WAVE ON STRING

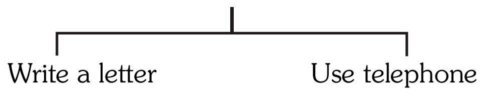
KEY CONCEPTS

INTRODUCTION OF WAVES

What is wave motion ?

- When a particle moves through space, it carries KE with itself. Wherever the particle goes, the energy goes with it. (One way of transport energy from one place to another place)
- There is another way (wave motion) to transport energy from one part of space to other without any bulk motion of material together with it. Sound is transmitted in air in this manner.

Ex. You (Kota) want to communicate your friend (Delhi)



1st option involves the concept of particle & the second choice involves the concept of wave.

Ex. When you say "Namaste" to your friend no material particle is ejected from your lips to fall on your friends ear. Basically you create some disturbance in the part of the air close to your lips. Energy is transferred to these air particles either by pushing them ahead or pulling them back. The density of the air in this part temporarily increases or decreases. These disturbed particles exert force on the next layer of air, transferring the disturbance to that layer. In this way, the disturbance proceeds in air and finally the air near the ear of the listener gets disturbed.

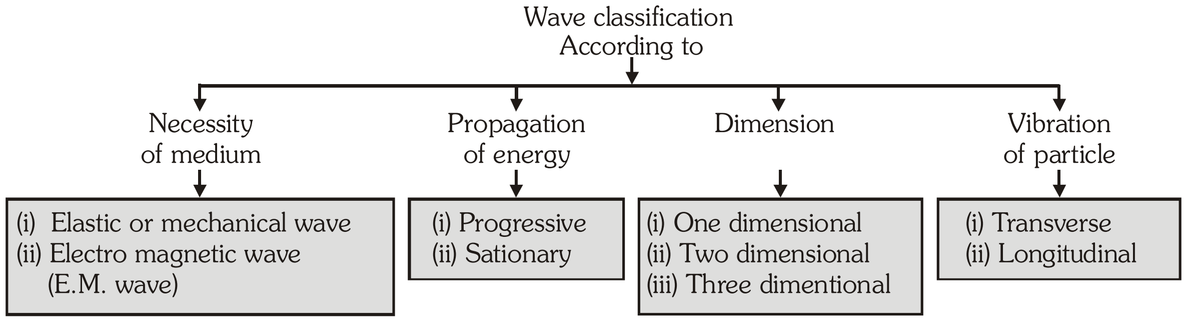
Note :- In the above example air itself does not move.

A **wave** is a disturbance that propagates in space, transports energy and momentum from one point to another without the transport of matter.

Few examples of waves :

The ripples on a pond (water waves), the sound we hear, visible light, radio and TV signals etc.

CLASSIFICATION OF WAVES



1. ***Based on medium necessity*** :- A wave may or may not require a medium for its propagation. The waves which do not require medium for their propagation are called non-mechanical, e.g. light, heat (infrared), radio waves etc. On the other hand the waves which require medium for their propagation are called mechanical waves. In the propagation of mechanical waves elasticity and density of the medium play an important role therefore mechanical waves are also known as **elastic waves**.

Example : Sound waves in water, seismic waves in earth's crust.

2. **Based on energy propagation :-** Waves can be divided into two parts on the basis of energy propagation (i) Progressive wave (ii) Stationary waves. The progressive wave propagates with fixed velocity in a medium. In stationary waves particles of the medium vibrate with different amplitude but energy does not propagate.

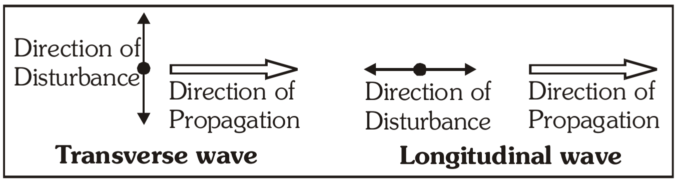
3. ***Based on direction of propagation*** :- Waves can be one, two or three dimensional according to the number of dimensions in which they propagate energy. Waves moving along strings are one-dimensional. Surface waves or ripples on water are two dimensional , while sound or light waves from a point source are three dimensional.

4. ***Based on the motion of particles of medium*** :

Waves are of two types on the basis of motion of particles of the medium.

(i) Longitudinal waves

(ii) Transverse waves



In the transverse wave the direction associated with the disturbance (i.e. motion of particles of the medium) is at right angle to the direction of propagation of wave while in the longitudinal wave the direction of disturbance is along the direction of propagation.

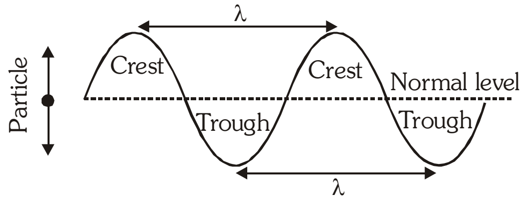
TRANSVERSE WAVE MOTION

Mechanical transverse waves produce in such type of medium which have shearing property, so they are known as shear wave or S-wave

Note :- Shearing is the property of a body by which it changes its shape on application of force.

⇒ Mechanical transverse waves are generated only in solids & surface of liquid.

In this individual particles of the medium execute SHM about their mean position in direction $\perp^r$ to the direction of propagation of wave motion.



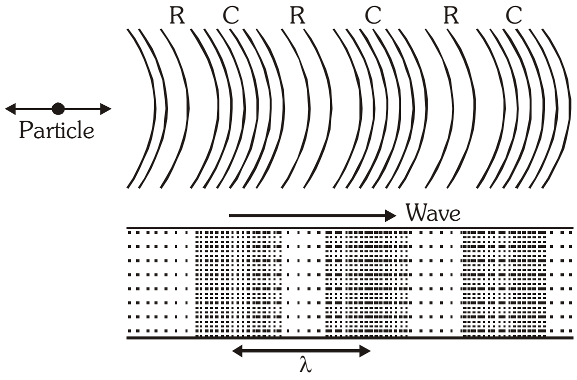
A **crest** is a portion of the medium, which is raised temporarily above the normal position of rest of particles of the medium , when a transverse wave passes.

A **trough** is a portion of the medium, which is depressed temporarily below the normal position of rest of particles of the medium , when a transverse wave passes.

LONGITUDINAL WAVE MOTION

In this type of waves, oscillatory motion of the medium particles produces regions of compression (high pressure) and rarefaction (low pressure) which propagated in space with time (see figure).

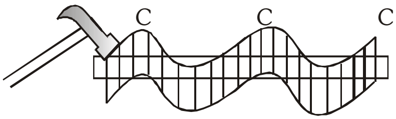
Note : The regions of high particle density are called compressions and regions of low particle density are called rarefactions.



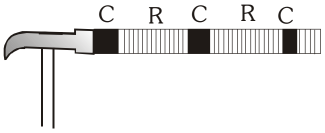
- The propagation of sound waves in air is visualized as the propagation of pressure or density fluctuations. The pressure fluctuations are of the order of 1 Pa, whereas atmospheric pressure is 10^5 Pa.

Mechanical Waves in Different Media

- A mechanical wave will be transverse or longitudinal depends on the nature of medium and mode of excitation.
- In strings mechanical waves are always transverse when string is under a tension. In gases and liquids mechanical waves are always longitudinal e.g. sound waves in air or water. This is because fluids cannot sustain shear.
- In solids, mechanical waves (may be sound) can be either transverse or longitudinal depending on the mode of excitation. The speed of the two waves in the same solid are different. (Longitudinal waves travels faster than transverse waves). e.g., if we struck a rod at an angle as shown in fig. (A) the waves in the rod will be transverse while if the rod is struck at the side as shown in fig. (B) or is rubbed with a cloth the waves in the rod will be longitudinal. In case of vibrating tuning fork waves in the prongs are transverse while in the stem are longitudinal.



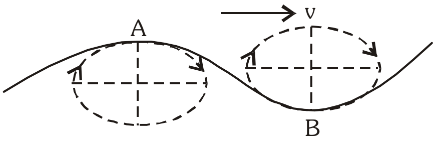
Transverse-wave
(A)



Longitudinal-wave
(B)

Further more in case of seismic waves produced by Earthquakes both S (shear) and P (pressure) waves are produced simultaneously which travel through the rock in the crust at different speeds [$v_S \cong 5$ km/s while $v_P \cong 9$ km/s] S–waves are transverse while P–waves longitudinal.

Some waves in nature are neither transverse nor longitudinal but a combination of the two. These waves are called 'ripple' and waves on the surface of a liquid are of this type. In these waves particles of the medium vibrate up and down and back and forth simultaneously describing ellipses in a vertical plane [Fig.]



Ripple

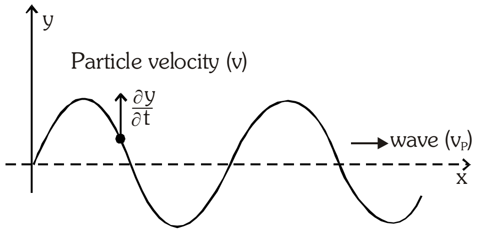
CHARACTERISTICS OF WAVE MOTION

Some of the important characteristics of wave motion are as follows :

- In a wave motion, the disturbance travels through the medium due to repeated periodic oscillations of the particles of the medium about their mean positions.
- The energy is transferred from place to another without any actual transfer of the particles of the medium.
- Each particle receives disturbance a little later than its preceding particle i.e., there is a regular phase difference between one particle and the next.
- The velocity with which a wave travels is different from the velocity of the particles with which they vibrate about their mean positions.
- The wave velocity remains constant in a given medium while the particle velocity changes continuously during its vibration about the mean position. It is maximum at the mean position and zero at the extreme position.
- For the propagation of a mechanical wave, the medium must possess the properties of inertia, elasticity and minimum friction amongst its particles.

SOME IMPORTANT TERMS CONNECTED WITH WAVE MOTION

- **Wavelength (λ) [length of one wave]**
  Distance travelled by the wave during the time, any one particle of the medium completes one vibration about its mean position. We may also define wavelength as the distance between any two nearest particles of the medium, vibrating in the same phase.
- **Frequency (n)** :Number of vibrations (Number of complete wavelengths) complete by a particle in one second.
- **Time period (T)** : Time taken by wave to travel a distance equal to one wavelength.
- **Amplitude (A)** : Maximum displacement of vibrating particle from its equilibrium position.
- **Angular frequency (ω) :** It is defined as $\omega = \frac{2\pi}{T} = 2\pi n$
- **Phase :** Phase is a quantity which contains all information related to any vibrating particle in a wave. For equation $y = A \sin(\omega t - kx)$; $(\omega t - kx)$ = phase.
- **Angular wave number (k) :** It is defined as $k = \frac{2\pi}{\lambda}$
- **Wave number ($\vec{v}$) :** It is defined as $\vec{v} = \frac{1}{\lambda} = \frac{k}{2\pi}$ = number of waves in a unit length of the wave pattern.
- **Particle velocity, wave velocity and particle's acceleration :** In plane progressive harmonic wave particles of the medium oscillate simple harmonically about their mean position. Therefore, all the formulae what we have read in SHM apply to the particles here also. For example, maximum particle velocity is $\pm A\omega$ at mean position and it is zero at extreme positions etc. Similarly maximum particle acceleration is $\pm\omega^2 A$ at extreme positions and zero at mean position. However the wave velocity is different from the particle velocity. This depends on certain characteristics of the medium. Unlike the particle velocity which oscillates simple harmonically (between $+A\omega$ and $-A\omega$) the wave velocity is constant for given characteristics of the medium.
- **Particle velocity in wave motion :**
  The individual particles which make up the medium do not travel through the medium with the waves. They simply oscillate about their equilibrium positions. The instantaneous velocity of an oscillating particle of the medium, through which a wave is travelling, is known as "Particle velocity".



- **Wave velocity :** The velocity with which the disturbance, or planes of equal phase (wave front), travel through the medium is called wave (or phase) velocity.
- **Relation between particle velocity and wave velocity :**

Wave equation :- $y = A \sin(\omega t - kx)$, Particle velocity $v = \frac{\partial y}{\partial t} = A\omega \cos(\omega t - kx)$.

Wave velocity = $v_P = \frac{\lambda}{T} = \lambda \frac{\omega}{2\pi} = \frac{\omega}{k}$, $\frac{\partial y}{\partial x} = -Ak\cos(\omega t - kx) = -\frac{A}{\omega}\omega k \cos(\omega k - kx) = -\frac{1}{v_p}\frac{\partial y}{\partial t}$

$$\Rightarrow \frac{\partial y}{\partial x} = -\frac{1}{v_P}\frac{\partial y}{\partial t}$$

Note : $\frac{\partial y}{\partial x}$ represent the slope of the string (wave) at the point x.

Particle velocity at a given position and time is equal to negative of the product of wave velocity with slope of the wave at that point at that instant.

- **Differential equation of harmonic progressive waves :**

$$\frac{\partial^2 y}{\partial t^2} = -A\omega^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = -Ak^2 \sin(\omega t - kx) \Rightarrow \frac{\partial^2 y}{\partial x^2} = \frac{1}{v_P^2}\frac{\partial^2 y}{\partial t^2}$$

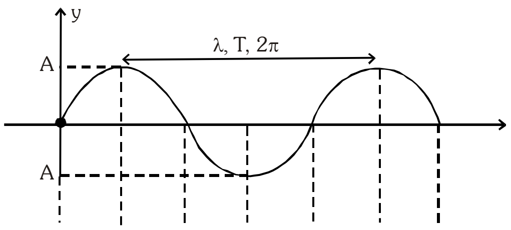
- **Particle velocity (v_p) and acceleration (a_p) in a sinusoidal wave :**

The acceleration of the particle is the second particle is the second partial derivative of y (x, t) with respect to t,

$$\therefore\ a_P = \frac{\partial^2 y(x,t)}{\partial t^2} = \omega^2 A \sin(kx - \omega t) = -\omega^2 y(x,t)$$

i.e., the acceleration of the particle equals $-\omega^2$ times its displacement, which is the result we obtained for SHM. Thus, $a_p = -w2$ (displacement)

- **Relation between Phase difference, Path difference & Time difference**



Phase (ϕ)	0	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π	$\frac{5\pi}{2}$	3π
Wave length (λ)	0	$\frac{\lambda}{4}$	$\frac{\lambda}{2}$	$\frac{3\lambda}{4}$	λ	$\frac{5\lambda}{4}$	$\frac{3}{2}\lambda$
Time-period (T)	0	$\frac{T}{4}$	$\frac{T}{2}$	$\frac{3T}{4}$	T	$\frac{5T}{4}$	$\frac{3T}{2}$

$$\Rightarrow \frac{\Delta\phi}{2\pi} = \frac{\Delta\lambda}{\lambda} = \frac{\Delta T}{T} \Rightarrow \text{Path difference} = \left(\frac{\lambda}{2\pi}\right) \text{Phase difference}$$

Ex. A progressive wave of frequency 500 Hz is travelling with a velocity of 360 m/s. How far apart are two points 60° out of phase.

Sol. We know that for a wave v = f λ So $\lambda = \frac{v}{f} = \frac{360}{500} = 0.72$ m

Phase difference $\Delta\phi = 60^\circ = (\pi/180) \times 60 = (\pi/3)$ rad,

so path difference $\Delta x = \frac{\lambda}{2\pi}(\Delta\phi) = \frac{0.72}{2\pi} \times \frac{\pi}{3} = 0.12$ m

THE GENERAL EQUATION OF WAVE MOTION

Some physical quantity (say y) is made to oscillate at one place and these oscillations of y propagate to other places. The y may be,

(i) displacement of particles from their mean position in case of transverse wave in a rope or longitudinal sound wave in a gas.

(ii) pressure difference (dP) or density difference (dρ) in case of sound wave or

(iii) electric and magnetic fields in case of electromagnetic waves.

The oscillations of y may or may not be simple harmonic in nature. Consider one-dimensional wave travelling along x-axis. In this case y is a function of x and t. i.e. y = f(x, t) But only those function of x & t, represent a wave motion which satisfy the differential equation. $\frac{\partial^2 y}{\partial t^2} = v^2 \frac{\partial^2 y}{\partial x^2}$ ...(i)

The general solution of this equation is of the form y (x, t) = f (ax ± bt) ...(ii)
Thus, any function of x and t and which satisfies equation (i) or which can be written as equation (ii) represents a wave. The only condition is that it should be finite everywhere and at all times, Further, if these conditions are satisfied, then speed of wave (v) is given by $v = \frac{\text{coefficient of t}}{\text{coefficient of x}} = \frac{b}{a}$

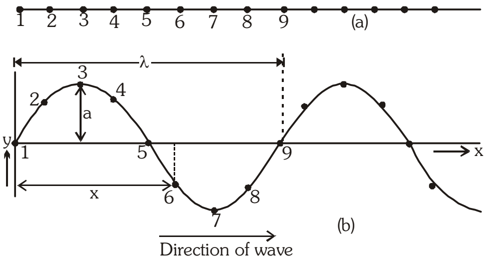
Ex. Which of the following functions represent a travelling wave ?

(a) $(x - vt)^2$ (b) $\ell n(x + vt)$ (c) $e^{-(x-vt)^2}$ (d) $\frac{1}{x + vt}$

Sol. Although all the four functions are written in the form f (ax $\pm$ bt), only (c) among the four functions is finite everywhere at all times. Hence only (c) represents a travelling wave.

Equation of a Plane Progressive Wave

If, on the propagation of wave in a medium, the particles of the medium perform simple harmonic motion then the wave is called a 'simple harmonic progressive wave'. Suppose, a simple harmonic progressive wave is propagating in a medium along the positive direction of the x-axis (from left to right). In fig. (a) are shown the equilibrium positions of the particles 1, 2, 3 ....



When the wave propagates, these particles oscillate about their equilibrium positions. In Fig. (b) are shown the instantaneous positions of these particles at a particular instant. The curve joining these positions represents the wave. Let the time be counted from the instant when the particle 1 situated at the origin starts oscillating. If y be the displacement of this particle after t seconds, then y = a sin ωt...(i)

where a is the amplitude of oscillation and ω = 2π n, where n is the frequency. As the wave reaches the particles beyond the particle 1, the particles start oscillating. If the speed of the wave be v, then it will reach particle 6, distant x from the particle 1, in x/v sec. Therefore, the particle 6 will start oscillating x/v sec after the particle 1. It means that the displacement of the particle 6 at a time t will be the same as that of the particle 1 at a time x/v sec earlier i.e. at time t – (x/v). The displacement of particle 1 at time t – (x/v) can be the particle 6, distant x from the origin (particle 1), at time t is given by

$y = a \sin \omega \left(t - \frac{x}{v}\right)$ But $\omega = 2\pi n$, $y = a \sin(\omega t - kx)$ $\left(k = \frac{\omega}{v}\right)$ ...(ii)

$y = a \sin \left[\frac{2\pi}{T} t - \frac{2\pi}{\lambda} x\right]$ Also $k = \frac{2\pi}{\lambda}$ ...(iii) $y = a \sin 2\pi \left[\frac{t}{T} - \frac{x}{\lambda}\right]$ ...(iv)

This is the equation of a simple harmonic wave travelling along +x direction. If the wave is travelling along the –x direction then inside the brackets in the above equations, instead of minus sign there will be plus sign. For example, equation (iv) will be of the following form : $y = a \sin 2\pi\left(\frac{t}{T}+\frac{x}{\lambda}\right)$. If ϕ be the phase difference between the above wave travelling along the +x direction and an other wave, then the equation of that wave will be

$$y = a \sin\left\{2\pi\left(\frac{t}{T}-\frac{x}{\lambda}\right)\pm\phi\right\}$$

Ex. The equation of a wave is, $y(x,t) = 0.05\sin\left[\frac{\pi}{2}(10x-40t)-\frac{\pi}{4}\right]$m

Find: (a) The wavelength, the frequency and the wave velocity

(b) The particle velocity and acceleration at x=0.5 m and t=0.05 s.

Sol. : (a) The equation may be rewritten as, $y(x,t) = 0.05\sin\left(5\pi x-20\pi t-\frac{\pi}{4}\right)$m

Comparing this with equation of plane progressive harmonic wave,

$y(x,t) = A\sin(kx-\omega t+\phi)$ we have, wave number $k = \frac{2\pi}{\lambda} = 5\pi\,\text{rad/m}$ $\therefore \lambda = 0.4\text{m}$

The angular frequency is, $\omega = 2\pi f = 20\pi\,\text{rad/s}$ $\therefore$ f = 10Hz

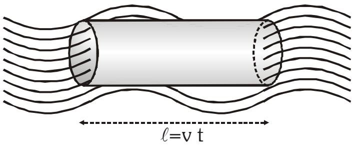
The wave velocity is, $v = f\lambda = \frac{\omega}{k} = 4\text{ms}^{-1}$ in + x direction

(b) The particle velocity and acceleration are, $v_p = \frac{\partial y}{\partial t} = -(20\pi)(0.05)\cos\left(\frac{5\pi}{2}-\pi-\frac{\pi}{4}\right)$ =2.22m/s

$a_p = \frac{\partial^2 y}{\partial t^2} = -(20\pi)^2(0.05)\sin\left(\frac{5\pi}{2}-\pi-\frac{\pi}{4}\right)$ =140 m/s^2

INTENSITY OF WAVE

The amount of energy flowing per unit area and per unit time is called the intensity of wave. It is represented by I. Its units are J/m^{2}s or watt/metre^2. $I = 2\pi^2 f^2 A^2 \rho v$ i.e. $I \propto A^2$ and $I \propto A^2$. If P is the power of an isotropic point source, then intensity at a distance r is given by,



$$I = \frac{P}{4\pi r^2} \text{ or } I \propto \frac{1}{r^2} \text{ (for a point source)}$$

If P is the power of a line source, then intensity at a distance r is given by,

$$I = \frac{P}{2\pi r\ell} \text{ or } I \propto \frac{1}{r}$$ (for a line source) As, $I \propto A^2$

Therefore, $A \propto \frac{1}{r}$ (for a point source) and $A \propto \frac{1}{\sqrt{r}}$ (for a line source)

SUPERPOSITION PRINCIPLE

Two or more waves can propagate in the same medium without affecting the motion of one another. If several waves propagate in a medium simultaneously, then the resultant displacement of any particle of the medium at any instant is equal to the vector sum of the displacements produced by individual wave. The phenomenon of intermixing of two or more waves to produce a new wave is called Superposition of waves. Therefore according to superposition principle.

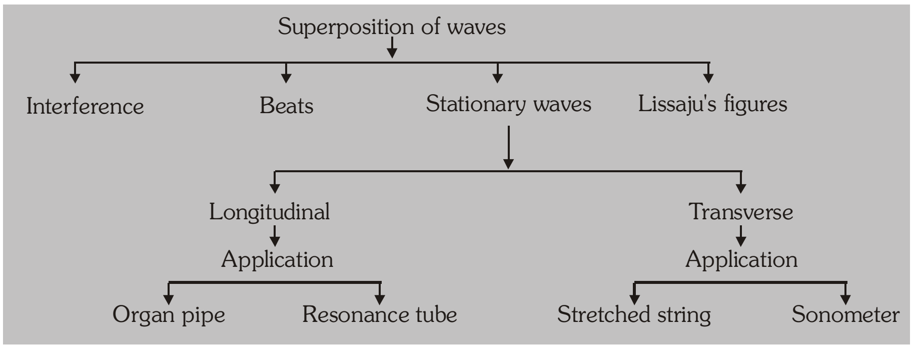
> The resultant displacement of a particle at any point of the medium, at any instant of time is the vector sum of the displacements caused to the particle by the individual waves.

If $\vec{y}_1, \vec{y}_2, \vec{y}_3, ...$ are the displacement of particle at a particular time due to individual waves, then the resultant displacement is given by $\vec{y} = \vec{y}_1 + \vec{y}_2 + \vec{y}_3 + ...$

Principle of superposition holds for all types of waves, i.e., mechanical as well as electromagnetic waves. But this principle is not applicable to the waves of very large amplitude.

Due to superposition of waves the following phenomenon can be seen

- **Interference :** Superposition of two waves having equal frequency and nearly equal amplitude.
- **Beats :** Superposition of two waves of nearly equal frequency in same direction.
- **Stationary waves :** Superposition of equal wave from opposite direction.
- **Lissajous' figure :** Superposition of perpendicular waves.



INTERFERENCE OF WAVES :

When two waves of equal frequency and nearly equal amplitude travelling in same direction having same state of polarisation in medium superimpose, then intensity is different at different points. At some points intensity is large, whereas at other points it is nearly zero.

Consider two waves $y_1 = A_1 \sin(\omega t - kx)$ and $y_2 = A_2 \sin(\omega t - kx + \phi)$

By principle of superposition $y = y_1 + y_2 = A \sin(\omega t - kx + \delta)$

$$\text{where} \qquad A^2 = A_1^2 + A_2^2 + 2A_1A_2\cos\phi \text{ and } \tan\delta = \frac{A_2\sin\phi}{A_1 + A_2\cos\phi}$$

As intensity $I \propto A^2$ so $I = I_1 + I_2 + 2\sqrt{I_1 I_2}\cos\phi$

- **Constructive interference (maximum intensity) :**

Phase difference $\phi = 2n\pi$ or path difference $= n\lambda$ where $n = 0, 1, 2, 3, ...$

$$\Rightarrow A_{max} = A_1 + A_2 \qquad \text{and} \qquad I_{max} = I_1 + I_2 + 2\sqrt{I_1 I_2}$$

- **Destructive interference (minimum intensity) :**

  Phase difference $\phi = (2n+1)\pi$, or path difference $= (2n-1)\,\dfrac{\lambda}{2}$ where n = 0, 1, 2, 3, ...

  $\Rightarrow A_{min} = A_1 - A_2$ and $I_{min} = I_1 + I_2 - 2\sqrt{I_1 I_2}$

KEY POINTS

- Maximum and minimum intensities in any interference wave form. $\dfrac{I_{Max}}{I_{Min}} = \left(\dfrac{\sqrt{I_1}+\sqrt{I_2}}{\sqrt{I_1}-\sqrt{I_2}}\right)^2 = \left(\dfrac{a_1+a_2}{a_1-a_2}\right)^2$
- Average intensity of interference wave form :– $< I >$or $I_{av} = \dfrac{I_{max}+I_{min}}{2} = I_1 + I_2$

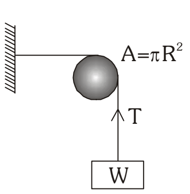
  if $a = a_1 = a_2$ and $I_1 = I_2 = I$ then $I_{max} = 4I$, $I_{min} = 0$ and $I_{AV} = 2I$
- Degree of interference Pattern (f) : Degree of hearing (Sound Wave) or

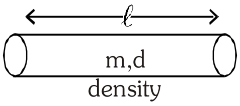
  Degree of visibility (Light Wave) $f = \dfrac{I_{max}-I_{min}}{I_{max}+I_{min}} \times 100$

  In condition of perfect interference degree of interference pattern is maximum $f_{max} = 1$ or 100%
- Condition of maximum contrast in interference wave form $a_1 = a_2$ and $I_1 = I_2$ then $I_{max} = 4I$ and $I_{min} = 0$

  For perfect destructive interference we have a maximum contrast in interference wave form.

VELOCITY OF TRANSVERSE WAVE





Mass of per unit length $m = \dfrac{\pi r^2 \ell \times d}{\ell}$, $m = \pi r^2 d$, where d = Density of matter

Velocity of transverse wave in any wire $v = \sqrt{\dfrac{T}{m}}$ or $\sqrt{\dfrac{T}{\pi r^2 d}} = \sqrt{\dfrac{T}{Ad}}$ $\because \pi r^2 = A$

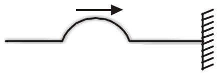
- If m is constant then, $v \propto \sqrt{T}$ it is called tension law.
- If tension is constant then $v \propto \sqrt{\dfrac{1}{m}}$ ← it is called law of mass
- If T is constant & take wire of different radius for same material then $v \propto \dfrac{1}{r}$ ← it is called law of radius
- If T is constant & take wire of same radius for different material. Then $v \propto \sqrt{\dfrac{1}{d}}$ ← law of density

REFLECTION FROM RIGID END

When the pulse reaches the right end which is clamped at the wall, the element at the right end exerts a force on the clamp and the clamp exerts equal and opposite force on the element. The element at the right end is thus acted upon by the force from the clamp. As this end remains fixed, the two forces are opposite to each other. The force from the left part of the string transmits the forward wave pulse and hence, the force exerted by the clamp sends a return pulse on the string whose shape is similar to a return pulse but is inverted. The original pulse tries to pull the element at the fixed end up and the return pulse sent by the clamp tries to pull it down, so the resultant displacement is zero. Thus, the wave is reflected from the fixed end and the reflected wave is inverted with respect to the original wave. The shape of the string at any time, while the pulse is being reflected, can be found by adding an inverted image pulse to the incident pulse.

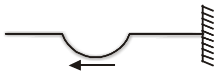
Equation of wave propagating in +ve x-axis

Incident wave $y_1 = a \sin(\omega t - kx)$



Reflected wave $y_2 = a \sin(\omega t + kx + \pi)$

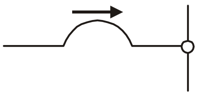
or $y_2 = -a \sin(\omega t + kx)$



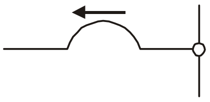
REFLECTION FROM FREE END

The right end of the string is attached to a light frictionless ring which can freely move on a vertical rod. A wave pulse is sent on the string from left. When the wave reaches the right end, the element at this end is acted on by the force from the left to go up. However, there is no corresponding restoring force from the right as the rod does not exert a vertical force on the ring. As a result, the right end is displaced in upward direction more than the height of the pulse i.e., it overshoots the normal maximum displacement. The lack of restoring force from right can be equivalent described in the following way. An extra force acts from right which sends a wave from right to left with its shape identical to the original one. The element at the end is acted upon by both the incident and the reflected wave and the displacements add. Thus, a wave is reflected by the free end without inversion.

Incident wave $y_1 = a \sin(\omega t - kx)$



Reflected wave $y_2 = a \sin(\omega t + kx)$



STATIONARY WAVES

* **Definition :** The wave propagating in such a medium will be reflected at the boundary and produce a wave of the same kind travelling in the opposite direction. The superposition of the two waves will give rise to a stationary wave. Formation of stationary wave is possible only and only in bounded medium.

ANALYTICAL METHOD FOR STATIONARY WAVES

- **From rigid end** : We know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$
  After reflection from rigid end $y_2 = a \sin(\omega t + kx + \pi) = -a \sin(\omega t + kx)$
  By principle of super position. $y = y_1 + y_2 = a \sin(\omega t - kx) - a \sin(\omega t + kx) = -2a \sin kx \cos \omega t$
  This is equation of stationary wave reflected from rigid end

Amplitude $= 2a \sin kx$ **Velocity of particle** $v_{pa} = \frac{dy}{dt} = 2a\,\omega \sin kx \sin \omega t$

Strain $\frac{dy}{dx} = -2ak \cos kx \cos \omega t$ **Elasticity** $E = \frac{\text{stress}}{\text{strain}} = \frac{dp}{\frac{dy}{dx}}$ **Change in pressure** $dp = E\frac{dy}{dx}$

• **Node** $x = 0, \frac{\lambda}{2}, \lambda$ .... $A = 0$, $V_{pa} = 0$, strain → max **Change in pressure** → max

• **Antinode** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}$ .... $A \to$ max, $-V_{pa} \to$ max. strain = 0 **Change in pressure** = 0

- **From free end** : we know equation for progressive wave in positive x-direction $y_1 = a \sin(\omega t - kx)$
  After reflection from free end $y_2 = a \sin(\omega t + kx)$

By Principle of superposition $y = y_1 + y_2 = a\sin(\omega t - kx) + a\sin(\omega t + kx) = 2a\sin\omega t\cos kx$

Amplitude = $2a\cos kx$, **Velocity of particle** = $v_{Pa} = \frac{dy}{dt} = 2a\omega\cos\omega t\cos kx$

Strain $\frac{dy}{dx} = -2ak\sin\omega t\sin kx$ **Change in pressure** $dp = E\frac{dy}{dx}$

- **Antinode :** $x = 0, \frac{\lambda}{2}, \lambda$ .... $A \to$ Max, $V_{pa} = \frac{dy}{dt} \to$ max. Strain = 0, dp = 0
- **Node :** $x = \frac{\lambda}{4}, \frac{3\lambda}{4}, \frac{5\lambda}{4}$ .... $A = 0$, $V_{pa} = \frac{dy}{dt} = 0$, strain → max, dp → max

PROPERTIES OF STATIONARY WAVES

The stationary waves are formed due to the superposition of two identical simple harmonic waves travelling in opposite direction with the same speed.

Important characteristics of stationary waves are:–

(i) Stationary waves are produced in the bounded medium and the boundaries of bounded medium may be rigid or free.

(ii) In stationary waves nodes and antinodes are formed alternately. Nodes are the points which are always in rest having maximum strain. Antinodes are the points where the particles vibrate with maximum amplitude having minimum strain.

(iii) All the particles except at the nodes vibrate simple harmonically with the same period.

(iv) The distance between any two successive nodes or antinodes is λ/2.

(v) The amplitude of vibration gradually increases from zero to maximum value from node to antinode.

(vi) All the particles in one particular segment vibrate in the same phase, but the particle of two adjacent segments differ in phase by 180°

(vii) All points of the medium pass through their mean position simultaneously twice in each period.

(viii) Velocity of the particles while crossing mean position varies from maximum at antinodes to zero at nodes.

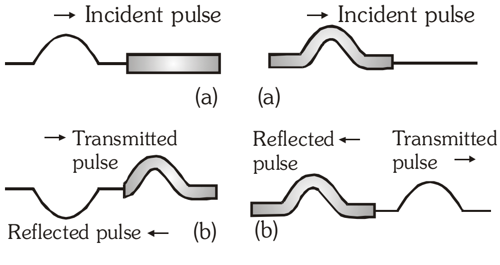
(ix) In a stationary wave the medium is splited into segments and each segment is vibrating up and down as a whole.

(x) In longitudinal stationary waves, condensation (compression) and refraction do not travel forward as in progressive waves but they appear and disappear alternately at the same place.

(xi) These waves do not transfer energy in the medium. Transmission of energy is not possible in a stationary wave.

TRANSMISSION OF WAVES

We may have a situation in which the boundary is intermediate between these two extreme cases, that is, one in which the boundary is neither rigid nor free. In this case, part of the incident energy is transmitted and part is reflected. For instance, suppose a light string is attached to a heavier string as in (figure). When a pulse travelling on the light reaches the knot, same part of it is reflected and inverted and same part of it is transmitted to the heavier string.



As one would expect, the reflected pulse has a smaller amplitude than the incident pulse, since part of the incident energy is transferred to the pulse in the heavier string. The inversion in the reflected wave is similar to the behaviour of a pulse meeting a rigid boundary, when it is totally reflected. When a pulse travelling on a heavy string strikes the boundary of a lighter string, as in (figure), again part is reflected and part is transmitted. However, in this case the reflected pulse is not inverted. In either case, the relative height of the reflected and transmitted pulses depend on the relative densities of the two string. In the previous section, we found that the speed of a wave on a string increases as the density of the string decreases. That is, a pulse travels more slowly on a heavy string than on a light string, if both are under the same tension. The following general rules apply to reflected waves. When a wave pulse travels from medium A to medium B and $v_A > v_B$ (that is, when B is denser than A), the pulse will be inverted upon reflection. When a wave pulse travels from medium A to medium B and $v_A < v_B$ (A is denser than B), it will not be inverted upon reflection.

KEY POINTS

Phenomenon of reflection and transmission of waves obeys the laws of reflection and refraction.

The frequency of these wave remains constant i.e. does not change. $\omega_i = \omega_r = \omega_t = \omega$

From rarer to denser medium $y_i = a_i \sin(\omega t - k_1x)$ $y_r = -a_i \sin(\omega t + k_1x)$ $y_t = a_t \sin(\omega t - k_2x)$

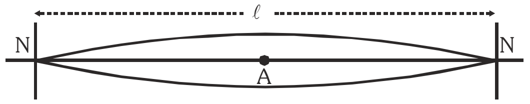
From denser to rarer medium $y_i = a_i \sin(\omega t - k_1x)$ $y_r = a_i \sin(\omega t + k_1x)$ $y_t = a_t \sin(\omega t - k_2x)$

STATIONARY WAVE ARE OF TWO TYPES :

(i) Transverse st. wave (stretched string) **(ii)** Longitudinal st. wave (organ pipes)

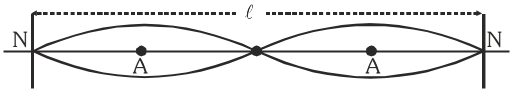
(i) Transverse Stationary wave

First Harmonic



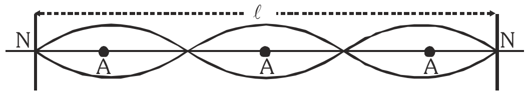
$$\frac{\lambda}{2} = \ell \Rightarrow \lambda = 2\ell$$

Second Harmonic



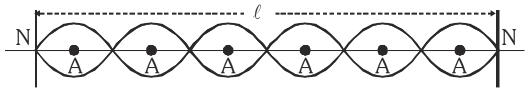
$$\frac{2\lambda}{2} = \ell \Rightarrow \lambda = \ell$$

Third Harmonic



$$\frac{3\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{3}$$

p th harmonic

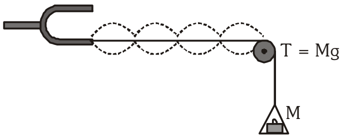


$$\frac{p\lambda}{2} = \ell \Rightarrow \lambda = \frac{2\ell}{p}$$

- **Law of length :** For a given string, under a given tension, the fundamental frequency of vibration is inversely proportional to the length of the string, i.e, $n \propto \frac{1}{\ell}$ (T and m are constant)
- **Law of tension :** The fundamental frequency of vibration of stretched string is directly proportional to the square root of the tension in the string, provided that length and mass per unit length of the string are kept constant. $n \propto \sqrt{T}$ (ℓ and m are constant)

- **Law of mass :** The fundamental frequency of vibration of a stretched string is inversely proportional to the square root of its mass per unit length provided that length of the string and tension in the string are kept constant, i.e., $n \propto \frac{1}{\sqrt{m}}$ (ℓ and T are constant)
- **Melde's experiment :** In Melde's experiment, one end of a flexible piece of thread is tied to the end of a tuning fork. The other end passed over a smooth pulley carries a pan which can be loaded. There are two arrangements to vibrate the tied fork with thread.

Transverse arrangement :

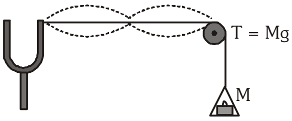


Case 1. In a vibrating string of fixed length, the product of number of loops and square root of tension are constant or $p\sqrt{T}$ = constant.

Case 2. When the tuning fork is set vibrating as shown in fig. then the prong vibrates at right angles to the thread. As a result the thread is set into motion. The frequency of vibration of the thread (string) is equal to the frequency of the tuning fork. If length and tension are properly adjusted then, standing waves are formed in the string. (This happens when frequency of one of the normal modes of the string matched with the frequency of the tuning fork). Then, if p loops are formed in the thread, then the frequency of the tuning fork is given by $n = \frac{p}{2\ell}\sqrt{\frac{T}{m}}$

Case 3. If the tuning fork is turned through a right angle, so that the prong vibrates along the length of the thread, then the string performs only a half oscillation for each complete vibrations of the prong. This is because the thread only makes node at the midpoint when the prong moves towards the pulley i.e. only once in a vibration.

Longitudinal arrangement :

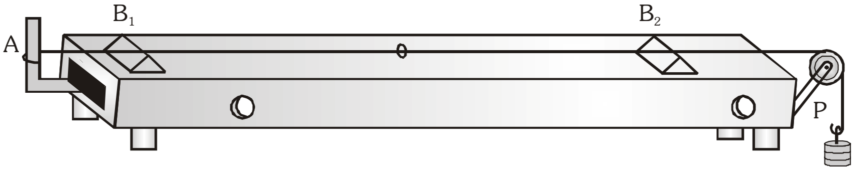


The thread performs sustained oscillations when the natural frequency of the given length of the thread under tension is half that of the fork.

Thus if p loops are formed in the thread, then the frequency of the tuning fork is $n = \frac{2p}{2\ell}\sqrt{\frac{T}{m}}$

SONOMETER :

Sonometer consists of a hollow rectangular box of light wood. One end of the experimental wire is fastened to one end of the box. The wire passes over a frictionless pulley P at the other end of the box. The wire is stretched by a tension T.



The box serves the purpose of increasing the loudness of the sound produced by the vibrating wire. If the length the wire between the two bridges is ℓ, then the frequency of vibration is $n = \frac{1}{2\ell}\sqrt{\frac{T}{m}}$

To test the tension of a tuning fork and string, a small paper rider is placed on the string. When a vibrating tuning fork is placed on the box, and if the length between the bridges is properly adjusted, then when the two frequencies are exactly equal, the string quickly picks up the vibrations of the fork and the rider is thrown off the wire. There are three laws of vibration of a wire.

COMPARISON OF PROGRESSIVE AND STATIONARY WAVES

	Progressive waves	**Stationary waves**
1.	These waves travels in a medium with definite velocity.	These waves do not travel and remain confined between two boundaries in the medium.
2.	These waves transmit energy in the medium.	These waves do not transmit energy in the medium.
3.	The phase of vibration varies continuously from particle to particle.	The phase of all the particles in between two nodes is always same. But particles of two Adjacent nodes differ in phase by 180°
4.	No particle of medium is Permanently at rest.	Particles at nodes are permanently at rest.
5.	All particles of the medium vibrate and amplitude of vibration is same.	The amplitude of vibration changes from particle to particle. The amplitude is zero for all at nodes and maximum at antinodes.
6.	All the particles do not attain the maximum displacement position simultaneously.	All the particles attain the maximum displacement

WORKED OUT EXAMPLES
GIVE THE SOLUTION

Ex.1 A wave is propagating along x-axis. The displacement of particles of the medium in z-direction at t = 0 is given by: $z = \exp[-(x+2)^2]$, where 'x' is in meters. At t = 1s, the same wave disturbance is given by: $z = \exp[-(2-x)^2]$. Then, the wave propagation velocity is

(A) 4 m/s in + x direction
(B) 4 m/s in –x direction
(C) 2 m/s in + x direction
(D) 2 m/s in – x direction

Ans. (D)

Ex.2 A transverse wave is propagating along +x direction. At t=2 sec, the particle at x=4m is at y = 2 mm. With the passage of time its y coordinate increases and reaches to a maximum of 4 mm. The wave equation may be (using ω and k with their usual meanings)

(A) $y = 4\sin(\omega(t+2)+k(x-2)+\frac{\pi}{6})$
(B) $y = 4\sin(\omega(t+2)+k(x)+\frac{\pi}{6})$
(C) $y = 4\sin(\omega(t-2)-k(x-4)+\frac{5\pi}{6})$
(D) $y = 4\sin(\omega(t-2)-k(x-4)+\frac{\pi}{6})$

Ans. (D)

Ex.3 $Y(x,t) = 0.05 / [(4x + 2t)^2 + 5]$ represents a moving wave pulse, where x and y are in meters and t is in seconds. Then which statement(s) are **CORRECT**:

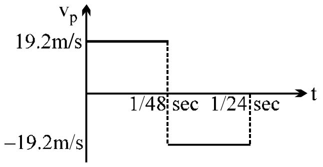
(A) Pulse is moving in – x direction (B) Wave speed is 0.5 m/s

(C) Maximum particle displacement is 1 cm (D) It is a symmetric pulse

Ans. **(A,B,C,D)**

Ex.4 A symmetrical triangular pulse of maximum height 0.4 m and total length 1 m is moving in the positive x-direction on a string on which the wave speed is 24 m/s. At t = 0 the pulse is entirely located between x = 0 and x = 1 m. Draw a graph of the transverse velocity of particle of string versus time at x = +1m.

Ans.



Ex.5 A transverse harmonic disturbance is produced in a string. The maximum transverse velocity is 3 m/s and maximum transverse acceleration is 90 m/s^2. If the wave velocity is 20 m/s then find the waveform. **[IIT-JEE 2005]**

Ans. $y = (10\text{ cm}) \sin\left(30\,t \pm \frac{3}{2}x + f\right)$

Ex.6 A non-uniform rope of mass M and length L has a variable linear mass density given by $\mu = kx$ where x is the distance from one end of the wire and k is a constant.

(a) Show that $M = kL^2/2$

(b) Show that the time required for a pulse generated at one end of the wire to travel to the other end is given by $t = \sqrt{8ML/9F}$ where F (constant)is the tension in the wire.

Ex.7 One end of a long string of linear mass density 10^{-2} kg m^{-1} is connected to an electrically driven tuning fork of frequency 150 Hz. The other end passes over a pulley and is tied to a pan containing a mass of 90 kg. The pulley end absorbs all the incoming energy so that reflected waves from this end have negligible amplitude. At t = 0, the left end (fork end) of the string is at x = 0 has a transverse displacement of 2.5 cm and is moving along positive y-direction. The amplitude of the wave is 5 cm. Write down the transverse displacement y (in cm) as function of x (in m) and t (in sec) that describes the wave on the string.

Ans. $y = 5\sin\left\{\pi\left(300t - x\right) + \frac{\pi}{6}\right\}$

Ex.8 If the tension in a stretched string fixed at both ends is changed by 20%, the fundamental frequency is found to increase by 15Hz. then the

(A) original frequency is 157 Hz

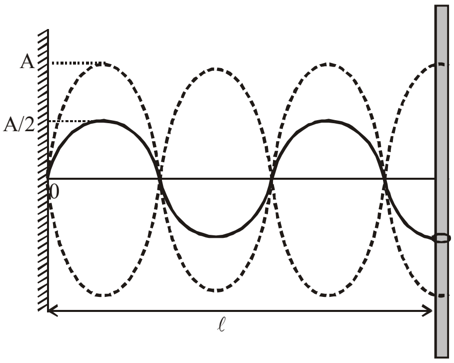
(B) velocity of propagation of the transverse wave along the string changes by 5%

(C) velocity of propagation of the transverse wave along the string changes by 10%.

(D) fundamental wave length on the string does not change.

Ans. **(A,C,D)**

Ex.9 Here given snap shot at $t = \frac{T}{12}$ of a standing wave. Then the equations of the wave will be when particles are moving towards their extreme and when particles are moving towards the mean position respectively $\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$.



(A) $y = A \sin kx \sin \omega t$, $y = A \sin\left(\omega t + \frac{2\pi}{3}\right) \sin kx$

(B) $y = A \sin kx \cos \omega t$, $y = A \sin\left(\omega t + \frac{2\pi}{3}\right) \sin kx$

(C) $y = A \cos kx \cos \omega t$, $y = A \cos\left(\omega t + \frac{2\pi}{3}\right) \sin kx$

(D) $y = A \cos kx \sin \omega t$, $y = A \cos\left(\omega t + \frac{2\pi}{3}\right) \cos kx$

Ans. **(A)**

Ex.10 A standing wave of time period T is set up in a string clamped between two rigid supports. At t = 0 antinode is at its maximum displacement 2A.

(A) The energy density of a node is equal to energy density of an antinode for the first time at t = T/4.

(B) The energy density of node and antinode becomes equal after T/2 second.

(C) The displacement of the particle at antinode at $t = \frac{T}{8}$ is $\sqrt{2}A$

(D) The displacement of the particle at node is zero

Ans. **(C,D)**

SOUND WAVE

KEY CONCEPTS

Sound is a mechanical three dimensional and longitudinal wave that is created by a vibrating source such as a guitar string, the human vocal cords, the prongs of a tuning fork or the diaphragm of a loudspeaker.

Sound waves are the most common example of longitudinal waves. They travel through any material waves. They travel through any material medium with a speed that depends on the properties of the medium. As the waves travel through air, the elements of air vibrate to produce changes in density and pressure along the direction of motion of the wave. If the source of the sound waves vibrates sinusoidally, the pressure variations are also sinusoidal. The mathematical description of sinusoidally, the pressure variations are also sinusoidal sound waves is very similar to that of sinusoidal string waves.

EQUATION OF SOUND WAVES

As the piston oscillates sinusoidally, regions of compression and rarefaction are continuously set up. The distance between two successive compressions (or two successive rarefactions) equals the wavelength λ. As these regions travel through the tube, any small element of the medium moves with simple harmonic motion parallel to the direction of the wave . If s(x, t) is the position os a small element relative to its equilibrium position, We can express this harmonic position function as

$$s(x, t) = s_{max} \cos(kx - \omega t)$$

Where s_{max} is the maximum position of the element relative to equilibrium. This is often called the displacement amplitude of the wave. The parameter k is the wave number and ω is the angular frequency of the piston . Note that the displacement of the element is along s, in the direction of propagation of the sound wave, which means we are describing a longitudinal wave.

Consider a thin disk-shaped element of gas whose circular cross section is parallel to the piston in figure. This element will undergo changes in position, pressure, and density as a sound wave propagates through the gas. From the definition of bulk modulus , the pressure variation in the gas is

$$\Delta P = -B\frac{\Delta V}{V_i}$$

The element has a thickness Δx in the horizontal direction and a cross-sectional area A, so its volume is $V_i = A\Delta x$. The change in volume ΔV accompanying the pressure change is equal to $A\Delta s$, where Δs is the difference between the value os s at $x + \Delta x$ and the value of s at x. Hence, we can express ΔP as

$$\Delta P = -B\frac{\Delta V}{V_i} = -B\frac{A\Delta s}{A\Delta x} = -B\frac{\Delta s}{\Delta x}$$

As Δx approaches zero , the ratio $\Delta s/\Delta x$ becomes $\partial s/\partial x$ (The partial derivative indicates that we are interested in the variation of s with position at a fixed time.) Therefore,

$$\Delta P = -B\frac{\partial s}{\partial x}$$

If the position function is the simple sinusoidal function given by Equation , we find that

$$\Delta P = -B\,\frac{\partial}{\partial x}[s_{max}\cos(kx-\omega t)] = Bs_{max}\,k\sin(kx-\omega t)$$

$$\Delta P = \Delta P_{max}\sin(kx-\omega t)$$

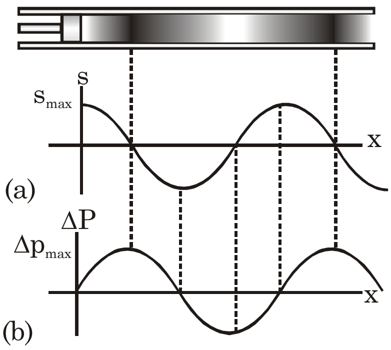


Figure : (a) Displacement amplitude and (b) pressure amplitude versus position for a sinusoidal longitudinal wave

Thus we can describe sound waves either in terms of excess pressure (equation 1.1) or in terms of the longitudinal displacement suffered by the particles of the medium.

If $s = s_0 \sin \omega(t - x/v)$ represents a sound wave where

s = displacement of medium particle from its mean position at x, then it can be proved that

$P = P_0 \sin\{w(t - x/v) + \pi/2\}$ ... (3.2)

represents that same sound wave where, P is excess pressure at position x, over and above the average atmospheric pressure and the pressure amplitude P_0 is given by

$$P_0 = \frac{B\omega s_0}{v} = BKs_0 \qquad \text{... (3.3)}$$

(B = Bulk modulus of the medium, K = angular wave number)

Note from equation (3.1) and (3.2) that the displacement of a medium particle and excess pressure at any position are out of phase by $\frac{\pi}{2}$. Hence a displacement maxima corresponds to a pressure minima and vice-versa.

Ex. The equation of a sound wave in air is given by

$P = 0.2 \sin[3000\,t - 9x]$, where all variables are in S.I. units.

(a) Find the frequency, wavelength and the speed of sound wave in air.

(b) If the equilibrium pressure of air is 1.0×10^5 N/m^2, what are the maximum and minimum pressures at a point as the wave passes through that point?

Sol. (a) Comparing with the standard form of a travelling wave

$P = P_0 \sin[\omega(t - x/v)]$

we see that ω 3000 s^{-1}. The frequency is

$$t = \frac{\omega}{2\pi} = \frac{3000}{2\pi}\text{ Hz}$$

Also from the same comparison, $\omega/v = 9.0$ m^{-1}.

$$\text{or,}\quad v = \frac{\omega}{9.0\,\text{m}^{-1}} = \frac{3000\,\text{s}^{-1}}{9.0\,\text{m}^{-1}} \qquad \frac{1000}{3}\text{ m/s}^{-1}$$

$$\text{The wavelength is } \lambda = \frac{v}{f} = \frac{1000/3\,\text{m/s}}{3000/2\pi\,\text{Hz}} = \frac{2\pi}{9}\text{ m}$$

(b) The pressure amplitude is $P_0 = 0.02$ N/m^2. Hence, the maximum and minimum pressures at a point in the wave motion will be $(1.01 \times 10^5 \pm 0.02)$ N/m^2.

Ex. A sound wave of wavelength 40cm travels in air. If the difference between the maximum and minimum pressure at a given point is 4.0×10^{-3} N/m^2, find the amplitude of vibration of the particles of the medium. The bulk modulus of air is 1.4×10^5 N/m^2.

Sol. The pressure amplitude is

$$p_0 = \frac{4.0 \times 10^{-3}\ \mathrm{N/m^2}}{2} = 2 \times 10^{-3}\ \mathrm{N/m^2}$$

The displacement amplitude s_0 is given by

$$p_0 = Bks_0$$

or

$$s_0 = \frac{p_0}{Bk} = \frac{p_0\lambda}{2\pi B}$$

$$= \frac{2\times 10^{-3}\ \mathrm{N/m^2} \times (40 \times 10^{-2}\ \mathrm{m})}{2 \times \pi \times 14 \times 10^4\ \mathrm{N/m^2}} = \frac{200}{7\pi}\ \mathrm{A}$$

$$= 13.2\ \mathrm{A}$$

SPEED OF SOUND WAVES :

Velocity of sound waves in a linear solid medium is given by

$$v = \sqrt{\frac{Y}{\rho}} \qquad (4.1)$$

where Y = young's modulus of elasticity and ρ = density

Velocity of sound wave in a fluid medium (liquid or gas) is given by

$$v = \sqrt{\frac{B}{\rho}} \qquad (4.2)$$

where, ρ = density of the medium and B = Bulk modulus of the medium given by,

$$B = -V\frac{dP}{dV} \qquad (4.3)$$

Newton's formula : Newton assumed propagation of sound through a gaseous medium to be an isothermal process.

$$PV = \text{constant}$$

$$\Rightarrow \quad \frac{dP}{dV} = \frac{-P}{V}$$

and hence B = P

and thus he obtained for velocity of sound in a gas.

$$v = \sqrt{\frac{P}{\rho}} = \sqrt{\frac{RT}{M}}$$ where M = molar mass

Laplace's correction : Later Laplace established that propagation of sound in a gas is not an isothermal but an adiabatic process and hence $PV' = \text{constant}$

$$\Rightarrow \frac{dP}{dV} = -\gamma \frac{P}{V}$$

where, $B = -V \dfrac{dP}{dV} = \gamma P$

and hence speed of sound in a gas,

$$v = \sqrt{\frac{\gamma P}{\rho}} = \sqrt{\frac{\gamma RT}{M}} \qquad (4.4)$$

FACTORS AFFECTING SPEED OF SOUND IN ATMOSPHERE

(a) Effect of temperature : as temperature (T) increases velocity (v) increases. For small change in temperature above room temperature v increases linearly by 0.6 m/s for every 1°C rise in temp.

(b) Effect of humidity : With increase in humidity density decreases. This is because the molar mass of water vapour is less than the molar mass of air.

(c) Effect of pressure :

The speed of sound in a gas is given by $v = \sqrt{\dfrac{\gamma P}{P}} = \sqrt{\dfrac{\gamma RT}{M}}$

So at constant temperature, if P changes then ρ also changes in such a way that P/ρ remains constant. Hence pressure does not have any effect on velocity of sound as long as temperature is constant.

Ex. The constant γ for oxygen as well as for hydrogen is 1.40. If the speed of sound in oxygen is 450 m/s, what will be the speed of hydrogen at the same temperature and pressure?

Sol. $v = \sqrt{\dfrac{\gamma RT}{M}}$

since temperature, T is constant

$$\therefore \frac{v_{(H_2)}}{v_{(O_2)}} = \sqrt{\frac{M_{O_2}}{M_{H_2}}} = \sqrt{\frac{32}{2}} = 4$$

$\Rightarrow v(H_2) = 4 \times 450 = 1800$ m/s

Aliter : The speed of sound in a gas is given by $u = \sqrt{\dfrac{\gamma P}{\rho}}$. At STP, 22.4 litres of oxygen has a mass of 32g whereas the same volume of hydrogen has a mass of 2g. Thus, the density of oxygen is 16 times the density of hydrogen at the same temperature and pressure. As γ is same for both the gases.

$$\frac{f_{(\text{hydrogen})}}{f_{(\text{oxygen})}} = \sqrt{\frac{\rho_{(\text{oxygen})}}{\rho_{(\text{hydrogen})}}}$$

or $\quad f_{(\text{hydrogen})} = 4f_{(\text{oxygen})} = 4 \times 450 \text{ m/s} = 1800 \text{ m/s}$

INTENSITY OF PERIODIC SOUND WAVES

Like any other progressive wave, sound waves also carry energy from one point of space to the other. This energy can be used to work, for example, forcing the eardrums to vibrate or in the extreme case of a sonic boom created by supersonic jet, can even cause glass panes of windows to crack.

The amount of energy carried per unit by a wave is called its power and power per unit area held perpendicular a sound wave travelling along positive x-axis described by the equation.

We define the intensity I of a wave, or the power per unit area , to be the rate at which the energy being transported by the wave transfers through a unit area A perpendicular to the direction of travel of the wave :

$$I = \frac{P}{A}$$

In the present case, therefore, the intensity is

$$I = \frac{P}{A} = \frac{1}{2}\rho A v(\omega s_{max})^2$$

Thus, we see that the intensity of a periodic sound wave is proportional to the square of the displacement amplitude and to the square of the angular frequency (as in the case of a periodic string wave). this can also be written in terms of the pressure amplitude ΔP_{max} ; in this case, we use Equation to obtain

$$I = \frac{\Delta P_{max}^2}{2\rho v}$$

Ex. The pressure amplitude in a sound wave from a radio receiver is 4.0×10^{-3} N/m^2 and the intensity at a point is 10^{-6} W/m^2. If by fuming the "Volume" knob the pressure amplitude is increased to 6×10^{-3} N/m^2, evaluate the intensity.

Sol. The intensity is proportional to the square of the pressure amplitude.

Thus, $$\frac{I'}{I} = \left(\frac{p_0'}{p_0}\right)^2$$

or $$I' = \left(\frac{p_0'}{p_0}\right)^2 I = \left(\frac{p_0'}{p_0}\right)^2 \times 10^{-6}\ \text{W/m}^2 = 2.25 \times 10^{-16}\ \text{W/m}^2.$$

APPEARANCE OF SOUND TO HUMAN EAR

Pitch and Frequency

Frequency as we have discussed till now is an objective property measured its units of Hz and which can be assigned a unique value. However a person's perception of frequency is subjective. The brain interprets frequency primarily in terms of a subjective quality called Pitch. A pure note of high frequency is interpreted as high-pitched sound and a pure note of low frequency as low-pitched sound.

Pitch of a sound is that sensation by which we differentiate a buffalo voice, a male voice is of low pitch, a male voice has higher pitch and a female voice has (generally) still higher pitch. This sensation primarily depends on the dominant frequency present in the sound. Higher the frequency, higher will be the pitch and vice versa. The dominant frequency of a buffalo voice is smaller than that of a male voice which in turn is smaller than that of a female voice.

Loudness and Intensity

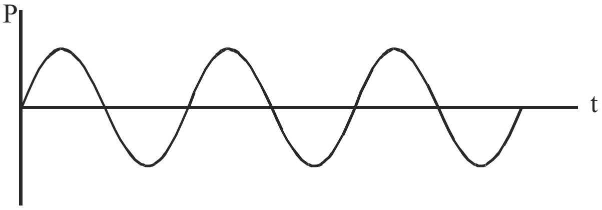
The loudness that we sense is related to the intensity of sound through it is not directly proportional to it. Our perception of loudness is better correlated with the sound level measured in decibels (abbreviated and dB) and defined as follows.

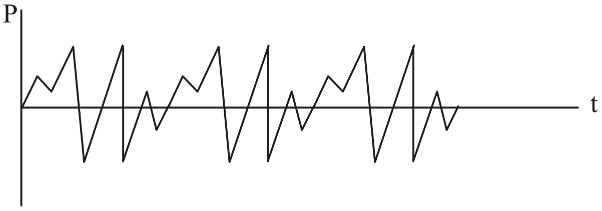
$$\beta = 10\log_{10}\left(\frac{I}{I_0}\right),$$

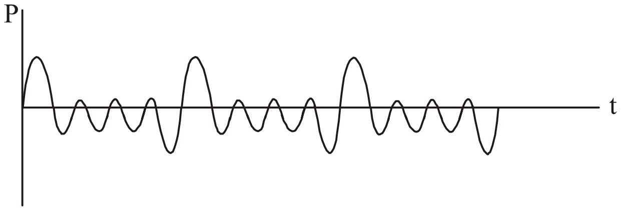
where I is the intensity of the sound and I_0 is a constant reference intensity 10^{-12} W/m^2. The reference intensity represents roughly the minimum intensity that is just audible at intermediate frequencies. For $I = I_0$, the sound level $\beta = 0$. Table shows the approximate sound levels of some of the sounds commonly encountered.

Quality and Waveform

A sound generated by a source may contain a number of frequency components in it. Different frequency components have different amplitudes and superposition of them results in the actual waveform. The appearance of sound depends on this waveform apart from the dominant frequency and intensity. Figure shows waveforms for a tuning fork, a clarinet and a cornet playing the same note (fundamental frequency = 440 Hz) with equal loudness.







We differentiate between the sound from a table and that from a mridang by saying that they have different quality. A musical sound has certain well-defined frequencies which have considerable amplitude. These frequencies are generally harmonics of a fundamental frequency. Such a sound is particularly pleasant to the ear. On the other hand, a noise has frequencies that do not bear well-defined relationship among themselves.

Ex. A bird is singing on a tree. A person approaches the tree and perceives that the intensity has increased by 10 dB. Find the ratio of initial and final separation between the man and the bird.

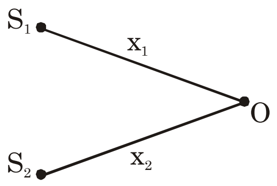
Sol. $\beta_1 = 10 \log \frac{I_1}{I_0}$

$\beta_2 = 10 \log \frac{I_2}{I_0} \Rightarrow \beta_2 - \beta_1 = 10 \log \frac{I_2}{I_1}$ or $10 = 10 \log_{10}\left(\frac{I_2}{I_1}\right)$

$\Rightarrow \frac{I_2}{I_1} = 10^1 = 10$

for point source $I \propto \frac{1}{r^2} \Rightarrow \frac{r_1}{r_2} = \sqrt{\frac{I_2}{I_1}} = \sqrt{10}$

INTERFERENCE OF SOUND WAVES :



If $p_1 = p_{m1} \sin(\omega t - kx_1 + \theta_1)$
and $p_2 = p_{m2} \sin(\omega t - kx_2 + \theta_1)$
resultant excess pressure at point O is

$p = p_1 + p_2$

$\Rightarrow p = p_0 \sin(\omega t - kx + \theta)$

where, $p_0 = \sqrt{p_{m_1}^2 + p_{m_2}^2 + 2p_{m_1}p_{m_2}\cos\phi}$, $\phi = |\,k(x_1 - x_2) + (\theta_1 - \theta_2)\,|$ ... (6.1)

(i) for constructive interference

$\phi = 2n\pi \qquad \Rightarrow p_0 = p_{m1} + p_{m2}$

(ii) for destructive interference

$\phi = (2n + 1)\pi \qquad \Rightarrow p_0 = |\,p_{m1} - p_{m2}\,|$

If ϕ is only due to path difference, then $\phi = \frac{2\pi}{\lambda}\Delta x$, and

Condition for constructive interference : $\Delta x = n\lambda$, $n = 0, \pm 1, \pm 2$

Condition for destructive interference : $\Delta x = (2n + 1)\frac{\lambda}{2}$, $n = 0 \pm 1, \pm 2$

from equation (6.1)

$$P_0{}^2 = P_{m_1}^2 + P_{m_2} + 2P_{m_1} P_{m_2} \cos\phi$$

Since intensity, $I \propto$ (Pressure amplitude)^2,

we have, for resultant intensity, $I = I_1 + I_2 + 2\sqrt{I_2 I_2}\cos\phi$ ... (6.2)

If $I_1 = I_2 = I_0$

$I = 2I_0(1 + \cos\phi)$

$\Rightarrow I = 4I_0 \cos^2\frac{\phi}{2}$ ... (6.3)

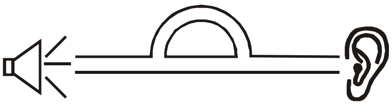
Hence in this case,

for constructive interference : $\phi = 0\ 2\pi, 4\pi$ ..... and $I_{max} = 4I_0$

and for destructive interference : $\phi = \pi, 3\pi$ .... and $I_{min} = 0$

Coherence : Two sources are said to be coherent if the phase difference between them does not change with time. In this case their resultant intensity at any point in space remains constant with time. Two independent sources of sound are generally incoherent in nature, i.e. phase difference between them changes with time and hence the resultant intensity due to them at any point in space changes with time.

Ex. Figure shows a tube structure in which a sound signal is sent from one end and is received at the other end. The semicircular part has a radius of 10.0 cm. The frequency of the sound source can be varied from 1 to 10 kHz. Find the frequencies at which the ear perceives maximum intensity. The speed of sound in air = 342 m/s.



Sol. The sound wave bifurcates at the junction of the straight and the semicircular parts. The wave through the straight part travels a distance $s_1 = 2 \times 10$ cm and the wave through the curved part travels a distance $s_2 = \pi\ 10$ cm = 31.4 cm before they meet again and travel to the receiver. The path difference between the two waves received is, therefore.

$$\Delta s = s_2 - s_1 = 31.4\text{ cm} - 20\text{ cm} = 11.4\text{ cm}$$

The wavelength of either wave is $\frac{v}{\upsilon} = \frac{330\text{ m/s}}{\upsilon}$. For constructive interference, $\Delta p = n\lambda$, where n is an integer.

$$\text{or , } \Delta p = n \cdot \frac{v}{v} \qquad \Rightarrow\ v = \frac{n \cdot v}{\Delta p} \Rightarrow \frac{n \cdot 342}{(0.114)} = 3000\ n$$

Thus, the frequencies within the specified range which cause maximum of intensity are

3000 × 1 3000 × 2 and 3000 × 3 Hz

LONGITUDINAL STANDING WAVES :

Two longitudinal waves of same frequency and amplitude travelling in opposite directions interfere to produce a standing wave.

If the two interfering wave are given by

$$p_1 = p_0 \sin(\omega t - kx)$$

and $p_2 = p_0 \sin(\omega t + kx + \phi)$

then the equation of the resultant standing wave would be given by

$$p + p_1 + p_2 = 2p_0 \cos\left(kx + \frac{\phi}{2}\right) \sin\left(\omega t + \frac{\phi}{2}\right)$$

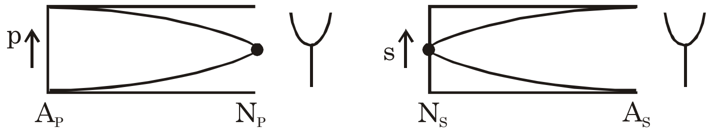
$$\Rightarrow\ p = p_0' \sin\left(\omega t + \frac{\phi}{2}\right) \qquad \text{... (8.1)}$$

VIBRATION OF AIR COLUMNS

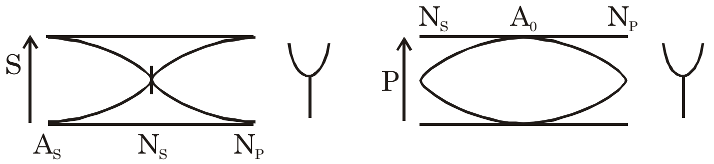
Standing waves can be set up in air-columns trapped inside cylindrical tubes if frequency of the tuning fork sounding the air column matches one of the natural frequency of air columns. In such a case the sound of the tuning fork becomes markedly louder, and we say there is resonance between the tuning fork and air column. To determine the natural frequency of the air column, notice that there is a displacement node (pressure antinode) at each closed end of the tube as air molecules there are not free to move, and a displacement antinode (pressure-node) at each open end of the air-column. In reality antinodes do not occurs exactly at the open end but a little distance outside. However if diameter of tube is small compared to its length, this end correction can be neglected.

Closed organ pipe

(In the diagram, A_P = Pressure antinode, A_S = displacement antinode, N_P = pressure node, N_s = displacement node.)

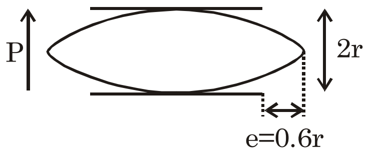


Open organ pipe :



End correction : As mentioned earlier the displacement antinode at an open end of an organ pipe lies slightly outside the open lend. The distance of the antinode from the open end is called end correction and its value is given by

$$e = 0.6\,r$$



where r = radius of the organ pipe.

with end correction, the fundamental frequency of a closed pipe (f_c) and an open argon pipe (f_0) will be given by

$$f_c = \frac{v}{4(\ell + 0.6r)} \text{ and}$$

$$f_0 = \frac{v}{2(\ell + 1.2r)} \qquad \text{......... (9.5)}$$

Ex. A tuning fork is vibrating at frequency 200 Hz. When another tuning fork is sounded simultaneously, 6 beats per second are heard. When some mass is added to the tuning fork of 200 Hz, beat frequency decreases. Find the frequency of the other tuning fork.

Sol. $|f - 200| = 6$

$\Rightarrow$ f = 194 or 206

when 1st tuning fork is loaded its frequency decreases and so does beat frequency

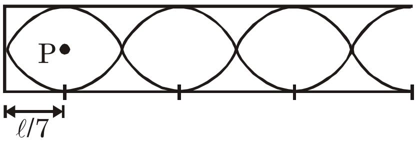
$\Rightarrow$ $200 > f$

$\Rightarrow$ f = 194 Hz

Ex. A closed organ pipe has length 'ℓ'. The air in it is vibrating in 3rd overtone with maximum amplitude 'a'. Find the amplitude at a distance of $\ell/7$ from closed end of the pipe.

Sol. The figure shows variation of displacement of particles in a closed organ pipe for 3rd overtone.

For third overtone $\ell = \dfrac{7\lambda}{4}$ or $\lambda = \dfrac{4\ell}{7}$ or $\dfrac{\lambda}{4} = \dfrac{\ell}{7}$

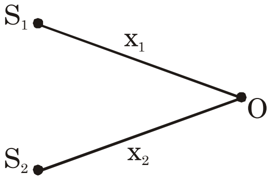


Hence the amplitude at P at a distance $\dfrac{\ell}{7}$ from closed end is 'a' because there is an antinode at that point.

BEATS

When two sound waves of same amplitude and different frequency superimpose, then intensity at any point in space varies periodically with time. This effect is called beats.

If the equation of the two interfering sound waves emitted by s_1 and s_2 at point O are,



$$p_1 = p_0 \sin (2\pi f_1 t - kx_1 + \theta_1)$$

$$p_2 = p_0 \sin (2\pi f_2 t - kx_2 + \theta_2)$$

By principle of superposition

$$p = p_1 + p_2$$

$$= 2\pi_0 \cos \{\pi(f_1 - f_2)t + \frac{\theta_1 + \theta_2}{2}\} \sin \{\pi(f_1 + f_2)t + \frac{\theta_1 + \theta_2}{2}\}$$

i.e., the resultant sound at point O has frequency $\left(\dfrac{f_1 + f_2}{2}\right)$ while pressure amplitude p'_0 (t) variex with time as

$$p_0 (t) = 2p_0 \cos \left\{\pi(f_1 - f_2)t + \frac{\phi_1 + \phi_2}{2}\right\}$$

Hence pressure amplitude at point O varies with time with a frequency of $\left(\dfrac{f_1 - f_2}{2}\right)$.

Hence sound intensity will vary with a frequency $f_1 - f_2$.

This frequency is called beat frequency (f_B) and the time interval between two successive intensity maxima (or minima) is called beat time period (T_B)

$$f_B = f_1 - f_2$$

$$T_B = \frac{1}{f_1 - f_2}$$

IMPORTANT POINTS :

(i) The frequency $|f_1 - f_2|$ should be less than 16 Hz, for it to be audible.

(ii) Beat phenomenon can be used for determining an unknown frequency by sounding it together with a sound of known frequency.

Ex. Two strings X and Y of a sitar produces a beat of frequency 4Hz. When the tension of string Y is slightly increased, the beat frequency is found to be 2Hz. If the frequency of X is 300 Hz, then the original frequency of Y was.

Ans. 296 Hz

DOPPLER EFFECT

We can express the general relationship for the observed frequency when a source is moving and an observer is at rest as equation , with the same sign convention applied to vs as was applied to v0 : a positive value is substituted for vs when the source moves toward the observer and a negative value is substituted when the source moves away from the observer.

Finally, we find the following general relationship for the observed frequency :

$$f' = \left(\frac{v + v0}{v - vs}\right) f$$

In this expression, the signs for the values substituted for v0 and vs depend on the direction of the velocity. A positive value is used for motion of the observer or the source toward the other, and a negative sign for motion of one away from the other.

A convenient rule concerning signs for you to remember when working with all Doppler effect problems is as follows :

The word toward is associated with an increase in observed frequency. The words away from are associated with a decrease in observed frequency.

Ex. A submarine (sub A) travels through water at a speed of 8.00 m/s, emitting a sonar wave at a frequency of 1400 Hz. The speed of sound in the water is 1533 m/s. A second submarine (sub B) is located such that both submarines are traveling directly toward one another. The second submarine is moving at 9.00 m/s.

(A) What frequency is detected by an observer riding on sub B as the subs approach each other?

(B) The subs barely miss each other and pass. What frequency is detected by an observer riding on sub B as the subs recede from each other?

Sol. (A) We use equation to find the Doppler - shifted frequency. As the two submarines approach each other, the observer in sub B hears the frequency

$$f' = \left(\frac{v + v0}{v - vs}\right) f = \left(\frac{1533 m/s + (+9.00 m/s)}{1533 m/s - (+8.00 m/s)}\right) (1400 \text{ Hz}) = 1416 \text{ Hz}$$

(B) As the two submarines recede from each other, the observer in sub B hears the frequency

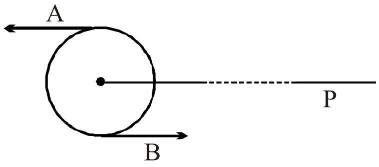
$$f' = \left(\frac{v+v0}{v-vs}\right)f$$

$$= \left(\frac{1533m/s+(-9.00m/s)}{1533m/s-(-8.00m/s)}\right)(1400\ Hz) = 1385\ Hz$$

Ex. A whistle of frequency 540 Hz is moving in a circle of radius 2m at a constant angular speed of 15 rad/s. What are the lowest and height frequencies heard by a listener standing at rest, a long distance away from the centre of the circle ? (velocity of sound in air is 330 m/s ft/sec.)

Sol. The whistle is moving along a circular path with constant angular velocity ω. The linear velocity of the whistle is given by

$v_s = \omega R$



where, R is radius of the circle

At points A and B, the velocity v_s of whistle is parallel to line OP, i.e. with respect to observer at P, whistle has maximum velocity v_s away from P at point A, and towards P at point B. (Since distance OP is large compared to radius R, whistle may be assumed to be moving along line OP)

Observer, therefore, listens maximum frequency when source is at B moving towards observer.

$$f_{max} = f\frac{v}{v-v_s} = 540 \times \frac{330}{330-30} = 540 \times \frac{330}{300} = 594\ Hz$$

where, v is speed of sound in air. Similarly, observer listens minimum frequency when source is at A, moving away from observer.

$$f_{min} = \frac{f\ v}{v+v_s} = 540 \times \frac{330}{360} = 495\ Hz$$

1. Vibrating air columns :

(i) In a pipe of length L closed at one end, the fundamental note has a frequency $f_1 = \frac{v}{4L}$, where v is the velocity of sound in air.

(ii) The first overtone $f_2 = \frac{v}{L} = 2f_1$

2. Propagation of sound in solids :

(i) The velocity of propagation of a longitudinal wave in a rod of Young's modulus Y and density ρ is given by $v = \sqrt{\frac{Y}{\rho}}$

(ii) In a sonometer wire of length L and mass per unit length m under tension T vibrating in n loops

$$f_n = \frac{n}{2L}\sqrt{\frac{T}{m}}$$

(iii) Propagation of sound in gases

Laplace formula $v = \sqrt{\frac{\gamma P}{\rho}}$ where γ is the ratio of specific heats, P is the pressure and ρ is the density.

$$\frac{v_t}{v_0} = \sqrt{\frac{T}{T_0}} = \sqrt{\frac{273+t}{273}}$$

3. Doppler Effect :

(i) When a source of sound moves with a velocity v_s in a certain direction, the wavelength decreases in front of the source and increases behind the source.

$$\lambda' \text{ (in front)} = \frac{v-v_s}{f_s} \; ; f' = \frac{v}{\lambda'} = \frac{v}{v-v_s} f_s$$

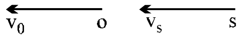
$$\lambda'' \text{(behind)} = \frac{v+v_s}{f_s} \; ; f'' = \frac{v}{\lambda''} = \frac{v}{v+v_s} f_s$$

Here v is the velocity of sound in air.

(ii) The apparent frequency $= \frac{v-v_0}{v} f_s$

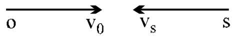
(a) When the source is moving towards the observer and the observer is moving away from the source, the apparent frequency

$$f_a = \frac{v-v_0}{v-v_s} f_s$$



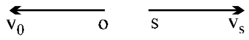
(b) When the source and the observer are moving towards each other.

$$f_a = \frac{v+v_0}{v-v_s} f_s$$



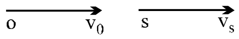
(c) When the source and observer are moving away from each other,

$$f_a = \frac{v-v_0}{v+v_s} f_s$$



(d) When the source is moving away from the observer and the observer is moving towards the source

$$f_a = \frac{v+v_0}{v+v_s} f_s$$



Here all velocities are relation to the medium.

4. Loudness of sound :

The loudness level B of sound is expressed in decibels, $B = 10\log\frac{I}{I_0}$ where I is the intensity, I_0 is a reference intensity.

5. Beats :

When two tuning forks of close but different frequencies f_1 and f_2 are vibrating simultaneously at nearby places, a listener observes a fluctuation in the intensity of sound, called beats. The number of beats heard per second is $f_1 - f_2$.

WORKED OUT EXAMPLES

Ex.1 It is found that an increase in pressure of 100 kPa causes a certain volume of water to decrease by 5×10^{-3} percent of its original volume. Then the speed of sound in the water is about (density of water 10^3 kg/m^3)

(A) 330 m/s (B) 1414 m/s (C) 1732 m/s (D) 2500 m/s

Ans. **(B)**

Sol. Bulk modulus $\beta = -\frac{\Delta P}{\frac{\Delta v}{v}} = \frac{100\times10^3}{\frac{5\times10^{-3}}{100}} = 2\times10^9$

speed $v = \sqrt{\frac{\beta}{\rho}} = \sqrt{\frac{2\times10^9}{10^3}} \approx 1414\ \text{m/s}$

Ex.2 A sound wave is travelling in a uniform pipe with gas of adiabatic exponent γ. If u is the particle velocity at any point in medium and c is the wave velocity, then relative change in pressure $\frac{dP}{P}$ while wave passes through this point is :-

(A) $\frac{u}{\gamma c}$ (B) $\gamma\sqrt{\frac{u}{c}}$ (C) $\gamma\frac{u}{c}$ (D) $\frac{u^2}{\gamma c^2}$

Ans. **(C)**

Sol. $\Delta P = -B\left(\frac{dv}{v}\right)$

$\Delta P = -B\left(\frac{\delta y}{\delta x}\right)$

$\Delta P = -\gamma P\frac{\delta y}{\delta x}$

$\frac{\Delta P}{P} = \gamma\left(\frac{u}{c}\right)$

Ex.3 A point source emits sound equally in all directions in a non-absorbing medium. Two points P and Q are at a distance of 9 meters and 25 meters respectively from the source. The ratio of the amplitudes of the waves at P and Q is :-

(A) 5 : 3 (B) 3 : 5 (C) 25 : 9 (D) 625 : 81

Ans. **(C)**

Sol. $I \propto A^2, I \propto \frac{1}{r^2}, \frac{A_1}{A_2} = \frac{r_2}{r_1}$

Ex.4 In a resonance tube experiment, an 80 cm air column is in resonance with a turning fork in first overtone. Which equation can represent correct pressure variation in the air column (x = 0 is the top point of the tube, neglect end correction, speed of sound = 320 m/sec) :-

(A) $A \sin \frac{15\pi}{8} x \cos 600\pi t$ (B) $A \cos \frac{15\pi}{8} x \sin 600\,\pi t$

(C) $A \cos \frac{15\pi}{8} x \sin 300\,\pi t$ (D) $A \sin \frac{15\pi}{8} x \sin 300\,\pi t$

Ans. **(A)**

Sol. $\frac{\omega}{k} = 320$

Ex.5 The displacement of the medium in a sound wave is given by the equation ; $y_1 = A\cos(ax + bt)$ where A, a & b are positive constants. The wave is reflected by an obstacle situated at x = 0. The intensity of the reflected wave is 0.64 times that of the incident wave.

(a) what are the wavelength & frequency of the incident wave.

(b) write the equation for the reflected wave.

(c) in the resultant wave formed after reflection, find the maximum & minimum values of the particle speeds in the medium.

Ans. (a) $2\pi/a$, $b/2\pi$, (b) $y_2 = \pm 0.8 A \cos(ax - bt)$, (c) max.=1.8 bA, min. = 0,

Sol. (a) $\omega = b$ & $k = a$

$$f = \frac{2\pi}{\omega} \;\&\; \lambda = \frac{2\pi}{k}$$

(b) $I \propto A^2$

So $A_r = 0.8$ A

(c) $(A_{net})_{max} = A + 0.8A = 1.8A$

Ex.6 An observer moves towards a stationary source of sound, with a velocity one-fifth of the velocity of sound. what is the percentage increase in the apparent frequency ? **[AIEEE - 2005]**

(1) zero (2) 0.5% (3) 5% (4) 20%

Ans. **(4)**

Sol. $$\frac{f_{app}}{f} = \left(\frac{v + \frac{v}{5}}{v}\right) = \frac{6}{5}$$

Ex.7 The equation of a longitudinal standing wave due to superposition of the progressive waves produced by two sources of sound is $s = -20 \sin 10\pi x \sin 100\pi t$ where s is the displacement from mean position measured in mm ; x is in meters and t in seconds. The specific gravity of the medium is 10^{-3}. Find

(a) wavelength, frequency and velocity of the progressive waves.

(b) Bulk modulus of the medium and the pressure amplitude of the progressive waves.

(c) minimum distance between pressure antinode and the displacement antinode.

Ans. (a) 1/5 m, 50 Hz, 10 m/s; (b) 100 Pa, 10π Pa, (c) 1/20 m

Sol. $k = 10\pi$

$\omega = 100\,\pi$

$$\because\ \lambda = \frac{1}{5}\ \&\ f = 50\ \text{Hz}$$

$v = \lambda f = 10$ m/s

$B = \rho v^2$

$P_0 = BKs$

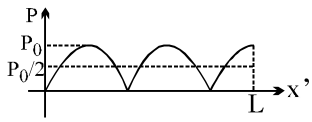
Minimum distance between pressure antinode and displacment antinode $= \dfrac{\lambda}{4}$

Ex.8 The air column in a pipe closed at one end and open to atmosphere at the other end is made to vibrate in its fifth harmonic by a tuning fork of frequency 470 Hz. The length of air column is $\dfrac{15}{16}$ m. Neglect end correction. Let p_0 denote the maximum gauge pressure at the closed end

(a) Find the speed of sound in air.

(b) Draw the graph of pressure amplitude vs distance from the open end of the tube.

(c) Find the points where the maximum gauge pressure is $\dfrac{p_0}{2}$.

Ans. (a) 352.5 m/s, (b) 

, (c) $\dfrac{l}{15}, \dfrac{5l}{15}, \dfrac{7l}{15}, \dfrac{11l}{15}, \dfrac{13l}{15}$

Sol. $$\frac{5\lambda}{4} = \frac{15}{16}$$

$\lambda = 0.75$ m

$f = 470$

$v = \lambda f = 352.5$ m/s

$$P = P_0 \sin\left(\frac{2\pi}{\lambda}x\right)$$

where $\lambda = \dfrac{4\ell}{5}$

$$P = P_0 \sin\left(\frac{5\pi}{2\ell}x\right)$$

Ex.9 A metal rod of length $l = 100$ cm is clamped at two points. Distance of each clamp from nearer end is a = 30cm. If density and Young's modulus of elasticity of rod material are $\rho = 9000$ kg m^{-3} and Y = 144 GPa respectively, calculate minimum and next higher frequency of natural longitudinal oscillations of the rod.

Ans. 10kHz, 30kHz

Sol. $v = \sqrt{\frac{Y}{\rho}} = 4 \times 10^3$ m/s

$\lambda = 0.4$ m

$f_0 = \frac{v}{\lambda} = 10\text{kHz}$

Ex.10 When two tuning forks (fork 1 and fork 2) are sounded simultaneously, 4 beats per second are heard. Now, some tape is attached on the prong of the fork 2. When the tuning forks are sounded again, 6 beats per second are heard. If the frequency of fork 1 is 200 Hz, then what was the original frequency of fork 2 ? **[AIEEE - 2005]**

(1) 200 Hz (2) 202 Hz (3) 196 Hz (4) 204 Hz

Ans. **(3)**

Sol. $f_1 - f_2 = 4$ or $f_2 - f_1 = 4$

But according to question

$f_1 - f_2 = 4$

So $f_2 = 196$

Ex.11 A whistling train approaches a junction. An observer standing at junction observers the frequency to be 2.2 KHz and 1.8 KHz of the approaching and the receding train. Find the speed of the train (speed sound = 300 m/s). **[JEE 2005]**

Ans. $V_s = 30$ m/s

Sol. $\left(\frac{v}{v - v_S}\right) f_0 = 2.2 \times 10^3$

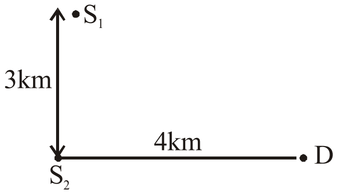
$\left(\frac{v}{v + v_S}\right) f_0 = 1.8 \times 10^3$

Dividing both we get

$\frac{v + v_S}{v - v_S} = \frac{11}{9}$

$v_S = 30$

Ex.12 Two point sound source S_1 and S_2 are both have the same power and send out sound waves in the same phase. The wavelength of both the waves is $\frac{48}{5}$ m. The intensity due to S_2 alone at D is 25 W/m^2. The resultant intensity at D is :



(A) 59 W/m^2 (B) 61 W/m^2 (C) 65 W/m^2 (D) None of these

Ans. **(B)**

Sol. $I \propto \frac{1}{r^2}$

Intensity due to S_1 alone at D = 16 W/m^2

Phase difference = $\frac{2\pi}{\left(\frac{48}{5}\right)} \times 1000 = \frac{625}{3}\pi$

$I_{res} = I_1 + I_2 + 2\sqrt{I_1}\sqrt{I_2}\cos\phi$

$= 25 + 16 + 2\,(5)\,(4)\left(\frac{1}{2}\right)$

Ex.13 Spherical sound waves are emitted uniformly in all directions from a point source. The variation in sound level SL as a function of distance r(> 0) from the source can be written as :- (where a and b are positive constant)

(A) $SL = -b\log r^a$ (B) $SL = a - b\log r$ (C) $SL = a - b(\log r)^2$ (D) $SL = a - \frac{b}{r^2}$

Ans. **(B)**

Sol. $I \propto \frac{1}{r^2}$

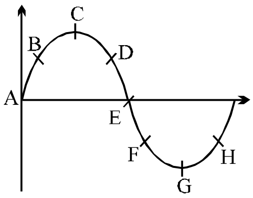
$SL = 20\log_{10} = \frac{I}{I_0}$

EXERCISE (S)

Wave on string

Wave equation

1. A transverse wave is travelling along a string from left to right. The fig. represents the shape of the string (snap-shot) at a given instant. At this instant (a) which points have an upward velocity (b) which points will have downward velocity (c) which points have zero velocity (d) which points have maximum magnitude of velocity.



WA0001

2. A light pointer fixed to one prong of a tuning fork touches a vertical plate. The fork is set vibrating and plate is allowed to fall freely. 8 complete oscillations are counted when the plate falls through 10 cm. What is the frequency of the tuning fork ? **WA0003**

3. A long uniform string of mass density 0.1 kg/m is stretched with a force of 40 N. One end of the string (x = 0) is oscillated transversely (sinusoidally) with an amplitude of 0.02 m and a period of 0.1 sec, so that travelling waves in the +x direction are set up.

(a) What is the velocity of the waves?

(b) What is their wavelength?

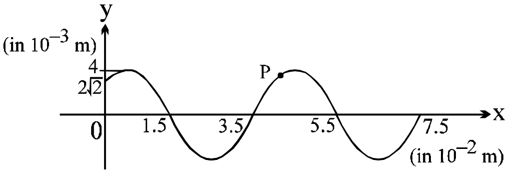
(c) If at the driving end (x = 0) the displacement (y) at t = 0 is 0.01 m with dy/dt negative, what is the equation of the travelling waves? **WA0004**

4. The figure shows a snap photograph of a vibrating string at t = 0. The particle P is observed moving up with velocity 20π cm/s. The angle made by string with x-axis at P is 6°.

(a) Find the direction in which the wave is moving

(b) the equation of the wave

(c) the total energy carried by the wave per cycle of the string, assuming that μ, the mass per unit length of the string = 50 gm/m.



WA0005

5. A plane progressive wave of frequency 25 Hz, amplitude 2.5×10^{-5} m and initial phase zero propagates along the negative x-direction with a velocity of 300 m/s. At any instant, the phase difference between the oscillations at two points 6m apart along the line of propagation is ...., and the corresponding amplitude difference is ...... m. **[IIT-1997C]**

WA0018

Velocity of wave

6. The extension in a string, obeying Hooke's law is x. The speed of wave in the stretched string is v. If the extension in the string is increased to 1.5 x find the new speed of wave. **WA0006**

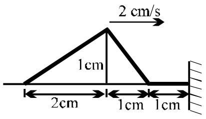
7. A copper wire is held at the two ends by rigid supports. At 30°C, the wire is just taut, with negligible. Find the speed of transverse waves in this wire at 10°C. **[IIT-1979]**
Given : Young modulus of copper = 1.3×10^{11} N/m^2.
Coefficient of linear expansion of copper = 1.7×10^{-5} °C^{-1}.
Density of copper = 9×10^3 kg/m^3. **WA0008**

Energy of wave

8. A steel wire has a mass of 5g/m and is under tension 450 N. Find the maximum average power that can be carried by the transverse wave in the wire if the amplitude is not to exceed 20% of the wavelength. **WA0009**

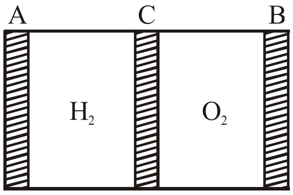
Superposition of waves

9. The figure shown a triangular pulse on a rope at t = 0. It is approaching a fixed end at 2 cm/s
(a) Draw the pulse at t = 2 sec.
(b) The particle speed on the leading edge at the instant depicted is _____.



WA0010

10. AB is a cylinder of length 1m fitted a thin flexible diaphragm C at the middle and other thin flexible diaphragms A and B at the ends. The portions AC and BC contain hydrogen and oxygen gases respectively. The diaphragms A and B are set into vibrations of same frequency. What is the minimum frequency of these vibration for which diaphragm C is a node? (Under the conditions of experiment $v_{H_2} = 1100$ m/s, $v_{O_2} = 300$ m/s). **[IIT-1978]**



WA0019

11. A 40 cm long wire having a mass 3.2 gm and area of cross-section 1 mm^2 is stretched between the supports 40.05 cm apart. In its fundamental mode it vibrate with a frequency 1000/64 Hz. Find the young's modulus of the wire. **WA0011**

12. A plane wave given by equation $y = 0.04 \sin(0.5\pi x - 100\pi t)$, where x and y are in meter and t in sec is incident normally on a boundary between two media beyond which wave speed becomes doubled. State boundary condition and find the equation of the reflected and transmitted waves. Take x = 0 as the boundary between two media. **WA0012**

13. A string between x = 0 and x = l vibrates in fundamental mode. The amplitude A, tension T and mass per unit length μ is given. Find the total energy of the string. **[IIT-JEE 2003(Scr)]**

WA0013

Sound Wave

Sound basics

14. Find the intensity of sound wave whose frequency is 250 Hz. The displacement amplitude of particles of the medium at this position is 1×10^{-8} m. The density of the medium is 1 kg/m^3, bulk modulus of elasticity of the medium is 400 N/m^2. **SW0001**

15. In a mixture of gases, the average number of degrees of freedom per molecule is 6. The rms speed of the molecules of the gas is c. Find the velocity of sound in the gas. **SW0002**

16. The loudness level at a distance R from a long linear source of sound is found to be 40dB. At this point, the amplitude of oscillations of air molecules is 0.01 cm. Then find the loudness level & amplitude at a point located at a distance '10R' from the source. **SW0003**

Superposition of sound

17. The first overtone of a pipe closed at one end resonates with the third harmonic of a string fixed at its ends. The ratio of the speed of sound to the speed of transverse wave travelling on the string is 2 :1. Find the ratio of the length of pipe to the length of string. **SW0004**

18. In a resonance-column experiment, a long tube, open at the top, is clamped vertically. By a separate device, water level inside the tube can be moved up or down. The section of the tube from the open end to the water level act as a closed organ pipe. A vibrating tuning fork is held above the open end, first and the second resonances occur when the water level is 24.1 cm and 74.1 cm respectively below the open end. Find the diameter of the tube. **SW0005**

19. A tuning fork of frequency 480 Hz resonates with a tube closed at one end of length, 16 cm and diameter 5 cm in fundamental mode. Calculate velocity of sound in air. **[JEE 2003]**
SW0006

20. An open organ pipe filled with air has a fundamental frequency 500Hz. The first harmonic of another organ pipe closed at one end and filled with carbon dioxide has the same frequency as that of the first harmonic of the open organ pipe. Calculate the length of each pipe. Assume that the velocity of sound in air and in carbondioxide to be 330 and 264 m/s respectively. **SW0007**

21. A steel rod having a length of 1 m is fastened at its middle. Assuming young's modulus to be 2×10^{11} Pa, and density to be 8 gm/cm^3 find the fundamental frequency of the longitudinal vibration and frequency of first overtone. **SW0008**

22. Two narrow cylindrical pipes A and B have the same length. Pipe A is open at both ends and is filled with a monoatomic gas of molar mass M_A. Pipe B is open at one end and closed at the other end, and is filled with a diatomic gas of molar mass M_B. Both gases are at the same temperature.
[JEE 2002]

(a) If the frequency of the second harmonic of the fundamental mode in pipe A is equal to the frequency of the third harmonic of the fundamental mode in pipe B, determine the value of M_A/M_B.

(b) Now the open end of pipe B is also closed (so that the pipe is closed at both ends). Find the ratio of the fundamental frequency in pipe A to that in pipe B. **SW0009**

23. A tube of a certain diameter and of length 48 cm is open at both ends. Its fundamental frequency of resonance is found to be 320 Hz. The velocity of sound in air is 320m/sec. Estimate the diameter of the tube. **[IIT-1980]**
SW0010

Beats

24. A stretched uniform wire of a sonometer between two fixed knife edges, when vibrates in its second harmonic gives 1 beat per second with a vibrating tuning fork of frequency 200 Hz. Find the percentage change in the tension of the wire to be in unison with the tuning fork. **SW0011**

25. A, B and C are three tuning forks. Frequency of A is 350Hz. Beats produced by A and B are 5 per second and by B and C are 4 per second. When a wax is put on A beat frequency between A and B is 2Hz and between A and C is 6Hz. Then, find the frequency of B and C respectively.

SW0012

26. A source of sound of frequency 256 Hz is moving rapidly towards wall with a velocity of 5 m/sec. How many beats per second will be heard if sound travels at a speed of 330 m/sec? **[IIT-1981]**

SW0013

27. Two tuning forks with natural frequencies of 340 Hz each move relative to a stationary observer. One fork moves away from the observer, while the other moves towards him at the same speed. The observer hears beats of frequency 3 Hz. Find the speed of the tuning fork **[IIT-1986]**

SW0014

Doppler effect

28. Two tuning forks A and B lying on opposite sides of observer 'O' and of natural frequency 85 Hz move with velocity 10 m/s relative to stationary observer O. Fork A moves away from the observer while the fork B moves towards him. A wind with a speed 10 m/s is blowing in the direction of motion of fork A. Find the beat frequency measured by the observer in Hz. [Take speed of sound in air as 340 m/s] **SW0015**

29. A car is moving towards a huge wall with a speed = c/10 , where c = speed of sound in still air. A wind is also blowing parallel to the velocity of the car in the same direction and with the same speed. If the car sounds a horn of frequency f, then what is the frequency of the reflected sound of the horn heard by driver of the car? **SW0016**

30. A bus is moving towards a huge wall with a velocity of 5 ms^{-1}. The driver sounds a horn of frequency 200 Hz. The frequency of the beats heard by a passenger of the bus will be..... Hz (speed of sound in air = 342 ms^{-1}) **[IIT-1994]**

SW0018

31. A sonometer wire under tension of 64 Newtons vibrating in its fundamental mode is in resonance with a vibrating tuning fork. The vibrating portion of the sonometer wire has a length of 10 cm and a mass of 1 gm. The vibrating tuning fork is now moved away of 1 gm. The vibrating wire with a constant speed and an observer standing near the sonometer hears oen beat per second. Calculate the speed with which the tuning fork is moved if the speed of sound in air is 300 m/s. **[IIT-1983]**

SW0026

EXERCISE (O)

Wave on string

Wave equation

1. The function of x and t that does not represent a progressive wave is :-

(A) $y = 2\sin(4t - 3x)$ (B) $y= e^{(4+(4t-3x)]}$ (C) $y = [4t - 3x]^{-1}$ (D) $y = [4t-3x]$

WA0023

2. At x = 0 particle oscillate by law $y = \frac{3}{2t^2+1}$. If wave is propagating along –ve x axis with velocity 2m/s. Find equation of wave

(A) $y = \frac{3}{2\left(t-\frac{x}{2}\right)^2+1}$ (B) $y = \frac{3}{2\left(t+\frac{x}{2}\right)^2+1}$ (C) $y = \frac{3}{2\left(t-\frac{z}{2}\right)^2+1}$ (D) $y = \frac{3}{2\left(t+\frac{z}{2}\right)^2+1}$

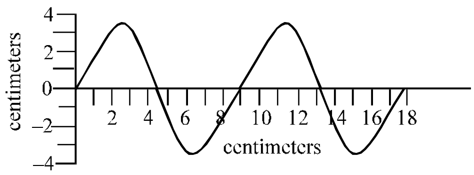
WA0024

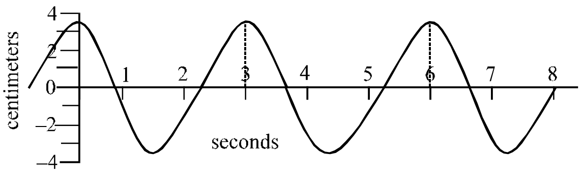
3. The shape of a wave propagating in the positive x or negative x- direction is given $y = \frac{1}{\sqrt{1+x^2}}$ at t =0 and $y = \frac{1}{\sqrt{2-2x+x^2}}$ at t = 1s where x and y are in meters. The shape the wave disturbance does not change during propagation. Find the velocity of the wave.

(A) 1 m/s in positive x direction (B) 1 m/s in negative x direction

(C) $\frac{1}{2}$ m/s in positive x direction (D) $\frac{1}{2}$ m/s in negative x direction **WA0025**

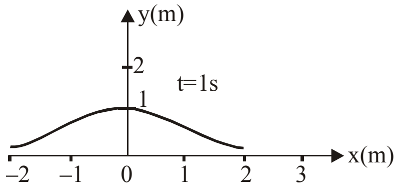
4. A transverse wave is travelling along a horizontal string. The first picture shows the shape of the string at an instant of time. This picture is superimposed on a coordinate system to help you make any necessary measurements. The second picture is a graph of the vertical displacement of *one* point along the string as a function of time. How far does this wave travel along the string in one second?

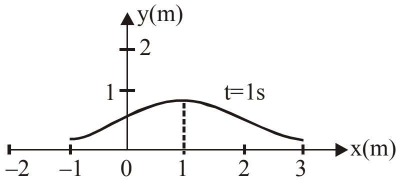


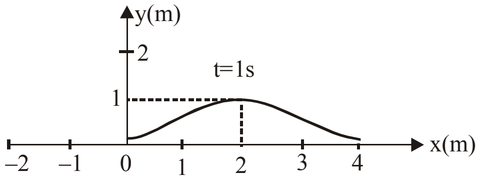


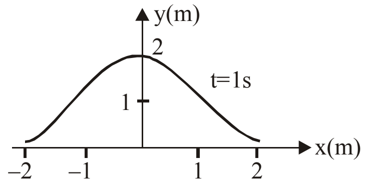
(A) 0.3 cm (B) 3.0 cm (C) 9.0 cm (D) 27 cm **WA0026**

5. A wave pulse is given by the equation $y = f(x, t) = A \exp(-B(x-vt)^2)$. Given $A = 1.0\text{m}$, $B = 1.0\ \text{m}^{-2}$ and $v = +2.0$ m/s, which of the following graph shows the correct wave profile at the instant $t = 1$ s?

(A)


(B)


(C)


(D)


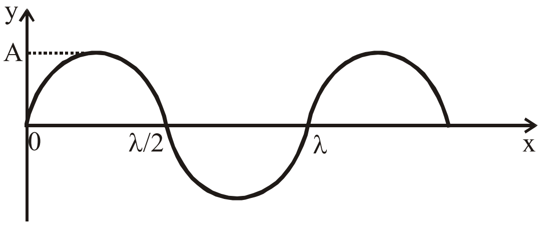
WA0027

6. The displacement from the position of equilibrium of a point 4 cm from a source of sinusoidal oscillations is half the amplitude at the moment $t = T/6$ (T is the time period). Assume that the source was at mean position at $t = 0$. The wavelength of the running wave is

(A) 0.96 m (B) 0.48 m (C) 0.24 m (D) 0.12 m **WA0028**

7. Here given snap shot of a progressive wave at $t = 0$ with time period = T. Then the equation of the wave if wave is going in +ve x-direction and if wave is going in –ve x-direction will be respectively.

$$\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$$



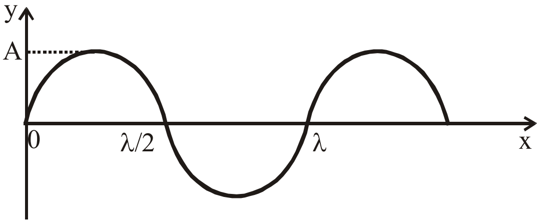
(A) $y = A \sin(kx + \omega t)$, $y = A \sin(kx - \omega t)$ (B) $y = A \cos(kx + \omega t)$, $y = A \cos(kx - \omega t)$

(C) $y = A \sin(\omega t - kx)$, $y = A \sin(\omega t + kx)$ (D) $y = A \sin(kx - \omega t)$, $y = A \sin(kx + \omega t)$

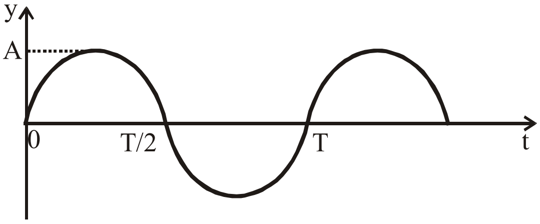
WA0029

8. Here given figure (i) shows snap shot at t = T/4 and figure (ii) shows motion of particle at x = λ/4. Then the possible equations of the wave will be $\left(\text{Here, } T = \frac{2\pi}{\omega}\right)$:-

(i)



(ii)



(A) $y = A\sin\left(\omega t + kx - \frac{\pi}{2}\right)$

(B) $y = A\sin\left(\omega t - kx + \frac{\pi}{2}\right)$

(C) Both $y = A\cos\left(\omega t + kx - \frac{\pi}{2}\right)$ and $y = A\cos\left(\omega t - kx + \frac{\pi}{2}\right)$

(D) Both $y = A\sin\left(\omega t + kx - \frac{\pi}{2}\right)$ and $y = A\sin\left(\omega t - kx + \frac{\pi}{2}\right)$ **WA0044**

9. A sinusoidal wave travelling in the positive direction of x on a stretched string has amplitude 2.0 cm, wavelength 1 m and wave velocity 5.0 m/s. At x = 0 and t = 0, it is given that displacement y = 0 and $\frac{\partial y}{\partial t} < 0$. Express the wave function correctly in the form y = f(x, t) :-

(A) $y = (0.02\text{ m})\sin 2\pi(x - 5t)$

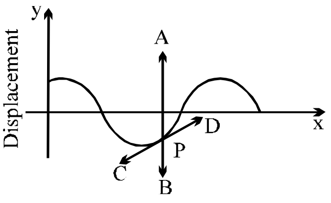
(B) $y = (0.02\text{ cm})\cos 2\pi(x - 5t)$

(C) $y = (0.02\text{ m})\sin 2\pi\left(x - 5t + \frac{1}{4}\right)$

(D) $y = (0.02\text{ cm})\cos 2\pi\left(x - 5t + \frac{1}{4}\right)$

WA0030

10. The figure below shows a snap photograph of a simple harmonic progressive wave, progressing in the negative X-axis, at a given instant. The direction of the velocity of the particle at the stage P on the figure is best represented by the arrow.



(A) $\overrightarrow{PA}$

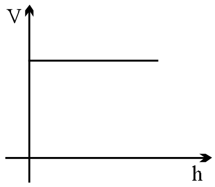
(B) $\overrightarrow{PB}$

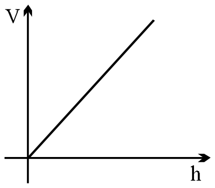
(C) $\overrightarrow{PC}$

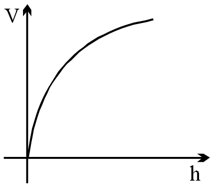
(D) $\overrightarrow{PD}$ **WA0031**

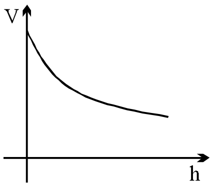
Velocity of wave

11. A uniform rope having some mass hanges vertically from a rigid support. A transverse wave pulse is produced at the lower end. The speed (v) of the wave pulse varies with height (h) from the lower end as:

(A) 

(B) 

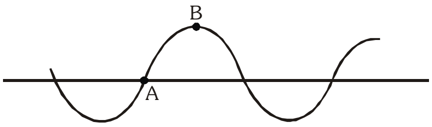
(C) 

(D) 

WA0032

Energy of wave

12. A progressive wave is travelling in a string as shown. Then which of the following statement about KE and potential energy of the elements A and B is true?



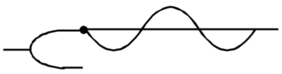
(A) For point A : kinetic energy is maximum and potential energy is min.

(B) For point B : kinetic energy is minimum and potential energy is min.

(C) For point A : kinetic energy is minimum and potential energy is max.

(D) For point B : kinetic energy is minimum and potential energy is max. **WA0033**

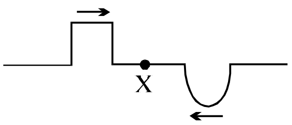
13. The prong of a electrically operated tuning fork is connected to a long string of $\mu = 1$ kg/m and tension 25N. The maximum velocity of the prong is 1 cm/s, then the average power needed to drive the prong is:

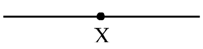


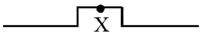
(A) 5×10^{-4} W (B) 2.5×10^{-4} W (C) 1×10^{-4} W (D) 10^{-3} W **WA0034**

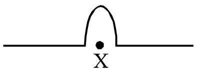
Superposition of waves

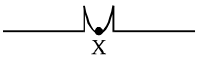
14. The diagram below shows two pulses traveling towards each other in a uniform medium with same speed. Pulses in the figure are at the same distance from X and has same height & width. Which diagram best represents the medium when the pulses meet at point X ?



(A) 

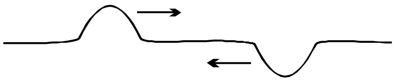
(B) 

(C) 

(D) 

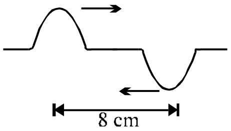
WA0035

15. Two symmetric, identical pulses of opposite amplitude travel along a stretched string in opposite directions as shown in the figure below. Which one of the following statements most fully describes the situation ?



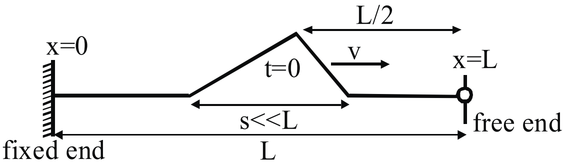
(A) There is an instant when the string is straight
(B) When the two pulses interfere completely, the energy of the wave is zero
(C) There is a point on the string that does not move up or down
(D) Both A and C
(E) Both A and B **WA0036**

16. Two pulses in a stretched string whose centres are initially 8 cm apart are moving towards each other as shown in figure. The speed of each pulse is 2 cm/s. After 2 seconds, the total energy of the pulses will be :- **[IIT-JEE 2001(Scr)]**

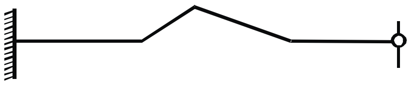


(A) zero (B) purely kinetic
(C) purely potential (D) partly kinetic and partly potential **WA0037**

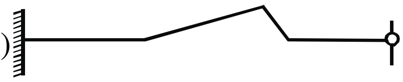
17. A small pulse travelling with speed v in a string is shown at t=0, moving towards free end. Which of these is not **CORRECTLY** matched.



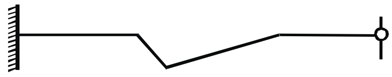
(i) $t=\frac{L}{v}$ (P)



(ii) $t=\frac{2L}{v}$ (Q)



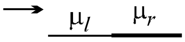
(iii) $t=\frac{3L}{v}$ (R)



(A) (i) (B) (ii) (C) (iii) (D) None of these

WA0038

18. A string consists of two parts attached at x = 0. The right part of the string (x > 0) has mass μ_r per unit length and the left part of the string (x < 0) has mass μ_l per unit length. The string tension is T. If a wave of unit amplitude travels along the left part of the string, as shown in the figure, what is the amplitude of the wave that is transmitted to the right part of the string ?



(A) 1 (B) $\frac{2}{1+\sqrt{\mu_l/\mu_r}}$ (C) $\frac{2\sqrt{\mu_l/\mu_r}}{1+\sqrt{\mu_l/\mu_r}}$ (D) $\frac{\sqrt{\mu_l/\mu_r}-1}{\sqrt{\mu_l/\mu_r}+1}$

WA0039

19. A wave travels on a light string .The equation of the wave is Y= Asin(kx–wt+30°) .It is reflected from a heavy string tied to an end of the light string at x = 0.If 64% of the incident energy is reflected the equation of the reflected wave is

(A)$Y = 0.8A\sin(kx-wt+30^0+180^0)$

(B)$Y = 0.8A\sin(kx+wt+30^0+180^0)$

(C)$Y = 0.8A\sin(kx+wt-30^0)$

(D)$Y = 0.8A\sin(kx+wt+30^0)$ **WA0040**

20. A wave is represented by the equation $y = 10 \sin 2\pi(100t-0.02x) + 10 \sin 2\pi(100t+0.02x)$. The maximum amplitude and loop length are respectively

(A) 20 units and 30 units (B) 20 units and 25 units

(C) 30 units and 20 units (D) 25 units and 20 units **WA0041**

21. A wave represented by the equation $y = A \cos (kx - \omega t)$ is superimposed with another wave to form a statioary wave such that the point x = 0 is a node. The equation of the other wave is:

(A) $-A \sin (kx + \omega t)$ (B) $- A \cos (kx + \omega t)$

(C) $A \sin (kx + \omega t)$ (D) $A \cos (kx + \omega t)$ **WA0042**

22. Five waveforms moving with equal speeds on the x-axis

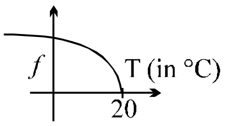
$y_1 = 8 \sin (\omega t + kx)$; $y_2 = 6 \sin (\omega t + \frac{\pi}{2} + kx)$; $y_3 = 4 \sin (\omega t + \pi + kx)$; $y_4 = 2 \sin (\omega t + \frac{3\pi}{2} + kx)$;

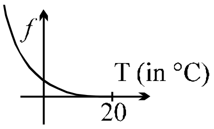
$y_5 = 4\sqrt{2} \sin (\omega t - kx + \frac{\pi}{4})$ are superimposed on each other. The resulting wave is :

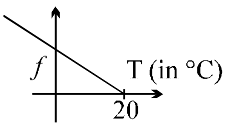
(A) $8\sqrt{2} \cos kx \sin (\omega t + \frac{\pi}{4})$ (B) $8\sqrt{2} \sin (\omega t - kx + \frac{\pi}{4})$

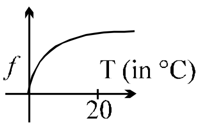
(C) $8\sqrt{2} \sin kx \cos (\omega t + \frac{\pi}{4})$ (D) $8 \sin (\omega t + kx)$ **WA0043**

23. A metal wire is clamped between two vertical walls. At 20 °C the unstrained length of the wire is exactly equal to the separation between walls. If the temperature of the wire is decreased the graph between fundamental frequency (*f*) and temperature (T) of the wire is

(A) 

(B) 

(C) 

(D) 

WA0045

24. What is the fractional change in the tension necessary in a sonometer of fixed length to produce a note one octave lower (half of original frequency) than before

(A) 1/4 (B) 1/2 (C) 2/3 (D) 3/4

(E) 2/1 **WA0046**

25. A string clamped at both ends is vibrating. At the moment the string looks flat, the instantaneous transverse velocity of points along the string, excluding its end-points, must be

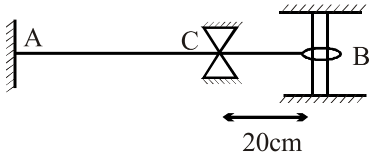
(A) zero everywhere

(B) dependent on the location along the string

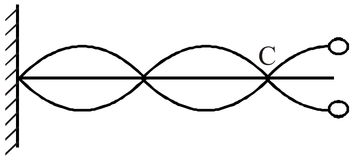
(C) non zero everywhere

(D) non-zero and in the same direction everywhere **WA0047**

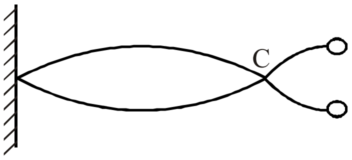
26. A 1m long wire having tension T is fixed at A and free at B. The point C, 20 cm from B is constrained to be stationary. What is shape of string for fundamental mode?



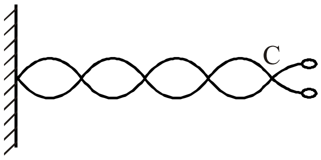
(A)



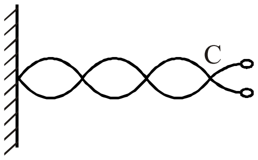
(B)



(C)



(D)



WA0048

27. The ends of a stretched wire of length L are fixed at $x = 0$ and $x = L$. In one experiment, the displacement of the wire is $y_1 = A \sin(\pi x/L) \sin \omega t$ and energy is E_1 and in another experiment its displacement is $y_2 = A\sin(2\pi x/L) \sin 2\omega t$ and energy is E_2. Then **[JEE 2001 (Scr)]**

(A) $E_2 = E_1$ (B) $E_2 = 2E_1$ (C) $E_2 = 4E_1$ (D) $E_2 = 16E_1$ **WA0049**

Sound wave

Sound basics

28. A sound wave has a wavelength of 3.0 m. The distance from a compression center to the adjacent rarefaction center is :-

(A) 0.75 m (B) 1.5 m

(C) 3.0 m (D) need to know wave speed **SW0030**

29. You are listening to an "A" note played on a violin string. Let the subscript "s" refer to the violin string and "a" refer to the air. Then :-

(A) $f_s = f_a$ but $\lambda_s \neq \lambda_a$ (B) $f_s = f_a$ and $\lambda_s = \lambda_a$

(C) $\lambda_s = \lambda_a$ but $f_s \neq f_a$ (D) $\lambda_s \neq \lambda_a$ and $f_s \neq f_a$ **SW0031**

30. The fig.(i) shows the graphical representation of the air molecules in a tube of air (length = L) at atmospheric pressure on the absolute pressure P(x) graph. Which one of the following pictures corresponds to the absolute pressure P(x) graph of fig. (ii).

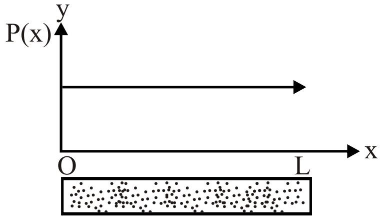


Figure (i)

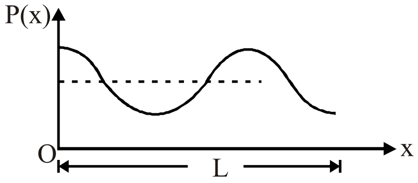
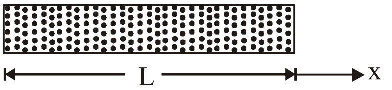
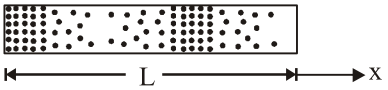
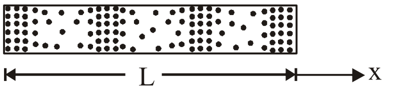
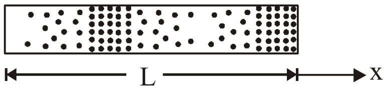


Figure (ii)

(A)


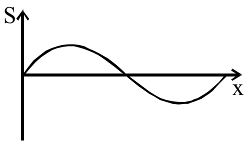
(B)


(C)


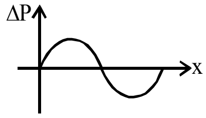
(D)


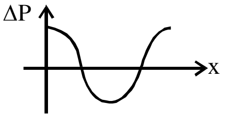
SW0032

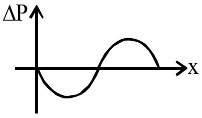
31. If a sound wave is travelling and snap shot at t = 0 is as shown in figure.

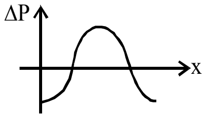


Choose snapshot of pressure variation.

(i)


(ii)


(iii)


(iv)


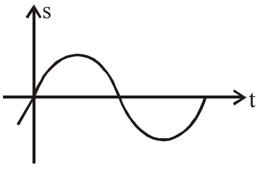
(A) For wave travelling towards right or left (i) is correct.

(B) For wave travelling towards right graph (iv) and for wave travelling towards left graph (iv) is correct.

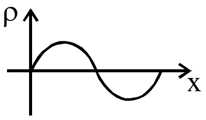
(C) For wave travelling towards right graph (i) and for wave travelling towards left graph (iii) is correct

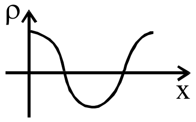
(D) For wave travelling towards right or left (ii) is correct. **SW0033**

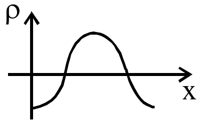
32. A sound waves is travelling towards right and its s-t graph is as shown for x = 0.

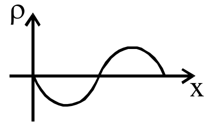


What will be density vs x graph at t = T/4 :-

(A)


(B)


(C)


(D)


SW0034

33. A point source of sound is located somewhere along the x-axis. Experiments show that the same wave front simultaneously reaches listeners at x = –8 m and x = +2.0 m.

A third listener is positioned along the positive y-axis. What is her y-coordinate (in m) if the same wave front reaches her at the same instant as it does the first two listeners ?

(A) 4 (B) 3 (C) 2 (D) 5 **SW0035**

34. Two monatomic ideal gases 1 and 2 of molecular masses m_1 and m_2 respectively are enclosed in separate container kept at the same temperature. The ratio of the speed of sound in gas 1 to that in gas 2 is given by **[JEE 2000 (Scr)]**

(A) $\sqrt{\frac{m_1}{m_2}}$ (B) $\sqrt{\frac{m_2}{m_1}}$ (C) $\frac{m_1}{m_2}$ (D) $\frac{m_2}{m_1}$ **SW0036**

35. A firecracker exploding on the surface of a lake is heard as two sounds a time interval *t* apart by a man on a boat close to water surface. Sound travels with a speed *u* in water and a speed *v* in air. The distance from the exploding firecracker to the boat is

(A) $\frac{uvt}{u+v}$ (B) $\frac{t(u+v)}{uv}$ (C) $\frac{t(u-v)}{uv}$ (D) $\frac{uvt}{u-v}$ **SW0037**

36. The speed of longitudinal wave is 100 times the speed of transverse wave in a taut brass wire. If the Young's modulus of brass is 1.0×10^{11} N/m^2, the stress in wire is :-

(A) 1.0×10^7 N/m^2 (B) 1.0×10^6 N/m^2

(C) 1.0×10^5 N/m^2 (D) 1.0×10^8 N/m^2 **SW0038**

37. The equations of S.H.M. of medium particle due to sound waves propagating in a medium are given by $s_1 = 2\sin(200\pi t)$ and $s_2 = 5\sin(150\pi t)$. The ratio of average intensities of sound at these points is:

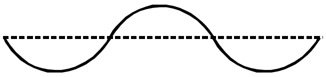
(A) 4 : 25 (B) 9 : 100 (C) 8 : 15 (D) 64 : 225 **SW0039**

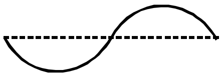
38. A is singing a note and at the same time B is also singing a note with 1/8th the frequency of A. The energies of the two sounds are equal. The displacement amplitude of the note of B is:

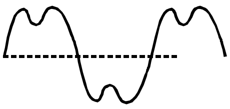
(A) same as that of A (B) twice that of A

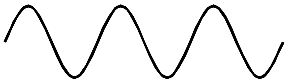
(C) four times that of A (D) eight times that of A **SW0040**

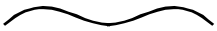
39. A microphone is connected to an oscilloscope. The diagram shows the trace on the screen when the microphone receives a pure note. Which trace can be obtained when a musical instrument produces a note of the same pitch but of a different quality ?



(A) 

(B) 

(C) 

(D) 

SW0041

40. Choose **correct** statement?

(A) Two different acoustic musical instruments can not have same loudness

(B) Two different acoustic musical instruments can not have same pitch

(C) Two different acoustic musical instruments can not have same quality

(D) Two different acoustic musical instruments can have more than two characteristics same

SW0042

41. A plane transverse wave is propagating in a direction making an angle of 30° with positive x-axis in the x-y plane. Find phase difference between points (0, 0, 0) and (1, 1, 1). Wavelength of the wave is 1m :-

(A) 2π rad (B) $\left(\sqrt{3}+1\right)\pi$ rad (C) $\left(\sqrt{2}+1\right)\pi$ rad (D) None **SW0043**

42. Which of the following is the equation of a spherical wave :-

(A) $S = S_0 \sin(Kx - \omega t)$ (B) $S = S_0 \cos(Kx - \omega t)$

(C) $S = (S_0/x)\sin(\omega t - Kx)$ (D) $S = (S_0/x^2)\sin(\omega t - Kx)$ **SW0044**

43. A note is produced when you blow air across the top of a test tube. Two students were asked about the effect of blowing harder.

Student-A : The pitch of sound would increase.

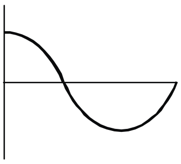
Student-B : The intensity of sound would increase

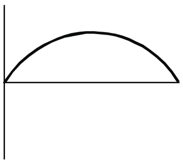
(A) A is correct, B is wrong (B) B is correct, A is wrong

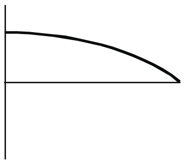
(C) both are correct (D) both are wrong **SW0045**

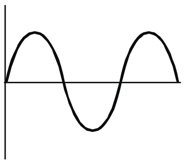
Standing waves

44. Which of the figures, shows the pressure difference from regular atmospheric pressure for an organ pipe of length L closed at one end, corresponds to the 1st overtone for the pipe?

(A)


(B)


(C)


(D)


SW0047

45. In an organ pipe whose one end is at x = 0, the pressure is expressed by

$$p = p_0 \cos\frac{3\pi x}{2}\ \sin 300\pi t$$ where x is in meter and t in sec. The organ pipe can be

(A) closed at one end, open at another with length = 0.5m

(B) open at both ends, length = 1m

(C) closed at both ends, length = 2m

(D) closed at one end, open at another with length = $\frac{2}{3}$m **SW0048**

46. If l_1 and l_2 are the lengths of air column for the first and second resonance when a tuning fork of frequency n is sounded on a resonance tube, then the distance of the displacement antinode from the top end of the resonance tube is:

(A) $2(l_2 - l_1)$ (B) $\frac{1}{2}(2l_1 - l_2)$ (C) $\frac{l_2 - 3l_1}{2}$ (D) $\frac{l_2 - l_1}{2}$ **SW0049**

47. A student is experimenting with resonance tube apparatus in Physics lab to find the speed of sound at room temperature. He got resonating lengths of air column as 17 cm and 51 cm, using tuning fork of frequency 512 Hz. Find speed of sound at room temperature and specify, whether the side water reservoir was moved upward or downward to obtain the second resonance (51 cm)?

(A) 348 m/s, downwards (B) 348 m/s, upwards

(C) 332 m/s, downwards (D) 332 m/s, upwards **SW0050**

Interference

48. The ratio of maximum to minimum intensity due to superposition of two waves is $\frac{49}{9}$. Then the ratio of the intensity of component waves is :-

(A) $\frac{25}{4}$ (B) $\frac{16}{25}$ (C) $\frac{4}{49}$ (D) $\frac{9}{49}$ **SW0051**

49. Two waves of sound having intensities I and 4I interfere to produce interference pattern. The phase difference between the waves is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is

(A) 2I (B) 4I (C) 5I (D) 7I **SW0052**

50. Three coherent waves of equal frequencies having amplitude 10 μm, 4μm and 7 μm respectively, arrive at a given point with successive phase difference of $\pi/2$. The amplitude of the resulting wave in mm is given by

(A) 5 (B) 6 (C) 3 (D) 4 **SW0053**

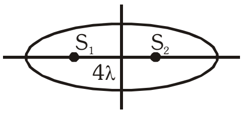
51. The ratio of intensities between two coherent soud sources is 4 : 1. The differenmce of loudness in dB between maximum and minimum intensities when they interfere in space is:

(A) 10 log 2 (B) 20 log 3 (C) 10 log 3 (D) 20 log 2 **SW0054**

52. In Quincke's tube a detector detects minimum intensity. Now one of the tube is displaced by 5 cm. During displacement detector detects maximum intensity 10 times, then finally a minimum intensity (when displacement is complete). The wavelength of sound is:

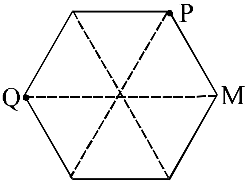
(A) 10/9 cm (B) 1 cm (C) 1/2 cm (D) 5/9 cm **SW0055**

53. S_1, S_2 are two coherent sources of sound located along x – axis separated by 4λ where λ is wavelength of sound emitted by them. Number of maxima located on the elliptical boundary around it will be :



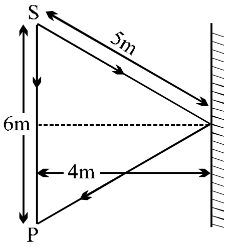
(A) 16 (B) 12 (C) 8 (D) 4 **SW0056**

54. Two sound source emitting sound of wavelength 1 m are located at points P and Q as shown in figure. All sides of the polygon are equal and of length 1m. The intensity of sound at M due to both the individual sources is I_0. What will be the intensity of sound at point M when both the sources are on.



(A) $4I_0$ (B) $2I_0$ (C) I_0 (D) $(1/2)I_0$ **SW0057**

55. A person standing at a distance of 6 m from a source of sound receives sound wave in two ways, one directly from the source and other after reflection from a rigid boundary as shown in the figure. The maximum wavelength for which, the person will receive maximum sound intensity, is



(A) 4 m (B) $\frac{16}{3}$ m (C) 2 m (D) $\frac{8}{3}$ m **SW0058**

Beats

56. Beats are heard when the A strings of two violins are played. The beat frequency decreases as the tension in the A string of violin 1 is slowly increased. Which of the following statement is correct ?

(A) the fundamental frequency of the A string in violin 1 is less than that for violin 2

(B) the fundamental frequency of the A string in violin 1 is greater than that for violin 2

(C) the fundamental frequency of the A string in violin 1 may be greater or less than that for violin 2 depending on the linear mass densities of the two strings.

(D) None of these **SW0059**

Doppler effect

57. A source when at rest in a medium produces waves with a velocity v and a wavelength of λ . If the source is set in motion with a velocity v_s what would be the wavelengths produced directly in front of the source?

(A) $\lambda\left(1-\frac{v_s}{v}\right)$ (B) $\lambda\left(1+\frac{v_s}{v}\right)$ (C) $\lambda\left(1+\frac{v}{v_s}\right)$ (D) $\frac{\lambda v}{v+v_s}$ **SW0061**

58. A source of sound S having frequency f. Wind is blowing from source to observer O with velocity u. If speed of sound with respect to air is C, the wavelength of sound detected by O is:

(A) $\frac{C+u}{f}$ (B) $\frac{C-u}{f}$ (C) $\frac{C(C+u)}{(C-u)f}$ (D) $\frac{C}{f}$ **SW0062**

59. A train moving towards a hill at a speed of 72 km / hr sounds a whistle of frequency 500 Hz. A wind is blowing from the hill at a speed of 36 km / hr. If the speed of sound in air is 340 m / s, the frequency heard by a man on the hill is

(A) 532.25 Hz. (B) 565.0 Hz. (C) 516.1 Hz. (D) none of the above.

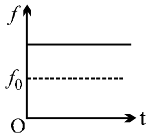
SW0063

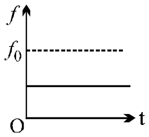
60. A source is moving with constant speed $v_s = 20$ m/sec towards a stationary observer due east of source. Wind is blowing at the speed of 20 m/sec due to 60° north of east. The source is generating of frequency 500 Hz. Then frequency registered by observer is:

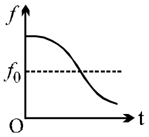
[Speed of sound in still air = 330 m/sec.]

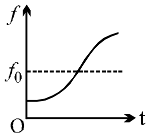
(A) 500 Hz (B) 532 Hz (C) 531 Hz (D) 530 Hz **SW0064**

61. Source and observer both start moving simultaneously from origin, one along x-axis and the other along y-axis with speed of source = twice the speed of observer. The graph between the apparent frequency observed by observer f and time t would approximately be :

(A) 

(B) 

(C) 

(D) 

SW0065

62. A siren placed at a railway platform is emitting sound of frequency 5 kHz. A passenger sitting in a moving train A records a frequency of 5.5 kHz while the train approaches the siren. During his return journey in a different train B he records a frequency of 6.0 kHz while approaching the same siren. The ratio of the velocity of train B to that of train A is :-

(A) 242/252 (B) 2 (C) 5/6 (D) 11/6 **SW0066**

63. A police van moving with velocity 22 m/s and emitting sound of frequency 176 Hz, follows a motor cycle in turn is moving towards a stationary car and away from the police van. The stationary car is emitting frequency 165 Hz. If motorcyclist does not hear any beats then his velocity is ($v_s = 330$ m/s)

[JEE 2003 (Scr)]

(A) 22 m/s (B) 24 m/s (C) 20 m/s (D) 18 m/s **SW0067**

WAVE OPTICS

KEY CONCEPTS

WAVE THEORY OF LIGHT

This theory was enunciated by Hygen in a hypothetical medium known as luminiferrous ether.

Ether is that imaginary medium which prevails in all space, in isotropic, perfectly elastic and massless.

The different colours of light are due to different wave lengths of these waves.

The velocity of light in a medium is constant but changes with change of medium.

This theory is valid for all types of waves.

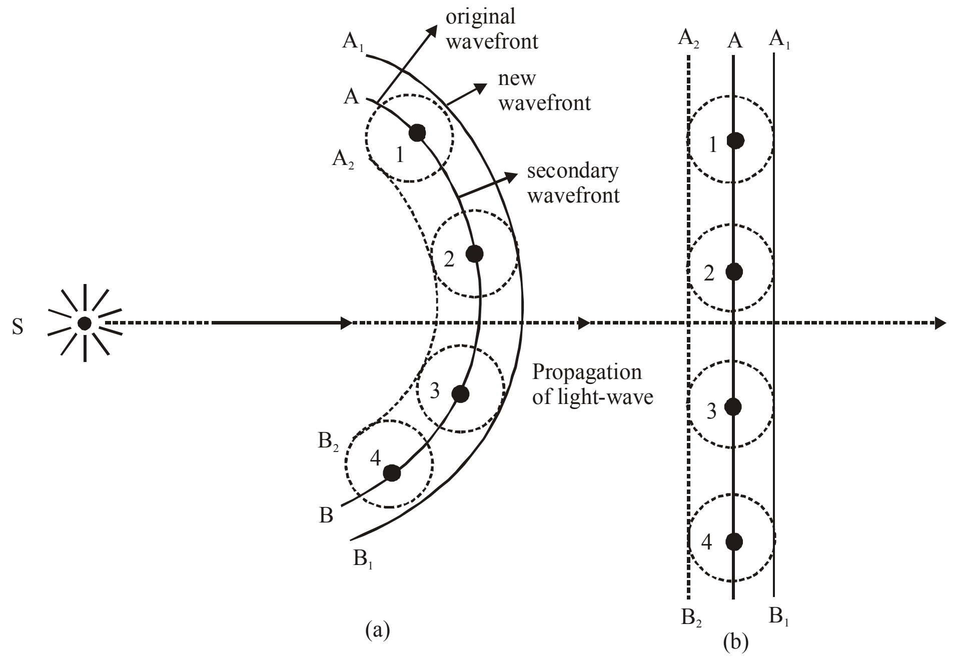
(i) The locus of all ether particles vibrating in same phase is known as wavefront.

(ii) Light travels in the medium in the form of wavefront.

(iii) When light travels in a medium then the particles of medium start vibrating and consequently a disturbance is created in the medium.

(iv) Every point on the wave front becomes the source of secondary wavelets. It emits secondary wavelets in all directions which travel with the speed of light (v),

The tangent plane to these secondary wavelets represents the new position of wave front.



(a) (b)

The phenomena explained by this theory

(i) Reflection, refraction, interference, diffraction, polarisation and double refraction.

(ii) Rectilinear propagation of light.

(iii) Velocity of light in rarer medium being grater than that in denser medium.

Phenomena not explained by this theory

(i) Photoelectric effect, Compton effect and Raman effect.

(ii) Backward propagation of light.

WAVE FRONT, VARIOUS TYPES OF WAVE FRONT AND RAYS

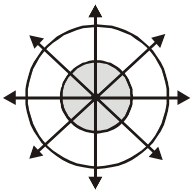
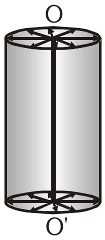
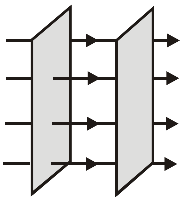
- **Wavefront**

  The locus of all the particles vibrating in the same phase is known as wavefront.

- **Types of wavefront**

  The shape of wavefront depends upon the shape of the light source originating that wavefront. On the basis of there are three types of wavefront.

Comparative study of three types of wavefront

S.No.	Wavefront	Shape of light source	Diagram of shape of wavefront	Variation of amplitude with distance	Variation of intensity with distance
1.	Spherical	Point source		$A \propto \frac{1}{d}$ or $A \propto \frac{1}{r}$	$I \propto \frac{1}{r^2}$
2.	cylindrical	Linear or slit		$A \propto \frac{1}{\sqrt{d}}$ or $A \propto \frac{1}{\sqrt{r}}$	$I \propto \frac{1}{r}$
3.	Plane	Extended large source situated at very large distance		A = constant	I = constant

CHARACTERISTIC OF WAVEFRONT

(a) The phase difference between various particles on the wavefront is zero.

(b) These wavefronts travel with the speed of light in all directions in an isotropic medium.

(c) A point source of light always gives rise to a spherical wavefront in an isotropic medium.

(d) In an anisotropic medium it travels with different velocities in different directions.

(e) Normal to the wavefront represents a ray of light.

(f) It always travels in the forward direction of the medium.

RAY OF LIGHT

The path of the light energy from one point to another is known as a ray of light.

(a) A line drawn at right angles to the wavefront is defined as a ray of light, which is shown by arrows in previous diagram of shape of wavefront.

(b) It represents the direction of propagation of light.

1. INTERFERENCE OF LIGHT

When two light waves of same frequency with zero initial phase difference or constant phase difference superimpose over each other, then the resultant intensity in the region of superposition is different from the sum of intensity of individual waves.

This modification in intensity in the region of superposition is called interference.

(a) Constructive interference

When resultant intensity is greater than the sum of two individual wave intensities $[I > (I_1 + I_2)]$, then the interference is said to be constructive.

(b) Destructive interference

When the resultant intensity is less than the sum of two individual wave intensities $[I < (I_1 + I_2)]$, then the interference is said to destructive. There is no violation of the law of conservation of energy in interference. Here, the energy from the points of minimum energy is shifted to the points of maximum energy.

2. TYPES OF SOURCES

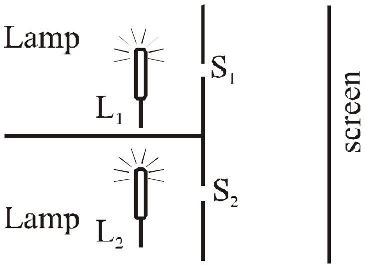
(a) Coherent sources

Two sources are said to be coherent if they emit light waves of the same wave length and start with same phase or have a constant phase difference.

Note : Laser is a source of monochromatic light waves of high degree of coherence.

(b) Incoherent sources

Two independent monochromatic sources, emit waves of same wavelength. But the waves are not in phase. So they are incoherent. This is because, atoms cannot emit light waves in same phase and these sources are said to be incoherent sources.

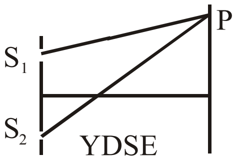


independent source
(noncoherent source)

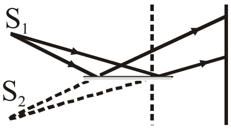
3. METHOD FOR OBTAINING COHERENT SOURCES

(a) Division of wavefront

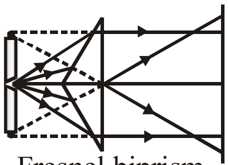
In this method, the wavefront is divided into two or more parts by use of mirrors, lenses or prisms.
Example : Young's double slit experiment. Fresnel's Biprism and Lloyd's single mirror method.



YDSE



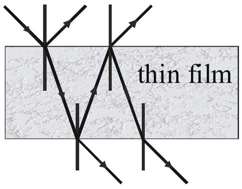
Lloyd's mirror

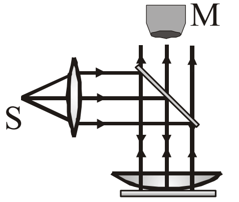


Fresnel biprism

(b) Division of amplitude

The amplitude of incoming beam is divided into two or more parts by partial reflection or refraction. These divided parts travel different paths and are finally brought together to produce interference. Example : The brilliant colour seen in a thin film of transparent material like soap film, oil film, Michelson's Interferro Meter, Newtons' ring etc.





Newton's rings

4. If two coherent waves with intensity I_1 and I_2 are superimposed with a phase difference of ϕ, the resulting wave intensity is

$$I = I_1 + I_2 + 2\sqrt{I_1 I_2}\ \cos\phi$$

5. The phase difference between two waves at a point will depend upon

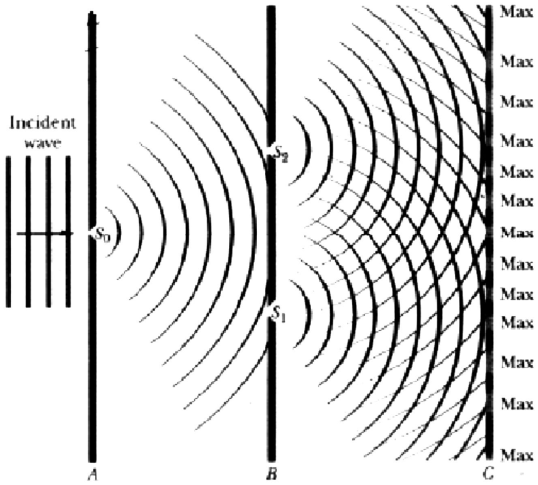
(i) the difference in path lengths of two waves from their respective sources.(geometrical path difference)

(ii) the refractive index of the medium (media)

(iii) phase difference at source (if any).

(iv) In case, the waves suffer reflection, the reflected wave differs in phase by π with respect to the incident wave if the incidence occurs in rarer medium. There would be no phase difference if incidence occurs in denser medium.

6. YOUNG'S DOUBLE SLIT EXPERIMENT

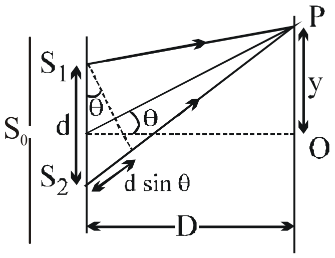


(i) If $d << D$

$\Delta x = S_2P - S_1P = d\sin\theta$

If $\lambda << d$ then $\sin\theta \approx \theta \approx \tan\theta$ as when P is close to D so θ is small.

$$\Delta x = d\left(\frac{y}{D}\right) = \frac{yd}{D}$$



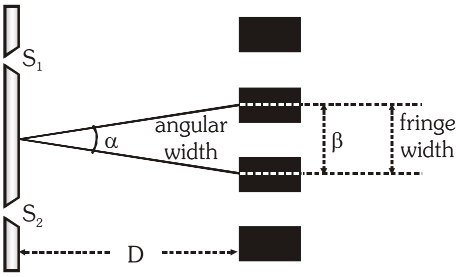
(ii) For maxima $\frac{yd}{D} = n\lambda$

or $y = 0, \pm\frac{D\lambda}{d}, \pm\frac{2D\lambda}{d}$

(iii) For minima $\frac{yd}{D} = [n + (1/2)]\lambda$ or $y = \pm\frac{D\lambda}{2d}, \pm\frac{3D\lambda}{2d}, \pm\frac{5D\lambda}{2d}$, so on

(iv) Fringe width = the distance between two successive maximas or minima $\beta = \frac{\lambda D}{d}$

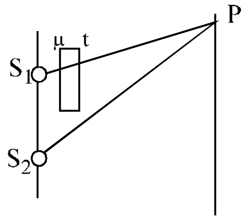
(v) **Angular Fringe width**



Angular Fringe Width $\alpha = \frac{\beta}{D}, \ \alpha = \frac{\lambda}{d} \ \left[\because \frac{\beta}{D} = \frac{\lambda}{d}\right]$

7. DISPLACEMENT OF FRINGE PATTERN

When a film of thickness t and refractive index μ is introduced in the path of one of the source's of light, then fringe shift occurs as the optical path difference changes.



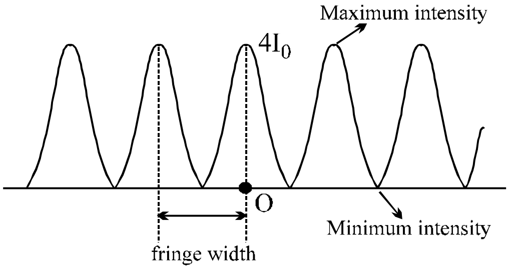
Optical path difference at P is

$\Delta x = S_2P - [S_1P + \mu t - t] = S_2P - S_1P - (\mu - 1)\,t = y.\,(d/D) - (\mu - 1)t$

$\Rightarrow$ The fringe shift is given by $\Delta y = \frac{D(\mu - 1)t}{d}$

8. INTENSITY VARIATION ON SCREEN

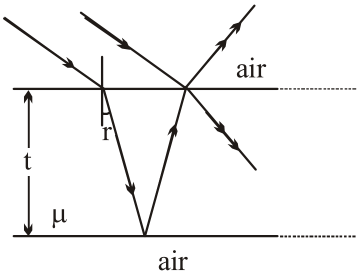
If I_0 is the intensity of light beam coming from each slit, the resultant intensity at a point where they have a phase difference of ϕ is $I = 4I_0\cos^2\frac{\phi}{2}$, where $\phi = \frac{2\pi(d\sin\theta)}{\lambda}$



9. INTERFERENCE AT THIN FILM

optical path difference $= 2\mu t \cos r$

$= 2\mu t$ (in case of near normal incidence)



(a) For interference in reflected light

- **(i)** Condition of minima $2\mu t \cos r = n\lambda$
- **(ii)** Condition of maxima $2\mu t \cos r = \left(n+\frac{1}{2}\right)\lambda$

(b) For interference in transmitted light

- **(i)** Condition of maxima $2\mu t \cos r = n\lambda$
- **(ii)** Condition of minima $2\mu t \cos r = \left(n+\frac{1}{2}\right)\lambda$

Ampere Law :

The general form of Ampere's law (sometimes called the **Ampere-Maxwell law**) as

$$\oint \vec{B}\cdot d\vec{\ell} = \mu_0\left(I+I_0\right) = \mu_0\left(I+\epsilon_0\frac{d\Phi_E}{dt}\right)$$

Hence, the displacement current through any surface is given by

$$I_d = \epsilon_0\frac{d\Phi_E}{dt}$$

By considering surface S_2, we can identify the displacement current as the source ofthe magnetic field

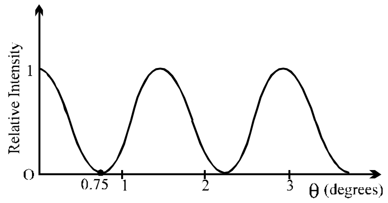
EXERCISE (S)

1. In Young's double slit experiment the slits are 0.5 mm apart and the interference is observed on a screen at a distance of 100 cm from the slit. It is found that the 9th bright fringe is at a distance of 7.5mm from the second dark fringe from the centre of the fringe pattern on same side. Find the wavelength of the light used. **WO0002**

2. In a YDSE apparatus, d = 1mm, λ = 600 nm and D = 1m. The slits individually produce same intensity on the screen. Find the minimum distance between two points on the screen having 75% intensity of the maximum intensity. **WO0005**

3. The distance between two slits in a YDSE apparatus is 3mm. The distance of the screen from the slits is 1m. Microwaves of wavelength 1 mm are incident on the plane of the slits normally. Find the distance of the first maxima on the screen from the central maxima. Also find the total number of maxima on the screen. **WO0006**

4. Light of wavelength 520 nm passing through a double slit, produces interference pattern of relative intensity versus angular position θ as shown in the figure. Find the separation d between the slits.



WO0003

5. In a Young's double slit experiment, two wavelengths of 500 nm and 700 nm were used. What is the minimum distance from the central maximum where their maximas coincide again? (Take $D/d = 10^3$. Symbols have their usual meanings.) **[IIT-JEE 2004]**

WO0004

6. In a Young's double slit experiment for interference of light, the slits are 0.2 cm apart and are illuminated by yellow light (λ = 600 nm). What would be the fringe width on a screen placed 1 m from the plane of slits if the whole system is immersed in water of index 4/3? **WO0001**

7. A monochromatic light of λ = 5000 Å is incident on two slits separated by a distance of 5×10^{-4} m. The interference pattern is seen on a screen placed at a distance of 1 m from the slits. A thin glass plate of thickness 1.5×10^{-6} m & refractive index μ = 1.5 is placed between one of the slits & the screen. Find the intensity at the centre of the screen, if the intensity there is I_0 in the absence of the plate. Also find the lateral shift of the central maximum. **WO0008**

8. One slit of a double slit experiment is covered by a thin glass plate of refractive index 1.4 and the other by a thin glass plate of refractive index 1.7. The point on the screen, where central bright fringe was formed before the introduction of the glass sheets, is now occupied by the 5th bright fringe. Assuming that both the glass plates have same thickness and wavelength of light used is 4800 Å, find their thickness. **WO0007**

9. In Young's experiment, the source is red light of wavelength 7×10^{-7} m . When a thin glass plate of refractive index 1.5 at this wavelength is put in the path of one of the interfering beams, the central bright fringe shifts by 10^{-3} m to the position which was previously occupied by the 5th bright fringe. Find the thickness of the plate. When the source is now changed to green light of wavelength 5×10^{-7} m, the central fringe shifts to a position initially occupied by the 6th bright fringe due to red light without the plate. Find the refractive index of glass for the green light. Also estimate the change in fringe width due to the change in wavelength. **[IIT-JEE 1997 C]**

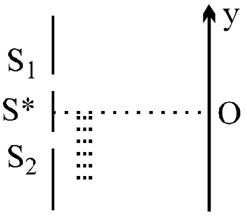
WO0015

10. The Young's double slit experiment is done in a medium of refractive index 4/3. A light of 600 nm wavelength is falling on the slits having 0.45 mm separation. The lower slit S_2 is covered by a thin glass sheet of thickness 10.4 μm and refractive index 1.5. The interference pattern is observed on a screen placed 1.5 m from the slits as shown **[IIT-JEE'99]**

(i) Find the location of the central maximum (bright fringe with zero path difference) on the y-axis.

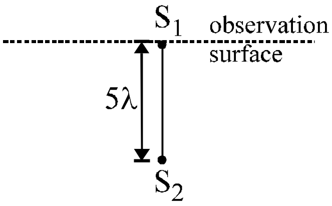
(ii) Find the light intensity at point O relative to the maximum fringe intensity.

(iii) Now, if 600 nm light is replaced by white light of range 400 to 700 nm, find the wavelengths of the light that form maxima exactly at point O. [All wavelengths in this problem are for the given medium of refractive index 4/3. Ignore dispersion]



WO0017

11. Two microwave coherent point sources emitting waves of wavelength λ are placed at 5λ distance apart. The interference is being observed on a flat non-reflecting surface along a line passing through one source, in a direction perpendicular to the line joining the two sources (refer figure). Considering λ as 4 mm, calculate the positions of maxima and draw shape of interference pattern. Take initial phase difference between the two sources to be zero.



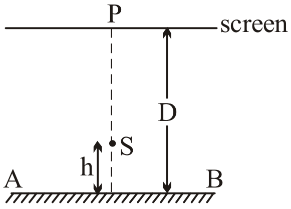
WO0011

12. A point source *S* emitting light of wavelength 600 nm is placed at a very small height h above the flat reflecting surface *AB* (see figure). The intensity of the reflected light is 36% of the incident intensity. Interference fringes are observed on a screen placed parallel to the reflecting surface at a very large distance *D* from it. **[IIT-JEE 2002]**

(i) What is the shape of the interference fringes on the screen?

(ii) Calculate the ratio of the minimum to the maximum intensities in the interference fringes formed near the point P (shown in the figure).

(iii) If the intensities at point P corresponds to a maximum, calculate the minimum distance through which the reflecting surface AB should be shifted so that the intensity at P again becomes max.



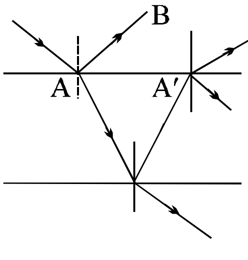
WO0012

13. A long narrow horizontal slit lies 1mm above a plane mirror as in Lloyd's mirror. The interference pattern produced by the slit and its image is viewed on a screen distant 1m from the slit. The wavelength of light is 600nm. Find the distance of first maximum above the mirror. **WO0010**

14. In a biprism experiment with sodium light, bands of width 0.0195 cm are observed on screen at 100cm from slit. On introducing a convex lens 30 cm away from the slit between biprism and screen, two images of the slit are seen 0.7 cm apart on screen. Calculate the wavelength of sodium light.

WO0009

15. A ray of light of intensity I is incident on a parallel glass-slab at a point as shown in figure. It undergoes partial reflection and refraction. At each reflection 20% of incident energy is reflected. The rays AB and A′ B′ undergo interference. Find the ratio I_{max}/I_{min}. [Neglect the absorption of light]

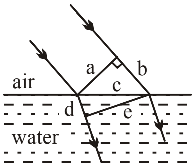


WO0013

EXERCISE (O)

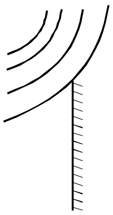
SINGLE CORRECT TYPE QUESTIONS

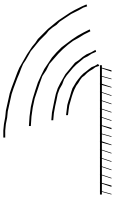
1. Figure shows plane waves refracted from air to water using Huygen's principle (where a, b, c, d, e are lengths on the diagram). The refractive index of water wrt air is the ratio :-

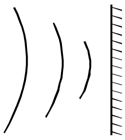


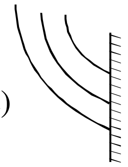
(A) a/e (B) b/e (C) b/d (D) d/b **WO0026**

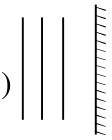
2. Spherical wavefronts shown in figure, strike a plane mirror. Reflected wavefronts will be as shown in



(A) 

(B) 

(C) 

(D) 

WO0027

3. When light is refracted into a denser medium,

(A) its wavelength and frequency both increase

(B) its wavelength increases but frequency remains unchanged

(C) its wavelength decreases but frequency remains unchanged

(D) its wavelength and frequency both decrease. **WO0031**

4. Two beams of light having intensifies I and $4I$ interfere to produce a fringe pattern on a screen. The phase difference between the beams is $\pi/2$ at point A and π at point B. Then the difference between the resultant intensities at A and B is : **[IIT-JEE (Scr.) 2001]**

(A) 2I (B) 4I (C) 5I (D) 7I **WO0028**

5. In a young double slit experiment, 12 fringes are observed to be formed in a certain segment of the screen when light of wavelength 600 nm is used. If the wavelength of light is changed to 400 nm, number of fringes observed in the same segment of the screen is given by **[IIT-JEE (Scr.) 2001]**

(A) 12 (B) 18 (C) 24 (D) 30 **WO0029**

6. Two coherent monochromatic light beams of intensities I and $4I$ are superposed. The maximum and minimum possible intensities in the resulting beam are :

(A) $5I$ and I (B) 5I and 3I (C) $9I$ and I (D) $9I$ and $3I$ **WO0030**

7. In YDSE how many maxima can be obtained on the screen if wavelength of light used is 200nm and $d = 700$ nm:

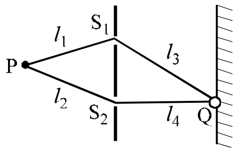
(A) 12 (B) 7 (C) 18 (D) none of these

WO0032

8. In a YDSE bi-chromatic light of wavelengths 400 nm and 560 nm are used. The distance between the slits is 0.1 mm and the distance between the plane of the slits and the screen is 1 m. The minimum distance between two successive regions of complete darkness is :- **[IIT-JEE' 2004 (Scr)]**

(A) 4 mm (B) 5.6 mm (C) 14 mm (D) 28 mm **WO0042**

9. Two identical narrow slits S_1 and S_2 are illuminated by light of wavelength λ from a point source P. If, as shown in the diagram, the light is then allowed to fall on a screen, and if n is a positive integer, the condition for destructive interference at Q is :-



(A) $(\ell_1 - \ell_2) = (2n + 1)\lambda/2$ (B) $(\ell_3 - \ell_4) = (2n + 1)\lambda/2$

(C) $(\ell_1 + \ell_2) - (\ell_3 + \ell_4) = n\lambda$ (D) $(\ell_1 + \ell_3) - (\ell_2 + \ell_4) = (2n + 1)\lambda/2$ **WO0036**

10. In Young's double slit experiment, the two slits act as coherent sources of equal amplitude A and wavelength λ. In another experiment with the same setup the two slits are sources of equal amplitude A and wavelength λ but are incoherent. The ratio of the average intensity of light at the midpoint of the screen in the first case to that in the second case is :-

(A) 1 : 1 (B) 2 : 1 (C) 4 : 1 (D) none of these

WO0037

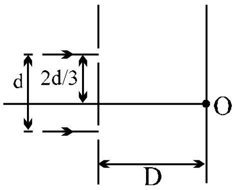
11. In a Young's double slit experiment, a small detector measures an intensity of illumination of I units at the centre of the fringe pattern. If one of the two (identical) slits is now covered, the measured intensity will be :-

(A) 2I (B) I (C) I/4 (D) I/2 **WO0038**

12. In a Young's double slit experiment D equals the distance of screen and d is the separation between the slit. The distance of the nearest point to the central maximum where the intensity is same as that due to a single slit, is equal to :-

(A) $\frac{D\lambda}{d}$ (B) $\frac{D\lambda}{2d}$ (C) $\frac{D\lambda}{3d}$ (D) $\frac{2D\lambda}{d}$ **WO0039**

13. In the figure shown if a parallel beam of white light is incident on the plane of the slits then the distance of the white spot on the screen from O is [Assume $d << D$, $\lambda << d$]



(A) 0 (B) $d/2$ (C) $d/3$ (D) $d/6$ **WO0040**

14. In the above question if the light incident is monochromatic and point O is a maxima, then the wavelength of the light incident cannot be :-

(A) $d^2/3D$ (B) $d^2/6D$ (C) $d^2/12D$ (D) $\mathrm{d}^2/18\mathrm{D}$ **WO0041**

15. In a double slit experiment, when the width of one slit is made twice as wide as the other in compared to normal YDSE having slits of equal width. Then, in the interference pattern **[IIT-JEE(Scr.) 2000]**

(A) the intensities of both the maxima and the minima increase.

(B) the intensity of the maxima increases and the minima has zero intensity.

(C) the intensity of the maxima decreases and that of the minima increases.

(D) the intensity of the maxima decreases and the minima has zero intensity. **WO0050**

16. In a double slit experiment, the separation between the slits is $d = 0.25$ cm and the distance of the screen D = 100 cm from the slits. If the wavelength of light used is $\lambda = 6000\text{Å}$ and I_0 is the intensity of the central bright fringe, the intensity at a distance $x = 4 \times 10^{-5}$ m from the central maxima is :-

(A) I_0 (B) $I_0/2$ (C) $3I_0/4$ (D) $I_0/3$ **WO0056**

17. Imagine a Young's double slit interference experiment performed with waves associated with fast moving electrons produced from an electron gun. The distance between successive maxima will decrease the most if :-

(A) the accelerating voltage in the electron gun is decreased

(B) the accelerating voltage is increased and the distance of the screen from the slits is decreased

(C) the distance of the screen from the slits is increased.

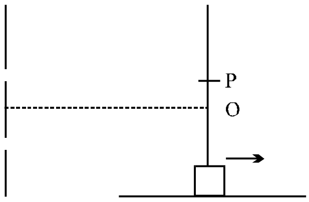
(D) the distance between the slits is decreased. **WO0057**

18. Two monochromatic and coherent point sources of light are placed at a certain distance from each other in the horizontal plane. The locus of all those points in the horizontal plane which have constructive interference will be

(A) a hyperbola (B) family of hyperbolas

(C) family of straight lines (D) family of parabolas **WO0058**

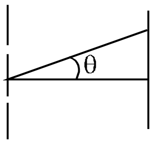
19. In a Young's Double slit experiment, first maxima is observed at a fixed point *P* on the screen. Now the screen is continuously moved away from the plane of slits. The ratio of intensity at point P to the intensity at point *O* (centre of the screen)



(A) remains constant
(B) keeps on decreasing
(C) first decreases and then increases
(D) First decreases and then becomes constant

WO0100

20. Two slits are separated by 0.3 mm. A beam of 500 nm light strikes the slits producing an interference pattern. The number of maxima observed in the angular range – 30° < θ < 30°.



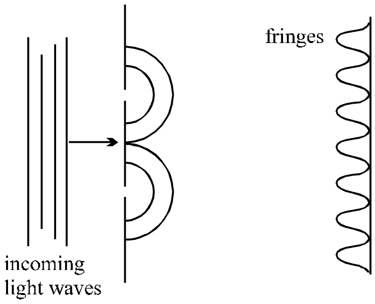
(A) 300
(B) 150
(C) 599
(D) 601

WO0101

21. In a YDSE, the central bright fringe can be identified :
(A) as it has greater intensity than the other bright fringes.
(B) as it is wider than the other bright fringes.
(C) as it is narrower than the other bright fringes.
(D) by using white light instead of single wavelength light.

WO0021

22. In a Young's double slit experiment, green light is incident on the two slits. The interference pattern is observed on a screen. Which of the following changes would cause the observed fringes to be more closely spaced?



(A) Reducing the separation between the slits
(B) Using blue light instead of green light
(C) Used red light instead of green light
(D) Moving the light source further away from the slits.

WO0022

23. In Young's double slit experiment, the wavelength of red light is 7800 Å and that of blue light is 5200 Å. The value of *n* for which n^{th} bright band due to red light coincides with $(n+1)^{th}$ bright band due to blue light, is :

(A) 1 (B) 2 (C) 3 (D) 4 **WO0023**

24. In Young's double slit arrangement, water is filled in the space between screen and slits. Then :

(A) fringe pattern shifts upwards but fringe width remains unchanged.

(B) fringe width decreases and central bright fringe shifts upwards.

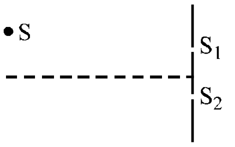
(C) fringe width increases and central bright fringe does not shift.

(D) fringe width decreases and central bright fringe does not shift. **WO0043**

25. Light of wavelength λ in air enters a medium of refractive index μ. Two points in this medium, lying along the path of this light, are at a distance x apart. The phase difference between these points is :

(A) $\dfrac{2\pi\mu x}{\lambda}$ (B) $\dfrac{2\pi x}{\mu\lambda}$ (C) $\dfrac{2\pi(\mu-1)x}{\lambda}$ (D) $\dfrac{2\pi x}{(\mu-1)\lambda}$ **WO0044**

26. In YDSE, the source placed symmetrically with respect to the slit is now moved parallel to the plane of the slits so that it is closer to the upper slit, as shown. Then,



(A) the fringe width will increase and fringe pattern will shift down.

(B) the fringe width will remain same but fringe pattern will shift up.

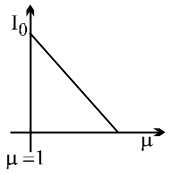
(C) the fringe width will decrease and fringe pattern will shift down.

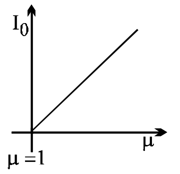
(D) the fringe width will remain same but fringe pattern will shift down. **WO0045**

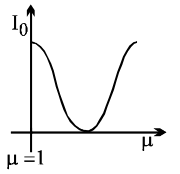
27. In the ideal double–slit experiment, when a glass–plate (refractive index 1.5) of thickness t is introduced in the path of one of the interfering beams (wavelength λ), the intensity at the position where the central maximum occurred previously remains unchanged. The minimum thickness of the glass–plate is :– **[IIT-JEE 2002]**

(A) 2λ (B) $\dfrac{2\lambda}{3}$ (C) $\dfrac{\lambda}{3}$ (D) λ **WO0046**

28. In a YDSE experiment if a slab whose refractive index can be varied is placed in front of one of the slits then the variation of resultant intensity at mid-point of screen with μ (μ ≥ 1) will be best represented by [Assume slits of equal width and there is no absorption by slab]

(A)


(B)


(C)


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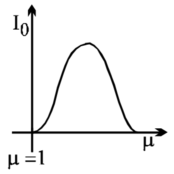


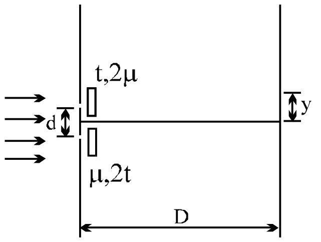
WO0047

29. In a Young's double-slit experiment, let A and B be the two slits. Films of thicknesses t_A and t_B and refractive indices μ_A and μ_B, are placed in front of A and B respectively. If $\mu_A t_A = \mu_B t_B$, the central maximum will :

(A) not shift
(B) shift towards A
(C) shift towards B
(D) option (B), if $t_B > t_A$; option (C) if $t_B < t_A$

WO0048

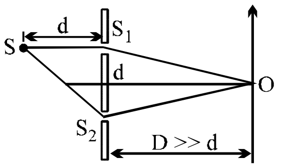
30. In the YDSE shown the two slits are covered with thin sheets having thickness t & $2t$ and refractive index 2μ and μ. Find the position (y) of central maxima



(A) zero
(B) $\dfrac{tD}{d}$
(C) $-\dfrac{tD}{d}$
(D) None

WO0049

31. To make the central fringe at the centre O, a mica sheet of refractive index 1.5 is introduced. Choose the correct statements (s).



(A) The thickness of sheet is $2(\sqrt{2}-1)d$ in front of S_1.

(B) The thickness of sheet is $(\sqrt{2}-1)d$ in front of S_2.

(C) The thickness of sheet is $2\sqrt{2}\,d$ in front of S_1.

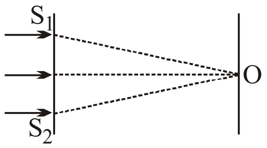
(D) The thickness of sheet is $(2\sqrt{2}-1)d$ in front of S_1.

WO0051

32. **Statement-1 :** In YDSE, as shown in figure, central bright fringe is formed at O. If a liquid is filled between plane of slits and screen, the central bright fringe is shifted in upward direction.

and

Statement-2 : If path difference at O increases, y-coordinate of central bright fringe will change.



(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

WO0052

33. **Statement-1 :** In glass, red light travels faster than blue light.

and

Statement-2 : Red light has a wavelength longer than blue.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true. **WO0053**

34. **Statement-1 :**In standard YDSE set up with visible light, the position on screen where phase difference is zero appears bright.

and

Statement-2 : In YDSE set up magnitude of electromagnetic field at central bright fringe is not varying with time.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true. **WO0054**

35. **Statement-1 :**In YDSE, the spacing between any two successive points having intensity half of the maximum intensity is same.

and

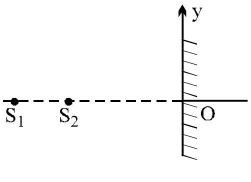
Statement-2 : The intensity on the screen in YDSE varies uniformly with distance from central maximum.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true. **WO0055**

36. Two point monochromatic and coherent sources of light of wavelength λ are placed on the dotted line in front of a large screen. The sources emit waves in phase with each other. The distance between S_1 and S_2 is d while their distance from the screen is much larger.

(1) If $d = 7\lambda/2$, O will be a minima
(2) If $d = 4.3\lambda$, there will be a total of 8 minima on y-axis.
(3) If $d = 7\lambda$, O will be a maxima.
(4) If $d = \lambda$, there will be only one maxima on the screen.

Which is the set of correct statement :



(A) 1, 2 & 3 (B) 2, 3 & 4 (C) 1, 2, 3 & 4 (D) 1, 3 & 4 **WO0105**

EXERCISE - JM

1. The equation of a wave on a string of linear mass density 0.04 kg m^{-1} is given by $y = 0.02(m)\sin\left[2\pi\left(\frac{t}{0.04(s)} - \frac{x}{0.50(m)}\right)\right]$. The tension in the string is : **[AIEEE - 2010]**

(1) 6.25 N (2) 4.0 N (3) 12.5 N (4) 0.5 N **WA0059**

Direction : Questions number 2 to 4 are based on the following paragraph.

An initially parallel cylindrical beam travels in a medium of refractive index $\mu(I) = \mu_0 + \mu_2 I$, where μ_0 and μ_2 are positive constants and I is the intensity of the light beam. The intensity of the beam is decreasing with increasing radius.

2. The initial shape of the wavefront of the beam is :- **[AIEEE-2010]**

(1) planar

(2) convex

(3) concave

(4) convex near the axis and concave near the periphery **WO0112**

3. The speed of the light in the medium is :- **[AIEEE-2010]**

(1) maximum on the axis of the beam

(2) minimum on the axis of the beam

(3) the same everywhere in the beam

(4) directly proportional to the intensity I **WO0112**

4. As the beam enters the medium, it will :- **[AIEEE-2010]**

(1) travel as a cylindrical beam

(2) diverge

(3) converge

(4) diverge near the axis and converge near the periphery **WO0112**

5. The transverse displacement y(x, t) of a wave on a string is given by $y(x,t) = e^{-\left(ax^2+bt^2+2\sqrt{ab}xt\right)}$. This represents a :- **[AIEEE - 2011]**

(1) standing wave of frequency $\sqrt{b}$

(2) standing wave of frequency $\frac{1}{\sqrt{b}}$

(3) wave moving in +x directionwith speed $\sqrt{\frac{a}{b}}$

(4) wave moving in –x direction with speed $\sqrt{\frac{b}{a}}$ **WA0060**

6. **Statement-1:** Two longitudinal waves given by equations : $y_1(x, t) = 2a \sin(\omega t - kx)$ and $y_2(x, t) = a \sin(2\omega t - 2kx)$ will have equal intensity.

Statement-1: Intensity of waves of given frequency in same medium is proportional to square of amplitude only. **[AIEEE - 2011]**

(1) Statement-1 is false, statement-2 is true

(2) Statement-1 is ture, statement-2 is false

(3) Statement-1 is ture, statement-2 true; statement-2 is the correct explanation of statement-1

(4) Statement-1 is true, statement-2 is true; statement -2 is not correct explanation of statement-1.

SW0082

7. A travelling wave represented by $y = A \sin(\omega t - kx)$ is superimposed on another wave represented by $y = A \sin(\omega t + kx)$. The resultant is :- **[AIEEE-2011]**

(1) A standing wave having nodes at $x = \left(n + \frac{1}{2}\right)\frac{\lambda}{2}, n = 0, 1, 2$

(2) A wave travelling along + x direction

(3) A wave travelling along –x direction

(4) A standing wave having nodes at $x = \frac{n\lambda}{2}$; n = 0, 1, 2 **WA0061**

8. At two points P and Q on screen in Young's double slit experiment, waves from slits S_1 and S_2 have a path difference of 0 and $\frac{\lambda}{4}$ respectively. the ratio of intensities at P and Q will be : **[AIEEE-2011]**

(1) 3 : 2 (2) 2 : 1 (3) $\sqrt{2}$: 1 (4) 4 : 1 **WO0113**

9. In a Young's double slit experiment, the two slits act as coherent sources of waves of equal amplitude A and wavelength λ. In another experiment with the same arrangement the two slits are made to act as incoherent sources of waves of same amplitude and wavelength. If the intensity at the middle point of the screen in the first case is I_1 and in the second case I_2, then the ratio $\frac{I_1}{I_2}$ is :- **[AIEEE-2011]**

(1) 4 (2) 2 (3) 1 (4) 0.5 **WO0115**

10. Direction :

The question has a paragraph followed by two statement, Statement-1 and statement-2. Of the given four alternatives after the statements, choose the one that describes the statements.

A thin air film is formed by putting the convex surface of a plane-convex lens over a plane glass plate. With monochromatic light, this film gives an interference pattern due to light reflected from the top (convex) surface and the bottom (glass plate) surface of the film **[AIEEE-2011]**

Statement-1:

When light reflects from the air-glass plate interface, the reflected wave suffers a phase change of π.

Statement-2: The centre of the interference pattern is dark :-

(1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is false

(4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

WO0116

11. In Young's double slit experiment, one of the slit is wider than other, so that the amplitude of the light from one slit is double of that from other slit. If I_m be the maximum intensity, the resultant intensity I when they interfere at phase difference ϕ is given by : **[AIEEE-2012]**

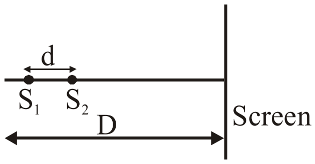
(1) $\frac{I_m}{9}(1 + 8\cos^2\frac{\phi}{2})$ (2) $\frac{I_m}{9}(4 + 5\cos\phi)$ (3) $\frac{I_m}{3}(1 + 2\cos^2\frac{\phi}{2})$ (4) $\frac{I_m}{5}(1 + 4\cos^2\frac{\phi}{2})$

WO0117

12. A sonometer wire of length 1.5m is made of steel. The tension in it produces an elastic strain of 1%. What is the fundamental frequency of steel if density and elasticity of steel are 7.7×10^3 kg/m^3 and 2.2×10^{11} N/m^2 respectively ? **[JEE Main 2013]**

(1) 188.5 Hz (2) 178.2 Hz (3) 200.5 Hz (4) 770 Hz **WA0062**

13. Two coherent point sources S_1 and S_2 are separated by a small distance 'd' as shown. The fringes obtained on the screen will be : **[JEE-Main 2013]**



(1) points (2) straight lines (3) semicircles (4) concentric circles

WO0121

14. A pipe of length 85 cm is closed from one end. Find the number of possible natural oscillations of air column in the pipe whose frequencies lie below 1250 Hz. The velocity of sound in air is 340 m/s.

[JEE Main 2014]

(1) 6 (2) 4 (3) 12 (4) 8 **SW0083**

15. A train is moving on a straight track with speed 20 ms^{-1}. It is blowing its whistle at the frequency of 1000 Hz. The percentage change in the frequency heard by a person standing near the track as the train passes him is (speed of sound = 320 ms^{-1}) close to :- **[JEE Main 2015]**

(1) 18% (2) 24% (3) 6% (4) 12% **SW0084**

16. On a hot summer night, the refractive index of air is smallest near the ground and increases with height from the ground. When a light beam is directed horizontally, the Huygens' principle leads us to conclude that as it travels, the light beam : **[JEE-Main 2015]**

(1) bends downwards (2) bends upwards
(3) becomes narrower (4) goes horizontally without any deflection

WO0127

17. A uniform string of length 20m is suspended from a rigid support. A short wave pulse is introduced at its lowest end. It starts moving up the string. The time taken to reach the support is :-
(take g = 10 ms^{-2}) **[JEE Main 2016]**

(1) $\sqrt{2}$ s (2) $2\pi\sqrt{2}$ s (3) 2s (4) $2\sqrt{2}$ s **WA0063**

18. A pipe open at both ends has a fundamental frequency f in air. The pipe is dipped vertically in water so that half of it is in water. The fundamental frequencty of the air column is now :-

[JEE Main 2016]

(1) f (2) $\frac{f}{2}$ (3) $\frac{3f}{4}$ (4) 2f **SW0085**

19. Arrange the following electromagnetic radiations per quantum in the order of increasing energy:-

A : Blue light B : Yellow light C : X–ray D : Radiowave

[JEE-Main 2016]

(1) B, A, D, C (2) D, B, A, C (3) A, B, D, C (4) C, A, B, D

WO0129

20. In a Young's double slit experiment, slits are separated by 0.5 mm, and the screen is placed 150 cm away. A beam of light consisting of two wavelengths, 650 nm and 520 nm, is used to obtain interference fringes on the screen. The least distance from the common central maximum to the point where the bright fringes due to both the wavelengths coincide is :- **[JEE-Main 2017]**

(1) 9.75 mm (2) 15.6 mm (3) 1.56 mm (4) 7.8 mm

WO0130

21. A granite rod of 60 cm length is clamped at its middle point and is set into longitudinal vibrations. The density of granite is 2.7×10^3 kg/m^3 and its Young's modulus is 9.27×10^{10} Pa. What will be the fundamental frequency of the longitudinal vibrations ? **[JEE-Main 2018]**

(1) 2.5 kHz (2) 10 kHz (3) 7.5 kHz (4) 5 kHz **WA0064**

22. A musician using an open flute of length 50 cm produces second harmonic sound waves. A person runs towards the musician from another end of a hall at a speed of 10 km/h. If the wave speed is 330m/s, the frequency heard by the running person shall be close to : **[JEE Main-2019_Jan]**

(1) 753 Hz (2) 500 Hz (3) 333 Hz (4) 666 Hz **WA0070**

23. Two coherent sources produce waves of different intensities which interfere. After interference, the ratio of the maximum intensity to the minimum intensity is 16. The intensity of the waves are in the ratio: **[JEE Main-2019_Jan]**

(1) 4 : 1 (2) 25 : 9 (3) 16 : 9 (4) 5 : 3 **WO0141**

24. A heavy ball of mass M is suspended from the ceiling of a car by a light string of mass m (m<<M). When the car is at rest, the speed of transverse waves in the string is 60 ms^{-1}. When the car has acceleration a, the wave-speed increases to 60.5 ms^{-1}. The value of a, in terms of gravitational acceleration g, is closest to : **[JEE Main-2019_Jan]**

(1) $\frac{g}{5}$ (2) $\frac{g}{20}$ (3) $\frac{g}{10}$ (4) $\frac{g}{30}$ **WA0071**

25. A closed organ pipe has a fundamental frequency of 1.5 kHz. The number of overtones that can be distinctly heard by a person with this organ pipe will be : (Assume that the highest frequency a person can hear is 20,000 Hz) **[JEE Main-2019_Jan]**

(1) 7 (2) 5 (3) 6 (4) 4 **SW0099**

26. A string of length 1 m and mass 5 g is fixed at both ends. The tension in the string is 8.0 N. The siring is set into vibration using an external vibrator of frequency 100 Hz. The separation between successive nodes on the string is close to : **[JEE Main-2019_Jan]**

(1) 16.6 cm (2) 20.0 cm (3) 10.0 cm (4) 33.3 cm **WA0072**

27. A train moves towards a stationary observer with speed 34 m/s. The train sounds a whistle and its frequency registered by the observer is f_1. If the speed of the train is reduced to 17 m/s, the frequency registered is f_2. If speed of sound is 340 m/s, then the ratio f_1/f_2 is : **[JEE Main-2019_Jan]**

(1) 18/17 (2) 19/18 (3) 20/19 (4) 21/20 **SW0100**

28. Equation of travelling wave on a stretched string of linear density 5 g/m is $y = 0.03 \sin(450\,t - 9x)$; where distance and time are measured is SI units. The tension in the string is : **[JEE Main-2019_Jan]**

(1) 10 N (2) 12.5 N (3) 7.5 N (4) 5 N **WA0073**

29. A resonance tube is old and has jagged end. It is still used in the laboratory to determine velocity of sound in air. A tuning fork of frequency 512 Hz produces first resonance when the tube is filled with water to a mark 11 cm below a reference mark, near the open end of the tube. The experiment is repeated with another fork of frequency 256 Hz which produces first resonance when water reaches a mark 27 cm below the reference mark. The velocity of sound in air, obtained in the experiment, is close to: **[JEE Main-2019_Jan]**

(1) 328ms^{-1} (2) 322ms^{-1} (3) 341ms^{-1} (4) 335ms^{-1} **SW0101**

30. A travelling harmonic wave is represented by the equation $y(x, t) = 10^{-3} \sin(50\,t + 2x)$, where x and y are in meter and t is in seconds. Which of the following is a correct statement about the wave? The wave is propagating along the **[JEE Main-2019_Jan]**

(1) negative x-axis with speed 25ms^{-1}

(2) The wave is propagating along the positive x-axis with speed 25 ms^{-1}

(3) The wave is propagating along the positive x-axis with speed 100 ms^{-1}

(4) The wave is propagating along the negative x-axis with speed 100 ms^{-1} **WA0074**

31. A person standing on an open ground hears the sound of a jet aeroplane, coming from north at an angle 60° with ground level. But he finds the aeroplane right vertically above his position. If υ is the speed of sound, speed of the plane is : **[JEE Main-2019_Jan]**

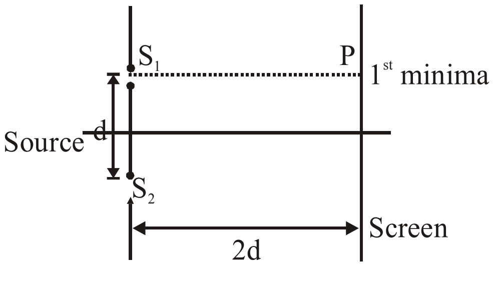
(1) $\frac{2\upsilon}{\sqrt{3}}$ (2) υ (3) $\frac{\upsilon}{2}$ (4) $\frac{\sqrt{3}}{2}\upsilon$ **SW0102**

32. In a Young's double slit experiment, the slits are placed 0.320 mm apart. Light of wavelength $\lambda = 500$ nm is incident on the slits. The total number of bright fringes that are observed in the angular range $-30^\circ \le \theta \le 30^\circ$ is: **[JEE Main-2019_Jan]**

(1) 320 (2) 641 (3) 321 (4) 640 **WO0142**

33. Consider a Young's double slit experiment as shown in figure. What should be the slit separation d in terms of wavelength λ such that the first minima occurs directly in front of the slit (S_1) ?

[JEE Main-2019_Jan]



(1) $\frac{\lambda}{2(5-\sqrt{2})}$ (2) $\frac{\lambda}{(5-\sqrt{2})}$ (3) $\frac{\lambda}{(\sqrt{5}-2)}$ (4) $\frac{\lambda}{2(\sqrt{5}-2)}$ **WO0143**

34. In a Young's double slit experiment with slit separation 0.1 mm, one observes a bright fringe at angle $\frac{1}{40}$ rad by using light of wavelength λ_1. When the light of wavelength λ_2 is used a bright fringe is seen at the same angle in the same set up. Given that λ_1 and λ_2 are in visible range (380 nm to 740nm), their values are : **[JEE Main-2019_Jan]**

(1) 380 nm, 500 nm (2) 625 nm, 500 nm (3) 380 nm, 525 nm (4) 400 nm, 500 nm

WO0144

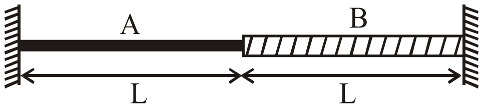
35. In a double-slit experiment, green light (5303 Å) falls on a double slit having a separation of 19.44 μm and a width of 4.05 μm. The number of bright fringes between the first and the second diffraction minima is :- **[JEE Main-2019_Jan]**

(1) 09 (2) 10 (3) 04 (4) 05 **WO0145**

36. In a Young's double slit experiment, the path different, at a certain point on the screen, between two interfering waves is $\frac{1}{8}$th of wavelength. The ratio of the intensity at this point to that at the centre of a brigth fringe is close to : **[JEE Main-2019_Jan]**

(1) 0.94 (2) 0.74 (3) 0.85 (4) 0.80 **WO0146**

37. A wire of length 2L, is made by joining two wires A and B of same length but different radii r and 2r and made of the same material. It is vibrating at a frequency such that the joint of the two wires forms a node. If the number of antinodes in wire A is p and that in B is q then the ratio p : q is : **[JEE Main-2019_April]**



(1) 4 : 9 (2) 3 : 5 (3) 1 : 4 (4) 1 : 2 **WA0075**

38. A string 2.0 m long and fixed at its ends is driven by a 240 Hz vibrator. The string vibrates in its third harmonic mode. The speed of the wave and its fundamental frequency is :-**[JEE Main-2019_April]**

(1) 320m/s, 120 Hz (2) 180m/s, 80 Hz (3) 180m/s, 120 Hz (4) 320m/s, 80 Hz

WA0076

39. The pressure wave, $P = 0.01 \sin [1000t - 3x]$ Nm^{-2}, corresponds to the sound produced by a vibrating blade on a day when atmospheric temperature is 0°C. On some other day, when temperature is T, the speed of sound produced by the same blade and at the same frequency is found to be 336 ms^{-1}. Approximate value of T is : **[JEE Main-2019_April]**

(1) 15°C (2) 12°C (3) 4°C (4) 11°C **SW0103**

40. A string is clamped at both the ends and it is vibrating in its 4th harmonic. The equation of the stationary wave is $Y = 0.3 \sin(0.157x) \cos(200\pi t)$. The length of the string is : (All quantities are in SI units.) **[JEE Main-2019_April]**

(1) 20 m (2) 80 m (3) 60 m (4) 40 m **WA0077**

41. A source of sound S is moving with a velocity of 50 m/s towards a stationary observer. The observer measures the frequency of the source as 1000 Hz. What will be the apparent frequency of the source when it is moving away from the observer after crossing him ?
(Take velocity of sound in air is 350 m/s) **[JEE Main-2019_April]**

(1) 857 Hz (2) 807 Hz (3) 750 Hz (4) 1143 Hz **SW0104**

42. A stationary source emits sound waves of frequency 500 Hz. Two observers moving along a line passing through the source detect sound to be of frequencies 480 Hz and 530 Hz. Their respective speeds are, in ms^{-1}, (Given speed of sound = 300 m/s) **[JEE Main-2019_April]**

(1) 16, 14 (2) 12, 18 (3) 12, 16 (4) 8, 18 **SW0105**

43. A tuning fork of frequency 480 Hz is used in an experiment for measuring speed of sound (ν) in air by resonance tube method. Resonance is observed to occur at two successive lengths of the air column, $l_1 = 30$ cm and $l_2 = 70$ cm. Then ν is equal to : **[JEE Main-2019_April]**

(1) 332 ms^{-1} (2) 379 ms^{-1} (3) 384 ms^{-1} (4) 338 ms^{-1} **SW0106**

44. Two sources of sound S_1 and S_2 produce sound waves of same frequency 660 Hz. A listener is moving from source S_1 towards S_2 with a constant speed u m/s and he hears 10 beats/s. The velocity of sound is 330 m/s. Then, u equals : **[JEE Main-2019_April]**

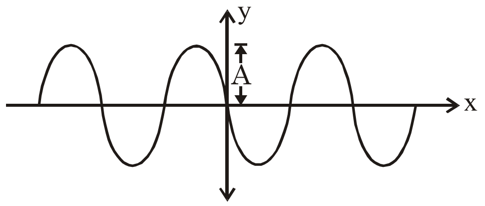
(1) 2.5 m/s (2) 15.0 m/s (3) 5.5 m/s (4) 10.0 m/s **SW0107**

45. A small speaker delivers 2 W of audio output. At what distance from the speaker will one detect 120dB intensity sound ? [Given reference intensity of sound as $10^{-12} W/m^2$]

[JEE Main-2019_April]

(1) 10 cm (2) 30 cm (3) 40 cm (4) 20 cm **SW0108**

46. A progressive wave travelling along the positive x-direction is represented by $y(x, t) = A \sin(kx - \omega t + \phi)$. Its snapshot at t = 0 is given in the figure: **[JEE Main-2019_April]**



For this wave, the phase ϕ is :

(1) 0 (2) $-\frac{\pi}{2}$ (3) π (4) $\frac{\pi}{2}$ **WA0078**

47. A submarine (A) travelling at 18 km/hr is being chased along the line of its velocity by another submarine (B) travelling at 27 km/hr. B sends a sonar signal of 500 Hz to detect A and receives a reflected sound of frequency ν. The value of ν is close to : (Speed of sound in water = 1500 ms^{-1})

[JEE Main-2019_April]

(1) 499 Hz (2) 502 Hz (3) 507 Hz (4) 504 Hz **SW0109**

48. Young's moduli of two wires A and B are in the ratio 7 : 4. Wire A is 2 m long and has radius R. Wire B is 1.5 m long and has radius 2 mm. If the two wires stretch by the same length for a given load, then the value of R is close to :- **[JEE Main-2019_April]**

(1) 1.9 mm (2) 1.7 mm (3) 1.5 mm (4) 1.3 mm **WA0079**

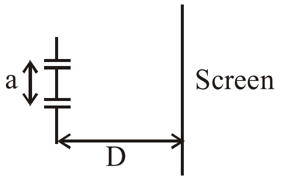
49. In an interference experiment the ratio of amplitudes of coherent waves is $\frac{a_1}{a_2} = \frac{1}{3}$. The ratio of maximum and minimum intensities of fringes will be : **[JEE Main-2019_April]**

(1) 4 (2) 2 (3) 9 (4) 18 **WO0147**

50. Two cars A and B are moving away from each other in opposite directions. Both the cars are moving with a speed of 20 ms^{-1} with respect to the ground. If an observer in car A detects a frequency 2000Hz of the sound coming from car B, what is the natural frequency of the sound source in car B? (speed of sound in air = 340 ms^{-1}) :- **[JEE Main-2019_April]**

(1) 2250 Hz (2) 2060 Hz (3) 2150 Hz (4) 2300 Hz **SW0110**

51. The figure shows a Young's double slit experimental setup. It is observed that when a thin transparent sheet of thickness t and refractive index μ is put in front of one of the slits, the central maximum gest shifted by a distance equal to n fringe widths. If the wavelength of light used is λ, t will be : **[JEE Main-2019_April]**

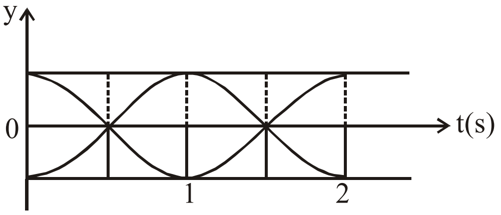


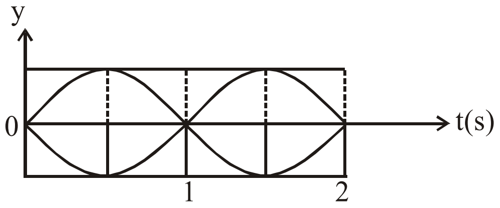
(1) $\frac{2D\lambda}{a(\mu-1)}$ (2) $\frac{D\lambda}{a(\mu-1)}$ (3) $\frac{2nD\lambda}{a(\mu-1)}$ (4) $\frac{nD\lambda}{a(\mu-1)}$ **WO0148**

52. In a Young's doubble slit experiment, the ratio of the slit's width is 4 : 1. The ratio of the intensity of maxima to minima, close to the central fringe on the screen, will be : **[JEE Main-2019_April]**

(1) $(\sqrt{3}+1)^4 : 16$ (2) 9 : 1 (3) 4 : 1 (4) 25 : 9 **WO0149**

53. The correct figure that shows, schematically, the wave pattern produced by superposition of two waves of frequencies 9 Hz and 11 Hz is : **[JEE Main-2019_April]**

(1)


(2)


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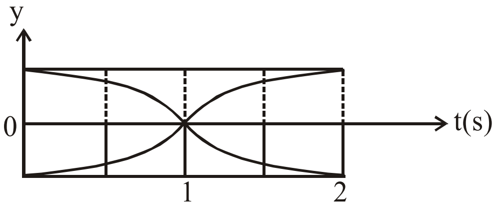

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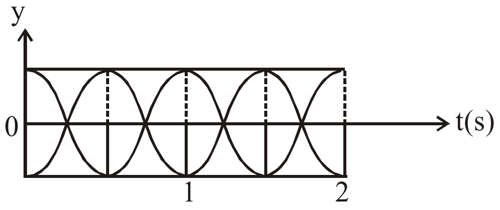


SW0111

54. In a double slit experiment, when a thin film of thickness t having refractive index μ is introduced in front of one of the slits, the maximum at the centre of the fringe pattern shifts by one fringe width. The value of t is (λ is the wavelength of the light used) : **[JEE Main-2019_April]**

(1) $\frac{\lambda}{2(\mu-1)}$ (2) $\frac{\lambda}{(2\mu-1)}$ (3) $\frac{2\lambda}{(\mu-1)}$ (4) $\frac{\lambda}{(\mu-1)}$ **WO0150**

ANSWER KEY

01_MECHANICAL WAVES

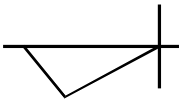
EXERCISE (S)

1. Ans. (a) DEF, (b) ABH, (c) CG, (D) AE **2. Ans.** $40\sqrt{2}$

3. Ans. (a) 20 m/s; (b) 2 m; (c) $y(x,t) = 0.02\sin(\pi x - 20\pi t + \pi/6)]$

4. Ans. (a) negative x ; (b) $y = 4\times10^{-3}\sin 100\pi\left(3t + 0.5x + \frac{1}{400}\right)$ (x , y in meter) ; (c) $144\pi^2\times10^{-5}$ J

5. Ans. π, 0 **6. Ans.** 1.22 v **7. Ans.** 70 m/s **8. Ans.** 10.8×10^4W

9. Ans. (a)  , (b) 2 cm/s **10. Ans.** 1650 Hz **11. Ans.** 1×10^9 Nm^2

12. Ans. $A_t = \frac{4}{3}A_i$, $A_r = \frac{1}{3}A_i$, $y_r = -\frac{0.04}{3}\sin(0.5\,\pi x + 100\,\pi t)$; $y_t = +\frac{0.16}{3}\sin(0.25\pi x - 100\,\pi t)$

13. Ans. $E = \frac{A^2\pi^2T}{4l}$ **14. Ans.** $\frac{\pi^2\times10^{-9}}{4}$ W/m^2 **15. Ans.** 2c/3

16. Ans. 30 dB, $10\sqrt{10}$ μm **17. Ans.** 1 : 1 **18. Ans.** 3 cm **19. Ans.** 336 m/s

20. Ans. 33 cm and 13.2 cm **21. Ans.** 2.5 kHz, 7.5 kHz **22. Ans.** (a) 2.116, (b) $\frac{3}{4}$

23. Ans. 3.33 cm, 163 Hz **24. Ans.** 1% **25. Ans.** 345, 341 or 349 Hz **26. Ans.** 8

27. Ans. 1.5 m/s **28. Ans.** 5 **29. Ans.** 11f / 9 **30. Ans.** 6 Hz

31. Ans. 0.75 m/s

EXERCISE (O)

1. Ans. (C) 2. Ans. (B) 3. Ans. (A) 4. Ans. (B) 5. Ans. (C) 6. Ans. (B)
7. Ans. (C) 8. Ans. (D) 9. Ans. (A) 10. Ans. (A) 11. Ans. (C) 12. Ans. (B)
13. Ans. (B) 14. Ans. (D) 15. Ans. (D) 16. Ans. (B) 17. Ans. (B) 18. Ans. (C)
19. Ans. (C) 20. Ans. (B) 21. Ans. (B) 22. Ans. (A) 23. Ans. (A) 24. Ans. (D)
25. Ans. (B) 26. Ans. (A) 27. Ans. (C) 28. Ans. (B) 29. Ans. (A) 30. Ans. (B)
31. Ans. (B) 32. Ans. (A) 33. Ans. (A) 34. Ans. (B) 35. Ans. (D) 36. Ans. (A)
37. Ans. (D) 38. Ans. (D) 39. Ans. (B) 40. Ans. (C) 41. Ans. (B) 42. Ans. (C)
43. Ans. (B) 44. Ans. (A) 45. Ans. (C) 46. Ans. (C) 47. Ans. (A) 48. Ans. (A)
49. Ans. (B) 50. Ans. (A) 51. Ans. (B) 52. Ans. (B) 53. Ans. (A) 54. Ans. (A)
55. Ans. (A) 56. Ans. (A) 57. Ans. (A) 58. Ans. (A) 59. Ans. (A) 60. Ans. (C)
61. Ans. (B) 62. Ans. (B) 63. Ans. (A)

02_WAVE OPTICS

EXERCISE (S)

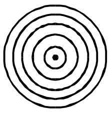
1. Ans. 5000Å **2. Ans.** 0.2 mm **3. Ans.** 35.35 cm approximate, 5

4. Ans. 1.99×10^{-2} mm **5. Ans.** 3.5 mm **6. Ans.** 0.225 mm **7. Ans.** 0, 1.5 mm

8. Ans. 8 μm **9. Ans.** 7 μm, 1.6, $\frac{400}{7}$ μm (decrease)

10. Ans. (i) y = – 13/3 mm, (ii) intensity at O = $0.75I_{max}$ (iii) 650 nm, 433.33 nm

11. Ans. 48, 21, $\frac{32}{3}$, $\frac{9}{2}$, 0 m.m.;



12. Ans. (i) circular, (ii) $\frac{1}{16}$, (C) 3000Å

13. Ans. 0.15 mm **14. Ans.** λ = 5850 Å **15. Ans.** 81 : 1

EXERCISE (O)

1. Ans. (C) **2. Ans. (C)** **3. Ans. (C)** **4. Ans. (B)** **5. Ans. (B)** **6. Ans. (C)**
7. Ans. (B) **8. Ans. (D)** **9. Ans. (D)** **10. Ans. (B)** **11. Ans. (C)** **12. Ans. (C)**
13. Ans. (D) **14. Ans. (A)** **15. Ans. (A)** **16. Ans. (C)** **17. Ans. (B)** **18. Ans. (B)**
19. Ans. (C) **20. Ans. (C)** **21. Ans. (D)** **22. Ans. (B)** **23. Ans. (B)** **24. Ans. (D)**
25. Ans. (A) **26. Ans. (D)** **27. Ans. (A)** **28. Ans. (C)** **29. Ans. (D)** **30. Ans. (B)**
31. Ans. (A) **32. Ans. (D)** **33. Ans. (A)** **34. Ans. (C)** **35. Ans. (C)** **36. Ans. (C)**

EXERCISE - JM

1. Ans. (1) **2. Ans. (1)** **3. Ans. (2)** **4. Ans. (3)** **5. Ans. (4)** **6. Ans. (2)**
7. Ans. (1) **8. Ans. (2)** **9. Ans. (2)** **10. Ans. (1)** **11. Ans. (1)** **12. Ans. (2)**
13. Ans. (4) **14. Ans. (1)** **15. Ans. (4)** **16. Ans. (2)** **17. Ans. (4)** **18. Ans. (1)**
19. Ans. (2) **20. Ans. (4)** **21. Ans. (4)** **22. Ans. (4)** **23. Ans. (2)** **24. Ans. (1)**
25. Ans. (1) **26. Ans. (2)** **27. Ans. (2)** **28. Ans. (2)** **29. Ans. (1)** **30. Ans. (1)**
31. Ans. (3) **32. Ans. (2)** **33. Ans. (4)** **34. Ans. (2)** **35. Ans. (4)** **36. Ans. (3)**
37. Ans. (4) **38. Ans. (4)** **39. Ans. (3)** **40. Ans. (2)** **41. Ans. (3)** **42. Ans. (2)**
43. Ans. (3) **44. Ans. (1)** **45. Ans. (3)** **46. Ans. (3)** **47. Ans. (2)** **48. Ans. (2)**
49. Ans. (1) **50. Ans. (1)** **51. Ans. (4)** **52. Ans. (2)** **53. Ans. (4)** **54. Ans. (4)**