MODERN PHYSICS-2

KEY CONCEPT

NUCLEAR PHYSICS

ATOMIC NUCLEUS

The atomic nucleus consists of two types of elementary particles, viz. protons and neutrons. These particles are called nucleons. The proton (denoted by p) has a charge +e and a mass $m_p = 1.6726 \times 10^{-27}$ kg, which is approximately 1840 times larger than the electron mass. The proton is the nucleus of the simplest atom with Z=1, viz the hydrogen atom.

The neutron (denoted by n) is an electrically neutral particle (its charge is zero). The neutron mass is 1.6749×10^{-27} kg. The fact that the mass of a neutron exceeds the mass of a proton by about 2.5 times the electronic masses is of essential importance. It follows from this that the neutron in free state (outside the nucleus) is unstable (radioactive). With half life equal to 12 min, the neutron spontaneously transforms into a proton by emitting an electron (e^-) and a particle called the antineutrino $(\bar{\nu})$.

This process can be schematically written as follows : ${}_0n^1 \rightarrow {}_1p^1 + {}_{-1}e^0 + \bar{\nu}$

The most important characteristics of the nucleus are the charge number Z (coinciding with atomic number of the element) and mass number A. The charge number Z is equal to the number of protons in the nucleus, and hence it determines the nuclear charge equal to Ze. The mass number A is equal to the number of nucleons in the nucleus (i.e., to the total number of protons and neutrons). Nuclei are symbolically designated as X_Z^A or ${}_ZX^A$ where X stands for the symbol of a chemical element.

For example, the nucleus of the oxygen atom is symbolically written as O_8^{16} or ${}_8O^{16}$.

The shape of nucleus is approximately spherical and its radius is approximately related to the mass number by $R = 1.2\ A^{1/3} \times 10^{-15}\ m = 1.2 \times 10^{-15} \times A^{1/3}\ m$

Most of the chemical elements have several types of atoms differing in the number of neutrons in their nuclei. These varieties are called isotopes. For example carbon has three isotopes ${}_6C^{12}$, ${}_6C^{13}$, ${}_6C^{14}$. In addition to stable isotopes, there also exist unstable (radioactive) isotopes. Atomic masses are specified in terms of the atomic mass unit or unified mass unit (u). The mass of a neutral atom of the carbon ${}_6C^{12}$ is defined to be exactly 12 u. $1u = 1.66056 \times 10^{-27}$ kg = 931.5 MeV.

BINDING ENERGY

The rest mass of the nucleus is smaller than the sum of the rest masses of nucleons constituting it. This is due to the fact that when nucleons combine to form a nucleus, some energy (binding energy) is liberated. The binding energy is equal to the work that must be done to split the nucleus into the particles constituting it.

The difference between the total mass of the nucleons and mass of the nucleus is called the mass defect of the nucleus represented by $\Delta m = [Zm_p + (A-Z)m_n] - m_{nuc}$

Multiplying the mass defect by the square of the velocity of light, we can find the binding energy of the nucleus.

$$BE = \Delta mc^2 = [(Zm_p + (A-Z)m_n)-m_{nuc}]c^2$$

If the masses are taken in atomic mass unit, the binding energy is given by

$$BE = [(Zm_p + (A-Z)m_n)-m_{nuc}]\ 931.5\ \text{MeV}$$

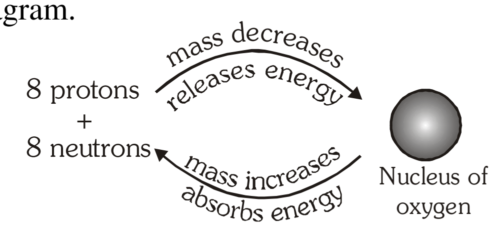
Let us take example of oxygen nucleus. It contains 8 protons and 8 neutrons.

We can discuss concept of binding energy by following diagram.

$8m_p + 8m_n >$ mass of nucleus of oxygen

For nucleus we apply mass energy conservation,

$$8m_p + 8m_n = \text{mass of nucleus} + \frac{B.E.}{c^2}$$



For general nucleus ${}^{A}_{Z}X$, mass defect = difference between total mass of nucleons and mass of the nucleus

$$\Delta m = [Zm_p + (A-Z)m_n]-M$$

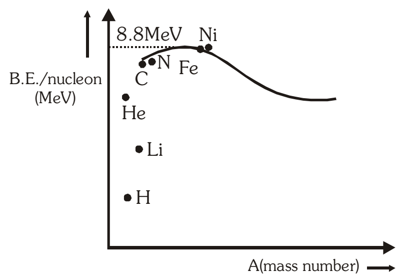
$$B.E. = \Delta mc^2\ (\text{joules}) = (\Delta m)_{\text{in amu}} \times\ 931.5\ \text{MeV}$$

Binding Energy per Nucleon

Stability of a nucleus does not depend upon binding energy of a nucleus but it depends upon binding energy per nucleon

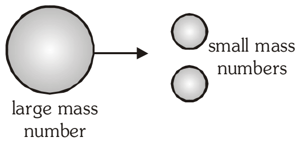
$$B.E./\text{nucleon} = \frac{B.E.}{\text{mass number}}$$

$$\text{Stability} \propto \frac{B.E.}{A}$$

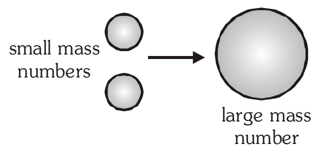


(i) B.E./A is maximum for A =62 (Ni), It is 8.79460 ± 0.00003 MeV/nucleon, means most stable nuclei are in the region of A = 62.

(ii) Heavy nuclei achieve stability by breaking into two smaller nuclei and this reaction is called fission reaction.



(iii) Nuclei achieve stability by combining and resulting into heavy nucleus and this reaction is called fusion reaction.



(iv) In both reactions products are more stable in comparison to reactants and Q value is positive.

NUCLEAR COLLISIONS

We can represent a nuclear collision or reaction by the following notation, which means X (a,b) Y

$$\underset{\text{(bombarding particle)}}{a} + \underset{\text{(at rest)}}{X} \rightarrow Y + b$$

We can apply :

(i) Conservation of momentum (ii) Conservation of charge (iii) Conservation of mass–energy

For any nuclear reaction $\underset{K_1}{a} + \underset{K_2}{X} \rightarrow \underset{K_3}{Y} + \underset{K_4}{b}$

By mass energy conservation

(i) $K_1 + K_2 + (m_\alpha + m_x)c^2 = K_3 + K_4 + (m_Y + m_b)c^2$

(ii) Energy released in any nuclear reaction or collision is called Q value of the reaction

(iii) $Q = (K_3 + K_4) - (K_1 + K_2) = \Sigma K_P - \Sigma K_R = (\Sigma m_R - \Sigma m_P)c^2$

(iv) If Q is positive, energy is released and products are more stable in comparison to reactants.

(v) If Q is negative, energy is absorbed and products are less stable in comparison to reactants.

$Q = \Sigma(\text{B.E.})_{\text{product}} - \Sigma(\text{B.E.})_{\text{reactants}}$

Ex. Let us find the Q value of fusion reaction

$^4He + {}^4He \rightarrow {}^8Be$, if $\frac{\text{B.E.}}{A}$ of He = X and $\frac{\text{B.E.}}{A}$ of Be = Y $\Rightarrow Q = 8Y - 8X$

Q value for α decay $\quad {}_ZX^A \rightarrow {}_{Z-2}Y^{A-4} + {}_2He^4 \Rightarrow Q = K_\alpha + K_Y$ ....(i)

Momentum conservation, $p_Y = p_\alpha$ ...(ii)

$$K_\alpha = \frac{p^2}{2 \times m \times 4} \qquad K_Y = \frac{p^2}{2m(A-4)} = \frac{4K_\alpha}{A-4}$$

$$Q = K_\alpha + \frac{4K_\alpha}{A-4} = \frac{A}{A-4}K_\alpha \qquad K_\alpha = \frac{A-4}{A}Q$$

For α decay A > 210 which means maximum part of released energy is associated with K.E. of α. If Q is negative, the reaction is endoergic. The minimum amount of energy that a bombarding particle must have in order to initiate an endoergic reaction is called Threshold energy E_{th},

given by $E_{th} = -Q\left(\frac{m_1}{m_2} + 1\right)$ where m_1 = mass of the projectile.

E_{th} = minimum kinetic energy of the projectile to initiate the nuclear reaction

m_2 = mass of the target

Ex. How much energy must a bombarding proton possess to cause the reaction ${}_3Li^7 + {}_1H^1 \rightarrow {}_4Be^7 + {}_0n^1$ (Mass of ${}_3Li^7$ atom is 7.01600, mass of ${}_1H^1$ atom is 1.0783, mass of ${}_4Be^7$ atom is 7.01693)

Sol. Since the mass of an atom includes the masses of the atomic electrons, the appropriate number of electron masses must be subtracted from the given values.

Reactants : Total mass = $(7.01600 - 3\,m_e) + (1.0783 - 1\,m_e) = 8.0943 - 4m_e$

Products : Total mass = $(7.01693 - 4m_e) + 1.0087 = 8.02563 - 4m_e$

The energy is supplied as kinetic energy of the bombarding proton. The incident proton must have more than this energy because the system must possess some kinetic energy even after the reaction, so that momentum is conserved with momentum conservation taken into account, the minimum kinetic energy that the incident particle must possess can be found with the formula.

where, $Q = -[(8.02563 - 4m_e) - (8.0943 - 4m_e)]\ 931.5\text{ MeV} = -63.96\text{ MeV}$

$$E_{th} = -\left(1+\frac{m}{M}\right) Q = -\left(1+\frac{1}{7}\right)(-63.96) = 73.1\text{ MeV}$$

NUCLEAR FISSION

In 1938 by Hahn and Strassmann. By attack of a particle splitting of a heavy nucleus (A > 230) into two or more lighter nuclei. In this process certain mass disappears which is obtained in the form of energy (enormous amount)

$$A + p \rightarrow \text{excited nucleus} \rightarrow B + C + Q$$

Hahn and Strassmann done the first fision of nucleus of U^{235}).

When U^{235} is bombarded by a neutron it splits into two fragments and 2 or 3 secondary neutrons and releases about 190 MeV ($\simeq$ 200 MeV) energy per fission (or from single nucleus)

Fragments are uncertain but each time energy released is almost same.

Possible reactions are

$$U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3\,{}_0n^1 + 200\text{ MeV or } U^{235} + {}_0n^1 \rightarrow Xe + Sr + 2\,{}_0n^1 + 200\text{ MeV}$$

and many other reactions are possible.

- The average number of secondary neutrons is 2.5.
- Nuclear fission can be explained by using "liquid drop model" also.
- The mass defect Δm is about 0.1% of mass of fissioned nucleus
- About 93% of released energy (Q) is appear in the form of kinetic energies of products and about 7% part in the form of γ – rays.

NUCLEAR CHAIN REACTION :

The equation of fission of U^{235} is $U^{235} + {}_0n^1 \rightarrow Ba + Kr + 3\,{}_0n^1 + Q$.

These three secondary neutrons produced in the reaction may cause of fission of three more U^{235} and give 9 neutrons, which in turn, may cause of nine more fission of U^{235} and so on.

Thus a continuous 'Nuclear Chain reaction' would start.

If there is no control on chain reaction then in a short time ($\simeq 10^{-6}$ sec.) a huge amount of energy will be released. (This is the principle of 'Atom bomb'). If chain is controlled then produced energy can be used for peaceful purposes. For example nuclear reactor (Based on fission) are generating electricity.

NATURAL URANIUM :

It is mixture of U^{235} (0.7%) and U^{238} (99.3%). U^{235} is easily fissionable, by slow neutron (or thermal neutrons) having K.E. of the order of 0.03 eV. But U^{238} is fissionable with fast neutrons.

Note : Chain reaction in natural uranium can't occur To improve the quality, percentage of U^{235} is increased to 3%. The proposed uranium is called 'Enriched Uranium' (97% U^{238} and 3% U^{235})

LOSSES OF SECONDARY NEUTRONS :

Leakage of neutrons from the system : Due to their maximum K.E. some neutrons escape from the system.

Absorption of neutrons by U^{238} : Which is not fissionable by these secondary neutrons.

CRITICAL SIZE (OR MASS) :

In order to sustain chain reaction in a sample of enriched uranium, it is required that the number of lost neutrons should be much smaller than the number of neutrons produced in a fission process. For it the size of uranium block should be equal or greater than a certain size called **critical size**.

REPRODUCTION FACTOR :

$$(K) = \frac{\text{rate of production of neutrons}}{\text{rate of loss of neutrons}}$$

(i) If size of Uranium used is 'Critical' then K = 1 and the chain reaction will be steady or sustained (As in nuclear reaction)

(ii) If size of Uranium used is 'Super critical' then K > 1 and chain reaction will accelerate resulting in a explosion (As in atom bomb)

(iii) If size of Uranium used is 'Sub Critical' then K < 1 and chain reaction will retard and will stop.

NUCLEAR REACTOR (K = 1) : Credit $\rightarrow$ To Enricho Fermi

Construction :

- **Nuclear Fuel :** Commonly used are U^{235} , Pu^{239} .Pu^{239} is the **best**. Its critical size is less than critical size of U^{235}. But Pu^{239} is not naturally available and U^{235} is used in most of the reactors.
- **Moderator** : Its function is to slow down the fast secondary neutrons. Because only slow neutrons can bring the fission of U^{235}. The moderator should be light and it should not absorb the neutrons. Commonly, Heavy water (D_2O, molecular weight 20 gm.) Graphite etc. are used. These are rich of protons. Neutrons collide with the protons and interchange their energy. Thus neutrons get slow down.
- **Control rods** :They have the ability to capture the slow neutrons and can control the chain reaction at any stage. Boron and Cadmium are best absorber of neutrons.
- **Coolant** : A substance which absorb the produced heat and transfers it to water for further use. Generally coolant is water at high pressure

FAST BREADER REACTORS

The atomic reactor in which fresh fissionable fuel (Pu^{239}) is produced along with energy. The amount of produced fuel (Pu^{239}) is more than consumed fuel (U^{235})

- **Fuel :** Natural Uranium.
- **Process**: During fission of U^{235}, energy and secondary neutrons are produced. These secondary neutrons are absorbed by U^{238} and U^{239} is formed. This U^{239} converts into Pu^{239} after two beta decay. This Pu^{239} can be separated, its half life is 2400 years.

$${}_{92}U^{238} + {}_{0}n^{1} \longrightarrow {}_{92}U^{239} \xrightarrow{2\beta^-} {}_{94}Pu^{239} \text{ (best fuel of fission)}$$

This Pu^{239} can be used in nuclear weapons because of its small critical size than U^{235}.

- **Moderator :** Are not used in these reactors.
- **Coolant :** Liquid sodium

NUCLEAR FUSION :

It is the phenomenon of fusing two or more lighter nuclei to form a single heavy nucleus.

$$A + B \rightarrow C + Q \text{ (Energy)}$$

The product (C) is more stable then reactants (A and B) and $m_c < (m_a + m_b)$

and mass defect $\Delta m = [(m_a + m_b) - m_c]$ amu

Energy released is $E = \Delta m$ 931 MeV

The total binding energy and binding energy per nucleon C both are more than of A and B.

$$\Delta E = E_c - (E_a + E_b)$$

Fusion of four hydrogen nuclei into helium nucleus :

$$4({}_1H^1) \rightarrow {}_2He^4 + 2\,{}_{+}\beta^0 + 2\nu + 26 \text{ MeV}$$

- Energy released per fission >> Energy released per fusion
- Energy per nucleon in fission $\left[= \frac{200}{235} \simeq 0.85\text{MeV}\right]$ << energy per nucleon in fusion $\left[= \frac{24}{4} \simeq 6\text{MeV}\right]$

REQUIRED CONDITION FOR NUCLEAR FUSION

- **High temperature** :

  Which provide kinetic energy to nuclei to overcome the repulsive electrostatic force between them.
- **High Pressure (or density)** :

  Which ensure frequent collision and increases the probability of fusion. The required temperature and pressure at earth (lab) are not possible. These condition exist in the sun and in many other stars. The source of energy in the sun is nuclear fusion, where hydrogen is in plasma state and there protons fuse to form helium nuclei.

HYDROGEN BOMB

It is based on nuclear fusion and produces more energy than an atom bomb.

Pair production	**Pair Annihilation**
A γ–photon of energy more than 1.02 MeV,when interact with a nucleus produces pair of electron (e^-) and positron (e^+). The energy equivalent to rest mass of e^- (or e^+)=0.51 MeV. The energy equivalent to rest mass of pair ($e^- + e^+$) = 1.02 MeV. For pair production Energy of photon $\geq$ 1.02 MeV. If energy of photon is more than 1.02 MeV, the extra energy (E–1.02) MeV devides approximately in equal amount to each particle as the kinetic energy or $(KE)_{e^- \text{ or } e^+} = \left[\frac{E_{Ph} - 1.02}{2}\right]$MeV If E < 1.02 MeV, pair will not produce.	When electron and positron combines they annihilates to each other and only energy is released in the form of two gama photons.If the energy of electron and positron are negligible then energy of each γ–photon is 0.5/MeV

Ex. In a nuclear reactor, fission is produced in 1 g for U^{235} (235.0439) in 24 hours by slow neutrons (1.0087 u). Assume that ${}_{35}Kr^{92}$ (91.8973 u) and ${}_{56}Ba^{141}$ (140.9139 amu) are produced in all reactions and no energy is lost.

(i) Write the complete reaction (ii) Calculate the total energy produced in kilowatt hour.

Given 1u = 931 MeV.

Sol. The nuclear fission reaction is ${}_{92}U^{235} + {}_{0}n^{1} \rightarrow {}_{56}Ba^{141} + {}_{36}Kr^{92} + 3{}_{0}n^{1}$

Mass defect $\Delta m = [(m_u + m_n) - (m_{Ba} + m_{Kr} + 3m_n)] = 256.0526 - 235.8373 = 0.2153$ u

Energy released $Q = 0.2153 \times 931 = 200$ MeV. Number of atoms in 1 g = $\frac{6.02 \times 10^{23}}{235} = 2.56 \times 10^{21}$

Energy released in fission of 1 g of U^{235} is $E = 200 \times 2.56 \times 10^{21} = 5.12 \times 10^{23}$ MeV

$= 5.12 \times 10^{23} \times 1.6 \times 10^{-13} = 8.2 \times 10^{10}$ J

$= \frac{8.2 \times 10^{10}}{3.6 \times 10^{6}}$ kWh $= 2.28 \times 10^{4}$ kWh

Ex. It is proposed to use the nuclear fusion reaction : ${}_{1}H^{2} + {}_{1}H^{2} \rightarrow {}_{2}He^{4}$ in a nuclear reactor of 200 MW rating. If the energy from above reaction is used with at 25% efficiency in the reactor, how many grams of deuterium will be needed per day. (Mass of ${}_{1}H^{2}$ is 2.0141 u and mass of ${}_{2}He^{4}$ is 4.0026 u)

Sol. Energy released in the nuclear fusion is $Q = \Delta mc^2 = \Delta m(931)$ MeV (where Δm is in amu)

$Q = (2 \times 2.0141 - 4.0026) \times 931$ MeV $= 23.834$ MeV $= 23.834 \times 10^{6}$ eV

Since efficiency of reactor is 25%

So effective energy used $= \frac{25}{100} \times 23.834 \times 10^{6} \times 1.6 \times 10^{-19}$ J $= 9.534 \times 10^{-13}$ J

Since the two deuterium nuclei are involved in a fusion reaction,

therefore, energy released per deuterium is $\frac{9.534 \times 10^{-13}}{2}$.

For 200 MW power per day, number of deuterium nuclei required

$$= \frac{200 \times 10^{6} \times 86400}{\frac{9.534 \times 10^{-23}}{2}} = 3.624 \times 10^{25}$$

Since 2g of deuterium constitute 6×10^{23} nuclei, therefore amount of deuterium required is

$$= \frac{2 \times 3.624 \times 10^{25}}{6 \times 10^{23}} = 120.83 \text{ g/day}$$

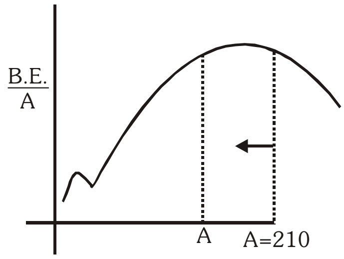
RADIOACTIVITY

The process of spontaneous disintegration shown by some unstable atomic nuclei is known as natural radioactivity. This property is associated with the emission of certain types of penetrating radiations, called radioactive rays, or Becquerel rays (α, β, γ–rays). The elements or compounds, whose atoms disintegrate and emit radiations are called radioactive elements. Radioactivity is a continuous, irreversible nuclear phenomenon.

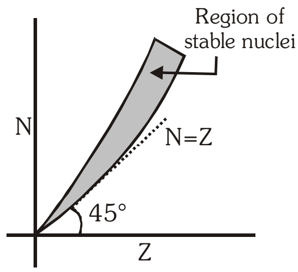
Radioactive Decays

Generally, there are three types of radioactive decays (i) α decay (ii) β^- and β^+ decay (iii) γ decay

- **α decay :** In α decay, the unstable nucleus emits an α particle. By emitting α particle, the nucleus decreases it's mass energy number and move towards stability. Nucleus having A>210 shows α decay.By releasing α particle, it can attain higher stability and Q value is positive.



- **β decay :** In beta decay (N/Z) ratio of nucleus is changed. This decay is shown by unstable nuclei. In beta decay, either a neutron is converted into proton or proton is converted into neutron. For better understanding we discuss N/Z graph. There are two type of unstable nuclides

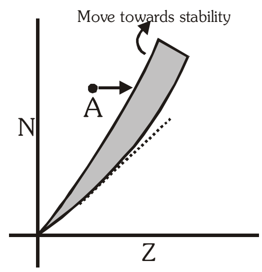


  - **A type**

    For A type nuclides $(N/Z)_A > (N/Z)$ stable

    To achieve stability, it increases Z by conversion of neutron into proton

    $_0n^1 \rightarrow {_1p^1} + e^{-1} + \bar{\nu}$, $_ZX^A \rightarrow {_{Z+1}Y^A} + \underset{(\beta \text{ particle})}{e^{-1}} + \bar{\nu}$



    This decay is called β^{-1} decay.

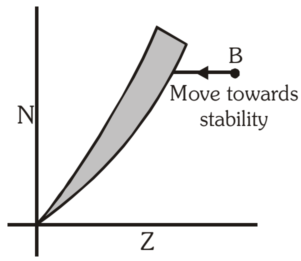
    Kinetic energy available for e^{-1} and $\bar{\nu}$ is, $Q = K_\beta + K_{\bar{\nu}}$

    K.E. of β satisfies the condition $0 < K_\beta < Q$

  - **B type**

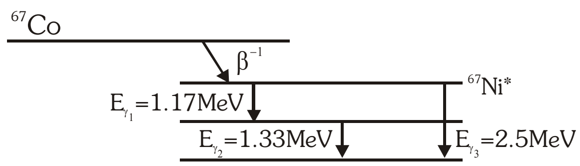
    For B type nuclides $(N/Z)_B < (N/Z)$ stable

    To achieve stability it decreases Z by the conversion of a proton into neutron. That is, $p \rightarrow n + \underset{(\text{positron})}{e^+} + \underset{(\text{neutrino})}{\nu}$, $_ZX^A \rightarrow {_{Z-1}Y^A} + \underset{(\beta^+ \text{particle})}{e^+} + \nu$



- **γ decay :** When an α or β decay takes place, the daughter nucleus is usually in higher energy state, such a nucleus comes to ground state by emitting a photon or photons.

  Order of energy of γ photon is 100 KeV



  e.g. ${}^{67}_{27}Co \rightarrow \underset{(\text{higher energy state})}{{}^{67}_{28}Ni^*} + \beta^- + \bar{\nu}$, ${}^{67}_{28}Ni^* \rightarrow {}^{67}_{28}Ni + \gamma$ photon

Properties of α, β and γ rays

Features	**α–particles**	**β–particles**	**γ–rays**
Identity	Helium nucleus or doubly ionised helium atom (${}_2He^4$)	Fast moving electrons (${}_{-}\beta^0$ or β^-)	Electromagnetic wave (photons)
Charge Mass	Twice of proton (+2e) ≈ $4m_p$ m_p–mass of proton	Electronic (– e) (rest mass of β) = (rest mass of electron)	Neutral rest mass = 0
Speed	1.4×10^7 m/s. to 2.2×10^7 m/s. (Only certain value between this range). Their speed depends on nature of the nucleus. So that it is a characteristic speed.	1% of c to 99% of c (All possible values between this range) β–particles come out with different speeds from the same type of nucleus. So that it can not be a characteristic speed.	Only $c = 3 \times 10^8$ m/s γ–photons come out with same speed from all types of nucleus. So, can not be a characteristic speed.
K.E.	≈ MeV	≈ MeV	≈ MeV
Energy spectrum	Line and discrete (or linear)	Continuous (or linear)	Line and discrete
Ionization power (α>β>γ)	10,000 times of γ–rays	100 times of γ–rays (or $\frac{1}{100}$ times of α)	1 (or $\frac{1}{100}$ times of β)
Penetration power (γ>β>α)	$\frac{1}{10000}$ times of γ–rays	$\frac{1}{100}$ times of γ–rays (100 times of α)	1(100 times of β)
Effect of electric or magnetic field	Deflection	Deflection (More than α)	No deflection
Explanation of emission	By Tunnel effect (or quantum mechanics)	By weak nuclear interactions	With the help of energy levels in nucleus

Laws of Radioactive Decay

1. The radioactive decay is a spontaneous process with the emission of α, β and γ rays. It is not influenced by external conditions such as temperature, pressure, electric and magnetic fields.

2. The rate of disintegration is directly proportional to the number of radioactive atoms present at that time i.e., rate of decay $\propto$ number of nuclei.

Rate of decay = λ (number of nuclei) i.e. $\frac{dN}{dt} = -\lambda N$

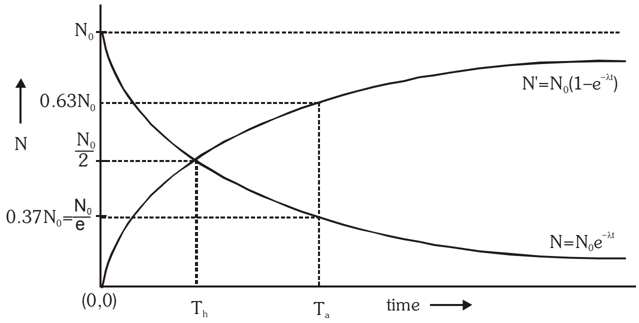
where λ is called the decay constant. This equation may be expressed in the form $\frac{dN}{N} = -\lambda dt$.

$$\int_{N_0}^{N}\frac{dN}{N} = -\lambda\int_0^t dt \Rightarrow \ell n\left(\frac{N}{N_0}\right) = -\lambda t$$

where N_0 is the number of parent nuclei at t=0. The number that survives at time t is therefore

$N=N_0e^{-\lambda t}$ and $t = \frac{2.303}{\lambda}\log_{10}\left(\frac{N_0}{N_t}\right)$ this function is plotted in figure.

Graph : Time versus N (or N')



- **Half life (T_h)** : It is the time during which number of active nuclei reduce to half of initial value.

  If at t = 0 no. of active nuclei N_0 then at t = T_h number of active nuclei will be $\frac{N_0}{2}$

  From decay equation $N = N_0e^{-\lambda t}$

  $$\frac{N_0}{2} = N_0e^{-\lambda T_h} \Rightarrow T_h = \frac{\ell n(2)}{\lambda} = \frac{0.693}{\lambda} \approx \frac{0.7}{\lambda}$$

- **Mean or Average Life (T_a)** : It is the average of age of all active nuclei i.e.

  $$T_a = \frac{\text{sum of times of existance of all nuclei in a sample}}{\text{initial number of active nuclei in that sample}} = \frac{1}{\lambda}$$

  (i) At t = 0, number of active nuclei = N_0 then number of active nuclei at

  $t = T_a$ is $N = N_0e^{-\lambda T_a} = N_0e^{-1} = \frac{N_0}{e} = 0.37N_0 = 37\%$ of N_0

  (ii) Number nuclei which have been disintegrated within duration T_a is
  $N' = N_0 - N = N_0 - 0.37\,N_0 = 0.63\,N_0 = 63\%$ of N_0

  - $T_a = \frac{1}{\lambda} = \frac{T_h}{\ell n(2)} = \frac{T_h}{0.693} = 1.44\,T_h$
  - Within duration $T_h \Rightarrow 50\%$ of N_0 decayed and 50% of N_0 remains active
  - Within duration $T_a \Rightarrow 63\%$ of N_0 decayed and 37% of N_0 remains active

ACTIVITY OF A SAMPLE (A OR R) (OR DECAY RATE)

It is the rate of decay of a radioactive sample $R = -\frac{dN}{dt} = N\lambda$ or $R = R_0e^{-\lambda t}$

- Activity of a sample at any instant depends upon number of active nuclei at that instant.
  $R \propto N$ (or active mass) , $R \propto m$
- R also decreases exponentially w.r.t. time same as the number of active nuclei decreases.

- R is not a constant with N, m and time while λ, T_h and T_a are constant
- At $t = 0$, $R = R_0$ then at $t = T_h \Rightarrow R = \frac{R_0}{2}$ and at $t = T_a \Rightarrow R = \frac{R_0}{e}$ or $0.37\ R_0$
- Similarly active mass of radioactive sample decreases exponentially. $m = m_0 e^{-\lambda t}$
- Activity of m gm active sample (molecular weight M_w) is $R = \lambda N = \frac{0.693}{T_h}\left[\frac{N_{AV}}{M_W}\right]m$

here N_{AV} = Avogadro number = 6.023×10^{23}
SI UNIT of R : 1 becquerel (1 Bq)= 1 decay/sec
Other Unit is curie : 1 Ci = 3.70×10^{10} decays/sec
1 Rutherford : (1 Rd) =10^6 decays/s
Specific activity : Activity of 1 gm sample of radioactive substance. Its unit is Ci/gm
e.g. specific activity of radium (226) is 1 Ci/gm.

Ex. The half–life of cobalt–60 is 5.25 yrs. After how long does its activity reduce to about one eight of its original value?

Sol The activity is proportional to the number of undecayed atoms: In each half–life, the remaining sample decays to half of its initial value. Since $\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)\times\left(\frac{1}{2}\right)=\frac{1}{8}$, therefore, three half–lives or 15.75 years are required for the sample to decay to 1/8th its original strength.

Ex. A count rate meter is used to measure the activity of a given sample. At one instant the meter shows 4750 counts per minute. Five minutes later it shows 2700 counts per minute.
(i) Find the decay constant. (ii) Also, find the half–life of the sample.

Sol. Initial activity $A_i = \left.\frac{-dN}{dt}\right|_{t=0} = \lambda N_0 = 4750$ ...(i) Final activity $A_f = \left.\frac{-dN}{dt}\right|_{t=5} = \lambda N = 2700$ ...(ii)

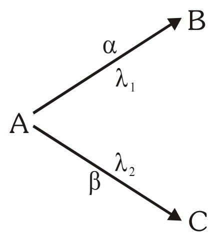
Dividing (i) by (ii), we get $\frac{4750}{2700} = \frac{N_0}{N_t}$

The decay constant is given by $\lambda = \frac{2.303}{t}\log\frac{N_0}{N_t} = \frac{2.303}{5}\log\frac{4750}{2700} = 0.113\ \text{min}^{-1}$

Half–life of the sample is $T = \frac{0.693}{\lambda} = \frac{0.693}{0.113} = 6.14$ min

- **Parallel radioactive disintegration**

Let initial number of nuclei of A is N_0 then at any time number of nuclei of A, B & C are given by $N_0 = N_A + N_B + N_C \Rightarrow \frac{dN_A}{dt} = \frac{-d}{dt}(N_B + N_C)$



A disintegrates into B and C by emitting α, β particle.

Now, $\frac{dN_B}{dt} = -\lambda_1 N_A$ and $\frac{dN_C}{dt} = -\lambda_2 N_A \Rightarrow \frac{d}{dt}(N_B + N_C) = -(\lambda_1 + \lambda_2)N_A$

$$\Rightarrow \frac{dN_A}{dt} = -(\lambda_1 + \lambda_2)N_A \Rightarrow \lambda_{eff} = \lambda_1 + \lambda_2 \Rightarrow t_{eff} = \frac{t_1 t_2}{t_1 + t_2}$$

Ex. The mean lives of a radioactive substances are 1620 and 405 years for α–emission and β–emission respectively. Find out the time during which three fourth of a sample will decay if it is decaying both by α–emission and β–emission simultaneously.

Sol. When a substance decays by α and β emission simultaneously, the average rate of disintegration λ_{av} is given by

$\lambda_{av} = \lambda_\alpha + \lambda_\beta$ when λ_α = disintegration constant for α–emission only λ_β = disintegration constant for β–emission only

Mean life is given by $T_m = \frac{1}{\lambda}$, $\lambda_{av} = \lambda_\alpha + \lambda_\beta \Rightarrow \frac{1}{T_m} = \frac{1}{T_\alpha} + \frac{1}{T_\beta} = \frac{1}{1620} + \frac{1}{405} = \frac{1}{324}$

$\lambda_{av} \times t = 2.303 \log \frac{N_0}{N_t}$, $\frac{1}{324} t = 2.303 \log \frac{100}{25} \Rightarrow t = 2.303 \times 324 \log 4 = 449$ years.

Ex. A radioactive decay is given by $A \xrightarrow[t_{1/2}=8\text{ yrs}]{} B$

Only A is present at t=0. Find the time at which if we are able to pick one atom out of the sample, then probability of getting B is 15 times of getting A.

Sol.

	A	⟶	B
at t = 0	N_0		0
at t = t	N		$N_0 - N$

Probability of getting A, $P_A = \frac{N}{N_0}$

Probability of getting B, $P_B = \frac{N_0 - N}{N_0} \Rightarrow P_B = 15\, P_A \Rightarrow \frac{N_0 - N}{N_0} = 15 \frac{N}{N_0} \Rightarrow N_0 = 16N \Rightarrow N = \frac{N_0}{16}$

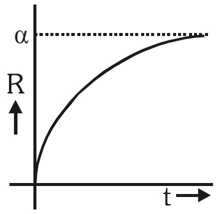
Remaining nuclei are $\frac{1}{16}$ th of initial nuclei, hence required time t = 4 half lives = 32 years

Radioactive Disintegration with Successive Production

$\xrightarrow[(\alpha=\text{rate of production})]{\alpha} A \xrightarrow{\lambda} B$

$\frac{dN_A}{dt} = \alpha = \lambda N_A$ ....(i)

when N_A in maximum $\frac{dN_A}{dt} = 0 = \alpha - \lambda N_A = 0$, N_A max = $\frac{\alpha}{\lambda} = \frac{\text{rate of production}}{\lambda}$



By equation (i) $\int_0^t \frac{dN_A}{\alpha - \lambda N_A} = \int_0^t dt$, Number of nuclei is $N_A = \frac{\alpha}{\lambda}\left(1 - e^{-\lambda t}\right)$

Ex. $\xrightarrow{10^{21}\text{ per sec}} A \xrightarrow{\lambda=\frac{1}{30}} B$

A shows radioactive disintegration and it is continuously produced at the rate of 10^{21} per sec. Find maximum number of nuclei of A.

Sol. At maximum, $r_{production} = r_{decay} \Rightarrow 10^{21} = \frac{1}{30}N \Rightarrow N = 30 \times 10^{21}$

Soddy and Fajan's Group Displacement Laws :

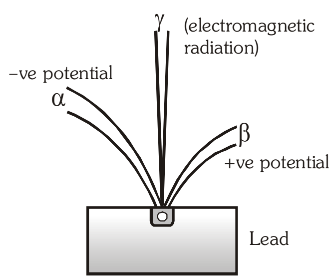
(i) **α–decay :** The emission of one α–particle reduces the mass number by 4 units and atomic number by 2 units. If parent and daughter nuclei are represented by symbols X and Y respectively then, ${}_{Z}X^{A} \rightarrow {}_{Z-2}Y^{A-4} + {}_{2}He^{4}(\alpha)$

(ii) **β–decay :** Beta particles are said to be fast moving electrons coming from the nucleus of a radioactive substance. Does it mean that a nucleus contains electrons? No, it is an established fact that nucleus does not contain any electrons. When a nucleus emits a beta particle, one of its neutrons breaks into a proton, an electron (i.e., β–particle) and an antineutrino $n \rightarrow p + e + \bar{\nu}$

where n= neutron p = proton e = β–particle

Thus emission of a beta particle is caused by the decay of a neutron into a proton. The daughter nucleus thus has an atomic number greater than one (due to one new proton in the nucleus) but same mass number as that of parent nucleus. Therefore, representing the parent and daughter nucleus by symbols X and Y respectively, we have ${}_{Z}X^{A} \rightarrow {}_{Z+1}Y^{A} + \beta + \bar{\nu}$

(iii) **γ–decay :** When parent atoms emit gamma rays, no charge is involved as these are neutral rays. Thus there is no effect on the atomic number and mass number of the parent nucleus. However the emission of γ–rays represents energy. Hence the emission of these rays changes the nucleus from an excited (high energy) state to a less excited (lower energy) state.



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EXERCISE (S)

Nuclear Physics :

1. Highly energetic electrons are bombarded on a target of an element containing 30 neutrons. The ratio of radii of nucleus to that of helium nucleus is $(14)^{1/3}$. Find **[JEE 2005]**

(a) atomic number of the nucleus

(b) the frequency of K_α line of the X-ray produced. ($R = 1.1\times 10^7\ m^{-1}$ and $c = 3 \times 10^8$ m/s)

MP0192

2. When two deutrons ($_1H^2$) fuse to form a helium nucleus $_2He^4$, 23.6 MeV energy is released. Find the binding energy of helium if it is 1.1 MeV for each nucleon of deuterium.

MP0191

Radioactivity :

3. The kinetic energy of an α – particle which flies out of the nucleus of a Ra^{226} atom in radioactive disintegration is 4.78 MeV. Find the total energy evolved during the escape of the α – particle.

MP0193

4. The positron is a fundamental particle with the same mass as that of the electron and with a charge equal to that of an electron but of opposite sign. When a positron and an electron collide, they may annihilate each other. The energy corresponding to their mass appears in two photons of equal energy. Find the wavelength of the radiation emitted. [Take : mass of electron = $(0.5/C^2)$MeV and $hC = 1.2 \times 10^{-12}$ MeV.m where h is the Plank's constant and C is the velocity of light in air]

MP0202

5. The element Curium $^{248}_{96}Cm$ has a mean life of 10^{13} seconds. Its primary decay modes are spontaneous fission and α decay, the former with a probability of 8% and the latter with a probability of 92%. Each fission releases 200 MeV of energy. The masses involved in α decay are as follows :

$^{248}_{96}Cm = 248.072220u,\ ^{244}_{94}Pu = 244.064100u\ \&\ ^{4}_{2}He = 4.002603\,u.$

Calculate the power output from a sample of 10^{20} Cm atoms. ($1\ u = 931\ MeV/c^2$)

MP0200

6. Suppose that the Sun consists entirely of hydrogen atom and releases the energy by the nuclear reaction, $4\,{}_1^1H \longrightarrow {}_2^4He$ with 26 MeV of energy released. If the total output power of the Sun is assumed to remain constant at 3.9×10^{26} W, find the time it will take to burn all the hydrogen. Take the mass of the Sun as 1.7×10^{30} kg.

MP0190

7. A radioactive sample emits n β-particles in 2 sec. In next 2 sec it emits 0.75 n β-particles, what is the mean life of the sample? **[JEE 2003]**

MP0205

8. The age of a rock containing lead and uranium is equal to 1.5×10^9 yrs. The uranium is decaying into lead with half life equal to 4.5×10^9 yrs. Find the ratio of lead to uranium present in the rock, assuming initially no lead was present in the rock. (Given $2^{1/3} = 1.259$). **[JEE 2004]**

MP0206

9. U^{238} and U^{235} occur in nature in an atomic ratio 140 : 1. Assuming that at the time of earth's formation the two isotopes were present in equal amounts. Calculate the age of the earth.
(Half life of $u^{238} = 4.5 \times 10^9$ yrs & that of $U^{235} = 7.13 \times 10^8$ yrs)

MP0194

10. At t = 0, a sample is placed in a reactor. An unstable nuclide is produced at a constant rate R in the sample by neutron absorption. This nuclide β^- decays with half life τ. Find the time required to produce 80% of the equilibrium quantity of this unstable nuclide.

MP0195

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

Nuclear Physics :

1. In Rutherford's famous gold foil scattering experiment, he found that most alpha particles would pass through the foil undeflected. Which one of the following nuclear properties can be inferred from this observation?

(A) The nucleus must have a positive charge

(B) Most of the mass of an atom is in the nucleus

(C) The nucleus contains both protons neutrons

(D) The diameter of the nucleus is small compared to the diameter of the atom

(E) None of these

MP0207

2. Let u be denote one atomic mass unit. One atom of an element of mass number A has mass exactly equal to Au

(A) for any value of A (B) only for A = 1

(C) only for A = 12 (D) for any value of A provided the atom is stable

MP0208

3. If radius of the ${}^{27}_{13}\text{Al}$ nucleus is estimated to be 3.6 fermi, then the radius of ${}^{125}_{52}\text{Te}$ nucleus be nearly- **[AIEEE - 2005]**

(A) 6 fermi (B) 8 fermi (C) 4 fermi (D) 5 fermi

MP0209

4. The surface area of a nucleus varies with mass number A as

(A) $A^{2/3}$ (B) $A^{1/3}$ (C) A (D) None

MP0210

5. A nucleus disintegrates into two nuclear parts which have their velocities in the ratio 2 : 1 The ratio of their nuclear sizes will be- **[AIEEE - 2004]**

(A) $2^{1/3} : 1$ (B) $1 : 3^{1/2}$ (C) $3^{1/2} : 1$ (D) $1 : 2^{1/3}$

MP0211

6. The binding energy per nucleon for C^{12} is 7.68 MeV and that for C^{13} is 7.5 MeV. The energy required to remove a neutron from C^{13} is

(A) 5.34 MeV (B) 5.5 MeV (C) 9.5 MeV (D) 9.34 MeV

MP0212

7. The following nuclear reaction is an example of ${}^{12}_{6}\text{C} + {}^{4}_{2}\text{H} \rightarrow {}^{16}_{8}\text{O}$ + energy

(A) fission (B) fusion (C) alpha decay (D) beta decay

MP0213

8. A nuclear transformation is denoted by $X(n, \alpha) \rightarrow {}^{7}_{3}\text{Li}$. Which of the following is the nucleus of element X ? **[AIEEE - 2005]**

(A) ${}^{12}_{6}\text{C}$ (B) ${}^{10}_{5}\text{B}$ (C) ${}^{9}_{5}\text{B}$ (D) ${}^{11}_{4}\text{Be}$

MP0215

9. A certain radioactive nuclide of mass number m_x disintegrates, with the emission of an electron and γ radiation only, to give second nuclide of mass number m_y. Which one of the following equation correctly relates m_x and m_y?

(A) $m_y = m_x + 1$ (B) $m_y = m_x - 2$ (C) $m_y = m_x - 1$ (D) $m_y = m_x$

MP0216

10. A nucleus with Z = 92 emits the following in a sequence : $\alpha, \alpha, \beta^-, \beta^-, \alpha, \alpha, \alpha, \alpha, \beta^-, \beta^-, \alpha, \beta^+, \beta^+, \alpha$. The Z of the resulting nucleus is- **[AIEEE - 2003]**

(A) 76 (B) 78 (C) 82 (D) 74

MP0217

11. The rest mass of the deuteron, ${}^{2}_{1}\text{H}$, is equivalent to an energy of 1876 MeV, the rest mass of a proton is equivalent to 939 MeV and that of a neutron to 940 MeV. A deuteron may disintegrate to a proton and a neutron if it :

(A) emits a γ - ray photon of energy 2 MeV (B) captures a γ - ray photon of energy 2 MeV
(C) emits a γ - ray photon of energy 3 MeV (D) captures a γ - ray photon of energy 3 MeV

MP0251

12. In an α-decay the Kinetic energy of α particle is 48 MeV and Q-value of the reaction is 50 MeV. The mass number of the mother nucleus is: (Assume that daughter nucleus is in ground state)

(A) 96 (B) 100 (C) 104 (D) none of these

MP0218

13. When U^{238} nucleus originally at rest, decays by emitting an alpha particle having a speed u, the recoil speed of the residual nucleus is- **[AIEEE - 2003]**

(A) $\frac{4u}{238}$ (B) $-\frac{4u}{234}$ (C) $\frac{4u}{234}$ (D) $-\frac{4u}{238}$

MP0219

14. If a star converts all of its Helium into oxygen nucleus, find the amount of energy released per nucleus of oxygen. O = 15.9994 amu and He = 4.0026 amu **[JEE' 2005 (Scr)]**

(A) 7.26 MeV (B) 7 MeV (C) 10.24 MeV (D) 5.12 MeV

MP0220

15. The nucleus of element X (A = 220) undergoes α-decay. If Q-value of the reaction is 5.5 MeV, then the kinetic energy of α-particle is : **[JEE 2003 (Scr)]**

(A) 5.4 MeV (B) 10.8 MeV (C) 2.7 MeV (D) None

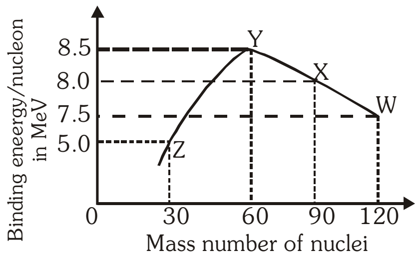
MP0221

16. In the uranium radioactive series the initial nucleus is ${}_{92}U^{238}$, and the final nucleus is ${}_{82}Pb^{206}$. When the uranium nucleus decays to lead, the number of α – particles emitted is.. and the number of β-particles emitted...

(A) 6, 8 (B) 8, 6 (C) 16, 6 (D) 32, 12

MP0222

17. Binding energy per nucleon versus mass number curve for nuclei is shown in figure. W, X, Y and Z are four nuclei indicated on the curve. The process that would release energy is :



(A) $Y \rightarrow 2Z$ (B) $W \rightarrow X + Z$ (C) $W \rightarrow 2Y$ (D) $X \rightarrow Y + Z$

MP0223

Radoactivity :

18. The half–life of ^{131}I is 8 days. Given a sample of ^{131}I at time t = 0, we can assert that :
(A) no nucleus will decay before t = 4 days
(B) no nucleus will decay before t = 8 days
(C) all nuclei will decay before t = 16 days
(D) a given nucleus may decay at any time after t = 0

MP0228

19. In a radioactive element the fraction of initial amount remaining after its mean life time is

(A) $1 - \frac{1}{e}$ (B) $\frac{1}{e^2}$ (C) $\frac{1}{e}$ (D) $1 - \frac{1}{e^2}$

MP0253

20. Activity of a radioactive substance is R_1 at time t_1 and R_2 at time $t_2 (t_2 > t_1)$. Then the ratio $\frac{R_2}{R_1}$ is:

(A) $\frac{t_2}{t_1}$ (B) $e^{-\lambda(t_1+t_2)}$ (C) $e\left(\frac{t_1 - t_2}{\lambda}\right)$ (D) $e^{\lambda(t_1-t_2)}$

MP0229

21. Two radioactive material A_1 and A_2 have decay constants of $10\,\lambda_0$ and λ_0. If initially they have same number of nuclei, the ratio of number of their undecayed nuclei will be (1/e) after a time

(A) $\frac{1}{\lambda_0}$ (B) $\frac{1}{9\lambda_0}$ (C) $\frac{1}{10\lambda_0}$ (D) 1

MP0254

22. A particular nucleus in a large population of identical radioactive nuclei did survive 5 half lives of that isotope. Then the probability that this surviving nucleus will survive the next half life :

(A) $\frac{1}{32}$ (B) $\frac{1}{5}$ (C) $\frac{1}{2}$ (D) $\frac{1}{10}$

MP0230

23. The binding energy per nucleon of deuteron $({}_1^2H)$ and helium nucleus $({}_2^4He)$ is 1.1 MeV and 7 MeV respectively. If two deuteron nuclei react to form a single helium nucleus, then the energy released is-

[AIEEE - 2004]

(A) 13.9 MeV (B) 26.9 MeV (C) 23.6 MeV (D) 19.2 MeV

MP0264

24. At time t = 0, N_1 nuclei of decay constant λ_1 & N_2 nuclei of decay constant λ_2 are mixed . The decay rate of the mixture is :

(A) $N_1N_2e^{-(\lambda_1+\lambda_2)t}$ (B) $+\left(\frac{N_1}{N_2}\right)e^{-(\lambda_1-\lambda_2)t}$

(C) $+\left(N_1\lambda_1e^{-\lambda_1 t}+N_2\lambda_2e^{-\lambda_2 t}\right)$ (D) $+N_1\lambda_1 N_2\lambda_2 e^{-(\lambda_1+\lambda_2)t}$

MP0258

25. A certain radio active substance has a half life of 5 years. Thus for a nucleus in a sample of the element, the probability of decay in ten years is

(A) 50% (B) 75% (C) 100% (D) 60%

MP0231

26. The activity of a sample reduces from A_0 to $A_0/\sqrt{3}$ in one hour. The activity after 3 hours more will be :-

(A) $\frac{A_0}{3\sqrt{3}}$ (B) $\frac{A_0}{9}$ (C) $\frac{A_0}{9\sqrt{3}}$ (D) $\frac{A_0}{27}$

MP0232

27. Half life of radium is 1620 years. How many radium nuclei decay in 5 hours in 5 gm radium? (Atomic weight of radium = 223)

(A) 9.1×10^{12} (B) 3.23×10^{15} (C) 1.72×10^{20} (D) 3.3×10^{17}

MP0233

28. The activity of a sample of radioactive material is A_1 at time t_1 and A_2 at time t_2 ($t_2 > t_1$). Its mean life is T.

(A) $A_1t_1 = A_2t_2$ (B) $\frac{A_1 - A_2}{t_2 - t_1}$ = constant

(C) $A_2 = A_1 e^{(t_1-t_2)/T}$ (D) $A_2 = A_1 e^{(t_1/Tt_2)}$

MP0234

29. The decay constant of the end product of a radioactive series is

(A) zero (B) infinite

(C) finite (non zero) (D) depends on the end product.

MP0235

30. A radioactive nuclide can decay simultaneously by two different processes which have decay constants λ_1 and λ_2. The effective decay constant of the nuclide is λ, then :

(A) $\lambda = \lambda_1 + \lambda_2$ (B) $\lambda = 1/2(\lambda_1 + \lambda_1)$ (C) $\frac{1}{\lambda} = \frac{1}{\lambda_1} + \frac{1}{\lambda_2}$ (D) $\lambda = \sqrt{\lambda_1\lambda_2}$

MP0259

31. A radioactive substance is dissolved in a liquid and the solution is heated. The activity of the solution

(A) is smaller than that of element

(B) is greater than that of element

(C) is equal to that of element

(D) will be smaller or greater depending upon whether the solution is weak or concentrated.

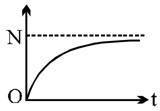
MP0236

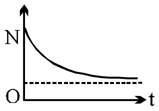
32. In a certain nuclear reactor, a radioactive nucleus is being produced at a constant rate = 1000 /s. The mean life of the radionuclide is 40 minutes. At steady state, the number of radionuclide will be

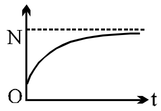
(A) 4×10^4 (B) 24×10^4 (C) 24×10^5 (D) 24×10^6

MP0237

33. In the above question, if there were 20×10^5 radionuclide at t = 0, then the graph of N v/s t is

(A) 

(B) 

(C) 

(D) 

MP0238

34. Which of the following cannot be emitted by radioactive substances during their decay ? **[AIEEE - 2003]**

(A) Protons (B) Neutrinos (C) Helium nuclei (D) Electrons

MP0239

35. Fast neutrons may most easily be slowed down by which one of the following methods?

(A) passing them through a substance rich in hydrogen

(B) allowing them to collide elastically with heavy nuclei

(C) using lead shielding

(D) passing them through an increasing potential gradient space

MP0214

36. A 280 days old radioactive substance shows an activity of 6000 dps, 140 days later it's activity becomes 3000dps. What was its initial activity. **[JEE 2004 (Scr)]**

(A) 20000 dps (B) 24000 dps (C) 12000 dps (D) 6000 dps

MP0240

37. In the nuclear fusion reaction,

$$^2_1H + ^3_1H \rightarrow ^4_2He + n$$

given that the repulsive potential energy between the two nuclei is 7.7×10^{-14} J, the temperature at which the gases must be heated to initiate the reaction is nearly [Boltzmann's constant $k = 1.38 \times 10^{-23}$ J/K]- **[AIEEE - 2003]**

(A) 10^7 K (B) 10^5 K (C) 10^3 K (D) 10^9 K

MP0263

EXERCISE - JM

Directions : Questions number 1 – 2 are based on the following paragraph.

A nucleus of mass M + Δm is at rest and decays into two daughter nuclei of equal mass $\frac{M}{2}$ each. Speed of light is c.

1. The speed of daughter nuclei is :- **[AIEEE-2010]**

(1) $c\sqrt{\frac{\Delta m}{M+\Delta m}}$ (2) $c\frac{\Delta m}{M+\Delta m}$

(3) $c\sqrt{\frac{2\Delta m}{M}}$ (4) $c\sqrt{\frac{\Delta m}{M}}$

MP0278

2. The binding energy per nucleon for the parent nucleus is E_1 an that for the daughter nuclei is E_2. Then:- **[AIEEE-2010]**

(1) $E_1 = 2E_2$ (2) $E_2 = 2E_1$ (3) $E_1 > E_2$ (4) $E_2 > E_1$

MP0279

3. A radioactive nucleus (initial mass number A and atomic number Z) emits 3 α-particles and 2 positrons. The ratio of number of neutrons to that of protons in the final nucleus will be:- **[AIEEE-2010]**

(1) $\frac{A-Z-4}{Z-2}$ (2) $\frac{A-Z-8}{Z-4}$ (3) $\frac{A-Z-4}{Z-8}$ (4) $\frac{A-Z-12}{Z-4}$

MP0280

4. The half life of a radioactive substance is 20 minutes. The approximate time interval $(t_2 - t_1)$ between the time t_2 when $\frac{2}{3}$ of it has decayed and time t_1 when $\frac{1}{3}$ of it had decayed is :- **[AIEEE-2011]**

(1) 20 min (2) 28 min (3) 7 min (4) 14 min

MP0281

5. After absorbing a slowly moving neutron of mass m_N (momentum ~0) a nucleus of mass M breaks into two nuclei of masses m_1 and $5m_1(6m_1 = M + m_N)$, respectively. If the de Broglie wavelength of the nucleus with mass m_1 is λ, then de Broglie wavelength of the other nucleus will be:- **[AIEEE-2011]**

(1) 25 λ (2) 5λ (3) $\frac{\lambda}{5}$ (4) λ

MP0282

6. **Statement-1:** A nucleus having energy E_1 decays be β^- emission to daughter nucleus having energy E_2, but the β^- rays are emitted with a continuous energy spectrum having end point energy $E_1 - E_2$.

Statement-1: To conserve energy and momentum in β-decay at least three particles must take part in the transformation. **[AIEEE-2011]**

(1) Statement-1 is incorrect, statement-2 is correct

(2) Statement-1 is correct, statement-2 is incorrect

(3) Statement-1 is correct, statement-2 correct; statement-2 is the correct explanation of statement-1

(4) Statement-1 is correct, statement-2 is correct; statement -2 is not the correct explanation of statement-1.

MP0283

7. Assume that a neutron breaks into a proton and an electron. The energy released during this process is : **[AIEEE-2012]**

(Mass of neutron = 1.6725×10^{-27} kg, Mass of proton = 1.6725×10^{-27} kg

Mass of electron = 9×10^{-31} kg)

(1) 5.4 MeV (2) 0.73 MeV (3) 7.10 MeV (4) 6.30 MeV

MP0284

8. Half-lives of two radioactive elements A and B are 20 minutes and 40 minutes, respectively. Initially, the samples have equal number of nuclei. After 80 minutes , the ratio of decayed numbers of A and B nuclei will be :- **[JEE-Main-2016]**

(1) 5 : 4 (2) 1 : 16 (3) 4 : 1 (4) 1 : 4

MP0287

9. A radioactive nucleus A with a half life T, decays into a nucleus B. At t = 0, there is no nucleus B. At sometime t, the ratio of the number of B to that of A is 0.3. Then, t is given by : **[JEE-Main-2017]**

(1) $t = T\log(1.3)$ (2) $t = \dfrac{T}{\log(1.3)}$ (3) $t = \dfrac{T}{2}\dfrac{\log 2}{\log 1.3}$ (4) $t = T\dfrac{\log 1.3}{\log 2}$

MP0288

10. An electron from various excited states of hydrogen atom emit radiation to come to the ground state. Let λ_n, λ_g be the de Broglie wavelength of the electron in the n^{th} state and the ground state respectively. Let Λ_n be the wavelength of the emitted photon in the transition from the n^{th} state to the ground state. For large n, (A, B are constants) **[JEE-Main-2018]**

(1) $\Lambda_n \approx A + B\lambda_n$ (2) $\Lambda_n^2 \approx A + B\lambda_n^2$ (3) $\Lambda_n^2 \approx \lambda$ (4) $\Lambda_n \approx A + \dfrac{B}{\lambda_n^2}$

MP0289

11. The ratio of mass densities of nuclei of ^{40}Ca and ^{16}O is close to :- **[JEE-Main (Online)-2019]**

(1) 1 (2) 2 (3) 0.1 (4) 5

MP0344

12. A damped harmonic oscillator has a frequency of 5 oscillations per second. The amplitude drops to half its value for every 10 oscillations. The time it will take to drop to $\frac{1}{1000}$ of the original amplitude is close to :- **[JEE-Main (Online)-2019]**

(1) 100 s (2) 20 s (3) 10 s (4) 50 s

MP0345

13. Two radioactive substances A and B have decay constants 5λ and λ respectively. At t = 0, a sample has the same number of the two nuclei. The time taken for the ratio of the number of nuclei to become $\left(\frac{1}{e}\right)^2$ will be : **[JEE-Main (Online)-2019]**

(1) 1 / 4λ (2) 1 / λ (3) 1 / 2λ (4) 2 / λ

MP0346

14. Two radioactive materials A and B have decay constants 10λ and λ, respectively. It initially they have the same number of nuclei, then the ratio of the number of nuclei of A to that of B will be 1/e after a time : **[JEE-Main (Online)-2019]**

(1) $\frac{11}{10\lambda}$ (2) $\frac{1}{9\lambda}$ (3) $\frac{1}{10\lambda}$ (4) $\frac{1}{11\lambda}$

MP0347

15. Half lives of two radioactive nuclei A and B are 10 minutes and 20 minutes, respectively. If, initially a sample has equal number of nuclei, then after 60 minutes, the ratio of decayed numbers of nuclei A and B will be : **[JEE-Main (Online)-2019]**

(1) 9 : 8 (2) 1 : 8 (3) 8 : 1 (4) 3 : 8

MP0348

16. At a given instant, say t = 0, two radioactive substances A and B have equal activities. The ratio $\frac{R_B}{R_A}$ of their activities after time t itself decays with time t as e^{-3t}. If the half-life of A is $\ell n2$, the half-life of B is : **[JEE-Main (Online)-2019]**

(1) $\frac{ln2}{2}$ (2) $2ln2$ (3) $\frac{ln2}{4}$ (4) $4ln2$

MP0349

17. A sample of radioactive material A, that has an activity of 10 mCi(1 Ci = 3.7×10^{10} decays/s), has twice the number of nuclei as another sample of a different radioactive maternal B which has an activity of 20 mCi. The correct choices for hall-lives of A and B would then be respectively : **[JEE-Main (Online)-2019]**

(1) 20 days and 5 days (2) 20 days and 10 days
(3) 5 days and 10 days (4) 10 days and 40 days

MP0350

18. Condiser the nuclear fission

$Ne^{20} \rightarrow 2He^{4} + C^{12}$

Given that the binding energy/nucleon of Ne^{20}, He^{4} and C^{12} are, respectively, 8.03 MeV, 7.07 MeV and 7.86 MeV, identify the correct statement : **[JEE-Main (Online)-2019]**

(1) 8.3 MeV energy will be released
(2) energy of 12.4 MeV will be supplied
(3) energy of 11.9 MeV has to be supplied
(4) energy of 3.6 MeV will be released

MP0351

19. Using a nuclear counter the count rate of emitted particles from a radioactive source is measured. At t = 0 it was 1600 counts per second and t = 8 seconds it was 100 counts per second. The count rate observed, as counts per second, at t = 6 seconds is close to : **[JEE-Main (Online)-2019]**

(1) 150 (2) 360 (3) 200 (4) 400

MP0352

20. In an electron microscope, the resolution that can be achieved is of the order of the wavelength of electrons used. To resolve a width of 7.5×10^{-12}m, the minimum electron energy required is close to : **[JEE-Main (Online)-2019]**

(1) 100 keV (2) 500 keV (3) 25 keV (4) 1 keV

MP0353

21. In a radioactive decay chain, the initial nucleus is $^{232}_{90}Th$. At the end there are 6 α-particles and 4 β-particles which are emitted. If the end nucleus, If $^{A}_{Z}X$, A and Z are given by :

[JEE-Main (Online)-2019]

(1) A = 208; Z = 80 (2) A = 202; Z = 80 (3) A = 200; Z = 81 (4) A = 208; Z = 82

MP0354

22. An alpha-particle of mass m suffers 1-dimensional elastic coolision with a nucleus at rest of unknown mass. It is scattered directly backwards losing, 64% of its initial kinetic energy. The mass of the nucleus is :- **[JEE-Main (Online)-2019]**

(1) 4 m (2) 3.5 m (3) 2 m (4) 1.5 m

MP0355

ANSWER KEY

EXERCISE (S)

1. Ans. $\nu = 1.546 \times 10^{18}$ Hz; Z = 26 **2. Ans.** 28 MeV **3. Ans.** 4.87 MeV

4. Ans. 2.4×10^{-12} m **5. Ans.** $\cong 33.298\ \mu W$

6. Ans. $8/3 \times 10^{18}$ sec **7. Ans.** $1.75n = N_0(1 - e^{-4\lambda})$, 6.95 sec, $\dfrac{2}{ln\left(\dfrac{4}{3}\right)}$

8. Ans. 0.259 **9. Ans.** 6.04×10^{9} yrs **10. Ans.** $t = \left(\dfrac{ln 5}{ln 2}\right)\tau$

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (D)	**2. Ans. (C)**	**3. Ans. (A)**	**4. Ans. (A)**	**5. Ans. (D)**	**6. Ans. (A)**
7. Ans. (B)	**8. Ans. (B)**	**9. Ans. (D)**	**10. Ans. (B)**	**11. Ans. (D)**	**12. Ans. (B)**
13. Ans. (C)	**14. Ans. (C)**	**15. Ans. (A)**	**16. Ans. (B)**	**17. Ans. (C)**	**18. Ans. (D)**
19. Ans. (C)	**20. Ans. (D)**	**21. Ans. (B)**	**22. Ans. (C)**	**23. Ans. (C)**	**24. Ans. (C)**
25. Ans. (B)	**26. Ans. (B)**	**27. Ans. (B)**	**28. Ans. (C)**	**29. Ans. (A)**	**30. Ans. (A)**
31. Ans. (C)	**32. Ans. (C)**	**33. Ans. (C)**	**34. Ans. (A)**	**35. Ans. (A)**	**36. Ans. (B)**
37. Ans. (D)					

EXERCISE (JM)

1. Ans. (3)	**2. Ans. (4)**	**3. Ans. (3)**	**4. Ans. (1)**	**5. Ans. (4)**	**6. Ans. (3)**
7. Ans. (Bonus)	**8. Ans. (1)**	**9. Ans. (4)**	**10. Ans. (4)**	**11. Ans. (1)**	**12. Ans. (2)**
13. Ans. (3)	**14. Ans. (2)**	**15. Ans. (1)**	**16. Ans. (3)**	**17. Ans. (1)**	**18. Ans. (3)**
19. Ans. (3)	**20. Ans. (3)**	**21. Ans. (4)**	**22. Ans. (1)**		

Important Notes