SIMPLE HARMONIC MOTION

KEY CONCEPTS

PERIODIC MOTION

- Any motion which repeats itself after regular interval of time is called periodic motion.
- The constant interval of time after which the motion is repeated is called time period.

Examples : (i) Motion of planets around the sun. (ii) Motion of the pendulum of wall clock.

OSCILLATORY MOTION

- The motion of body is said to be oscillatory if it moves back and forth (to and fro) about a fixed point after regular interval of time. Oscillation of very high frequency & small amplitude is called vibration.
- The fixed point about which the body oscillates is called mean position or equilibrium position.

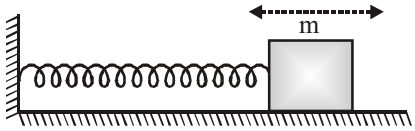
Examples : (i) Vibration of the wire of 'Sitar'.(ii) Oscillation of the mass suspended from spring.

SIMPLE HARMONIC MOTION (S.H.M.)

Simple harmonic motion is the simplest form of oscillatory motion.

(i) S.H.M. are of two types

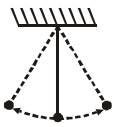
- **Linear S.H.M. :** When a particle moves to and fro about a fixed point (called equilibrium position) along a straight line then its motion is called linear simple harmonic motion.



Example : Motion of a mass connected to spring.

- **Angular S.H.M.**

When a system oscillates angularly with respect to a fixed axis then its motion is called angular simple harmonic motion.
Example :- Motion of a bob of simple pendulum.



(ii) Necessary Condition to execute S.H.M.

- **In linear S.H.M. :** The restoring force (or acceleration) acting on the particle should always be proportional to the displacement of the particle and directed towards the equilibrium position

$\therefore \quad F \propto -x \quad \text{or} \quad a \propto -x$

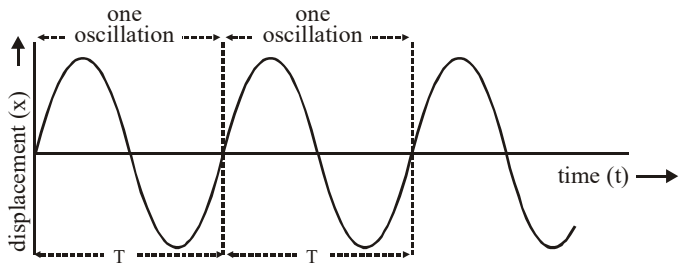
Negative sign shows that direction of force and acceleration is towards equilibrium position and x is displacement of particle from equilibrium position.

- **In angular S.H.M. :** The restoring torque (or angular acceleration) acting on the particle should always be proportional to the angular displacement of the particle and directed towards the equilibrium position

$\therefore \quad \tau \propto -\theta \quad \text{or} \quad \alpha \propto -\theta$

SOME BASIC TERMS

- **Mean Position :** The point at which the restoring force on the particle is zero and potential energy is minimum, is known as its mean position.
- **Restoring Force**
  - The force acting on the particle which tends to bring the particle towards its mean position, is known as restoring force.
  - Restoring force always acts in a direction opposite to that of displacement. Displacement is measured from the mean position.
- **Amplitude :** The maximum (positive or negative) value of displacement of particle from mean position is define as amplitude.
- **Time period (T)**
  - The minimum time after which the particle keeps on repeating its motion is known as time period.
  - The smallest time taken to complete one oscillation or vibration is also define as time period.
  - It is given by $T = \frac{2\pi}{\omega} = \frac{1}{n}$ where ω is angular frequency and n is frequency.
- **Oscillation :** When a particle goes on one side from mean position and returns back and then it goes to other side and again returns back to mean position, then this process is known as one oscillation.



- **Frequency (n or f)**
  - The number of oscillations per second is define as frequency.
  - It is given by $n = \frac{1}{T}$, $n = \frac{\omega}{2\pi}$
  - **SI UNIT :** hertz (Hz), 1 hertz = 1 cycle per second (cycle is a number not a dimensional quantity).
  - **Dimensions :** $M^0 L^0 T^{-1}$.
- **Phase** :
  - Phase of a vibrating particle at any instant is the state of the vibrating particle regarding its displacement and direction of vibration at that particular instant.
  - In the equation $x = A \sin(\omega t + \theta)$, $(\omega t + \theta)$ is the phase of the particle.
  - The phase angle at time t = 0 is known as initial phase or epoch.
  - The difference of total phase angles of two particles executing S.H.M. with respect to the mean position is known as phase difference.

- Two vibrating particles are said to be in same phase if the phase difference between them is an even multiple of π, i.e., $\Delta\phi = 2n\pi$ Where n = 0, 1, 2, 3,....
- Two vibrating particle are said to be in opposite phase if the phase difference between them is an odd multiple of π i.e., $\Delta\phi = (2n + 1)\pi$ Where n = 0, 1, 2, 3,....

- **Angular frequency (ω)** :

  The rate of change of phase angle of a particle with respect to time is define as its angular frequency.

  SI unit **:** radian/second, *Dimensions* **:** $M^0 L^0 T^{-1}$,

DISPLACEMENT IN S.H.M.

(i) The displacement of a particle executing linear S.H.M. at any instant is defined as the distance of the particle from the mean position at that instant.

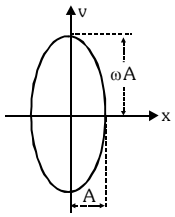
(ii) It can be given by relation $x = A\sin\omega t$ or $x = A\cos\omega t$.

The first relation is valid when the time is measured from the mean position and the second relation is valid when the time is measured from the extreme position of the particle executing S.H.M. along a straight line path.

VELOCITY IN SHM

(i) It is define as the time rate of change of the displacement of the particle at the given instant.

(ii) Velocity in S.H.M. is given by $v = \frac{dx}{dt} = \frac{d}{dt}(A\sin\omega t) \Rightarrow v = A\omega\cos\omega t$



$$v = \pm A\omega\sqrt{1-\sin^2\omega t} \Rightarrow v = \pm A\omega\sqrt{1-\frac{x^2}{A^2}} = \pm\omega\sqrt{(A^2-x^2)} \left[\because x = A\sin\omega t\right]$$

Squaring both the sides $v^2 = \omega^2 (A^2 - x^2) \Rightarrow \frac{v^2}{\omega^2} = A^2 - x^2 \Rightarrow \frac{v^2}{\omega^2 A^2} = 1 - \frac{x^2}{A^2} \Rightarrow \frac{x^2}{A^2} + \frac{v^2}{A^2\omega^2} = 1$

This is equation of ellipse. So curve between displacement and velocity of particle executing S.H.M. is ellipse.

(iii) The graph between velocity and displacement is shown in figure. If particle oscillates with unit angular frequency (ω = 1) then curve between v and x will be circular.

Note :

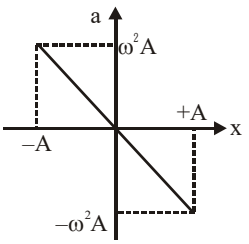
(i) The direction of velocity of a particle in S.H.M. is either towards or away from the mean position.

(ii) At mean position (x = 0), velocity is maximum (=Aω) and at extreme position (x = ±A), the velocity of particle executing S.H.M. is zero (minimum).

ACCELERATION IN SHM

(i) It is define as the time rate of change of the velocity of the particle at given instant.

(ii) Acceleration in S.H.M. is given by $a = \frac{dv}{dt} = \frac{d}{dt}(A\omega\cos\omega t)$



$a = -\omega^2 A \sin \omega t \Rightarrow \quad a = -\omega^2 x$

(iii) The graph between acceleration and displacement as shown in figure

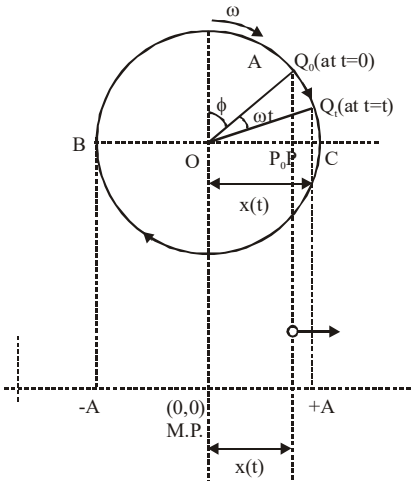
Note

(i) The acceleration of a particle executing S.H.M. is always directed towards the mean position.

(ii) The acceleration of the particle executing S.H.M. is maximum at extreme position (= $\omega^2 A$) and minimum at mean position (= zero)

SHM AS A PROJECTION OF UNIFORM CIRCULAR MOTION

Consider a particle Q, moving on a circle of radius A with constant angular velocity ω. The projection of Q on a diameter BC is P. It is clear from the figure that as Q moves around the circle the projection P executes a simple harmonic motion on the x-axis between B and C. The angle that the radius OQ makes with the +ve vertical in clockwise direction in at t = 0 is equal to phase constant (ϕ).



Let the radius OQ_0 makes an angle ωt with the OQ_t at time t. Then $x(t)=A\sin(\omega t+\phi)$

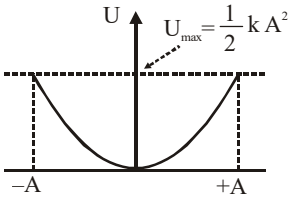
In the above discussion the foot of projection is x-axis so it is called horizontal phasor. Similarly the foot of perpendiuclar on y-axis will also executes SHM of amplitude A and angular frequency ω[$y(t)=A\cos\omega t$]. This is called vertical phasor. The phaser of the two SHM differ by π/2.

ENERGY OF PARTICLE IN S.H.M.

- **Potential Energy (U or P.E.)**

(i) In terms of displacement

The potential energy is related to force by the relation $F = -\frac{dU}{dx} \Rightarrow \int dU = -\int F dx$

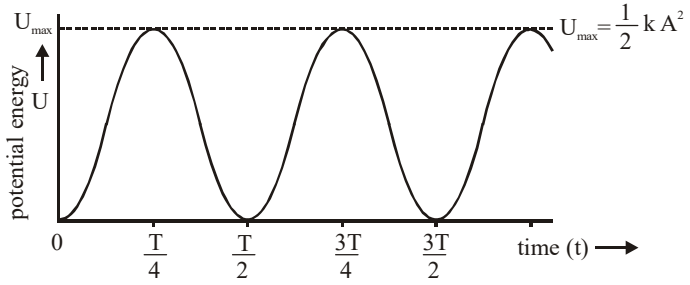


For S.H.M. $F = -kx$ so $\int dU = -\int (-kx)dx = \int kx\,dx \Rightarrow U = \frac{1}{2}kx^2 + C$

At $x = 0, U = U_0 \Rightarrow C = U_0$ So $U = \frac{1}{2}kx^2 + U_0$

Where the potenital energy at equilibrium position = U_0 when $U_0 = 0$ then $U = \frac{1}{2}kx^2$

(ii) In terms of time



Since $x = A\sin(\omega t + \phi)$, $U = \frac{1}{2}kA^2\sin^2(\omega t + \phi)$

If initial phase (ϕ) is zero then $U = \frac{1}{2}kA^2\sin^2\omega t = \frac{1}{2}m\omega^2A^2\sin^2\omega t$

Note :

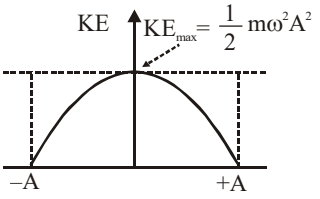
(i) In S.H.M. the potential energy is a parabolic function of displacement, the potential energy is minimum at the mean position (x = 0) and maximum at extreme position ($x = \pm A$)

(ii) The potential energy is the periodic function of time. For $x = A \sin(\omega t)$, it is minimum at $t = 0, \frac{T}{2}, T, \frac{3T}{2}\ldots$ and maximum at $t = \frac{T}{4}, \frac{3T}{4}, \frac{5T}{4}\ldots$

- **Kinetic Energy (K)**

(i) In terms of displacement

If mass of the particle executing S.H.M. is m and its velocity is v then kinetic energy at any instant.



$$K = \frac{1}{2}mv^2 = \frac{1}{2}m\omega^2(A^2 - x^2) = \frac{1}{2}k(A^2 - x^2)$$

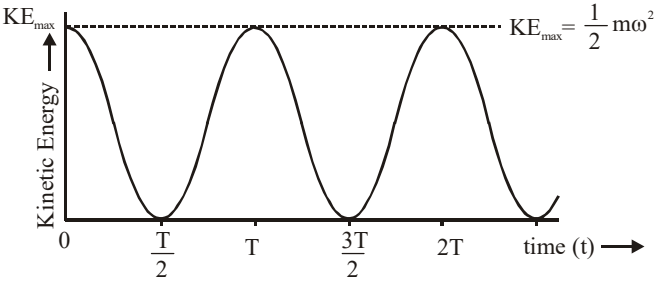
(ii) In terms of time

$$v = A\omega\cos(\omega t + \phi)$$

$$K = \frac{1}{2}m\omega^2A^2\cos^2(\omega t + \phi)$$

If initial phase ϕ is zero

$$K = \frac{1}{2}m\omega^2A^2\cos^2\omega t$$



Note :

(i) In S.H.M. the kinetic energy is a inverted parabolic function of displacement. The kinetic energy is maximum ($\frac{1}{2}kA^2$) at mean position (x = 0) and minimum (zero) at extreme position ($x = \pm A$)

(ii) The kinetic energy is the periodic function of time. For x = A sin (ωt), it is maximum at t = 0, T, 2T, 3T.....and minimum at t = $\frac{T}{2}, \frac{3T}{2}, \frac{5T}{2}$...

- **Total energy (E)**

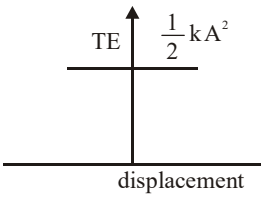
Total energy in S.H.M. is given by ; E = potential energy + kinetic energy = U + K

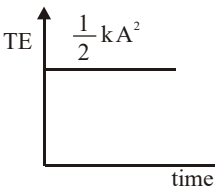
(i) w.r.t. position $E = \frac{1}{2}kx^2 + \frac{1}{2}k(A^2 - x^2) \Rightarrow E = \frac{1}{2}kA^2 = \text{constant}$

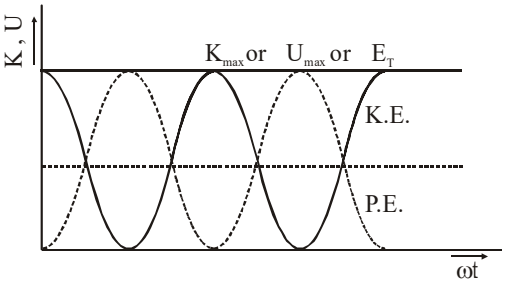
(ii) w.r.t. time

$$E = \frac{1}{2}m\omega^2A^2\sin^2\omega t + \frac{1}{2}m\omega^2A^2\cos^2\omega t = \frac{1}{2}m\omega^2A^2\left(\sin^2\omega t + \cos^2\omega t\right) = \frac{1}{2}m\omega^2A^2$$

$$= \frac{1}{2}kA^2 = \text{constant}$$







Note :

(i) Total energy of a particle in S.H.M. is same at all instant and at all displacement.

(ii) Total energy depends upon mass, amplitude and frequency of vibration of the particle executing S.H.M.

SIMPLE PENDULUM

If a heavy point mass is suspended by a weightless, inextensible and perfectly flexible string from a rigid support, then this arrangement is called a simple pendulum

- **Second's pendulum**

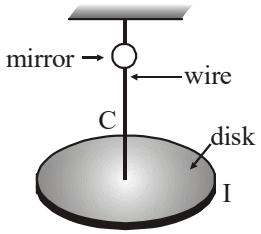
If the time period of a simple pendulum is 2 second then it is called second's pendulum. Second's pendulum take one second to go from one extreme position to other extreme position.

COMPOUND PENDULUM

Any rigid body which is free to oscillate in a vertical plane about a horizontal axis passing through a point, is define compound pendulum

- **Torsional Oscillator :** (Angular SHM)

$$T = 2\pi\sqrt{\frac{I}{C}} \text{ where } C = \frac{\eta\pi r^4}{2\ell}$$



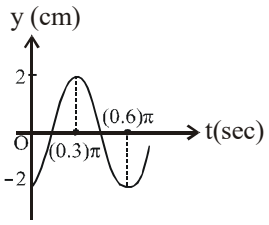
η = modulus of elasticity of the wire ; r = radius of the wire

L = length of the wire ; I = Moment of inertia of the disc

EXERCISE (S)

Kinematics of SHM :

1. Part of a simple harmonic motion is graphed in the figure, where y is the displacement from the mean position. The correct equation describing this S.H.M is :-



SH0001

2. The displacement of a body executing SHM is given by $x = A \sin(2\pi t + \pi/3)$. The first time from $t = 0$ when the velocity is maximum is.

SH0002

3. A body undergoing SHM about the origin has its equation given by $x = 0.2 \cos 5\pi t$. Find its average speed from $t = 0$ to $t = 0.7$ sec.

SH0003

Energy of SHM :

4. An object of mass 0.2 kg executes SHM along the x-axis with frequency of $(25/\pi)$ Hz. At the point $x = 0.04$m the object has KE 0.5 J and PE 0.4 J. The amplitude of oscillation is _____.

SH0004

5. A point particle of mass 0.1kg is executing SHM with amplitude of 0.1m. When the particle passes through the mean position, its K.E. is 8×10^{-3}J. Obtain the equation of motion of this particle if the initial phase of oscillation is 45°.

SH0005

6. Potential Energy (U) of a body of unit mass moving in a one-dimension conservative force field is given by $U = (x^2 - 4x + 3)$. All units are in S.I.

(i) Find the equilibrium position of the body.

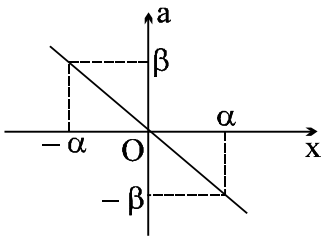
(ii) Show that oscillations of the body about this equilibrium position is simple harmonic motion & find its time period.

(iii) Find the amplitude of oscillations if speed of the body at equilibrium position is $2\sqrt{6}$ m/s.

SH0006

Time Period :

7. The acceleration-displacement (a – x) graph of a particle executing simple harmonic motion is shown in the figure. Find the frequency of oscillation.

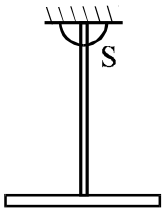


SH0007

8. A body is in SHM with period T when oscillated from a freely suspended spring. If this spring is cut in two parts of length ratio 1 : 3 & again oscillated from the two parts separately, then the periods are T_1 & T_2 then find T_1/T_2.

SH0009

9. Two identical rods each of mass m and length L, are rigidly joined and then suspended in a vertical plane so as to oscillate freely about an axis normal to the plane of paper passing through 'S' (point of suspension). Find the time period of such small oscillations.



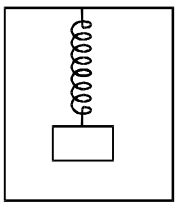
SH0010

10. A mass M attached to a spring, oscillates with a period of 2 sec. If the mass is increased by 2 kg the period increases by one sec. Find the initial mass M assuming that Hook's Law is obeyed.

SH0012

Complex situations :

11. A spring mass system is hanging from the ceiling of an elevator in equilibrium. Elongation of spring is *l*. The elevator suddenly starts accelerating downwards with acceleration g/3, find
(a) the frequency and (b) the amplitude of the resulting SHM.



SH0013

12. (a) Find the time period of oscillations of a torsional pendulum, if the torsional constant of the wire is $K = 10\pi^2$ J/rad. The moment of inertia of rigid body is 10 kg m^2 about the axis of rotation.

(b) A simple pendulum of length l = 0.5 m is hanging from ceiling of a car. The car is kept on a horizontal plane. The car starts accelerating on the horizontal road with acceleration of 5 m/s^2. Find the time period of oscillations of the pendulum for small amplitudes about the mean position.

SH0014

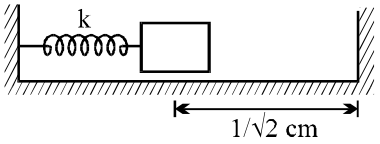
13. A physical pendulum has the shape of a disk of radius R. The pendulum swings about an axis perpendicular to the plane of the disk and at distance ℓ from the center of the disk.

(a) Show that the frequency of the oscillations of this pendulum is $\omega = \sqrt{\dfrac{g\ell}{\frac{1}{2}R^2 + \ell^2}}$

(b) For what value of ℓ is this frequency at a maximum ?

SH0016

14. A block of mass 0.9 kg attached to a spring of force constant k is lying on a frictionless floor. The spring is compressed to $\sqrt{2}$ cm and the block is at a distance $1/\sqrt{2}$ cm from the wall as shown in the figure. When the block is released, it makes elastic collision with the wall and its period of motion is 0.2 sec. Find the approximate value of k.



SH0017

15. A body of mass 1 kg is suspended from a weightless spring having force constant 600N/m. Another body of mass 0.5 kg moving vertically upwards hits the suspended body with a velocity of 3.0m/s and get embedded in it. Find the frequency of oscillations and amplitude of motion.

SH0018

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

Kinematics of SHM :

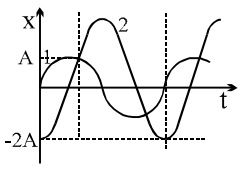
1. The equation of motion of a particle is $x = a \cos(\alpha t)^2$. The motion is
(A) periodic but not oscillatory. (B) periodic and oscillatory.
(C) oscillatory but not periodic. (D) neither periodic nor oscillatory.

SH0037

2. A simple harmonic motion having an amplitude A and time period T is represented by the equation :
$y = 5 \sin \pi(t + 4)$ m. Then the values of A (in m) and T (in sec) are :
(A) A = 5; T = 2 (B) A = 10 ; T = 1 (C) A = 5 ; T = 1 (D) A = 10 ; T = 2

SH0039

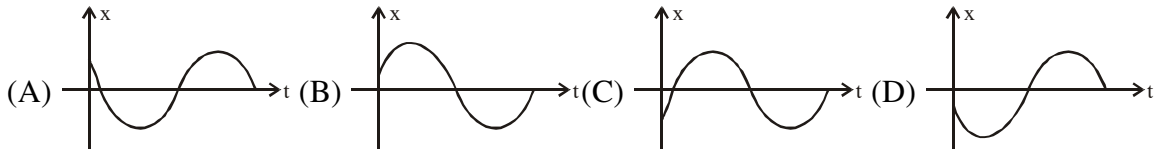
3. The oscillations represented by curve 1 in the graph are expressed by equation $x = A \sin\omega t$. The equation for the oscillations represented by curve 2 is expressed as:



(A) $x = 2A \sin(\omega t - \pi/2)$ (B) $x = 2A \sin(\omega t + \pi/2)$
(C) $x = -2A \sin(\omega t - \pi/2)$ (D) $x = A \sin(\omega t - \pi/2)$

SH0040

4. A particle performing S.H.M. about mean position x = 0 and at t = 0 it is at position $x = \frac{A}{\sqrt{2}}$ and moving towards the origin. Then which of the following is its possible graph between position (x) and time (t)



SH0041

5. The phase difference between two SHMs $Y_1 = 10\sin(10\pi t+\pi/3)$ and $Y_2 = 12\sin(8\pi t+\pi/4)$ at t=0.5s is----.

(A) 195º (B) 200° (C) 180° (D) 170°

SH0133

6. A particle is performing S.H.M. and at $t = \frac{3T}{4}$, is at position $\frac{\sqrt{3}A}{2}$ and moving towards the origin. Equilibrium position of the particle is at $x = 0$. Then what was the position and direction of particle at $t = 0$?

(A) $-\frac{A}{2}$, away from mean position (B) $\frac{A}{2}$, away from mean position

(C) $\frac{A}{2}$, towards mean position (D) $-\frac{A}{2}$, towards mean position

SH0044

7. The time taken by a particle performing SHM to pass from point A to B where its velocities are same is 2 seconds. After another 2 seconds it returns to B. The time period of oscillation is (in seconds)

(A) 2 (B) 8 (C) 6 (D) 4

SH0045

8. A bob is attached to a long, light string. The string is deflected by 3° initially with respect to vertical. The length of the string is 1 m . The value of θ at any time t after the bob released can be approximately written as (Use : $g = \pi^2$)

(A) $3°\cos \pi t$ (B) $3°\sin \pi t$ (C) $3°\sin(\pi t + \frac{\pi}{6})$ (D) $3°\cos(\pi t + \frac{\pi}{6})$

SH0046

9. A particle performing SHM is found at its equilibrium at t = 1sec. and it is found to have a speed of 0.25 m/s at t = 2 sec. If the period of oscillation is 6 sec. Calculate amplitude of oscillation

(A) $\frac{3}{2\pi}$ m (B) $\frac{3}{4\pi}$ m (C) $\frac{6}{\pi}$ m (D) $\frac{3}{8\pi}$

SH0047

10. A particle performs SHM with a period T and amplitude a. The mean velocity of the particle over the time interval during which it travels a distance a/2 from the extreme position is

(A) a/T (B) 2a/T (C) 3a/T (D) a/2T

SH0048

11. Two particles execute SHM of same amplitude of 20 cm with same period along the same line about the same equilibrium position. The maximum distance between the two is 20 cm. Their phase difference in radians is :-

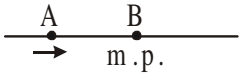
(A) $\frac{2\pi}{3}$ (B) $\frac{\pi}{2}$ (C) $\frac{\pi}{3}$ (D) $\frac{\pi}{4}$

SH0049

12. Two pendulums have time periods T and 5T/4. They start SHM at the same time from the mean position. After how many oscillations of the smaller pendulum they will be again in the same phase:
(A) 5 (B) 4 (C) 11 (D) 9

SH0051

13. A particle is oscillating simple harmonically with angular frequency ω and amplitude A . It is at a point (A) at certain instant (shown in figure). At this instant it is moving towards mean position (B). It takes time t to reach mean position (B). If time period of oscillation is T, the average speed between A and B is :-



(A) $\frac{A\sin\omega t}{t}$ (B) $\frac{A\cos\omega t}{t}$ (C) $\frac{A\sin\omega t}{T}$ (D) $\frac{A\cos\omega t}{T}$

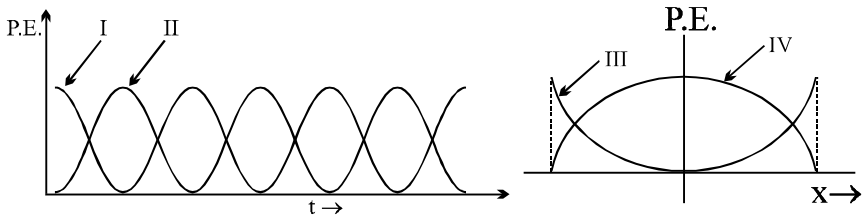
SH0052

14. Speed v of a particle moving along a straight line, when it is at a distance x from a fixed point on the line is given by $v^2 = 108 - 9x^2$ (all quantities in S.I. unit). Then
(A) The motion is uniformly accelerated along the straight line
(B) The magnitude of the acceleration at a distance 3 cm from the fixed point is 0.27 m/s^2.
(C) The motion is simple harmonic about x = $\sqrt{12}$ m.
(D) The maximum displacement from the fixed point is 4 cm.

SH0054

Energy of SHM :

15. A particle is executing SHM according to $x = a\cos\omega t$. Then which of the graphs represents variations of potential energy : **[IIT JEE (Scr) 2003]**



(A) (I) & (III) (B) (II) & (IV) (C) (I) & (IV) (D) (II) & (III)

SH0055

16. Potential energy of a particle is given as $U(x) = 2x^3 - 9x^2 + 12x$ where U is in joule and x is in metre. If the motion of a particle is S.H.M., then find the approx potential energy of the particle :-
(A) – 36 J (B) 4 J (C) 5 J (D) None of these

SH0056

17. If the potential energy of a harmonic oscillator of mass 2 kg on its equilibrium position is 5 joules and the total energy is 9 joules. If the amplitude is one meter then period of the oscillator (in sec) is:
(A) 1.5 (B) 3.14 (C) 6.28 (D) 4.67

SH0057

18. Kinetic energy of a particle executing simple harmonic motion in straight line is pv^2 and potential energy is qx^2, where v is speed at distance x from the mean position. It time period is given by the expression :-

(A) $2\pi\sqrt{\frac{q}{p}}$ (B) $2\pi\sqrt{\frac{p}{q}}$ (C) $2\pi\sqrt{\frac{q}{p+q}}$ (D) $2\pi\sqrt{\frac{p}{p+q}}$

SH0058

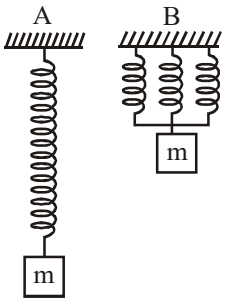
Time Period :

19. The angular frequency of motion whose equation is $4\frac{d^2y}{dt^2} + 9y = 0$ is (y = displacement and t = time)

(A) $\frac{9}{4}$ (B) $\frac{4}{9}$ (C) $\frac{3}{2}$ (D) $\frac{2}{3}$

SH0059

20. The springs in figure A and B are identical but length of spring in A is three times than the length of each spring in B. The ratio of period T_A/T_B is :-



(A) $\sqrt{3}$ (B) 1/3 (C) 3 (D) $1/\sqrt{3}$

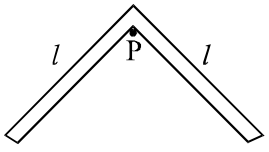
SH0060

21. A rod whose ends are A & B of length 25 cm is hanged in vertical plane. When hanged from point A and point B the time periods calculated are 3 sec & 4 sec respectively. Given the moment of inertia of rod about axis perpendicular to the rod is in ratio 9 : 4 at points A and B. Find the distance of the centre of mass from point A.

(A) 9 cm (B) 5 cm (C) 25 cm (D) 20 cm

SH0061

22. A system of two identical rods (L-shaped) of mass *m* and length *l* are resting on a peg P as shown in the figure. If the system is displaced in its plane by a small angle θ, find the period of oscillations:



(A) $2\pi\sqrt{\frac{\sqrt{2}l}{3g}}$ (B) $2\pi\sqrt{\frac{2\sqrt{2}l}{3g}}$ (C) $2\pi\sqrt{\frac{2l}{3g}}$ (D) $3\pi\sqrt{\frac{l}{3g}}$

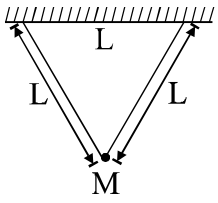
SH0062

23. A ring of diameter 2m oscillates as a compound pendulum about a horizontal axis passing through a point at its rim. It oscillates such that its centre move in a plane which is perpendicular to the plane of the ring. The equivalent length of the simple pendulum is :-

(A) 2m (B) 4m (C) 1.5m (D) 3m

SH0063

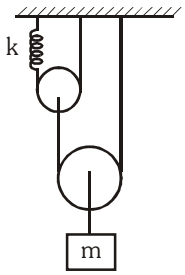
24. A man is swinging on a swing made of 2 ropes of equal length L and in direction perpendicular to the plane of paper. The time period of the small oscillations about the mean position is :-



(A) $2\pi\sqrt{\frac{L}{2g}}$ (B) $2\pi\sqrt{\frac{\sqrt{3}L}{2g}}$ (C) $2\pi\sqrt{\frac{L}{2\sqrt{3}g}}$ (D) $\pi\sqrt{\frac{L}{g}}$

SH0064

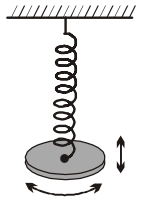
25. What is the period of small oscillations of the block of mass m if the springs are ideal and pulleys are massless ?



(A) $\frac{\pi}{2}\sqrt{\frac{m}{k}}$ (B) $\frac{\pi}{2}\sqrt{\frac{m}{2k}}$ (C) $\frac{\pi}{2}\sqrt{\frac{2m}{k}}$ (D) $\pi\sqrt{\frac{m}{2k}}$

SH0065

26. A solid disk of radius R is suspended from a spring of linear spring constant k and torsional constant c, as shown in figure. In terms of k and c, what value of R will give the same period for the vertical and torsional oscillations of this system?



(A) $\sqrt{\frac{2c}{k}}$ (B) $\sqrt{\frac{c}{2k}}$ (C) $2\sqrt{\frac{c}{k}}$ (D) $\frac{1}{2}\sqrt{\frac{c}{k}}$

SH0067

Superposition :

27. The amplitude of the vibrating particle due to superposition of two SHMs,

$y_1 = \sin\left(\omega t + \frac{\pi}{3}\right)$ and $y_2 = \sin \omega t$ is :

(A) 1 (B) $\sqrt{2}$ (C) $\sqrt{3}$ (D) 2

SH0068

28. Two simple harmonic motions $y_1 = A \sin \omega t$ and $y_2 = A \cos \omega t$ are superimposed on a particle of mass m. The total mechanical energy of the particle is:

(A) $\frac{1}{2} m\omega^2 A^2$ (B) $m\omega^2 A^2$ (C) $\frac{1}{4} m\omega^2 A^2$ (D) zero

SH0069

29. The displacement of a particle varies with time according to the relation $y = a \sin \omega t + b \cos \omega t$.

(A) The motion is oscillatory but not S.H.M.

(B) The motion is S.H.M. with amplitude $a + b$.

(C) The motion is S.H.M. with amplitude $a^2 + b^2$.

(D) The motion is S.H.M. with amplitude $\sqrt{a^2 + b^2}$

SH0070

30. Equations $y = 2A \cos^2 \omega t$ and $y = A (\sin \omega t + \sqrt{3} \cos \omega t)$ represent the motion of two particles.

(A) Only one of these is S.H.M. (B) Ratio of maximum speeds is 2 : 1

(C) Ratio of maximum speeds is 1 : 1 (D) Ratio of maximum accelerations is 1 : 4

SH0071

Complex situations :

31. A plank with a small block on top of it is under going vertical SHM. Its period is 2 sec. The minimum amplitude at which the block will separate from plank is :

(A) $\frac{10}{\pi^2}$ (B) $\frac{\pi^2}{10}$ (C) $\frac{20}{\pi^2}$ (D) $\frac{\pi}{10}$

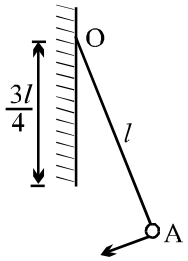
SH0072

32. Two bodies P & Q of equal mass are suspended from two separate massless springs of force constants k_1 & k_2 respectively. If the maximum velocity of them are equal during their motion, the ratio of amplitude of P to Q is :

(A) $\frac{k_1}{k_2}$ (B) $\sqrt{\frac{k_2}{k_1}}$ (C) $\frac{k_2}{k_1}$ (D) $\sqrt{\frac{k_1}{k_2}}$

SH0074

33. A small bob attached to a light inextensible thread of length l has a periodic time T when allowed to vibrate as a simple pendulum. The thread is now suspended from a fixed end O of a vertical rigid rod of length $\frac{3l}{4}$ (as in figure). If now the pendulum performs periodic oscillations in this arrangement, the periodic time will be



(A) $\frac{3T}{4}$ (B) $\frac{T}{2}$ (C) T (D) 2T

SH0075

34. Vertical displacement of a plank with a body of mass 'm' on it is varying according to law $y = \sin \omega t + \sqrt{3} \cos \omega t$. The minimum value of ω for which the mass just breaks off the plank and the moment it occurs first after t = 0 are given by: (y is positive vertically upwards)

(A) $\sqrt{\frac{g}{2}}, \frac{\sqrt{2}}{6}\frac{\pi}{\sqrt{g}}$ (B) $\frac{g}{\sqrt{2}}, \frac{2}{3}\sqrt{\frac{\pi}{g}}$ (C) $\sqrt{\frac{g}{2}}, \frac{\pi}{3}\sqrt{\frac{2}{g}}$ (D) $\sqrt{2g}, \sqrt{\frac{2\pi}{3g}}$

SH0076

EXERCISE (JM)

1. A mass M, attached to a horizontal spring, executes S.H.M. with amplitude A_1. When the mass M passes through its mean position then a smaller mass m is placed over it and both of them move together with amplitude A_2. The ratio of $\left(\frac{A_1}{A_2}\right)$ is :- **[AIEEE-2011]**

(1) $\left(\frac{M}{M+m}\right)^{1/2}$ (2) $\left(\frac{M+m}{M}\right)^{1/2}$ (3) $\frac{M}{M+m}$ (4) $\frac{M+m}{M}$

SH0116

2. Two particles are executing simple harmonic motion of the same amplitude A and frequency ω along the x-axis. Their mean position is separated by distance $X_0(X_0 > A)$. If the maximum separation between them is $(X_0 + A)$, the phase difference between their motion is :- **[AIEEE-2011]**

(1) $\frac{\pi}{4}$ (2) $\frac{\pi}{6}$ (3) $\frac{\pi}{2}$ (4) $\frac{\pi}{3}$

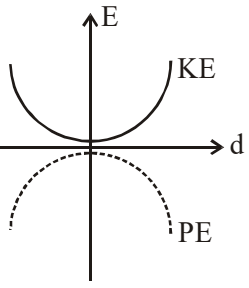
SH0117

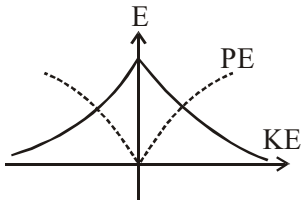
3. A particle moves with simple harmonic motion in a straight line. In first τ s, after starting from rest it travels a distance a, and in next τ s it travels 2a, in same direction, then : **[JEE Main-2014]**

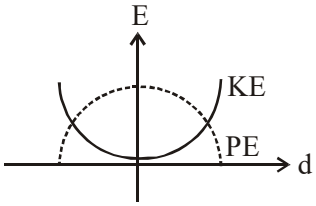
(1) Amplitude of motion is 4a
(2) Time period of oscillation is 6τ
(3) Amplitude of motion is 3a
(4) Time period of oscillation is 8τ

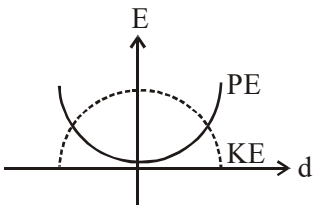
SH0119

4. For a simple pendulum, a graph is plotted between its kinetic energy (KE) and potential energy (PE) against its displacement d. Which one of the following represents these correctly ? (graphs are schematic and not drawn to scale) **[JEE Main-2015]**

(1) 

(2) 

(3) 

(4) 

SH0120

5. A particle performs simple harmonic motion with amplitude A. Its speed is trebled at the instant that it is at a distance $\frac{2A}{3}$ from equilibrium position. The new amplitude of the motion is :-

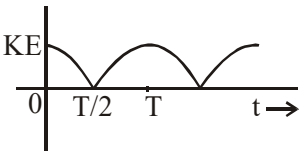
[JEE Main-2016]

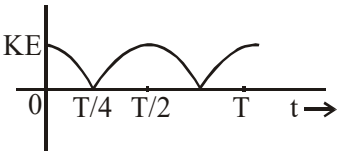
(1) $\frac{7A}{3}$ (2) $\frac{A}{3}\sqrt{41}$ (3) 3A (4) $A\sqrt{3}$

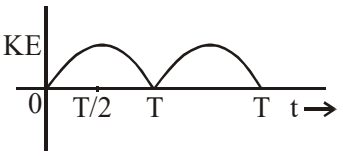
SH0121

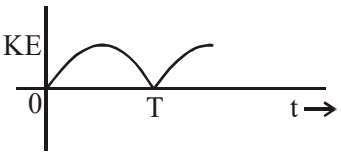
6. A particle is executing simple harmonic motion with a time period T. AT time t = 0, it is at its position of equilibrium. The kinetic energy-time graph of the particle will look like

[JEE Main-2017]

(1) 

(2) 

(3) 

(4) 

SH0122

7. A rod of mass 'M' and length '2L' is suspended at its middle by a wire. It exhibits torsional oscillations; If two masses each of 'm' are attached at distance 'L/2' from its centre on both sides, it reduces the oscillation frequency by 20%. The value of ratio m/M is close to :

[JEE Main-2019]

(1) 0.17 (2) 0.37 (3) 0.57 (4) 0.77

SH0134

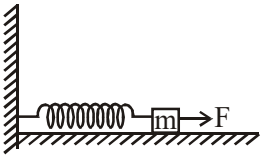
8. A particle is executing simple harmonic motion (SHM) of amplitude A, along the x-axis, about x = 0. When its potential Energy (PE) equals kinetic energy (KE), the position of the particle will be :

[JEE Main-2019]

(1) $\frac{A}{2}$ (2) $\frac{A}{2\sqrt{2}}$ (3) $\frac{A}{\sqrt{2}}$ (4) A

SH0135

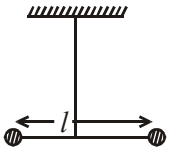
9. A block of mass m, lying on a smooth horizontal surface, is attached to a spring (of negligible mass) of spring constant k. The other end of the spring is fixed, as shown in the figure. The block is initally at rest in its equilibrium position. If now the block is pulled with a constant force F, the maximum speed of the block is : **[JEE Main-2019]**



(1) $\frac{\pi F}{\sqrt{mk}}$ (2) $\frac{2F}{\sqrt{mk}}$ (3) $\frac{F}{\sqrt{mk}}$ (4) $\frac{F}{\pi\sqrt{mk}}$

SH0136

10. Two masses m and $\frac{m}{2}$ are connected at the two ends of a massless rigid rod of length *l*. The rod is suspended by a thin wire of torsional constant k at the centre of mass of the rod-mass system(see figure). Because of torsional constant k, the restoring torque is $\tau=k\theta$ for angular displacement θ. If the rod is rotated by θ_0 and released, the tension in it when it passes through its mean position will be: **[JEE Main-2019]**



(1) $\frac{3k\theta_0^2}{l}$ (2) $\frac{k\theta_0^2}{2l}$ (3) $\frac{2k\theta_0^2}{l}$ (4) $\frac{k\theta_0^2}{l}$

SH0137

11. A particle executes simple harmonic motion with an amplitude of 5 cm. When the particle is at 4 cm from the mean position, the magnitude of its velocity in SI units is equal to that of its acceleration. Then, its periodic time in seconds is : **[JEE Main-2019]**

(1) $\frac{7}{3}\pi$ (2) $\frac{3}{8}\pi$ (3) $\frac{4\pi}{3}$ (4) $\frac{8\pi}{3}$

SH0138

12. A pendulum is executing simple harmonic motion and its maximum kinetic energy is K_1. If the length of the pendulum is doubled and it performs simple harmonic motion with the same amplitude as in the first case, its maximum kinetic energy is K_2. Then :- **[JEE Main-2019]**

(1) $K_2 = \frac{K_1}{4}$ (2) $K_2 = \frac{K_1}{2}$ (3) $K_2 = 2K_1$ (4) $K_2 = K_1$

SH0139

13. A simple harmonic motion is represented by: **[JEE Main-2019]**

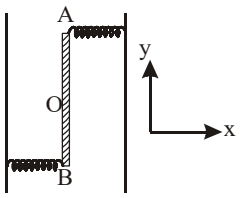
$y = 5(\sin 3\pi t + \sqrt{3}\cos 3\pi t)$ cm

The amplitude and time period of the motion are:

(1) 5cm, $\frac{3}{2}$s (2) 5cm, $\frac{2}{3}$s (3) 10cm, $\frac{3}{2}$s (4) 10cm, $\frac{2}{3}$s

SH0140

14. Two light identical springs of spring constant k are attached horizontally at the two ends of a uniform horizontal rod AB of length ℓ and mass m. The rod is pivoted at its centre 'O' and can rotate freely in horizontal plane. The other ends of the two springs are fixed to rigid supports as shown in figure. The rod is gently pushed through a small angle and released. The frequency of resulting oscillation is: **[JEE Main-2019]**



(1) $\frac{1}{2\pi}\sqrt{\frac{6k}{m}}$ (2) $\frac{1}{2\pi}\sqrt{\frac{2k}{m}}$ (3) $\frac{1}{2\pi}\sqrt{\frac{k}{m}}$ (4) $\frac{1}{2\pi}\sqrt{\frac{3k}{m}}$

SH0141

ANSWER KEY

EXERCISE (S)

1. Ans. $y = 2\sin\left(\frac{10}{3}t - \frac{\pi}{2}\right)$ **2. Ans.** 0.33 sec **3. Ans.** 2 m/s **4. Ans.** 0.06m

5. Ans. $y = 0.1\sin(4t + \pi/4)$ **6. Ans.** (i) $x_0 = 2$m; (ii) T = $\sqrt{2}\ \pi$ sec.; (iii) $2\sqrt{3}$

7. Ans. $\frac{1}{2\pi}\sqrt{\frac{\beta}{\alpha}}$ **8. Ans.** $1/\sqrt{3}$ **9. Ans.** $2\pi\sqrt{\frac{17L}{18g}}$ **10. Ans. M = 1.6 kg**

11. Ans. (a) $\frac{1}{T} = \frac{1}{2\pi}\sqrt{\frac{g}{L}}$, (b) $\frac{L}{3}$ **12. Ans.** (a) 2 sec, (b) T = $\frac{2}{5^{1/4}}$ sec

13. Ans. (b) $\frac{R}{\sqrt{2}}$ **14. Ans.** 100 Nm^{-1} **15. Ans.** $10/\pi$ Hz, $\frac{5\sqrt{37}}{6}$cm

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (C)	**2. Ans. (A)**	**3. Ans. (A)**	**4. Ans. (A)**	**5. Ans. (A)**	**6. Ans. (A)**
7. Ans. (B)	**8. Ans. (A)**	**9. Ans. (A)**	**10. Ans. (C)**	**11. Ans. (C)**	**12. Ans. (A)**
13. Ans. (A)	**14. Ans. (B)**	**15. Ans. (A)**	**16. Ans. (B)**	**17. Ans. (B)**	**18. Ans. (B)**
19. Ans. (C)	**20. Ans. (C)**	**21. Ans. (D)**	**22. Ans. (B)**	**23. Ans. (C)**	**24. Ans. (B)**
25. Ans. (A)	**26. Ans. (A)**	**27. Ans. (C)**	**28. Ans. (B)**	**29. Ans. (D)**	**30. Ans. (C)**
31. Ans. (A)	**32. Ans. (B)**	**33. Ans. (A)**	**34. Ans. (A)**		

EXERCISE (JM)

1. Ans. (2)	**2. Ans. (4)**	**3. Ans. (2)**	**4. Ans. (4)**	**5. Ans. (1)**	**6. Ans. (2)**
7. Ans. (2)	**8. Ans. (3)**	**9. Ans. (3)**	**10. Ans. (4)**	**11. Ans. (4)**	**12. Ans. (3)**
13. Ans. (4)	**14. Ans. (1)**				

Important Notes