2.0 cm right and 0.9 cm above 1 kg mass

8. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is :

(A) $\frac{3}{4}mu^2$ (B) $\frac{1}{8}mu^2$ (C) $\sqrt{\frac{2}{3}}mu^2$ (D) $\frac{1}{3}mu^2$

9. A rod of length L has non-uniform linear mass density given by $\rho(x) = a + b\left(\frac{x}{L}\right)^2$, where a and b are constants and $0 \le x \le L$. The value of x for the centre of mass of the rod is at :

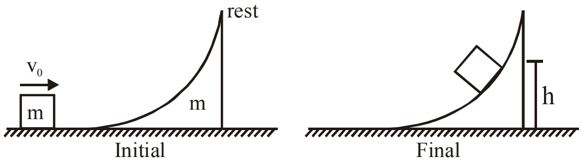
(A) $\frac{4}{3}\left(\frac{a+b}{2a+3b}\right)L$ (B) $\frac{3}{2}\left(\frac{a+b}{2a+b}\right)L$ (C) $\frac{3}{2}\left(\frac{2a+b}{3a+b}\right)L$ (D) $\frac{3}{4}\left(\frac{2a+b}{3a+b}\right)L$

Multiple Correct Answer Type Question

10. Two charges moving under their only own mutual attraction separated by large distance initially. Then choose the correct statement(s)

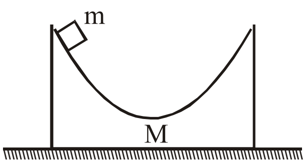
(A) If both are free, mechanical energy is conserved.
(B) If one is fixed and other is free, mechanical energy is conserved.
(C) If one is fixed and other is free, momentum is conserved.
(D) If both are free momentum is conserved.

11. In the arrangement shown, horizontal surface is smooth, but friction is present between the block and the surface of the wedge. Block is given velocity v_0 at t = 0. After achieving height 'h' on the wedge, block comes to rest with respect to wedge at $t = t_0$. Then from t = 0 to $t = t_0$:-



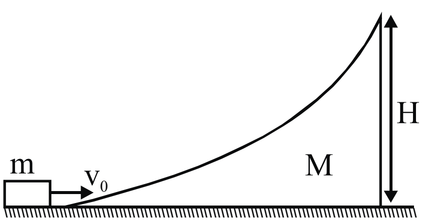
(A) Work done by friction on the block is negative
(B) Work done by friction on the wedge is negative
(C) Work done by block on the wedge is positive
(D) Work done by wedge on the block is positive

12. Figure shows a wedge on which a small block is released from rest. All the surfaces are smooth system comprises of wedge and blocks. Mark the correct statement(s) regarding motion of block on wedge till block attains maximum height on wedge.



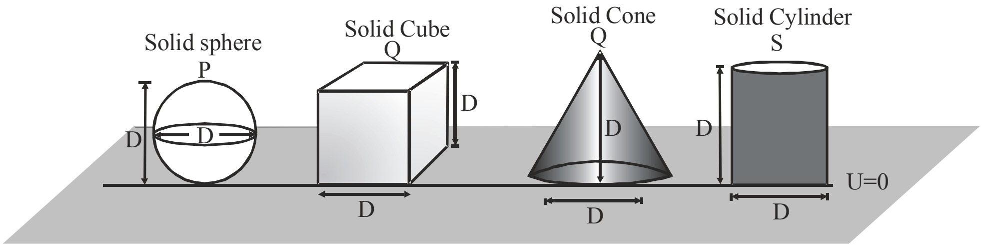
(A) Acceleration of centre of mass of system is initially vertically down then vertically up.
(B) Initially centre of mass moves down and then up.
(C) At the maximum height block and wedge move with common velocity.
(D) Centre of mass of wedge moves towards left then right

13. Figure shows a block of mass m projected with velocity v_0 towards a wedge. Consider all the surfaces to be smooth. Block does not have sufficient energy to negotiate (over come) wedge. Mark the correct option(s)



(A) when block is at the maximum height on wedge, block and wedge have velocity equal to velocity of centre of mass of block wedge system
(B) wedge acquires maximum speed with respect to ground when block returns to lowest point on wedge.
(C) momentum of wedge and block is conserved at all times
(D) centre of mass of wedge and block remains stationary

14. Assuming potential energy 'U' at ground level to be zero.



All objects are made up of same material.

U_P = Potential energy of solid sphere　　U_Q = Potential energy of solid cube

U_R = Potential energy of solid cone　　U_S = Potential energy of solid cylinder

(A) $U_S > U_P$　　(B) $U_Q > U_S$　　(C) $U_P > U_Q$　　(D) $U_S > U_R$

15. A blast breaks a body initially at rest of mass 0.5 kg into three pieces, two smaller pieces of equal mass and the third double the mass of either of small piece. After the blast the two smaller masses move at right angles to one another with equal speed. Find the statements that is/are true for this case assuming that the energy of blast is totally transferred to masses.

(A) All the three pieces share the energy of blast equally

(B) The speed of bigger mass is $\sqrt{2}$ times the speed of either of the smaller mass

(C) The direction of motion of bigger mass makes an angle of 135° with the direction of smaller pieces

(D) The bigger piece carries double the energy of either piece.

16. A particle moving with kinetic energy = 3 joule makes an elastic head on collision with a stationary particle which has twice its mass during the impact.

(A) The minimum kinetic energy of the system is 1 joule.

(B) The maximum elastic potential energy of the system is 2 joule.

(C) Momentum and total kinetic energy of the system are conserved at every instant.

(D) The ratio of kinetic energy to potential energy of the system first decreases and then increases.

17. In a one dimensional collision between two identical particles A and B, B is stationary and A has momentum p before impact. During impact, B gives impulse J to A.

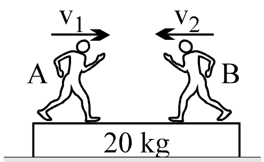
(A) The total momentum of the 'A plus B' system is p before and after the impact, and (p-J) during the impact.

(B) During the impact A gives impulse of magnitude J to B

(C) The coefficient of restitution is $\frac{2J}{p} - 1$

(D) The coefficient of restitution is $\frac{J}{p} + 1$

18. In the figure shown the system is at rest initially. Two persons 'A' and 'B' of masses 40 kg each move with speeds v_1 and v_2 respectively towards each other on a plank lying on a smooth horizontal surface as shown in figure. Plank travels a distance of 20 m towards right direction in 5 sec. (Here v_1 and v_2 are given with respect to the plank). Then the possible condition(s) can be



(A) $v_1 = 0$ m/s, $v_2 = 10$ m/s (B) $v_1 = 5$ m/s, $v_2 = 15$ m/s
(C) $v_1 = 10$ m/s, $v_2 = 20$ m/s (D) $v_1 = 2$ m/s, $v_2 = 12$ m/s

Linked Comprehension Type Question

Paragraph for Question No. 19 and 20 (2 Question)

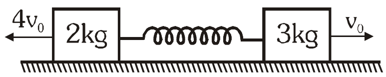
A projectile of mass "m" is projected from ground with a speed of 50 m/s at an angle of 53° with the horizontal. It breaks up into two equal parts at the highest point of the trajectory. One particle coming to rest immediately after the explosion.

19. The ratio of the radii of curvatures of the moving particle just before and just after the explosion are:
(A) 1 : 4 (B) 1 : 3 (C) 2 : 3 (D) 4 : 9

20. The distance between the pieces of the projectile when they reach the ground are:
(A) 240 (B) 360 (C) 120 (D) none

Paragraph for Question 21 to 23 (3 Question)

2 kg and 3 kg blocks are placed on a smooth horizontal surface and connected by spring which is unstretched initially. The blocks are imparted velocities as shown in the figure.



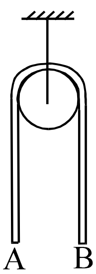
21. The maximum energy stored in the spring in the subsequent motion will be
(A) $5v_0^2$ (B) $15v_0^2$ (C) zero (D) $10v_0^2$

22. Maximum speed of 3 kg block in the subsequent motion will be
(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

23. Maximum speed of 2 kg block in the subsequent motion will be
(A) v_0 (B) $2v_0$ (C) $3v_0$ (D) $4v_0$

Paragraph for Question No. 24 and 25 (2 Question)

A uniform chain of length 2L is hanging in equilibrium position, if end B is given a slightly downward displacement the imbalance causes an acceleration. Here pulley is small and smooth & string is inextensible



24. The acceleration of end B when it has been displaced by distance x, is

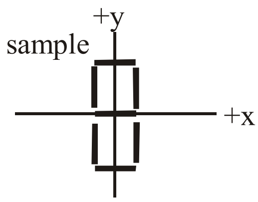
(A) $\frac{x}{L}g$ (B) $\frac{2x}{L}g$ (C) $\frac{x}{2}g$ (D) g

25. The velocity v of the string when it slips out of the pulley (height of pulley from floor > 2L)

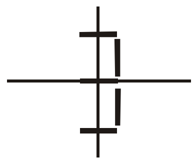
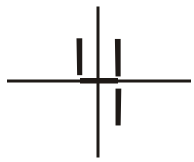
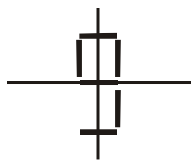
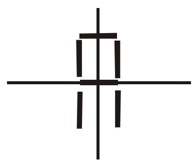
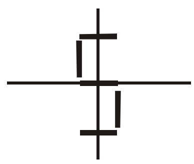
(A) $\sqrt{\frac{gL}{2}}$ (B) $\sqrt{2gL}$ (C) $\sqrt{gL}$ (D) none of these

Matrix Match Type Question

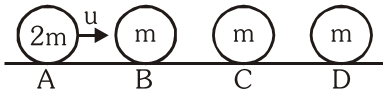
26. On the left are statements about the location of the center of mass of the objects depicted on the right. The objects on the right are symbols constructed out of sticks of equal length and mass. The location of the center of mass is described using the coordinate system depicted in the sample.



The centre of mass lies at x = 0, y = 0

Column I		Column II	
(A)	The center of mass is at x > 0 and y = 0	(P)	
(B)	The center of mass is at x = 0 and y > 0	(Q)	
(C)	The center of mass is at x > 0 and y > 0	(R)	
(D)	The center of mass is at x = 0 and y = 0	(S)	
		(T)	

27. Four balls A,B,C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity u towards B- (Assume each collision to be elastic)



Column-I		Column-II
(A)	Total impulse of all collisions on A	(P) $\frac{4mu}{9}$
(B)	Total impulse of all collisions on B	(Q) $\frac{4mu}{27}$
(C)	Total impulse of all collision on C	(R) $\frac{4mu}{3}$
(D)	Total impulse of all collisions on D	(S) $\frac{52}{27}mu$

28. In Column-I, 4 situations are depicted and in column-II, 4 possible kinds of collision are listed. Match the situation with type of collision.

Column-I **Before** | **Column-II** **After**

(A) 

(P) Elastic

(B) 

(Q) Perfectly Inelastic

(C) 

(R) Partially elastic

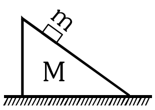
(S) Collision is not possible

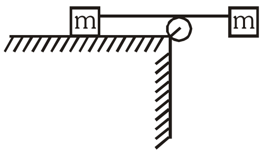
29. A particle of mass m, kinetic energy K and momentum p collides head on elastically with another particle of mass 2 m at rest. After collision :

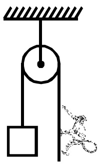
Column I		Column II
(A)	Momentum of first particle	(P) 3/4 p
(B)	Momentum of second particle	(Q) – K/9
(C)	Kinetic energy of first particle	(R) – p/3
(D)	Kinetic energy of second particle	(S) $\frac{8K}{9}$
		(T) None

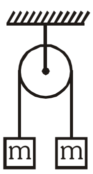
30. In each situation of column–I, a system involving two bodies is given. All strings and pulleys are light and friction is absent everywhere. Initially each body of every system is at rest. Consider the system in all situation of column I from rest till any collision occurs. Then match the statements in column – I with the corresponding results in column–II

Column I		Column II
(A) The block plus wedge system is placed over smooth horizontal surface. After the system is released from rest, the centre of mass of system	(P)	Shifts towards right
(B) The string connecting both the blocks of mass m is horizontal. Left block is placed over smooth horizontal table as shown. After the two block system is released from rest, the centre of mass of system	(Q)	Shifts downwards
(C) The block and monkey have same mass. The monkey starts climbing up the rope. After the monkey starts climbing up, the centre of mass of monkey+block system	(R)	Shifts upwards
(D) Both block of mass m are initially at rest. The left block is given initial velocity u downwards. Then, the centre of mass of two block system afterwards	(S)	Does not shift

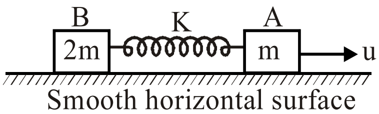
(A) 

(B) 

(C) 

(D) 

31. Two blocks A and B of mass m and 2m respectively are connected by a massless spring of spring constant K. This system lies over a smooth horizontal surface. At t = 0 the block A has velocity u towards right as shown while the speed of block B is zero, and the length of spring is equal to its natural length at that instant.

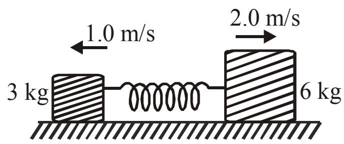


Column-I	Column-II
(A) The velocity of block A	(P) can never be zero
(B) The velocity of block B	(Q) may be zero at certain instants of time
(C) The kinetic energy of system of two block	(R) is minimum at maximum compression of spring
(D) The potential energy of spring	(S) is maximum at maximum extension of spring

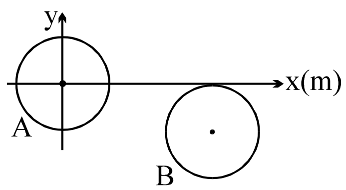
Advanced Subjective (AS)

1. From a circle of radius a, an isosceles right angled triangle with the hypotenuse as the diameter of the circle is removed. The distance of the centre of gravity of the remaining position from the centre of the circle is

2. Two blocks of mass 3 kg and 6 kg respectively are placed on a smooth horizontal surface. They are connected by a light spring of force constant k = 200 N/m. Initially the spring is unstretched. The indicated velocities are imparted to the blocks. The maximum extension of the spring will be



3. Two smooth balls A and B, each of mass m and radius R, have their centres at (0,0,R) and at (5R,–R,R) respectively, in a coordinate system as shown. Ball A, moving along positive x axis, collides with ball B. Just before the collision, speed of ball A is 4 m/s and ball B is stationary. The collision between the balls is elastic. Find Velocity of the ball A just after the collision and impulse of the force exerted by A on B during the collision.



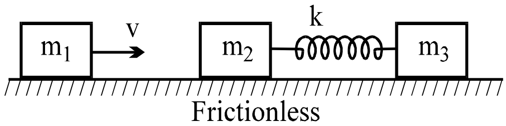
4. Mass m_1 hits & sticks with m_2 while sliding horizontally with velocity v along the common line of centres of the three equal masses ($m_1 = m_2 = m_3 = m$). Initially masses m_2 and m_3 are stationary and the spring is unstretched. Find the

(i) velocities of m_1, m_2 and m_3 immediately after impact.

(ii) maximum kinetic energy of m_3.

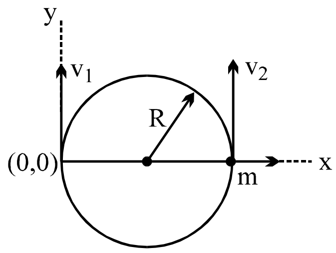
(iii) minimum kinetic energy of m_2.

(iv) maximum compression of the spring.

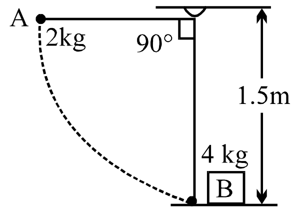


5. A particle of mass m, moving in a circular path of radius R with a constant speed v_2 is located at point (2 R, 0) at time t = 0 and a man starts moving with a velocity v_1 along the +ve y-axis from origin at time t = 0. Calculate the linear momentum of the particle w.r.t. the man as a function of time.

[IIT-JEE' 2003]



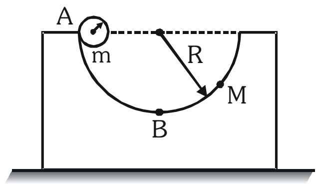
6. A sphere A is released from rest in the position shown and strikes the block B which is at rest. If e = 0.75 between A and B and μ_k = 0.5 between B and the support, determine



(i) the velocity of A just after the impact

(ii) the maximum displacement of B after the impact.

7. A block of mass M with a semicircular track of radius R, rests on a horizontal frictionless surface. A uniform cylinder of radius r and mass m is released from rest at the top point A (see Fig). The cylinder slips on the semicircular frictionless track. How far has the block moved when the cylinder reaches the bottom (point B) of the track ? How fast is the block moving when the cylinder reaches the bottom of the track?



EXERCISE (J-A)

1. **Statement–I :** In an elastic collision between two bodies, the relative speed of the bodies after collision is equal to the relative speed before the collision. **[IIT-JEE 2007]**

Because :

Statement–II : In an elastic collision, the linear momentum of the system is conserved.

2. Two balls, having linear momenta $\vec{p}_1 = p\hat{i}$ and $\vec{p}_2 = -p\hat{i}$, undergo a collision in free space. There is no external force acting on the balls. Let $\vec{p}'_1$ and $\vec{p}'_2$ be their final momenta. The following option(s) is(are) NOT ALLOWED for any non-zero value of p, a_1, a_2, b_1, b_2, c_1 and c_2.

[IIT-JEE 2008]

(A) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}'_2 = a_2\hat{i} + b_2\hat{j}$

(B) $\vec{p}'_1 = c_1\hat{k}$, $\vec{p}'_2 = c_2\hat{k}$

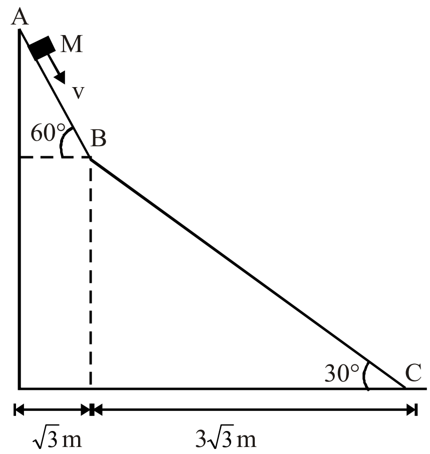
(C) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j} + c_1\hat{k}$, $\vec{p}'_2 = a_2\hat{i} + b_2\hat{j} - c_1\hat{k}$

(D) $\vec{p}'_1 = a_1\hat{i} + b_1\hat{j}$, $\vec{p}'_2 = a_2\hat{i} + b_1\hat{j}$

Paragraph for Question no. 3 to 5 (3 Question)

A small block of mass M moves on a frictionless surface of an inclined plane, as shown in figure. The angle of the incline suddenly changes from 60° to 30° at point B. The block is initially at rest at A. Assume that collisions between the block and the incline are totally inelastic ($g = 10\ m/s^2$)

[IIT-JEE 2008]



3. The speed of the block at point B immediately after it strikes the second incline is :

(A) $\sqrt{60}$ m/s (B) $\sqrt{45}$ m/s (C) $\sqrt{30}$ m/s (D) $\sqrt{15}$ m/s

4. The speed of the block at point C, immediately before it leaves the second incline is :

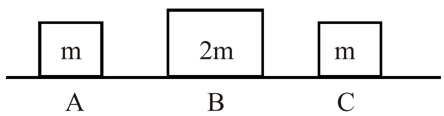
(A) $\sqrt{120}$ m/s (B) $\sqrt{105}$ m/s (C) $\sqrt{90}$ m/s (D) $\sqrt{75}$ m/s

5. If collision between the block and the incline is completely elastic, then the vertical (upward) component of the velocity of the block at point B, immediately after it strikes the second incline is :

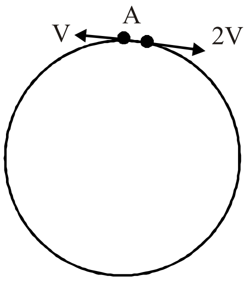
(A) $\sqrt{30}$ m/s (B) $\sqrt{15}$ m/s (C) 0 (D) $-\sqrt{15}$ m/s

6. Three objects A, B and C are kept in a straight line on a frictionless horizontal surface. These have masses m, 2m and m, respectively. The object A moves towards B with a speed 9 m/s and makes an elastic collision with it. Thereafter, B makes completely inelastic collision with C. All motions occur on the same straight line. Find the final speed (in m/s) of the object C.

[IIT-JEE-2009]

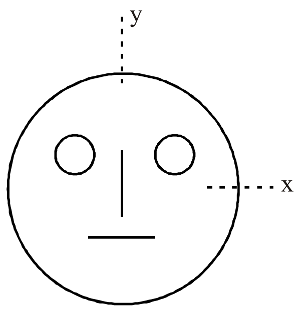


7. Two small particles of equal masses start moving in opposite directions from a point A in a horizontal circular orbit. Their tangential velocities are v and 2v, respectively, as shown in the figure. Between collisions, the particles move with constant speeds. After making how many elastic collisions, other than that at A, these two particles will again reach the point A? **[IIT-JEE-2009]**



(A) 4 (B) 3 (C) 2 (D) 1

8. Look at the drawing given in the figure which has been drawn with ink of uniform line-thickness. The mass of ink used to draw each of the two inner circles, and each of the two line segments is m. The mass of the ink used to draw the outer circle is 6m. The coordinates of the centres of the different parts are: outer circle (0, 0), left inner circle (–a, a), right inner circle (a, a), vertical line (0, 0) and horizontal line (0, –a). The y-coordinate of the centre of mass of the ink in this drawing is **[IIT-JEE-2009]**



(A) $\frac{a}{10}$ (B) $\frac{a}{8}$ (C) $\frac{a}{12}$ (D) $\frac{a}{3}$

9. A point mass of 1 kg collides elastically with a stationary point mass of 5 kg. Aftr their collision, the 1 kg mass reverses its direction and moves with a speed of 2 m/s. Which of the following statemet(s) is (are) correct for the system of these two masses ? **[IIT-JEE 2010]**

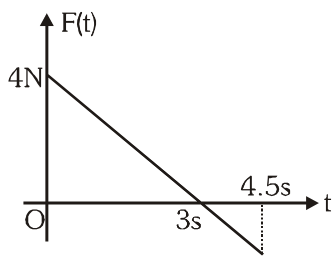
(A) Total momentum of the system is 3 kg m/s.

(B) Momentum of 5 kg mass after collision is 4 kg m/s.

(C) Kinetic energy of the centre of mass is 0.75 J.

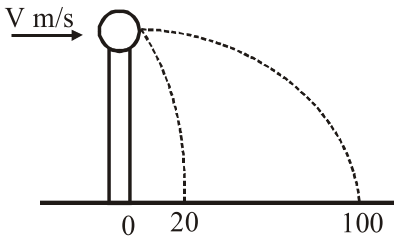
(D) Total kinetic energy of the system is 4 J.

10. A block of mass 2 kg is free to move along the x–axis. It is at rest and from t=0 onwards it is subjected to a time–dependent force F(t) in the x–direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 second is **[IIT-JEE-2010]**



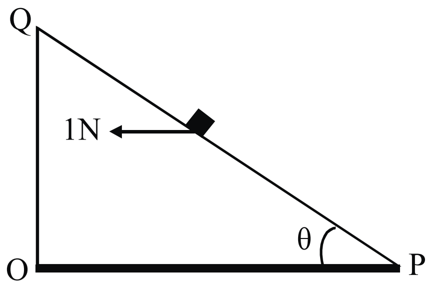
(A) 4.50 J (B) 7.50 J (C) 5.06 J (D) 14.06 J

11. A ball of mass 0.2 kg rests on a vertical post of height 5m. A bullet of mass 0.01 kg, traveling with a velocity V m/s in a horizontal direction, hits the centre of the ball. After the collision, the ball and bullet travel independently. The ball hits the ground at a distance of 20 m and the bullet at a distance of 100 m from the foot of the post. The initial velocity V of the bullet is **[IIT-JEE 2011]**



(A) 250 m/s
(B) $250\sqrt{2}$ m/s
(C) 400 m/s
(D) 500 m/s

12. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2) **[IIT-JEE 2012]**



(A) θ = 45°

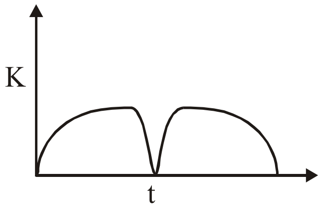
(B) θ > 45° and a frictional force acts on the block towards P

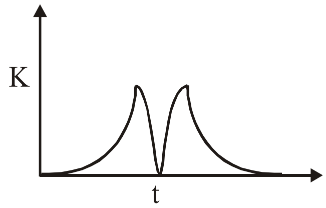
(C) θ > 45° and a frictional force acts on the block towards Q

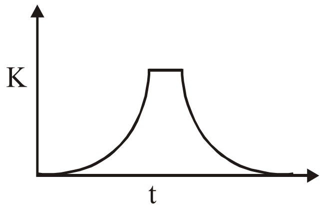
(D) θ < 45° and a frictional force acts on the block towards Q

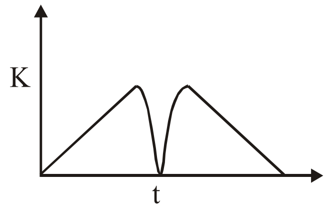
13. A bob of mass m, suspended by a string of length ℓ_1 is given a minimum velocity required to complete a full circle in the vertical plane. At the highest point, it collides elastically with another bob of mass m suspended by a string of length ℓ_2, which is initially at rest. Both the strings are mass-less and inextensible. If the second bob, after collision acquires the minimum sped required to complete a full circle in the vertical plane, the ratio $\frac{\ell_1}{\ell_2}$ is. **[JEE Advanced-2013]**

14. A tennis ball is dropped on a horizontal smooth surface. It bounces back to its original position after hitting the surface. The force on the ball during the collision is proportional to the length of compression of the ball. Which one of the following sketches describes the variation of its kinetic energy K with time t most appropriately ? The figures are only illustrative and not to the scale. **[JEE Advanced-2014]**

(A) 

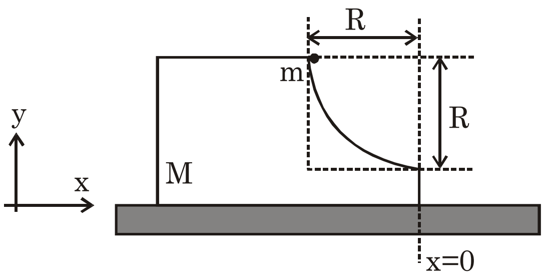
(B) 

(C) 

(D) 

15. A block of mass M has a circular cut with a frictionless surface as shown. The block rests on the horizontal frictionless surface of a fixed table. Initially the right edge of the block is at x = 0, in a *co-ordinate system fixed to the table*. A point mass m is released from rest at the topmost point of the path as shown and it slides down. When the mass loses contact with the block, its position is x and the velocity is v. At that instant, which of the following options is/are **correct** ?

[JEE Advanced-2017]



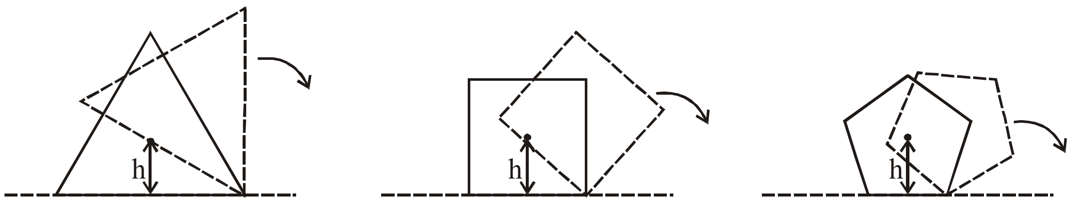
(A) The x component of displacement of the centre of mass of the block M is : $-\frac{mR}{M+m}$

(B) The position of the point mass is : $x=-\sqrt{2}\,\frac{mR}{M+m}$

(C) The velocity of the point mass m is : $v=\sqrt{\frac{2gR}{1+\frac{m}{M}}}$

(D) The velocity of the block M is : $V=-\frac{m}{M}\sqrt{2gR}$

16. Consider regular polygons with number of sides n = 3, 4, 5 ..... as shown in the figure. The center of mass of all the polygons is at height h from the ground. They roll on a horizontal surface about the leading vertex without slipping and sliding as depicted. The maximum increase in height of the locus of the center of mass for each polygon is Δ. Then Δ depends on n and h as : **[JEE Advanced-2017]**



(A) $\Delta = h\sin^2\left(\frac{\pi}{n}\right)$

(B) $\Delta = h\sin\left(\frac{2\pi}{n}\right)$

(C) $\Delta = h\left(\frac{1}{\cos\left(\frac{\pi}{n}\right)}-1\right)$

(D) $\Delta = h\tan^2\left(\frac{\pi}{2n}\right)$

17. A solid horizontal surface is covered with a thin layer of oil. A rectangular block of mass m = 0.4kg is at rest on this surface. An impulse of 1.0 N s is applied to the block at time to t = 0 so that it starts moving along the x-axis with a velocity $v(t) = v_0e^{-t/\tau}$, where v_0 is a constant and τ = 4 s. The displacement of the block, in metres, at t = τ is.................. Take $e^{-1} = 0.37$? **[JEE Advanced-2018]**

18. A small particle of mass m moving inside a heavy, hollow and straight tube along the tube axis undergoes elastic collision at two ends. The tube has no friction and it is closed at one end by a flat surface while the other end is fitted with a heavy movable flat piston as shown in figure. When the distance of the piston from closed end is $L = L_0$ the particle speed is $v = v_0$. The piston is moved inward at a very low speed V such that $V \ll \frac{dL}{L}v_0$, where dL is the infinitesimal displacement of the piston. Which of the following statement(s) is/are correct ? **[JEE Advanced-2019]**

(1) The rate at which the particle strikes the piston is v/L

(2) After each collision with the piston, the particle speed increases by 2V

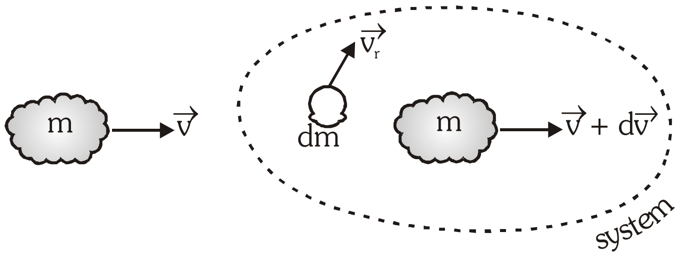
(3) The particle's kinetic energy increases by a factor of 4 when the piston is moved inward from L_0 to $\frac{1}{2}L_0$

(4) If the piston moves inward by dL, the particle speed increases by $2v\frac{dL}{L}$

THEORY

Variable mass system:

In previous discussion of the conservation of linear momentum, we assume that system's mass remains constant. Now we are consider those system whose mass is variable i.e. those in which mass enters or leaves the system. Suppose at some moment t = t mass of a body is m and its velocity is $\vec{v}$. After some time at t = t + dt its mass becomes (m – dm) and velocity becomes $\vec{v}+d\vec{v}$. The mass dm is ejected with relative velocity $\vec{v}_r$.

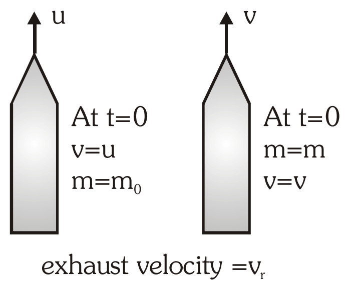


If no forces are acting on the system then the linear momentum of the system will remain conserved.

$$\Rightarrow \vec{F}_{ex}dt = (m - dm)\left(\vec{v}+d\vec{v}\right) + dm\left(\vec{v}_r+\vec{v}+d\vec{v}\right) - m\vec{v}$$

$$\because F_{ex} = 0 \Rightarrow md\vec{v} = -\vec{v}_r dm \Rightarrow m\left(\frac{d\vec{v}}{dt}\right) = \vec{v}_r\left(-\frac{dm}{dt}\right)$$

Rocket propulsion :



Thrust force on the rocket = $v_r\left(-\frac{dm}{dt}\right)$

So for motion of rocket $m\frac{dv}{dt} = v_r\left(-\frac{dm}{dt}\right) - mg$

$$\Rightarrow dv = v_r\left(-\frac{dm}{m}\right) - gdt \Rightarrow \int_{u_0}^{v} dv = -v_r\int_{m_0}^{m}\frac{dm}{m} - g\int_0^t dt$$

$$\Rightarrow v-u = v_r \ell n\left(\frac{m_0}{m}\right) - gt \quad \Rightarrow v = u - gt + v_r\ell n\left(\frac{m_0}{m}\right)$$

Ex An open topped rail road car of mass M has an initial velocity v_0 along a straight horizontal frictionless track. It suddenly starts raining at time t=0. The rain drops fall vertically with velocity u and add a mass m kg/sec of water. Find the velocity of car after t second (assuming that it is not completely filled with water).

Ex. A uniform chain of mass m and length ℓ hangs on a thread and touches the surface of a table by its lower end. Find the force exerted by the chain on the table when half of its length has fallen on the table. The fallen part does not form heap.

Ex. A uniform chain of mass M and length L is held vertically in such a way that its lower end just touches the horizontal floor. The chain is released from rest in this position. Any portion that strikes the floor comes to rest. Assuming that the chain does not form a heap on the floor, calculate the force exerted by it on the floor when a length x has reached the floor.

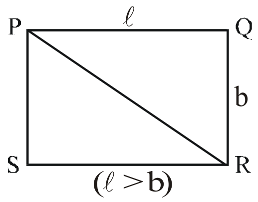
Ex. A rocket ejects a steady jet whose velocity is equal to u relative to the rocket. The gas discharge rate equals μ kg/s. Demonstrate that the rocket motion equation in this case takes the form $ma = F - \mu u$, where m is the mass of the rocket at a given moment, a is its acceleration, and F is the external force.

ROTATIONAL MECHANICS

Advanced Objective (AO)

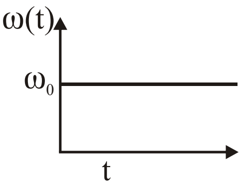
Single Correct Answer Type Question

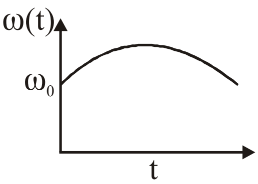
1. Moment of inertia of a rectangular plate about an axis passing through P and perpendicular to the plate is I. Then moment of PQR about an axis perpendicular to the plane of the plate:

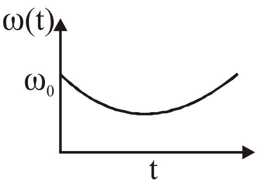


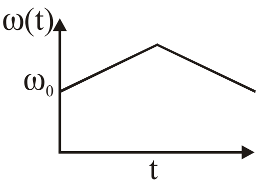
(A) about P = I/2 (B) about R = I/2 (C) about P > I/2 (D) about R > I/2

2. A circular platform is free to rotate in a horizontal plane about a vertical axis passing through its centre. A tortoise is sitting at the edge of the platform. Now, the platform is given an angular velocity ω_0 . When the tortoise move along a chord of the platform with a constant velocity (with respect to the platform), the angular velocity of the platform ω(t). will vary with time t as **[IIT-JEE 2002]**

(A) 

(B) 

(C) 

(D) 

3. A cylinder rolls up an inclined plane, reaches some height, and then rolls down (without slipping throughout these motions). The directions of the frictional force acting on the cylinder are **[IIT-JEE 2002]**

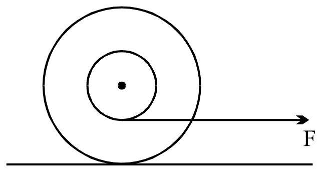
(A) Up the incline while ascending and down the incline while descending

(B) Up the incline while ascending as well as descending

(C) down the incline while ascending and up the incline while descending

(D) down the incline while ascending as well as descending

4. Inner and outer radii of a spool are r and R respectively. A thread is wound over its inner surface and placed over a rough horizontal surface. Thread is pulled by a force F as shown in fig. then in case of pure rolling



(A) Thread unwinds, spool rotates anticlockwise and friction act leftwards

(B) Thread winds, spool rotates clockwise and friction acts leftwards

(C) Thread winds, spool moves to the right and friction act rightwards

(D) Thread winds, spool moves to the right and friction does not come into existence.

5. A hollow sphere of radius R and mass m is fully filled with non viscous liquid of mass m. It is rolled down a horizontal plane such that its centre of mass moves with a velocity v. If it purely rolls

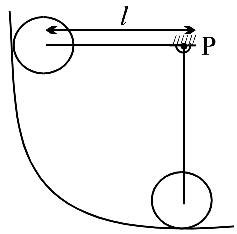
(A) Kinetic energy of the sphere is $\frac{5}{6}mv^2$

(B) Kinetic energy of the sphere is $\frac{4}{5}mv^2$

(C) Angular momentum of the sphere about a fixed point on ground is $\frac{8}{3}mvR$

(D) Angular momentum of the sphere about a fixed point on ground is $\frac{14}{5}mvR$

6. A sphere of mass M and radius R is attached by a light rod of length ℓ to a point P. The sphere rolls without slipping on a circular track as shown. It is released from the horizontal position. The angular momentum of the system about P when the rod becomes vertical is :



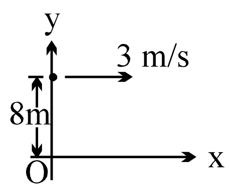
(A) $M\sqrt{\frac{10}{7}gl}\ [l+R]$

(B) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{2}{5}R\right]$

(C) $M\sqrt{\frac{10}{7}gl}\left[l+\frac{7}{5}R\right]$

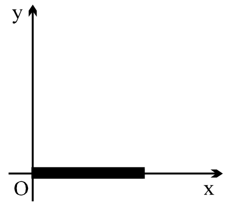
(D) $M\sqrt{\frac{10}{7}gl}\left[l-\frac{2}{5}R\right]$

7. A particle starts from the point (0m, 8m) and moves with uniform velocity of $3\hat{i}$ m/s. After 5 seconds, the angular velocity of the particle about the origin will be :



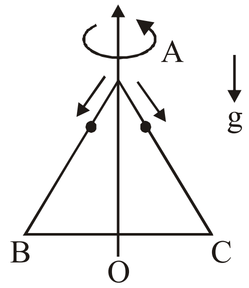
(A) $\frac{8}{289}$ rad/s (B) $\frac{3}{8}$ rad/s (C) $\frac{24}{289}$ rad/s (D) $\frac{8}{17}$ rad/s

8. The figure shows a uniform rod lying along the x-axis. The locus of all the points lying on the xy-plane, about which the moment of inertia of the rod is same as that about O is (assume axis is always parallel to z axis):-



(A) an ellipse (B) a circle (C) a parabola (D) a straight line

9. An equilateral triangle ABC formed from a uniform wire has two small identical beads initially located at A. The triangle is set rotating about the vertical axis AO. Then the beads are released from rest simultaneously and allowed to slide down, one along AB and the other along AC as shown. Neglecting frictional effects, the quantities that are conserved as the beads slide down, are



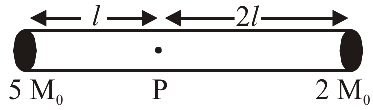
(A) Angular velocity and total energy (kinetic and potential)

(B) Total angular momentum and total energy

(C) Angular velocity and moment of inertia about the axis of rotation.

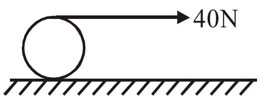
(D) Total angular momentum and moment of inertia about the axis of rotation.

10. A rigid massless rod of length $3l$ has two masses attached at each end as shown in the figure. The rod is pivoted at point P on the horizontal axis (see figure). When released from initial horizontal position, its instantaneous angular acceleration will be :



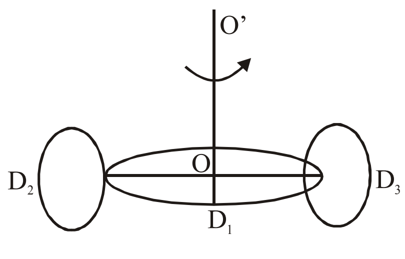
(A) $\frac{g}{2l}$ (B) $\frac{7g}{3l}$ (C) $\frac{g}{13l}$ (D) $\frac{g}{3l}$

11. A string is wound around a hollow cylinder of mass 5 kg and radius 0.5 m. If the string is now pulled with a horizontal force of 40 N, and the cylinder is rolling without slipping on a horizontal surface (see figure), then the angular acceleration of the cylinder will be (Neglect the mass and thickness of the string) :-



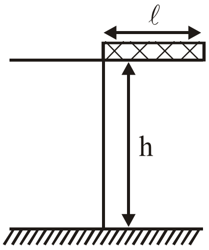
(A) 12 rad/s^2 (B) 16 rad/s^2 (C) 10 rad/s^2 (D) 20 rad/s^2

12. A circular disc D_1 of mass M and radius R has two identical discs D_2 and D_3 of the same mass M and radius R attached rigidly at its opposite ends (see figure). The moment of inertia of the system about the axis OO', passing through the centre of D_1, as shown in the figure, will be:-



(A) $3MR^2$ (B) $\frac{2}{3}MR^2$ (C) MR^2 (D) $\frac{4}{5}MR^2$

13. A rectangular solid box of length 0.3 m is held horizontally, with one of its sides on the edge of a platform of height 5m. When released, it slips off the table in a very short time $\tau = 0.01$s, remaining essentially horizontal. The angle by which it would rotate when it hits the ground will be (in radians) close to :-



(A) 0.02 (B) 0.28 (C) 0.5 (D) 0.3

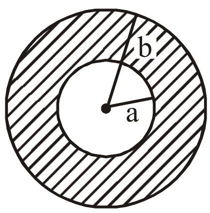
14. Two particles of equal mass m have respective initial velocities $u\hat{i}$ and $u\left(\frac{\hat{i}+\hat{j}}{2}\right)$. They collide completely inelastically. The energy lost in the process is :

(A) $\frac{3}{4}mu^2$ (B) $\frac{1}{8}mu^2$ (C) $\sqrt{\frac{2}{3}}mu^2$ (D) $\frac{1}{3}mu^2$

15. Two coaxial discs, having moments of inertia I_1 and $\frac{I_1}{2}$, are rotating with respective angular velocities ω_1 and $\frac{\omega_1}{2}$, about their common axis. They are brought in contact with each other and thereafter they rotate with a common angular velocity. If E_f and E_i are the final and initial total energies, then $(E_f - E_i)$ is :

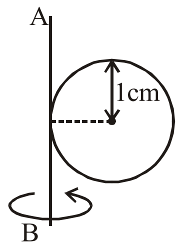
(A) $\frac{I_1\omega_1^2}{12}$ (B) $\frac{3}{8}I_1\omega_1^2$ (C) $\frac{I_1\omega_1^2}{6}$ (D) $\frac{I_1\omega_1^2}{24}$

16. A circular disc of radius b has a hole of radius a at its centre (see figure). If the mass per unit area of the disc varies as $\left(\frac{\sigma_0}{r}\right)$, then the radius of gyration of the disc about its axis passing through the centre is :



(A) $\frac{a+b}{2}$ (B) $\frac{a+b}{3}$ (C) $\sqrt{\frac{a^2+b^2+ab}{2}}$ (D) $\sqrt{\frac{a^2+b^2+ab}{3}}$

17. A metal coin of mass 5 g and radius 1 cm is fixed to a thin stick AB of negligible mass as shown in the figure. The system is initially at rest. The constant torque, that will make the system rotate about AB at 25 rotations per second in 5 s, is close to :



(A) 4.0×10^{-6} Nm (B) 2.0×10^{-5} Nm (C) 1.6×10^{-5} Nm (D) 7.9×10^{-6} Nm

18. Mass per unit area of a circular disc of radius a depends on the distance r from its centre as $\sigma(r) = A + Br$. The moment of inertia of the disc about the the axis, perpendicular to the plane and passing through its centre is :

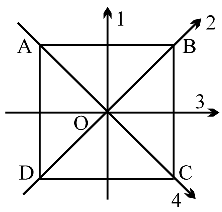
(A) $2\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (B) $\pi a^4\left(\frac{A}{4}+\frac{aB}{5}\right)$ (C) $2\pi a^4\left(\frac{aA}{4}+\frac{B}{5}\right)$ (D) $2\pi a^4\left(\frac{A}{4}+\frac{B}{5}\right)$

19. The following bodies are made to roll up (without slipping) the same inclined plane from a horizontal plane. : (i) a ring of radius R, (ii) a solid cylinder of radius $\frac{R}{2}$ and (iii) a solid sphere of radius $\frac{R}{4}$. If in each case, the speed of the centre of mass at the bottom of the incline is same, the ratio of the maximum heights they climb is :

(A) 4 : 3 : 2 (B) 14 : 15 : 20 (C) 10 : 15 : 7 (D) 2 : 3 : 4

Multiple Correct Answer Type Question

20. ABCD is a square plate with centre O. The moments of inertia of the plate about the perpendicular axis through O is I and about the axes 1, 2, 3 & 4 are I_1, I_2, I_3 & I_4 respectively. It follows that :



(A) $I_2 = I_3$ (B) $I = I_1 + I_4$ (C) $I = I_2 + I_4$ (D) $I_1 = I_3$

21. A rod of weight w is supported by two parallel knifes at end points A and B and is in equilibrium in a horizontal position. The knives are at a distance 'd' from each other. The centre of mass of the rod is at a distance 'x' from A.

(A) the normal reaction at A is $\frac{wx}{d}$ (B) the normal reaction at A is $\frac{w(d-x)}{d}$

(C) the normal reaction at B is $\frac{wx}{d}$ (D) the normal reaction at B is $\frac{w(d-x)}{d}$

22. A block with a square base measuring 'a' × 'a' and height h, is placed on an inclined plane. The coefficient of friction is μ. The angle of inclination (θ) of the plane is gradually increased. The block will

(A) topple before sliding if $\mu > \frac{a}{h}$
(B) topple before sliding if $\mu < \frac{a}{h}$
(C) slide before toppling if $\mu > \frac{a}{h}$
(D) slide before toppling if $\mu < \frac{a}{h}$

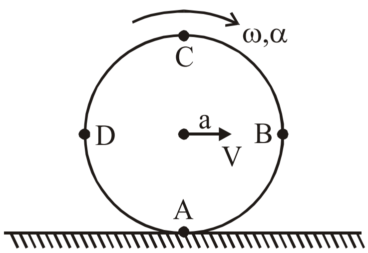
23. A particle falls freely near the surface of the earth. Consider a fixed point O (not vertically below the particle) on the ground. (axis is perpendicular to plane of O and particle)

(A) Angular momentum of the particle about O is increasing .
(B) Torque of the gravitational force on the particle about O is decreasing.
(C) The moment of inertia of the particle about O is decreasing
(D) The angular velocity of the particle about O is increasing.

24. A man spinning in free space changes the shape of his body, eg. by spreading his arms or curling up. By doing this, he can change his

(A) moment of inertia
(B) angular momentum
(C) angular velocity
(D) rotational kinetic energy

25. A circular disc of radius R rolls without slipping on a rough horizontal surface. At the instant shown its linear velocity is V, linear acceleration a, angular velocity ω and angular acceleration α. Four points A, B, C and D lie on its circumference such that the diameter AC is vertical & BD horizontal then choose the **CORRECT** option(s).



(A) $V_B = \sqrt{V^2 + (R\omega)^2}$
(B) $V_C = V + R\omega$
(C) $a_A = \sqrt{(a - R\alpha)^2 + (\omega^2 R)^2}$
(D) $a_D = \sqrt{(a + \omega^2 R)^2 + (R\alpha)^2}$

26. The torque $\vec{\tau}$ on a body about a given point is found to be equal to $\vec{A} \times \vec{L}$ where $\vec{A}$ is a constant vector and $\vec{L}$ is the angular momentum of the body about that point. From this it follows that

(A) $d\vec{L}/dt$ is perpendicular to $\vec{L}$ at all instants of time
(B) the components of $\vec{L}$ in the direction of $\vec{A}$ does not change with time
(C) the magnitude of $\vec{L}$ does not change with time
(D) $\vec{L}$ does not change with time

27. A block of mass m moves on a horizontal rough surface with initial velocity v. The height of the centre of mass of the block is h from the surface. Consider a point A on the surface.

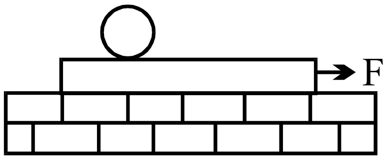
(A) angular momentum about A is mvh initially

(B) the velocity of the block decreases at time passes.

(C) torque of the forces acting on block is zero about A

(D) angular mometum is not conserved about A.

28. A plank with a uniform sphere placed on it rests on a smooth horizontal plane. Plank is pulled to right by a constant force F. If sphere does not slip over the plank. Which of the following is correct.



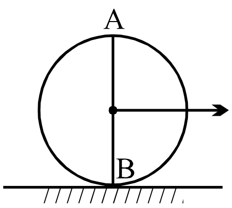
(A) Acceleration of the centre of sphere is less than that of the plank.

(B) Work done by friction acting on the sphere is equal to its total kinetic energy.

(C) Total kinetic energy of the system is equal to work done by the force F

(D) None of the above

29. A uniform disc is rolling on a horizontal surface. At a certain instant B is the point of contact and A is at height 2R from ground, where R is radius of disc.



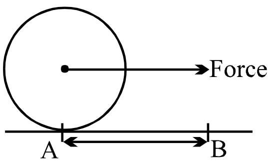
(A) The magnitude of the angular momentum of the disc about B is thrice that about A.

(B) The angular momentum of the disc about A is anticlockwise.

(C) The angular momentum of the disc about B is clockwise

(D) The angular momentum of the disc about A is equal to that about B.

30. A disc of circumference s is at rest at a point A on a horizontal surface when a constant horizontal force begins to act on its centre. Between A and B there is sufficient friction to prevent slipping, and the surface is smooth to the right of B. AB = s. The disc moves from A to B in time T. To the right of B,



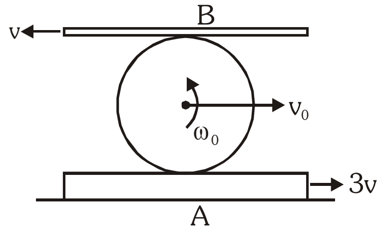
(A) the angular acceleration of the disc will disappear, linear acceleration will remain unchanged

(B) linear acceleration of the disc will increase

(C) the disc will make one rotation in time T/2

(D) the disc will cover a distance greater than s in further time T.

31. The disc of radius r is confined to roll without slipping at A and B. If the plates have the velocities shown, then



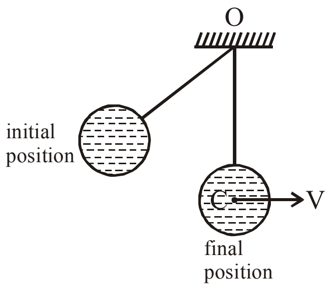
(A) linear velocity $v_0 = v$

(B) angular velocity of disc is $\frac{3v}{2r}$

(C) angular velocity of disc is $\frac{2v}{r}$

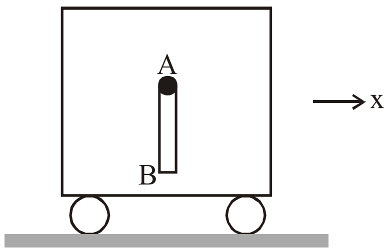
(D) None of these

32. A massles rod has a massless hollow sphere attached to it. This sphere can be fully filled either with a liquid (non viscous) or with a solid (rigidly fitted into sphere) of same mass. System is released from rest from initial position (as shown). When it reaches final position which of the following is/are true for the system.



(A) Kinetic energy in case of liquid will be more than in case of solid.
(B) Velocity of centre (V) in case of liquid will be more than in case of solid.
(C) About C angular momentum in case of liquid will be more than in case of solid.
(D) About C angular momentum in case of liquid will be less than in case of solid.

33. A uniform rod AB of length ℓ and mass M hangs from point A at which it is freely hinged in a car moving with velocity v_0. The rod can rotate in vertical plane about the axis at A. If the car suddenly stops,



(A) The angular speed ω with which the rod starts rotating is $\frac{3v_0}{2\ell}$

(B) The minimum value of v_0 so that the rod completes the rotation $\sqrt{\frac{8}{3}g\ell}$

(C) Loss of energy during the process $\frac{1}{8}Mv_0^2$

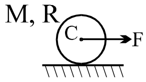
(D) There is no loss of energy

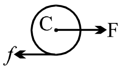
Linked Comprehension Type Question

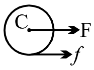
Paragraph for Question No. 34 to 36 (4 Question)

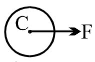
In the following problems, indicate the correct direction of friction force acting on the cylinder, which is pulled on a rough surface by a constant force F.

34. A cylinder of mass M and radius R is pulled horizontally by a force F. The friction force can be given by which of the following diagrams :-



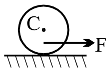
(A) 

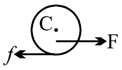
(B) 

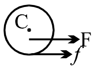
(C) 

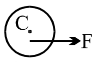
$f = 0$
(D) cannot be interpreted

35. A cylinder is pulled horizontally by a force F acting at a point below the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams :-



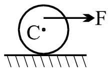
(A) 

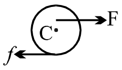
(B) 

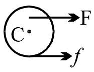
(C) 

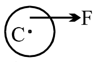
$f = 0$
(D) cannot be interpreted

36. A cylinder is pulled horizontally by a force F acting at a point above the centre of mass of the cylinder, as shown in figure. The friction force can be given by which of the following diagrams



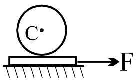
(A) 

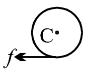
(B) 

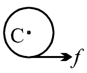
(C) 

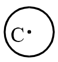
$f = 0$
(D) cannot be interpreted

37. A cylinder is placed on a rough plank which in turn is placed on a smooth surface. The plank is pulled with a constant force F. The friction force can be given by which of the following diagrams



(A) 

(B) 

(C) 

$f = 0$
(D) canot be interpreted

Paragraph for Question No. 38 to 40 (3 Question)

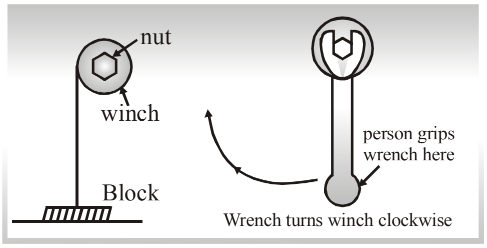
In figure, the winch is mounted on an axle, and the 6-sided nut is welded to the winch. By turning the nut with a wrench, a person can rotate the winch. For instance, turning the nut clockwise lifts the block off the ground, because more and more rope gets wrapped around the winch.

Three students agree that using a longer wrench makes it easier to turn the winch. But they disagree about why. All three students are talking about the case where the winch is used, over a 10 s time interval, to lift the block one metre off the ground.

Student 1 : By using a longer wrench, the person decreases the average force he must exert on the wrench, in order to lift the block one metre in 10 s.

Student 2 : Using a longer wrench reduces the work done by the person as he uses the winch to lift the block 1m in 10s.

Student 3 : Using a longer wrench reduces the power that the person must exert to lift the block 1m in 10s.



38. Student 1 is :-

(A) correct, because the torque that the wrench must exert to lift the block doesn't depend on the wrench's length

(B) correct, because using a longer wrench decreases the torque it must exert on the winch

(C) incorrect, because the torque that the wrench must exert to lift the block doesn't depend on the wrench's length

(D) Incorrect, because using a longer wrench decreases the torque it must exert on the winch.

39. Which of the following is true about student 2 and 3 :-

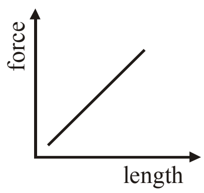
(A) Student 2 and 3 are both correct

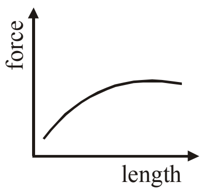
(B) Student 2 is correct, but student 3 is incorrect

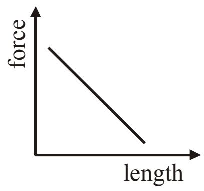
(C) Student 3 is correct, but student 2 is incorrect

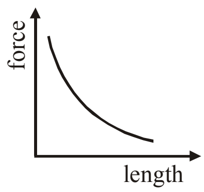
(D) Student 2 and 3 are both incorrect

40. If several wrenches all apply the same torque to a nut, which graph best expresses the relationship between the force the person must apply to the wrench, and the length of the wrench :-

(1) 

(2) 

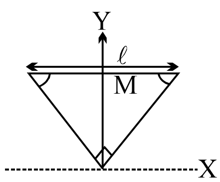
(3) 

(4) 

(A) 1 (B) 2 (C) 3 (D) 4

Paragraph for Question No. 41 to 44 (4 Question)

The figure shows an isosceles triangular plate of mass M and base L. The angle at the apex is 90°. The apex lies at the origin and the base is parallel to X–axis



41. The moment of inertia of the plate about the z-axis is :-

(A) $\frac{ML^2}{12}$ (B) $\frac{ML^2}{24}$ (C) $\frac{ML^2}{6}$ (D) none of these

42. The moment of inertia of the plate about the x-axis is :-

(A) $\frac{ML^2}{8}$ (B) $\frac{ML^2}{32}$ (C) $\frac{ML^2}{24}$ (D) $\frac{ML^2}{6}$

43. The moment of inertia of the plate about its base parallel to the x-axis is :-

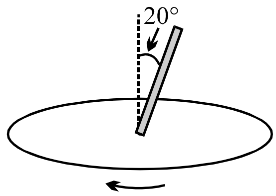
(A) $\frac{ML^2}{18}$ (B) $\frac{ML^2}{36}$ (C) $\frac{ML^2}{24}$ (D) none of these

44. The moment of inertia of the plate about the y-axis is :-

(A) $\frac{ML^2}{6}$ (B) $\frac{ML^2}{8}$ (C) $\frac{ML^2}{24}$ (D) none of these

Paragraph for Question No. 45 and 46 (2 Question)

A uniform rod is fixed to a rotating turntable so that its lower end is on the axis of the turntable and it makes an angle of 20° to the vertical. (The rod is thus rotating with uniform angular velocity about a vertical axis passing through one end.) If the turntable is rotating clockwise as seen from above.



45. What is the direction of the rod's angular momentum vector (calculated about its lower end)?

(A) vertically downwards (B) down at 20° to the horizontal

(C) up at 20° to the horizontal (D) vertically upwards

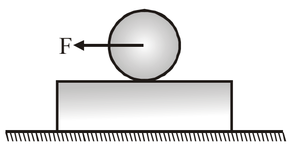
46. Is there a torque acting on it, and if so in what direction?

(A) yes, vertically (B) yes, horizontally

(C) yes at 20° to the horizontal (D) no

Paragraph for Question No. 47 and 48 (2 Question)

A disc of mass m and radius R is placed over a plank of same mass m. There is sufficient friction between disc and plank to prevent slipping. A force F is applied at the centre of the disc.



47. Acceleration of the plank is :-

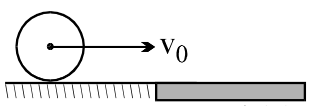
(A) $\frac{F}{2m}$ (B) $\frac{3F}{4m}$ (C) $\frac{F}{4m}$ (D) $\frac{3F}{2m}$

48. Force of friction between the disc and the plank is :-

(A) $\frac{F}{2}$ (B) $\frac{F}{4}$ (C) $\frac{F}{3}$ (D) $\frac{2F}{3}$

Paragraph for Question No. 49 to 52 (4 Question)

A ring of mass M and radius R sliding with a velocity v_0 suddenly enters into rough surface where the coefficient of friction is μ, as shown in figure.



49. Choose the correct statement(s)

(A) As the ring enters on the rough surface, the limiting friction force acts on it

(B) The direction of friction is opposite to the direction of motion

(C) The friction force accelerates the ring in the clockwise sense about its centre of mass

(D) As the ring enters on the rough surface it starts rolling

50. Choose the correct statement(s)

(A) The momentum of the ring is conserved

(B) The angular momentum of the ring is conserved about its centre of mass

(C) The angular momentum of the ring conserved about any point on the horizontal surface in line of friction.

(D) The mechanical energy of the ring is conserved

51. Choose the correct statement(s)

(A) The ring starts its rolling motion when the centre of mas stationary

(B) The ring starts rolling motion when the point of contact becomes stationary

(C) The time after which the ring starts rolling is $\frac{v_0}{2\mu g}$

(D) The rolling velocity is $\frac{v_0}{2}$

52. Choose the correct alternative(s) :-

(A) The linear distance moved by the centre of mass before the ring starts rolling is $\frac{3v_0^2}{8\mu g}$

(B) The net work done by friction force is $-\frac{3}{8}mv_0^2$

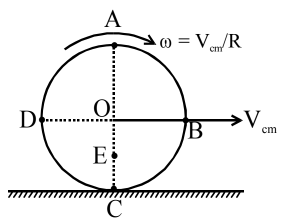
(C) The loss is kinetic energy of the ring is $\frac{mv_0^2}{4}$

(D) The gain in rotational kinetic energy is $+\frac{mv_0^2}{8}$

Matrix Match Type Question

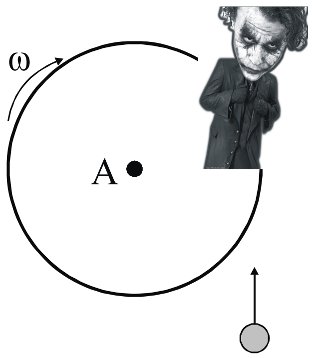
53. Consider a body rolling on a horizontal surface as shown in figure (Symbols have their usual meaning)

Column-I		Column-II
(A) Velocity is zero	(P)	Point A
(B) Speed in maximum	(Q)	Point B
(C) $0 <$ speed $< v_{cm}$	(R)	Point C
(D) $1.3v_{cm} <$ speed $< 2\ v_{cm}$	(S)	Point D
	(T)	Point E



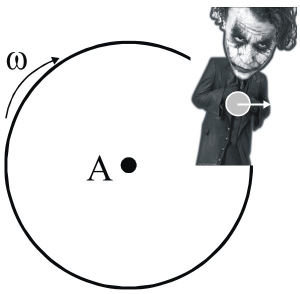
54. Column-I depicts various situations where some sudden events are taking place. Column-II describes changes in various parameters of systems immediately after the events taking place in column-I.

Column-I		**Column-II**
(A)	Joker is standing on revolving platform and batman throws the ball and joker catches the ball while it was moving horizontally.	(P) Linear momentum remains conserved.



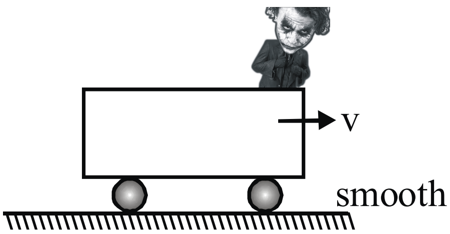
Joker, ball and platform is system.

(B)	Joker throws the ball horizontally and perpendicular to his motion while standing on the revolving platform.	(Q) Mechanical energy is conserved



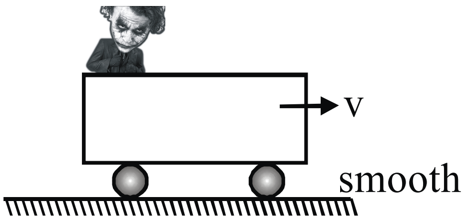
Joker, ball and platform is system.

(C)	Joker jumps horizontally towards right from the cart which is moving at speed v on smooth horizontal floor.	(R) Mechanical energy increases.



Joker and cart is the system

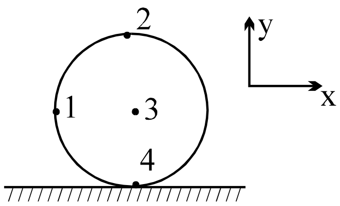
(D)	Joker drops himself vertically from the moving cart with no horizontal velocity relative to cart.	(S) Mechanical energy decreases.



(T) v or ω changes

Joker and cart is the system

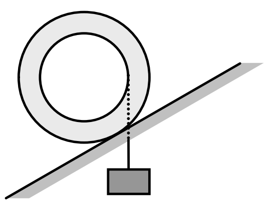
55. A rigid cylinder is kept on a smooth horizontal surface as shown. If **Column-I** indicates velocities of various points (3-centre of cylinder, 2- top point, 4-bottom point, 1- on the level of 3 at the rim) on it shown, choose correct state of motion from **Column-II**.



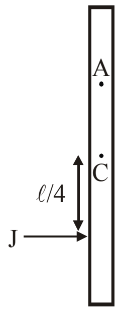
	Column-I		**Column-II**
(A)	$\vec{v}_1 = \hat{i} + \hat{j}, \vec{v}_2 = 2\hat{i}$	(P)	Pure rotation about centre
(B)	$\vec{v}_1 = \hat{i} + \hat{j}, \vec{v}_3 = -\hat{i}$	(Q)	Rolling without slipping to left
(C)	$\vec{v}_2 = \hat{i}, \vec{v}_3 = 0$	(R)	Rolling without slipping to right
(D)	$\vec{v}_4 = 0, \vec{v}_1 = -\hat{i} - \hat{j}$	(S)	Not possible

Advanced Subjective (AS)

1. A 3.0 kg bobbin consists of a central cylinder of radius 5.0 cm and two end plates each of radius 6.0 cm. It is placed on a slotted incline, where friction is sufficient to prevent sliding. A block of mass 4.5 kg is suspended from a cord wound around the bobbin and passing through the slot under the incline. If the bobbin is in static equilibrium, what is the angle of tilt of the incline?

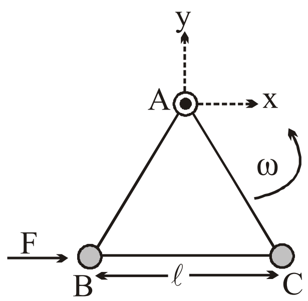


2. A uniform rod of mass m and length ℓ is placed in gravity free space and linear impulse J is given to the rod at a distance x = ℓ/4 from centre 'C' and perpendicular to the rod. Point A is at a distance ℓ/3 from centre as shown in the figure. Then find



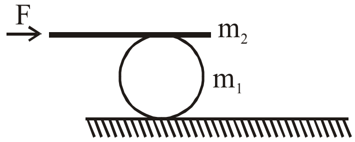
(i) Speed of centre of rod
(ii) Speed of point A
(iii) Speed of upper end of rod
(iv) Speed of lower end of rod

3. Three particles A,B, and C each of mass m, are connected to each other by three massless rigid rods to form a rigid, equilateral triangular body of side ℓ. This body is placed on a horizontal frictionless table (x-y plane) and is hinged to it at the point A, so that it can move without friction about the vertical axis through A (see figure). The body is set into rotational motion on the table about A with a constant angular velocity ω.



(a) Find the magnitude of the horizontal force exerted by the hinge on the body.
(b) At time T, when the side BC is parallel to the x-axis, a force F is applied on B along BC (as shown).Obtain the x–component and the y–component of the force exerted by the hinge on the body, immediately after time T. **[IIT-JEE' 2001]**

4. A man pushes a cylinder of mass m_1 with help of a plank of mass m_2 as shown. There is no slipping at any contact. the horizontal component of the force applied by the man is F. Find:



(a) the accelerations of the plank and the centre of mass of the cylinder, and
(b) the magnitudes and directions of frictional forces at contact points. **[JEE '99 , 6 + 4]**

5. A uniform disk of mass m and radius R is projected horizontally with velocity v_0 on a rough horizontal floor so that it starts off with a purely sliding motion at t = 0. After t_0 seconds it acquires a purely rolling motion as shown in figure.

(i) Calculate the velocity of the centre of mass of the disk at t_0.

(ii) Assuming the coefficient of friction to be μ calculate t_0. Also calculate the work done by the frictional force as a function of time and the total work done by it over a time t much longer than t_0.

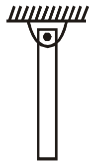
6. A rod AB of mass M and length L is lying on a horizontal frictionless surface. A particle of mass m travelling along the surface hits the end 'A' of the rod with a velocity v_0 in the direction perpendicular to AB. The collision is completely elastic. After the collision the particle comes to rest.

(a) Find the ratio m/M .

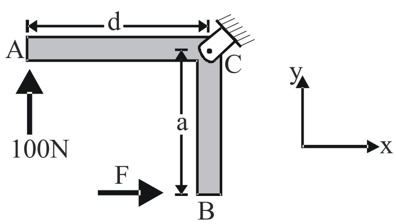
(b) A point P on the rod is at rest immediately after the collision. Find the distance AP.

(c) Find the linear speed of the point P at a time $\pi L/(3v_0)$ after the collision.

7. A rod of mass m & length ℓ is hinged at it's upper end. It can rotate in vertical plane. It is given angular velocity ω so that it can complete vertical circle. Find (a) ω (b) Tension at centre of rod at initial moment.

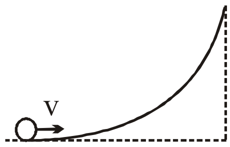


8. Find the force F required to keep the system in equilibrium. The dimensions of the system are d = 0.3 m and a = 0.2 m. Assume the rods to have negligible mass.



EXERCISE (JA)

1. A small object of uniform density rolls up a curved surface with an initial velocity v. It reaches upto maximum height of $\frac{3v^2}{4g}$ with respect to the initial position. The object is **[JEE 2007]**



(A) ring (B) solid sphere (C) hollow sphere (D) disc

Paragraph for Question no. 2 to 4

Two discs A and B are mounted coaxially on a vertical axle. The discs have moments of inertia I and 2I respectively about the common axis. Disc A is imparted an initial angular velocity 2 ω using the entire potential energy of a spring compressed by a distance x_1. Disc B is imparted an angular velocity ω by a spring having the same spring constant and compressed by a distance x_2. Both the discs rotate in the clockwise direction. **[JEE 2007]**

2. The ratio x_1/x_2 is

(A) 2 (B) $\frac{1}{2}$ (C) $\sqrt{2}$ (D) $\frac{1}{\sqrt{2}}$

3. When disc B is brought in contact with disc A, they acquire a common angular velocity in time t. The average frictional torque on one disc by the other during this period is

(A) $\frac{2I\omega}{3t}$ (B) $\frac{9I\omega}{3t}$ (C) $\frac{9I\omega}{4t}$ (D) $\frac{3I\omega}{2t}$

4. The loss of kinetic energy the above process is

(A) $\frac{I\omega^2}{2}$ (B) $\frac{I\omega^2}{3}$ (C) $\frac{I\omega^2}{4}$ (D) $\frac{I\omega^2}{6}$

5. **Statement–I :** If there is no external torque on a body about its centre of mass, then the velocity of the centre of mass remains constant. **[IIT-JEE 2007]**

Because :

Statement–II : The linear momentum of an isolated system remains constant.

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement–1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

6. **Statement-1 :** Two cylinders, one hollow (metal) and the other solid (wood) with the same mass and identical dimensions are simultaneously allowed to roll without slipping down an inclined plane from the same height. The hollow cylinder will reach the bottom of the inclined plane first.

and

Statement-2 : By the principle of conservation of energy, the total kinetic energies of both the cylinders are identical when they reach the bottom of the incline. **[JEE 2008]**

(A) Statement-1 is True, Statement-2 is True ; statement-2 is a correct explanation for statement-1

(B) Statement-1 is True, Statement-2 is True ; statement-2 is NOT a correct explanation for statement-1

(C) Statement-1 is True, Statement-2 is False

(D) Statement-1 is False, Statement-2 is True

7. A block of base 10 cm × 10 cm and height 15 cm is kept on an inclined plane. The coefficient of friction between them is $\sqrt{3}$. The inclination θ of this inclined plane from the horizontal plane is gradually increased from 0°. Then :- **[IIT-JEE 2009]**

(A) at θ =30°, the block will start sliding down the plane

(B) the block will remain at rest on the plane up to certain θ and then it will topple

(C) at θ = 60°, the block will start sliding down the plane and continue to do so at higher angles

(D) at θ = 60°, the block will start sliding down the plane and on further increasing θ, it will topple at certain θ

8. If the resultant of the external forces acting on a system of particles is zero, then from an inertial frame, one can surely say that **[IIT-JEE 2009]**

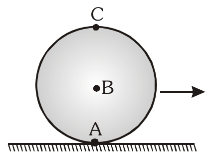
(A) linear momentum of the system does not change in time

(B) kinetic energy of the system does not change in time

(C) angular momentum of the system does not change in time

(D) potential energy of the system does not change in time

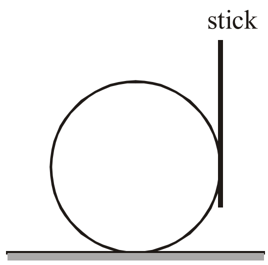
9. A sphere is rolling without slipping on a fixed horizontal plane surface. In the figure, A is the point of contact, B is the centre of the sphere and C is its topmost point. Then **[IIT-JEE 2009]**



(A) $\vec{v}_C - \vec{v}_A = 2(\vec{v}_B - \vec{v}_C)$ (B) $\vec{v}_C - \vec{v}_B = \vec{v}_B - \vec{v}_A$

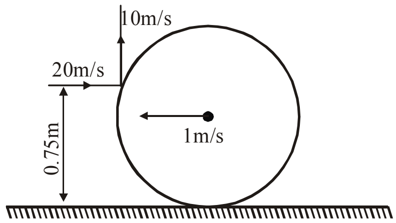
(C) $|\vec{v}_C - \vec{v}_A| = 2|\vec{v}_B - \vec{v}_C|$ (D) $|\vec{v}_C - \vec{v}_A| = 4|\vec{v}_B|$

10. A boy is pushing a ring of mass 2 kg and radius 0.5 m with a stick as shown in the figure. The stick applies a force of 2N on the ring and rolls it without slipping with an acceleration of 0.3 m/s^2. The coefficient of friction between the ground and the ring is large enough that rolling always occurs and the coefficient of friction between the stick and the ring is (P/10). The value of P is **[IIT-JEE 2011]**



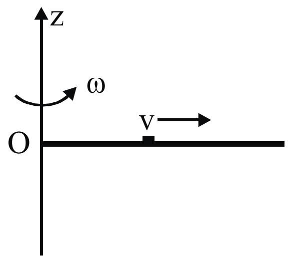
11. Four solid spheres each of diameter $\sqrt{5}$ cm and mass 0.5 kg are placed with their centers at the corners of a square of side 4 cm. The moment of inertia of the system about the diagonal of the square is $N \times 10^{-4}$ kg-m^2, then N is **[IIT-JEE 2011]**

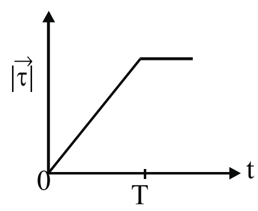
12. A thin ring of mass 2 kg and radius 0.5 m is rolling without slipping on a horizontal plane with velocity 1 m/s. A small ball of mass 0.1 kg, moving with velocity 20 m/s in the opposite direction, hits the ring at a height of 0.75 m and goes vertically up with velocity 10 m/s. Immediately after the collision **[IIT-JEE 2011]**

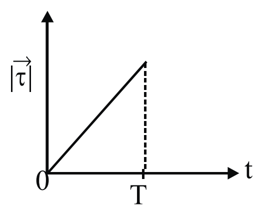


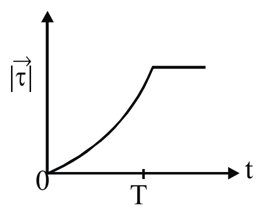
(A) the ring has pure rotation about its stationary CM
(B) the ring comes to a complete stop
(C) friction between the ring and the ground is to the left
(D) there is no friction between the ring and the ground

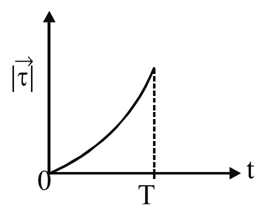
13. A thin uniform rod, pivoted at OP, is rotating in the horizontal plane with constant angular speed ω. as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. If reaches the end of the rod at t = T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\tau|)$ on the system about O, as a function of time is best represented by which plot? **[IIT-JEE 2012]**



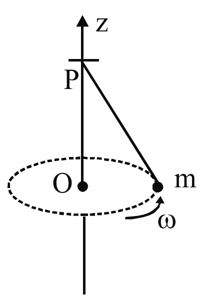
(A) 

(B) 

(C) 

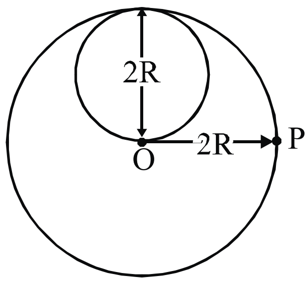
(D) 

14. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then **[IIT-JEE 2012]**



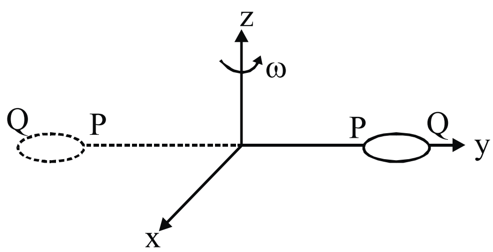
(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time
(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant
(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time
(D) $|\vec{L}_O|$ and $|\vec{L}_P|$ both do not vary with time

15. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes assing through O and P is I_0 and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is **[IIT-JEE 2012]**

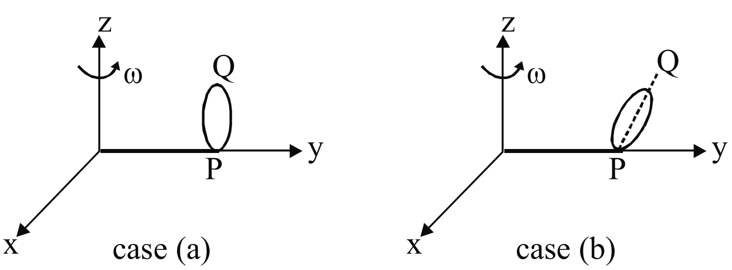


Paragraph for Questions 16 and 17 (2 Question)

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure : case (A) the disc with its face vertical and parallel to x-z plane; Case (B) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis. **[IIT-JEE 2012]**



case (a) case (b)

16. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2\omega}$ for both the cases. (B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).

(C) It is ω for case (a); and $\sqrt{2\omega}$ for case (b). (D) It is ω for both the cases.

17. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

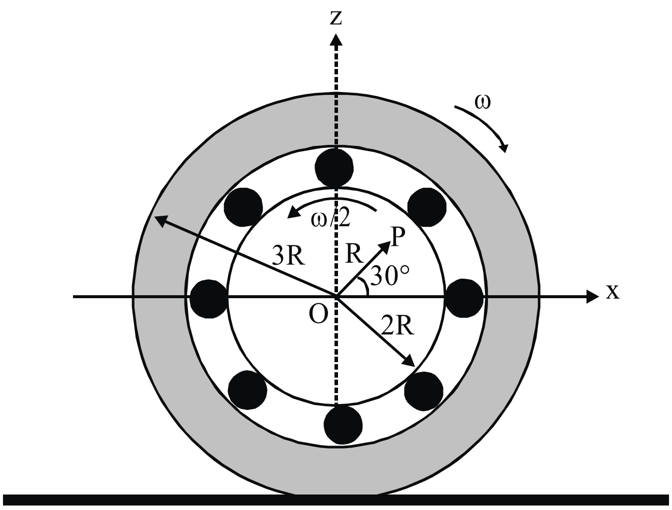
(A) It is vertical for both the cases (a) and (b).

(B) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).

(C) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

(D) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

18. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed ω/2. The ring and disc are separated by frictionless ball bearing. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface, **[IIT-JEE 2012]**



(A) the point O has a linear velocity $3R\omega\hat{i}$

(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) the point P has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

19. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

(A) Both cylinders P and Q reach the ground at the same time. **[IIT-JEE 2012]**

(B) Cylinder P has larger acceleration than cylinder Q.

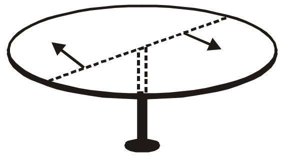
(C) Both cylinders reach the ground with same translational kinetic energy.

(D) Cylinder Q reaches the ground with larger angular speed.

20. A uniform circular disc of mass 50 kg and radius 0.4 m is rotating with an angular velocity of 10 rad s^{-1} about its own axis, which is vertical. Two uniform circular rings, each of mass 6.25 kg and radius 0.2 m, are gently placed symmetrically on the disc in such a manner that they are touching each other along the axis of the disc and are horizontal. Assume that the friction is large enough such that the rings are at rest relative to the disc and the system rotates about the original axis. The new angular velocity (in rad s^{-1}) of the system is **[IIT-JEE 2013]**

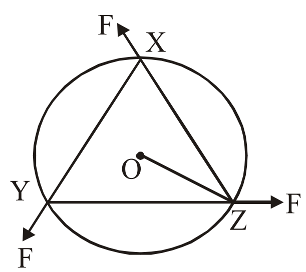
21. A horizontal circular platform of radius 0.5 m and mass 0.45 kg is free to rotate about its axis. Two massless spring toy-guns, each carrying a steel ball of mass 0.05 kg are attached to the platform at a distance 0.25 m from the centre on its either sides along its diameter (see figure). Each gun simultaneously fires the balls horizontally and perpendicular to the diameter in opposite directions. After leaving the platform, the balls have horizontal speed of 9 ms^{-1} with respect to the ground. The rotational speed of the platform in rad s^{-1} after the balls leave the platform is

[JEE Advanced-2014]



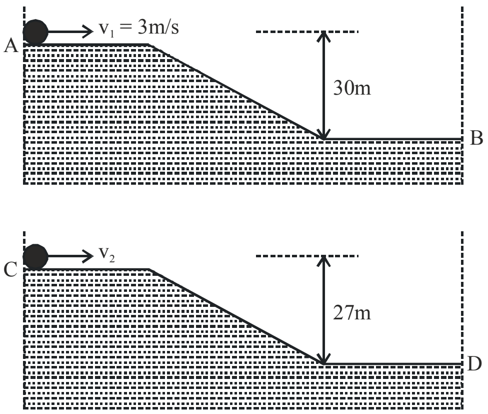
22. A uniform circular disc of mass 1.5 kg and radius 0.5 m is initially at rest on a horizontal frictionless surface. Three forces of equal magnitude F = 0.5 N are applied simultaneously along the three sides of an equilateral triangle XYZ with its vertices on the perimeter of the disc (see figure). One second after applying the forces, the angular speed of the disc in rad s^{-1} is

[JEE Advanced-2014]



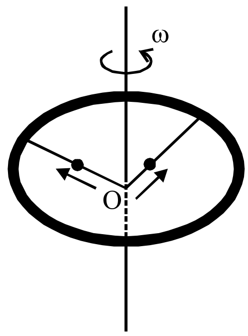
23. Two identical uniform discs roll without slipping on two different, surfaces AB and CD (see figure) starting at A and C with linear speeds v_1 and v_2 respectively, and always remain in contact with the surfaces. If they reach B and D with the same linear speed and $v_1 = 3$m/s, then v_2 in m/s is ($g = 10$ m/s^2)

[JEE Advanced-2015]



24. A ring of mass M and radius R is rotating with angular speed ω about a fixed vertical axis passing through its centre O with two point masses each of mass $\frac{M}{8}$ at rest at O. These masses can move radially outwards along two massless rods fixed on the ring as shown in the figure. At some instant the angular speed of the system is $\frac{8}{9}\omega$ and one of the masses is at a distance of $\frac{3}{5}$ R from O. At this instant the distance of the other mass from O is :

[JEE Advanced-2015]



(A) $\frac{2}{3}R$ (B) $\frac{1}{3}R$ (C) $\frac{3}{5}R$ (D) $\frac{4}{5}R$

25. The densities of two solid spheres A and B of the same radii R vary with radial distance r as $\rho_A(r) = k\left(\frac{r}{R}\right)$ and $\rho_B(r) = k\left(\frac{r}{R}\right)^5$, respectively, where k is a constant. The moments of inertia of the indivisual spheres about axes passing through their centres are I_A and I_B, respectively. If $\frac{I_B}{I_A} = \frac{n}{10}$, the value of n is.

[JEE Advanced-2015]

26. A uniform wooden stick of mass 1.6 kg and length ℓ rests in an inclined manner on a smooth, vertical wall of height h ($<\ell$) such that a small portion of the stick extends beyond the wall. The reaction force of the wall on the stick is perpendicular to the stick. The stick makes an angle of 30° with the wall and the bottom of the stick is on a rough floor. The reaction of the wall on the stick is equal in magnitude to the reaction of the floor on the stick. The ratio h/ℓ and the frictional force f at the bottom of the stick are: ($g = 10$ ms^{-2})

[JEE Advanced-2016]

(A) $\frac{h}{\ell}=\frac{\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}N$
(B) $\frac{h}{\ell}=\frac{3}{16}, f=\frac{16\sqrt{3}}{3}N$

(C) $\frac{h}{\ell}=\frac{3\sqrt{3}}{16}, f=\frac{8\sqrt{3}}{3}N$
(D) $\frac{h}{\ell}=\frac{3\sqrt{3}}{16}, f=\frac{16\sqrt{3}}{3}N$

27. The position vector $\vec{r}$ of a particle of mass m is given by the following equation $\vec{r}(t)=\alpha t^3\hat{i}+\beta t^2\hat{j}$, where $\alpha=\frac{10}{3}ms^{-3}$, $\beta = 5\ ms^{-2}$ and m = 0.1 kg. At t = 1 s, which of the following statement(s) is(are) true about the particle? **[JEE Advanced-2016]**

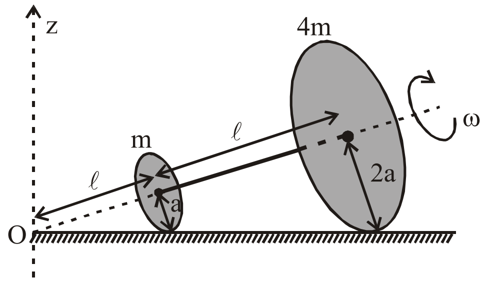
(A) The velocity $\vec{v}$ is given by $\vec{v}=(10\hat{i}+10\hat{j})ms^{-1}$

(B) The angular momentum $\vec{L}$ with respect to the origin is given by $\vec{L}=-\left(\frac{5}{3}\right)\hat{k}$ Nms

(C) The force $\vec{F}$ is given by $\vec{F}=(\hat{i}+2\hat{j})N$

(D) The torque $\vec{\tau}$ with respect to the origin is given by $\vec{\tau}=-\left(\frac{20}{3}\right)\hat{k}$ Nm

28. Two thin circular discs of mass m and 4m, having radii of a and 2a, respectively, are rigidly fixed by a massless, right rod of length $\ell=\sqrt{24}a$ throught their center. This assembly is laid on a firm and flat surface, and set rolling without slipping on the surface so that the angular speed about the axis of the rod is ω. The angular momentum of the entire assembly about the point 'O' is $\vec{L}$ (see the figure). Which of the following statement(s) is(are) true ? **[JEE Advanced-2016]**



(A) The magnitude of angular momentum of the assembly about its center of mass is $17ma^2\omega/2$

(B) The magnitude of the z-component of $\vec{L}$ is $55\ ma^2\omega$

(C) The magnitude of angular momentum of center of mass of the assembly about the point O is $81ma^2\omega$

(D) The center of mass of the assembly rotates about the z-axis with an angular speed of ω/5

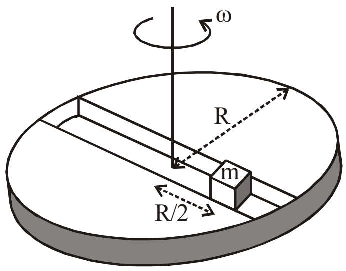
Paragraph for Question No. 29 and 30

A frame of reference that is accelerated with respect to an inertial frame of reference is called a non-inertial frame of reference. A coordinate system fixed on a circular disc rotating about a fixed axis with a constant angular velocity ω is an example of a non-inertial frame of reference. The relationship between the force $\vec{F}_{rot}$ experienced by a particle of mass m moving on the rotating disc and the force $\vec{F}_{in}$ experienced by the particle in an inertial frame of reference is

$$\vec{F}_{rot}=\vec{F}_{in}+2m(\vec{\upsilon}_{rot}\times\vec{\omega})+m(\vec{\omega}\times\vec{r})\times\vec{\omega},$$

where $\vec{\upsilon}_{rot}$ is the velocity of the particle in the rotating frame of reference and $\vec{r}$ is the position vector of the particle with respect to the centre of the disc.

Now consider a smooth slot along a diameter of a disc of radius R rotating counter-clockwise with a constant angular speed ω about its vertical axis through its center. We assign a coordinate system with the origin at the centre of the disc, the x-axis along the slot, the y-axis perpendicular to the slot and the z-axis along the rotation axis $(\vec{\omega} = \omega\hat{k})$. A small block of mass m is gently placed in the slot at $\vec{r} = (R/2)\hat{i}$ at $t = 0$ and is constrained to move only along the slot. **[JEE Advanced-2016]**



29. The distance r of the block at time t is :

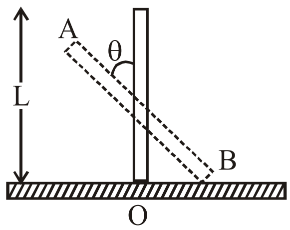
(A) $\frac{R}{4}(e^{2\omega t}+e^{-2\omega t})$ (B) $\frac{R}{2}\cos 2\omega t$ (C) $\frac{R}{2}\cos \omega t$ (D) $\frac{R}{4}(e^{\omega t}+e^{-\omega t})$

30. The net reaction of the disc on the block is :

(A) $-m\omega^2 R\cos\omega t\hat{j} - mg\hat{k}$ (B) $m\omega^2 R\sin\omega t\hat{j} - mg\hat{k}$

(C) $\frac{1}{2}m\omega^2 R(e^{\omega t}-e^{-\omega t})\hat{j} + mg\hat{k}$ (D) $\frac{1}{2}m\omega^2 R(e^{2\omega t}-e^{-2\omega t})\hat{j} + mg\hat{k}$

31. A rigid uniform bar AB of length L is slipping from its vertical position on a frictionless floor (as shown in the figure). At some instant of time, the angle made by the bar with the vertical is θ. Which of the following statements about its motion is/are correct ? **[JEE Advanced-2017]**



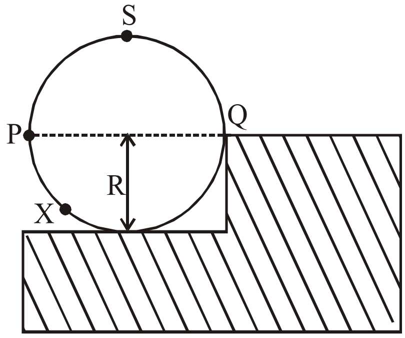
(A) When the bar makes an angle θ with the vertical, the displacement of its midpoint from the initial position is proportional to $(1 - \cos\theta)$

(B) The midpoint of the bar will fall vertically downward

(C) Instantaneous torque about the point in contact with the floor is proportional to $\sin\theta$

(D) The trajectory of the point A is a parabola

32. A wheel of radius R and mass M is placed at the bottom of a fixed step of height R as shown in the figure. A constant force is continuously applied on the surface of the wheel so that it just climbs the step without slipping. Consider the torque τ about an axis normal to the plane of the paper passing through the point Q. Which of the following options is/are correct ? **[JEE Advanced-2017]**



(A) If the force is applied normal to the circumference at point X then τ is constant

(B) If the force is applied tangentially at point S then $\tau \neq 0$ but the wheel never climbs the step

(C) If the force is applied normal to the circumference at point P then τ is zero

(D) If the force is applied at point P tangentially then τ decreases continuously as the wheel climbs

Paragraph for Question no. 33 and 34

One twirls a circular ring (of mass M and radius R) near the tip of one's finger as shown in Figure 1. In the process the finger never loses contact with the inner rim of the ring. The finger traces out the surface of a cone, shown by the dotted line. The radius of the path traced out by the point where the ring and the finger is in contact is r. The finger rotates with an angular velocity ω_0. The rotating ring rolls without slipping on the outside of a smaller circle described by the point where the ring and the finger is in contact (Figure 2). The coefficient of friction between the ring and the finger is μ and the acceleration due to gravity is g.

[JEE Advanced-2017]

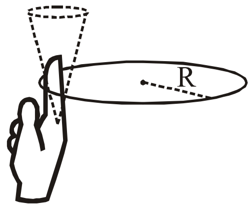


Figure 1

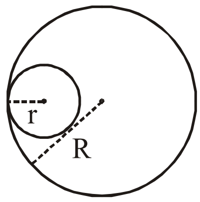


Figure 2

33. The total kinetic energy of the ring is :-

(A) $M\omega_0^2R^2$ (B) $M\omega_0^2(R-r)^2$ (C) $\frac{1}{2}M\omega_0^2(R-r)^2$ (D) $\frac{3}{2}M\omega_0^2(R-r)^2$

34. The minimum value of ω_0 below which the ring will drop down is :-

(A) $\sqrt{\frac{3g}{2\mu(R-r)}}$ (B) $\sqrt{\frac{g}{\mu(R-r)}}$ (C) $\sqrt{\frac{2g}{\mu(R-r)}}$ (D) $\sqrt{\frac{2g}{2\mu(R-r)}}$

35. The potential energy of a particle of mass m at a distance r from a fixed point O is given by $V(r) = kr^2/2$, where k is a positive constant of appropriate dimensions. This particle is moving in a circular orbit of radius R about the point O. If v is the speed of the particle and L is the magnitude of its angular momentum about O, which of the following statements is (are) true ?

[JEE Advanced-2018]

(A) $v = \sqrt{\frac{k}{2m}}R$ (B) $v = \sqrt{\frac{k}{m}}R$ (C) $L = \sqrt{mk}R^2$ (D) $L = \sqrt{\frac{mk}{2}}R^2$

36. Consider a body of mass 1.0 kg at rest at the origin at time t = 0. A force $\vec{F} = (\alpha t\hat{i} + \beta\hat{j})$ is applied on the body, where $\alpha = 1.0 \text{ Ns}^{-1}$ and $\beta = 1.0$ N. The torque acting on the body about the origin at time t = 1.0 s is $\vec{\tau}$. Which of the following statements is (are) true? **[JEE Advanced-2018]**

(A) $|\vec{\tau}| = \frac{1}{3}\text{Nm}$

(B) The torque $\vec{\tau}$ is in the direction of the unit vector $+\hat{k}$

(C) The velocity of the body at t = 1 s is $\vec{v} = \frac{1}{2}(\hat{i} + 2\hat{j})\text{ms}^{-1}$

(D) The magnitude of displacement of the body at t = 1 s is $\frac{1}{6}\text{m}$

37. A ring and a disc are initially at rest, side by side, at the top of an inclined plane which makes an angle 60° with the horizontal. They start to roll without slipping at the same instant of time along the shortest path. If the time difference between their reaching the ground is $(2-\sqrt{3})/\sqrt{10}\text{s}$, then the height of the top of the inclined plane, in meters, is _____. Take g = 10 ms^{-2}. **[JEE Advanced-2018]**

38. A thin and uniform rod of mass M and length L is held vertical on a floor with large friction. The rod is released from rest so that it falls by rotating about its contact-point with the floor without slipping. Which of the following statement(s) is/are correct, when the rod makes an angle 60° with vertical ? [g is the acceleration due to gravity] **[JEE Advanced-2019]**

(1) The radial acceleration of the rod's center of mass will be $\frac{3g}{4}$

(2) The angular acceleration of the rod will be $\frac{2g}{L}$

(3) The angular speed of the rod will be $\sqrt{\frac{3g}{2L}}$

(4) The normal reaction force from the floor on the rod will be $\frac{Mg}{16}$

SIMPLE HARMONIC MOTION

Advanced Objective (AO)

Single Correct Answer Type Question

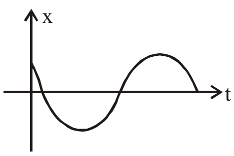
1. Statement 1 : Position–time equation of a particle moving along x–axis is $x = 4 + 6 \sin\omega t$. Under this situation, motion of particle is not simple harmonic.

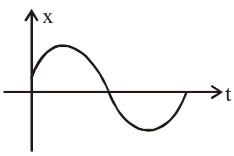
and

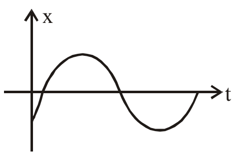
Statement 2 : $\frac{d^2x}{dt^2}$ for the given equation is not proportional to $-x$.

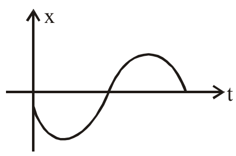
(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1.
(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1.
(C) Statement–1 is True, Statement–2 is False.
(D) Statement–1 is False, Statement–2 is True.

2. A particle is performing S.H.M. and at $t = \frac{3T}{4}$, is at position $= \frac{A}{\sqrt{2}}$ and moving towards the origin. Equilibrium position of the particle is at x = 0 . After $t = \frac{3T}{2}$ what will be the graph of the particle :-

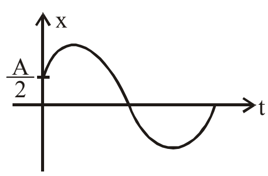
(A)


(B)


(C)


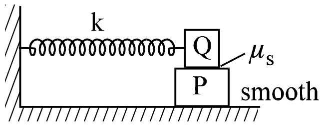
(D)


3. Following graph shows a particle performing S.H.M. about mean position x = 0. The equation of particle if $t = \frac{T}{4}$ is taken as starting time is (Notations have usual meanings)



(A) $A\sin\left(\omega t + \frac{2\pi}{3}\right)$ (B) $A\sin\left(\omega t + \frac{\pi}{3}\right)$ (C) $A\sin\left(\omega t + \frac{\pi}{6}\right)$ (D) $A\cos\left(\omega t + \frac{2\pi}{3}\right)$

4. A block P of mass m is placed on a frictionless horizontal surface. Another block *Q* of same mass is kept on P and connected to the wall with the help of a spring of spring constant k as shown in the figure. μ_s is the coefficient of friction between *P* and *Q*. The blocks move together performing SHM of amplitude *A*. The maximum value of the friction force between *P* and *Q* is :-



(A) kA (B) $\frac{kA}{2}$ (C) zero (D) $\mu_s mg$

5. A silver atom in a solid oscillates in simple harmonic motion in some direction with a frequency of 10^{12}/sec. What is the force constant of the bonds connecting one atom with the other ? (Mole wt. of silver =108 and Avagadro number = 6.02×10^{23} gm mole^{-1})

(A) 7.1 N/m (B) 2.2 N/m (C) 5.5 N/m (D) 6.4 N/m

6. A simple pendulum oscillating in air has period T. The bob of the pendulum is completely immersed in a non-viscous liquid. The density of the liquid is $\frac{1}{16}$th of the material of the bob. If the bob is inside liquid all the time, its period of oscillation in this liquid is :

(A) $4T\sqrt{\frac{1}{15}}$ (B) $2T\sqrt{\frac{1}{10}}$ (C) $4T\sqrt{\frac{1}{14}}$ (D) $2T\sqrt{\frac{1}{14}}$

7. A simple pendulum of length 1 m is oscillating with an angular frequency 10 rad/s. The support of the pendulum starts oscillating up and down with a small angular frequency of 1 rad/s and an amplitude of 10^{-2} m. The relative change in the angular frequency of the pendulum is best given by :-

(A) 10^{-3} rad/s (B) 10^{-1} rad/s (C) 1 rad/s (D) 10^{-5} rad/s

8. A particle undergoing simple harmonic motion has time dependent displacement given by $x(t) = A\sin\frac{\pi t}{90}$. The ratio of kinetic to potential energy of this particle at t = 210 s will be :

(A) 2 (B) $\frac{1}{9}$ (C) 3 (D) 1

9. For a particle acceleration is defined as $\vec{a} = \frac{-5x\hat{i}}{|x|}$ for $x \neq 0$ and $\vec{a} = 0$ for x = 0. If the particle is initially at rest at (a, 0), what is period of motion of the particle.

(A) $4\sqrt{2a/5}$ sec. (B) $8\sqrt{2a/5}$ sec.

(C) $2\sqrt{2a/5}$ sec. (D) cannot be determined

10. A particle executing a simple harmonic motion of period 2s. When it is at its extreme displacement from its mean position, it receives an additional energy equal to what it had in its mean position. Due to this, in its subsequent motion,

(A) its amplitude will change and become equal to $\sqrt{2}$ times its previous amplitude

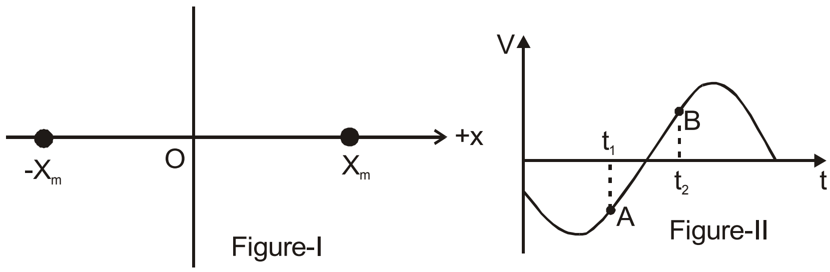
(B) its periodic time will become doubled i.e. 4s

(C) its potential energy will be decreased

(D) it will continue to execute simple harmonic motion of the same amplitude and period as before receiving the additional energy.

Multiple Correct Answer Type Question

11. A particle is executing SHM between points $-X_m$ and X_m, as shown in figure-I. The velocity V(t) of the particle is partially graphed and shown in figure-II. Two points A and B corresponding to time t_1 and time t_2 respectively are marked on the V(t) curve.



(A) At time t_1 , it is going towards X_m.

(B) At time t_1, its speed is decreasing.

(C) At time t_2, its position lies in between $-X_m$ and O.

(D) The phase difference $\Delta\phi$ between points A and B must be expressed as $90° < \Delta\phi < 180°$.

12. For a body executing SHM with amplitudes A, time period T, max velocity v_{max} and phase constant zero, which of the following statements are correct?

(A) At $y = (A/2)$, $v > (v_{max} / 2)$ (B) $v = (v_{max}/2)$ for $|y| > (A/2)$

(C) For $t = (T/8)$, $y > (A/2)$ (D) For $y = (A/2)$, $t < (T/8)$

13. The position vector of a particle that is moving in space is given by

$$\vec{r} = (1 + 2\cos 2\omega t)\hat{i} + (3\sin^2 \omega t)\hat{j} + (3)\hat{k}$$

in the ground frame. All units are in SI. Choose the correct statement (s) :

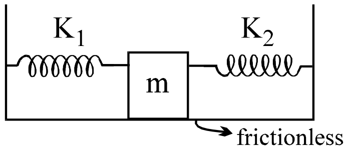
(A) The particle executes SHM in the ground frame about the mean position $\left(1, \frac{3}{2}, 3\right)$

(B) The particle executes SHM in a frame moving along the z-axis with a velocity of 3 m/s.

(C) The amplitude of the SHM of the particle is $\frac{5}{2}$ m.

(D) The direction of the SHM of the particle is given by the vector $\left(\frac{4}{5}\hat{i} - \frac{3}{5}\hat{j}\right)$

14. Two springs with negligible masses and force constant of $K_1 = 200\ Nm^{-1}$ and $K_2 = 160\ Nm^{-1}$ are attached to the block of mass m = 10 kg as shown in the figure. Initially the block is at rest, at the equilibrium position in which both springs are neither stretched nor compressed. At time t = 0, a sharp impulse of 50 Ns is given to the block with a hammer.



(A) Period of oscillations for the mass m is $\frac{\pi}{3}$ s.

(B) Maximum velocity of the mass m during its oscillation is 5 ms^{-1}.

(C) Data are insufficient to determine maximum velocity.

(D) Amplitude of oscillation is 0.42 m.

15. Two particles are in SHM with same amplitude A and same angular frequency ω. At time t = 0, one is at $x = +\frac{A}{2}$ and other is at $x = -\frac{A}{2}$. Both are moving in same direction.

(A) Phase difference between the two particle is $\frac{\pi}{3}$

(B) Phase difference between the two particle is $\frac{2\pi}{3}$

(C) They will collide after time $t = \frac{\pi}{2\omega}$

(D) They will collide after time $t = \frac{3\pi}{\omega}$

16. Three simple harmonic motions in the same direction having the same amplitude a and same period are superposed. If each differs in phase from the next by 45°, then :-

(A) the resultant amplitude is $(1 + \sqrt{2})$ a

(B) the phase of the resultant motion relative to the first is 90°

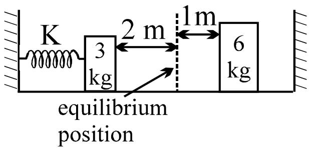
(C) the energy associated with the resulting motion is $(3 + 2\sqrt{2})$ times the energy associated with any single motion

(D) the resulting motion is not simple harmonic

Linked Comprehension Type Question

Paragraph for Question No. 17 and 18 (2 Question)

Two blocks of masses 3 kg and 6 kg rest on on a horizontal frictionless surface. The 3 kg block is attached to a spring with a force constant K = 900 N/m which is compressed 2 m initially from its equilibrium position. When 3 kg mass is released, it strikes the 6 kg mass and the two stick together.



17. The common velocity of the blocks after collision is :-

(A) 10 m/s (B) 30 m/s (C) 15 m/s (D) 2 m/s

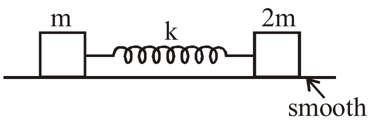
18. The amplitude of resulting oscillation after the collision is :-

(A) $\frac{1}{\sqrt{2}}$ m (B) $\frac{1}{\sqrt{3}}$ m (C) $\sqrt{2}$ m (D) $\sqrt{3}$ m

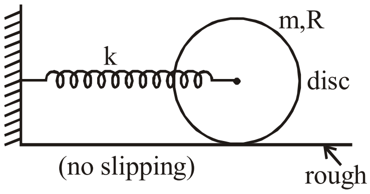
Matrix Match Type Question

19. In list-I, the systems are performing SHM and in list-II, the time period of SHM is shown then match list-I with list-II.

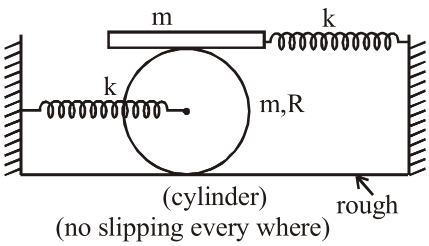
List-I / **List-II**

(P)


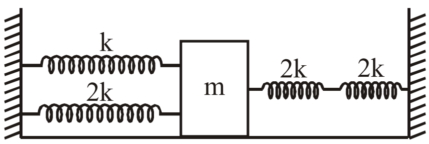
(1) $\left[2\pi\sqrt{\frac{11m}{10k}}\right]$

(Q)


(2) $2\pi\sqrt{\frac{m}{4k}}$

(R)


(3) $2\pi\sqrt{\frac{3m}{2k}}$

(S)


(4) $2\pi\sqrt{\frac{2m}{3k}}$

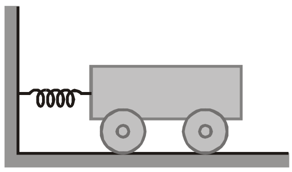
Codes :

	P	Q	R	S
(A)	3	2	1	4
(B)	4	3	2	1
(C)	3	4	2	1
(D)	4	3	1	2

Advanced Subjective (AS)

1. A small body of mass m is fixed to the middle of a stretched string of length 2ℓ. In the equilibrium position the string tension is equal to T_0. Find the angular frequency of small oscillations of the body in the transverse direction. The mass of the string is negligible, the gravitational field is absent.

2. A cart consists of a body and four wheels on frictionless axles. The body has a mass m. The wheels are uniform disks of mass M and radius R. The cart rolls, without slipping, back and forth on a horizontal plane under the influence of a spring attached to one end of the cart (figure). The spring constant is k. Taking into account the moment of inertia of the wheels, find a formula for the frequency of the back and forth motion of the cart.



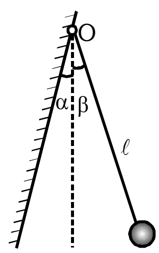
3. The motion of a simple pendulum is given by

$$\theta = A\cos\left(\sqrt{\frac{g}{\ell}}\,t\right) \text{ (symbols have their usual meaning)}$$

(a) Find the tension in the string of this pendulum as a function of time assume that $\theta << A$.

(b) At what time is the tension maximum? What is the value of this maximum tension?

4. A ball is suspended by a thread of length ℓ at the point O on the wall, forming a small angle α with the vertical as shown in figure. Then the thread with the ball was deviated through a small angle β ($\beta > \alpha$) and set free. Assuming the collision of the ball against the wall to be perfectly elastic, find the oscillation period of such a pendulum.

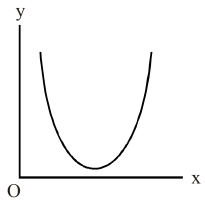


EXERCISE (JA)

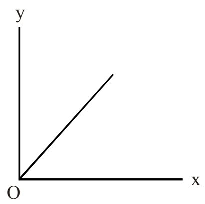
1. **Column-I** gives a list of possible set of parameters measured in some experiments. The variations of the parameters in the form of graphs are shown in **Column-II**. Match the set of parameters given in **Column-I** with the graphs given in **Column-II**. **[IIT JEE 2008]**

Column-I — **Column-II**

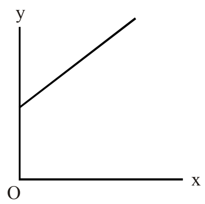
(A) Potential energy of a simple pendulum (*y* axis) as a function of displacement (*x* axis)

(P) 

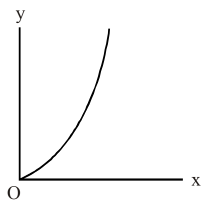
(B) Displacement (*y* axis) as a function of time (*x* axis) for an one dimensional motion at zero or constant acceleration when the body is moving along the positive x-direction.

(Q) 

(C) Range of a projectile (*y* axis) as a function of its velocity (*x* axis) when projected at a fixed angle.

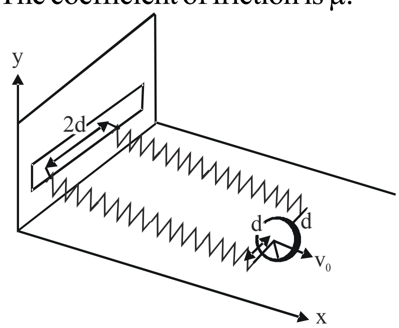
(R) 

(D) The square of the time period (*y* axis) of a simple pendulum as a function of its length (*x* axis)

(S) 

Paragraph for Question no. 2 to 4

A uniform thin cylindrical disk of mass M and radius R is attached to two identical massless springs of spring constant *k* which are fixed to the wall as shown in the figure. The springs are attached to the axle of the disk symmetrically on either side at a distance d from its centre. The axle is massless and both the springs and the axle are in a horizontal plane. The unstretched length of each spring is *L*. The disk is initially at its equilibrium position with its centre of mass (CM) at a distance *L* from the wall. The disk rolls without slipping with velocity $\vec{v}_0 = v_0\hat{i}$. The coefficient of friction is μ. **[IIT JEE 2008]**



2. The net external force acting on the disk when its centre of mass is at displacement *x* with respect to its equilibrium position, is :

(A) $-kx$ (B) $-2kx$ (C) $-\frac{2kx}{3}$ (D) $-\frac{4kx}{3}$

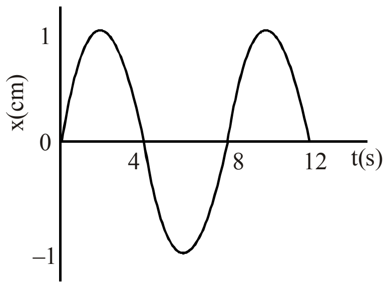
3. The centre of mass of the disk undergoes simple harmonic motion with angular frequency ω, equal to

(A) $\sqrt{\frac{k}{M}}$ (B) $\sqrt{\frac{2k}{M}}$ (C) $\sqrt{\frac{2k}{3M}}$ (D) $\sqrt{\frac{4k}{3M}}$

4. The maximum value of v_0 for which the disk will roll without slipping, is :

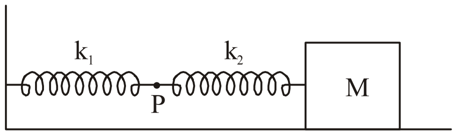
(A) $\mu g\sqrt{\frac{M}{k}}$ (B) $\mu g\sqrt{\frac{M}{2k}}$ (C) $\mu g\sqrt{\frac{3M}{k}}$ (D) $\mu g\sqrt{\frac{5M}{2k}}$

5. The x-t graph of a particle undergoing simple harmonic motion is shown below. The acceleration of the particle at $t = 4/3$ s is **[IIT JEE 2009]**



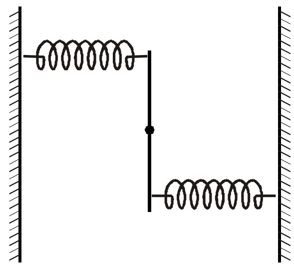
(A) $\frac{\sqrt{3}}{32}\pi^2\ cm/s^2$ (B) $\frac{-\pi^2}{32}\ cm/s^2$ (C) $\frac{\pi^2}{32}\ cm/s^2$ (D) $-\frac{\sqrt{3}}{32}\pi^2\ cm/s^2$

6. The mass *M* shown in the figure oscillates in simple harmonic motion with amplitude *A*. The amplitude of the point *P* is **[IIT JEE 2009]**



(A) $\frac{k_1A}{k_2}$ (B) $\frac{k_2A}{k_1}$ (C) $\frac{k_1A}{k_1+k_2}$ (D) $\frac{k_2A}{k_1+k_2}$

7. A uniform rod of length *L* and mass *M* is pivoted at the centre. Its two ends are attached to two springs of equal spring constants *k*. The springs are fixed to rigid supports as shown in the figure, and the rod is free to oscillate in the horizontal plane. The rod is gently pushed through a small angle θ in one direction and released. The frequency of oscillation is :- **[IIT JEE 2009]**

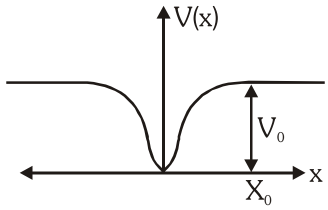


(A) $\frac{1}{2\pi}\sqrt{\frac{2k}{M}}$ (B) $\frac{1}{2\pi}\sqrt{\frac{k}{M}}$ (C) $\frac{1}{2\pi}\sqrt{\frac{6k}{M}}$ (D) $\frac{1}{2\pi}\sqrt{\frac{24k}{M}}$

Paragraph for Question No. 8 to 10

When a particle of mass m moves on the x–axis in a potential of the form $V(x) = kx^2$, it performs simple harmonic motion. The corresponding time period is proportional to $\sqrt{\frac{m}{k}}$, as can be seen easily using dimensional analysis. However, the motion of a particle can be periodic even when its potential energy increases on both sides of $x = 0$ in a way different from kx^2 and its total energy is such that the particle does not escape to infinity. Consider a particle of mass m moving on the x–axis. Its potential energy is $V(x) = \alpha x^4$ $(\alpha > 0)$ for $|x|$ near the origin and becomes a constant equal to V_0 for $|x| \geq X_0$ (see figure)

[IIT-JEE 2010]



8. If the total energy of the particle is E, it will perform periodic motion only if :-

(A) $E < 0$ (B) $E > 0$ (C) $V_0 > E > 0$ (D) $E > V_0$

9. For periodic motion of small amplitude A, the time period T of this particle is proportional to :-

(A) $A\sqrt{\frac{m}{\alpha}}$ (B) $\frac{1}{A}\sqrt{\frac{m}{\alpha}}$ (C) $A\sqrt{\frac{\alpha}{m}}$ (D) $\frac{1}{A}\sqrt{\frac{\alpha}{m}}$

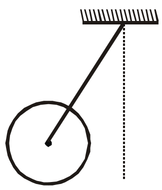
10. The acceleration of this particle for $|x| > X_0$ is :-

(A) proportional to V_0 (B) proportional to $\frac{V_0}{mX_0}$ (C) proportional to $\sqrt{\frac{V_0}{mX_0}}$ (D) Zero

11. A point mass is subjected to two simultaneous sinusoidal displacements in x-direction, $x_1(t) = A\sin\omega t$ and $x_2(t) = A\sin\left(\omega t + \frac{2\pi}{3}\right)$. Adding a third sinusoidal displacement $x_3(t) = B\sin(\omega t + \phi)$ brings the mass to a complete rest. The values of B and ϕ are :- **[IIT-JEE 2011]**

(A) $\sqrt{2}A, \frac{3\pi}{4}$ (B) $A, \frac{4\pi}{3}$ (C) $\sqrt{3}A, \frac{5\pi}{6}$ (D) $A, \frac{\pi}{3}$

12. A metal rod of length 'L' and mass 'm' is pivoted at one end. A thin disk of mass 'M' and radius 'R' (< L) is attached at its center to the free end of the rod. Consider two ways the disc is attached: (case A). The disc is not free to rotate about its center and (case B) the disc is free to rotated about its center. The rod-disc system performs SHM in vertical plane after being released from the same displaced position. Which of the following statement(s) is(are) true? **[IIT-JEE 2011]**



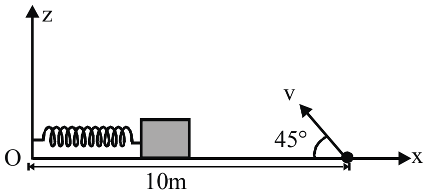
(A) Restoring torque in case A = Restoring torque in case B

(B) Restoring torque in case A < Restoring torque in case B

(C) Angular frequency for case A > Angular frequency for case B

(D) Angular frequency for case A < Angular frequency for case B

13. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. They system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10m from O. If the pebble hits the block at t = 1s, the value of v is:- (take g = 10 m/s^2) **[IIT-JEE 2012]**



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

14. A particle of mass m is attached to one end of a mass-less spring of force constant k, lying on a frictionless horizontal plane. The other end of the spring is fixed. The particle starts moving horizontally from its equilibrium position at time t = 0 with an initial velocity u_0. When the speed of the particle is 0.5 u_0, it collides elastically with a rigid wall. After this collision :- **[JEE-Advanced-2013]**

(A) the speed of the particle when it returns to its equilibrium position is u_0

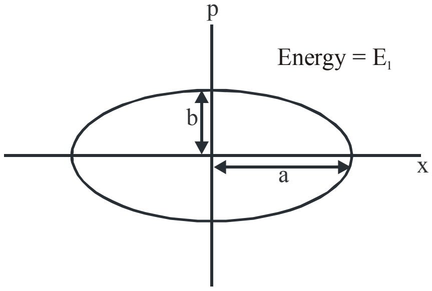
(B) the time at which the particle passes through the equilibrium position for the first time is $t = \pi\sqrt{\frac{m}{k}}$

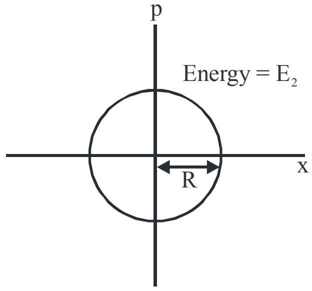
(C) the time at which the maximum compression of the spring occurs is $t = \frac{4\pi}{3}\sqrt{\frac{m}{k}}$

(D) the time at which the particle passes through the equilibrium position for the second time is $t = \frac{5\pi}{3}\sqrt{\frac{m}{k}}$

15. Two independent harmonic oscillators of equal mass are oscillating about the origin with angular frequencies ω_1 and ω_2 and have total energies E_1 and E_2, respectively. The variations of their momenta p with positions x are shown in the figures. If $\frac{a}{b} = n^2$ and $\frac{a}{R} = n$, then the correct equation (s) is (are) :-

[JEE-Advanced-2015]





(A) $E_1\omega_1 = E_2\omega_2$ (B) $\frac{\omega_2}{\omega_1} = n^2$ (C) $\omega_1\omega_2 = n^2$ (D) $\frac{E_1}{\omega_1} = \frac{E_2}{\omega_2}$

16. A particle of unit mass is moving along the x-axis under the influence of a force and its total enegy is conserved. Four possible forms of the potential energy of the particle are given in column I (a and U_0 are constants). Match the potential energies in column I to the corresponding statement(s) in column-II

[JEE-Advanced-2015]

Column-I		**Column-II**	
(A)	$U_1(x) = \frac{U_0}{2}\left[1-\left(\frac{x}{a}\right)^2\right]^2$	(P)	The force acting on the particle is zero at x = a.
(B)	$U_2(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2$	(Q)	The force acting on the particle is zero at x = 0.
(C)	$U_3(x) = \frac{U_0}{2}\left(\frac{x}{a}\right)^2 \exp\left[-\left(\frac{x}{a}\right)^2\right]$	(R)	The force acting on the particle is zero at x =–a.
(D)	$U_4(x) = \frac{U_0}{2}\left[\frac{x}{a} - \frac{1}{3}\left(\frac{x}{a}\right)^3\right]$	(S)	The particle experiences an attractive force towards x = 0 in the region $\lvert x \rvert < a$.
		(T)	The particle with total enegy $\frac{U_0}{4}$ can oscillate about the point x = –a.

17. A block with mass M is connected by a massless spring with stiffness constant k to a rigid wall and moves without friction on a horizontal surface. The block oscillates with small amplitude A about an equilibrium position x_0. Consider two cases : (i) when the block is at x_0; and (ii) when the block is at $x = x_0 + A$. In both the cases, a particle with mass m (<M) is softly placed on the block after which they stick to each other. Which of the following statement(s) is(are) true about the motion after the mass m is placed on the mass M? **[JEE-Advanced-2016]**

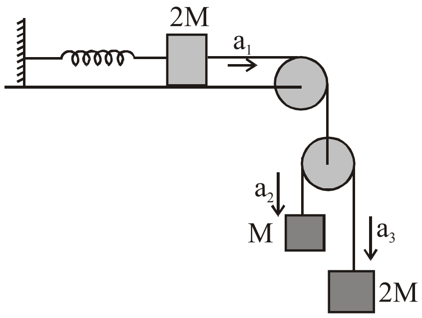
(A) The amplitude of oscillation in the first case changes by a factor of $\sqrt{\frac{M}{m+M}}$, whereas in the second case it remains unchanged

(B) The final time period of oscillation in both the cases is same

(C) The total energy decreases in both the cases

(D) The instantaneous speed at x_0 of the combined masses decreases in both the cases.

18. A block of mass 2M is attached to a massless spring with spring-constant k. This block is connected to two other blocks of masses M and 2M using two massless pulleys and strings. The accelerations of the blocks are a_1, a_2 and a_3 as shown in figure. The system is released from rest with the spring in its unstretched state. The maximum extension of the spring is x_0. Which of the following option(s) is/are correct ?
[g is the acceleration due to gravity. Neglect friction] **[JEE-Advance-2019]**



(1) $x_0 = \frac{4Mg}{k}$

(2) When spring achieves an extension of $\frac{x_0}{2}$ for the first time, the speed of the block connected to the spring is $3g\sqrt{\frac{M}{5k}}$

(3) $a_2 - a_1 = a_1 - a_3$

(4) At an extension of $\frac{x_0}{4}$ of the spring, the magnitude of acceleration of the block connected to the spring is $\frac{3g}{10}$

ANSWER KEY

01_KINEMATICS

Advanced Objective (AO)

1. Ans. (A) **2. Ans. (B)** **3. Ans. (D)** **4. Ans. (B)**

Numerical Grid Type Question

5. Ans. 3

Matrix Match Type Question

6. Ans. (A) → (T); (B) → (P,Q); (C) → (R); (D) → (S)

02_NLM & FRICTION

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (C) **2. Ans. (A)** **3. Ans. (B)**

Multiple Correct Answer Type Question

4. Ans. (C, D) **5. Ans. (A,B,C,D)** **6. Ans. (A,B,C)** **7. Ans. (A,B,D)**

Advanced Subjective (AS)

1. Ans. 5 m/s^2 downwards, 0 m/s^2, 10 N

03_Circular Motion

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (B) **2. Ans. (D)** **3. Ans. (A)**

Multiple Correct Answer Type Question

4. Ans. (A, B, C)

Advanced Subjective (AS)

1. Ans. (i) 36N, (ii) 11.66rad/sec, (iii) 0.1m, 0.2m

04_Work Power & Energy

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (D)

Matrix Match Type Question

2. Ans. (A) Q,S (B) P, S (C) R, S ; (D) P,S

Advanced Subjective (AS)

1. Ans. 2.5cm **2. Ans. 25 m** **3. Ans. 27R/28**

05_Center of Mass & Collision

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (D) **2. Ans. (B)** **3. Ans. (D)** **4. Ans. (C)** **5. Ans. (B)** **6. Ans. (B)**
7. Ans. (B) **8. Ans. (B)** **9. Ans. (D)**

Multiple Correct Answer Type Question

10. Ans. (A, B,D) **11. Ans. (A, C)** **12. Ans. (B,C)** **13. Ans. (AB)** **14. Ans. (A,B,D)**
15. Ans. (A,C) **16. Ans. (A,B,D)** **17. Ans. (B,C)** **18. Ans. (A,B,C,D)**

Linked Comprehension Type Question

19. Ans. (A) **20. Ans. (A)** **21. Ans. (B)** **22. Ans. (C)** **23. Ans. (D)** **24. Ans. (A)**
25. Ans. (C)

Matrix Match Type Question

26. Ans. (A)-P; (B)-S; (C)-Q, R; (D)-T **27. Ans. A-(S), B-(Q), C-(P), D-(R)**
28. Ans. (A)-Q (B)-S (C)-P **29. Ans. (A) R, (B) T, (C) T, (D)S**
30. Ans. (A)-Q; (B)-P, Q; (C)-R; (D) S **31. Ans. (A)-Q; (B)-Q; (C)-P,R; (D)-Q,S**

Advanced Subjective (AS)

1. Ans. $\frac{a}{3(\pi-1)}$ **2. Ans.** 30 cm **3. Ans.** $(\hat{i}+\sqrt{3}\hat{j})$ m/s, $(3m\hat{i}-\sqrt{3}m\hat{j})$ kg-m/s

4. Ans. (i) v/2, v/2, 0; (ii) $2mv^2/9$; (iii) $mv^2/72$; (d) x = $\sqrt{m/6k}$ v

5. Ans. $\vec{P}_{PM} = m\vec{v}_{PM}$
$= -mv_2 \sin\omega t\,\hat{i} + m(v_2\cos\omega t - v_1)\,\hat{j}$

6. Ans. (i) $v_A = \sqrt{g/12}$ m/s, (ii) $S_{max} = 49/48$ m

7. Ans. $\frac{m(R-r)}{M+m}$, $m\sqrt{\frac{2g(R-r)}{M(M+m)}}$

EXERCISE (J-A)

1. Ans. (B or D) 2. Ans. (A,D) 3. Ans. (B) 4. Ans. (B) 5. Ans. (C)
6. Ans. 4m/s 7. Ans. (C) 8. Ans. (A) 9. Ans. (A,C) 10. Ans. (C)
11. Ans. (D) 12. Ans. (A,C) 13. Ans. 5 14. Ans. (B) 15. Ans. (A,C)
16. Ans. (C) 17. Ans. 6.3 [6.29, 6.31] 18. Ans. (2,3)

06_Rotational Mechanics

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (C) 2. Ans. (B) 3. Ans. (B) 4. Ans. (B) 5. Ans. (C) 6. Ans. (D)
7. Ans. (C) 8. Ans. (B) 9. Ans. (B) 10. Ans. (C) 11. Ans. (B) 12. Ans. (A)
13. Ans. (C) 14. Ans.(B) 15. Ans. (D) 16. Ans. (D) 17. Ans. (B) 18. Ans. (A)
19. Ans. (B)

Multiple Correct Answer Type Question

20. Ans. (A,B,C,D) 21. Ans. (B,C) 22. Ans. (A,D) 23. Ans. (A,C,D)
24. Ans. (A,C,D) 25. Ans. (A,B,C,D) 26. Ans. (A,B,C) 27. Ans. (A,B,D)
28. Ans. (A,B,C) 29. Ans. (A,B,C) 30. Ans. (B,C,D) 31. Ans. (A,C) 32. Ans. (B,D)
33. Ans. (A, B, C)

Linked Comprehension Type Question

34. Ans. (A) 35. Ans. (A) 36. Ans. (D) 37. Ans. (B) 38. Ans. (A) 39. Ans. (D)
40. Ans. (D) 41. Ans. (C) 42. Ans. (A) 43. Ans. (C) 44. Ans. (C) 45. Ans. (B)
46. Ans. (B) 47. Ans. (C) 48. Ans. (B) 49. Ans. (A,B,C) 50. Ans. (C)
51. Ans. (B,C,D) 52. Ans. (A,C,D)

Matrix Match Type Question

53. Ans. (A)→(R); (B)→(P); (C)→(T); (D)→(Q,S) 54. Ans. (A) ST (B) R (C) PRT (D) PQ
55. Ans. (A)-R, (B)-S, (C)-P, (D)-Q

Advanced Subjective (AS)

1. Ans. 30° **2. Ans.** (i) $\frac{J}{m}$ (ii) zero (iii) $\frac{J}{2m}$ (iv) $\frac{5}{2}\frac{J}{m}$

3. Ans. (a) $\sqrt{3}\ m l\omega^2$ (b) $(F_{net})_x = -\frac{F}{4}$, $(F_{net})_y = \sqrt{3}\ m l\omega^2$

4. Ans. (a) $a_c = \frac{4F}{3m_1 + 8m_2}$, $a_p = 2a_c$

(b) friction at the top of the cylinder = $3m_1F(3m_1+8m_2)$ towards right; friction at the bottom = $m_1F/(3m_1+8m_2)$ towards right.

5. Ans. (i) $2v_0/3$, (ii) $t = v_0/3\mu g$, $W = \frac{1}{2}[3\mu^2 m g^2 t^2 - 2\mu m g t v_0]$ $(t < t_0)$, $W = -\frac{1}{6} m v_0^2$ $(t > t_0)$

6. Ans. (a) $\frac{m}{M} = \frac{1}{4}$; (b) $x = \frac{2L}{3}$; (c) $\frac{v_0}{2\sqrt{2}}$ **7. Ans.** (a) $\omega = \sqrt{\frac{6g}{\ell}}$, (b) $T = \frac{11mg}{4}$

8. Ans. $150(+\hat{i})$

EXERCISE (JA)

1. Ans. (D) 2. Ans. (C) 3. Ans. (A) 4. Ans. (B) 5. Ans. (D)
6. Ans. (D) 7. Ans. (B) 8. Ans. (A) 9. Ans. (B,C) 10. Ans. 4
11. Ans. 9 12. Ans. (AC or C) 13. Ans. (B) 14. Ans. (C,D)
15. Ans. 3 16. Ans. (D) 17. Ans. (A) 18. Ans. (A,B) 19. Ans. (D)
20. Ans. 8 21. Ans. 4 22. Ans. 2 23. Ans. 7 24. Ans. (C, D)
25. Ans. 6 26. Ans. (D) 27. Ans. (A, B, D) 28. Ans. (D)
29. Ans. (D) 30. Ans. (C) 31. Ans. (A), (B), (C) 32. Ans. (B,C or C)
33. Ans. (Bonus) 34. Ans. (B) 35. Ans. (B,C) 36. Ans. (A,C) 37. Ans. 0.75 [0.74, 0.76]
38. Ans. (1,3,4)

07_Simple Harmonic Motion

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (D) 2. Ans. (B) 3. Ans. (A) 4. Ans. (B) 5. Ans. (A) 6. Ans. (A)
7. Ans. (A) 8. Ans. (C) 9. Ans. (A) 10. Ans.(A)

Multiple Correct Answer Type Question

11. Ans. (B, C) 12. Ans. (A,B,C) 13. Ans. (A,C,D) 14. Ans. (A,B) 15. Ans. (A, C) 16. Ans. (A, C)

Linked Comprehension Type Question

17. Ans. (A) 18. Ans. (C)

Matrix Match Type Question

19. Ans. (D)

Advanced Subjective (AS)

1. Ans. $\omega = \sqrt{\frac{2T_0}{m\ell}}$ **2. Ans.** $\omega = \sqrt{\frac{k}{(m+6M)}}$

3. Ans. (a) $T = mg + mgA^2 \sin^2 \sqrt{\frac{g}{\ell}}\, t$ (b) $t = (2n+1)\frac{\pi}{2}\sqrt{\frac{\ell}{g}}$; $n \in I$ $T_{max} = mg + mgA^2$

4. Ans. $T = 2\sqrt{\ell/g}\,[\pi/2 + \sin^{-1}(\alpha/\beta)]$

EXERCISE (JA)

1. Ans. (A) P,S (B) Q,R,S (C) S (D) Q 2. Ans. (D) 3. Ans. (D) 4. Ans. (C)
5. Ans. (D) 6. Ans. (D) 7. Ans. (C) 8. Ans. (C) 9. Ans. (B) 10. Ans. (D)
11. Ans. (B) 12. Ans. (A,D) 13. Ans. (A) 14. Ans. (A,D) 15. Ans. (B,D)
16. Ans. (A)-P,Q,R,T; (B)-Q,S; (C)-P,Q,R,S; (D)-P,R,T 17. Ans. (A,B,D)
18. Ans. (3)

ALLEN CAREER INSTITUTE KOTA (RAJASTHAN)

Chapter Contents 02

COMPENDIUM-2

Important Notes

ELECTROSTATICS

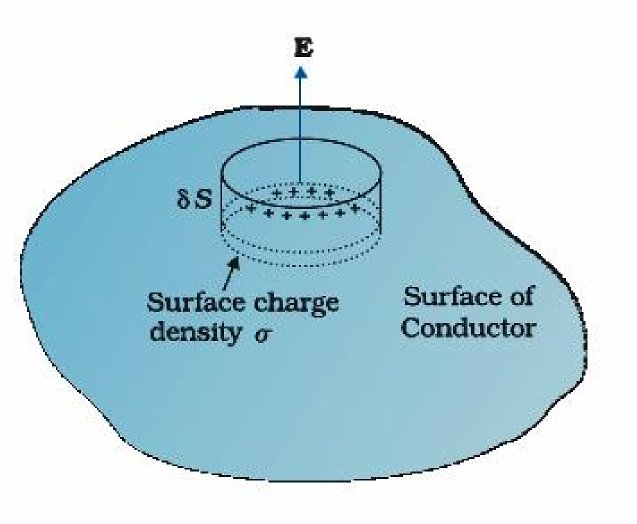
THEORY

Electric field at the surface of a charged conductor

$$E = \frac{\sigma}{\varepsilon_0}\hat{n}$$

where σ is the surface charge density and $\hat{n}$ is a unit vector normal to the surface in the outward direction.

To derive the result, choose a pill box (a short cylinder) as the Gaussian surface about any point P on the surface, as shown in figure.



The pill box is partly inside and partly outside the surface of the conductor. It has a small area of cross section δS and negligible height.

Just inside the surface, the electrostatic field is zero; just outside, the field is normal to the surface with magnitude E. Thus, the contribution to the total flux through the pill box comes only from the outside (circular) cross-section of the pill box. This equals $\pm E\delta S$ (positive for $\sigma > 0$, negative for $\sigma < 0$), since over the small area δS, **E** may be considered constant and **E** and δS are parallel or antiparallel. The charge enclosed by the pill box is $\sigma\delta S$.

By Gauss's law

$$E\delta S = \frac{|\sigma|\delta S}{\varepsilon_0}$$

$$E = \frac{|\sigma|}{\varepsilon_0}$$

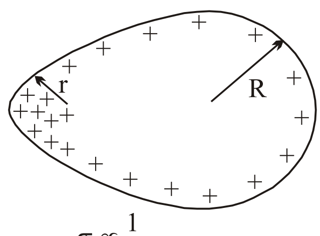
Including the fact that electric field is normal to the surface, we get the vector relation, Eq. $E = \frac{\sigma}{\varepsilon_0}\hat{n}$,

which is true for both signs of σ. For $\sigma > 0$, electric field is normal to the surface outward; for $\sigma < 0$, electric field is normal to the surface inward.

Electrostatic potential is constant throughout the volume of the conductor and has the same value (as inside) on its surface

Since **E** = 0 inside the conductor and has no tangential component on the surface, no work is done in moving a small test charge within the conductor and on its surface. That is, there is no potential difference between any two points inside or on the surface of the conductor. Hence, the result. If the conductor is charged, electric field normal to the surface exists; this means potential will be different for the surface and a point just outside the surface. In a system of conductors of arbitrary size, shape and charge configuration, each conductor is characterised by a constant value of potential, but this constant may differ from one conductor to the other.

Charge density on a conductor's surface in abscence of an external field



$\sigma \propto \frac{1}{r}$

Now consider a single conductor with a nonspherical shape. If a charge is given to this conductor figure, the charge density will not be uniform on the entire surface. A portion where the surface is more "flat" may be considered as part of a sphere of larger radius. The charge density at such a portion will be smaller from equation (1). At portions where the surface is more curved, the charge density will be larger. More precisely, the charge density will be larger where the radius of curvature is small. Thus the distribution of charge Q on surface of conductor is "Non-uniform

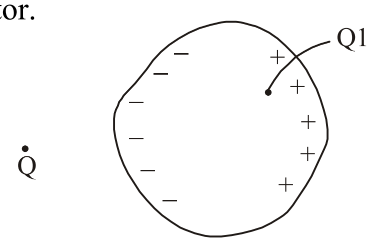
and $\sigma \quad \alpha \quad \frac{1}{R_{curvature}}$

Charged conductor in external field :

Figure shows a charge Q placed infront of a neutral conductor.

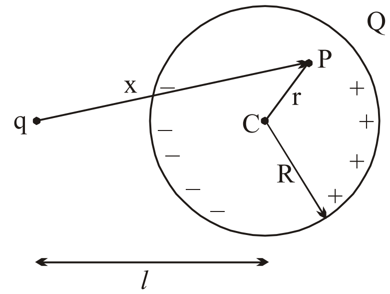
Here the charge density also depends on external field and

$$\vec{E}_{out} + \vec{E}_{ind.int} = 0$$



Note : So here charge Q_1 must be distributed non uniformly unrelated to Radius of curvature.

Ex. A point charge q is kept at a distance l from neutral sphere of radius R. Find electric potential V at P, solid conducting sphere.



Sol. Potential due to induced charges is zero as centre C is equidistant from all induced charges
Volume of solid conductor is equipotential volume

$$\therefore \quad V_c = V_p = \frac{Kq}{l} + \frac{KQ}{R}$$

Also at P

$$V_P = V_{\text{due to q}} + V_{\text{due to induced charges}} + V_{\text{due to Q}}$$

$$\frac{Kq}{l} + \frac{KQ}{R} = \frac{Kq}{x} + \frac{KQ}{R} + V_{\text{due to induced charges}}$$

$$V_{\text{induced charge}} = \frac{Kq}{l} - \frac{Kq}{x}$$

$$\vec{E}_{\text{external}} + \vec{E}_{\text{conductor}} + \vec{E}_{\text{induced}} = 0$$

Resultant electric field inside material is zero

Ex. Find electric field due to induced charges at P

$$|\vec{E}_p| \text{ by induced charges} = -\frac{Kq}{x^2}$$

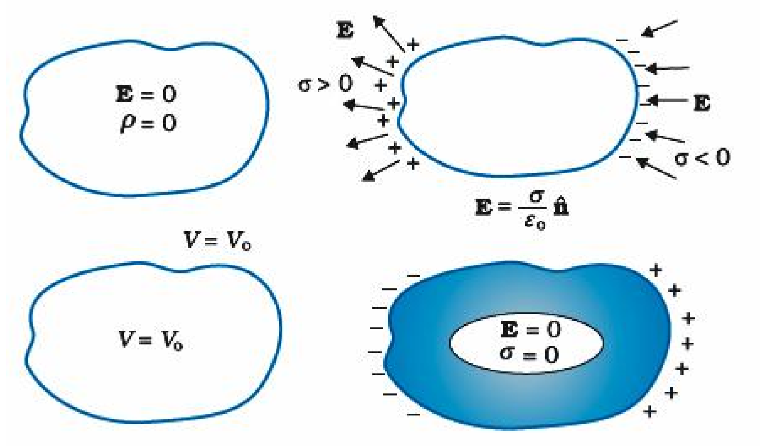
Cavity inside a conductor :

Cavity is aplace surrounded from all sides by the conductor such that without touching the body we can't reach cavity.

Electrostatic shielding

Consider a conductor with a cavity, with no charges inside the cavity. A remarkable result is that the electric field inside the cavity is zero, whatever be the size and shape of the cavity and whatever be the charge on the conductor and the external fields in which it might be placed. We have proved a simple case of this result already: the electric field inside a charged spherical shell is zero. The proof of the result for the shell makes use ofthe spherical symmetry of the shell. But the vanishing of electric field in the (charge-free) cavity of a conductor is, as mentioned above, is a general result. A related result is that even if the conductor is charged or charges are induced on a neutralconductor by an external field, all charges reside only on the outer surface of a conductor with cavity.

Whatever be the charge and field configuration outside, any cavity in a conductor remains shielded from outside electric influence: thefield inside the cavity is always zero. This is known as electrostatic shielding. The effect can be made use of in protecting sensitive instruments from outside electrical influence. Figure gives a summary of the important electrostatic propertiesof a conductor.



1. If there is no charge present inside the cavity than field inside it is zero.

(This does not mean that we are implying "no net charge". If there is some charge present here and there in cavity such that there sum total is zero then the following disscussion will not be valid)

Proof wrong sol.

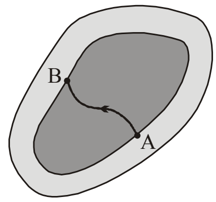
Let us consider a gauesion surface just near to cavity.

$$\therefore \quad \oint \vec{E}\cdot d\vec{A} = 0$$

$$\Rightarrow \quad q\ in = 0$$

Sol. From this we don't have proved that no charge resides on cavity but have proved that net charge on cavity surface is O.

Now suppose a conductor of arbitrary shape contains a cavity as shown in figure.



Let us assume that no charges are inside the cavity. In this case, the electric field inside the cavity must be zero regardless of the charge distribution on the outside surface of the conductor. Furthermore, the field in the cavity is zero even if an electric field exists outside the conductor.

To prove this point, we use the fact that every point on the conductor is at the same electric potential, and therefore any two points A and B on the surface of the cavity must be at the same potential. Now imagine that a field E exists in the cavity and evaluate the potential difference $V_B - V_A$ defined by equation.

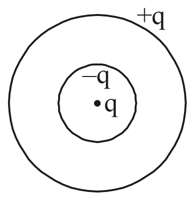
$$\Delta V \equiv \frac{\Delta U}{q_0} = -\int_A^B E\cdot ds$$

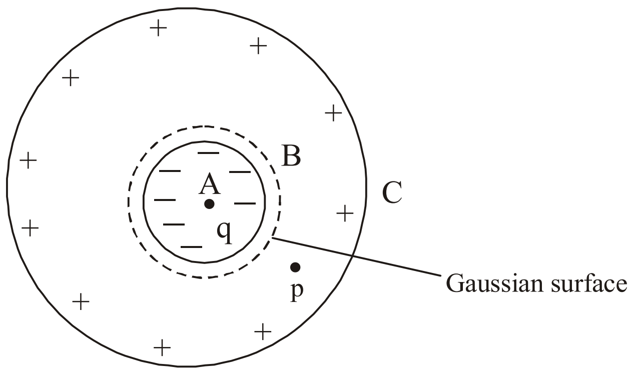
$$V_B - V_A = -\int_A^B E\cdot ds$$

Because $V_B - V_A = 0$, the integral of $E \cdot ds$ must be zero for all paths between any two points A and B on the conductor. The only way that this can be true for all paths is if E is zero everywhere in the cavity. Thus, we conclude that a cavity surrounded by conducting walls is a field-free region as along no charges are inside the cavity.

2. Equal and opposite charge is induced on the inner surface of cavity.

Figure shows a conductor with a cavity inside it. A charge q is placed inside cavity. In electrostatic equilibrium charge distribution will be as shown in figure charge on inner surface of cavity is – q since material of conductor is initially neutral, equal and opposite charge appears on outer surface.





Proof: Let us consider a gaussian surface just outside cavity inside material of conductor. As $\vec{E}$ in material of conductor is zero.

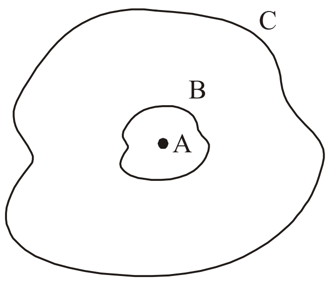
$$\oint E \cdot dA = 0$$

$\therefore \quad q_{enclosed} = 0$

Thus equal and opposite charge is induced on the inner surface of cavity.

3. The electric field due to charges on the outer surface of conductor is zero for all the points inside the conductor seperately

Consider a charged conductor having charge $+q_1$ and Q is kept inside the cavity. Lets call charge Q inside cavity as A, the induced charge -Q on the surface of the cavity as B and the charge on the surface of the conductor $Q + q_1$ as C.

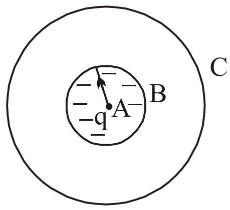


Now field inside the conductor is, $\vec{E}_{net}$ and $\vec{E}_A$, $\vec{E}_B$ and $\vec{E}_C$ are fields due to charge A,B and C inside the conductor.

and, $\vec{E}_{net} = \vec{E}_A + \vec{E}_B + \vec{E}_C = 0$

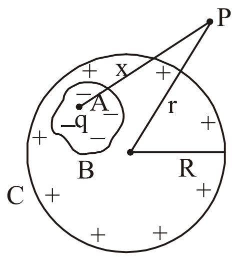
Now the electric field due to charges on the outer surface of conductor is zero for all the points inside the conductor seperately and the $\vec{E}_B + \vec{E}_A$ is zero seperately.

Special case : When spherical cavity in present inside spherical conductor and charge is at the centre. Then field inside the cavity is just due to charge A only as new field lines are already ⊥ to B and E due to B = 0. [Symmetrical distribution]



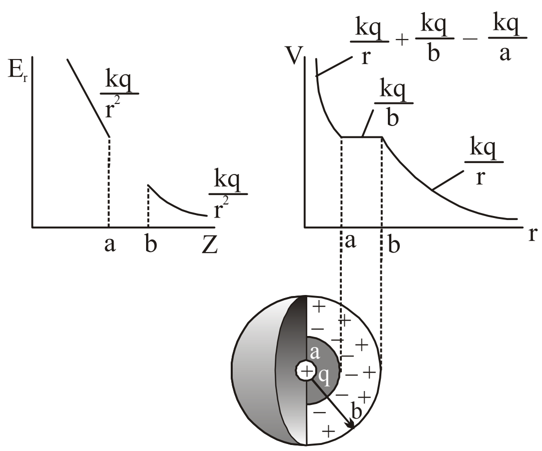
Note : When if we displace the charge q inside the cavity, it will only affect the charge distribution at B the charge distribution at C will remain unaffected.

Ex. Find field at P, the conductor is neutral. and cavityis having a charge q inside the cavity.



Sol. ∴ $E_p = \dfrac{RQ}{r^2}$ as $E_A + E_B = O$

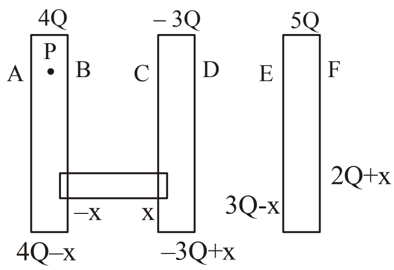
Ex. A hollow, uncharged spherical conductor has inner radius a and outer radius b. A positive point charge +q is in the cavity at the centre of the sphere. Make the graph E and potential V(r) everywhere, assuming that V = 0 at r = ∞.



Ex. Find charge on all surfaces :

Sol. Applying Gauss' law on opposite faces.

$$\oint \vec{E}\cdot d\vec{s} = 0$$



as through two lids E = 0 as part of conductor and through lateral surface $\vec{E}\cdot d\vec{s} = 0$

$\therefore$ $q_{enclosed} = 0$

Important : (i) Thus facing surfaces have equal and opposite charges.

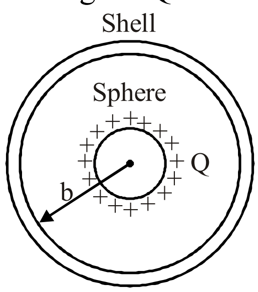
(ii) Using this also prove that outer faces of the two last plates have equal charges.

$$\frac{(4Q-x)/\text{Area}}{2\varepsilon_0} - \frac{(2Q+x)/\text{Area}}{2\varepsilon_0} = 0 \qquad \text{[as no field because of equal is cancelled]}$$

$4Q - x - 2Q - x = 0$

$\Rightarrow$ $2x = 2Q$ $\Rightarrow$ $x = Q$.

Ex. A solid conducting sphere having a charge Q is surrounded by an uncharged concentric conducting hollow spherical shell. Let the potential difference between the surface of the solid sphere and that of the outer surface of hollow shell be V. What will be the new potential difference between the same two surfaces if the shell given a charge –3Q ?



Sol. In case of a charged conducting sphere,

$$V_{in} = V_C = V_S = \frac{1}{4\pi\varepsilon_0}\frac{q}{R}$$

and $\quad V_{out} = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{r}$

So if a and b are the radii of sphere and spherical shell respectively, potential at their surfaces will be,

$$V_{sphere} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{a} \text{ and } V_{shell} = \frac{1}{4\pi\varepsilon_0}\frac{Q}{b}$$

And so according to given problem

$$V = V_{sphere} - V_{shell} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right] \quad \text{.......... (i)}$$

Now when the shell is given a charge (–3Q) the potential at its surface and also inside will change

by $V_0 = \frac{1}{4\pi\varepsilon_0}\left[\frac{-3Q}{b}\right]$

So that now

$$V'_{sphere} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{a}\right] + V_0$$

and $\quad V'_{shell} = \frac{1}{4\pi\varepsilon_0}\left[\frac{Q}{b}\right] + V_0$

And hence

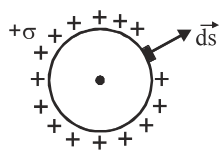
$$V'_{sphere} - V'_{shell} = \frac{Q}{4\pi\varepsilon_0}\left[\frac{1}{a}-\frac{1}{b}\right] = V \text{ [from Eq.(1)]}$$

i.e., if any charge is given to external shell, the potential difference between sphere and shell will not change. This is because by presence of charge on outer shell, potential everywhere inside and on the surface of shell will change by same amount and hence potential difference between sphere and shell will remain unchanged.

ELECTROSTATIC PRESSURE

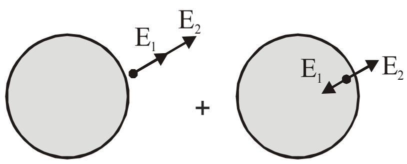
Force due to electrostatic pressure is directed normally outwards to the surface

Force on small element ds of charged conductor



dF = (Charge on ds) x Electric field = (σ ds) $\frac{\sigma}{2\epsilon_0}$ $dF = \frac{\sigma^2}{2\epsilon_0}ds$

Inside $E_1 - E_2 = 0 \Rightarrow E_1 = E_2$



Just outside $E = E_1 + E_2 = 2E_2 \Rightarrow E_2 = \frac{\sigma}{2\epsilon_0}$

(E_1 is field due to point charge on the surface and E_2 is field due to rest of the sphere).

The electric force acting per unit area of charged surface is defined as electrostatic pressure.

$$P_{eleectrostatic} = \frac{dF}{dS} = \frac{\sigma^2}{2\epsilon_0}$$

Advanced Objective (AO)

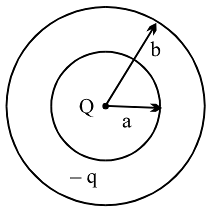
Single Correct Answer Type Question

1. From a point if we move in a direction making an angle θ measured from +ve x-axis, the potential gradient is given as $\frac{dv}{dr} = 2\cos\theta$. Find the direction and magnitude of electric field at that point.

(A) $2\hat{i}$ (B) $-2\hat{i}$ (C) $\hat{i} + \hat{j}$ (D) $-\hat{i} + \hat{j}$

2. Both question (a) and (b) refer to the system of charges as shown in the figure. A spherical shell with an inner radius 'a' and an outer radius 'b' is made of conducting material. A point charge +Q is placed at the centre of the spherical shell and a total charge – q is placed on the shell.

a. Charge – q is distributed on the surfaces as

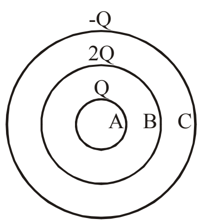


(A) – Q on the inner surface, – q on outer surface
(B) – Q on the inner surface, – q + Q on the outer surface
(C) +Q on the inner surface, –q – Q on the outer surface
(D) The charge –q is spread uniformly between the inner and outer surface.

b. Assume that the electrostatic potential is zero at an infinite distance from the spherical shell. The electrostatic potential at a distance R (a < R < b) from the centre of the shell is (where $K = \frac{1}{4\pi\varepsilon_0}$)

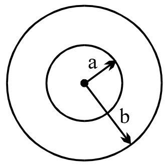
(A) 0 (B) $\frac{KQ}{a}$ (C) $K\frac{Q-q}{R}$ (D) $K\frac{Q-q}{b}$

3. Charge Q, 2Q and –Q are given to three concentric conducting spherical shells A, B and C respectively. The ratio of charges on the inner and the outer surfaces of the shell 'C' will be :-



(A) $\frac{3}{4}$ (B) $-\frac{3}{4}$ (C) $\frac{3}{2}$ (D) $-\frac{3}{2}$

4. If the electric potential of the inner metal sphere is 10 volt & that of the outer shell is 5 volt, then the potential at the centre will be :

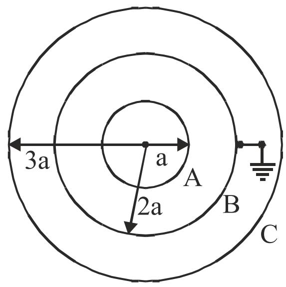


(A) 10 volt (B) 5 volt (C) 15 volt (D) 0

5. There are two uncharged identical metallic spheres 1 and 2 of radius r separated by a distance d (d >> r). A charged metallic sphere of same radius having charge q is touched with one of the sphere. After some time it is moved away from the system. Now the uncharged sphere is earthed. Charge on earthed sphere is

(A) q (B) – q (C) – qr/2d (D) 0

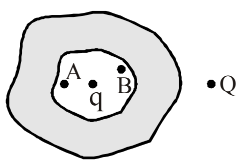
6. Figure shows a system of three concentric metal shells A, B and C with radii a, 2a and 3a respectively. Shell B is earthed and shell C is given a charge Q. Now if shell C is connected to shell A, then the final charge on the shell B, is equal to :



(A) $-\frac{4Q}{13}$ (B) $-\frac{8Q}{11}$ (C) $-\frac{5Q}{3}$ (D) $-\frac{3Q}{7}$

7. **Statement-1 :** A point charge q is placed inside a cavity of conductor as shown. Another point charge Q is placed outside the conductor as shown. Now as the point charge Q is pushed away from conductor, the potential difference $(V_A - V_B)$ between two points A and B within the cavity of sphere remains constant.

Statement-2 : The electric field due to charges on outer surface of conductor and outside the conductor is zero at all points inside the conductor.



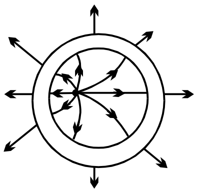
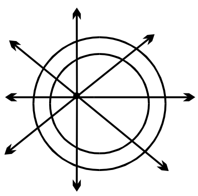
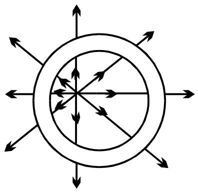
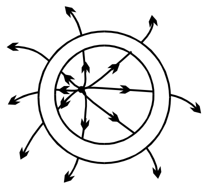
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true.

8. A point charge q is placed at a point inside a hollow conducting sphere. Which of the following electric force pattern is correct ? **[IIT-JEE'2003 (scr)]**

(A)  (B)  (C)  (D) 

9. Two point charges $q_1(\sqrt{10}\ \mu C)$ and $q_2(-25\ \mu C)$ are placed on the x-axis at x = 1 m and x = 4 m respectively. The electric field (in V/m) at a point y = 3 m on y-axis is, **[JEE-Main(online)-2019]**

$\left[\text{take}\ \frac{1}{4\pi\varepsilon_0} = 9\times10^9\ Nm^2C^{-2}\right]$

(A) $(-63\hat{i}+27\hat{j})\times10^2$ (B) $(81\hat{i}-81\hat{j})\times10^2$ (C) $(63\hat{i}-27\hat{j})\times10^2$ (D) $(-81\hat{i}+81\hat{j})\times10^2$

10. Charge is distributed within a sphere of radius R with a volume charge density $\rho(r) = \frac{A}{r^2}e^{-2r/a}$, where A and a are constants. If Q is the total charge of this charge distribution, the radius R is :
[JEE-Main(online)-2019]

(A) $\frac{a}{2}\log\left(1-\frac{Q}{2\pi aA}\right)$ (B) $a\log\left(1-\frac{Q}{2\pi aA}\right)$

(C) $a\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$ (D) $\frac{a}{2}\log\left(\frac{1}{1-\frac{Q}{2\pi aA}}\right)$

11. Three charges +Q, q, + Q are placed respectively, at distance, 0, d/2 and d from the origin, on the x-axis. If the net force experienced by + Q, placed at x= 0, Ls zero, then value of q is :
[JEE-Main(online)-2019]

(A) +Q/2 (B) –Q/2 (C) –Q/4 (D) +Q/4

12. A charge Q is distributed over three concentric spherical shells of radii a, b, c (a < b < c) such that their surface charge densities are equal to one another. The total potential at a point at distance r from their common centre, where r < a, would be : **[JEE-Main(online)-2019]**

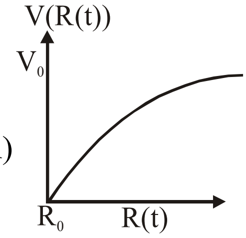
(A) $\frac{Q}{4\pi\varepsilon_0(a+b+c)}$ (B) $\frac{Q(a+b+c)}{4\pi\varepsilon_0(a^2+b^2+c^2)}$

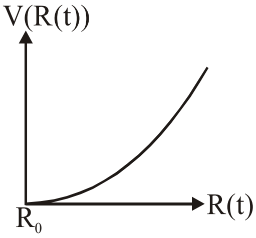
(C) $\frac{Q}{12\pi\varepsilon_0}\frac{ab+bc+ca}{abc}$ (D) $\frac{Q}{4\pi\varepsilon_0}\frac{(a^2+b^2+c^2)}{(a^3+b^3+c^3)}$

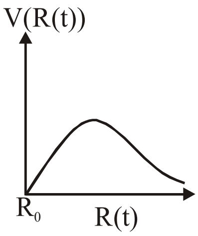
13. An electric field of 1000 V/m is applied to an electric dipole at angle of 45°. The value of electric dipole moment is 10^{-29} C.m. What is the potential energy of the electric dipole ?
[JEE-Main(online)-2019]

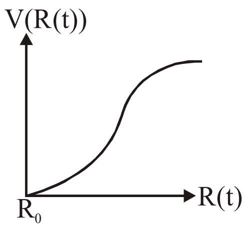
(A) -9×10^{-20} J (B) -7×10^{-27} J (C) -10×10^{-29} J (D) -20×10^{-18} J

14. There is a uniform spherically symmetric surface charge density at a distance R_0 from the origin. The charge distribution is initially at rest and starts expanding because of mutual repulsion. The figure that represents best the speed V(R(t)) of the distribution as a function of its instantaneous radius R (t) is : **[JEE-Main(online)-2019]**

(A)


(B)


(C)


(D)


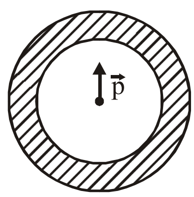
15. Four point charges –q, +q, +q and –q are placed on y-axis at y = –2d, y = –d, y = +d and y = +2d, respectively. The magnitude of the electric field E at a point on the x-axis at x = D, with D >> d, will behave as :- **[JEE-Main(online)-2019]**

(A) $E \propto \frac{1}{D}$ (B) $E \propto \frac{1}{D^3}$ (C) $E \propto \frac{1}{D^2}$ (D) $E \propto \frac{1}{D^4}$

16. Let a total charge 2Q be distributed in a sphere of radius R, with the charge density given by $\rho(r) = kr$, where r is the distance from the centre. Two charges A and B, of –Q each, are placed on diametrically opposite points, at equal distance, a, from the centre. If A and B do not experience any force, then : **[JEE-Main(online)-2019]**

(A) $a = \frac{3R}{2^{1/4}}$ (B) $a = R/\sqrt{3}$ (C) $a = 8^{-1/4}R$ (D) $a = 2^{-1/4}$ R

17. Shown in the figure is a shell made of a conductor. It has inner radius a and outer radius b, and carries charge Q. At its centre is a dipole $\vec{p}$ as shown. In this case : **[JEE-Main(online)-2019]**



(A) Electric field outside the shell is the same as that of a point charge at the centre of the shell.

(B) Surface charge density on the inner surface of the shell is zero everywhere.

(C) Surface charge density on the inner surface is uniform and equal to $\frac{(Q/2)}{4\pi a^2}$.

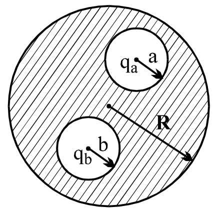
(D) Surface charge density on the outer surface depends on $|\vec{p}|$

18. A point dipole $\vec{p} = -p_0\hat{x}$ is kept at the origin. The potential and electric field due to this dipole on the y-axis at a distance d are, respectively: (Take V = 0 at infinity) : **[JEE-Main(online)-2019]**

(A) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$ (B) $0, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$ (C) $\frac{|\vec{p}|}{4\pi\varepsilon_0 d^2}, \frac{\vec{p}}{4\pi\varepsilon_0 d^3}$ (D) $0, \frac{-\vec{p}}{4\pi\varepsilon_0 d^3}$

Advanced Subjective (AS)

1. A conducting sphere of radius R has two spherical cavities of radius a and b. The cavities have charges q_a and q_b respectively at their centres. Find:



(a) The electric field and electric potential at a distance r

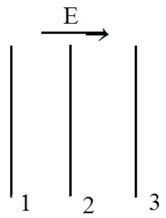
(i) r (distance from O, the centre of sphere > R)

(ii) r (distance from B, the centre of cavity b) < b

(b) Surface charge densities on the surface of radius R, radius a and radius b.

(c) What is the force on q_a and q_b?

2. Three uncharge conducing large plates are placed parallel to each other in a uniform electric field. Find the induced charge density on each surface of each plate.



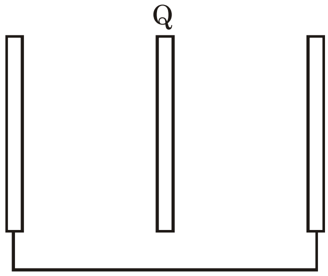
3. Four point charges $+8\mu C, -1\mu C, -1\mu C$ and $+8\mu C$, are fixed at the points, $-\sqrt{\frac{27}{2}}$ m, $-\sqrt{\frac{3}{2}}$ m, $+\sqrt{\frac{3}{2}}$ m and $+\sqrt{\frac{27}{2}}$ m respectively on the y-axis. A particle of mass 6×10^{-4} kg and of charge $+0.1\ \mu C$ moves along the $-x$ direction. Its speed at $x = +\infty$ is v_0. Find the least value of v_0 for which the particle will cross the origin . Find also the kinetic energy of the particle at the origin. Assume that space is gratity free. (Given : $1/(4\pi\varepsilon_0) = 9 \times 10^9$ Nm^2/C^2)

4. A non-conducting disc of radius a and uniform positive surface charge density σ is placed on the ground, with its axis vertical. A particle of mass m & positive charge q is dropped, along the axis of the disc, from a height H with zero initial velocity. The particle has $\frac{q}{m} = \frac{4\varepsilon_0 g}{\sigma}$.

(a) Find the value of H if the particle just reaches the disc .

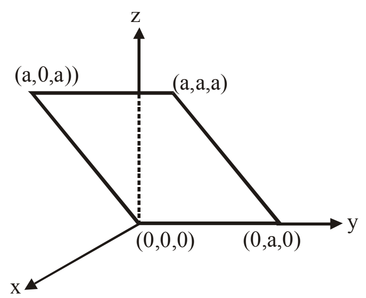
(b) Sketch the potential energy of the particle as a function of its height and find its equilibrium position.

5. Three large metallic plates are placed parallel to each other with separation between adjacent plates equal to d. Middle plate is given a charge Q and extreme plates are connected by a conducting wire as shown. If charge appearing on right surface of middle plate in steady state is $\frac{Q}{n}$, then find the value of n.



EXERCISE-(JA)

1. Consider an electric field $\vec{E} = E_0\hat{x}$, where E_0 is a constant. The flux through the shaded area (as shown in the figure) due to this field is **[IIT-JEE 2011]**



(A) $2E_0a^2$ (B) $\sqrt{2}E_0a^2$

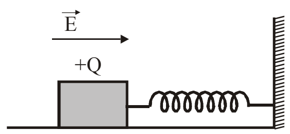
(C) E_0a^2 (D) $\dfrac{E_0a^2}{\sqrt{2}}$

2. A spherical metal shell A of radius R_A and a solid metal sphere B of radius $R_B(<R_A)$ are kept far apart and each is given charge $+Q$. Now they are connected by a thin metal wire. Then **[IIT-JEE 2011]**

(A) $E_A^{inside} = 0$ (B) $Q_A > Q_B$

(C) $\dfrac{\sigma_A}{\sigma_B} = \dfrac{R_B}{R_A}$ (D) $E_A^{on\ \text{surface}} < E_B^{on\ \text{surface}}$

3. A wooden block performs SHM on a frictionless surface with frequency, ν_0. The block carries a charge $+Q$ on its surface. If now a uniform electric field $\vec{E}$ is switched-on as shown, then the SHM of the block will be :- **[IIT-JEE 2011]**



(A) of the same frequency and with shifted mean position
(B) of the same frequency and with the same mean position
(C) of changed frequency and with shifted mean position
(D) of changed frequency and with the same mean position

4. Which of the following statement(s) is/are correct? **[IIT-JEE 2011]**

(A) If the electric field due to a point charge varies as $r^{-2.5}$ instead of r^{-2}, then the Gauss law will still be valid.

(B) The Gauss law can be used to calculate the field distribution around an electric dipole.

(C) If the electric field between two point charges is zero somewhere, then the sign of the two charges is the same.

(D) The work done by the external force in moving a unit positive charge from point A at potential V_A to point B at potential V_B is $(V_B - V_A)$

5. Four point charges, each of +q, are rigidly fixed at the four corners of a square planar soap film of side 'α'. The surface tension of the soap film is γ. The system of charges and planar film are in equilibrium, and $a = k\left[\frac{q^2}{\gamma}\right]^{1/N}$, where 'k' is a constant. Then N is **[IIT-JEE 2011]**

Paragraph for Question Nos. 6 and 7

A dense collection of equal number of electrons and positive ions is called neutral plasma. Certain solids containing fixed positive ions surrounded by free electrons can be treated as neutral plasma. Let 'N' be the number density of free electrons, each of mass 'm'. When the electrons are subjected to an electric field, they are displaced relatively away from the heavy positive ions. If the electric field becomes zero, the electrons begins to oscillate about the positive ions with a natural angular frequency 'ω_ρ', which is called the plasma frequency. To sustain the oscillations, a time varying electric field needs to be applied that has an angular frequency ω, where a part of the energy is absorbed and a part of it is reflected. As ω approaches ω_ρ, all the free electrons are set to resonance together and all the energy is reflected. This is the explanation of high reflectivity of metals. **[IIT-JEE 2011]**

6. Taking the electronic charge as 'e' and the permittivity as 'ε_0', use dimensional analysis to determine the correct expression for ω_ρ.

(A) $\sqrt{\frac{Ne}{m\varepsilon_0}}$ (B) $\sqrt{\frac{m\varepsilon_0}{Ne}}$

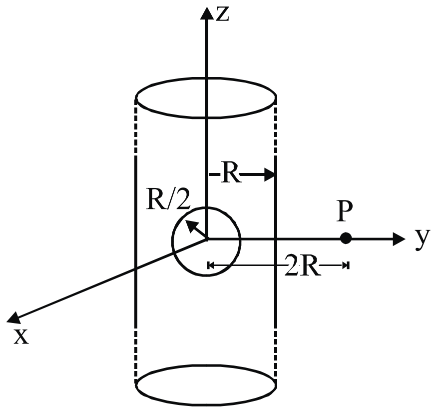
(C) $\sqrt{\frac{Ne^2}{m\varepsilon_0}}$ (D) $\sqrt{\frac{m\varepsilon_0}{Ne^2}}$

7. Estimate the wavelength at which plasma reflection will occur for a metal having the density of electrons $N \approx 4 \times 10^{27} m^{-3}$. Take $\varepsilon_0 \approx 10^{-11}$ and $m \approx 10^{-30}$, where these quantities are in proper SI units

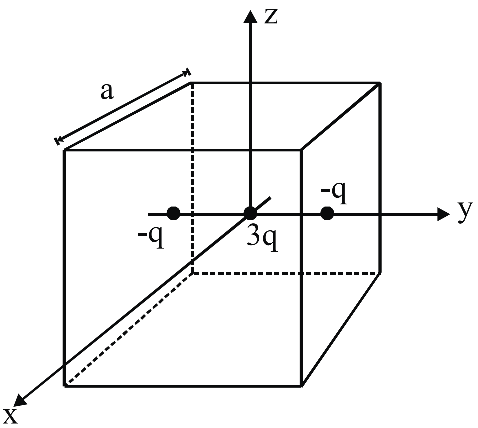
(A) 800 nm (B) 600 nm

(C) 300 nm (D) 200 nm

8. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is **[IIT-JEE 2012]**



9. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, –q at (0, –a/4, 0), +3q at (0,0,0) and –q at (0, +a/4, 0). Choose the correct option(s). **[IIT-JEE 2012]**



(A) The net electric flux crossing the plane x = +a/2 is equal to the net electric flux crossing the plane x = – a/2

(B) The net electric flux crossing the plane y = +a/2 is more than the net electric flux crossing the plane y = –a/2.

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

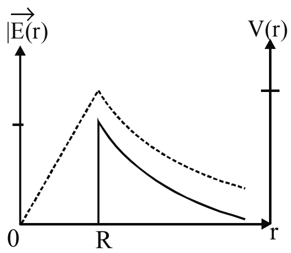
(D) The net electric flux crossing the plane z = +a/2 is equal to the net electric flux crossing the plane x = +a/2.

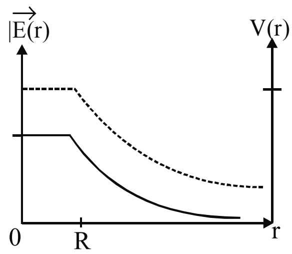
10. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly **[IIT-JEE 2012]**

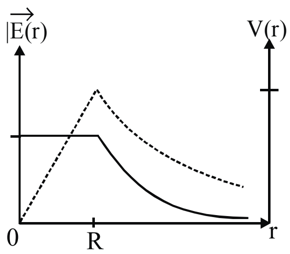
(A) 1×10^{-5} V (B) 1×10^{-7} V (C) 1×10^{-9} V (D) 1×10^{-10} V

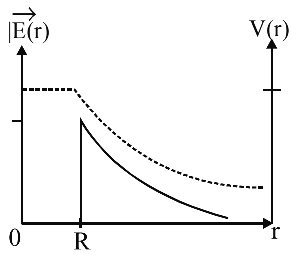
11. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

[IIT-JEE 2012]

(A)


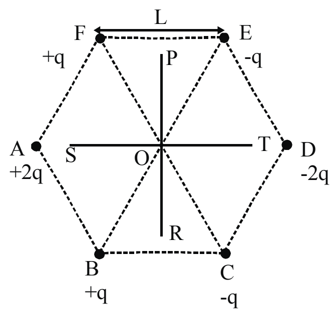
(B)


(C)


(D)


12. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in figure. Given that $K = \frac{1}{4\pi \epsilon_0} \frac{q}{L^2}$, which of the following statement(s) is(are) correct?

[IIT-JEE 2012]



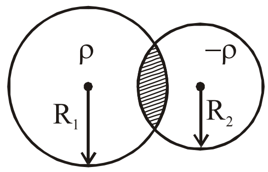
(A) The electric field at O is 6 K along OD.
(B) The potential at O is zero.
(C) The potential at all points on the line PR is same.
(D) The potential at all points on the line ST is same.

13. Two non-conducting solid spheres of radii R and 2R, having uniform volume charge densities ρ_1 and ρ_2 respectively, touch each other. The net electric field at a distance 2R from the centre of the smaller sphere, along the line joining the centres of the spheres, is zero. The ratio $\frac{\rho_1}{\rho_2}$ can be

[JEE-Advance-2013]

(A) –4 (B) $-\frac{32}{25}$ (C) $\frac{32}{25}$ (D) 4

14. Two non-conducting spheres of radii R_1 and R_2 and carrying uniform volume charge densities $+\rho$ and $-\rho$, respectively, are placed such that they partially overlap, as shown in the figure. At all points in the overlapping region :- **[JEE-Advance-2013]**



(A) the electrostatic field is zero
(B) the electrostatic potential is constant
(C) the electrostatic field is constant in magnitude
(D) the electrostatic field has same direction

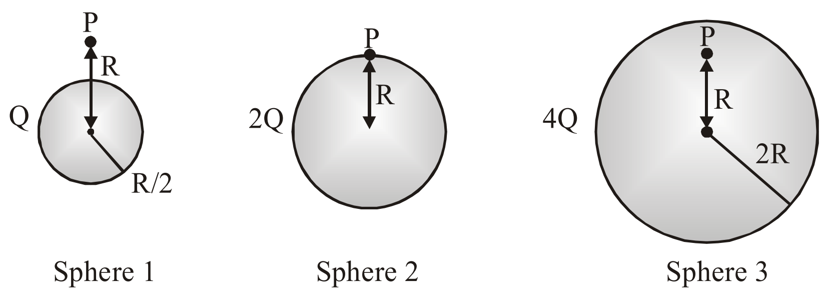
15. Let $E_1(r)$, $E_2(r)$ and $E_3(r)$ be the respective electric fields at a distance r from a point charge Q, an infinitely long wire with constant linear charge density λ, and an infinite plane with uniform surface charge density σ. If $E_1(r_0) = E_2(r_0) = E_3(r_0)$ at a given distance r_0, then :- **[JEE-Advance-2014]**

(A) $Q = 4\sigma\pi r_0^2$ (B) $r_0 = \frac{\lambda}{2\pi\sigma}$

(C) $E_1(r_0/2) = 2E_2(r_0/2)$ (D) $E_2(r_0/2) = 4E_3(r_0/2)$

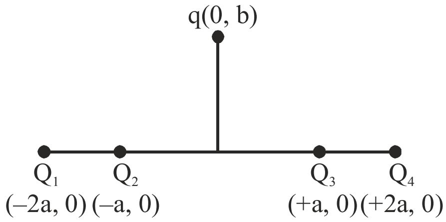
16. Charges Q, 2Q and 4Q are uniformly distributed in three dielectric solid spheres 1, 2 and 3 of radii R/2, R and 2R respectively, as shown in figure. If magnitudes of the electric fields at point P at a distance R from the centre of spheres 1, 2 and 3 are E_1, E_2 and E_2 respectively, then

[JEE-Advance-2014]



(A) $E_1 > E_2 > E_3$ (B) $E_3 > E_1 > E_2$ (C) $E_2 > E_1 > E_3$ (D) $E_3 > E_2 > E_1$

17. Four charges Q_1, Q_2, Q_3 and Q_4 of same magnitude are fixed along the x axis at x = –2a, –a, +a and +2a, respectively. A positive charge q is placed on the positive y axis at a distance b > 0. Four options of the signs of these charges are given in List I. The direction of the forces on the charge q is given in List II. Match List I with List II and select the correct answer using the code given below the lists. **[JEE-Advance-2014]**

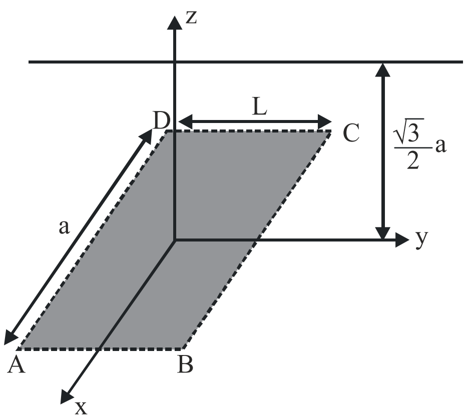


	List-I		**List-II**
(P)	Q_1, Q_2, Q_3, Q_4 all positive	(1)	+x
(Q)	Q_1, Q_2 positive ; Q_3, Q_4 negative	(2)	–x
(R)	Q_1, Q_4 positive ; Q_2, Q_3 negative	(3)	+y
(S)	Q_1, Q_3 positive ; Q_2, Q_4 negative	(4)	–y

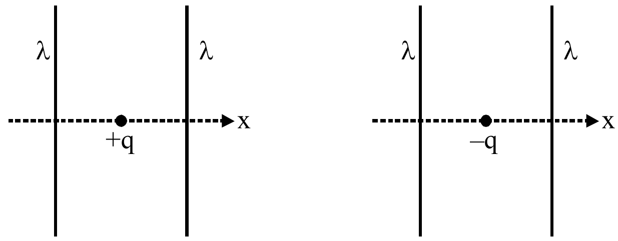
Code :

(A) P-3, Q-1, R-4, S-2
(B) P-4, Q-2, R-3, S-1
(C) P-3, Q-1, R-2, S-4
(D) P-4, Q-2, R-1, S-3

18. An infinitely long uniform line charge distribution of charge per unit length λ lies parallel to the y-axis in the y-z plane at $z = \frac{\sqrt{3}}{2}a$ (see figure). If the magnitude of the flux of the electric field through the rectangular surface ABCD lying in the x-y plane with its centre at the origin is $\frac{\lambda L}{n\varepsilon_0}$ (ε_0 = perimittivity of free space) then the value of n is. **[JEE-Advance-2015]**



19. The figures below depict two situations in which two infinitely long static line charges of constant positive line charge density λ are kept parallel to each other. In their resulting electric field, point charges q and –q are kept in equilibrium between them .The point charges are confined to move in the x direction only. If they are given a small displacement about their equlibrium positions, then the correct statement(s) is (are) : **[JEE-Advance-2015]**



(A) Both charges execute simple harmonic motion

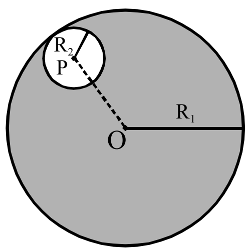
(B) Both charges will continue moving in the direction of their displacement

(C) Charge +q executes simple harmonic motion while charge –q continues moving in the direction of its displacement.

(D) Charge –q executes simple harmonic motion while charge +q continues moving in the direction of its displacement.

20. Consider a uniform spherical distirbution of radius R_1 centred at the origin O. In this distibution, a spherical cavity of radius R_2, centred at P with distance OP = a = $R_1 - R_2$ (see figure) is made. If the electric field inside the cavity at position $\vec{r}$ is $\vec{E}(\vec{r})$, then the correct statement(s) is(are) :

[JEE-Advance-2015]



(A) $\vec{E}$ is uniform, its magnitude is independent of R_2 but its direction depends on $\vec{r}$

(B) $\vec{E}$ is uniform, its magnitude depends on R_2 and its direction depends on $\vec{r}$

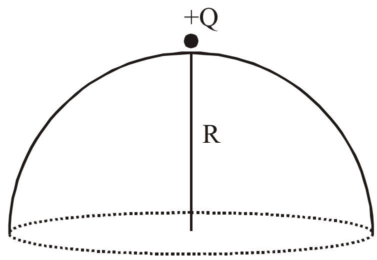
(C) $\vec{E}$ is uniform, its magnitude is independent of a but its direction depends on $\vec{a}$

(D) $\vec{E}$ is uniform and both its magnitude and direction depend on $\vec{a}$

21. A length-scale (ℓ) depends on the permittivity (ε) of a dielectric material, Boltzmann constant k_B, the absolute temperature T, the number per unit volume (n) of certain charged particles, and the charge (q) carried by each of the particles, Which of the following expressions(s) for ℓ is(are) dimensionally correct? **[JEE-Advance-2016]**

(A) $\ell = \sqrt{\left(\dfrac{nq^2}{\varepsilon k_B T}\right)}$ (B) $\ell = \sqrt{\left(\dfrac{\varepsilon k_B T}{nq^2}\right)}$ (C) $\ell = \sqrt{\left(\dfrac{q^2}{\varepsilon n^{2/3} k_B T}\right)}$ (D) $\ell = \sqrt{\left(\dfrac{q^2}{\varepsilon n^{1/3} k_B T}\right)}$

22. A point charge +Q is placed just outside an imaginary hemispherical surface of radius R as shown in the figure. Which of the following statements is/are correct ? **[JEE-Advance-2017]**



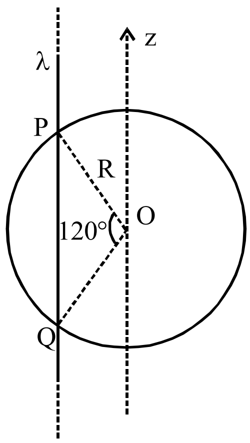
(A) The circumference of the flat surface is an equipotential

(B) The electric flux passing through the curved surface of the hemisphere is $-\frac{Q}{2\varepsilon_0}\left(1-\frac{1}{\sqrt{2}}\right)$

(C) Total flux through the curved and the flat surfaces is $\frac{Q}{\varepsilon_0}$

(D) The component of the electric field normal to the flat surface is constant over the surface.

23. An infinitely long thin non-conducting wire is parallel to the z-axis and carries a uniform line charge density λ. It pierces a thin non-conducting spherical shell of radius R in such a way that the arc PQ subtends an angle 120° at the centre O of the spherical shell, as shown in the figure. The permittivity of free space is ε_0. Which of the following statements is (are) true ? **[JEE-Advance-2018]**



(A) The electric flux through the shell is $\sqrt{3}\,R\lambda/\varepsilon_0$

(B) The z-component of the electric field is zero at all the points on the surface of the shell

(C) The electric flux through the shell is $\sqrt{2}\,R\lambda/\varepsilon_0$

(D) The electric field is normal to the surface of the shell at all points

24. A particle, of mass 10^{-3} kg and charge 1.0 C, is initially at rest. At time t = 0, the particle comes under the influence of an electric field $\vec{E}(t) = E_0 \sin\omega t\,\hat{i}$ where E_0 = 1.0 N C^{-1} and $\omega=10^3$ rad s^{-1}. Consider the effect of only the electrical force on the particle. Then the maximum speed, in ms^{-1}, attained by the particle at subsequent times is.......................... **[JEE-Advance-2018]**

25. The electric field E is measured at a point P(0,0,d) generated due to various charge distributions and the dependence of E on d is found to be different for different charge distributions. List-I contains different relations between E and d. List-II describes different electric charge distributions, along with their locations. Match the functions in List-I with the related charge distributions in List-II.

[JEE-Advance-2018]

List–I		**List–II**	
P.	E is indpendent of d	**1.**	A point charge Q at the origin
Q.	$E \propto \frac{1}{d}$	2.	A small dipole with point charges Q at $(0,0,\ell)$ and $-Q$ at $(0,0,-\ell)$. Take $2\ell << d$
R.	$E \propto \frac{1}{d^2}$	3.	An infinite line charge coincident with the x-axis, with uniform linear charge density λ.
S.	$E \propto \frac{1}{d^3}$	4.	Two infinite wires carrying uniform linear Charge density parallel to the x - axis. The one along $(y = 0, z = \ell)$ has a charge density $+\lambda$ and the one along $(y = 0, z = -\ell)$ has a charge density $-\lambda$. Take $2\ell << d$
		5.	Infinite plane charge coincident with the xy-plane with uniform surface charge density

(A) $P \to 5$; $Q \to 3, 4$; $R \to 1$; $S \to 2$
(B) $P \to 5$; $Q \to 3$, ; $R \to 1,4$; $S \to 2$
(C) $P \to 5$; $Q \to 3$, ; $R \to 1,2$; $S \to 4$
(D) $P \to 4$; $Q \to 2, 3$; $R \to 1$; $S \to 5$

26. A thin spherical insulating shell of radius R carries a uniformly distributed charge such that the potential at its surface is V_0. A hole with a small area $\alpha 4\pi R^2$ ($\alpha<<1$) is made on the shell without affecting the rest of the shell. Which one of the following statements is correct ? **[JEE-Advance-2019]**

(1) The ratio of the potential at the center of the shell to that of the point at $\frac{1}{2}$R from center towards the hole will be $\frac{1-\alpha}{1-2\alpha}$

(2) The magnitude of electric field at the center of the shell is reduced by $\frac{\alpha V_0}{2R}$

(3) The magnitude of electric field at a point, located on a line passing through the hole and shell's center on a distance 2R from the center of the spherical shell will be reduced by $\frac{\alpha V_0}{2R}$

(4) The potential at the center of the shell is reduced by $2\alpha V_0$

27. A charged shell of radius R carries a total charge Q. Given Φ as the flux of electric field through a closed cylindrical surface of height h, radius r and with its center same as that of the shell. Here, center of the cylinder is a point on the axis of the cylinder which is equidistant from its top and bottom surfaces. Which of the following option(s) is/are correct ? [$\in_0$ is the permittivity of free space]

[JEE-Advance-2019]

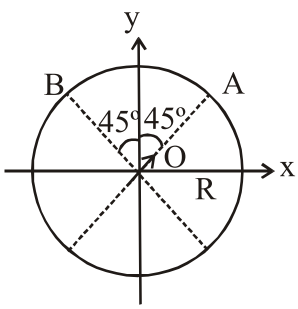
(1) If h > 2R and r > R then $\Phi = \frac{Q}{\in_0}$

(2) If $h < \frac{8R}{5}$ and $r = \frac{3R}{5}$ then $\Phi = 0$

(3) If h > 2R and $r = \frac{4R}{5}$ then $\Phi = \frac{Q}{5\in_0}$

(4) If h > 2R and $r = \frac{3R}{5}$ then $\Phi = \frac{Q}{5\in_0}$

28. An electric dipole with dipole moment $\frac{p_0}{\sqrt{2}}(\hat{i}+\hat{j})$ is held fixed at the origin O in the presence of an uniform electric field of magnitude E_0. If the potential is constant on a circle of radius R centered at the origin as shown in figure, then the correct statement(s) is/are:
(ε_0 is permittivity of free space, R >> dipole size) **[JEE-Advance-2019]**



(1) $R = \left(\frac{p_0}{4\pi\varepsilon_0 E_0}\right)^{1/3}$

(2) The magnitude of total electric field on any two points of the circle will be same

(3) Total electric field at point A is $\vec{E}_A = \sqrt{2}E_0(\hat{i}+\hat{j})$

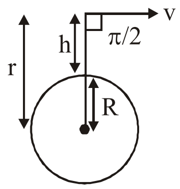
(4) Total electric field at point B is $\vec{E}_B = 0$

GRAVITATION

THEORY

Conditions for different trajectroy

For a body being projected tangentially from above earth's surface, say at a distance r from earth's center, the trajectory would depend on the velocity of projection v.



Velocity	Orbit
1. velocity, $v < \sqrt{\frac{GM}{r}\left(\frac{2R}{r+R}\right)}$	Body returns to earth
2. $\sqrt{\frac{GM}{r}} > v > \sqrt{\frac{GM}{r}\left(\frac{2R}{r+R}\right)}$	Body acquires an elliptical orbit with earth as the far-focus w.r.t. the point of projection.
3. Velocity is equal to the critical velocity of the orbit, $v = \sqrt{\frac{GM_e}{r}}$	Circular orbit with radius r
4. Velocity is between the critical and escape velocity of the orbit $\sqrt{\frac{2GM_e}{r}} > v > \sqrt{\frac{GM_e}{r}}$	Body acquires an elliptical orbit with earth as the near focus w.r.t. the point of projection.
5. $v = v_{esc} = \sqrt{\frac{2GM_e}{r}}$	Body just escapes earth's gravity, along a parabolic path.
6. $v > v_{esc} = \sqrt{\frac{2GM_e}{r}}$	Body escape earth's gravity along a hyperbolic path.

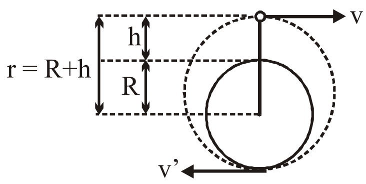
Gravitaional pressure :

A uniform sphere has a mass M and radius R. The pressure p inside the sphere, caused by gravitational compression, as a function of the distance r from its centre can be found to be $p = \frac{3}{8}(1 - r^2/R^2)\gamma M^2/\pi R^4$.

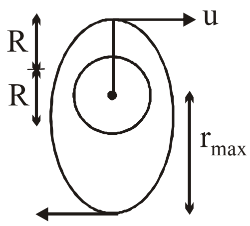
Launching of an artifical satellite around earth

Ex. A satellite is launched tangentially from a height h above earth's surface as shown.

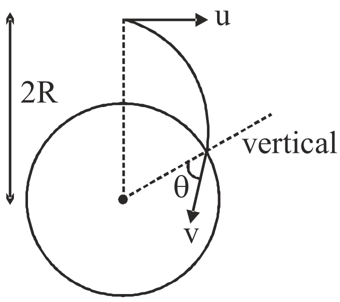
I. Find minimum launch speed 'v' so that it just touches the earth's surface



II. If h = R and satellite is launched tangentially with speed = $\sqrt{\frac{3GM}{5R}}$ find the mximum distance of satellite from earth's center



(III) If h = R and satellite is launched tangentially with a speed $u = \sqrt{\frac{GM}{7R}}$. Find the angle w.r.t. vertical at which the satellite will crash on earth's surface.



Sol.

(I) Angular momentum conservation

$mv(R + h) = mv'R$

Energy concervation

$$\frac{-GMm}{R+h}+\frac{1}{2}mv^2=-\frac{GMm}{R}+\frac{1}{2}mv'^2$$

Solving , $v=\sqrt{\frac{2GMR}{r(R+r)}}$

(II) Angular momentum conservation $mu\cdot 2R = mvr_{max}$.

Energy conservation :

$$\frac{-GMm}{2R}+\frac{1}{2}mu^2=\frac{-GMm}{r_{max}}+\frac{1}{2}mv^2$$

$$\Rightarrow\ GMm\left(\frac{1}{2R}-\frac{1}{r_{max}}\right)=\frac{1}{2}mu^2\left(1-\left(\frac{2R}{r_{max}}\right)^2\right)$$

$$\Rightarrow\ GMm\left(\frac{1}{2R}-\frac{1}{r_{max}}\right)=\frac{3GM}{10R}\left(1-\frac{4R^2}{r_{max}^2}\right)$$

$$\Rightarrow\ 2r_{max}^2-10Rr_{max}+12R^2=0$$

$$\Rightarrow\ (r_{max}-2R)(r_{max}-3R)=0$$

$$\Rightarrow\ r_{max}=3R$$

(III) Energy conservation

$$\frac{1}{2}mu^2-\frac{GMm}{2R}=\frac{1}{2}mv^2-\frac{GMm}{R}$$

$$\Rightarrow\ \frac{1}{2}m(v^2-u^2)=\frac{GMm}{2R}$$

$$v^2=u^2+\frac{GM}{R}\ \Rightarrow\ v=\sqrt{\frac{8GM}{7R}}$$

Angular momentum conservation

$mu\cdot 2R = mvR\sin\theta$

$$\Rightarrow \sin\theta=\frac{2u}{v}$$

$$\Rightarrow \theta = 45^\circ$$

Advanced Objective (AO)

Single Correct Answer Type Question

1. A planet revolves about the sun in elliptical orbit. The arial velocity $\left(\frac{dA}{dt}\right)$ of the planet is 4.0×10^{16} m^2/s. The least distance between planet and the sun is 2×10^{12} m. Then the maximum speed of the planet in km/s is :

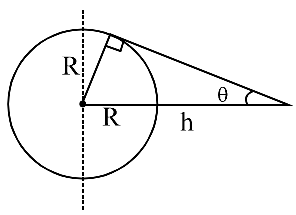
(A) 10 (B) 20 (C) 40 (D) None of these

2. The Sun travels in approximately circular orbit of radius R around the center of the galaxy and completes one revolution in time T. The Earth also revolves around the Sun in time t. Assume orbit of the Earth to be a circle of radius r (r << R) and whole mass of the galaxy centered on its center. By using only these given informations, find an expression for the ratio of the mass of the galaxy to that of the Sun.

(A) $\left(\frac{R}{r}\right)^3\left(\frac{t}{T}\right)^2$ (B) $\left(\frac{R}{r}\right)^3\left(\frac{T}{t}\right)^2$ (C) $\left(\frac{R}{r}\right)^2\left(\frac{t}{T}\right)^3$ (D) $\left(\frac{R}{r}\right)^2\left(\frac{T}{t}\right)^3$

Multiple Correct Answer Type Question

3. A geostationary satellite is at a height h above the surface of earth. If earth radius is R



(A) The minimum colatitude on earth upto which the satellite can be used for communication is $\sin^{-1}(R/R+h)$.

(B) The maximum latitudes on earth upto which the satellite can be used for communication is $\cos^{-1}(R/R+h)$.

(C) The area on earth escaped from this satellite is given as $2\pi R^2(1+\sin\theta)$

(D) The area on earth escaped from this satellite is given as $2\pi R^2(1+\cos\theta)$

4. Two satellites s_1 & s_2 of equal masses revolve in the same sense around a heavy planet in coplanar circular orbit of radii R & 4R

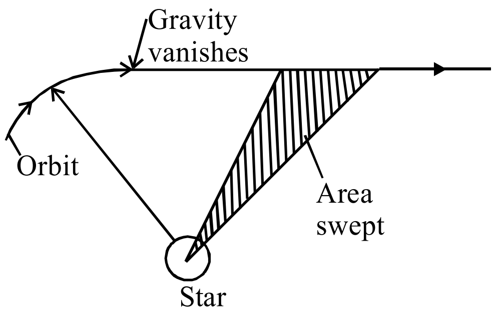
(A) the ratio of period of revolution of s_1 & s_2 is 1 : 8.

(B) their velocities are in the ratio 2 : 1

(C) their angular momentum about the planet are in the ratio 2 : 1

(D) the ratio of angular velocities of s_2 w.r.t. s_1 when all three are in the same line is 9 : 5.

5. A planet is orbiting a star when for no apparent reason the star's gravity suddenly vanishes. After which planet moves in a straight line. Mark the **CORRECT** statement(s) :

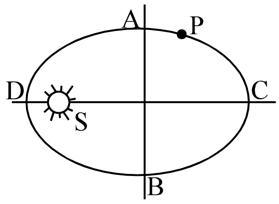


(A) Newton's first law is obeyed on planet after gravity vanishes
(B) Kepler's law of areas is obeyed only till the planet is in gravity of star
(C) Kepler's law of areas is obeyed even after gravity vanishes
(D) Angular momentum of planet about centre of star is conserved through out its motion

Linked Comprehension Type Question

Paragraph for Question No. 6 and 7

Figure shows the orbit of a planet P around the sun S. AB and CD are the minor and major axes of the ellipse.



6. If t_1 is the time taken by the planet to travel along ACB and t_2 the time along BDA, then
(A) $t_1 = t_2$ (B) $t_1 > t_2$
(C) $t_1 < t_2$ (D) nothing can be concluded

7. If U is the potential energy and K kinetic energy then |U| > |K| at
(A) Only D (B) Only C (C) both D & C (D) neither D nor C

Advanced Subjective (AS)

1. A satellite close to the earth is in orbit above the equator with a period of rotation of 1.5 hours. If it is above a point P on the equator at some time, it will be above P again after time________.

2. A hypothetical spherical planet of radius R and its density varies as $\rho = Kr$, where K is constant and r is the distance from the center. Determine the pressure caused by gravitational pull inside (r < R) the planet at a distance r measured from its center.

EXERCISE (JA)

1. A double star system consists of two stars A and B which have time period T_A and T_B. Radius R_A and R_B and mass M_A and M_B. Choose the correct option :– **[IIT-JEE 2006]**

(A) If $T_A > T_B$ then $R_A > R_B$

(B) If $T_A > T_B$ then $M_A > M_B$

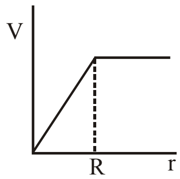
(C) $\left(\frac{T_A}{T_B}\right)^2 = \left(\frac{R_A}{R_B}\right)^3$

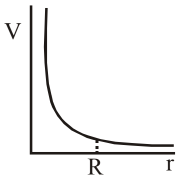
(D) $T_A = T_B$

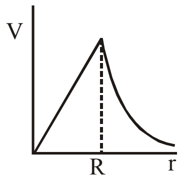
2. Column-I describes some situations in which a small object moves. Column-II describes some characteristics of these motions. Match the situations in column I with the characteristics in column-II. **[IIT-JEE 2007]**

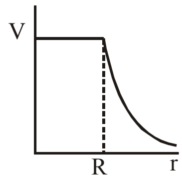
Column-I	**Column-II**
(A) The object moves on the x-axis under a conservative force in such a way that its "speed" and "position" satisfy $v = c_1\sqrt{c_2 - x^2}$, where c_1 and c_2 are positive constants.	(p) The object executes a simple harmonic motion.
(B) The object moves on the x-axis in such a way that its velocity and its displacement from the origin satisfy $v = -kx$, where k is a positive constant.	(q) The object does not change its direction.
(C) The object is attached to one end of a mass-less spring of a given spring constant. The other end of the spring is attached to the ceiling of an elevator. Initially everything is at rest. The elevator starts going upwards with a constant acceleration a. The motion of the object is observed from the elevator during the period it maintains this acceleration.	(r) The kinetic energy of the object keeps on decreasing
(D) The object is projected from the earth's surface vertically upwards with a speed $2\sqrt{\frac{GM_e}{R_e}}$, where M_e is the mass of the earth and R_e is the radius of the earth. Neglect forces from objects other than the earth.	(s) The object can change its direction only once.

3. A spherically symmetric gravitational system of particles has a mass density $\rho = \begin{cases} \rho_0 & \text{for } r \le R \\ 0 & \text{for } r > R \end{cases}$ where ρ_0 is a constant. A test mass can undergo circular motion under the influence of the gravitational field of particles. Its speed V as a function of distance r ($0 < r < \infty$) from the centre of the system is represented by :- **[IIT-JEE 2008]**

(A) 

(B) 

(C) 

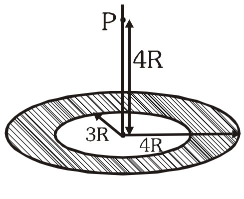
(D) 

4. **Statement – 1 :** An astronaut in an orbiting space station above the Earth experiences weightlessness.
and **[IIT-JEE 2008]**
Statement – 2 : An object moving around the Earth under the influence of Earth's gravitational force is in a state of 'free-fall'.
(A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
(B) Statement-1 is True, Statement-2 is True; Statement-2 is Not a correct explanation for Statement-1
(C) Statement-1 is True, Statement-2 is False
(D) Statement-1 is False, Statement-2 is True

5. Gravitational acceleration on the surface of a planet is $\frac{\sqrt{6}}{11}g$, where g is the gravitational acceleration on the surface of the earth. The average mass density of the planet is $\frac{2}{3}$ times that of the Earth. If the escape speed on the surface of the earth is taken to be 11 kms^{-1}, the escape speed on the surface of the planet in kms^{-1} will be **[IIT-JEE 2010]**

6. A binary star consists of two stars A (mass 2.2 M_S) and B (mass 11 M_S), where M_S is the mass of the sun. They are separated by distance d and are rotating about their centre of mass, which is stationary. The ratio of the total angular momentum of the binary star to the angular momentum of star B about the centre of mass is :- **[IIT-JEE 2010]**

7. A thin uniform annular disc (see figure) of mass M has outer radius 4R and inner radius 3R. The work required to take a unit mass from point P on its axis to infinity is **[IIT-JEE 2010]**



(A) $\frac{2GM}{7R}(4\sqrt{2}-5)$ (B) $-\frac{2GM}{7R}(4\sqrt{2}-5)$ (C) $\frac{GM}{4R}$ (D) $\frac{2GM}{5R}(\sqrt{2}-1)$

8. A satellite is moving with a constant speed V in a circular orbit about the earth. An object of mass 'm' is ejected from the satellite such that it just escapes from the gravitational pull of the earth. At the time of its ejection, the kinetic energy of the object is :- **[IIT-JEE 2011]**

(A) $\frac{1}{2}mV^2$ (B) mV^2 (C) $\frac{3}{2}mV^2$ (D) $2mV^2$

9. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then **[IIT-JEE 2012]**

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P/V_Q = 1/2$

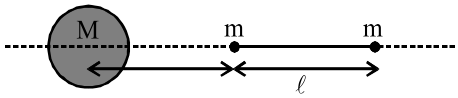
10. A planet of radius $R = \frac{1}{10} \times$ (radius of Earth) has the same mass density as Earth. Scientists dig a well of depth $\frac{R}{5}$ on it and lower a wire of the same length and of linear mass density 10^{-3} kgm^{-1} into it. If the wire is not touching anywhere, the force applied at the top of the wire by a person holding it in place is (take the radius of Earth = 6×10^6 m and the acceleration due to gravity on Earth is 10 ms^{-2}) **[JEE-Advance 2014]**

(A) 96 N (B) 108 N (C) 120 N (D) 150 N

11. A bullet is fired vertically upwards with velocity v from the surface of a spherical planet. When it reaches its maximum height, its acceleration due to the planet's gravity is 1/4th of its value at the surface of the planet. If the escape velocity from the planet is $v_{esc} = v\sqrt{N}$, then the value of N is (ignore energy loss due to atmosphere) **[JEE-Advance 2015]**

12. A large spherical mass M is fixed at one position and two identical point masses m are kept on a line passing through the centre of M (see figure). The point masses are connected by a rigid massless rod of length ℓ and this assembly is free to move along the line connecting them. All three masses interact only through their mutual gravitational interaction. When the point mass nearer to M is at a distance $r = 3\ell$ from M, the tension in the rod is zero for $m = k\left(\frac{M}{288}\right)$. The value of k is :

[JEE-Advance 2015]



13. A spherical body of radius R consists of a fluid of constant density and is in equilibrium under its own gravity. If P(r) is the pressure at r(r < R), then the correct option(s) is(are) :- **[JEE-Advance 2015]**

(A) $P(r = 0) = 0$ (B) $\frac{P(r = 3R/4)}{P(r = 2R/3)} = \frac{63}{80}$

(C) $\frac{P(r = 3R/5)}{P(r = 2R/5)} = \frac{16}{21}$ (D) $\frac{P(r = R/2)}{P(r = R/3)} = \frac{20}{27}$

14. A rocket is launched normal to the surface of the Earth, away from the Sun, along the line joining the sun and the Earth. The Sun is 3×10^5 times heavier than the Earth and is at a distance 2.5×10^4 times larger than the radius of the Earth. The escape velocity from Earth's gravitational field is $v_e = 11.2 \text{ km s}^{-1}$. The minimum initial velocity (v_s) required for the rocket to be able to leave the *Sun-Earth system* is closest to (*Ignore the rotation and revolution of the Earth and the presence of any other planet*) **[JEE-Advance 2017]**

(A) $v_s = 22 \text{ km s}^{-1}$ (B) $v_s = 72 \text{ km s}^{-1}$ (C) $v_s = 42 \text{ km s}^{-1}$ (D) $v_s = 62 \text{ km s}^{-1}$

15. A planet of mass M, has two natural satellites with masses m_1 and m_2. The radii of their circular orbits are R_1 and R_2 respectively. Ignore the gravitational force between the satellites. Define v_1, L_1, K_1 and T_1 to be, respectively, the orbital speed, angular momentum, kinetic energy and time period of revolution of satellite 1 ; and v_2, L_2, K_2 and T_2 to be the corresponding quantities of satellite 2. Given $m_1/m_2 = 2$ and $R_1/R_2 = 1/4$, match the ratios in List-I to the numbers in List-II. **[JEE-Advance 2018]**

List–I		List–II	
P.	$\frac{v_1}{v_2}$	1.	$\frac{1}{8}$
Q.	$\frac{L_1}{L_2}$	2.	1
R.	$\frac{K_1}{K_2}$	3.	2
S.	$\frac{T_1}{T_2}$	4.	8

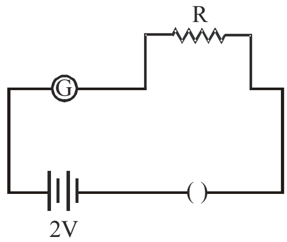
(A) P → 4 ; Q → 2 ; R → 1 ; S → 3
(B) P → 3 ; Q → 2 ; R → 4 ; S → 1
(C) P → 2 ; Q → 3 ; R → 1 ; S → 4
(D) P → 2 ; Q → 3 ; R → 4 ; S → 1

CURRENT ELECTRICITY

Advanced Objective (AO)

Single Correct Answer Type Question

1. A galvanometer has a 50 division scale. Battery has no internal resistance. It is found that there is deflection of 40 divisions when R = 2400 Ω. Deflection becomes 20 divisions when resistance taken from resistance box is 4900 Ω.Then we can conclude : **[JEE-Main (Online) 2016]**



(A) Resistance of galvanometer is 200 Ω.
(B) Resistance required on R.B.for a deflection of 10 divisions is 9800 Ω.
(C) Full scale deflection current is 2 mA.
(D) Current sensitivity of galvanometer is 20 μA/division.

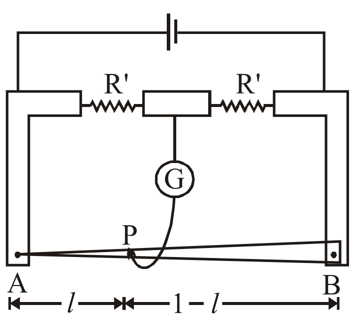
2. On interchanging the resistances, the balance point of a meter bridge shifts to the left by 10 cm. The resistance of their series combination is 1 kΩ. How much was the resistance on the left slot before interchanging the resistances ? **[JEE-Main-2018]**

(A) 505 kΩ (B) 550 kΩ (C) 910 kΩ (D) 990 kΩ

3. A current of 2 mA was passed through an unknown resistor which dissipated a power of 4.4 W. Dissipated power when an ideal power supply of 11V is connected across it is :
[JEE-Main(online)-2019]

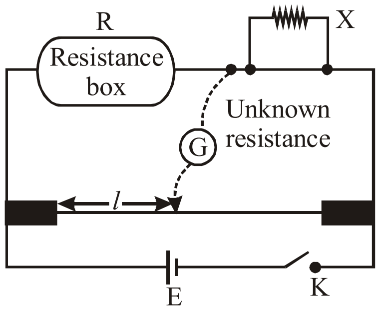
(A) 11×10^{-5} W (B) 11×10^{-4} W (C) 11×10^{5} W (D) 11×10^{-3} W

4. In a meter bridge, the wire of length 1 m has a non-uniform cross-section such that, the variation $\frac{dR}{d\ell}$ of its resistance R with length ℓ is $\frac{dR}{d\ell} \propto \frac{1}{\sqrt{\ell}}$. Two equal resistances are connected as shown in the figure. The galvanometer has zero deflection when the jockey is at point P. What is the length AP?
[JEE-Main(online)-2019]



(A) 0.25 m (B) 0.3m (C) 0.35 m (D) 0.2 m

5. In a meter bridge experiment, the circuit diagram and the corresponding observation table are shown in figure **[JEE-Main(online)-2019]**



SI. No.	R(Ω)	l(cm)
1.	1000	60
2.	100	13
3.	10	1.5
4.	1	1.0

Which of the readings is inconsistent?

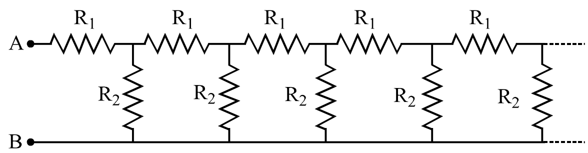
(A) 4 (B) 1 (C) 2 (D) 3

6. A moving coil galvanometer, having a resistance G, produces full scale deflection when a current I_g flows through it. This galvanometer can be converted into (i) an ammeter of range 0 to I_0 ($I_0 > I_g$) by connecting a shunt resistance R_A to it and (ii) into a voltmeter of range 0 to $V(V = GI_0)$ by connecting a series resistance R_V to it. Then, **[JEE-Main(online)-2019]**

(A) $R_A R_V = G^2\left(\frac{I_g}{I_0 - I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_0 - I_g}{I_g}\right)^2$ (B) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

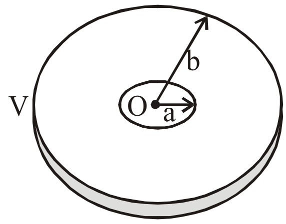
(C) $R_A R_V = G^2$ and $\frac{R_A}{R_V} = \frac{I_g}{(I_0 - I_g)}$ (D) $R_A R_V = G^2\left(\frac{I_0 - I_g}{I_g}\right)$ and $\frac{R_A}{R_V} = \left(\frac{I_g}{I_0 - I_g}\right)^2$

7. Consider an infinte ladder network shown in figure. A voltage V is applied between the points A and B. This applied value of voltage is halved after each section.



(A) $R_1/R_2 = 1$ (B) $R_1/R_2 = 1/2$ (C) $R_1/R_2 = 2$ (D) $R_1/R_2 = 3$

8. A circular portion is cut of a disc of thickness t, its resistivity is ρ and radii of disc are a and b (b > a). A potential difference is maintained between outer and inner cylindrical surfaces of the disc. What is resistance of the disc ?



(A) $\frac{\rho}{2\pi t}\ln\left(\frac{b}{a}\right)$ (B) $\rho\left(\frac{1}{a} - \frac{1}{b}\right)$ (C) $2\pi\rho t\left(\frac{1}{a^2} - \frac{1}{b^2}\right)$ (D) $\frac{\rho}{2\pi t}\left(\frac{b^2 - a^2}{ab}\right)$

9. The wire shown in figure has a uniform cross-section A.

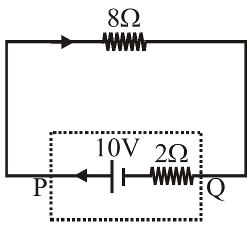
Resistivity of the material of wire is given by $\rho = \rho_0\left(\frac{L}{L+x}\right)$. A potential difference V is applied across the wire :-

(A) Resistance of wire is $\frac{\rho_0 L}{A}.\ln(2)$ (B) Current density is variable inside the wire

(C) Electric field at x = 0 is $\frac{2V}{(\ln 2).L}$ (D) Electric field at x = L is $\frac{V}{(\ln 2)L}$

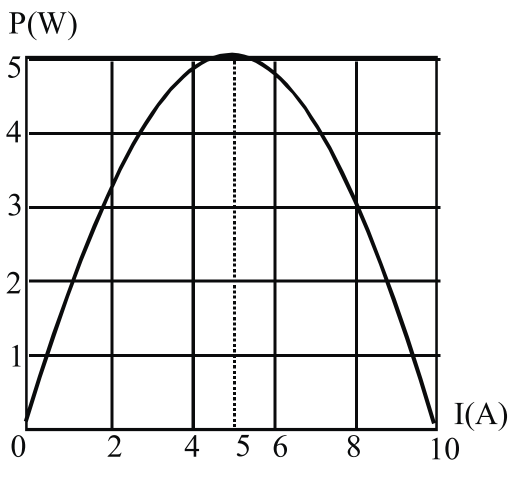
Multiple Correct Answer Type Question

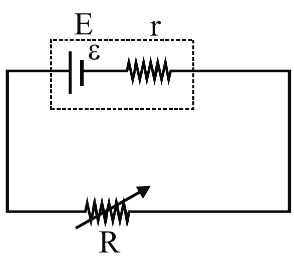
10. A battery of emf 10 volt and internal resistance 2Ω is connected to an external resistance 8Ω as shown in the figure :-



(A) Work done due to conservative electric field while a unit positive charge passes through battery from Q to P (along the arrow) is 8 Joule.
(B) Work done due to conservative electric field while a unit positive charge passes through battery from Q to P (along the arrow) is – 8 Joule.
(C) Work done due to conservative electric field while a unit positive charge passes through 8Ω along the arrow is –8 Joule.
(D) Work done due to non conservative electric field while a unit positive charge moves from Q to P (along the arrow) is 10 Joule.

11. Figure shows the net power dissipated in R versus the current in a simple circuit shown.





(A) The internal resistance of battery is 0.2Ω (B) The emf of battery is 2V
(C) R at which power is 5W is 2.5 Ω (D) At i = 2A, power is 3.2 W

12. In a potentiometer wire experiment the emf of a battery in the primary circuit is 20V and its internal resistance is 5Ω. There is a resistance box in series with the battery and the potentiometer wire, whose resistance can be varied from 120Ω to 170Ω. Resistance of the potentiometer wire is 75Ω. The following potential differences can be measured using this potentiometer.

(A) 5V (B) 6V (C) 7V (D) 8V

Advanced Subjective (AS)

1. A galvanometer (coil resistance 99Ω) is converted into a ammeter using a shunt of 1 Ω and connected as shown in the figure (i). The ammeter reads 3A. The same galvanometer is converted into a voltmeter by connected a resistance of 101Ω in series. This voltmeter is connected as shown in figure (ii). Its reading is found to be 4/5 of the full scale reading. Find

(i) internal resistance r of the cell

(ii) range of the ammeter and voltmeter

(iii) full scale deflection current of the galvanometer

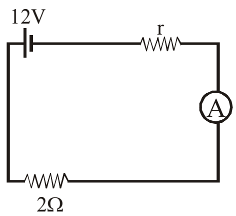


Fig. (i)

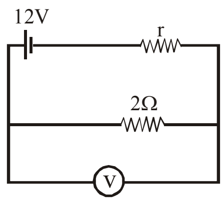
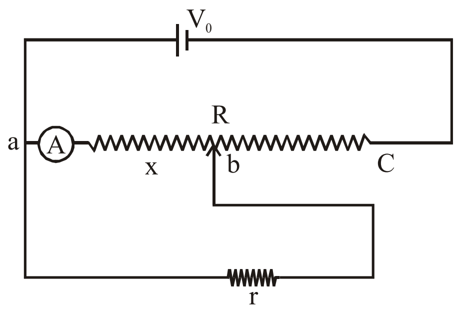


Fig. (ii)

2. An accumulator of emf 2 Volt and negligible internal resistance is connected across a uniform wire of length 10m and resistance 30Ω. The appropriate terminals of a cell of emf 1.5 Volt and internal resistance 1Ω is connected to one end of the wire, and the other terminal of the cell is connected through a sensitive galvanometer to a slider on the wire. What length of the wire will be required to produce zero deflection of the galvanometer ? How will the balancing change (a) when a coil of resistance 5Ω is placed in series with the accumulator, (b) the cell of 1.5 volt is shunted with 5Ω resistor?

3. A constant voltage V_0 (= 12V) is applied to a potential divider of resistance R (= 4Ω), connected to an ideal ammeter. A constant resistor r (= 2 Ω) is connected to the sliding contact of the potential divider (as shown). Find the minimum current (in A) measured by ammeter.



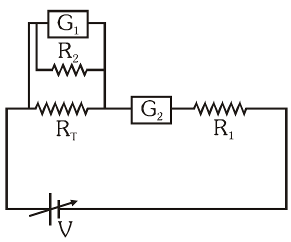
EXERCISE (JA)

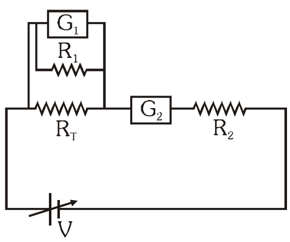
1. Incandescent bulbs are designed by keeping in mind that the resistance of their filament increases with the increase in temperature. If at room temperature, 100 W, 60 W and 40 W bulbs have filament resistance R_{100}, R_{60} and R_{40}, respectively, the relation between these resistances is **[IIT-JEE 2010]**

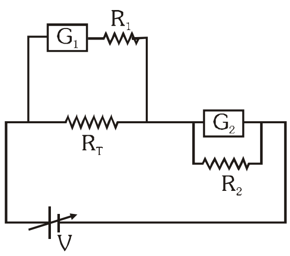
(A) $\frac{1}{R_{100}} = \frac{1}{R_{40}} + \frac{1}{R_{60}}$ (B) $R_{100} = R_{40} + R_{60}$

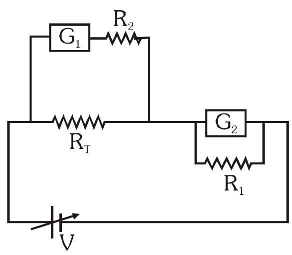
(C) $R_{100} > R_{60} > R_{40}$ (D) $\frac{1}{R_{100}} > \frac{1}{R_{60}} > \frac{1}{R_{40}}$

2. To verify Ohm's law, a student is provided with a test resistor R_T, a high resistance R_1, a small resistance R_2, two identical galvanometers G_1 and G_2, and a variable voltage source V. The correct circuit to carry out the experiment is :– **[IIT-JEE 2010]**

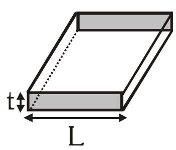
(A)


(B)


(C)


(D)


3. Consider a thin square sheet of side L and thickness t, made of a material of resistivity ρ. The resistance between two opposite faces, shown by the shaded areas in the figure is **[IIT-JEE 2010]**

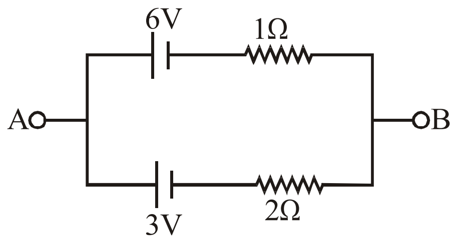


(A) directly proportional to L (B) directly proportional to t

(C) independent of L (D) independent of t

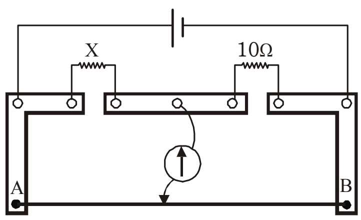
4. When two identical batteries of internal resistance 1Ω each are connected in series across a resistor R, the rate of heat produced in R is J_1. When the same batteries are connected in parallel across R, the rate is J_2. If $J_1 = 2.25\ J_2$ then the value of R in Ω is **[IIT-JEE 2010]**

5. Two batteries of different emfs and different internal resistances are connected as shown. The voltage across AB in volts is **[IIT-JEE 2011]**



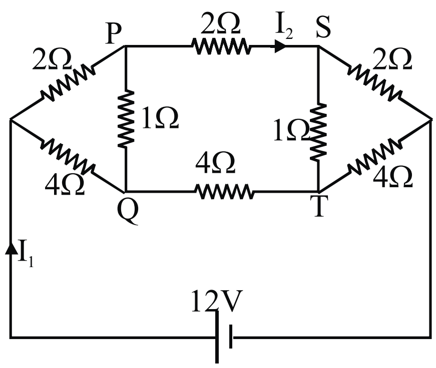
6. A meter bridge is set-up as shown, to determine an unknown resistance 'X' using a standard 10 ohm resistor. The galvanometer shows null point when tapping-key is at 52 cm mark. The end-corrections are 1 cm and 2 cm respectively for the ends A and B. The determined value of 'X' is

[IIT-JEE 2011]



(A) 10.2 ohm (B) 10.6 ohm (C) 10.8 ohm (D) 11.1 ohm

7. For the resistance network shown in the figure, choose the correct option(s). **[IIT-JEE 2012]**



(A) the current through PQ is zero (B) $I_1 = 3A$

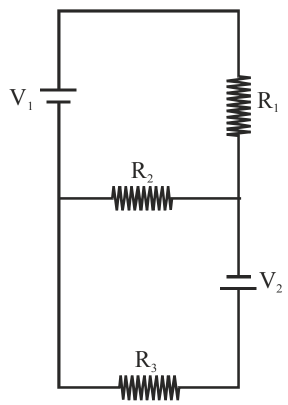
(C) The potential at S is less than that at Q (D) $I_2 = 2A$

8. Heater of an electric kettle is made of a wire of length L and diameter d. It takes 4 minutes to raise the temperature of 0.5 kg water by 40 K. This heater is replaced by a new heater having two wires of the same material, each of length L and diameter 2d. The way these wires are connected is given in the options. How much time in minutes will it take to raise the temperature of the same amount of water by 40 K ? **[JEE Advanced 2014]**

(A) 4 if wires are in parallel (B) 2 if wires are in series

(C) 1 if wires are in series (D) 0.5 if wires are in parallel

9. Two ideal batteries of emf V_1 and V_2 and three resistances R_1, R_2 and R_3 are connected as shown in the figure. The current in resistance R_2 would be zero if :- **[JEE Advanced 2014]**

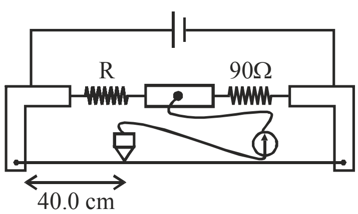


(A) $V_1 = V_2$ and $R_1 = R_2 = R_3$ (B) $V_1 = V_2$ and $R_1 = 2R_2 = R_3$

(C) $V_1 = 2V_2$ and $2R_1 = 2R_2 = R_3$ (D) $2V_1 = V_2$ and $2R_1 = R_2 = R_3$

10. A galvanometer gives full scale deflection with 0.006 A current. By connecting it to a 4990 Ω resistance, it can be converted into a voltmeter of range 0 - 30 V. If connected to a $\frac{2n}{249}\Omega$ resistance, it becomes an ammeter of range 0 - 1.5 A. The value of n is :- **[JEE Advanced 2014]**

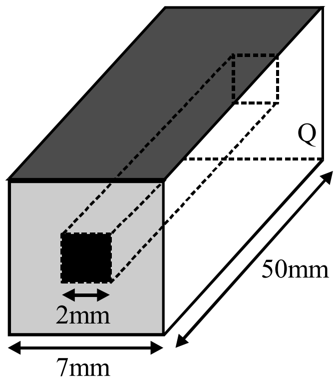
11. During an experiment with a metre bridge, the galvanometer shows a null point when the jockey is pressed at 40.0 cm using a standard resistance of 90Ω, as shown in the figure. The least count of the scale used in the metre bridge is 1mm. The unknown resistance is :- **[JEE Advanced 2014]**



(A) 60 ± 0.15 Ω (B) 135 ± 0.56 Ω (C) 60 ± 0.25 Ω (D) 135 ± 0.23 Ω

12. In an aluminum (Al) bar of square cross section, a square hole is drilled and is filled with iron (Fe) as shown in the figure. The electrical resistivities of Al and Fe are $2.7 \times 10^{-8}\ \Omega$ m and $1.0 \times 10^{-7}\Omega$m, respectively. The electrical resistance between the two faces P and Q of the composite bar is :

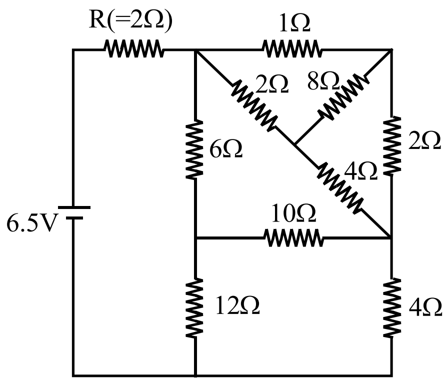
[JEE Advanced-2015]



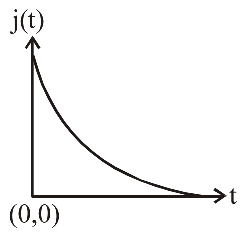
(A) $\frac{2475}{64}\mu\Omega$ (B) $\frac{1875}{64}\mu\Omega$ (C) $\frac{1875}{49}\mu\Omega$ (D) $\frac{2475}{132}\mu\Omega$

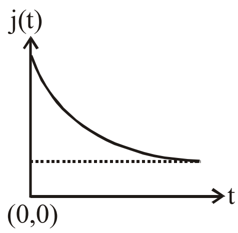
13. In the following circuit, the current through the resistor R (= 2Ω) is I Amperes. The value of I is

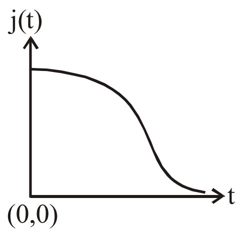
[JEE Advanced-2015]

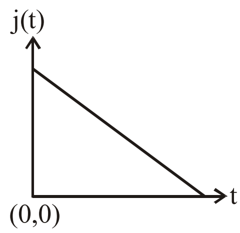


14. An infinite line charge of uniform electric charge density λ lies along the axis of an electrically conducting infinite cylindrical shell of radius R. At time t = 0, the space inside the cylinder is filled with a material of permittivity ε and electrical conductivity σ. The electrical conduction in the material follows Ohm's law. Which one of the following graphs best describes the subsequent variation of the magnitude of current density j(t) at any point in the material? **[JEE Advanced-2016]**

(A)


(B)


(C)


(D)


15. An incandescent bulb has a thin filament of tungsten that is heated to high temperature by passing an electric current. The hot filament emits black-body radiation. The filament is observed to break up at random locations after a sufficiently long time of operation due to non-uniform evaporation of tungsten from the filament. If the bulb is powered at constant voltage, which of the following statement(s) is(are) true? **[JEE Advanced-2016]**

(A) The temperature distribution over the filament is uniform

(B) The resistance over small sections of the filament decreases with time

(C) The filament emits more light at higher band of frequencies before it breaks up

(D) The filament consumes less electrical power towards the end of the life of the bulb

16. Consider two identical galvanometers and two identical resistors with resistance R. If the internal resistance of the galvanometers $R_C < R/2$, which of the following statement(s) about any one of the galvanometers is(are) true ? **[JEE Advanced-2016]**

(A) The maximum voltage range is obtained when all the components are connected in series

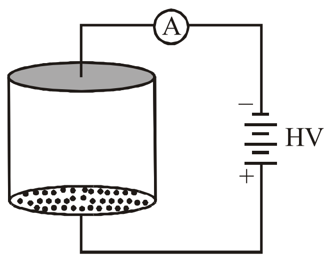
(B) The maximum voltage range is obtained when the two resistors and one galvanometer are connected in series, and the second galvanometer is connected in parallel to the first galvanometer

(C) The maximum current range is obtained when all the components are connected in parallel

(D) The maximum current range is obtained when the two galvanometers are connected in series and the combination is connected in parallel with both the resistors.

Paragraph for Questions No. 17 and 18

Consider an evacuated cylindrical chamber of height h having rigid conducting plates at the ends and an insulating curved surface as shown in the figure. A number of spherical balls made of a light weight and soft material and coated with a conducting material are placed on the bottom plate. The balls have a radius r <<h. Now a high voltage source (HV) is connected across the conducting plates such that the bottom plate is at $+V_0$ and the top plate at $-V_0$. Due to their conducting surface, the balls will get charged, will become equipotential with the plate and are repelled by it. The balls will eventually collide with the top plate, where the coefficient of restitution can be taken to be zero due to the soft nature of the material of the balls. The electric field in the chamber can be considered to be that of a parallel plate capacitor. Assume that there are no collision between the balls and the interaction between them is negligible. (Ignore gravity) **[JEE Advanced-2016]**



17. Which of the following statements is correct ?

(A) The balls will bounce back to the bottom plate carrying the opposite charge they went up with

(B) the balls will execute simple harmonic motion between the two plates

(C) The balls will bounce back to the bottom plate carrying the same charge they went up with

(D) The balls will stick to the top plate and remain there

18. The average current in the steady state registered by the ammeter in the circuit will be :

(A) Proportional to $V_0^{1/2}$

(B) Proportional to V_0^2

(C) Proportional to the potential V_0

(D) Zero

19. Two identical moving coil galvanometer have 10 Ω resistance and full scale deflection at 2 μA current. One of them is converted into a voltmeter of 100 mV full scale reading and the other into an Ammeter of 1 mA full scale current using appropriate resistors. These are then used to measure the voltage and current in the Ohm's law experiment with R = 1000 Ω resistor by using an ideal cell. Which of the following statement(s) is/are correct ? **[JEE Advanced-2019]**

(1) The measured value of R will be 978 Ω < R < 982 Ω.

(2) The resistance of the Voltmeter will be 100 kΩ.

(3) The resistance of the Ammeter will be 0.02 Ω (round off to 2nd decimal place)

(4) If the ideal cell is replaced by a cell having internal resistance of 5 Ω then the measured value of R will be more than 1000 Ω.

CAPACITANCE

Advanced Objective (AO)

Single Correct Answer Type Question

1. A capacitance of 2 μF is required in an electrical circuit across a potential difference of 1.0 kV. A large number of 1 μF capacitors are available which can withstand a potential difference of not more than 300 V.

The minimum number of capacitors required to achieve this is : **[JEE-Main-2017]**

(A) 24 (B) 32

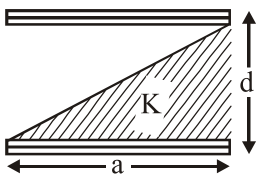
(C) 2 (D) 16

2. A parallel plate capacitor of capacitance 90 pF is connected to a battery of emf 20V. If a dielectric material of dielectric constant $K = \frac{5}{3}$ is inserted between the plates, the magnitude of the induced charge will be :- **[JEE-Main-2018]**

(A) 0.3 n C (B) 2.4 n C

(C) 0.9 n C (D) 1.2 n C

3. A parallel plate capacitor is made of two square plates of side 'a', separated by a distance d (d<<a). The lower triangular portion is filled with a dielectric of dielectric constant K, as shown in the figure. **[JEE-Main(online)-2019]**



Capacitance of this capacitor is : **[JEE-Main(online)-2019]**

(A) $\frac{1}{2}\frac{k \in_0 a^2}{d}$ (B) $\frac{k \in_0 a^2}{d}\ln K$

(C) $\frac{k \in_0 a^2}{d(K-1)}\ln K$ (D) $\frac{k \in_0 a^2}{2d(K+1)}$

4. A parallel plate capacitor has 1μF capacitance. One of its two plates is given +2μC charge and the other plate, +4μC charge. The potential difference developed across the capacitor is:- **[JEE-Main(online)-2019]**

(A) 5V (B) 2V

(C) 3V (D) 1V

5. Three long concentric conducting cylindrical shells have radii R, 2R and $2\sqrt{2}$ R. Inner and outer shells are connected to each other. The capacitance across middle and inner shells per unit length is:

(A) $\frac{\frac{1}{3}\in_0}{ln 2}$ (B) $\frac{6\pi \in_0}{ln 2}$

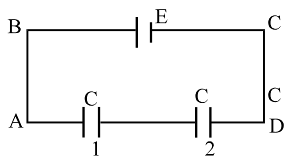
(C) $\frac{\pi \in_0}{2ln 2}$ (D) None

6. A conducting body 1 has some initial charge Q, and its capacitance is C. There are two other conducting bodies, 2 and 3, having capacitances : $C_2 = 2C$ and $C_3 \to \infty$. Bodies 2 and 3 are initially uncharged. "Body 2 is touched with body 1. Then, body 2 is removed from body 1 and touched with body 3, and then removed." This process is repeated N times. Then, the charge on body 1 at the end must be

(A) $Q/3^N$ (B) $Q/3^{N-1}$

(C) Q/N^3 (D) None

7. In the adjoining figure, capacitor (1) and (2) have a capacitance 'C' each. When the dielectric of dielectric consatnt K is inserted between the plates of one of the capacitor, the total charge flowing through battery is :-



(A) $\frac{KCE}{K+1}$ from B to C (B) $\frac{KCE}{K+1}$ from C to B

(C) $\frac{(K-1)CE}{2(K+1)}$ from B to C (D) $\frac{(K-1)CE}{2(K+1)}$ from C to B

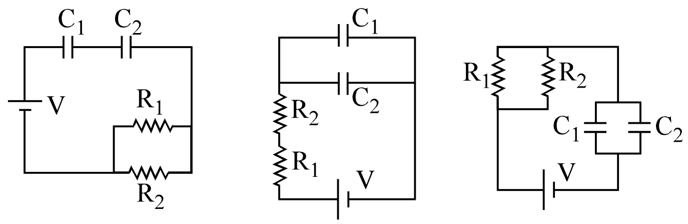
8. A parallel plate condenser with a dielectric of dielectric constant K between the plates has a capacity C and is charged to a potential V volts. The dielectric slab is slowly removed from between the plates and then re-inserted. The net work done by the system in this process is- **[AIEEE-2007]**

(A) $\frac{1}{2}(K-1)CV^2$ (B) $CV^2(K-1)/K$

(C) $(K-1)CV^2$ (D) zero

9. Given : $R_1 = 1\Omega$, $R_2 = 2\Omega$, $C_1 = 2\mu F$, $C_2 = 4\mu F$ **[IIT-JEE 2006]**

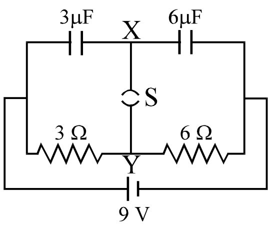
The time constants (in μS) for the circuits I, II, III are respectively



(I) (II) (III)

(A) 18, 8/9, 4
(B) 18, 4, 8/9
(C) 4, 8/9, 18
(D) 8/9, 18, 4

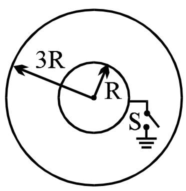
10. A circuit is connected as shown in the figure with the switch S open. When the switch is closed, the total amount of charge that flows from Y to X is :- **[IIT-JEE 2007]**



(A) 0
(B) 54 μC
(C) 27 μC
(D) 81 μC

Multiple Correct Answer Type Question

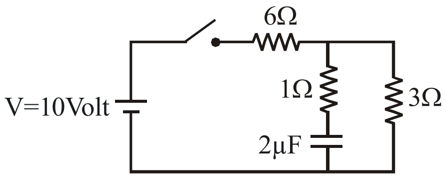
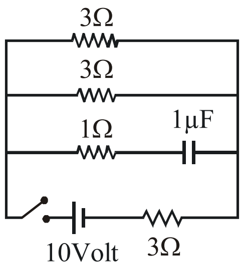
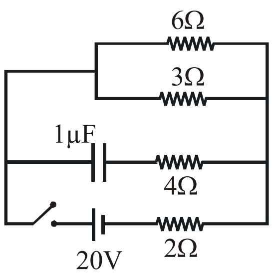
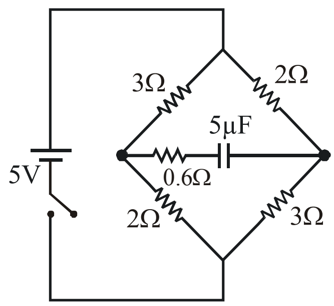
11. Two thin conducting shells of radii R and 3R are shown in the figure. The outer shell carries a charge +Q and the inner shell is neutral. The inner shell is earthed with the help of a switch S.



(A) With the switch S open, the potential of the inner sphere is equal to that of the outer.
(B) When the switch S is closed, the potential of the inner sphere becomes zero.
(C) With the switch S closed, the charge attained by the inner sphere is – Q/3.
(D) By closing the switch the capacitance of the system increases.

Matrix Match Type Question

12. Match the following. In each of the cases shown below, find the time constant of the circuit (in μs) after switch is closed.

List-I		List-II
(P)		(1) 6
(Q)		(2) 2
(R)		(3) 5
(S)		(4) 15

Codes :

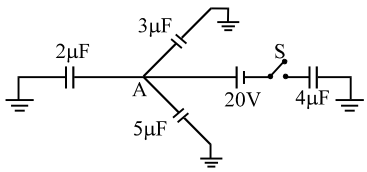
	P	**Q**	**R**	**S**
(A)	1	2	1	4
(B)	3	1	4	2
(C)	1	2	3	4
(D)	2	4	1	3

Advanced Subjective (AS)

1. Three capacitors of $2\mu F$, $3\mu F$ and $5\mu F$ are independently charged with batteries of emf's 5V, 20V and 10V respectively. After disconnecting from the voltage sources. These capacitors are connected as shown in figure with their positive polarity plates are connected to A and negative polarity is earthed. Now a battery of 20V and an uncharged capacitor of $4\mu F$ capacitance are connected to the junction A as shown with a switch S. When switch is closed, find :

(a) the potential of the junction A.

(b) final charges on all four capacitors.



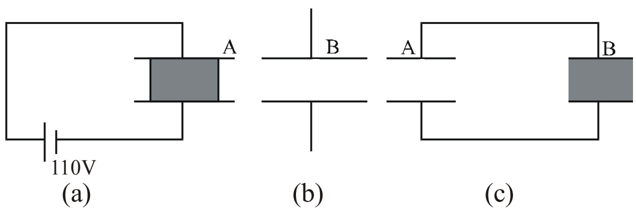
2. A potential difference of 300 V is applied between the plates of a parallel plate capacitor spaced 1 cm apart. A plane parallel glass plate with a thickness of 0.5 cm and a plane parallel paraffin plate with a thickness of 0.5 cm are placed in the space between the capacitor plates find :

(i) Intensity of electric field in each layer.

(ii) The drop of potential in each layer.

(iii) Surface charge density on the capacitor. Given that : $k_{glass} = 6$, $k_{paraffin} = 2$

3. Two parallel plate capacitors A & B have the same separation $d = 8.85 \times 10^{-4}$ m between the plates. The plate areas of A & B are $0.04\ m^2$ & $0.02\ m^2$ respectively. A slab of di-electric constant (relative permittivity) K = 9 has dimensions such that it can exactly fill the space between the plates of capacitor B.

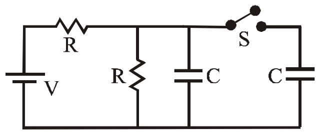


(i) The di-electric slab is placed inside A as shown in the figure (a) A is then charged to a potential difference of 110 volt. Calculate the capacitance of A and the energy stored in it.

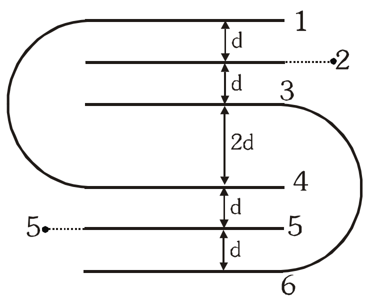
(ii) The battery is disconnected & then the di-electric slab is removed from A . Find the work done by the external agency in removing the slab from A .

(iii) The same di-electric slab is now placed inside B, filling it completely. The two capacitors A & B are then connected as shown in figure (c). Calculate the energy stored in the system.

4. In the figure shown initially switch is open for a long time. Now the switch is closed at t = 0. Find the charge on the rightmost capacitor as a function of time given that it was intially uncharged.

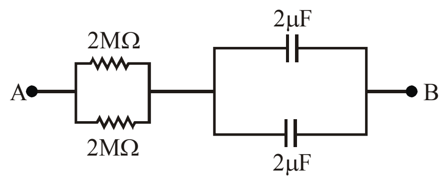


5. There are six plates of equal area A and separation between the plates is d (d<<A) are arranged as shown in figure. The equivalent capacitance between points 2 and 5, is $\alpha \frac{\epsilon_0 A}{d}$. Then find the value of α.

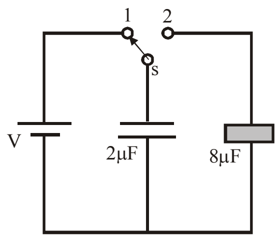


EXERCISE (JA)

1. At time t = 0, a battery of 10V is connected across points A and B in the given circuit. If the capacitors have no charge initially, at what time (in seconds) does the voltage across them become 4V?
[Take : $\ell n\ 5 = 1.6$, $\ell n 3 = 1.1$] **[IIT-JEE 2010]**

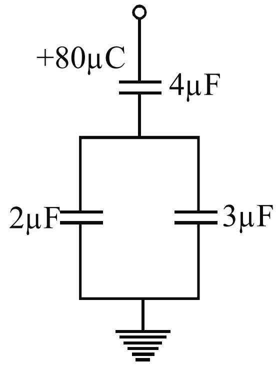


2. A 2 μF capacitor is charged as shown in figure. The percentage of its stored energy dissipated after the switch S is turned to position 2 is :- **[IIT-JEE 2011]**



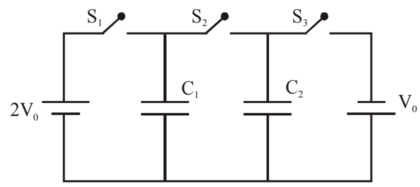
(A) 0% (B) 20% (C) 75% (D) 80%

3. In the given circuit, a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is :- **[IIT-JEE 2012]**



(A) + 32 μC (B) + 40μC (C) +48μC (D) + 80μC

4. In the circuit shown in the figure, there are two parallel plate capacitors each of the capacitance C. The switch S_1 is pressed first to fully charge the capacitor C_1 and then released. The switch S_2 is then pressed to charge the capacitor C_2. After some time, S_2 is released and then S_3 is pressed, After some time, **[JEE-Advance 2013]**



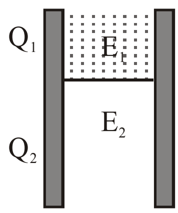
(A) the charge on the upper plate of C_1 is $2CV_0$

(B) the charge on the upper plate of C_1 is CV_0

(C) the charge on the upper plate of C_2 is 0.

(D) the charge on the upper plate of C_2 is $-CV_0$

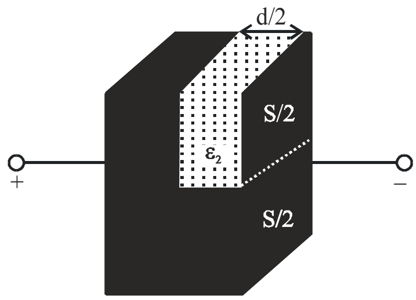
5. A parallel plate capacitor has a dielectric slab of dielectric constant K between its plates that covers 1/3 of the area of its plates, as shown in the figure. The total capacitance of the capacitor is C while that of the portion with dielectric in between is C_1. When the capacitor is charged, the plate area covered by the dielectric gets charge Q_1 and the rest of the area gets charge Q_2. The electric field in the dielectric is E_1 and that in the other portion is E_2. Choose the correct option/options, ignoring edge effects. **[JEE-Advance 2014]**



(A) $\frac{E_1}{E_2}=1$ (B) $\frac{E_1}{E_2}=\frac{1}{K}$ (C) $\frac{Q_1}{Q_2}=\frac{3}{K}$ (D) $\frac{C}{C_1}=\frac{2+K}{K}$

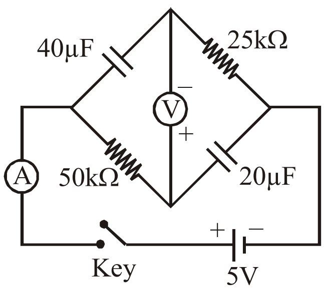
6. A parallel plate capacitor having plates of area S and plate separation d, has capacitance C_1 in air. When two dielectrics of different relative permittivities ($\varepsilon_1 = 2$ and $\varepsilon_2 = 4$) are introduced between the two plates as shown in the figure, the capacitance becomes C_2. The ratio $\frac{C_2}{C_1}$ is :-

[JEE-Advance 2015]



(A) $\frac{6}{5}$ (B) $\frac{5}{3}$ (C) $\frac{7}{5}$ (D) $\frac{7}{3}$

7. In the circuit shown below, the key is pressed at time t = 0. Which of the following statement(s) is(are) true? **[JEE-Advance 2016]**



(A) The voltmeter displays –5V as soon as the key is pressed, and displays +5V after a long time

(B) The voltmeter will display 0 V at time t = ln 2 seconds

(C) The current in the ammeter becomes 1/e of the initial value after 1 second

(D) The current in the ammeter becomes zero after a long time

PARAGRAPH–1

Consider a simple RC circuit as shown in figure 1.

Process 1 : In the circuit the switch S is closed at t = 0 and the capacitor is fully charged to voltage V_0 (i.e., charging continues for time T >> RC). In the process some dissipation (E_D) occurs across the resistance R. The amount of energy finally stored in the fully charged capacitor is E_C.

Process 2 : In a different process the voltage is first set to $\frac{V_0}{3}$ and maintained for a charging time $T >> RC$. Then the voltage is raised to $\frac{2V_0}{3}$ without discharging the capacitor and again maintained for a time $T >> RC$. The process is repeated one more time by raising the voltage to V_0 and the capacitor is charged to the same final voltage V_0 as in Process 1.

These two processes are depicted in Figure 2. **[JEE-Advance 2017]**

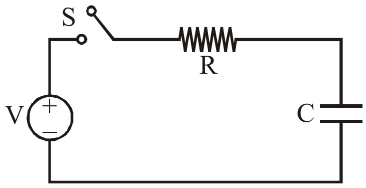


Figure 1

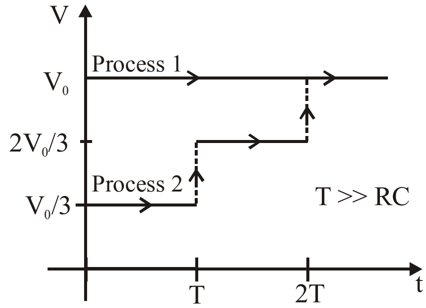


Figure 2

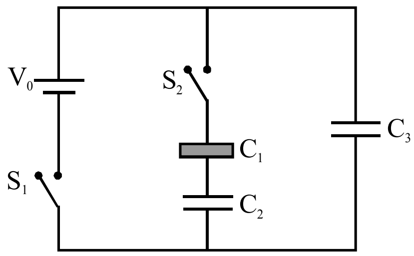
8. In Process 1, the energy stored in the capacitor E_C and heat dissipated across resistance E_D are related by :-

(A) $E_C = E_D$ (B) $E_C = 2E_D$ (C) $E_C = \frac{1}{2}E_D$ (D) $E_C = E_D \ln 2$

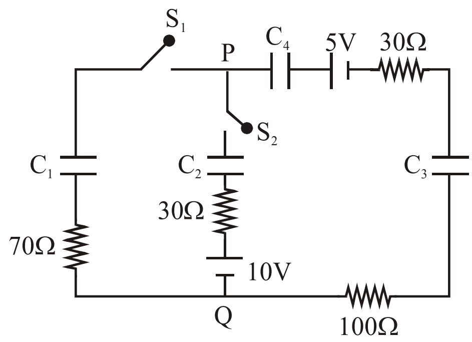
9. In Process 2, total energy dissipated across the resistance E_D is :-

(A) $E_D = \frac{1}{3}\left(\frac{1}{2}CV_0^2\right)$ (B) $E_D = 3\left(\frac{1}{2}CV_0^2\right)$ (C) $E_D = \frac{1}{2}CV_0^2$ (D) $E_D = 3CV_0^2$

10. Three identical capacitors C_1, C_2 and C_3 have a capacitance of 1.0 μF each and they are uncharged initially. They are connected in a circuit as shown in the figure and C_1 is then filled completely with a dielectric material of relative permittivity $\in_r$. The cell electromotive force (emf) $V_0 = 8V$. First the switch S_1 is closed while the switch S_2 is kept open. When the capacitor C_3 is fully charged, S_1 is opened and S_2 is closed simultaneously. When all the capacitors reach equilibrium, the charge on C_3 is found to be 5μC. The value of $\in_r$. **[JEE-Advance 2018]**



11. In the circuit shown, initially there is no charge on capacitors and keys S_1 and S_2 are open. The values of the capacitors are $C_1 = 10\ \mu F$, $C_2 = 30\ \mu F$ and $C_3 = C_4 = 80\ \mu F$. **[JEE-Advance 2019]**



Which of the statement(s) is/are correct ?

(1) The keys S_1 is kept closed for long time such that capacitors are fully charged. Now key S_2 is closed, at this time, the instantaneous current across 30 Ω resistor (between points P and Q) will be 0.2 A (round off to 1st decimal place).

(2) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage difference between points P and Q will be 10 V.

(3) At time t = 0, the key S_1 is closed, the instantaneous current in the closed circuit will be 25 mA.

(4) If key S_1 is kept closed for long time such that capacitors are fully charged, the voltage across the capacitors C_1 will be 4V.

12. A parallel plate capacitor of capacitance C has spacing d between two plates having area A. The region between the plates is filled with N dielectric layers, parallel to its plates, each with thickness $\delta = \frac{d}{N}$. The dielectric constant of the m^{th} layer is $K_m = K\left(1+\frac{m}{N}\right)$. For a very large N (> 10^3), the capacitance C is $\alpha\left(\frac{K \in_0 A}{d \ell n 2}\right)$. The value of α will be _____.

[$\in_0$ is the permittivity of free space] **[JEE-Advance 2019]**

MAGNETIC EFFECT OF CURRENT

Advanced Objective (AO)

Single Correct Answer Type Question

1. A ring like metallic conductor of resistance R and radius a, caries a constant current I. The ratio of the angular momentum L of the conduction electrons (about the axis of the ring) and the magnetic field B at the centre of the ring satisfy [where e and m represent the magnitudes of the electronic charge and mass]

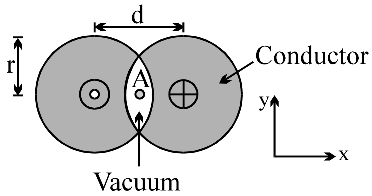
(A) $\frac{B}{L} \propto \frac{e^2}{m}$ (B) $\frac{B}{L} \propto e.m$ (C) $\frac{B}{L} \propto \frac{e}{m}$ (D) $\frac{B}{L} \propto \frac{m}{e}$

2. A long straight wire along the z-axis carries a current I in the negative z direction. The magnetic vector field $\vec{B}$ at a point having coordinates (x, y) in the z = 0 plane is :- **[JEE 2002 (screening)]**

(A) $\frac{\mu_0 I}{2\pi}\frac{(y\hat{i} - x\hat{j})}{(x^2 + y^2)}$ (B) $\frac{\mu_0 I}{2\pi}\frac{(x\hat{i} + y\hat{j})}{(x^2 + y^2)}$

(C) $\frac{\mu_0 I}{2\pi}\frac{(x\hat{j} - y\hat{i})}{(x^2 + y^2)}$ (D) $\frac{\mu_0 I}{2\pi}\frac{(x\hat{i} - y\hat{j})}{(x^2 + y^2)}$

3. Two long conductors are arranged as shown above to form overlapping cylinders, each of radius r, whose centers are separated by a distance d. Current of density J flows into the plane of the page along the shaded part of one conductor and an equal current flows out of the plane of the page along the shaded portion of the other, as shown. What are the magnitude and direction of the magnetic field at point A?



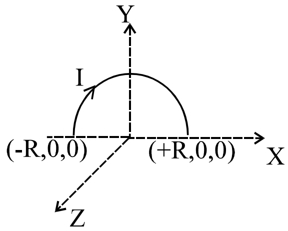
(A) $(\mu_0/2\pi)\pi dJ$, in the +y-direction (B) $(\mu_0/2\pi)d^2/r$, in the +y-direction
(C) $(\mu_0/2\pi)4d^2J/r$, in the –y-direction (D) $(\mu_0/2\pi)Jr^2/d$, in the –y-direction

4. A particle of specific charge (q/m) is projected from the origin of coordinates with initial velocity [u**i** – v**j**]. Uniform electric and magnetic fields exist in the region along the +y direction, of magnitude E and B. The particle will definitely return to the origin once if

(A) $[vB/2\pi E]$ is an integer (B) $(u^2 + v^2)^{1/2}\,[B/\pi E]$ is an integer

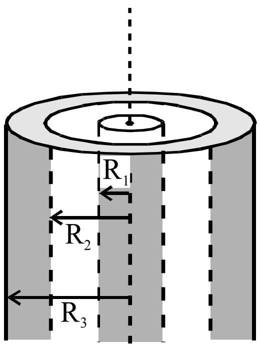
(C) $[vB/\pi E]$ in an integer (D) $[uB/\pi E]$ is an integer

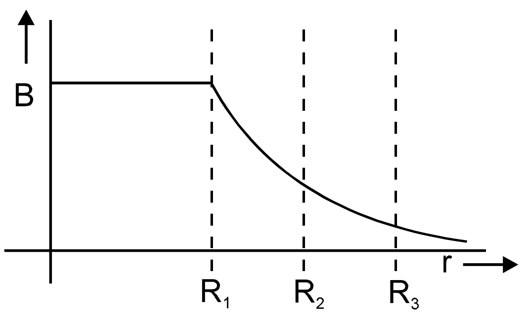
5. A semi circular current carrying wire having radius R is placed in x-y plane with its centre at origin 'O'. There is non-uniform magnetic field $\vec{B} = \frac{B_o x}{2R}\hat{k}$ (here B_o is +ve constant) is existing in the region. The magnetic force acting on semi circular wire will be along :-

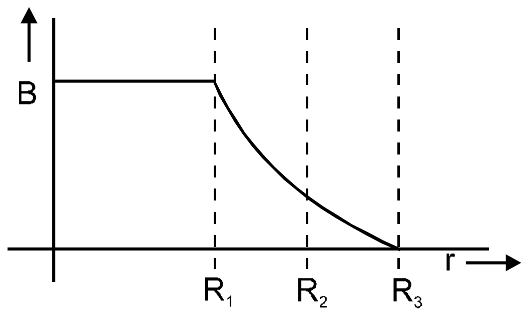


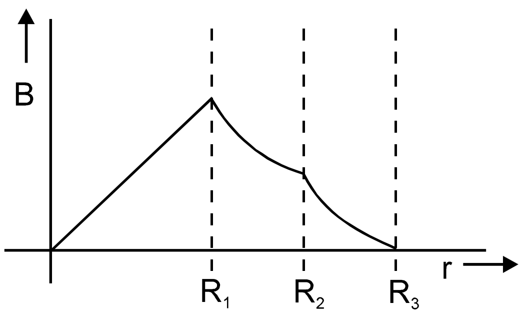
(A) – x-axis (B) + y-axis (C) – y-axis (D) + x-axis

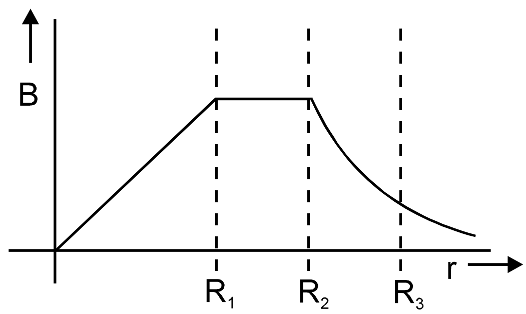
6. A coaxial cable is made up of two conductors. The inner conductor is solid and is of radius R_1 & the outer conductor is hollow of inner radius R_2 and outer radius R_3. The space between the conductors is filled with air. The inner and outer conductors are carrying currents of equal magnitudes and in opposite directions. Then the variation of magnetic field with distance from the axis is best plotted as:



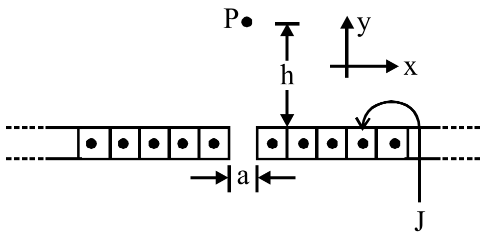
(A)


(B)


(C)


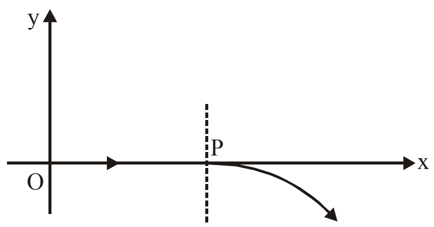
(D)


7. A small segment of length a is cut along z-axis from a infinite sheet having a surface current density J (current per unit width). The magnetic field at P is :-



(A) $2\mu_0 J\left(1-\frac{h}{a\pi}\right)\hat{i}$ (B) $\frac{\mu_0 Jh}{2a\pi}\hat{i}$ (C) $\frac{\mu_0 J}{2}\left(\frac{a}{h\pi}-1\right)\hat{i}$ (D) $-\frac{\mu_0 j}{2}\left(\frac{h}{a\pi}-1\right)\hat{i}$

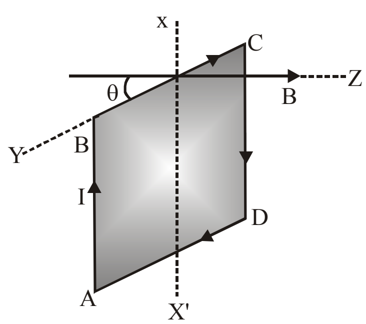
8. For a positively charged particle moving in a x – y plane initially along the x–axis, there is a sudden change in it path due to the presence of electric and/or magnetic field beyond P. The curved path is shown in the x – y plane and is found to be non–circular. Which one of the following combinations is possible? **[JEE 2004]**



(A) $\vec{E}=0;\ \vec{B}=b\hat{j}+c\hat{k}$ (B) $\vec{E}=a\hat{i};\ \vec{B}=c\hat{k}+a\hat{i}$

(C) $\vec{E}=0;\ \vec{B}=c\hat{j}+b\hat{k}$ (D) $\vec{E}=a\hat{i};\ \vec{B}=c\hat{k}+b\hat{j}$

9. The square loop ABCD, carrying a current I, is placed in a uniform magnetic field B, as shown. The loop can rotate about the axis XX'. The plane of the loop makes an angle θ (θ < 90°) with the direction of B. Through what angle will the loop rotate by itself before the torque on it becomes zero–



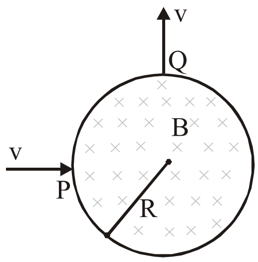
(A) θ (B) 90° – θ (C) 90° + θ (D) 180° – θ

10. A moving coil galvanometer has 150 equal divisions. Its current sensitivity is 10 divisions per milli ampere and voltage sensitivity is 2 divisions per millivolt. In order that each division reads 1 V, the resistance in Ohm's needed to be connected in series with the coil will be- **[AIEEE-2005]**

(A) 10^3 (B) 10^5 (C) 99995 (D) 9995

Multiple Correct Answer Type Question

11. A particle of charge 'q' and mass 'm' enters normally (at point P) in a region of magnetic field with speed v. It comes out normally from Q after time T as shown in figure. The magnetic field B is present only in the region of radius R and is uniform. Initial and final velocities are along radial direction and they are perpendicular to each other. For this to happen, which of the following expression(s) is/are correct :-



(A) $B = \dfrac{mv}{qR}$ (B) $T = \dfrac{\pi R}{2v}$ (C) $T = \dfrac{\pi m}{2qB}$ (D) None of these

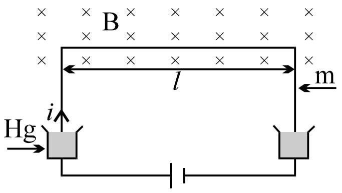
Matrix Match Type Question

12. A charged particle with some initial velocity is projected in a region where uniform electric and/or magnetic fields are present. In Column-I information about the existence of electric and/or magnetic field and direction of initial velocity of charged particle are given, while in column-II the possible paths of charged particle is mentioned. Match the entries of Column I with the entries of Column-II.

Column–I	**Column–II**
(A) $\vec{E} = 0, \vec{B} \neq 0$ and initial velocity is at an unknown angle with $\vec{B}$	(P) Straight line
(B) $\vec{E} \neq 0, \vec{B} = 0$ and initial velocity is at an unknown angle with $\vec{E}$	(Q) Parabola
(C) $\vec{E} \neq 0, \vec{B} \neq 0, \vec{E} \parallel \vec{B}$ and initial velocity is perpendicular to $\vec{E}$	(R) Circular
(D) $\vec{E} \neq 0, B \neq 0, \vec{E}$ is perpendicular to $\vec{B}$ and initial velocity is perpendicular to both $\vec{E}$ and $\vec{B}$	(S) Helical path with nonuniform pitch
	(T) Helical path with uniform pitch

Advanced Subjective (AS)

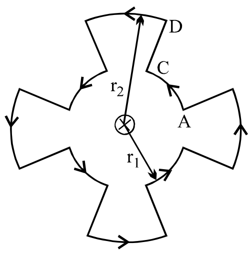
1. A U-shaped wire of mass m and length l is immersed with its two ends in mercury (see figure). The wire is in a homogeneous field of magnetic induction **B**. If a charge, that is, a current pulse $q = \int idt$, is sent through the wire, the wire will jump up. Calculate, the height h that the wire reaches, the size of the charge or current pulse, assuming that the time of the current pulse is very small in comparision with the time of flight. Make use of the fact that impulse of force equals $\int F dt$, which equals mv. Evaluate q for B = 0.1 Wb/m^2, m = 10gm, ℓ = 20cm & h = 3 meters. [g = 10 m/s^2]



2. A current of 10A flows around a closed path in a circuit which is in the horizontal plane as shown in the figure. The circuit consists of eight alternating arcs of radii r_1 = 0.08 m and r_2 = 0.12 m. Each arc subtends the same angle at the centre.

(a) Find the magnetic field produced by this circuit at the centre.

(b) An infinitely long straight wire carrying a current of 10A is passing through the centre of the above circuit vertically with the direction of the current being into the plane of the circuit. What is the force acting on the wire at the centre due to the current in the circuit? What is the force acting on the arc AC and the straight segment CD due to the current at the centre? **[JEE 2001]**

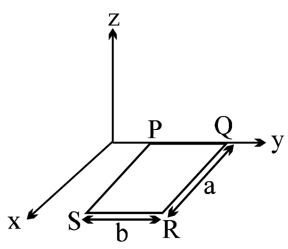


3. A rectangular loop PQRS made from a uniform wire has length a, width b and mass m. It is free to rotate about the arm PQ, which remains hinged along a horizontal line taken as the y-axis (see figure). Take the vertically upward direction as the z-axis. A uniform magnetic field $\vec{B} = (3\hat{i} + 4\hat{k})B_0$ exists in the region. The loop is held in the x-y plane and a current I is passed through it. The loop is now released and is found to stay in the horizontal position in equilibrium. **[JEE 2002]**

(a) What is the direction of the current I in PQ?

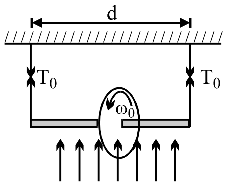
(b) Find the magnetic force on the arm RS.

(c) Find the expression for I in terms of B_0, a, b and m.



4. A wheel of radius R having charge Q, uniformly distributed on the rim of the wheel is free to rotate about a light horizontal rod. The rod is suspended by light inextensible stringe and a magnetic field B is applied as shown in the figure. The initial tensions in the strings are T_0. If the breaking tension of the strings are $\frac{3T_0}{2}$, find the maximum angular velocity ω_0 with which the wheel can be rotate.

[JEE 2003]



5. In a moving coil galvanometer, torque on the coil can be expressed as $\tau = ki$, where i is current through the wire and k is constant. The rectangular coil of the galvanometer having numbers of turns N, area A and moment of inertia I is placed in magnetic field B. Find

(a) k in terms of given parameters N, I, A and B.

(b) the torsional constant of the spring, if a current i_0 produces a deflection of $\pi/2$ in the coil in reaching equilibrium position.

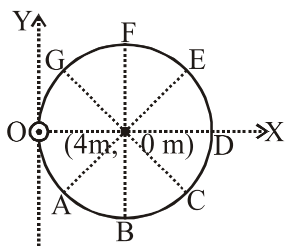
(c) the maximum angle through which coil is deflected, id charge Q is passed through the coil almost instantaneously. (Ignore the damping in mechanical oscillations) **[JEE 2005]**

6. 3 infinitely long thin wires each carrying current *i* in the same direction, are in the x-y plane of a gravity free space. The central wire is along the y-axis while the other two are along $x = \pm d$.

(i) Find the locus of the points for which the magnetic field B is zero .

(ii) If the central wire is displaced along the z-direction by a small amount & released, show that it will execute simple harmonic motion. If the linear density of the wires is λ, find the frequency of oscillation.

7. An infinite uniform current carrying wire is kept along z-axis, carrrying current I_0 in the direction of the positive z-axis. OABCDEFG represents a circle (where all the points are equally spaced), whose centre at point (4m, 0 m) and radius 4 m as shown in the figure. If $\int_{DEF} \vec{B}.d\vec{\ell} = \frac{\mu_0 I_0}{k}$ in S.I. unit, then find the value of k.



8. A particle of charge +q and mass m moving under the influence of a uniform electric field $E\,\hat{i}$ and a magnetic field $B\,\hat{k}$ enters in I quadrant of a coordinate system at a point (0, a) with initial velocity $v\,\hat{i}$ and leaves the quadrant at a point (2a, 0) with velocity $-2v\,\hat{j}$. Find

(a) Magnitude of electric field

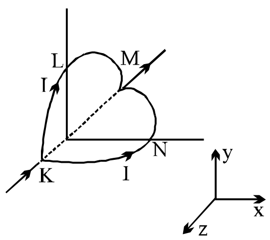
(b) Rate of work done by the electric field at point (0, a)

(c) Rate of work done by both the fields at (2a, 0).

9. A circular loop of radius R is bent along a diameter and given a shape as shown in the figure. One of the semicircles (KNM) lies in the $x-z$ plane and the other one (KLM) in the $y-z$ plane with their centers at the origin. Current I is flowing through each of the semicircles as shown in figure .

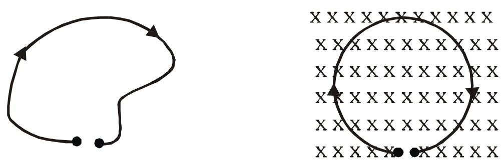
(i) A particle of charge q is released at the origin with a velocity $v = -v_0\,\hat{i}$. Find the instantaneous force f on the particle. Assume that space is gravity free.

(ii) If an external uniform magnetic field $B\,\hat{j}$ is applied, determine the forces F_1 and F_2 on the semicircles KLM and KNM due to this field and the net force F on the loop . **[JEE 2000]**



EXERCISE (JA)

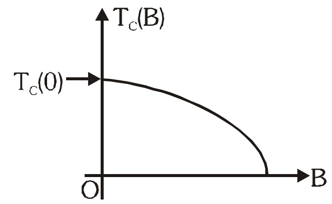
1. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is : **[2010, 3M]**



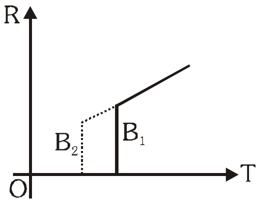
(A) IBL (B) $\dfrac{IBL}{\pi}$ (C) $\dfrac{IBL}{2\pi}$ (D) $\dfrac{IBL}{4\pi}$

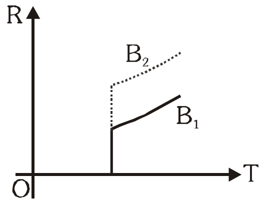
Paragraph for Question no. 2 and 3

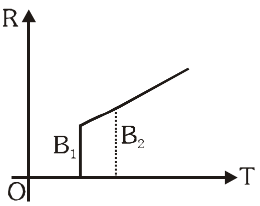
Electrical resistance of certain materials, known as superconductors, changes abruptly from a nonzero value to zero as their temperature is lowered below a critical temperature $T_C(0)$. An interesting property of superconductors is that their critical temperature becomes smaller than $T_C(0)$ if they are placed in a magnetic field, i.e., the critical temperature $T_C(B)$ is a function of the magnetic field strength B. The dependence of T_C (B) on B is shown in the figure. **[JEE2010]**

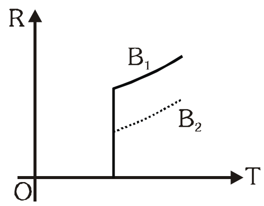


2. In the graphs below, the resistance R of a superconductor is shown as a function of its temperature T for two different magnetic fields B_1(sold line) and B_2 (dashed line). If B_2 is larger than B_1, which of the following graphs shows the correct variation of R with T in these fields?

(A)


(B)


(C)


(D)


3. A superconductor has $T_C(0) = 100$ K. When a magnetic field of 7.5 Tesla is applied, its T_C decreases to 75 K. For this material one can definitely say that when

(A) B = 5 Tesla, T_C (B) = 80 K

(B) B = 5 Tesla, 75 K < T_C(B) < 100 K

(C) B = 10 Tesla, 75 K < T_C(B) < 100K

(D) B = 10 Tesla, T_C(B) = 70 K

4. An electron and a proton are moving on straight parallel paths with same velocity. They enter a semi-infinite region of uniform magnetic field perpendicular to the velocity. Which of the following statement(s) is/are true? **[IIT-JEE 2011]**

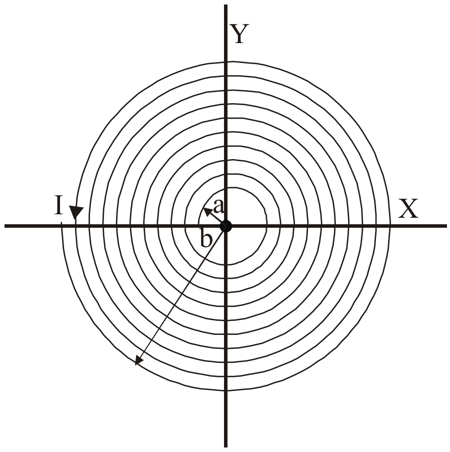
(A) they will never come out of the magnetic field region

(B) they will come out travelling along parallel paths

(C) they will come out of the same time

(D) they will come out at different times

5. A long insulated copper wire is closely wound as a spiral of 'N' turns. The spiral has inner radius 'a' and outer radius 'b'. The spiral lies in the X-Y plane and a steady current 'I' flows through the wire. The Z-component of the magnetic field at the center of the spiral is **[IIT-JEE 2011]**



(A) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b}{a}\right)$ (B) $\frac{\mu_0 NI}{2(b-a)}\ell n\left(\frac{b+a}{b-a}\right)$

(C) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b}{a}\right)$ (D) $\frac{\mu_0 NI}{2b}\ell n\left(\frac{b+a}{b-a}\right)$

6. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t =0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option (s) is (are) correct for time t > 0? **[IIT-JEE 2012]**

(A) If θ = 0°, the charge moves in a circular path in the x-z plane.

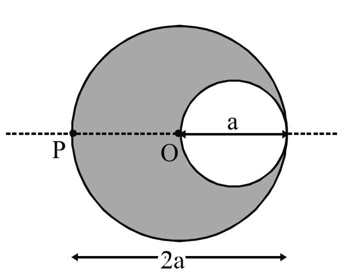
(B) If θ = 0°, the charge undergoes helical motion with constant pitch along the y-axis.

(C) If θ = 10°, the charge undergoes helical motion with its pitch increasing with time, along the y-axis

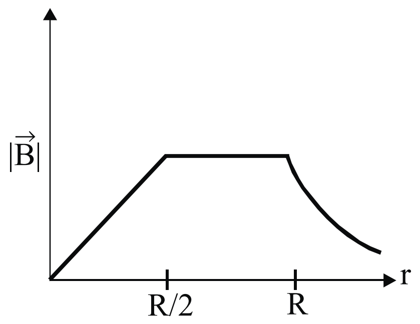
(D) If θ = 90°, the charge undergoes linear but accelerated motion along the y-axis

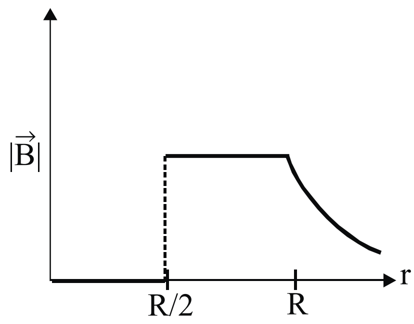
7. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is :

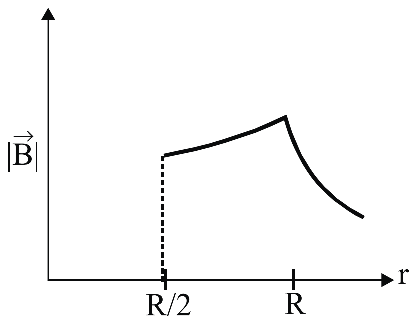
[IIT-JEE 2012]

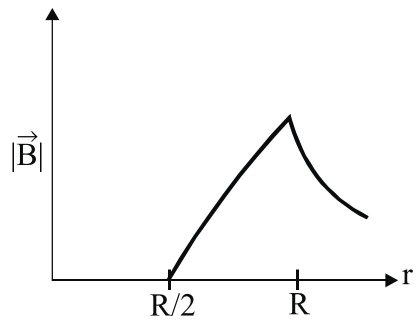


8. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by **[IIT-JEE 2012]**

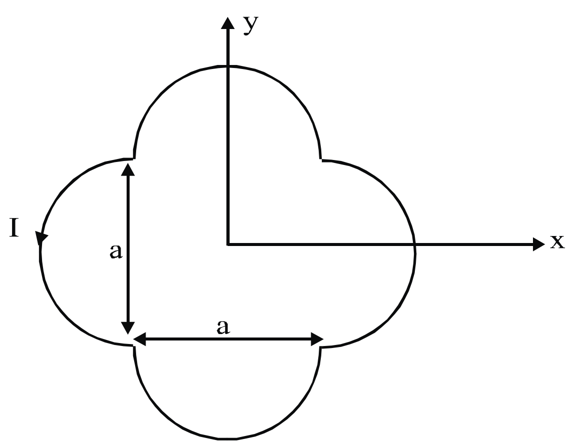
(A)


(B)


(C)


(D)


9. A loop carrying current 'I' lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is **[JEE 2012]**



(A) $a^2 I\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (C) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (D) $(2\pi+1)a^2 I\hat{k}$

10. A particle of mass M and positive charge Q, moving with a constant velocity $\vec{u}_1 = 4\hat{i}\,ms^{-1}$, enters a region of uniform static magnetic field normal to the x-y plane. The region of the magnetic field extends from x = 0 to x = L from all values of y. After passing through this region, the particle emerges on the other side after 10 milliseconds with a velocity $\vec{u}_2 = 2\left(\sqrt{3}\hat{i}+\hat{j}\right)ms^{-1}$. The correct statements(s) is (are) :- **[JEE-Advanced-2013]**

(A) The direction of the magnetic field is –z direction.

(B) The direction of the magnetic field is +z direction.

(C) The magnitude of the magnetic field $\frac{50\pi M}{3Q}$ units.

(D) The magnitude of the magnetic field is $\frac{100\pi M}{3Q}$ units.

11. A steady current I flows along an infinitely long hollow cylindrical conductor of radius R. This cylinder is placed coaxially inside an infinite solenoid of radius 2R. The solenoid has n turns per unit length and carries a steady current I. Consider a point P at a distance r from the common axis. The correct statement(s) is (are) :- **[JEE-Advanced-2013]**

(A) In the region $0 < r < R$, the magnetic field is non-zero

(B) In the region $R < r < 2R$, the magnetic field is along the common axis

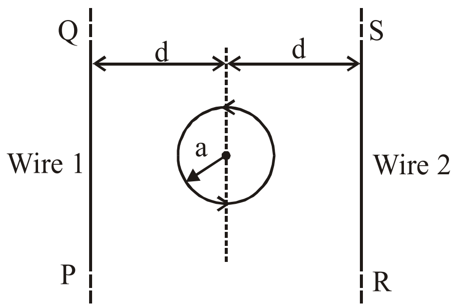
(C) In the region $R < r < 2R$, the magnetic field is tangential to the circle of radius r, centred on the axis

(D) In the region $r > 2R$, the magnetic field is non-zero

12. Two parallel wires in the plane of the paper are distance X_0 apart. A point charge is moving with speed u between the wires in the same plane at a distance X_1 from one of the wires. When the wires carry current of magnitude I in the same direction, the radius of curvature of the path of the point charge is R_1. In contrast, if the currents I in the two wires have directions opposite to each other, the radius of curvature of the path is R_2. If $\frac{X_0}{X_1}=3$, and value of $\frac{R_1}{R_2}$ is:- **[JEE-Advanced-2014]**

Paragraph for Questions 13 & 14

The figure shows a circular loop of radius a with two long parallel wires (numbered 1 and 2) all in the plane of the paper. The distance of each wire from the centre of the loop is d. The loop and the wires are carrying the same current I. The current in the loop is in the counterclockwise direction if seen from above. **[JEE-Advanced-2014]**



13. When d ≈ a but wires are not touching the loop, it is found that the net magnetic field on the axis of the loop is zero at a height h above the loop. In that case

(A) current in wire 1 and wire 2 is the direction PQ and RS, respectively and h ≈ a

(B) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ a

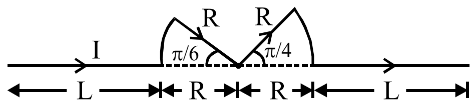
(C) current in wire 1 and wire 2 is the direction PQ and SR, respectively and h ≈ 1.2 a

(D) current in wire 1 and wire 2 is the direction PQ and RS, respectively and h ≈ 1.2 a

14. Consider d >> a, and the loop is rotated about its diameter parallel to the wires by 30° from the position shown in the figure. If the currents in the wires are in the opposite directions, the torque on the loop at its new position will be (assume that the net field due to the wires is constant over the loop) **[JEE-Advanced-2014]**

(A) $\frac{\mu_0 I^2 a^2}{d}$ (B) $\frac{\mu_0 I^2 a^2}{2d}$ (C) $\frac{\sqrt{3}\mu_0 I^2 a^2}{d}$ (D) $\frac{\sqrt{3}\mu_0 I^2 a^2}{2d}$

15. A conductor (shown in the figure) carrying constant current I is kept in the x-y plane in a unfirom magnetic field $\vec{B}$. If F is the magnitude of the total magnetic force acting on the conductor, then the correct statement(s) is (are) **[JEE-Advanced-2015]**



(A) If $\vec{B}$ is along $\hat{z}$, $F \propto (L + R)$

(B) If $\vec{B}$ is along $\hat{x}$, $F = 0$

(C) If $\vec{B}$ is along $\hat{y}$, $F \propto (L + R)$

(D) If $\vec{B}$ is along $\hat{z}$, $F = 0$

Answer Q.16, Q.17 and Q.18 by appropriately matching the information given in the three columns of the following table.

A charged particle (electron or proton) is introduced at the origin (x = 0, y = 0, z = 0) with a given initial velocity $\vec{v}$. A uniform electric field $\vec{E}$ and a uniform magnetic field $\vec{B}$ exist everywhere. The velocity $\vec{v}$, electric field $\vec{E}$ and magnetic field $\vec{B}$ are given in column 1, 2 and 3, respectively. The quantities E_0, B_0 are positive in magnitude. **[JEE-Advanced-2017]**

Column-1	**Column-2**	**Column-3**
(I) Electron with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(i) $\vec{E} = E_0\hat{z}$	(P) $\vec{B} = -B_0\hat{x}$
(II) Electron with $\vec{v} = \frac{E_0}{B_0}\hat{y}$	(ii) $\vec{E} = -E_0\hat{y}$	(Q) $\vec{B} = B_0\hat{x}$
(III) Proton with $\vec{v} = 0$	(iii) $\vec{E} = -E_0\hat{x}$	(R) $\vec{B} = B_0\hat{y}$
(IV) Proton with $\vec{v} = 2\frac{E_0}{B_0}\hat{x}$	(iv) $\vec{E} = E_0\hat{x}$	(S) $\vec{B} = B_0\hat{z}$

16. In which case will the particle move in a straight line with constant velocity ?

(A) (II) (iii) (S) (B) (IV) (i) (S) (C) (III) (ii) (R) (D) (III) (iii) (P)

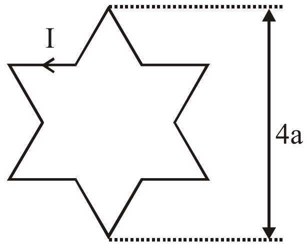
17. In which case will the particle describe a helical path with axis along the positive z-direction ?

(A) (II) (ii) (R) (B) (IV) (ii) (R) (C) (IV) (i) (S) (D) (III) (iii) (P)

18. In which case would the particle move in a straight line along the negative direction of y-axis (i.e., move along $-\hat{y}$) ?

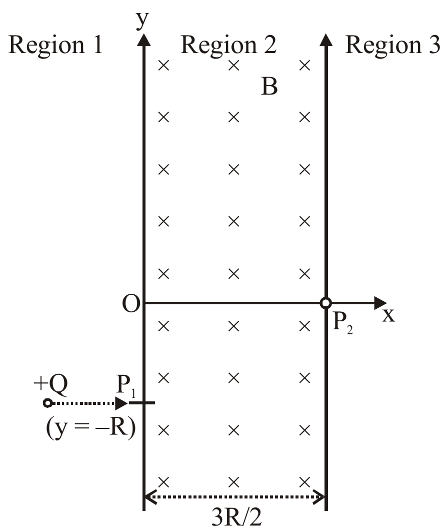
(A) (IV) (ii) (S) (B) (III) (ii) (P) (C) (II) (iii) (Q) (D) (III) (ii) (R)

19. A symmetric star shaped conducting wire loop is carrying a steady state current I as shown in the figure. The distance between the diametrically opposite vertices of the star is 4a. The magnitude of the magnetic field at the center of the loop is : **[JEE-Advanced-2017]**



(A) $\frac{\mu_0 I}{4\pi a}3\left[\sqrt{3}-1\right]$ (B) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}-1\right]$ (C) $\frac{\mu_0 I}{4\pi a}6\left[\sqrt{3}+1\right]$ (D) $\frac{\mu_0 I}{4\pi a}3\left[2-\sqrt{3}\right]$

20. A uniform magnetic field B exists in the region between x = 0 and x = $\frac{3R}{2}$ (region 2 in the figure) pointing normally into the plane of the paper. A particle with charge +Q and momentum p directed along x-axis enters region 2 from region 1 at point P_1(y = –R). Which of the following options(s) is/are correct ? **[JEE-Advanced-2017]**



(A) For $B = \frac{8}{13}\frac{p}{QR}$, the particle will enter region 3 through the point P_2 on x-axis

(B) For $B > \frac{2}{3}\frac{p}{QR}$, the particle will re-enter region 1

(C) For a fixed B, particle of same charge Q and same velocity v, the distance between the point P_1 and the point of re-entry into region 1 is inversely proportional to the mass of the particle.

(D) When the particle re-enters region 1 through the longest possible path in region 2, the magnitude of the chage in its linear momentum between point P_1 and the farthest point from y-axis is $\frac{p}{\sqrt{2}}$.

21. Two infinitely long straight wires lie in the xy-plane along the lines x = ±R. The wire located at x = +R carries a constant current I_1 and the wire located at x = –R carries a constant current I_2. A circular loop of radius R is suspended with its centre at $(0, 0, \sqrt{3}R)$ and in a plane parallel to the xy-plane. This loop carries a constant current I in the clockwise direction as seen from above the loop. The current in the wire is taken to be positive if it is in the $+\hat{j}$ direction. Which of the following statements regarding the magnetic field $\vec{B}$ is (are) true ? **[JEE-Advanced-2018]**

(A) If $I_1 = I_2$, then $\vec{B}$ cannot be equal to zero at the origin (0, 0, 0)

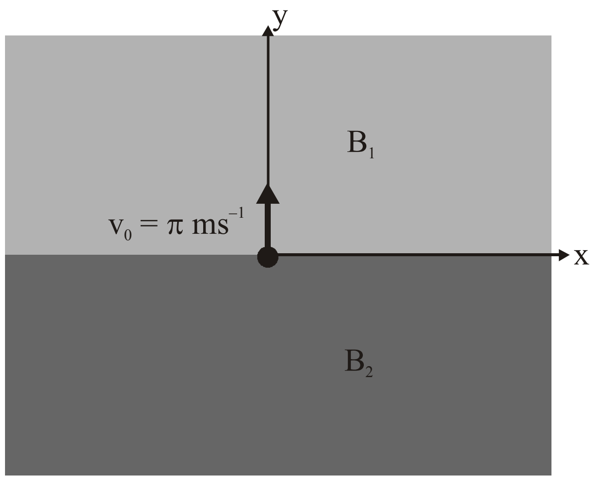
(B) If $I_1 > 0$ and $I_2 < 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(C) If $I_1 < 0$ and $I_2 > 0$, then $\vec{B}$ can be equal to zero at the origin (0, 0, 0)

(D) If $I_1 = I_2$, then the z-component of the magnetic field at the centre of the loop is $\left(-\frac{\mu_0 I}{2R}\right)$

22. In the x-y-plane, the region y > 0 has a uniform magnetic field $B_1\hat{k}$ and the region y < 0 has a another uniform magnetic field $B_2\hat{k}$. A positively charged particle is projected from the origin along the positive y-axis with speed $v_0 = \pi\,ms^{-1}$ at t = 0, as shown in the figure. Neglect gravity in this problem. Let t = T be the time when the particle crosses the x-axis from below for the first time. If $B_2 = 4B_1$, the average speed of the particle, in ms^{-1}, along the x-axis in the time interval T is _______.

[JEE-Advanced-2018]



ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

THEORY

Ex. Self-Inductance of a Toroid :

Calculate the self-inductance of a toroid which consists of *N* turns and has a rectangular cross section, with inner radius *a*, outer radius *b* and height *h*, as shown in Figure (a).

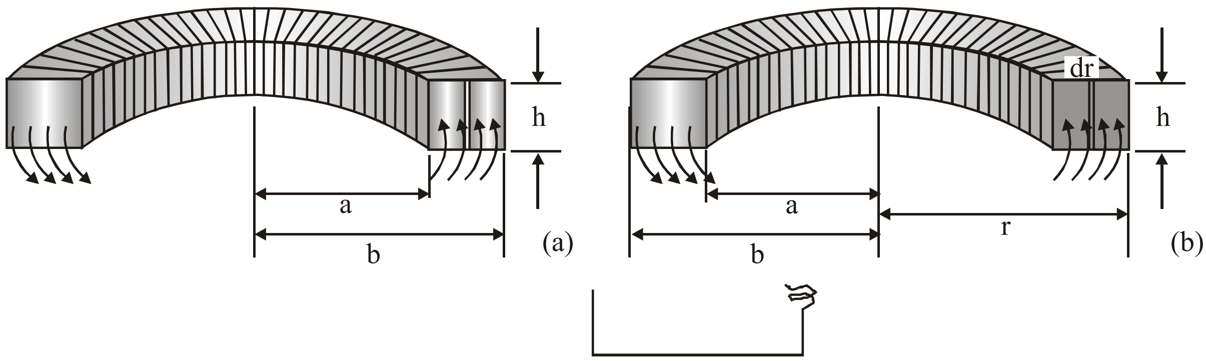


Figure : A toroid with *N* turns

Sol : According to Ampere's law discussed in section, the magnetic field is given by

$$\oint \vec{B}.d\vec{s} \oint Bds = B\oint ds = B\,(2\pi r) = \mu 0NI$$

or $$B = \frac{\mu_0 NI}{2\pi r}$$

The magnetic flux through one turn of the torid may be obtained by integrating over the rectangular cross section,k with dA = hdr as the differential area element (figure-b)

$$\Phi_B = \iint \vec{B}\ .\ d\vec{A} = \int_a^b \left(\frac{\mu_0 NI}{2\pi r}\right) hdr = \frac{\mu_0 NIh}{2\pi} \ln\left(\frac{b}{a}\right)$$

The total flux is $N\Phi_B$. Therefore, the self-inductance is

$$L = \frac{N\Phi_B}{I} = \frac{\mu_0 N^2 h}{2\pi} \ln\left(\frac{b}{a}\right)$$

Again, the self-inductance L depends only on the geometrical factors. Let's consider the situation where a >>b – a. In this limit, the logarithmic term in the equation above may be expanded as

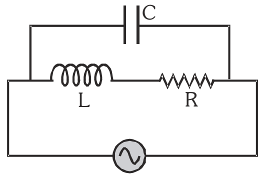
$$\ln\left(\frac{b}{a}\right) = \ln\left(1 + \frac{b-a}{a}\right) \approx \frac{b-a}{a}$$

and the self-inductance becomes

$$L \approx \frac{\mu_0 N^2 h}{2\pi} \cdot \frac{b-a}{a} = \frac{\mu_0 N^2 A}{2\pi a} = \frac{\mu_0 N^2 A}{\ell}$$

where A = h(b – a) is the cross-sectional area, and $\ell = 2\pi a$. We see that the self inductance of the torid in this limit has the same form as that of a solenoid.

PARALLEL RESONANCE



(a) At resonance

(i) $S_L = S_C$

(ii) $I_L = I_C$

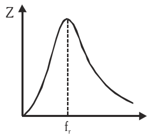
(iii) $\phi = 0$

(iv) $Z_{max} = R$ (impedance maximum)

(v) $I_{min} = \frac{V}{R}$ (current minimum)

(b) Resonant frequency $f_r = \frac{1}{2\pi\sqrt{LC}}$

(c)

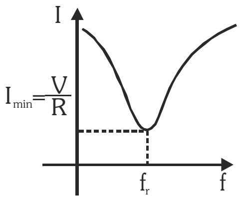


Variation of Z with f as f increases , Z first increases then decreases

◎ If $f < f_r$ then $S_L > S_C$, ϕ (positive), circuit nature is inductive

◎ If $f > f_r$ then $S_C > S_L$, ϕ (negative), circuit nature capacitive.

(d) Variation of I with f as f increases , I first decreases then increases

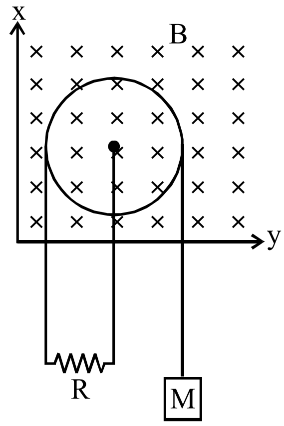


Note : For this circuit $f_r = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}} \Rightarrow Z_{max} = \frac{L}{RC}$ For resonance $\frac{1}{LC} > \frac{R^2}{L^2}$

Advanced Objective (AO)

Single Correct Answer Type Question

1. The block of mass (M) is connected by thread which is wound on a pulley, free to rotate about fixed horizontal axis as shown. A uniform magnetic field B exists in a horizontal plane. The disc is connected with the resistance R as shown. Calculate the terminal velocity of the block if it was released from rest. Treat pulley as uniform metallic disc of radius L.

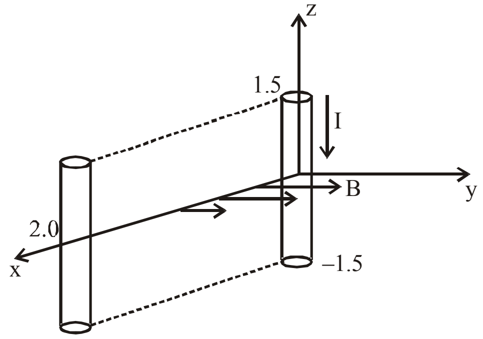


(A) $\frac{4mgR}{B^2L^2}$ (B) $\frac{3mgR}{4B^2L^2}$ (C) $\frac{2mgR}{B^2L^2}$ (D) $\frac{3mgR}{2B^2L^2}$

2. A current I flows in an infinity long wire with cross section in the from of a semicircular ring of radius R. the magnitude of the magnetic induction along its axis is :- **[AIEEE - 2011]**

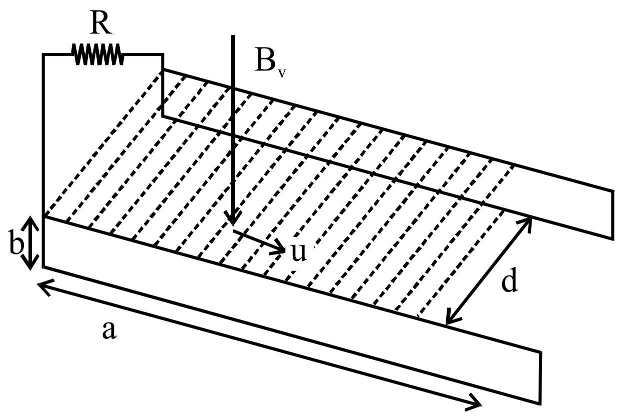
(A) $\frac{\mu_0 I}{2\pi R}$ (B) $\frac{\mu_0 I}{4\pi R}$ (C) $\frac{\mu_0 I}{\pi^2 R}$ (D) $\frac{\mu_0 I}{2\pi^2 R}$

3. A conductor lies along the z-axis at $-1.5 \le z < 1.5$ m and carries a fixed current of 10.0 A in $-\hat{a}_z$ direction (see figure). For a field $\vec{B} = 3.0\times10^{-4} e^{-0.2x}\hat{a}_y$ T, find the power required to move the conductor at constant speed to x = 2.0 m, y = 0 m in 5×10^{-3}s. Assume parallel motion along the x-axis. **[JEE(Mains) - 2014]**



(A) 14.85 W (B) 29.7 W (C) 1.57 W (D) 2.97 W

4. The figure shows an apparatus suggested by Faraday to generate electric current from a flowing river. Two identical conducting plates of length a and width b are placed parallel facing one another on opposite sides of the river following with velocity u at a distance d apart. Now both the plates are connected by a load resistance R. Then the current through the load R is :- (Consider vertical component of the magnetic field produced by earth is B_v and the resistivity of river water is ρ.)



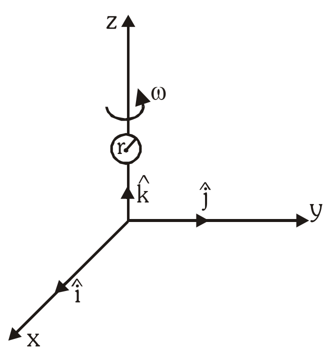
(A) $\frac{B_v ub}{R}$

(B) $\frac{B_v ub}{R+\frac{\rho d}{ab}}$

(C) $\frac{B_v ud}{R+\frac{\rho d}{ab}}$

(D) None of the above

5. A circular loop wire of radius r rotates about the z–axis with angular velocity ω. The normal to the loop is always perpendicular to the z–axis. At time t = 0, the normal is parallel to the y–axis. An external magnetic field $\vec{B} = B_y\hat{j} + B_z\hat{k}$ is applied. The EMF ε(t) induced in the loop is



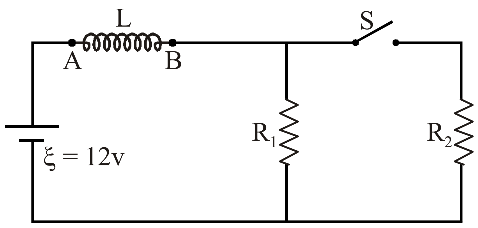
(A) $\pi r^2 \omega B_y \sin\omega t$ (B) $\pi r^2 \omega B_z \cos\omega t$ (C) $\pi r^2 \omega B_z \sin\omega t$ (D) $\pi r^2 \omega B_y \cos\omega t$

6. The magnetic field in a region is given by $\vec{B} = B_0\left(1+\frac{x}{a}\right)\hat{k}$. A square loop of edge - length d is placed with its edge along x & y axis. The loop is moved with constant velocity $\vec{V} = V_0\hat{i}$. The emf induced in the loop is

(A) $\frac{V_0B_0d^2}{a}$ (B) $\frac{V_0B_0d^2}{2a}$

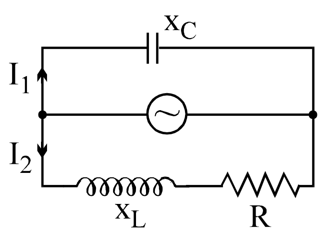
(C) $\frac{V_0B_0a^2}{d}$ (D) None

7. The circuit shown has been operating for a long time. The instant after the switch in the circuit labeled S is opened, what is the voltage across the inductor V_L and which labeled point (A or B) of the inductor is at a higher potential ? Take $R_1 = 4.0\ \Omega$, $R_2 = 8.0\ \Omega$, and L = 2.5 H.



(A) V_L = 12 V ; Point A is at the higher potential
(B) V_L = 12 V ; Point B is at the higher potential
(C) V_L = 6V ; Point A is at the higher potential
(D) V_L = 6V ; Point B is at the higher potential

8. In the shown AC circuit phase difference between currents I_1 and I_2 is

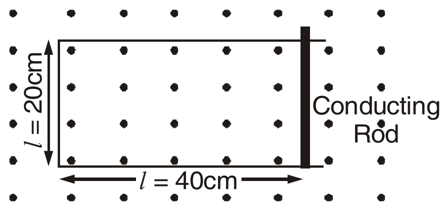


(A) $\frac{\pi}{2} - \tan^{-1}\frac{x_L}{R}$ (B) $\tan^{-1}\frac{x_L - x_C}{R}$

(C) $\frac{\pi}{2} + \tan^{-1}\frac{x_L}{R}$ (D) $\tan^{-1}\frac{x_L - x_C}{R} + \frac{\pi}{2}$

Multiple Correct Answer Type Question

9. Figure shows a conducting rod of negligible resistance that can slide on smooth U-shaped rail made of wire of resistance 1 Ω/m. Position of the conducting rod at t = 0 is shown. A time dependent magnetic field B = 2t Tesla is switched on at t = 0. After the magnetic field is switched on, the conducting rod is moved to the left perpendicular to the rails at constant speed 5 cm/s by some external agent.



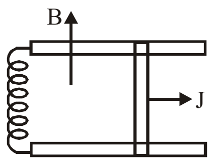
(A) The current in the loop at t = 0 due to induced emf is 0.16 A, clockwise

(B) At t = 2s, induced emf has magnitude 0.08 V

(C) The magnitude of the force required to move the conducting rod at constant speed 5 cm/s at t = 2s, is equal to 0.08 N

(D) The magnitude of the force required to move the conducting rod at constant speed 5 cm/s at t = 2s, is equal to 0.16 N

10. Two parallel resistanceless rails are connected by an inductor of inductance L at one end as shown in the figure. A magnetic field B exists in the space which is perpendicular to the plane of the rails. Now a conductor of length ℓ and mass m is placed transverse on the rails and given an impulse J towards the rightward direction. Then choose the **CORRECT** option (s).



(A) Velocity of the conductor is half of the initial velocity after a displacement of the conductor $d = \sqrt{\frac{3J^2L}{4B^2\ell^2 m}}$

(B) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\frac{3J^2}{4Lm}}$

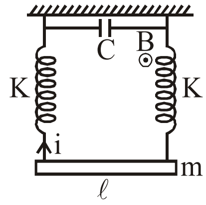
(C) Velocity of the conductor is half of the initial velocity after a displacement of the conductor $d = \sqrt{\frac{3J^2L}{B^2\ell^2 m}}$

(D) Current flowing through the inductor at the instant when velocity of the conductor is half of the initial velocity is $i = \sqrt{\frac{3J^2}{mL}}$

Linked Comprehension Type Question

Paragraph for Question Nos. 11 and 12

In the figure shown a uniform conducting rod of mass m and length ℓ is suspended in vertical plane by two conducting springs of spring constant K. Upper end of spring are connected to each other by capacitor of capacitance C. A uniform horizontal magnetic field (B_0) perpendicular to plane of spring exists in space. Initially rod is in equilibrium but if centre of rod is pulled down and released, it performs SHM. Assume that the spring is small and neglect the magnetic force of interaction between circular section of springs & self inductance of rod.



11. Find time period of oscillation of rod :-

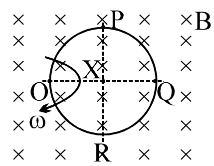
(A) $2\pi\sqrt{\frac{m}{k}}$ (B) $2\pi\sqrt{\frac{B^2\ell^2C}{K}}$ (C) $\pi\sqrt{\frac{m+B^2\ell^2C}{K}}$ (D) $2\pi\sqrt{\frac{B^2\ell^2C+m}{2K}}$

12. Choose correct options from following :-

(A) Electrical energy stored in capacitor is maximum when rod is at its lower extreme position
(B) Electrical energy stored in capacitor is maximum when rod is at its mean position
(C) Current in rod is maximum at mean position of rod
(D) If magnetic field is switched off then mean position of rod will change

Paragraph for Question No. 13 to 15

A conducting ring of radius a is rotated about a point O on its periphery as shown in the figure in a plane perpendicular to uniform magnetic field B which exists everywhere. The rotational velocity is ω.



13. Choose the correct statement(s) related to the potential of the points P, Q and R

(A) $V_P - V_O > 0$ and $V_R - V_O < 0$ (B) $V_P = V_R > V_O$
(C) $V_O > V_P = V_Q$ (D) $V_Q - V_P = V_P - V_O$

14. Choose the correct statement(s) related to the magnitude of potential differences

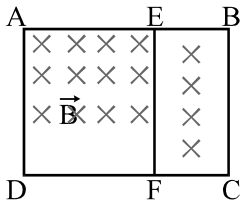
(A) $V_P - V_O = \frac{1}{2}B\omega a^2$ (B) $V_P - V_Q = \frac{1}{2}B\omega a^2$
(C) $V_Q - V_O = 2B\omega a^2$ (D) $V_P - V_R = 2B\omega a^2$

15. Choose the correct statement(s) related to the induced current in the ring

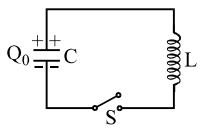
(A) Current flows from $Q \longrightarrow P \longrightarrow O \longrightarrow R \longrightarrow Q$
(B) Current flows from $Q \longrightarrow R \longrightarrow O \longrightarrow P \longrightarrow Q$
(C) Current flows from $Q \longrightarrow P \longrightarrow O$ and from $Q \longrightarrow R \longrightarrow O$
(D) No current flows

Advanced Subjective (AS)

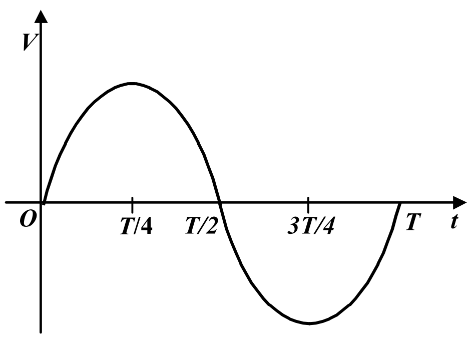
1. A rectangular frame ABCD made of a uniform metal wire has a straight connection between E & F made of the same wire as shown in the figure. AEFD is a square of side 1 m & EB = FC = 0.5 m. The entire circuit is placed in a steadily increasing uniform magnetic field directed into the plane of the paper & normal to it . The rate of change of the magnetic field is 1 T/s, the resistance per unit length of the wire is 1 Ω/m. Find the current in segments AE, BE & EF.



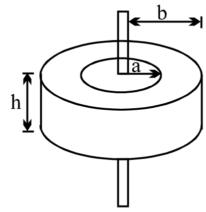
2. A capacitor C with a charge Q_0 is connected across an inductor through a switch S. If at t = 0, the switch is closed, then find the instantaneous charge q on the upper plate of capacitor.



3. In an LR series circuit, a sinusoidal voltage $V = V_o \sin \omega t$ is applied. It is given that L = 35 mH, $R = 11\ \Omega$, $V_{rms} = 220$ V, $\frac{\omega}{2\pi} = 50$ Hz and $\pi = 22/7$. Find the amplitude of current in the steady state and obtain the phase difference between the current and the voltage. Also plot the variation of current for one cycle on the given graph. **[JEE 2004]**



4. A long straight wire is arranged along the symmetry axis of a toroidal coil of rectangular cross–section, whose dimensions are given in the figure. The number of turns on the coil is N, and relative permeability of the surrounding medium is unity. Find the amplitude of the emf induced in this coil, if the current $i = i_m \cos \omega t$ flows along the straight wire.

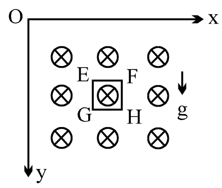


5. A magnetic field $B = (B_0 y / a)\ \hat{k}$ is into the plane of paper in the +z direction. B_0 and a are positive constants. A square loop EFGH of side a, mass m and resistance R, in x-y plane, starts falling under the influence of gravity. Note the directions of x and y axes in the figure. Find

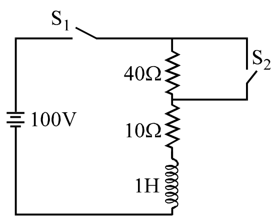
(a) the induced current in the loop and indicate its direction,

(b) the total Lorentz force acting on the loop and indicate its direction,

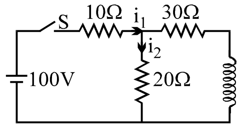
(c) an expression for the speed of the loop, v(t) and its terminal value.



6. In the circuit shown in the figure the switched S_1 and S_2 are closed at time t = 0. After time t = (0.1) ln 2 sec, switch S_2 is opened. Find the current in the circuit at time t = (0.2) ln 2 sec.



7. Find the values of i_1 and i_2



(i) immediately after the switch S is closed.

(ii) long time later, with S closed.

(iii) immediately after S is open.

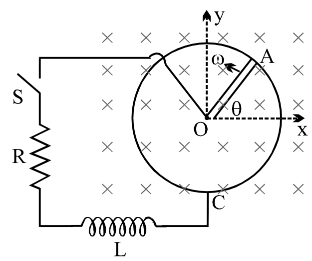
(iv) long time after S is opened.

8. A metal rod OA of mass m & length r is kept rotating with a constant angular speed ω in a vertical plane about a horizontal axis at the end O. The free end A is arranged to slide without friction along a fixed conducting circular ring in the same plane as that of rotation. A uniform & constant magnetic induction $\vec{B}$ is applied perpendicular & into the plane of rotation as shown in figure. An inductor L and an external resistance R are connected through a switch S between the point O & a point C on the ring to form an electrical circuit. Neglect the resistance of the ring and the rod. Initially, the switch is open.

(a) What is the induced emf across the terminals of the switch ?

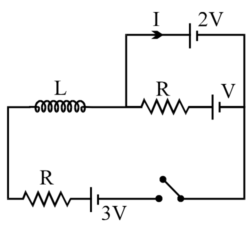
(b) (i) Obtain an expression for the current as a function of time after switch S is closed.

(ii) Obtain the time dependence of the torque required to maintain the constant angular speed, given that the rod OA was along the positive X-axis at t = 0.

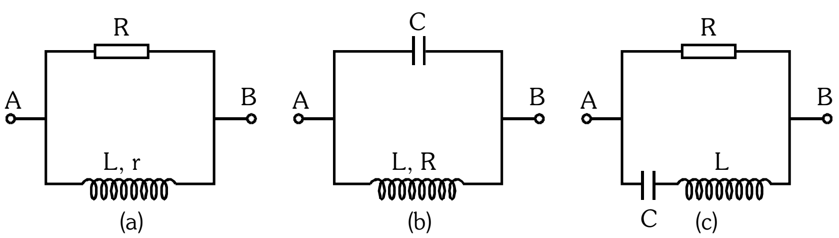


9. A zero resistance coil of inductance L connects the upper ends of two vertical parallel long conductors. A horizontal sliding conductor, free to slide up and down, always maintaining contact with the vertical conductors, starts falling from rest at t = 0, due to its own weight mg. A uniform magnetic field of magnitude B exists in the region horizontally and perpendicular to the plane of the conductors. The distance between the vertical conductors is 'l'. After what time does the conductor come back to its starting position? Also find maximum speed achieved.

10. In the LR circuit shown, what is the variation of the current I as a function of time? The switch is closed at time t = 0 sec.

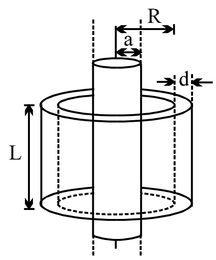


11. Draw the approximate vector diagrams of currents in the circuits shown in Fig. The voltage applied across the points A and B is assumed to be sinusoidal; the parameters of each circuit are so chosen that the total current I_0 lags in phase behind the external voltage by an angle ϕ.



12. A long solenoid of radius a and number of turns per unit length n is enclosed by cylindrical shell of radius R, thickness d (d <<R) and length L. A variable current $i = i_0 \sin \omega t$ flows through the coil. If the resistivity of the material of cylindrical shell is ρ, find the induced current in the shell.

[JEE 2005]



EXERCISE (JA)

1. An AC voltage source of variable angular frequency ω and fixed amplitude V_0 is connected in series with a capacitance C and an electric bulb of resistance R (inductance zero). When ω is increased

[JEE 2010]

(A) the bulb glows dimmer

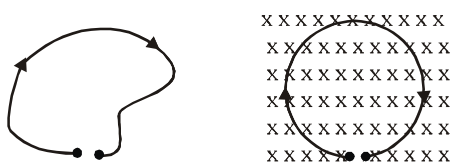
(B) the bulb glows brighter

(C) total impedance of the circuit is unchanged

(D) total impedance of the circuit increases

2. A thin flexible wire of length L is connected to two adjacent fixed points and carries a current I in the clockwise direction, as shown in the figure. When the system is put in a uniform magnetic field of strength B going into the plane of the paper, the wire takes the shape of a circle. The tension in the wire is :-

[JEE 2010]



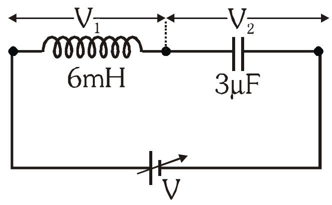
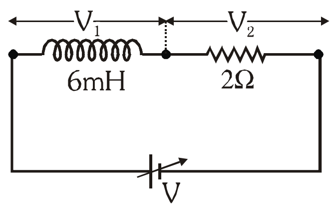
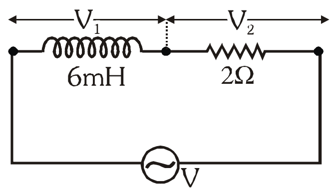
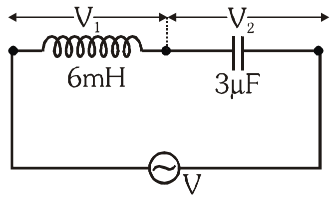
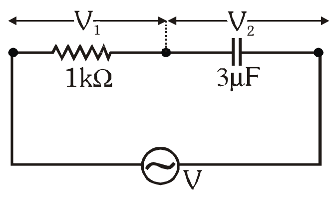
(A) IBL

(B) $\frac{IBL}{\pi}$

(C) $\frac{IBL}{2\pi}$

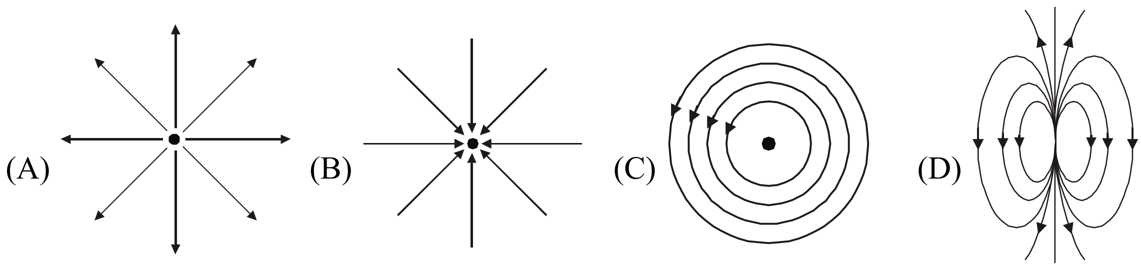
(D) $\frac{IBL}{4\pi}$

3. You are given many resistances, capacitors and inductors. These are connected to a variable DC voltage source (the first two circuits) or an AC voltage source of 50 Hz frequency (the next three circuits) in different ways as shown in **Column II**. When a current I (steady state for DC or rms for AC) flows through the circuit, the corresponding voltage V_1 and V_2 (indicated in circuits) are related as shown in **Column I**. Match the two **[JEE 2010]**

Column I	Column II
(A) $I \neq 0$, V_1 is proportional to I	(p) 
(B) $I \neq 0$, $V_2 > V_1$	(q) 
(C) $V_1 = 0$, $V_2 = V$	(r) 
(D) $I \neq 0$, V_2 is proportional to I	(s) 
	(t) 

4. Which of the field patterns given below is valid for electric field as well as for magnetic field?

[JEE 2011]

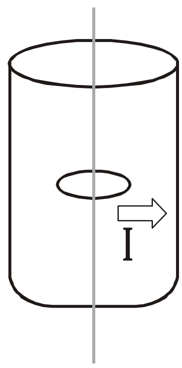


5. A series R-C circuit is connected to AC voltage source. Consider two cases ; (A) when C is without a dielectric medium and (B) when C is filled with dielectric of constant 4. The current I_R through the resistor and voltage V_C across the capacitor are compared in the two cases. Which of the following is/are true? **[JEE 2011]**

(A) $I_R^A > I_R^B$ (B) $I_R^A < I_R^B$

(C) $V_C^A > V_C^B$ (D) $V_C^A < V_C^B$

6. A long circular tube of length 10 m and radius 0.3 m carries a current I along its curved surface as shown. A wire-loop of resistance 0.005 ohm and of radius 0.1 m is placed inside the tube with its axis coinciding with the axis of the tube. The current varies as $I = I_0\cos(300t)$ where I_0 is constant. If the magnetic moment of the loop is $N\mu_0 I_0 \sin(300t)$, then 'N' is **[JEE 2011]**

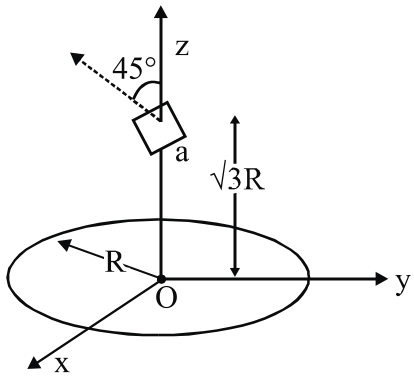


7. A series R-C combination is connected to an AC voltage of angular frequency ω = 500 radian/s. If the impedance of the R-C circuit is $R\sqrt{1.25}$, the time constant (in millisecond) of the circuit is :-

[JEE 2011]

8. A circular wire loop of radius R is placed in the x-y plane centred at the origin O. A square loop of side a (a<<R) having two turns is placed with its centre at z = $\sqrt{3}$ R along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is :-

[JEE 2012]



9. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is (are) **[JEE 2012]**

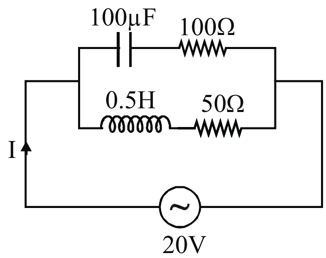
(A) The emf induced in the loop is zero if the current is constant.

(B) The emf induced in the loop is infinite if the current is constant.

(C) The emf induced in the loop is zero if the current decreases at a steady rate.

(D) The emf induced in the loop is finite if the current decreases at a steady rate.

10. In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice (s) is(are) **[JEE 2012]**



(A) The current through the circuit, I is 0.3 A.

(B) The current through the circuit, i is 0.3 √2A.

(C) The voltage across 100Ω resistor = 10√2V.

(D) The voltage across 50Ω resistor = 10V.

Paragraph for Questions 11 and 12

A point charge Q is moving in a circular orbit of radius R in the x-y plane with an angular velocity ω. This can be considered as equivalent to a loop carrying a steady current $\frac{Q\omega}{2\pi}$. A uniform magnetic field along the positive z-axis is now switched on, which increases at a constant rate from 0 to B in one second. Assume that the radius of the orbit remains constant. The application of the magnetic field induces an emf in the orbit. The induced emf is defined as the work done by an induced electric field in moving a unit positive charge around a closed loop. It is known that for an orbiting charge, the magnetic dipole moment is proportional to the angular momentum with a proportionality constant γ.

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11. The change in the magnetic dipole moment associated with the orbit, at the end of the time interval of the magnetic field change is **[JEE Advance-2013]**

(A) $-\gamma BQR^2$ (B) $-\gamma \frac{BQR^2}{2}$ (C) $\gamma \frac{BQR^2}{2}$ (D) γBQR^2

12. The magnitude of the induced electric field in the orbit at any instant of time during the time interval of the magnetic field change is

(A) $\frac{BR}{4}$ (B) $\frac{BR}{2}$ (C) BR (D) 2BR

Paragraph for Questions 13 and 14

A thermal power plant produces electric power of 600 kW and 4000 V, which is to be transported to a place 20 km away from the power plant for consumers' usage. It can be transported either directly with a cable of large current carrying capacity or by using a combination of step-up and step-down transformers at the two ends. The drawback of the direct transmission is the large energy dissipation. In the method using transformers, the dissipation is much smaller. In this method, a step-up transformer is used at the plant side so that the current is reduced to a smaller value. At the consumers' end, a step-down transformer is used to supply power to the consumers at the specified lower voltage. It is reasonable to assume that the power cable is purely resistive and the transformers are ideal with a power factor unity. All the currents and voltages mentioned are rms values. **[JEE Advance-2013]**

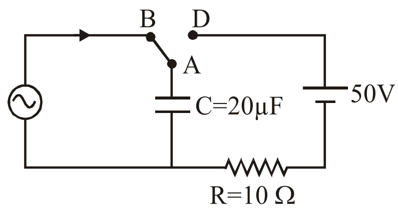
13. In the method using the transformers, assume that the ratio of the number of turns in the primary to that in the secondary in the step-up transformer is 1 : 10. If the power to the consumers has to be supplied at 200 V, the ratio of the number of turns in the primary to that in the secondary in the step-down transformer is

(A) 200 : 1 (B) 150 : 1 (C) 100 : 1 (D) 50 : 1

14. If the direct transmission method with a cable of resistance $0.4\ \Omega\ km^{-1}$ is used, the power dissipation (in %) during transmission is

(A) 20 (B) 30 (C) 40 (D) 50

15. At time t = 0, terminal A in the circuit shown in the figure is connected to B by a key and an alternating current $I(t) = I_0\cos(\omega t)$, with $I_0 = 1A$ and $\omega = 500\ rad\ s^{-1}$ starts flowing in it with the initial direction shown in the figure. At $t = \frac{7\pi}{6\omega}$, the key is switched from B to D. Now onwards only A and D are connected. A total charge Q flows from the battery to charge the capacitor fully. If $C = 20\ \mu F$, $R = 10\ \Omega$ and the battery is ideal with emf of 50 V, identify the correct statement (s). **[JEE Advance-2014]**



(A) Magnitude of the maximum charge on the capacitor before $t = \frac{7\pi}{6\omega}$ is 1×10^{-3}C.

(B) The current in the left part of the circuit just before $t = \frac{7\pi}{6\omega}$ is clockwise.

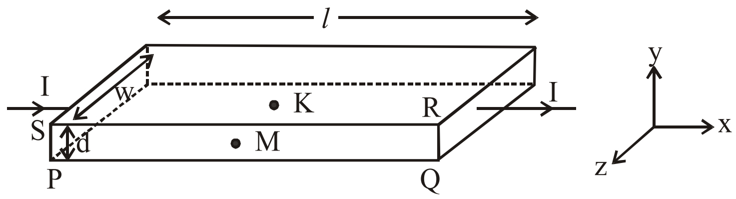
(C) Immediately after A is connected to D, the current in R is 10A

(D) $Q = 2 \times 10^{-3}$ C

Paragraph for Question No. 16 and 17

In a thin rectangular metallic strip a constant current I flows along the positive x-direction, as shown in the figure. The length, width the thickness of the strip are l, w and d, respectively.

A uniform magnetic field $\vec{B}$ is applied on the strip along the positive y-direction. Due to this, the charge carriers experience a net deflection along the z-direction. This results in accumulation of charge carriers on the surface PQRS and appearance of equal and opposite charges on the face opposite to PQRS. A potential difference along the z-direction is thus developed. Charge accumulation continues until the magnetic force is balanced by the electric force. The current is assumed to be uniformly distributed on the cross section of the strip and carried by electrons. **[JEE Advance-2015]**



16. Consider two different metallic strips (1 and 2) of the same material. Their lengths are the same, widths are w_1 and w_2 and thicknesses are d_1 and d_2, respectively. Two points K and M are symmetrically located on the opposite faces parallel to the x-y plane (see figure). V_1 and V_2 are the potential differences between K and M in strips 1 and 2, respectively. Then, for a given current I flowing through them in a given magnetic field strength B, the correct statement(s) is(are)

(A) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = 2V_1$ (B) If $w_1 = w_2$ and $d_1 = 2d_2$, then $V_2 = V_1$

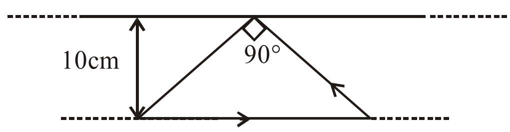
(C) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = 2V_1$ (D) If $w_1 = 2w_2$ and $d_1 = d_2$, then $V_2 = V_1$

17. Consider two different metallic strips (1 and 2) of same dimensions (length l, width w and thickness d) with carrier densities n_1 and n_2, respectively. Strip 1 is placed in magnetic field B_1 and strip 2 is placed in magnetic field B_2, both along positive y-direction. Then V_1 and V_2 are the potential differences developed between K and M in strips 1 and 2, respectively. Assuming that the current I is the same for both the strips, the correct option(s) is(are)

(A) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = 2V_1$ (B) If $B_1 = B_2$ and $n_1 = 2n_2$, then $V_2 = V_1$

(C) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = 0.5V_1$ (D) If $B_1 = 2B_2$ and $n_1 = n_2$, then $V_2 = V_1$

18. A conducting loop in the shape of right angled isosceles triangle of height 10 cm is kept such that the 90° vertex is very close to an infinitely long conducting wire (see the figure). The wire is electrically insulated from the loop. The hypotenuse of the triangle is parallel to the wire. The current in the triangular loop is in counterclockwise direction and increased at constant rate of $10\ \mathrm{A\,s^{-1}}$. Which of the following statement(s) is(are) true? **[JEE Advance-2016]**



(A) The induced current in the wire is in opposite direction to the current along the hypotenuse.

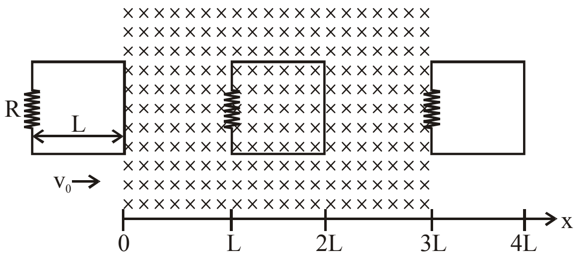
(B) There is a repulsive force between the wire and the loop

(C) If the loop is rotated at a constant angular speed about the wire, an additional emf of $\left(\frac{\mu_0}{\pi}\right)$ volt is induced in the wire

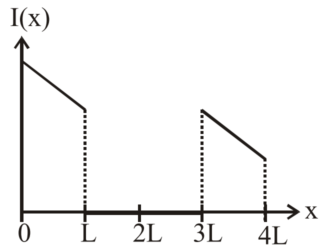
(D) The magnitude of induced emf in the wire is $\left(\frac{\mu_0}{\pi}\right)$ volt.

19. Two inductors L_1 (inductance 1 mH, internal resistance 3 Ω) and L_2 (inductance 2mH, internal resistance 4Ω), and a resistor R (resistance 12 Ω) are all connected in parallel across a 5V battery. The circuit is switched on at time t = 0. The ratio of the maximum to the minimum current (I_{max}/I_{min}) drawn from the battery is. **[JEE Advance-2016]**

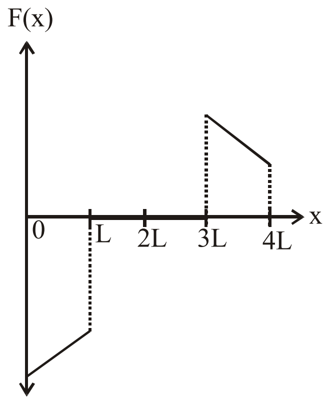
20. A rigid wire loop of square shape having side of length L and resistance R is moving along the x-axis with a constant velocity v_0 in the plane of the paper. At t = 0, the right edge of the loop enters a region of length 3L where there is a uniform magnetic field B_0 into the plane of the paper, as shown in the figure. For sufficiently large v_0, the loop eventually crosses the region. Let x be the location of the right edge of the loop. Let v(x), I(x) and F(x) represent the velocity of the loop, current in the loop, and force on the loop, respectively, as a function of x. Counter-clockwise current is taken as positive. Which of the following schematic plot(s) is(are) correct ? (Ignore gravity) **[JEE Advance-2016]**



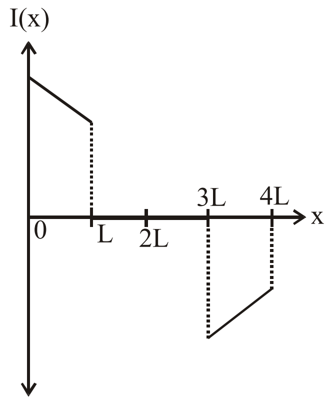
(A)



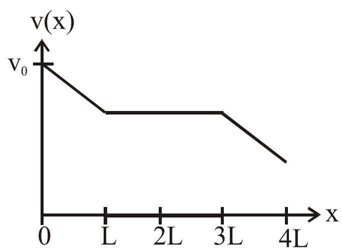
(B)



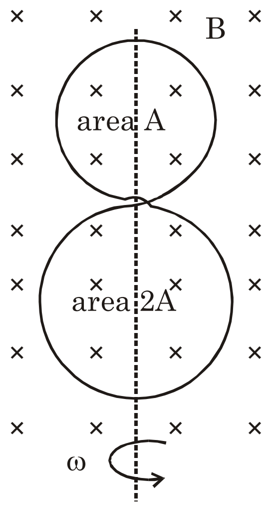
(C)



(D)



21. A circular insulated copper wire loop is twisted to form two loops of area A and 2A as shown in the figure. *At the point of crossing the wires remain electrically insulated from each other*. The entire loop lies in the plane (of the paper). A uniform magnetic field $\vec{B}$ points into the plane of the paper. At t = 0, the loop starts rotating about the common diameter as axis with a constant angular velocity ω in the magnetic field. Which of the following options is/are correct? **[JEE Advance-2017]**



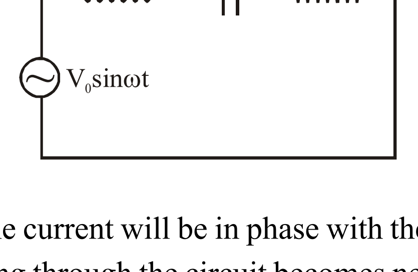
(A) The rate of change of the flux is maximum when the plane of the loops is perpendicular to plane of the paper

(B) The net emf induced due to both the loops is proportional to cos ωt

(C) The emf induced in the loop is proportional to the sum of the areas of the two loops

(D) The amplitude of the maximum net emf induced due to both the loops is equal to the amplitude of maximum emf induced in the smaller loop alone

22. In the circuit shown, L = 1 μH, C = 1 μF and R = 1 kΩ. They are connected in series with an a.c. source $V = V_0 \sin \omega t$ as shown. Which of the following options is/are correct ? **[JEE Advance-2017]**



(A) The frequency at which the current will be in phase with the voltage is independent of R.

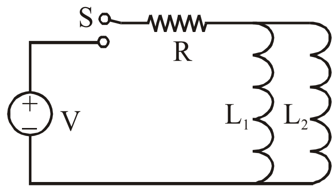
(B) At ω ~ 0 the current flowing through the circuit becomes nearly zero

(C) At $\omega >> 10^6$ rad.s^{-1}, the circuit behaves like a capacitor.

(D) The current will be in phase with the voltage if $\omega = 10^4$ rad.s^{-1}.

23. A source of constant voltage V is connected to a resistance R and two ideal inductors L_1 and L_2 through a switch S as shown. There is no mutual inductance between the two inductors. The switch S is initially open. At $t = 0$, the switch is closed and current begins to flow. Which of the following options is/are correct?

[JEE Advance-2017]



(A) The ratio of the currents through L_1 and L_2 is fixed at all times ($t > 0$)

(B) After a long time, the current through L_1 will be $\frac{V}{R}\frac{L_2}{L_1+L_2}$

(C) After a long time, the current through L_2 will be $\frac{V}{R}\frac{L_1}{L_1+L_2}$

(D) At $t = 0$, the current through the resistance R is $\frac{V}{R}$

24. The instantaneous voltages at three terminals marked X, Y and Z are given by

$V_X = V_0 \sin \omega t$

$V_Y = V_0 \sin\left(\omega t + \frac{2\pi}{3}\right)$ and

$V_Z = V_0 \sin\left(\omega t + \frac{4\pi}{3}\right)$

An ideal voltmeter is configured to read rms value of the potential difference between its terminals. It is connected between points X and Y and then between Y and Z. The reading(s) of the voltmeter will be:-

[JEE Advance-2017]

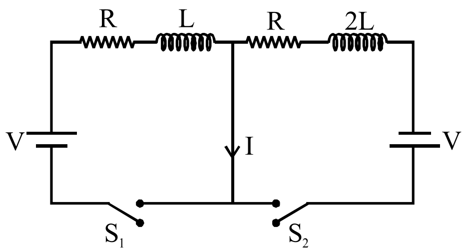
(A) $V_{XY}^{rms} = V_0$

(B) $V_{YZ}^{rms} = V_0\sqrt{\frac{1}{2}}$

(C) Independent of the choice of the two terminals

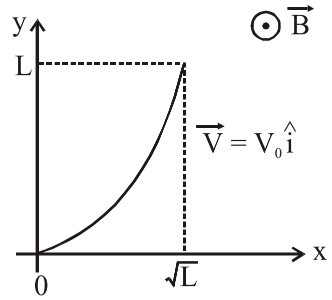
(D) $V_{XY}^{rms} = V_0\sqrt{\frac{3}{2}}$

25. In the figure below, the switches S_1 and S_2 are closed simultaneously at $t = 0$ and a current starts to flow in the circuit. Both the batteries have the same magnitude of the electromotive force (emf) and the polarities are as indicated in the figure. Ignore mutual inductance between the inductors. The current I in the middle wire reaches its maximum magnitude I_{max} at time $t = \tau$. Which of the following statement(s) is (are) true? **[JEE Advance-2018]**



(A) $I_{max} = \frac{V}{2R}$ (B) $I_{max} = \frac{V}{4R}$ (C) $\tau = \frac{L}{R}\ell n2$ (D) $\tau = \frac{2L}{R}\ell n2$

26. A conducting wire of parabolic shape, initially $y = x^2$, is moving with velocity $\vec{V} = V_0\hat{i}$ in a non-uniform magnetic field $\vec{B} = B_0\left(1+\left(\frac{y}{L}\right)^{\beta}\right)\hat{k}$, as shown in figure. If V_0, B_0, L and β are positive constants and $\Delta\phi$ is the potential difference developed between the ends of the wire, then the correct statement(s) is/are: **[JEE Advance-2019]**



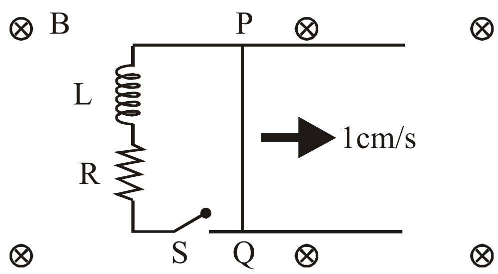
(1) $|\Delta\phi|$ remains the same if the parabolic wire is replaced by a straight wire, $y = x$ initially, of length $\sqrt{2}L$

(2) $|\Delta\phi|$ is proportional to the length of the wire projected on the y-axis.

(3) $|\Delta\phi| = \frac{1}{2}B_0V_0L$ for $\beta = 0$

(4) $|\Delta\phi| = \frac{4}{3}B_0V_0L$ for $\beta = 2$

27. A 10 cm long perfectly conducting wire PQ is moving, with a velocity 1cm/s on a pair of horizontal rails of zero resistance. One side of the rails is connected to an inductor L = 1 mH and a resistance R = 1Ω as shown in figure. The horizontal rails, L and R lie in the same plane with a uniform magnetic field B = 1 T perpendicular to the plane. If the key S is closed at certain instant, the current in the circuit after 1 millisecond is $x \times 10^{-3}$A, where the value of x is_______.
[Assume the velocity of wire PQ remains constant (1 cm/s) after key S is closed.
Given : $e^{-1} = 0.37$, where e is base of the natural logarithm] **[JEE Advance-2019]**



ANSWER KEY

01_Electrostatics

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (B) **2. a. Ans. (B); b. Ans. (D)** **3. Ans. (D)** **4. Ans. (A)** **5. Ans. (C)**
6. Ans. (B) **7. Ans. (A)** **8. Ans. (A)** **9. Ans. (C)** **10. Ans. (D)** **11. Ans. (C)**
12. Ans. (B) **13. Ans. (B)** **14. Ans. (A)** **15. Ans. (D)** **16. Ans. (C)** **17. Ans. (A)**
18. Ans. (D)

Advanced Subjective (AS)

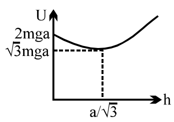
1. Ans. (a) (i) $V=\dfrac{k(q_a+q_b)}{r}$, $E=\dfrac{k(q_a+q_b)}{r^2}$; (ii) $\dfrac{k(q_a+q_b)}{R}+\dfrac{kq_b}{r}-\dfrac{kq_b}{b}$; $\dfrac{kq_b}{r^2}$

(b) $\sigma_R=\left(\dfrac{q_a+q_b}{4\pi R^2}\right)$, $\sigma_a=-\dfrac{q_a}{4\pi a^2}$; $\sigma_b=-\dfrac{q_b}{4\pi b^2}$; (c) f = 0]

2. Ans. $-\varepsilon_0 E, \varepsilon_0 E$ and so on

3. Ans. $v_0 = 3$ m/s ; K.E. at the origin $=\left(27-10\sqrt{6}\right)\times 10^{-4}$ J approx. 2.5×10^{-4} J

4. Ans. (a) $H=\dfrac{4a}{3}$, (b) $U=mg\left[2\sqrt{h^2+a^2}-h\right]$ equilibrium at $h=\dfrac{a}{\sqrt{3}}$,



5. Ans. 2

EXERCISE-(JA)

1. Ans. (C) **2. Ans. (A,B,C,D)** **3. Ans. (A)** **4. Ans. (C,D)** **5. Ans. 3**
6. Ans. (C) **7. Ans. (B)** **8. Ans. 6** **9. Ans. (A,C,D)** **10. Ans. (C)**
11. Ans. (D) **12. Ans. (A, B, C)** **13. Ans. (B, D)** **14. Ans. (C, D)** **15. Ans. (C)**
16. Ans. (C) **17. Ans. (A)** **18. Ans. 6** **19. Ans. (C)** **20. Ans. (D)**
21. Ans. (B,D) **22. Ans. (A) (B)** **23. Ans. (A,B)** **24. Ans. 2 [1.99, 2.01]**
25. Ans. (B) **26. Ans. (1)** **27. Ans. (1,2,4)** **28. Ans. (1,4)**

02_Gravitation

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (C) **2. Ans. (A)**

Multiple Correct Answer Type Question

3. Ans. (A,B,C) **4. Ans. (A,B,D)** **5. Ans. (A,C,D)**

Linked Comprehension Type Question

6. Ans. (B) **7. Ans. (C)**

Advanced Subjective (AS)

1. Ans. 1.6 hours if it is rotating from west to east, 24/17 hours if it is rotating from east to west

2. Ans. $\frac{1}{4}K^2G\pi(R^4 - r^4)$

EXERCISE (JA)

1. Ans. (D) **2. Ans. (A) -p, (B) -q,r (C) -p (D) -q,r** **3. Ans. (C)** **4. Ans. (A)**
5. Ans. 3 **6. Ans. 6** **7. Ans. (A)** **8. Ans. (B)** **9. Ans. (B,D)** **10. Ans. (B)**
11. Ans. 2 **12. Ans. 7** **13. Ans. (B,C)** **14. Ans. (C)** **15. Ans. (B)**

03_Current Electricity

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (D) **2. Ans. (B)** **3. Ans. (A)** **4. Ans. (A)** **5. Ans. (A)** **6. Ans. (B)**
7. Ans. (B) **8. Ans. (A)** **9. Ans. (A)**

Multiple Correct Answer Type Question

10. Ans. (B, D) **11. Ans. (A, B,D)** **12. Ans. (A,B,C)**

Advanced Subjective (AS)

1. Ans. (i) 1.01Ω (ii) 0-5 A, 0-10V, (ii) 0.05 A **2. Ans.** 7.5 m, 8.75m, 6.25m **3. Ans.** 2

EXERCISE (JA)

1. Ans. (D) **2. Ans. (C)** **3. Ans. (C)** **4. Ans. 4** **5. Ans. 5** **6. Ans. (B)**
7. Ans. (A,B,C,D) **8. Ans. (B,D)** **9. Ans. (A,B,D)** **10. Ans. 5**
11. Ans. (C) **12. Ans. (B)** **13. Ans. 1** **14. Ans. (A)** **15. Ans. (C,D)**
16. Ans. (B,C) **17. Ans. (A)** **18. Ans. (B)** **19. Ans. (1,3)**

04_Capacitance

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (B) **2. Ans. (D)** **3. Ans. (C)** **4. Ans. (D)** **5. Ans. (B)** **6. Ans. (A)**
7. Ans. (D) **8. Ans. (D)** **9. Ans. (D)** **10. Ans. (C)**

Multiple Correct Answer Type Question

11. Ans. (A,B,C,D)

Matrix Match Type Question

12. Ans. (C)

Advanced Subjective (AS)

1. Ans. (a) $\frac{100}{7}$ volts; (b) 28.56 μC, 42.84 μC, 71.4 μC, 22.88 μC

2. Ans. (i) 1.5×10^4 V/m, 4.5×10^4 V/m, (ii) 75 V, 225 V, (iii) 8×10^{-7} C/m^2

3. Ans. (i) 0.2×10^{-8} F, 1.2×10^{-5} J ; (ii) 4.84×10^{-5} J ; (iii) 1.1×10^{-5} J

4. Ans. $q = \frac{CV}{2}\left(1-\frac{1}{2}e^{-t/RC}\right)$ **5. Ans.** 1

EXERCISE (JA)

1. Ans. 2 **2. Ans. (D)** **3. Ans. (C)** **4. Ans. (B,D)** **5. Ans. (A,D)** **6. Ans. (D)**
7. Ans. (A,B,C,D) **8. Ans. (A)** **9. Ans. (A)** **10. Ans. 1.50** **11. Ans. (3,4)**
12. Ans. (1.00)

05_Magnetic Effect of Current

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (C) **2. Ans. (A)** **3. Ans. (A)** **4. Ans. (C)** **5. Ans. (A)** **6. Ans. (C)**
7. Ans. (C) **8. Ans. (B)** **9. Ans. (C)** **10. Ans. (D)**

Multiple Correct Answer Type Question

11. Ans. (A,B,C)

Matrix Match Type Question

12. Ans. (A) → (P,R, T) ; (B) → (P,Q) ; (C) → (S) ; (D) → (P)

Advanced Subjective (AS)

1. Ans. $\sqrt{15}$ C **2. Ans.** (a) 6.6×10^{-5}T, (b) 0, 0, $2 \times 10^{-5}\,\ell n\left(\frac{4}{3}\right)$N

3. Ans. (a) current in loop PQRS is clockwise from P to QRS., (b) $\vec{F} = BI_0b\,(3\hat{k}-4\hat{i})$, (c) $I = \frac{mg}{6bB_0}$

4. Ans. $\omega = \frac{dT_0}{QR^2B}$ **5. Ans.** (a) k = NAB, (b) $C = \frac{2i_0NAB}{\pi}$, (c) $Q \times \sqrt{\frac{NAB\pi}{2li_0}}$

6. Ans. $z = 0$, $x = \pm\frac{d}{\sqrt{3}}$, (ii) $\frac{I}{2\pi d}\sqrt{\frac{\mu_0}{\pi\lambda}}$ **7. Ans.** 8 **8. Ans.** (a) $\frac{3mv^2}{4qa}$, (b) $\frac{3mv^3}{4a}$, (c) zero

9. Ans. (i) $-\frac{\mu_0 I}{4R} q v_0\,\hat{k}$ (ii) $F_1 = 2IRB$ $F_2 = 2IRB$, Net force $= F_1 + F_2 = 4IRB\,\hat{i}$

EXERCISE (JA)

1. Ans. (C) **2. Ans. (A)** **3. Ans. (B)** **4. Ans. (BC,BD, BCD)** **5. Ans. (A)**
6. Ans. (C,D) **7. Ans. 5** **8. Ans. (D)** **9. Ans. (B)** **10. Ans. (A, C)**
11. Ans. (A, D) **12. Ans. 3** **13. Ans. (C)** **14. Ans. (B)** **15. Ans. (A,B,C)**
16. Ans. (A) **17. Ans. (C)** **18. Ans. (D)** **19. Ans. (B)** **20. Ans. (A) (B)**
21. Ans. (A,B,D) **22. Ans. 2 [1.99, 2.01]**

06_Electromagnetic induction & Alternating current

Advanced Objective (AO)

Single Correct Answer Type Question

1. Ans. (A) **2. Ans. (C)** **3. Ans. (D)** **4. Ans. (C)** **5. Ans. (A)** **6. Ans. (A)**
7. Ans. (D) **8. Ans. (C)**

Multiple Correct Answer Type Question

9. Ans. (A, B, C) **10. Ans. (A, B)**

Linked Comprehension Type Question

11. Ans. (D) **12. Ans. (B)** **13. Ans. (B, D)** **14. Ans. (C)** **15. Ans. (D)**

Advanced Subjective (AS)

1. Ans. $I_{EA}=\frac{7}{22}$ A ; $I_{BE}=\frac{3}{11}$ A ; $I_{FE}=\frac{1}{22}$ A

2. Ans. $q = Q_0\sin\left(\sqrt{\frac{1}{LC}}\,t+\frac{\pi}{2}\right)$

3. Ans. 20 A, $\pi/4$,

4. Ans. $\frac{\mu_0 h\omega i_m N}{2\pi}\ln\frac{b}{a}$

5. Ans. (a) $i=\frac{B_0av}{R}$ in anticlockwise direction, v = velocity at time t, (b) $F_{nett}=B_0^2a^2V/R$,

(c) $V=\frac{mgR}{B_0^2a^2}\left(1-e^{-\frac{B_0^2a^2t}{mR}}\right)$

6. Ans. 67/32 A

7. Ans. (i) $i_1=i_2=10/3$ A, (ii) $i_1=50/11$ A; $i_2=30/11$ A, (iii) $i_1=0$, $i_2=20/11$ A, (iv) $i_1=i_2=0$

8. Ans. (a) $E=\frac{1}{2}B\omega r^2$ (b) (i) $I=\frac{B\omega r^2\left[1-e^{-Rt/L}\right]}{2R}$, (ii) $\tau=\frac{mgr}{2}\cos\omega t+\frac{\omega B^2r^4}{4R}(1-e^{-Rt/L})$

9. Ans. $2\pi\frac{\sqrt{mL}}{lB}, g\frac{\sqrt{mL}}{lB}$

10. Ans. $-\frac{V}{R}e^{-\frac{Rt}{L}}$

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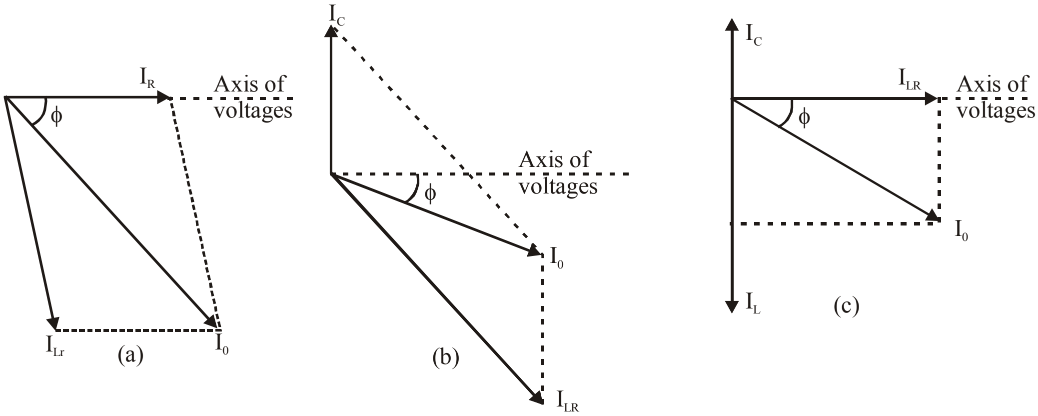
11. Ans.



12. Ans. $I = \dfrac{(\mu_0 n i_0 \omega \cos \omega t)\pi a^2 (Ld)}{\rho 2\pi R}$

EXERCISE (JA)

1. Ans. (B) **2. Ans. (C)** **3. Ans. (A)-R,S,T; (B)-Q,R,S,T; (C)-P,Q; (D)-Q,R,S,T**

4. Ans. (C) **5. Ans. (B,C)** **6. Ans. 6** **7. Ans. 4** **8. Ans. 7** **9. Ans. (A,C)**

10. Ans. (C) or (AC) **11. Ans. (B)** **12. Ans. (B)** **13. Ans. (A)** **14. Ans. (B)**

15. Ans. (C,D) **16. Ans. (A,D)** **17. Ans. (A,C)** **18. Ans. (B,D)** **19. Ans. 8** **20. Ans. (C, D)**

21. Ans. (A, D) **22. Ans. (A,B)** **23. Ans. (A), (B), (C)** **24. Ans. (C), (D)**

25. Ans. (B,D) **26. Ans. (1,2,4)** **27. Ans. (0.63)**