CENTER OF MASS, MOMENTUM & COLLISION

KEY CONCEPTS

CENTRE OF MASS :

For a system of particles centre of mass is that point at which its total mass is supposed to be concentrated.

The centre of mass of an object is a point that represents the entire body and moves in the same way as a point mass having mass equal to that of the object, when subjected to the same external forces that act on the object.

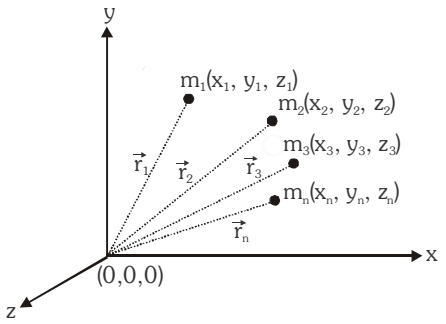
Centre of mass of system of discrete particles

Total mass of the body : $M = m_1 + m_2 + ... + m_n$

Then $\vec{R}_{CM} = \dfrac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...} = \dfrac{1}{M}\Sigma m_i\vec{r}_i$

co-ordinates of centre of mass :

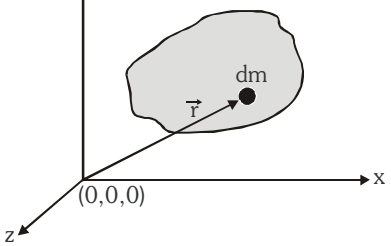
$$x_{cm} = \frac{1}{M}\Sigma m_i x_i,\ y_{cm} = \frac{1}{M}\Sigma m_i y_i \text{ and } z_{cm} = \frac{1}{M}\Sigma m_i z_i$$



Centre of mass of continuous distribution of particles

If the system has continuous distribution of mass, treating the mass element dm at position $\vec{r}$ as a point mass and replacing summation by integration. $\vec{R}_{CM} = \dfrac{1}{M}\int \vec{r}\,dm$

So that $x_{cm} = \dfrac{1}{M}\int x\,dm$, $y_{cm} = \dfrac{1}{M}\int y\,dm$ and $z_{cm} = \dfrac{1}{M}\int z\,dm$



If co-ordinates of particles of mass m_1, m_2, ... are

$(x_1, y_1, z_1), (x_2, y_2, z_2)....$

then position vector of their centre of mass is

$$\vec{R}_{CM} = x_{cm}\hat{i} + y_{cm}\hat{j} + z_{cm}\hat{k}$$

$$= \frac{m_1(x_1\hat{i} + y_1\hat{j} + z_1\hat{k}) + m_2(x_2\hat{i} + y_2\hat{j} + z_2\hat{k}) + m_3\left(x_3\hat{i} + y_3\hat{j} + z_3\hat{k}\right) + ...}{m_1 + m_2 + m_3 + ...}$$

$$= \frac{(m_1x_1 + m_2x_2 + ...)\hat{i} + (m_1y_1 + m_2y_2...)\hat{j} + (m_1z_1 + m_2z_2 + ..)\hat{k}}{m_1 + m_2 + m_3 + ..}$$

So, $x_{cm} = \left(\dfrac{m_1x_1 + m_2x_2 + ...}{m_1 + m_2 + m_3 + ...}\right)$, $y_{cm} = \left(\dfrac{m_1y_1 + m_2y_2 + ...}{m_1 + m_2 + ...}\right)$, $z_{cm} = \left(\dfrac{m_1z_1 + m_2z_2 + ...}{m_1 + m_2 + ...}\right)$

The centre of mass after removal of a part of a body

If a portion of a body is taken out, the remaining portion may be considered as,

Original mass (M) – mass of the removed part (m) = {original mass (M)} + { – mass of the removed part (m)}

The formula changes to : $x_{CM} = \dfrac{Mx - mx'}{M - m}$; $y_{CM} = \dfrac{My - my'}{M - m}$; $z_{CM} = \dfrac{Mz - mz'}{M - m}$

Where x', y' and z' represent the coordinates of the centre of mass of the removed part.

MOTION OF CENTRE OF MASS

As for a system of particles, position of centre of mass is $\vec{R}_{CM} = \frac{m_1\vec{r}_1 + m_2\vec{r}_2 + m_3\vec{r}_3 + ...}{m_1 + m_2 + m_3 + ...}$

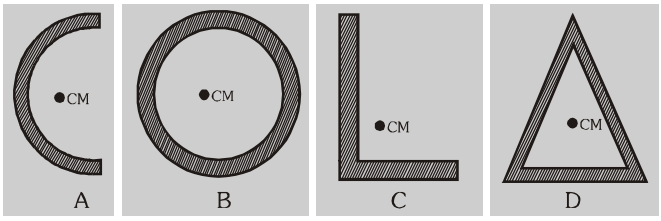
So $\frac{d}{dt}\left(\vec{R}_{CM}\right) = \frac{m_1\frac{d\vec{r}_1}{dt} + m_2\frac{d\vec{r}_2}{dt} + m_3\frac{d\vec{r}_3}{dt} + ...}{m_1 + m_2 + m_3 + ...} \Rightarrow$

Similarly acceleration $\vec{a}_{CM} = \frac{d}{dt}\left(\vec{v}_{CM}\right) = \frac{m_1\vec{a}_1 + m_2\vec{a}_2 + ...}{m_1 + m_2 + ...}$

We can write $M\vec{v}_{CM} = m_1\vec{v}_1 + m_2\vec{v}_2 + ... = \vec{p}_1 + \vec{p}_2 + \vec{p}_3 + ...$ [$\because \vec{p} = m\vec{v}$]

$M\vec{v}_{CM} = \vec{p}_{CM}$ [$\because \Sigma\vec{p}_i = \vec{p}_{CM}$]

IMPORTANT POINTS

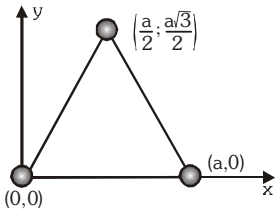


- There may or may not be any mass present physically at centre of mass (See Figure A, B, C)
- Centre of mass may be inside or outside of the body (See figure A, B, C)
- Position of centre of mass depends on the shape of the body. (See figure A, B, C)
- For a given shape it depends on the distribution of mass of within the body and is closer to massive part. (See figure A,C)
- For symmetrical bodies having homogeneous distribution of mass it coincides with centre of symmetry of geometrical centre. (See figure B,D).
- If we know the centre of mass of parts of the system and their masses, we can get the combined centre of mass by treating the parts as point particles placed at their respective centre of masses.
- It is independent of the co-ordinate system, e.g., the centre of mass of a ring is at its centre whatever be the co-ordinate system.
- If the origin of co-ordinate system is at centre of mass, i.e., $\vec{R}_{CM} = 0$, then by definition.

$\frac{1}{M}\Sigma m_i\vec{r}_i = 0 \Rightarrow \Sigma m_i\vec{r}_i = 0$

The sum of the moments of the masses of a system about its centre of mass is always zero.

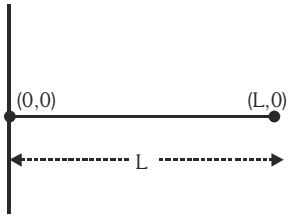
Ex. Three bodies of equal masses are placed at (0, 0), (a, 0) and at $\left(\frac{a}{2}, \frac{a\sqrt{3}}{2}\right)$.



Find out the co-ordinates of centre of mass.

Sol. $x_{CM} = \frac{0\times m + a\times m + \frac{a}{2}\times m}{m+m+m} = \frac{a}{2}$, $y_{CM} = \frac{0\times m + 0\times m + \frac{a\sqrt{3}}{2}\times m}{m+m+m} = \frac{a\sqrt{3}}{6}$

Ex. Calculate the position of the centre of mass of a system consisting of two particles of masses m_1 and m_2 separated by a distance L apart, from m_1.



Sol. Treating the line joining the two particles as x axis

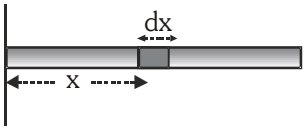
$$x_{CM} = \frac{m_1\times 0 + m_2\times L}{m_1+m_2} = \frac{m_2 L}{m_1+m_2}, \quad y_{CM} = 0 \qquad z_{CM} = 0$$

Ex. If the linear density of a rod of length L varies as $\lambda = A + Bx$, compute position of its centre of mass.

Sol. Let the x–axis be along the length of the rod and origin at one of its end as shown in figure. As rod is along x–axis, for all points on it y and z will be zero so, $y_{CM} = 0$ and $z_{CM} = 0$ i.e., centre of mass will be on the rod.

Now consider an element of rod of length dx at a distance x from the origin, mass of this element dm $= \lambda dx = (A + Bx)dx$

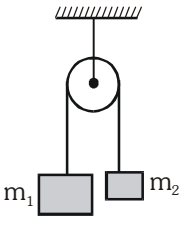
$$\text{so, } x_{CM} = \frac{\int_0^L x\,dm}{\int_0^L dm} = \frac{\int_0^L x(A+Bx)dx}{\int_0^L (A+Bx)dx} = \frac{\frac{AL^2}{2}+\frac{BL^3}{3}}{AL+\frac{BL^2}{2}} = \frac{L(3A+2BL)}{3(2A+BL)}$$



Note : (i) If the rod is of uniform density then $\lambda = A$ = constant & B = 0 then $X_{CM} = L/2$

(ii) If the density of rod varies linearly with x, then $\lambda = Bx$ and A = 0 then $X_{CM} = 2L/3$

Ex. Two bodies of masses m_1 and m_2 ($<m_1$) are connected to the ends of a massless cord and allowed to move as shown in. The pulley is both massless and frictionless. Calculate the acceleration of the centre of mass.



Sol. If $\vec{a}$ is the acceleration of m_1,then $-\vec{a}$ is the acceleration of m_2 then

acceleration of each body $a = \frac{\text{Net force}}{\text{Net mass}} = \left(\frac{m_1-m_2}{m_1+m_2}\right)g$

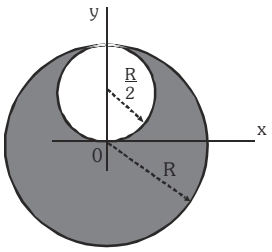
$$\vec{a}_{cm} = \frac{m_1\vec{a}+m_2(-\vec{a})}{m_1+m_2} = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{a}$$

But $\vec{a} = \left(\frac{m_1-m_2}{m_1+m_2}\right)\vec{g}$ so $\vec{a}_{cm} = \left(\frac{m_1-m_2}{m_1+m_2}\right)^2 \vec{g}$

Ex. A circle of radius R is cut from a uniform thin sheet of metal. A circular hole of radius $\frac{R}{2}$ is now cut out of the circle, with the hole tangent to the rim. Find the distance of centre of mass from the centre of the original uncut circle to the centre of mass.

Sol. We treat the hole as a 'negative mass' object that is combined with the original uncut circle. (When the two are added together, the hole region then has zero mass). By symmetry, the CM lies along the +y–axis in figure, so $x_{CM} = 0$. With the origin at the centre of the original circle whose mass is assumed to be m.

Mass of original uncut circle $m_1 = m$ & $(0, 0)$



Mass of hole of negative mass : $m_2 = \frac{m}{4}$; Location of CM $\left(0, \frac{R}{2}\right)$

Thus $y_{CM} = \frac{m_1y_1 + m_2y_2}{m_1 + m_2} = \frac{m(0) + \left(-\frac{m}{4}\right)\frac{R}{2}}{m + \left(-\frac{m}{4}\right)} = \frac{R}{6}$

So the centre of mass is at the point $\left(0, -\frac{R}{6}\right)$

Ex. Two particles of mass 1 kg and 0.5 kg are moving in the same direction with speeds of 2m/s and 6m/s, respectively, on a smooth horizontal surface. Find the speed of centre of mass of the system.

Sol. Velocity of centre of mass of the system $\vec{v}_{cm} = \frac{m_1\vec{v}_1 + m_2\vec{v}_2}{m_1 + m_2}$ Since the particles m_1 and m_2 are moving in same direction, $m_1\vec{v}_1$ and $m_2\vec{v}_2$ are parallel. $\Rightarrow$ $\left|m_1\vec{v}_1 + m_2\vec{v}_2\right| = m_1v_1 + m_2v_2$

Therefore, $v_{cm} = \frac{\left|m_1\vec{v}_1 + m_2\vec{v}_2\right|}{m_1 + m_2} = \frac{m_1v_1 + m_2v_2}{m_1 + m_2} = \frac{(1)(2) + \left(\frac{1}{2}\right)(6)}{\left(1 + \frac{1}{2}\right)} = 3.33 \text{ ms}^{-1}$

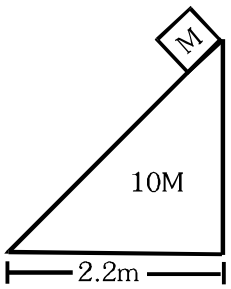
Ex. Two particles of masses 2 kg and 4 kg are approaching towards each other with acceleration 1 m/s^2 and 2 m/s^2, respectively, on a smooth horizontal surface. Find the acceleration of centre of mass of the system.

Sol. The acceleration of centre of mass of the system $\vec{a}_{cm} = \frac{m_1\vec{a}_1 + m_2\vec{a}_2}{m_1 + m_2} \Rightarrow a_{cm} = \frac{\left|m_1\vec{a}_1 + m_2\vec{a}_2\right|}{m_1 + m_2}$

Since $\vec{a}_1$ and $\vec{a}_2$ are anti–parallel, so $a_{cm} = \frac{\left|m_1a_1 - m_2a_2\right|}{m_1 + m_2} = \frac{\left|(2)(1) - (4)(2)\right|}{2 + 4} = 1 \text{ ms}^{-2}$

Since $m_2a_2 > m_1a_1$ so the direction of acceleration of centre of mass will be directed in the direction of a_2.

Ex. A block of mass M is placed on the top of a bigger block of mass 10 M as shown in figure. All the surfaces are frictionless. The system is released from rest. Find the distance moved by the bigger block at the instant the smaller block reaches the ground.



Sol. If the bigger block moves towards right by a distance (X), the smaller block will move towards left by a distance (2.2 –X) (taking the two blocks together as the system). The horizontal position of CM remains same $\Rightarrow$ M (2.2 – X) = 10 MX $\Rightarrow$ X = 0.2 m.

MOMENTUM

The total quantity of motion possessed by a moving body is known as the momentum of the body. It is the product of the mass and velocity of a body i.e. momentum $\vec{p} = m\vec{v}$

IMPULSE

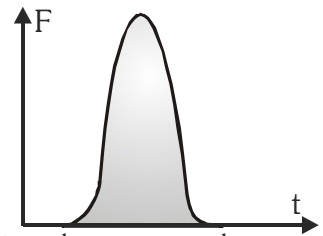
When a large force act for an extremely short duration, neither the magnitude of the force nor the time for which it acts is important. In such a case, the total effect of force is measured. The total effect of force is called impulse (measure of the action of force). This type of force is generally variable in magnitude and is sometimes called impulsive force.

If a large force act on a body or particle for a small time then

Impulse = product of force with time.

Suppose a force $\vec{F}$ acts for a short time dt then impulse = $\vec{F}dt$

For a finite interval of time from t_1 to t_2 then the impulse = $\int_{t_1}^{t_2} \vec{F}dt$

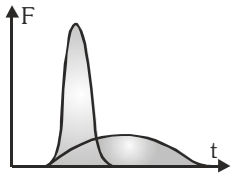


impulse = area under curve

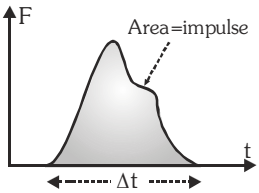
If constant force $\vec{F}$ acts for an interval Δt then Impulse = $\vec{F}\Delta t$

Impulse – Momentum theorem :

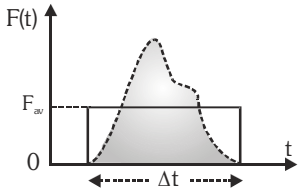
Impulse of a force is equal to the change of momentum $\boxed{\vec{F}\Delta t = \Delta\vec{p}}$



A large force acting for a short time and a small force acting for a long time may result in the same impulse, as indicated by the equal areas under the force-versus time curves.



The force F(t) involved in the collision between a ball and bat. The interaction occurs during the time Δt. The impulse is equal in magnitude to the shaded area under the curve.

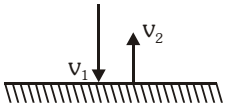


The constant force F_{av} acting for the same time interval Δt produces the same impulse as in figure since the areas under the two graphs are the same.

Ex. A ball of mass 50 g is dropped from a height h = 10 m. It rebounds losing 75 percent of its total mechanical energy. If it remains in contact with the ground for 0.01 s, find the impulse of the impact force.

Sol. Impulse = change in momentum = $m(v_1+v_2)$

Here $v_1 = \sqrt{2gh}$ and for v_2, $\frac{1}{2}mv_2^2 = \frac{1}{2}mv_1^2\left(1-\frac{75}{100}\right) \Rightarrow v_2 = \frac{v_1}{2}$



So impulse $= m\left(v_1+\frac{v_1}{2}\right) = \frac{3mv_1}{2} = \frac{3}{2}\,m \times \sqrt{2gh} = \frac{3}{2} \times 50 \times 10^{-3} \times \sqrt{2\times 9.8\times 10} = 1.05$ N–s

LAW OF CONSERVATION OF LINEAR MOMENTUM

According to Newton's Second law of motion the rate of change of momentum is equal to the applied force.

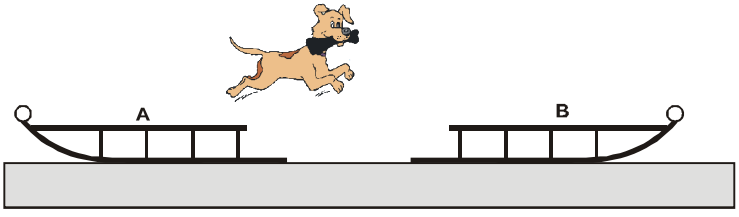
$$\vec{F} = \frac{d\vec{p}}{dt} \quad \text{if } \vec{F} = \vec{0} \text{ then } \frac{d\vec{p}}{dt} = \vec{0} \text{ i.e. } \vec{p} = \text{constant}$$

This leads to the law of conservation of momentum which is" In the absence of external forces, the total momentum of the system is conserved."

IMPORTANT POINTS

- For an isolated system, the initial momentum of the system is equal to the final momentum of the system. If the system consists of n bodies having momentum
  $\vec{p}_1, \vec{p}_2, \vec{p}_3, ...\vec{p}_n$, then $\vec{p}_1+\vec{p}_2+\vec{p}_3+.....+\vec{p}_n = \text{constant}$
- As linear momentum depends on frame of reference. Observers in different frames would find different values of linear momentum of a given system but each would agree that his own value of linear momentum does not change with time provided. But the system should be isolated and closed, i.e., law of conservation of linear momentum is independent of frame of reference though linear momentum depends on frame of reference.
- Conservation of linear momentum is equivalent to Newton's III law of motion for a system of two particles in absence of external force by law of conservation of linear momentum.
  $\Rightarrow \quad \vec{p}_1+\vec{p}_2 = \text{constant} \quad$ i.e. $m_1\vec{v}_1+m_2\vec{v}_2 = \text{constant}$
  Differentiating above with respect to time $m_1\frac{d\vec{v}_1}{dt}+m_2\frac{d\vec{v}_2}{dt}=\vec{0}$ [as m is constant]
  $\Rightarrow m_1\vec{a}_1+m_2\vec{a}_2=\vec{0}$ $\left[\because \frac{d\vec{v}_1}{dt}=\vec{a}\right] \Rightarrow \vec{F}_1+\vec{F}_2=\vec{0}$ $[\because \vec{F}=m\vec{a}]$ $\Rightarrow \vec{F}_1=-\vec{F}_2$
  i.e., for every action there is equal and opposite reaction which is Newton's III law of motion.
- This law is universal, i.e., it applies to body macroscopic as well as microscopic systems.

Ex. Two 22.7 kg ice sleds A and B are placed a short distance apart, one directly behind the other, as shown in figure. A 3.63 kg dog, standing on one sled, jumps across to the other and immediately back to the first. Both jumps are made at a speed of 3.05 ms^{-1} relative to the ice. Find the final speeds of the two sleds.



Sol. Total momentum imparted to B $p_B = 2 \times 3.63 \times 3.05$ kg ms^{-1}.

Velocity of $B = \frac{p_B}{m_B} = \frac{2 \times 3.63 \times 3.05}{22.7} = 0.975$ ms^{-1}.

Velocity of A when the dog jumps away from $A = \frac{p_A}{m_A} = \frac{3.63 \times 3.05}{22.7} = 0.4877$ ms^{-1}.

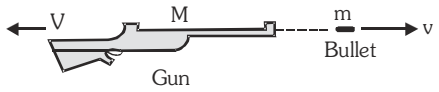
When the dog comes back to A, Velocity of $A = \frac{22.7 \times 0.4877 + 3.63 \times 3.05}{22.7 + 3.63} = 0.841$ ms^{-1}.

APPLICATIONS OF CONSERVATION OF LINEAR MOMENTUM

Firing a Bullet from a Gun :

- If the bullet is the system, the force exerted by trigger will be external and so the linear momentum of the bullet will change from 0 to mv. This is not the violation of law of conservation of linear momentum as linear momentum is conserved only in absence of external force .
- If the bullet and gun is the system, the force exerted by trigger will be internal so.

Total momentum of the system $\vec{p}_s = \vec{p}_B + \vec{p}_G$ = constant.



Now as initially both bullet and gun are at rest so $\vec{p}_B + \vec{p}_G = \vec{0}$ From this it is evident that :

- $\vec{p}_G = -\vec{p}_B$, i.e., if bullet acquires forward momentum, the gun will acquire equal and opposite (backward) momentum.
- As $\vec{p} = m\vec{v}$, $m\vec{v} + M\vec{V} = \vec{0}$, i.e, $\vec{V} = -\frac{m}{M}\vec{v}$ i.e, if the bullet moves forward, gun 'recoils' or 'kicks' backward. Heavier the gun lesser will be the recoil velocity V.
- Kinetic energy $K = \frac{p^2}{2m}$ and $|\vec{p}_B| = |\vec{p}_G| = p$ Kinetic energy of gun $K_G = \frac{p^2}{2M}$,

Kinetic energy of bullet $K_B = \frac{p^2}{2m}$ $\therefore$ $\frac{K_G}{K_B} = \frac{m}{M} < 1$ ($\because$ M $\gg$ m) Thus kinetic energy of gun is smaller than bullet i.e., kinetic energy of bullet and gun will not be equal.

- Initial kinetic energy of the system is zero as both are at rest initially.
  Final kinetic energy of the system $[(1/2)(mv^2 + MV^2)] > 0$.
  So, here kinetic energy of the system is not constant but increases. If PE is assumed to be constant then Mechanical energy = (kinetic energy + potential energy) will also increase. However, energy is always conserved. Here chemical energy of gun powder is converted into KE.

Ex. A bullet of mass 100g is fired by a gun of 10kg with a speed 2000 m/s. Find recoil velocity of gun.

Sol. According to conservation of momentum mv + MV = 0.

Velocity of gun $V = -\frac{mv}{M} = -\frac{0.1\times 2000}{10} = -20 m/s$

Block Bullet System :

(a) When bullet remains in the block

Conserving momentum of bullet and block mv + 0 = (M+m) V

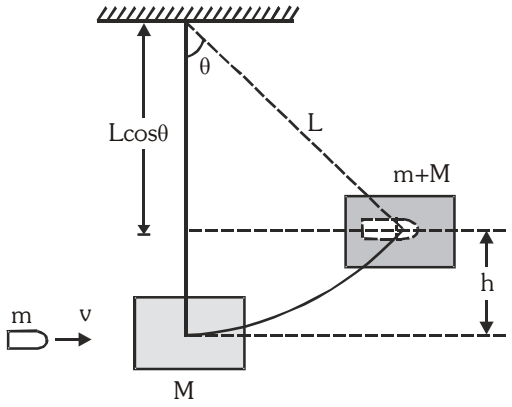
Velocity of block $V = \frac{mv}{M+m}$ ...(i)

By conservation of mechanical energy

$\frac{1}{2}(M+m)V^2 = (M+m)gh \Rightarrow V = \sqrt{2gh}$ ....(ii)

From eqⁿ. (i) and eqⁿ. (ii) $\frac{mv}{M+m} = \sqrt{2gh}$;

Speed of bullet $v = \frac{(M+m)\sqrt{2gh}}{m}$,



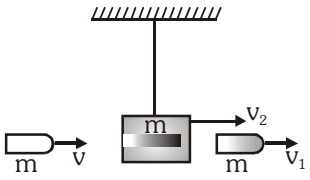
Maximum height gained by block $h = \frac{V^2}{2g} = \frac{m^2v^2}{2g(M+m)^2}$

$h = L - L\cos\theta \quad \therefore \cos\theta = 1-\frac{h}{L} \Rightarrow \theta = \cos^{-1}\left(1-\frac{h}{L}\right)$

(b) If bullet moves out of the block

Conserving momentum $mv + 0 = mv_1 + Mv_2$

$m(v - v_1) = Mv_2$ .....(i)



Conserving energy $\frac{1}{2}Mv_2^2 = Mgh \Rightarrow v_2 = \sqrt{2gh}$ .....(ii)

From eqⁿ. (i) & eqⁿ. (ii) $m(v - v_1) = M\sqrt{2gh} \Rightarrow h = \frac{m^2(v-v_1)^2}{2gM^2}$

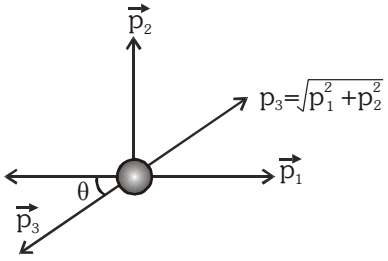
Explosion of a Bomb at rest

Conserving momentum

$\vec{p}_1 + \vec{p}_2 + \vec{p}_3 = \vec{0} \Rightarrow \vec{p}_3 = -(\vec{p}_1 + \vec{p}_2) \Rightarrow p_3 = \sqrt{p_1^2 + p_2^2}$ as $\vec{p}_1 \perp \vec{p}_2$

Angle made by $\vec{p}_3$ from $\vec{p}_1 = \pi + \theta$

Angle made by $\vec{p}_3$ from $\vec{p}_2 = \frac{\pi}{2} + \theta$

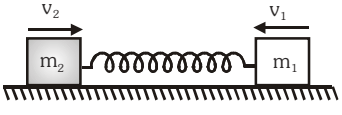


Energy released in explosion = $K_f - K_i = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3} - 0 = \frac{p_1^2}{2m_1} + \frac{p_2^2}{2m_2} + \frac{p_3^2}{2m_3}$

Motion of Two Masses Connected to a Spring

Consider two blocks, resting on a frictionless surface and connected by a massless spring as shown in figure. If the spring is stretched (or compressed) and then released from rest,

Then $F_{ext}=0$ so $\vec{p}_s = \vec{p}_1 + \vec{p}_2$ = constant



However, initially both the blocks were at rest so, $\vec{p}_1 + \vec{p}_2 = \vec{0}$

It is clear that :

- $\vec{p}_2 = -\vec{p}_1$, i.e., at any instant the two blocks will have momentum equal in magnitude but opposite in direction (Though they have different values of momentum at different positions).
- As momentum $\vec{p} = m\vec{v}$, $m_1\vec{v}_1 + m_2\vec{v}_2 = \vec{0} \Rightarrow \vec{v}_2 = -\left(\frac{m_1}{m_2}\right)\vec{v}_1$

  The two blocks always move in opposite directions with lighter block moving faster.
- Kinetic energy $KE = \frac{p^2}{2m}$ and $|\vec{p}_1| = |\vec{p}_2|$, $\frac{KE_1}{KE_2} = \frac{m_2}{m_1}$ or the kinetic energy of two blocks will not be equal but in the inverse ratio of their masses and so lighter block will have greater kinetic energy.
- Initially kinetic energy of the blocks is zero (as both are at rest) but after some time kinetic energy of the blocks is not zero (as both are in motion). So, kinetic energy is not constant but changes. Here during motion of blocks KE is converted into elastic potential energy of the spring and vice–versa but total mechanical energy of the system remain constant.

  Kinetic energy + Potential energy = Mechanical Energy = Constant

Note – If $\vec{F}$ is the average of the time varying force during collision and Δt is the duration of collision then impulse $\vec{I} = \vec{F}\Delta t$.

Conservation of Linear Momentum During Impact :

If two bodies of masses m_1 and m_2 collide in air, the total external force acting on the system of bodies (m_1 + m_2) is equal to $\vec{F}_1 + m_1\vec{g} + \vec{F}_2 + m_2\vec{g} \Rightarrow F_{total} = m_1\vec{g} + m_2\vec{g} + \vec{F}_1 + \vec{F}_2$

During collision the impact forces $\vec{F}_1$ and $\vec{F}_2$ are equal in magnitude and opposite in direction. According to Newton's 3rd law of motion, $\vec{F}_1 + \vec{F}_2 = \vec{0} \Rightarrow \vec{F}_{net} = m_1\vec{g} + m_2\vec{g}$

So Impulse = $\vec{F}_{net}\Delta t = \left(m_1\vec{g} + m_2\vec{g}\right)\Delta t$

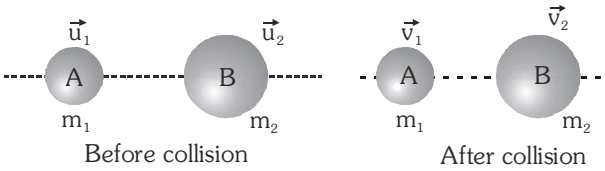
Since Δt is a very small time interval, the impulse $F(\Delta t)$ will be negligibly small. As impulse is equal to change in momentum of the system, a negligible impulse means negligible change of momentum. Let the change of momentum of 1 & 2 be $\Delta\vec{p}_1$ & $\Delta\vec{p}_2$, respectively then the total change in momentum of the system $\Rightarrow \Delta\vec{p} = \Delta\vec{p}_1 + \Delta p_2 = \vec{F}_{net}.dt \approx \vec{0} \Rightarrow \Delta\left(\vec{p}_1 + \vec{p}_2\right) = 0 \Rightarrow \vec{p}_1 + \vec{p}_2 =$ constant.

Therefore, the net or total momentum of the colliding bodies remains practically unchanged along the line of action (impact) during the collision. In other words, the momentum of the system remains constant or conserved during the period of impact. Therefore, we can conveniently equate the net momentum of the colliding bodies at the beginning and at the end of the collision (or just before and just after the impact).

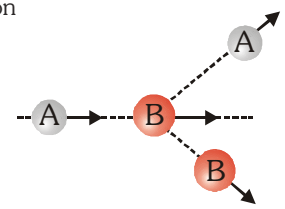
Note : Remember that the impact force F is not an external force for the system of colliding bodies. If no external force acts on the system, its momentum remains constant for all the times including the time of collision. Even if some external forces like gravitation and friction (known as non–impulsive forces in general) are present, we can conserved the momentum of the system during the impact, because the finite external forces cannot change the momentum of the system significantly in very short time. Therefore, the change in position of the system during infinitesimal time of impact can also be neglected.

- Types of collision according to the direction of collision :

(a) **Head on collision** : Direction of velocities of bodies is similar to the direction of collision.



(b) **Oblique collision** : Direction of velocities of bodies is not similar to the direction of collision.



- Types of collision according to the conservation law of kinetic energy :

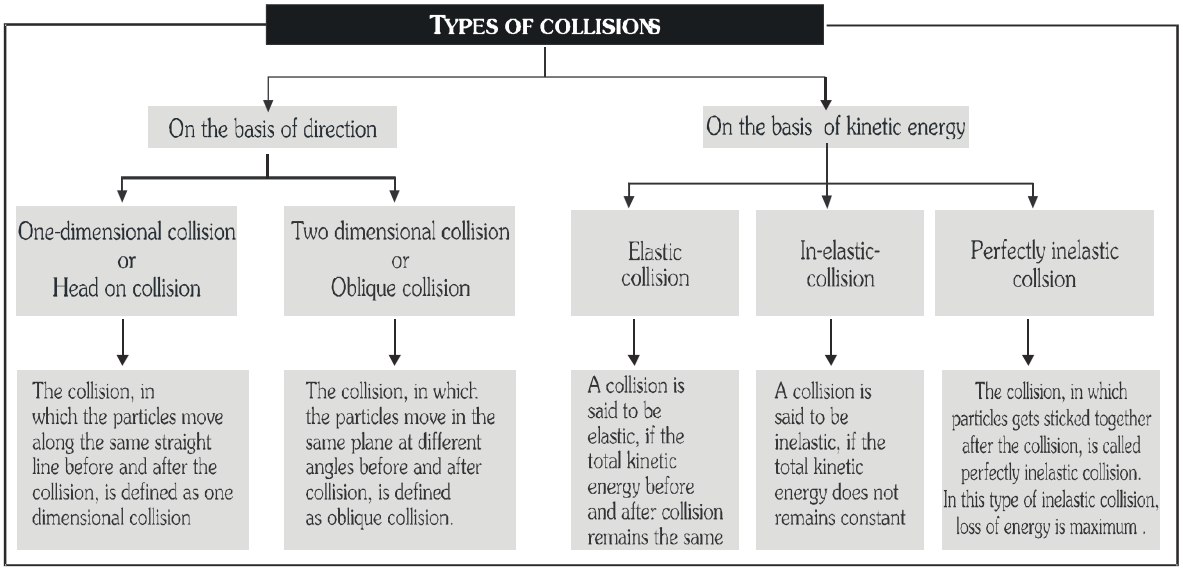
(a) **Elastic collision** : $KE_{\text{before collision}} = KE_{\text{after collision}}$

(b) **Inelastic collision** : kinetic energy is not conserved.

Some energy is lost in collision $KE_{\text{before collision}} > KE_{\text{after collision}}$

(c) **Perfect inelastic collision :** Two bodies stick together after the collision.

momentum remains conserved in all types of collisions.



Coefficient of restitution (e)

The coefficient of restitution is defined as the ratio of the impulses of recovery and deformation of either body.

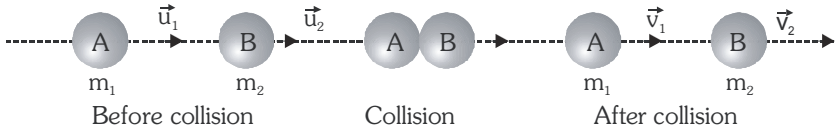
$$e = -\frac{\text{impulse of recovery}}{\text{impulse of deformation}}$$

$$e = \frac{\text{velocity of separation along line of impact}}{\text{velocity of approach along line of impact}}$$

Value of e is 1 for elastic collision, 0 for perfectly inelastic collision and $0 < e < 1$ for inelastic collision.

HEAD ON ELASTIC COLLISION

The elastic collision in which the colliding bodies move along the same straight line path before and after the collision.



Assuming initial direction of motion to be positive and $u_1 > u_2$ (so that collision may take place) and applying law of conservation of linear momentum

$$m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2 \Rightarrow m_1(u_1 - v_1) = m_2(v_2 - u_2) \quad ...(i)$$

For elastic collision, kinetic energy before collision must be equal to kinetic energy after collision,

i.e., $$\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2 = \frac{1}{2}m_1v_1^2 + \frac{1}{2}m_2v_2^2 \Rightarrow m_1(u_1^2 - v_1^2) = m_2(v_2^2 - u_2^2) \quad ... (ii)$$

Dividing equation (ii) by (i) $u_1 + v_1 = v_2 + u_2 \Rightarrow (u_1 - u_2) = (v_2 - v_1)$ ...(iii)

In 1–D elastic collision 'velocity of approach' before collision is equal to the 'velocity of recession' after collision, no matter what the masses of the colliding particles be.

This law is called **Newton's law for elastic collision**

Now if we multiply equation (iii) by m_2 and subtracting it from (i)

$$(m_1 - m_2)\,u_1 + 2m_2u_2 = (m_1 + m_2)v_1 \Rightarrow v_1 = \frac{m_1 - m_2}{m_1 + m_2}u_1 + \frac{2m_2}{m_1 + m_2}u_2 \quad ... (iv)$$

Similarly multiplying equation (iii) by m_1 and adding it to equation (i)

$$2m_1u_1 + (m_2 - m_1)u_2 = (m_2 + m_1)v_2 \Rightarrow v_2 = \frac{2m_1}{m_1 + m_2}u_1 + \frac{m_2 - m_1}{m_1 + m_2}u_2 \quad ...(v)$$

IMPORTANT POINTS

- **If the two bodies are of equal masses :** $m_1 = m_2 = m$, $v_1 = u_2$ and $v_2 = u_1$

  Thus, if two bodies of equal masses undergo elastic collision in one dimension, then after the collision, the bodies will exchange their velocities.

- **If two bodies are of equal masses and second body is at rest.**

  $m_1 = m_2$ and initial velocity of second body $u_2 = 0$, $v_1 = 0$, $v_2 = u_1$

  When body A collides against body B of equal mass at rest, the body A comes to rest and the body B moves on with the velocity of the body A. In this case transfer of energy is hundred percent

  e.g.. Billiard's Ball, Nuclear moderation.

- **If the mass of a body is negligible as compared to other.**

  If $m_1 >> m_2$ and $u_2 = 0$ then $v_1 = u_1$, $v_2 = 2u_1$

  When a heavy body A collides against a light body B at rest, the body A should keep on moving with same velocity and the body B will move with velocity double that of A.

  If $m_2 >> m_1$ and $u_2 = 0$ then $v_2 = 0$, $v_1 = -u_1$

  When light body A collides against a heavy body B at rest, the body A should start moving with same velocity just in opposite direction while the body B should practically remains at rest.

Ex. Two ball of mass 5kg each is moving in opposite directions with equal speed 5m/s. collides head on with each other. Find out the final velocities of the balls if collision is elastic.

Sol. Here $m_1 = m_2 = 5kg$, $u_1 = 5$ m/s, $u_2 = -5$ m/s

In such type of condition velocity get interchange so $v_2 = u_1 = 5$ m/s & $v_1 = u_2 = -5$ m/s

Ex. A ball of 0.1 kg makes an elastic head on collision with a ball of unknown mass that is initially at rest. If the 0.1kg ball rebounds at one third of its original speed. What is the mass of other ball ?

Sol. Here $m_1 = 0.1$ kg, $m_2 = ?$, $u_2 = 0$, $u_1 = u$, $v_1 = -u/3$

As $$v_1 = \left(\frac{m_1 - m_2}{m_1 + m_2}\right)u + \frac{2m_2u_2}{m_1 + m_2} \Rightarrow -\frac{u}{3} = \left(\frac{0.1 - m_2}{0.1 + m_2}\right)u \Rightarrow m_2 = 0.2 \text{ kg}$$

HEAD ON INELASTIC COLLISION OF TWO PARTICLES

Let the coefficient of restitution for collision is e

(i) Momentum is conserved $m_1u_1 + m_2u_2 = m_1v_1 + m_2v_2$ ...(i)

(ii) Kinetic energy is not conserved.

(iii) According to Newton's law $$\frac{v_2 - v_1}{u_2 - u_1} = -e \quad ...(ii)$$

By solving eq. (i) and (ii)

$$v_1 = \left(\frac{m_1 + em_2}{m_1 + m_2}\right)u_1 + \left(\frac{(1+e)m_2}{m_1 + m_2}\right)u_2 \ , \ v_2 = \left(\frac{m_2 + em_1}{m_1 + m_2}\right)u_2 + \left(\frac{(1+e)m_1}{m_1 + m_2}\right)u_1$$

PERFECT INELASTIC COLLISION

In case of inelastic collision, after collision two bodies move with same velocity (or stick together).

If two particles of masses m_1 and m_2, moving with velocity u_1 and u_2 ($u_2 < u_1$) respectively along the same line collide 'head on ' and after collision they have same common velocity v, then by conservation of linear momentum,

$$m_1u_1 + m_2u_2 = m_1v + m_2v \quad \Rightarrow \quad v = \frac{m_1u_1 + m_2u_2}{(m_1 + m_2)} \quad ...(i)$$

Kinetic energy of the system before collision is $KE_I = \frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2$

And after collision is $KE_f = \frac{1}{2}(m_1+m_2)v^2$

Loss in KE during collision

$$\Delta KE = KE_i - KE_f = \left[\frac{1}{2}m_1u_1^2 + \frac{1}{2}m_2u_2^2\right] - \frac{1}{2}(m_1 + m_2)v^2 \quad ...(ii)$$

Substituting the value of v from eq. (i),

$$\Delta KE = \frac{1}{2}\left[(m_1u_1^2 + m_2u_2^2) - \frac{(m_1u_1 + m_2u_2)^2}{(m_1 + m_2)}\right]$$

$$\Rightarrow \Delta KE = \frac{1}{2}\left[\frac{m_1m_2(u_1^2 + u_2^2 - 2u_1u_2)}{(m_1 + m_2)}\right] \Rightarrow \boxed{\Delta KE = \frac{1}{2}\frac{m_1m_2}{(m_1 + m_2)}(u_1 - u_2)^2}$$

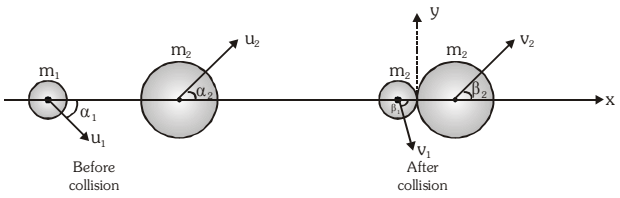
If the target is initially at rest $u_2 = 0$ and $u_1 = u$

$$\Delta KE = \frac{m_1m_2}{2(m_1 + m_2)}u^2, \quad \frac{\Delta KE}{KE_i} = \frac{m_2}{(m_1 + m_2)} \ [\because KE_i = \frac{1}{2}m_1u_1^2]$$

Now if target is massive, i.e., $m_2 >> m_1$ then $\frac{\Delta KE}{KE_i} \approx 1$ so percentage loss in KE = 100%

i.e., if a light moving body strikes a heavy target at rest and sticks to it, practically all its KE is lost.

Oblique Collision



In oblique impact the relative velocity of approach of the bodies doesn't coincide with the line of impact. Conserving the momentum of system in directions along normal (x axis in our case) and tangential (y axis in our case)

$m_1u_1\cos\alpha_1 + m_2u_2\cos\alpha_2 = m_1v_1\cos\beta_1 + m_2v_2\cos\beta_2$ and $m_2u_2\sin\alpha_2 - m_1u_1\sin\alpha_1 = m_2v_2\sin\beta_2 - m_1v_1\sin\beta_1$

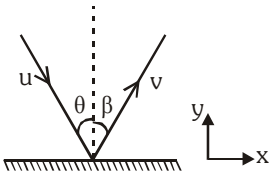
Since no force is acting on m_1 and m_2 along the tangent (i.e. y–axis) the individual momentum of m_1 and m_2 remains conserved. $m_1u_1\sin\alpha_1 = m_1v_1\sin\beta_1$ & $m_2u_2\sin\alpha_2 = m_2v_2\sin\beta_2$

By using Newton's experimental law along the line of impact $e = \dfrac{v_2\cos\beta_2 - v_1\cos\beta_1}{u_1\cos\alpha_1 - u_2\cos\alpha_2}$

Oblique Impact on a Fixed Plane

Let a small ball collides with a smooth horizontal floor with a speed u at an angle θ to the vertical as shown in the figure. Just after the collision, let the ball leaves the floor with a speed v at angle β to vertical.

It is quite clear that the line of action is perpendicular to the floor. Therefore, the impact takes place along the (normal) vertical. Now we can use Newton's experimental law as



$$e = \frac{\text{velocity of separation}}{\text{velocity of approach}}$$

$\Rightarrow$ e [velocity of approach] = velocity of separation

$\Rightarrow$ $e\,[u\cos\theta\,(-\hat{j})] = -[v\cos\beta\,(+\hat{j})] \Rightarrow v\cos\beta = e\,u\cos\theta$ ...(i)

Since impulsive force N acts on the body along the normal, we cannot conserve its momentum. Since along horizontal the component of N is zero, therefore we can conserve the horizontal momentum of the body.

Momentum $(p_x)_{body}$ = constant $\Rightarrow$ $(p_x)_{initial} = (p_x)_{final}$

$\Rightarrow$ $mu\sin\theta = mv\sin\beta \Rightarrow v\sin\beta = u\sin\theta$ ..(ii)

Squaring equations(i) and (ii) and adding, $v^2\cos^2\beta + v^2\sin^2\beta = e^2u^2\cos^2\theta + u^2\sin^2\theta$

$\Rightarrow$ $v^2 = u^2[e^2\cos^2\theta + \sin^2\theta] \Rightarrow v = u\sqrt{\sin^2\theta + e^2\cos^2\theta}$

Dividing equation (i) by (ii)

$$\Rightarrow \frac{v\cos\beta}{v\sin\beta} = \frac{eu\cos\theta}{u\sin\theta} \Rightarrow \cot\beta = e\cot\theta \Rightarrow \beta = \cot^{-1}(e\cot\theta)$$

Impulse of the blow = change of momentum of the body

$= \{(mv\sin\beta)\,\hat{i} + (mv\cos\beta)\,\hat{j}\} - \{(mu\sin\theta)\,\hat{i} - (mu\cos\theta)\,\hat{j}\}$

$= (mv\sin\beta - mu\sin\theta)\,\hat{i} + (mv\cos\beta + mu\cos\theta)\,\hat{j}$

Since $v\sin\beta = u\sin\theta \Rightarrow$ Impulse $= m(v\cos\beta + u\cos\theta)\,\hat{j}$

Putting $v\cos\beta = eu\cos\theta$ from eq. (i),

Impulse $= m(1+e)u\cos\theta\hat{j}$ $\therefore$ Magnitude of the impulse $= m(1+e)u\cos\theta$

Change in Kinetic energy: $\Delta K.E. = \frac{1}{2} mv^2 - \frac{1}{2} mu^2$

Putting the value of v we obtain

$$\Delta KE = \frac{1}{2} m \left[\left[\sqrt{u\left(\sin^2\theta + e^2\cos^2\theta\right)} \right]^2 - u^2 \right] = \frac{1}{2} mu^2 [\sin^2\theta + e^2\cos^2\theta - 1]$$

$$= -\frac{1}{2} mu^2 [\cos^2\theta - e^2\cos^2\theta] = -\frac{1}{2}(1 - e^2) mu^2 \cos^2\theta$$

Negative sign indicates the loss of kinetic energy

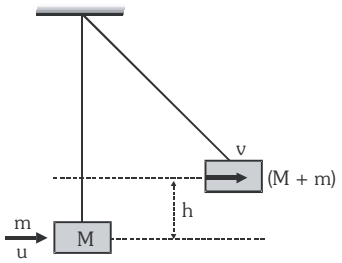
IMPORTANT POINTS

- Momentum remains conserved in all types of collisions.
- Total energy remains conserved in all types of collisions.
- Only conservative forces works in elastic collisions.
- In inelastic collisions all the forces are not conservative.

Ex. A simple pendulum of length 1m has a wooden bob of mass 1kg. It is struck by a bullet of mass 10^{-2}kg moving with a speed of 2×10^2 m/s. The bullet gets embedded into the bob. Obtain the height to which the bob rises before swinging back.

Sol. Applying principle of conservation of linear momentum

$$mu = (M + m)v \Rightarrow 10^{-2} \times (2 \times 10^2) = (1 + .01)v \Rightarrow v = \frac{2}{1.01}$$



KE_i of the block with bullet in it, is converted into P.E. as it rises through a height h

$$\frac{1}{2}(M + m)v^2 = (M + m)gh \Rightarrow v^2 = 2gh \Rightarrow h = \frac{v^2}{2g} = \left(\frac{2}{1.01}\right)^2 \times \frac{1}{2 \times 9.8} = 0.2 \text{ m}$$

Ex. A body falling on the ground from a height of 10m, rebounds to a height 2.5m calculate
(i) The percentage loss in K.E.
(ii) Ratio of the velocities of the body just before and just after the collision.

Sol. Let v_1 and v_2 be the velocity of the body just before and just after the collision

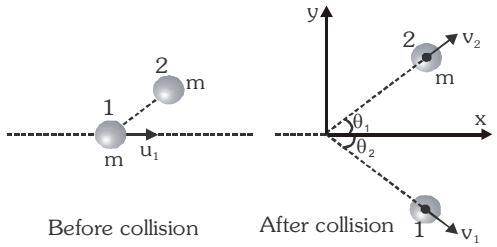
$$KE_1 = \frac{1}{2} mv_1^2 = mgh_1 ...(i) \qquad \text{and } KE_2 = \frac{1}{2} mv_2^2 = mgh_2 ...(ii)$$

$$\Rightarrow \quad \frac{v_1^2}{v_2^2} = \frac{h_1}{h_2} = \frac{10}{2.5} = 4 \Rightarrow \frac{v_1}{v_2} = 2$$

$$\text{Percentage loss in KE} = \frac{mg(h_1 - h_2)}{mgh_1} \times 100 = \frac{10 - 2.5}{10} \times 100 = 75\%$$

Ex. A body strikes obliquely with another identical stationary rest body elastically. Prove that they will move perpendicular to each other after collision.

Sol.



Conservation of linear momentum in x–direction gives

$mu_1 = mv_1\cos\theta_1 + mv_2\cos\theta_2 \Rightarrow u_1 = v_1\cos\theta_1 + v_2\cos\theta_2$ ... (i)

Conservation of linear momentum in y–direction gives

$0 = mv_1\sin\theta_1 - mv_2\sin\theta_2 \Rightarrow 0 = v_1\sin\theta_1 - v_2\sin\theta_2$ ...(ii)

Conservation of kinetic energy

$$\frac{1}{2}mu_1^2 = \frac{1}{2}mv_1^2 + \frac{1}{2}mv_2^2 \qquad \Rightarrow \quad u_1^2 = v_1^2 + v_2^2 \quad ...(iii)$$

$(i)^2 + (ii)^2$

$\Rightarrow \quad u_1^2 + 0 = v_1^2\cos^2\theta_1 + v_2^2\cos^2\theta_2 + 2v_1v_2\cos\theta_1\cos\theta_2 + v_1^2\sin^2\theta_1 + v_2^2\sin^2\theta_2 - 2v_1v_2\sin\theta_1\sin\theta_2$

$\Rightarrow \quad u_1^2 = v_1^2(\cos^2\theta_1 + \sin^2\theta_1) + v_2^2(\cos^2\theta_2 + \sin^2\theta_2) + 2v_1v_2(\cos\theta_1\cos\theta_2 - \sin\theta_1\sin\theta_2)$

$\Rightarrow \quad u_1^2 = v_1^2 + v_2^2 + 2v_1v_2\cos(\theta_1 + \theta_2)$ { $\because u_1^2 = v_1^2 + v_2^2$ }

$\Rightarrow \quad \cos(\theta_1 + \theta_2) = 0 \Rightarrow \theta_1 + \theta_2 = 90^\circ$

Ex. A steel ball is dropped on a smooth horizontal plane from certain height h. Assuming coefficient of restitution of impact as e, find the average speed of the ball till it stops.

Sol. Since the ball falls through a height h, just before the first impact its speed v will be given as $v = \sqrt{2gh}$. Let its speed the v_1 just after the first impact.Then, Newton's experimental formula yields, $\frac{0 - v_1}{v} = e \Rightarrow v_1 = ev$

Similarly, its speed just before 2nd impact, $v_1 = ev = e\sqrt{2gh}$

Speed just after n^{th} impact, $v_n = e^n v = e^n\sqrt{2gh}$

The maximum height attained after 1^{st} impact = $h_1 = \frac{v^2}{2g} = (e\sqrt{2gh})^2 = e^2h$. Similarly, the maximum height attained after 2nd impact, $h_2 = e^4h$. Hence, the maximum height attained after n^{th} impact $= e^{2n}h$

The ball experiences infinite impacts till it becomes stationary. $\Rightarrow$ The total distance covered, $d = h + 2h_1 + 2h_2 + ...d = h + 2e^2h + 2e^4h + ... = h[1+2(e^2 + e^4 + e^6 + ...)]$

$$= \left[1 + 2\left(\frac{e^2}{1-e^2}\right)\right]h = \left(\frac{1+e^2}{1-e^2}\right)h.$$

The total time taken by the ball till it stops bouncing $T = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2h_1}{g}} + 2\sqrt{\frac{2h_2}{g}} + ...$

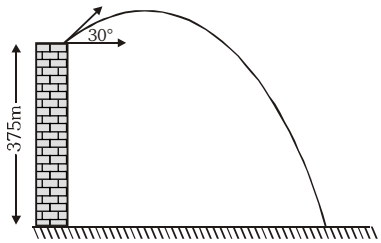
Putting $h_1 = e^2h$, $h_2 = e^4h$, $T = \sqrt{\frac{2h}{g}} + 2\sqrt{\frac{2e^2h}{g}} + 2\sqrt{\frac{2e^4h}{g}} + ...$

$\Rightarrow T = \sqrt{\frac{2h}{g}}\ [1 + 2(e + e^2 + ...)] = \sqrt{\frac{2h}{g}}\left[1 + \frac{2e}{1-e}\right] = \frac{1+e}{1-e}\sqrt{\frac{2h}{g}}$

Therefore, average speed of the ball for its total time of motion, $\vec{v} = \frac{\text{total distance}}{\text{total time}} = \frac{d}{T}$

Putting the values of d and T, we obtain $\vec{v} = \frac{1+e^2}{(1+e)^2}\sqrt{\frac{gh}{2}}$

Ex. A particle of mass 1 kg is projected from a tower of height 375m with initial velocity 100 ms^{-1} at an angle 30° with the horizontal. Find out its kinetic energy in joule just after collision with ground if collision is inelastic with $e = \frac{1}{2}$ ($g = 10\ ms^{-2}$)



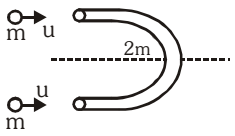
Sol. $v_y^2 = u_y^2 + 2gh \Rightarrow v_y = \sqrt{(50)^2 + 2\times10\times375} = 100 ms^{-1}$

Horizontal velocity just after collision = $50\sqrt{3}$ ms^{-1}

Vertical velocity just after collision = $100 \times \frac{1}{2} = 50\ ms^{-1}$

Kinetic energy just after collision = $\frac{1}{2}\times1\times\left[\left(50\sqrt{3}\right)^2 + (50)^2\right] = 5000J$

Ex A U shaped tube of mass 2m is placed on a horizontal surface. Two spheres each of diameter d (just less than the inner diameter of tube) and mass m enter into the tube with a velocity u as shown in figure. Taking all collisions to be elastic and all surfaces smooth. Match the following-



Column-I	**Column-II**
(A) The speed of the tube with respect to ground, when spheres are just about to collide inside the tube.	(p) u
(B) The speed of spheres when spheres are just about to collide.	(q) u/2
(C) The speed of the spheres when they comes out the tube.	(r) $\frac{\sqrt{3}}{2}u$
(D) The speed of the tube when spheres comes out the	(s) zero

Sol. **For (A)** From conservation of linear momentum $2mu = (m+m)v \Rightarrow v = \frac{u}{2}$

For (B) Let v_1 be the velocity of spheres w.r.t. tube when they are just about to collide then by using conservation of kinetic energy $\frac{1}{2}(2m)u^2 = \frac{1}{2}(4m)\left(\frac{u}{2}\right)^2 + 2\frac{1}{2}mv_1^2 \Rightarrow v_1 = \frac{u}{\sqrt{2}}$

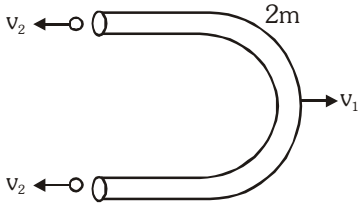
$\Rightarrow$ Required speed of spheres $= \sqrt{\left(\frac{u}{2}\right)^2 + \left(\frac{u}{\sqrt{2}}\right)^2} = \sqrt{\frac{u^2}{4} + \frac{u^2}{2}} = \frac{\sqrt{3}u}{2}$

For (C) $2mu = 2mv_1 - 2mv_2$

$2 \times \frac{1}{2}mu^2 = 2 \times \frac{1}{2}mv_2^2 + \frac{1}{2}(2m)v_1^2$

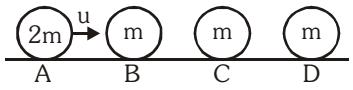
$\Rightarrow u^2 = v_1^2 + v_2^2 - 2v_1v_2$ & $u^2 = v_1^2 + v_2^2$

$\Rightarrow v_1v_2 = 0$ but $v_1 \neq 0$ so $v_2 = 0$



For (D) Speed of tube $v_1 = u$

Ex. Four balls A,B,C and D are kept on a smooth horizontal surface as shown in figure. Ball A is given velocity u towards B-



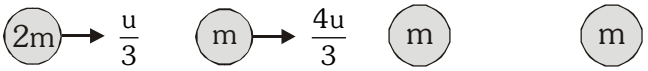
	Column-I		Column-II
(A)	Total impulse of all collisions on A	(p)	$\frac{4mu}{9}$
(B)	Total impulse of all collisions on B	(q)	$\frac{4mu}{27}$
(C)	Total impulse of all collision on C	(r)	$\frac{4mu}{3}$
(D)	Total impulse of all collisions on D	(s)	$\frac{52}{27}mu$

Sol. In 1st collision between A & B

$2mu = 2mv_A + 2mv_B$ & $e = 1 = \frac{v_B - v_A}{u} \Rightarrow v_A = \frac{u}{3}, v_B = \frac{4u}{3}$

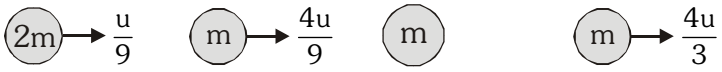
Situation of all collisions is shown in figure.

Initial position 

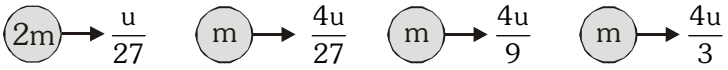
1st collision 

2nd collision 

3rd Collision 

4th Collision 

5th collision 

6th collision 

For (A) Total impulse on $A = 2m\left(u - \frac{u}{27}\right) = \frac{52}{27}mu$

For (B) Total impulse on $B = m\left(\frac{4u}{27} - 0\right) = \frac{4}{27}mu$

For (C) Total impulse on $C = m\left(\frac{4u}{9} - 0\right) = \frac{4}{9}mu$

For (D) Total impulse on $D = m\left(\frac{4u}{3} - 0\right) = \frac{4}{3}mu$

EXERCISE (S)

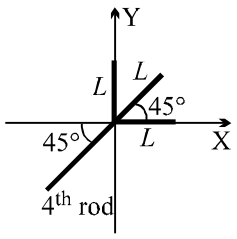
1. Four particles of mass 5, 3, 2, 4 kg are at the points (1, 6), (–1, 5), (2, –3), (–1, –4). Find the coordinates of their centre of mass.

CM0001

2. A rigid body consists of a 3 kg mass connected to a 2 kg mass by a massless rod. The 3kg mass is located at $\vec{r}_1 = (2\hat{i} + 5\hat{j})$ m and the 2 kg mass at $\vec{r}_2 = (4\hat{i} + 2\hat{j})$ m. Find the length of rod and the coordinates of the centre of mass.

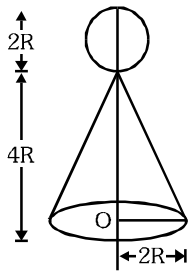
CM0002

3. Three identical uniform rods of the same mass *M* and length *L* are arranged in xy plane as shown in the figure. A fourth uniform rod of mass *3M* has been placed as shown in the xy plane. What should be the value of the length of the fourth rod such that the center of mass of all the four rods lie at the origin?



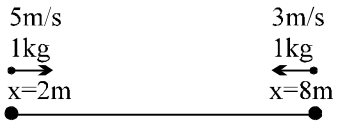
CM0003

4. A man has constructed a toy as shown in figure. If density of the material of the sphere is 12 times of the cone compute the position of the centre of mass. [Centre of mass of a cone of height h is at height of h/4 from its base.]



CM0005

5. The figure shows the positions and velocities of two particles. If the particles move under the mutual attraction of each other, then find the position of centre of mass at t = 1 s.

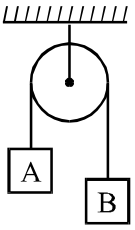


CM0006

6. Mass centers of a system of three particles of masses 1, 2, 3 kg is at the point (1 m, 2 m, 3 m) and mass center of another group of two particles of masses 2 kg and 3 kg is at point (–1 m, 3 m, –2 m). Where a 5 kg particle should be placed, so that mass center of the system of all these six particles shifts to mass center of the first system?

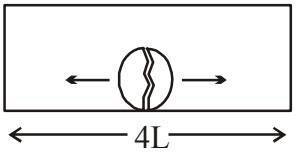
CM0007

7. In the arrangement shown in the figure, m_A = 2 kg and m_B = 1 kg. String is light and inextensible. Find the acceleration of centre of mass of both the blocks. Neglect friction everywhere.



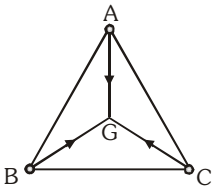
CM0008

8. A bomb of mass 3m is kept inside a closed box of mass 3m and length 4L at it's centre. It explodes in two parts of mass m & 2m. The two parts move in opposite direction and stick to the opposite side of the walls of box. Box is kept on a smooth horizontal surface. What is the distance moved by the box during this time interval.



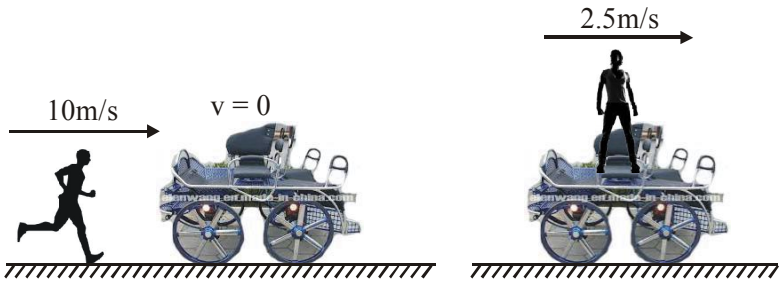
CM0009

9. Three particles A, B and C of equal mass move with equal speed v along the medians of an equilateral triangle as shown in fig. They collide at the centroid G of the triangle. After the collision, A comes to rest, B retraces its path with the speed v. What is the velocity of C ?



CM0010

10. A 50 kg boy runs at a speed of 10 m/s and jumps onto a cart as shown in the figure. The cart is initially at rest. If the speed of the cart with the boy on it is 2.50 m/s, what is the mass of the cart ? (Assuming friction is absent between cart and ground)



CM0011

11. Two cars initially at rest are free to move in the x direction. Car A has mass 4 kg and car B has mass 2 kg. They are tied together, compressing a spring in between them. When the spring holding them together is burned, car A moves off with a speed of 2 m/s.
(i) With what speed does car B leave.
(ii) How much energy was stored in the spring before it was burned.

CM0012

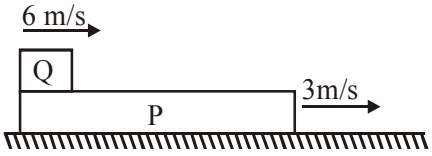
12. A 24 kg projectile is fired at an angle of 53° above the horizontal with an initial speed of 50 m/s. At the highest point in its trajectory, the projectile explodes into two fragments of equal mass, the first of which falls vertically with zero initial speed.
(i) How far from the point of firing does the second fragment strike the ground? (Assume the ground is level.)
(ii) How much energy was released during the explosion?

CM0013

13. A spaceship is moving with constant speed v_0 in gravity free space along +Y-axis suddenly shoots out one third of its part with speed $2v_0$ along + X-axis. Find the speed of the remaining part.

CM0015

14. A plank P and block Q are arranged as shown on a smooth table top. They are given velocities 3 m/s and 6 m/s respectively. The length of plank is 1m and block is of negligible size. After some time when the block has reached the other end of plank it stops slipping on plank. Find the coefficient of friction between plank P and block Q if mass of plank is double of block).

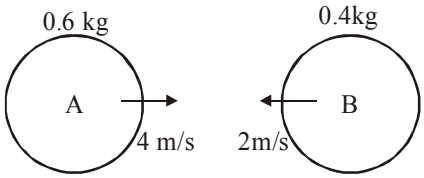


CM0017

15. A bullet of mass m strikes an obstruction and deviates off at 60° to its original direction. If its speed is also changed from u to v, find the magnitude of the impulse acting on the bullet.

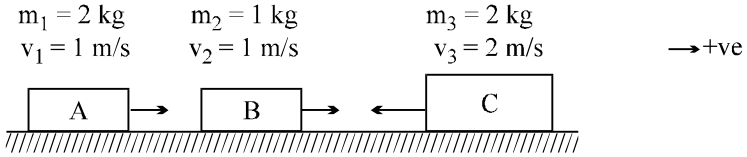
CM0018

16. The velocities of two steel balls before impact are shown. If after head on impact the velocity of ball B is observed to be 3 m/s to the right, the coefficient of restitution is



CM0019

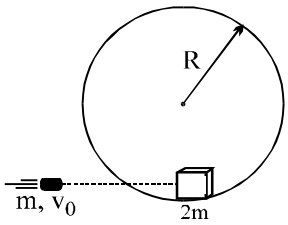
17. Three carts move on a frictionless track with inertias and velocities as shown. The carts collide and stick together after successive collisions.



(i) Find loss of mechanical energy when B & C stick together.
(ii) Find magnitude of impulse experienced by A when it sticks to combined mass (B & C).

CM0020

18. A small block of mass 2m initially rests at the bottom of a fixed circular, vertical track, which has a radius of R. The contact surface between the mass and the loop is frictionless. A bullet of mass m strikes the block horizontally with initial speed v_0 and remain embedded in the block as the block and the bullet circle the loop. Determine each of the following in terms of m, v_0, R and g.



(i) The speed of the masses immediately after the impact.
(ii) The minimum initial speed of the bullet if the block and the bullet are to successfully execute a complete ride on the loop

CM0021

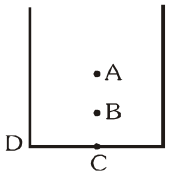
19. Bullets of mass 10 g each are fired from a machine gun at rate of 60 bullets/minute. The muzzle velocity of bullets is 100 m/s. The thrust force due to firing bullets experienced by the person holding the gun stationary is _____.

CM0024

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

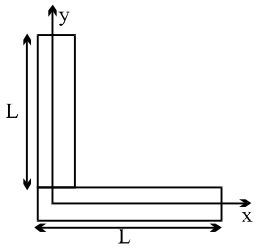
1. A thick uniform wire is bent into the shape of the letter "U" as shown. Which point indicates the location of the center of mass of this wire? A is the midpoint of the line joining mid points of two parallel sides of 'U' shaped wire.



(A) D (B) A (C) B (D) C

CM0035

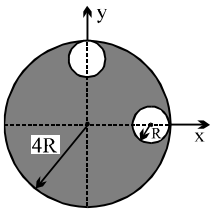
2. Centre of mass of two thin uniform rods of same length but made up of different materials & kept as shown , can be, if the meeting point is the origin of co-ordinates



(A) (L/2, L/2) (B) (2L/3, L/2) (C) (L/3, L/3) (D) (L/3, L/6)

CM0037

3. From the circular disc of radius 4R two small disc of radius R are cut off. The centre of mass of the new structure will be :



(A) $i\frac{R}{5}+j\frac{R}{5}$ (B) $-i\frac{R}{5}+j\frac{R}{5}$ (C) $\frac{-3R}{14}(\hat{i}+\hat{j})$ (D) None of these

CM0038

4. Seven identical birds are flying south together at constant velocity. A hunter shoots one of them, which immediately dies and falls to the ground. The other six continue flying south at the original velocity. After the one bird has hit the ground, the centre of mass of all seven birds

(A) continues south at the original speed, but is now located some distance behind the flying birds

(B) continues south, but at 6/7 the original velocity

(C) continues south, but at 1/7 the original velocity

(D) stops with the dead bird

CM0039

5. There are some passengers inside a stationary railway compartment. The track is frictionless. The centre of mass of the compartment itself (without the passengers) is C_1, while the centre of mass of the 'compartment plus passengers' system is C_2. If the passengers move about inside the compartment along the track.

(A) both C_1 and C_2 will move with respect to the ground.

(B) neither C_1 nor C_2 will move with respect to the ground.

(C) C_1 will move but C_2 will be stationary with respect to the ground.

(D) C_2 will move but C_1 will be stationary with respect to the ground.

CM0041

6. A non–zero external force acts on a system of particles. The velocity and acceleration of the centre of mass are found to be v_0 and a_C respectively at any instant t. It is possible that

(i) $v_0 = 0, a_C = 0$ (ii) $v_0 \neq 0, a_C = 0$ (iii) $v_0 = 0, a_C \neq 0$ (iv) $v_0 \neq 0, a_C \neq 0$

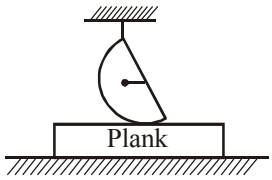
Then

(A) (iii) and (iv) are true. (B) (i) and (ii) are true.

(C) (i) and (iii) are true. (D) (ii), (iii) and (iv) are true.

CM0042

7. Lower surface of a plank is rough and lying at rest on a rough horizontal surface. Upper surface of the plank is smooth and has a smooth hemisphere placed over it through a light string as shown in the figure. After the string is burnt, trajectory of centre of mass of the sphere is :-



(A) a circle (B) an ellipse (C) a straight line (D) a parabola

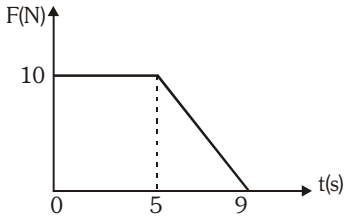
CM0043

8. Three interacting particles of masses 100 g, 200 g and 400 g each have a velocity of 20 m/s magnitude along the positive direction of x-axis, y-axis and z-axis. Due to force of interaction the third particle stops moving. The velocity of the second particle is $\left(10\hat{j} + 5\hat{k}\right)$. What is the velocity of the first particle?

(A) $20\hat{i} + 20\hat{j} + 70\hat{k}$ (B) $10\hat{i} + 20\hat{j} + 8\hat{k}$ (C) $30\hat{i} + 10\hat{j} + 7\hat{k}$ (D) $15\hat{i} + 5\hat{j} + 60\hat{k}$

CM0044

9. A body of mass 4 kg is acted on by a force which varies as shown in the graph below. The momentum acquired is : (Initially body is at rest)

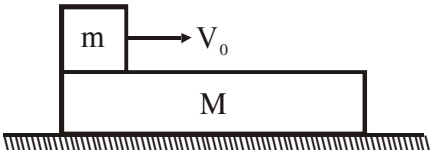


(A) 280 N–s (B) 140 N–s (C) 70 N–s (D) 210 N–s

CM0046

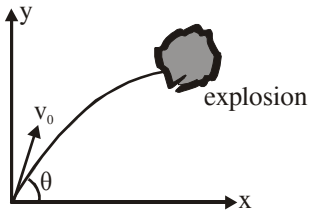
10. The coefficient of friction between the block and plank is μ and ground is smooth. The value of μ is such that block becomes stationary with respect to plank before it reaches the other end. Then which of the following statement is **incorrect.**

(A) The work done by friction on the block is negative.
(B) The work done by friction on the plank is positive .
(C) The net work done by friction is negative.
(D) Net work done by the friction is zero.



CM0047

11. A projectile is projected in x-y plane with velocity v_0. At top most point of its trajectory projectile explodes into two identical fragments. Both the fragments land simultaneously on ground and stick there. Taking point of projection as origin and R as range of projectile if explosion had not taken place. Which of the following can not be position vectors of two pieces, when they land on ground.



(A) $\frac{R}{2}\hat{i}, \frac{3R}{2}\hat{i}$ (B) $0\hat{i}, 2R\hat{i}$ (C) $R\hat{i} - R\hat{k}, R\hat{i} + R\hat{k}$ (D) $2R\hat{i} + \frac{R}{2}\hat{k}, R\hat{i} - \frac{R}{2}\hat{k}$

CM0048

12. A boy hits a baseball with a bat and imparts an impulse J to the ball. The boy hits the ball again with the same force, except that the ball and the bat are in contact for twice the amount of time as in the first hit. The new impulse equals:

(A) half the original impulse (B) the original impulse
(C) twice the original impulse (D) four times the original impulse

CM0049

13. Two balls of same mass are dropped from the same height h, on to the floor. The first ball bounces to a height h/4 ,after the collision & the second ball to a height h/16. The impulse applied by the first & second ball on the floor are I_1 and I_2 respectively. Then

(A) $5I_1 = 6I_2$ (B) $6I_1 = 5I_2$ (C) $I_1 = 2I_2$ (D) $2I_1 = I_2$

CM0050

14. Ball A of mass 5.0 kilograms moving at 20 m/s collides with ball B of unknown mass moving at 10m/s in the same direction. After the collision, ball A moves at 10 m/s and ball B at 15 m/s, both still in the same direction. What is the mass of ball B?

(A) 6.0 kg (B) 10. kg (C) 2.0 kg (D) 12 kg

CM0051

15. A smooth small spherical ball of mass m, moving with velocity u collides head on with another small spherical ball of mass 3m, which was initially at rest. Two-third of the initial kinetic energy of the system is lost. The coefficient of restitution between the spheres is

(A) $\frac{1}{3}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) zero

CM0052

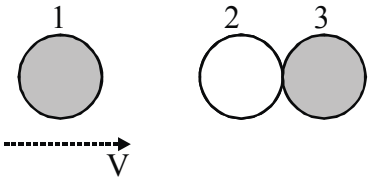
16. A ball strikes a smooth horizontal ground at an angle of 45° with the vertical. What cannot be the possible angle of its velocity with the vertical after the collision. (Assume $e \leq 1$).

(A) 45° (B) 30° (C) 53° (D) 60°

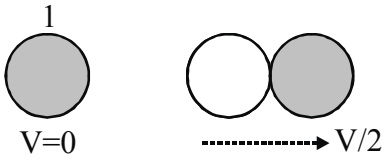
CM0053

17. Two identical ball bearings in contact with each other and resting on a frictionless table are hit head-on by another ball bearing of the same mass moving initially with a speed *V* as shown in figure (i). If the collision is elastic, which of the following is a possible result after collision?

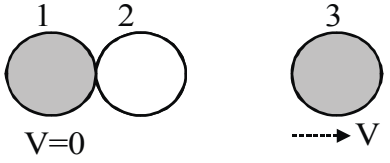
fig. (i)



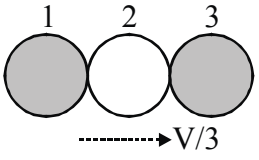
(A)



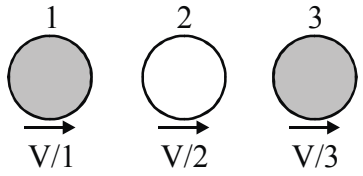
(B)



(C)



(D)



CM0054

18. A ball is projected from ground with a velocity V at an angle θ to the vertical. On its path it makes an elastic collison with a vertical smooth wall and returns to ground. The total time of flight of the ball is

(A) $\frac{2v\sin\theta}{g}$ (B) $\frac{2v\cos\theta}{g}$ (C) $\frac{v\sin 2\theta}{g}$ (D) $\frac{v\cos\theta}{g}$

CM0055

19. A ball is thrown downwards with initial speed = 6 m/s, from a point at height = 3.2 m above a horizontalfloor. If the ball rebounds back to the same height then coefficient of restitution equals to

(A) 1/2 (B) 0.75 (C) 0.8 (D) None

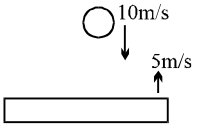
CM0056

20. A particle is projected from a smooth horizontal surface with velocity v at an angle θ from horizontal. Coefficient of restitution between the surface and ball is e. The distance of the point where ball strikes the surface second time from the point of projection is

(A) $\frac{v^2 \sin 2\theta(1+e^2)}{g}$ (B) $\frac{v^2 \sin 2\theta(1+e^4)}{g}$ (C) $\frac{v^2 \sin 2\theta(1+e^3)}{g}$ (D) $\frac{v^2 \sin 2\theta(1+e)}{g}$

CM0057

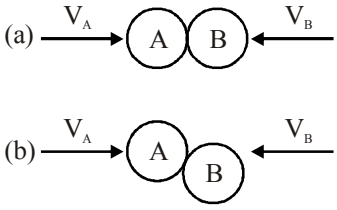
21. A ball of mass 1kg strikes a heavy platform, elastically, moving upwards with a velocity of 5m/s. The speed of the ball just before the collision is 10m/s downwards. Then the impulse imparted by the platform on the ball is :-



(A) 15 N – s (B) 10 N – s (C) 20 N – s (D) 30 N – s

CM0058

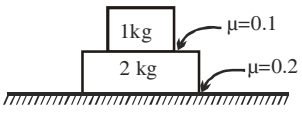
22. Two bodies, A and B, collide as shown in figures a and b below. Circle the true statement :



(A) They exert equal and opposite forces on each other in (a) but not in (b)
(B) They exert equal and opposite force on each other in (b) but not in (a)
(C) They exert equal and opposite force on each other in both (a) and (b)
(D) The forces are equal and opposite to each other in (a), but only the components of the forces parallel to the velocities are equal in (b).

CM0059

23. If both the blocks as shown in the given arrangement are given together a horizontal velocity towards right. If a_{cm} be the subsequent acceleration of the centre of mass of the system of blocks then a_{cm} equals



(A) 0 m/s^2 (B) $\frac{5}{3}$ m/s^2 (C) $\frac{7}{3}$ m/s^2 (D) 2 m/s^2

CM0075

KINEMATICS OF ROTATION MOTION

Rigid Body

A rigid body is an assemblage of a large number of material particles, which do not change their mutual distances under any circumstance or in other words, they are not deformed under any circumstance.

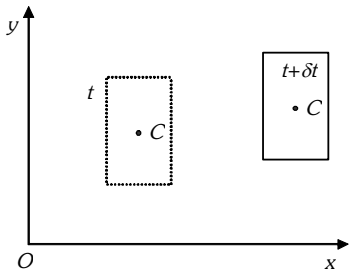
Actual material bodies are never perfectly rigid and are deformed under action of external forces. When these deformations are small enough to be considered during their course of motion, the body is assumed a rigid body. Hence, all solid objects such as stone, ball, vehicles etc are considered as rigid bodies while analyzing their translation as well as rotation motion.

To analyze rotation of a body relative motion between its particles cannot be neglected and size of the body becomes a considerable factor. This is why study of rotation motion is also known as *mechanics of rigid bodies*.

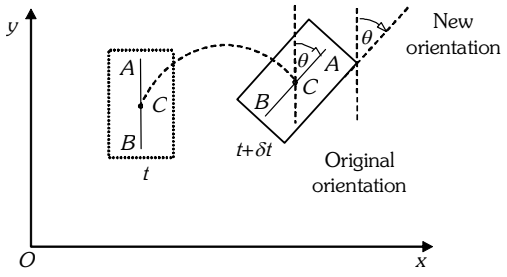
Rotation Motion of a Rigid Body

Any kind of motion of a body is identified by change in position or change in orientation or change in both. If a body changes its orientation during its motion it said to be in rotation motion.

In the following figures, a rectangular plate is shown moving in the *x*-*y* plane. The point *C* is its mass center. In the first case it does not changes orientation, therefore is in pure translation motion. In the second case it changes its orientation by during its motion. It is a combination of translation and rotation motion.



Pure Translation

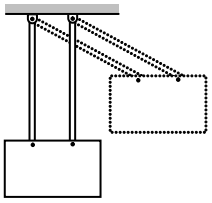


Combination of translation and rotation

Rotation i.e. change in orientation is identified by the angle through which a linear dimension or a straight line drawn on the body turns. In the figure this angle is shown by θ.

Ex. Identify Translation and rotation motion

A rectangular plate is suspended from the ceiling by two parallel rods each pivoted at one end on the plate and at the other end on the ceiling. The plate is given a side-push to oscillate in the vertical plane containing the plate. Identify motion of the plate and the rods.



Sol.

Neither of the linear dimensions of the plate turns during the motion. Therefore, the plate does not change its orientation. Here edges of the body easily fulfill our purpose to measure orientation; therefore, no line is drawn on it.

The plate is in *curvilinear translation motion* and the rods are in rotation motion.

Types of Motions involving Rotation

Motion of body involving rotation can be classified into following three categories.

I Rotation about a fixed axis.

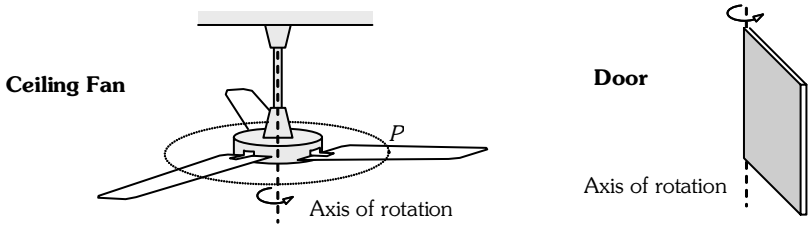
II Rotation about an axis in translation.

III Rotation about an axis in rotation

Rotation about a fixed axis

Rotation of ceiling fan, potter's wheel, opening and closing of doors and needles of a wall clock etc. come into this category.

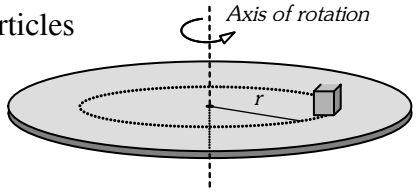
When a ceiling fan it rotates, the vertical rod supporting it remains stationary and all the particles on the fan move on circular paths. Circular path of a particle P on one of its blades is shown by dotted circle. Centers of circular paths followed by every particle are on the central line through the rod. This central line is known as *axis of rotation* and is shown by a dashed line. All the particles on the axis of rotation are at rest, therefore the axis is stationary and the fan is in rotation about this fixed axis.



A door rotates about a vertical line that passes through its hinges. This vertical line is the axis of rotation. In the figure, the axis of rotation is shown by dashed line.

Axis of rotation

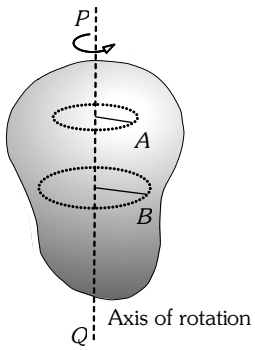
An imaginary line perpendicular to plane of circular paths of particles of a rigid body in rotation and containing the centers of all these circular paths is known as *axis of rotation*.



It is not necessary that the axis of rotation pass through the body. Consider system shown in the figure, where a block is fixed on a rotating disk. The axis of rotation passes through the center of the disk but not through the block.

Important observations

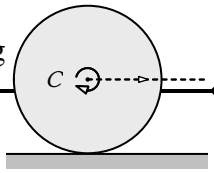
Let us consider a rigid body of arbitrary shape rotating about a fixed axis PQ passing through the body. Two of its particles A and B are shown moving on their circular paths.



- ⇨ All of its particles, not on the axis of rotation, move on circular paths with centers on the axis or rotation. All these circular paths are in parallel planes that are perpendicular to the axis of rotation.
- ⇨ All the particles of the body cover same angular displacement in the same time interval, therefore all of them move with the same angular velocity and angular acceleration.
- ⇨ Particles moving on circular paths of different radii move with different speeds and different magnitudes of linear acceleration. Furthermore, no two particles in the same plane perpendicular to the axis of rotation have same velocity and acceleration vectors.
- ⇨ All the particles on a line parallel to the axis of rotation move circular paths of the same radius therefore have same velocity and acceleration vectors.
- ⇨ Consider two particles in a plane perpendicular to the rotational axis. Every such particle on a rigid body in rotation motion moves on circular path relative to another one. Radius of the circular path equals to the distance between the particles. In addition, angular velocity and angular acceleration equals to that of rotation motion of the body.

Rotation about an axis in translation

Rotation about an axis in translation includes a broad category of motions. Rolling



is an example of this kind of motion. A rod lying on table when pushed from its one in its perpendicular direction also executes this kind of motion. To understand more let us discuss few examples.

Consider rolling of wheels of a vehicle, moving on straight level road. Relative to a reference frame, moving with the vehicle wheel appears rotating about its stationary axel. The rotation of the wheel from this frame is rotation about fixed axis. Relative to a reference frame fixed with the ground, the wheel appears rotating about the moving axel, therefore, rolling of a wheel is superposition two simultaneous but distinct motions – rotation about the axel fixed with the vehicle and translation of the axel together with the vehicle.

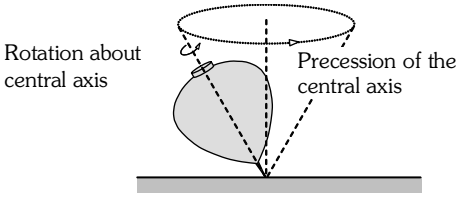
Important observations

- ⇨ Every particle of the body always remains in a plane perpendicular to the rotational axis. Therefore, this kind of motion is also known as *general plane motion*.
- ⇨ Relative to every particle another particle in a plane perpendicular to axis of rotation moves on circular path. Radius of the circular path equals to the distance between the particles and angular velocity and angular acceleration equals to that of rotation motion of the body.
- ⇨ Rotation about axis in translation is superposition of pure rotation about the axis and simultaneous translation motion of the axis.

Rotation about an axis in rotation.

In this kind of motion, the body rotates about an axis that also rotates about some other axis. Analysis of rotation about rotating axes is not in the scope of JEE, therefore we will discus it to have an elementary idea only.

As an example consider a rotating top. The top rotates about its central axis of symmetry and this axis sweeps a cone about a vertical axis. The central axis continuously changes its orientation, therefore is in rotation motion. This type of rotation in which the axis of rotation also rotates and sweeps out a cone is known as *precession*.

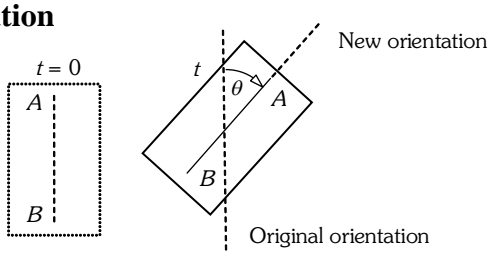


Another example of rotation about axis in rotation is a table-fan swinging while rotating. Table-fan rotates about its horizontal shaft along which axis of rotation passes. When the rotating table-fan swings, its shaft rotates about a vertical axis.

Angular displacement, angular velocity and angular acceleration

Rotation motion is the change in orientation of a rigid body with time.It is measured by turning of a linear dimension or a straight line drawn on the body.

In the figure is shown at two different instants $t = 0$ and t a rectangular plate moving in its own plane. Change in orientation during time t equals to the angle θ through which all the linear dimensions of the plate or a line AB turns.



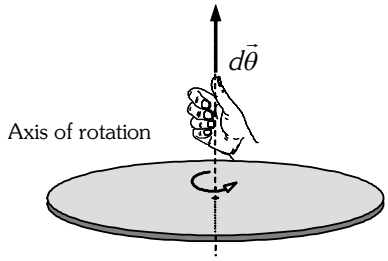
If the angle θ continuously changes with time t, instantaneous angular velocity ω and angular acceleration α for rotation of the body are defined by the following equations.

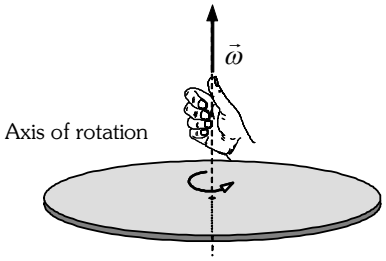
$$\omega = \frac{d\theta}{dt} \quad [1] \qquad \alpha = \frac{d^2\theta}{dt^2} = \frac{d\omega}{dt} = \omega\frac{d\omega}{d\theta} \quad [2]$$

Direction of angular motion quantities

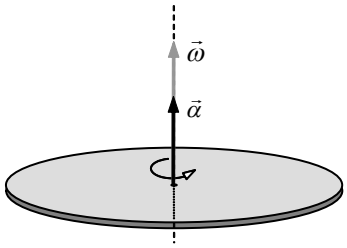
Angular displacement, angular velocity and angular acceleration are known as angular motion quantities. Infinitesimally small angular displacement, instantaneous angular velocity and angular

acceleration are vector quantities. Direction of infinitesimally small angular displacement and instantaneous angular velocity is given by the right hand rule. For a disk rotating as shown in the figure, the angular velocity points upwards along the axis of rotation.

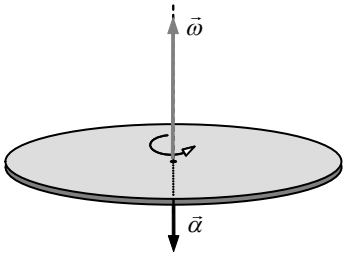




The direction of angular acceleration depends, whether angular velocity increases or decreases with time. For increasing angular velocity, the angular acceleration vector points in the direction of angular velocity vector and for decreasing angular velocity, the angular acceleration vector points opposite to the angular velocity vector.



Angular acceleration: Increasing angular speed



Angular acceleration: Decreasing angular speed

In rotation about fixed axis and rotation about axis in translation, the axis of rotation does not rotate and angular velocity and acceleration always point along the axis of rotation. Therefore, in dealing these kinds of motions, the angular motion quantities can used in scalar notations by assigning them positive sign for one direction and negative sign for the opposite direction.

These quantities have similar mathematical relations as position coordinate, velocity, acceleration and time have in rectilinear motion.

⇨ A body rotating with constant angular velocity ω and hence zero angular acceleration is said to be uniform rotation. Angular position θ is given by equation

$$\theta = \theta_o + \omega t \qquad [3]$$

⇨ Thus for a body rotating with uniform angular acceleration α, the angular position θ and angular velocity ω can be expressed by the following equation.

$$\omega = \omega_o + \alpha t \qquad [4]$$

$$\theta = \theta_o + \omega_o t + \tfrac{1}{2}\alpha t^2 = \theta_o + \tfrac{1}{2}(\omega_o + \omega)t \qquad [5]$$

$$\omega^2 = \omega_o^2 + 2\alpha(\theta - \theta_o) \qquad [6]$$

Ex. A disk rotates about a fixed axis. Its angular velocity ω varies with time according to equation $\omega = at + b$. At the instant $t = 0$ its angular velocity is 1.0 rad/s at angular position is 2 rad and at the instant $t = 2$ s, angular velocity is 5.0 rad/s. Determine angular position θ and angular acceleration α when $t = 4$ s.

Sol. The given equation $\omega = at + b$ has form similar to eq.[4], therefore motion is rotation with uniform angular acceleration. Initial angular velocity = $\omega_o = b = 1.0$ rad/s, Angular acceleration $\alpha = a$,

$$\theta = \tfrac{1}{2}at^2 + t + c$$

Since at $t = 0$, $\omega = 1.0$ rad/s, we obtain the constant c.

Initial angular position = $\theta_o = c = 2.0$ rad

Since at $t = 2.0$ s angular velocity is 5.0 rad/s, from given expression of angular velocity, we have

$\omega = at + b \rightarrow$ Substituting $b = 1.0$ rad/s, $t = 2.0$ s and $\omega = 5.0$ rad/s, we have $a = 2.0$ rad/s

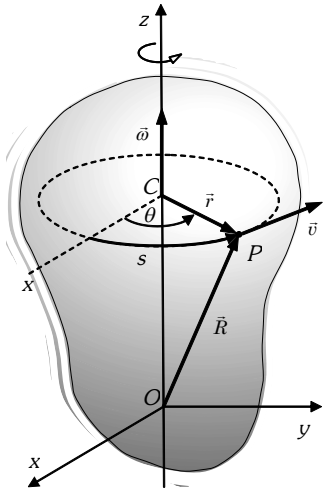
Now we can write expressions for angular position, angular velocity and angular acceleration.

$$\theta = t^2 + t + 2.0 \quad (1) \qquad \omega = 2.0t + 1.0 \quad (2)$$

From the above equations, we can calculate angular position, angular velocity and angular acceleration at $t = 4.0$ s

$\theta_4 = 22$ rad, $\omega_4 = 9.0$ rad/s, $\alpha = 2.0$ rad/s^2 **Ans.**

Kinematics of rotation about fixed axis



In figure is shown a rigid body of arbitrary shape rotating about the z-axis. In the selected frame (here the coordinate system) all the three axes are at rest, therefore the z-axis that is the axis of rotation is at rest and the body is in fixed axis rotation. All of its particles other than those on the z-axis move on circular paths with their centers on the z-axis. All these circular paths are parallel to the x-y plane. In the figure, one of its particles P is shown moving with velocity $\vec{v}$ on a circular path of radius r and center C. Its position vector is $\vec{R}$. It were at the line Cx at $t = 0$ and at the position shown at the instant t. During time interval t, it covers the circular arc of length s and its radius vector turns through angle θ.

In an infinitesimally small time interval dt let, the particle covers infinitesimally small distance ds along its circular path.

$$d\vec{s} = d\vec{\theta} \times \vec{r} = d\vec{\theta} \times \vec{R} \qquad [7]$$

$$\vec{v} = \frac{d\vec{s}}{dt} = \frac{d\vec{r}}{dt} = \frac{d\vec{R}}{dt} \qquad [8]$$

From eq. [7] and [8] we have

$$\vec{v} = \vec{\omega} \times \vec{r} = \vec{\omega} \times \vec{R} \qquad [9]$$

The above equation tells us the relation between the liner and angular velocity. Now we explore relation between the linear and angular accelerations. For the purpose, differentiate the above equation with respect to time.

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v} \qquad [10]$$

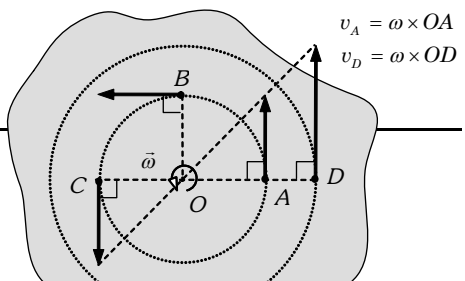
The first term on the RHS points along the tangent in the direction of the velocity vector and it is known as tangential acceleration $\vec{a}_T$ same as we have in circular motion. In addition, the second term point towards the center C. It is known as centripetal acceleration of normal component $\vec{a}_n$ of acceleration same as in circular motion. Now we have

Tangential acceleration $\vec{a}_T = \vec{\alpha} \times \vec{r}$ [11]

Normal acceleration $\vec{a}_n = \vec{\omega} \times \vec{v} = -\omega^2 \vec{r}$ [12]

How to Locate Axis of Rotation

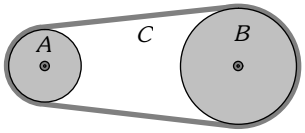
Every particle in a plane perpendicular to the axis of rotation move with different velocities and accelerations, moreover,



they all have the same angular velocity and angular acceleration. Such a section of a body in rotation is shown here. The particles A, B and C at equal distance from the axis of rotation move with equal speeds v_A and the particle D moves with speed v_D on concentric circular paths. The location of rotational axis can be determined by any of the two graphical techniques.

- ⇨ Lines perpendicular to velocity vectors and passing through the particles, whose velocity vectors are neither parallel nor antiparallel intersect at the axis of rotation. See pairs of particles A and B, B and C and B and D.
- ⇨ Lines perpendicular to velocity vectors and passing through the particles, whose velocity vectors are either parallel or antiparallel, coincide and intersect the line joining tips of their velocity vectors at the axis of rotation. Refer pairs of particles A and C, A and D and C and D.

Ex. A belt moves over two pulleys A and B as shown in the figure. The pulleys are mounted on two fixed horizontal axels. Radii of the pulleys A and B are 50 cm and 80 cm respectively. Pulley A is driven at constant angular acceleration 0.8 rad/s^2 until the pulley B acquires an angular velocity of 10 rad/s. The belt does not slide on either of the pulleys.



(a) Find acceleration of a point C on the belt and angular acceleration of the pulley B.

(b) How long after the pulley B achieve angular velocity of 10 rad/s.

Sol. Since the belt does not slide on the pulleys, magnitude of velocity and acceleration of any point on the belt are same as velocity tangential acceleration of any point on periphery of either of the pulleys.

Using the above fact with eq.[11], we have

$\vec{a}_T = \vec{\alpha} \times \vec{r} \rightarrow \quad a_C = \alpha_A r_A = \alpha_B r_B$

Substituting $r_A = 0.5$ m, $r_B = 0.8$ m and $\alpha = 0.8$ rad/s^2, we have

$a_C = 5$ m/s^2 and $\alpha_B = \dfrac{a_C}{r_B} = \dfrac{\alpha_A r_A}{r_B} = 0.5$ rad/s^2 **Ans.**

From eq. [4], we have

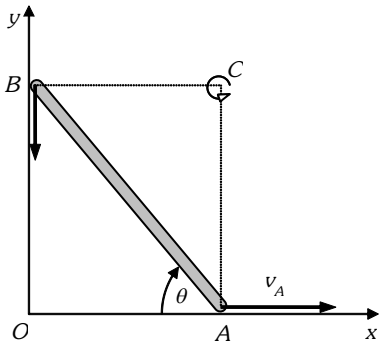
$\omega = \omega_o + \alpha t \rightarrow \quad t = \dfrac{\omega_B - \omega_{Bo}}{\alpha_B}$

Substituting $\omega_{Bo} = 0$, $\omega_B = 10$ rad/s and $\alpha_B = 0.5$ rad/s^2, we have $t = 20$ s **Ans.**

Kinematics of rotation about axis in translation

In this kind of motion, the body rotates about an axis and the axis moves without rotation. Rolling is a very common example of this kind of motion.

As an example consider a rod whose ends A and B are sliding on the x and y-axis as shown in the figure. Change in its orientation measured by change in angle θ indicates that the rod is in rotation. Perpendiculars drawn to velocity vector of its end points intersect at the axis of rotation, which is continuously changing it position.

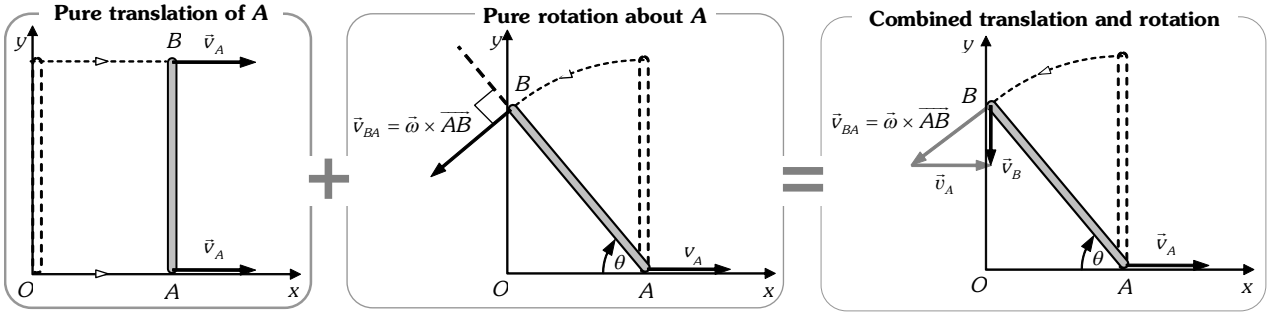


Instantaneous Axis of Rotation (IAR)

It is a mathematical line about that a body in combined translation and rotation can be conceived in pure rotation at an instant. It continuously changes its location.

Now we explore how the combined translation and rotational motion of the rod is supper position of translation motion of any of its particle and pure rotation about an axis through that particle.

Consider motion of the rod from beginning when it was parallel to the *y*-axis. In the following figure translation motion of point *A* is superimposed with pure rotation about *A*.



The motion of the rod can be conceived as superposition of translation of point *A* and simultaneous rotation about an axis through *A*.

The same experiment can be repeated to demonstrate that motion of the rod can be conceived as superposition of translation of any of its particle and simultaneous rotation about an axis through that particle.

Considering translation of *A* and rotation about *A* this fact can be expressed by the following equation.

Combined Motion = Translation of point A + Pure rotation about point A

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad [13]$$

Since point *B* moves relative *A* moving on circular path its velocity relative to *A* is given by the equation

$$\vec{v}_{BA} = \vec{\omega} \times \overrightarrow{AB}\,.$$

Now we have $\vec{v}_B = \vec{v}_A + \vec{\omega} \times \overrightarrow{AB}$ [14]

The above fact is true for any rigid body in combined translation and rotation motion.

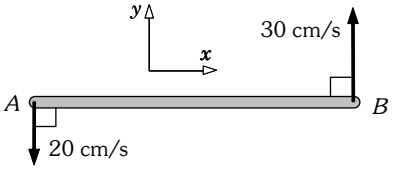
Rotation about an axis in translation of a rigid body can be conceived as well as analyzed as superposition of translation motion of any of its particle and simultaneous rotation about an axis passing through that particle provided that the axis is parallel to the actual one.

Similar to eq.[13], we can write equation for acceleration.

$$\left.\begin{aligned} \vec{a}_A &= \vec{a}_B + \vec{a}_{BA} \\ \vec{a}_A &= \vec{a}_B + \vec{a}_{BAT} + \vec{a}_{BAn} \\ \vec{a}_A &= \vec{a}_B + \vec{\alpha} \times \overrightarrow{AB} + -\omega^2 \overrightarrow{AB} \end{aligned}\right\} \qquad [15]$$

Ex. A 100 cm rod is moving on a horizontal surface. At an instant, when it is parallel to the *x*-axis its ends *A* and *B* have velocities 30 cm/s and 20 cm/s as shown in the figure.

(a) Find its angular velocity and velocity of its center.



(b) Locate its instantaneous axis of rotation.

Sol.

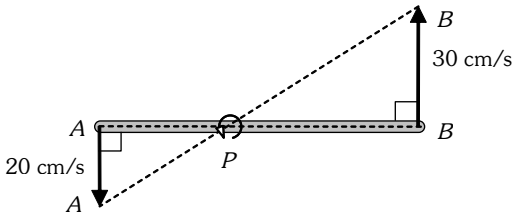
Let the rod is rotating anticlockwise, therefore its angular velocity is given by $\vec{\omega} = \omega\hat{k}$. Velocity vectors of all the points on the rod and its angular velocity must satisfy the relative motion eq.[14].

(a) Substituting velocities $\vec{v}_A = -20\hat{j}$ cm/s and $\vec{v}_A = 30\hat{j}$ cm/s and angular velocity $\vec{\omega}$ in eq.[14] , we have $\vec{v}_B = \vec{v}_A + \vec{\omega} \times \overrightarrow{AB} \rightarrow \quad \omega = 0.5$ rad/s **Ans.**

Velocity vector of the center *C* of the rod also satisfy the following equation.

$\vec{v}_C = \vec{v}_A + \vec{\omega} \times \overrightarrow{AC} \rightarrow \qquad \vec{v}_C = -20\hat{j} + 0.5\hat{k} \times 50\hat{i} = 5.0\hat{j}$ cm/s **Ans.**

(b) Here velocity vectors of the particles *A* and *B* are antiparallel, therefore the instantaneous axis of rotation passes through intersection of the common perpendicular to their velocity vectors and a line joining tips of the velocity vectors. The required geometrical construction is shown in the following figure.



Since triangles *AA'P* and *BB'P* are similar and $AB = 100$ cm, we have $AP = 40$ cm.

The instantaneous axis of rotation passes through the point *P*, which is 40 cm from *A*. **Ans.**

Analytical Approach.

The instantaneous center of rotation is at instantaneous rest. Using this fact in eq.[14], we have

$\vec{v}_P = \vec{v}_A + \vec{\omega} \times \overrightarrow{AP} \rightarrow \qquad \vec{0} = -20\hat{j} + 0.5\hat{k} \times (AP)\hat{j} \Rightarrow AP = 40$ cm **Ans.**

Concept of Rotational Inertia (Moment of inertia)

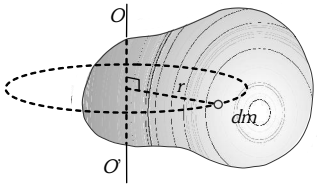
Total mass of a body in translation motion is the measure of its inertia to translation motion. Similarly if a point mass m is rotating about an axis at a distance r from the axis, then term mr^2 provides suitable measure of its inertia to rotation motion. *The inertia to rotation motion is known as rotational inertia or more commonly moment of inertia.*

Moment of inertia of a rigid body

A rigid body is continuous distribution of mass and can be assumed consisting of infinitely large number of point particles. If one of the point particle of infinitely small mass *dm* is at a distance *r* from the axis of rotation *OO*', the moment of inertia of this point particle is given by

$$dI_o = r^2 dm$$

The moment of inertia of the whole body about the axis *OO*' can now be obtained by integrating term of the above equation over the limits to cover whole of the body.

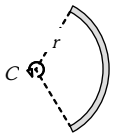
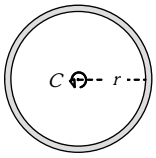
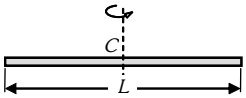
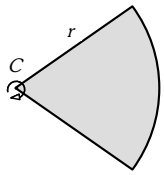
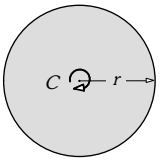
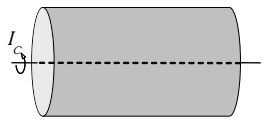
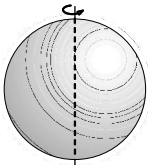
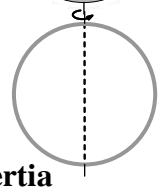


$$I_o = \int dI_o = \int r^2 dm$$

Expression for moment of inertia contains product of two terms. One of them is the mass of the body and the other is a characteristic dimension, which depends on the manner how mass of the body is distributed relative to the axis of rotation. Therefore moment of inertia of a rigid body depends on the mass of the body and distribution of the mass relative to the axis of rotation. Obviously for uniform bodies expression of moment of inertia depends on their shape and location and orientation of the axis of rotation. Based on these facts we can conclude

1. If mass distribution is similar for two bodies about an axis, expressions of their moment of inertia must be of the same form about that axis.
2. If the whole body or any of its portions is shifted parallel to the axis of rotation, moment of inertia remains unchanged.

Moment of Inertia for some commonly used bodies

Body	Axis	Moment of Inertia	
Uniform thin rod bent into shape of an arc of mass m		Passing through center and perpendicular to the plane containing the arc	$I_C = mr^2$
Uniform ring of mass m		Passing through center and perpendicular to the plane containing the arc or the centroidal axis.	$I_C = mr^2$
Straight uniform rod		Passing through center and perpendicular to the rod or the centroidal axis.	$I_C = \frac{mL^2}{12}$
Sector of a uniform disk of mass m		Passing through center and perpendicular to the plane containing the sector.	$I_C = \frac{mr^2}{2}$
Uniform disk of mass m		Passing through center and perpendicular to the plane containing the disk or the centroidal axis.	$I_C = \frac{mr^2}{2}$
Homogeneous cylinder of mass m		Axis of the cylinder or the centroidal axis.	$I_C = \frac{mr^2}{2}$
Homogeneous sphere of mass m		Diameter or the centroidal axis	$I_C = \frac{2}{5}mR^2$
Spherical shell of mass m		Diameter or the centroidal axis	$I = \frac{2}{3}mR^2$

Theorems on Moment of Inertia

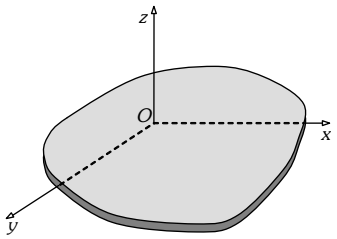
Moment of inertias of a rigid body about different axes may be different. There are two theorems known as *theorem of perpendicular axes* and *theorem of parallel axes*, which greatly simplify calculation of moment of inertia about an axis if moment of inertia of a body about another suitable axis is known.

Theorem of Perpendicular Axes

This theorem is applicable for a rigid body that lies entirely within a plane i.e. a laminar body or a rod bent into shape of a plane curve. The moment of inertia I_x, I_y and I_z of the body about the x, y and z-axis can be expressed by the following equations.

$$I_z = I_y + I_x$$

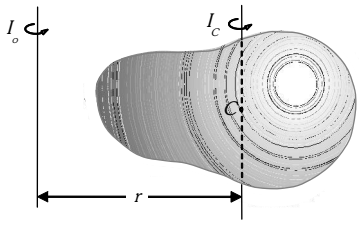
For a planar body, the moment of inertia about an axis perpendicular to the plane of the body is the sum of the moment of inertias about two perpendicular axes in the plane of the object provided that all the three axes are concurrent.



Theorem of Parallel Axes

This theorem also known as Steiner's theorem can be used to determine the moment of inertia of a rigid body about any axis, if the moment of inertia of the body about a parallel axis passing through mass center of the body and perpendicular distance between both the axes is known.

Consider a body of arbitrary shape and mass m shown in the figure. Its moment of inerta I_o and I_C are defined about two parallel axes. The axis about which moment of ienertia I_C is defined passes through the mass center C. Seperation between the axes is r. These two moment of inertias are related by the following equation.



$$I_O = I_C + Mx_C^2$$

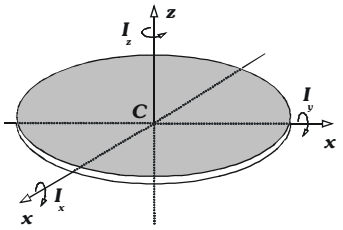
The above equation is known as the theorem of parallel axes or Steiner's theorem.

- ⇨ *The moment of inertia about any axis parallel to an axis through the mass center is given by sum of moment of inertia about the axis through the mass center and product term of mass of the body and square of the distance between the axes.*
- ⇨ *Among all the parallel axes the moment of inertia of a rigid body about the axis through the mass center is the minimum moment of inertia.*

The second term added to the moment of inertia I_C about the centroidal axis in the above equation can be recognized as the moment of inertia of a particle of mass equal to that of the body and located at its mass center. It again reveals that *the plane motion of a rigid body is superposition of pure rotation about the mass center or centroidal rotation and translation of its mass center.*

Ex. Find moment of inertia of a uniform disk of mass m and radius r about one of its diameter.

Sol. In the adjoining figure a disk is shown with two of its diameter perpendicular to each other. These diameters are along the x and the y-axis of a coordinate system. The x-axis is perpendicular to the plane of the disk and passes through its center is also shown. Since the disk is symmetric about both the diameters, moment of inertias about both the diameters must be equal. Thus substituting this in the theorem of perpendicular axes, we have



$$I_z = I_y + I_x \rightarrow \qquad I_z = 2I_x = 2I_y$$

Moment of inertia of the disk about the z-axis is $I_z = \frac{1}{2}mr^2$. Substituting it in the above equation, we have

$$I_x = I_y = \frac{1}{2}I_z = \frac{1}{4}mr^2 \qquad \textbf{Ans.}$$

Radius of Gyration

It is the radial distance from a rotation axis at which the mass of an object could be concentrated without altering the moment of inertia of the body about that axis.

If the mass m of the body were actually concentrated at a distance k from the axis, the moment of inertia about that axis would be mk^2.

$$k = \sqrt{\frac{I}{m}}$$

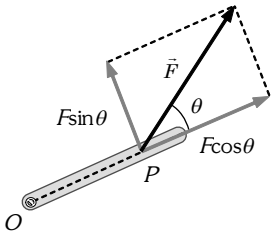
The radius of gyration has dimensions of length and is measured in appropriate units of length such as meters.

DYNAMICS OF RIGID BODY

Torque: Moment of a force

Torque is rotational analogue of force and expresses tendency of a force applied to an object to cause the object to rotate about a given point.

To investigate further let us discuss an experiment. Consider a rod pivoted at the point O. A force $\vec{F}$ is applied on it at the point P. The component $F\cos\theta$ of the force along the rod is counterbalanced by the reaction force of the pivot and cannot contribute in rotating the rod. It is the component $F\sin\theta$ of the force perpendicular to the rod, which is responsible for rotation of the rod. Moreover, farther is the point P from O, where the force is applied easier is to rotate the rod. This is why handle on a door is attached as far away as possible from the hinges.

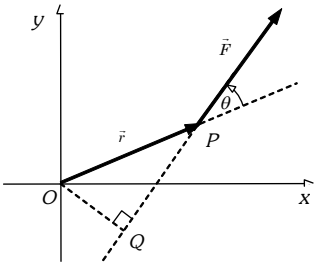


Magnitude of torque of a force is proportional to the product of distance of point of application of the force from the pivot and magnitude of the perpendicular component $F\sin\theta$ of the force. Denoting torque by symbol τ, the distance of point of application of force from the pivot by r, we can write

$$\tau_o \propto rF\sin\theta$$

Since rotation has sense of direction, torque should also be a vector. Its direction is given by right hand rule. Now we can express torque by the cross product of $\vec{r}$ and $\vec{F}$.

$$\vec{\tau}_o = \vec{r} \times \vec{F}$$



Here constant of proportionality has been assumed a dimensionless number unity because a unit of torque has been chosen as product of unit of force and unit of length.

The geometrical construction shown in figure suggests a simple way to calculate torque. The line OQ ($= r\sin\theta$) known as *moment arm*, is the length of perpendicular drawn from O on the line of action of the force. The magnitude of the torque equals to the product of OQ and magnitude of the force $\vec{F}$..

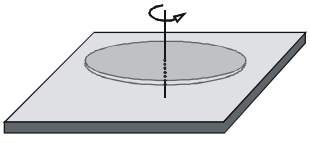
Torque about a Point and Torque about an Axes

We have defined *torque of a force about a point* as the moment of the force about that point. In dealing with rotation about a fixed axis we need to know torque about the axis rotation.

When a body is in plane motion the net torque of all the forces including the forces necessary to restrain rotation of the axis is along the axis of rotation. It is known as torque about the axis. Torque of a force about an axis of rotation equals to the moment of force about the point where plane of motion of the point of application of the force intersects the axis.

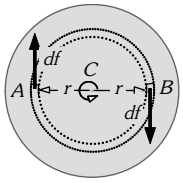
In analyzing plane motion we always consider torque about an axis under consideration and in rest of the book by the term torque of force we mean torque about an axis.

Ex. A uniform disk of mass M and radius R rotating about a vertical axis



passing through its center and perpendicular to its plane is placed gently on a rough horizontal ground, where coefficient of friction is μ. Calculate torque of the frictional forces.

Sol. When the disk rotates on the ground, kinetic friction acts at every contact point. Since the gravity acts uniformly everywhere and the disk is also uniform, the normal reaction form the ground is uniformly distributed over the entire contact area. Consider two diametrically opposite identical portions A and B of the disk each of mass dm at distance r from the center as shown in the adjacent figure. The normal reaction form the ground on each of these portions equals to their weights and hence frictional forces are $df = \mu dmg$.



Consider a ring of radius r and width dr shown by dashed lines. Net torque $d\tau_C$ of friction force on this ring can easily be expressed by the following equation.

$$d\tau_C = r\mu(\text{mass of the ring})g = r\mu\left(\frac{\text{Mass of the disk}}{\text{Area of the disk}} \times \text{Area of the ring}\right)g = r\mu\left(\frac{2Mrdr}{R^2}\right)g$$

Integrating both sides of the above equation, we have

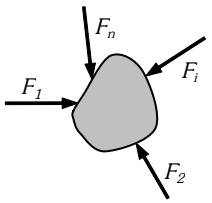
$$\tau_C = \frac{2\mu Mg}{R^2}\int_{r=0}^{R} r^2 dr = \tfrac{2}{3}\mu MgR \qquad \textbf{Ans.}$$

Rotational equilibrium

A rigid body is said to be state of rotational equilibrium if its angular acceleration is zero. Therefore a body in rotational equilibrium must either be in rest or rotation with constant angular velocity.

Since scope of JEE syllabus is confined only to rotation about a fixed axis or rotation about an axis in translation motion, the discussion regarding rotational equilibrium is limited here to situations involving only coplanar forces. Under these circumstances the necessary and sufficient condition for rotational equilibrium is

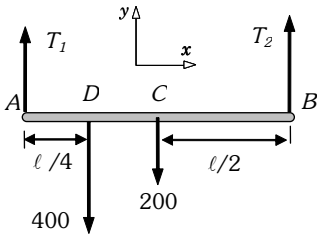
If a rigid body is in rotational equilibrium under the action of several coplanar forces, the resultant torque of all the forces about any axis perpendicular to the plane containing the forces must be zero.



In the figure a body is shown under the action of several external coplanar forces F_1, F_2, ... F_i, and F_n.

$$\sum \vec{\tau}_P = 0$$

Here P is a point in the plane of the forces about which we calculate torque of all the external forces acting on the body. The flexibility available in selection of the point P provides us with advantages that we can select such a point about which torques of several unknown forces will become zero or we can make as many number of equations as desired by selecting several different points. The first situation yields to a simpler equation to be solved and second situation though does not give independent equation, which can be used to determine additional unknowns yet may be used to check the solution.



The above condition reveals that a body cannot be in rotational equilibrium under the action of a single force unless the line of action passes through the mass center of the body.

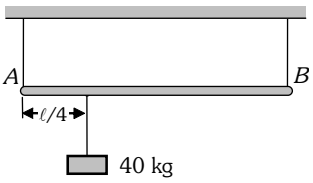
A case of particular interest arises where only three coplanar forces are involved and the body is in rotational equilibrium. It can be shown that *if a body is in rotational equilibrium under the action of three forces, the lines of action of the three forces must be either concurrent or parallel.* This condition provides us with a graphical technique to analyze rotational equilibrium.

Equilibrium of Rigid Bodies

A rigid body is said to be in equilibrium, if it is in translational as well as rotational equilibrium both. To analyze such problems conditions for both the equilibriums must be applied.

Ex. A uniform rod of 20 kg is hanging in horizontal position with the help of two threads. It also supports a 40 kg mass as shown in the figure. Find the tension developed in each thread.

Sol. Free body diagram of the rod is shown in the figure.



Translational equilibrium

$\Sigma F_y = 0 \rightarrow \quad T_1 + T_2 = 400 + 200 = 600\ N \qquad (1)$

Rotational equilibrium: Applying the condition about A, we get T_2.

$\Sigma \vec{\tau}_A = \vec{0} \rightarrow \quad 400(l/4) + 200(l/2) - T_2 l = 0$

$T_2 = 200\ \text{N}$ **Ans.**

Similarly writing torque equation about B, we have

$\Sigma \vec{\tau}_B = \vec{0} \rightarrow \quad T_1 = 400\ \text{N}.$ **Ans.**

Force and Torque equations in General Plane Motion

Motion of a rigid body either pure rotation or rotation about axis in translation can be thought and analyzed as superposition of translation of any of its particle and simultaneous rotation about an axis passing through that particle provided that the axis remain parallel to the original one. As far as kinematics in concerned this particle may or may not be the mass center. Whereas in dealing with kinetics, general plane motion is conceived as superposition of translation motion of the mass center and simultaneous centroidal rotation.

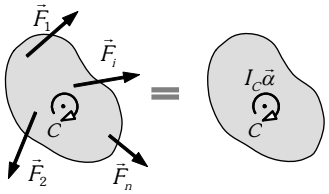
To make use the above idea and equations developed in the previous section we classify pure rotation i.e. rotation about fixed axis into two categories and deal with general plane motion as the third category.

Pure centroidal rotation: Rotation about fixed axis through mass centre

In this kind of rotation motion the axis of rotation passes through the mass center and remain fixed in space. Rotation of ceiling fan is a common example of this category. It is a subcategory of pure rotation. The axis of rotation passes through the mass center and remains fixed. In this kind of rotation the mass center of the body does not move.

In the figure, free body diagram and kinetic diagram of a body rotating about a fixed axis passing through its mass center C is shown. The mass center of the body does not accelerate; therefore we only need to write the torque equation.

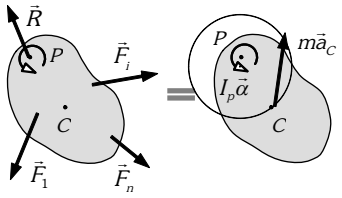
$\Sigma \vec{\tau}_C = I_C \vec{\alpha}$



Rotation about fixed axis not passing through mass center

In this kind of rotation the axis of rotation remains fixed and does not passes through the mass center. Rotation of door is a common example of this category. Doors are hinged about their edges; therefore their axis of rotation does not pass through the mass center. In this kind of rotation motion the mass center executes circular motion about the axis of rotation.

In the figure, free body diagram and kinetic diagram of a body rotating about a fixed axis through point P is shown. It is easy to conceive that as the body rotates its mass center moves on a circular path of radius $\vec{r}_{P/C}$. The mass center of the body is in translation motion with acceleration $\vec{a}_C$ on circular path of radius $r_{P/C}$. To deal with this kind of motion, we have to make use of both the force and the torque equations.



Translation of mass center $\Sigma\vec{F}_i = M\vec{a}_C = M\vec{\alpha}\times\vec{r}_{C/P} - M\omega^2\vec{r}_{C/P}$

Centroidal Rotation $\Sigma\vec{\tau}_C = I_C\vec{\alpha}$

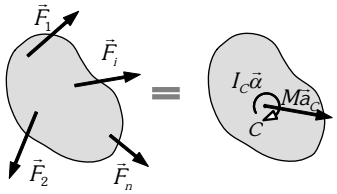
Making use of parallel axis theorem $\left(I_P = Mr_{P/C}^2 + I_C\right)$ and $\vec{a}_{C/P} = \vec{\alpha}\times\vec{r}_{C/P} - \omega^2\vec{r}_{C/P}$

we can write the following equation also.

Pure Rotation about P $\Sigma\vec{\tau}_P = I_P\vec{\alpha}$

General Plane Motion: Rotation about axis in translation motion

Rotation of bodies about an axis in translation motion can be dealt with either as superposition of translation of mass center and centroidal rotation or assuming pure rotation about the instantaneous axis of rotation. In the figure is shown the free body diagram and kinetic diagram of a body in general plane motion.



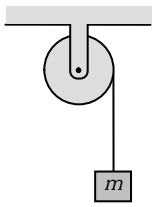
Translation of mass center $\sum_{i=0}^{n}\vec{F}_i = M\vec{a}_C$

Centroidal Rotation $\sum_{i-1}^{n}\vec{\tau}_C = I_C\vec{\alpha}$

This kind of situation can also be dealt with considering it rotation about IAR. It gives sometimes quick solutions, especially when IAR is known and forces if acting at the IAR are not required to be found.

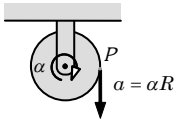
Example 10

A block of mass m is suspended with the help of a light cord wrapped over a cylindrical pulley of mass M and radius R as shown in the figure. The system is released from rest. Find the angular acceleration of the pulley and the acceleration of the block.



Solution.

After the system is released, the block is in translation motion and the pulley in rotation about an axis passing through its mass center i.e. in pure rotation.



Let the block moves vertically down with acceleration a pulling the cord down and causing the pulley to rotate clockwise. Since the cord is inextensible every point on its vertical portion and point of contact P of the pulley move down with acceleration a as shown in the adjacent figure. It is the

tangential acceleration of point P so the angular acceleration α of the pulley rotating in clockwise sense is given by

$$a = \alpha R \qquad (1)$$

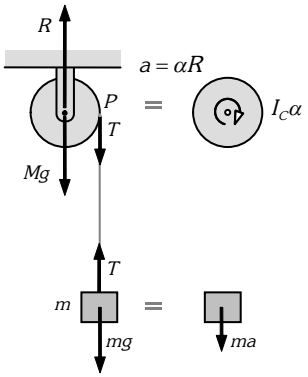
The forces acting on the pulley and on the block are shown in their free-body diagrams along with the effective torque $I_C\alpha$ of the pulley and effective force ma of the block. Here T is the tension in the string, R is the reaction by the axel of the pulley, Mg is weight of the pulley and mg is weight of the block.

The pulley is in rotation about fixed axis through its mass center so we use eq. .

$\sum \vec{\tau}_C = I_C\vec{\alpha} \rightarrow$ $\quad TR = I_C\alpha$

After substituting $I_C = \frac{1}{2}MR^2$ and α from eq. (1), we have

$$T = \tfrac{1}{2}Ma \qquad (2)$$



The block is in translation motion, so we use Newton's second law

$\sum \vec{F} = m\vec{a} \rightarrow$ $\quad mg - T = ma \qquad (3)$

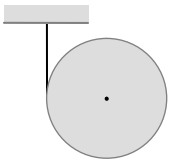
From equation (2) and (3), we have

Acceleration of the block $\quad a = \dfrac{2mg}{M+2m}$ **Ans.**

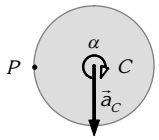
From eq. (1) and the above, we have

$$\alpha = \frac{2mg}{R(M+2m)} \qquad \textbf{Ans.}$$

Ex. A thread is wrapped around a uniform disk of radius r and mass m. One end of the thread is attached to a fixed support on the ceiling and the disk is held stationary in vertical plane below the fixed support as shown in the figure. When the disk is set free, it rolls down due to gravity. Find the acceleration of the center of the disk and tension in the thread.



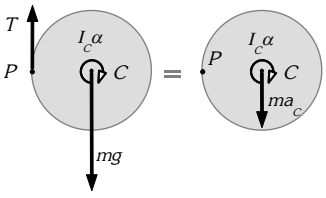
Sol. The point P, where the thread leaves the disk is always at instantaneous rest; therefore the disk can be assumed rolling without slipping with ICR at point P. Acceleration of the mass center $\vec{a}_C$ and angular acceleration of the disk are shown in the adjacent figure. Applying condition for rolling on stationary surface, we have



$\vec{a}_C = \vec{\alpha} \times \vec{r}_{C/P} \rightarrow$ $\quad a_C = \alpha r \qquad (1)$

The disk rolls down on the vertical stationary thread. Its motion can either be analyzed as superposition of translation of the mass center and simultaneous centroidal rotation or a pure rotation about ICR. Since tension, which acts at the ICR is asked; we prefer superposition of translation of the mass center and simultaneous centroidal rotation.

Forces acting on the disk are tension T applied by the thread at point P and weight of the disk. These forces and the effective force ma_C and effective torque $I_C\alpha$ are shown in the adjacent figure.



Applying Newton's second law for translation of mass center, we have

$\sum \vec{F}_i = M\vec{a}_C \rightarrow$ $\quad mg - T = ma_C \qquad (2)$

Applying torque equation for centroidal rotation, we have

$\sum \vec{\tau}_C = I_C\vec{\alpha} \rightarrow$ $\quad Tr = I_C\alpha$

Substituting $\frac{1}{2}mr^2$ for I_C and α from eq. (1), we have

$$T = \frac{1}{2}ma_C \qquad (3)$$

From eq. (2) and (3), we have

Acceleration of the mass center $a_C = \frac{2}{3}g$ **Ans.**

Tension in the string $T = \frac{1}{3}mg$ **Ans.**

Energy Methods

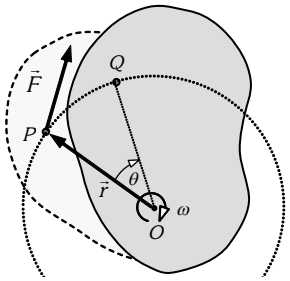
Newton's laws of motion tell us what is happening at an instant, while method of work and energy equips us to analyze what happens when a body moves from one place to other or a system changes its configuration. In this section, we introduce how to use methods of work and energy to analyze motion of rigid bodies.

Concept of Work in rotation motion

Work of a force is defined as the scalar product of the force vector and displacement vector of the point of application of the force. If during the action of a force $\vec{F}$ its point of application moves from position $\vec{r}_1$ to $\vec{r}_2$, the work $W_{1\to2}$ done by the force is expressed by the following equation.

$$W_{1\to2} = \int_{\vec{r}_1}^{\vec{r}_2} \vec{F}\cdot d\vec{r}$$

Either we can use of this idea to calculate work of a force or its modified version in terms of torque and angular displacement.

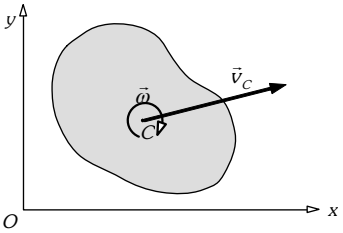


The work done by a torque during a finite rotation of the rigid body from initial value θ_i of the angle θ to final value θ_f, can be obtained by integrating both the sides of the equation given

$$W_{i\to f} = \int_{\theta_i}^{\theta_f} \vec{\tau}_o \cdot d\vec{\theta}$$

Kinetic Energy of a rigid body in rotation motion

A rigid body can be represented as a system of large number of particles, which keep their mutual distances unchanged in all circumstances. Kinetic energy of the whole body must be sum of kinetic energies of all of its particles. In this section we develop expressions for kinetic energy of a rigid body.



Kinetic Energy of a rigid body in plane motion

In the figure is shown a body in plane motion. Its mass center at an instant is moving with velocity $\vec{v}_C$ and rotating with angular velocity $\vec{\omega}$. Both these motions are shown superimposed in the given figure.

Kinetic energy too can be written as sum of kinetic energy $\left(\frac{1}{2}Mv_C^2\right)$ due to translation motion of the mass center and kinetic energy $\left(\frac{1}{2}I_C\omega^2\right)$ due to centroidal rotation.

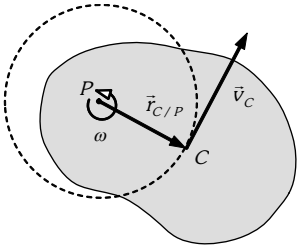
$$K = \frac{1}{2}Mv_C^2 + \frac{1}{2}I_C\omega^2$$

If location of the instantaneous axis of rotation (IAR) is known, making use of the parallel axis theorem we can write kinetic energy by the following equation also.

$$K = \tfrac{1}{2} I_{IAR}\omega^2$$

Kinetic Energy of a rigid body in rotation about fixed axis not passing through the mass centre

In this kind of motion the mass center is in circular motion about the axis of rotation. In the figure is shown a body rotation with angular velocity ω about a fixed axis through pint P and perpendicular to plane of the paper. Mass center moves with speed $v_C = \omega r$. Kinetic energy of the body can now be expressed by the following equation.



$$K = \tfrac{1}{2} M v_C^2 + \tfrac{1}{2} I_C \omega^2$$

Making use of the parallel axis theorem $\left(I_P = M r_{P/C}^2 + I_C\right)$ we can write kinetic energy by the following equation also.

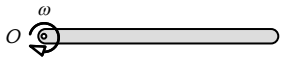
$$K = \tfrac{1}{2} I_P \omega^2$$

Kinetic Energy of a rigid body in pure centroidal rotation

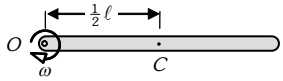
In pure centroidal rotation the mass center remain at rest; therefore kinetic energy due to translation of mass center vanishes.

$$K = \tfrac{1}{2} I_C \omega^2$$

Ex. A rod of mass m and length ℓ is pivoted to a fixed support at one of its ends O. It is rotating with constant angular velocity ω. Write expression for its kinetic energy.



Sol. If the point C is the mass center of the rod, from theorem of parallel axes, the moment of inertia I_O of the rod about the fixed axis is



$I_O = I_C + m(OC)^2 \rightarrow \quad I_O = I_C + \tfrac{1}{4} m\ell^2$

Substituting $\tfrac{1}{12} m\ell^2$ for I_C, we have

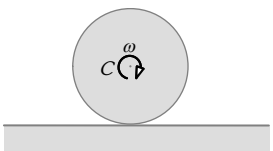
$I_O = \tfrac{1}{3} m\ell^2$

Kinetic energy of the rod equals to kinetic energy due to rotation about the fixed axis.

$K = \tfrac{1}{2} I_o \omega^2 \rightarrow$ Using above expression for I_O, we have

$K = \tfrac{1}{6} m\ell^2\omega^2$ **Ans.**

Ex. A uniform rigid body of mass m and round section of radius r is rolling on horizontal ground with angular velocity ω. Its radius of gyration about the centroidal axis is k.

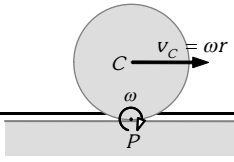


(a) Write expression of its kinetic energy.

(b) Also express the kinetic energy as sum of kinetic energy due to translation of mass center and kinetic energy due to simultaneous centroidal rotation.

Sol. *(a)* The point of contact with ground of a body rolling on the ground is its ICR. Let the point P is the ICR as shown in the adjacent figure. The geometrical center C of a uniform body and the mass center coincide. Therefore moment of inertia I_P of the body about the ICR can be written by using the theorem of parallel axes.

$I_P = I_C + m(PC)^2 \rightarrow \qquad I_P = I_C + mr^2$



Substituting $I_C = mk^2$, we have

$$I_P = m\left(k^2 + r^2\right) \qquad (1)$$

Kinetic energy of a rigid body equals to kinetic energy due to rotation about the ICR.

$K = \frac{1}{2} I_P \omega^2 \rightarrow$ Substituting I_P from eq. (1), we have

$$K = \frac{1}{2} m\left(k^2 + r^2\right)\omega^2 \qquad \textbf{Ans.}$$

(b) Kinetic energy of the body also equals to sum of kinetic energy due to translation of its mass center and kinetic energy due to simultaneous centroidal rotation.

$K = \frac{1}{2} m v_C^2 + \frac{1}{2} I_C \omega^2 \rightarrow$ Substituting condition for rolling $v_C = \omega r$ and $I_C = mk^2$, we have

$$K = \frac{1}{2} m(\omega r)^2 + \frac{1}{2} mk^2\omega^2 = \frac{1}{2} m\left(r^2 + k^2\right)\omega^2 \qquad \textbf{Ans.}$$

Rolling as rotation about an axis in translation

If the point of contact of the of the rolling body does not slide it is known as *rolling without slipping* or *pure rolling* or simply *rolling* and if the point of contact slides it is known as *rolling with slipping*. All kind of rolling motion is examples of rotation abut an axis in translation.

Rolling without slipping on stationary surface.

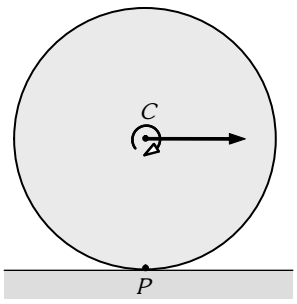
We first discuss velocity relations and thereafter accelerations relations of two points of a body of round section rolling on a stationary surface. For the purpose, we can use any of the following methods.

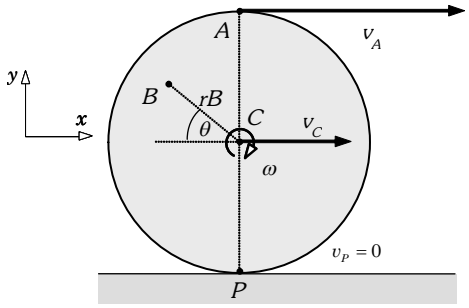
I Analytical Method: By using relative motion equations.

II Superposition Method: By superimposing translation of a point and pure rotation about that point.

Velocity relations by Analytical Method

Its point of contact *P* does not slide on the surface, therefore velocity of the point of contact relative to the surface is zero. In the next figure, velocity vectors of its center *C* and top point *A* are shown.





Velocity of the center *C* can be obtained with the help of relative motion equation.

$$\vec{v}_C = \vec{v}_P + \vec{\omega} \times \overrightarrow{PC} \rightarrow \quad \vec{v}_C = \vec{0} + \left(-\omega\hat{k}\right) \times R\hat{j}$$

$$\vec{v}_C = \omega R\hat{i} \qquad [16]$$

The above equation is used as condition of rolling without slipping on stationary surface.

Velocity of the top point *A* can be obtained by relative motion equation.

$$\vec{v}_A = \vec{v}_P + \vec{\omega} \times \overrightarrow{PA} \rightarrow \quad \vec{v}_C = \vec{0} + \left(-\omega\hat{k}\right) \times \left(2R\hat{j}\right)$$

$$\vec{v}_A = 2\omega R\hat{i} = 2\vec{v}_C \qquad [17]$$

Once velocity of the center is obtained, we can use relative motion between *A* and *C* as well.

$$\vec{v}_A = \vec{v}_C + \vec{\omega} \times \overrightarrow{CA} \rightarrow \quad \vec{v}_C = \omega R\hat{i} + \left(-\omega\hat{k}\right) \times \left(2R\hat{j}\right)$$

$$\vec{v}_A = 2\omega R\hat{i} = 2\vec{v}_C \qquad [18]$$

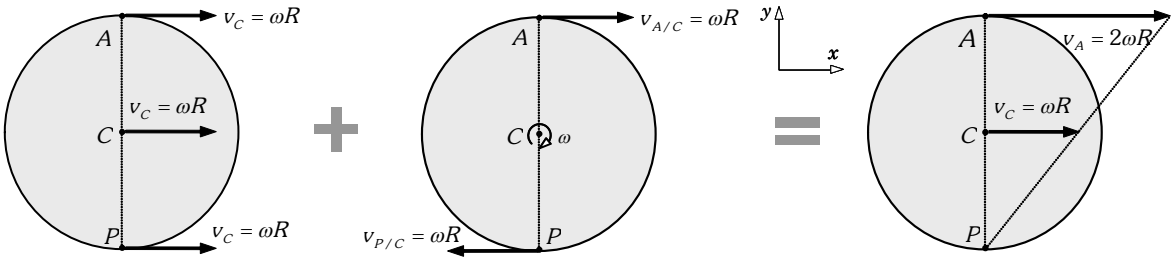
In similar fashion, velocity vector of an arbitrarily chosen point *B*

$$\vec{v}_B = \vec{v}_C + \vec{\omega} \times \overrightarrow{CB} \rightarrow \quad \vec{v}_B = v_C\hat{i} + (-\omega\hat{k}) \times (-r\cos\theta\hat{i} + r\sin\theta\hat{j})$$

$$\vec{v}_B = (v_C + \omega r\sin\theta)\hat{i} + \omega r\cos\theta\hat{j} \qquad [19]$$

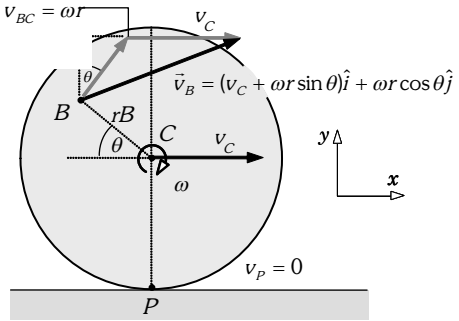
Velocity relations by Superposition Method

Now we will see that the above velocity relation can also be obtained by assuming rolling of the wheel as superposition of translation of its center and simultaneous rotation about the center.



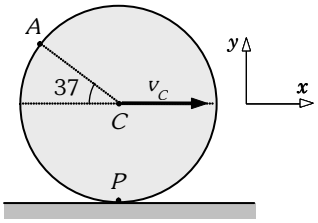
Translation of the center **Pure rotation about the center** **Rolling**

Velocity of an arbitrary point *B* as superposition of translation the center and rotation about the center.



Ex. A cylinder of radius 5 m rolls on a horizontal surface. Velocity of its center is 25 m/s. Find its angular velocity and velocity of the point *A*.

Sol. In rolling the angular velocity $\vec{\omega}$ and velocity of the center of a round section body satisfy condition described in the relative motion eq.[14]. So we have



$$\vec{v}_C = \vec{\omega} \times \vec{r}_{C/P} \rightarrow \qquad 25\hat{i} = \omega\hat{k} \times 5\hat{j} \Rightarrow \vec{\omega} = -5\hat{k}\ \text{rad/s}$$

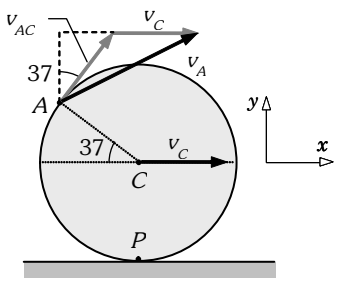
Angular velocity vector points in the negative *z*-axis so the cylinder rotates in clockwise sense.

Velocity of the point *A* can be calculated by either analytical method, superposition method.

Analytical Method

$$\vec{v}_A = \vec{v}_C + \vec{\omega} \times \overrightarrow{AC} \rightarrow \quad \vec{v}_A = 25\hat{i} + (-5\hat{k}) \times (-5\cos 37°\hat{i} + 5\sin 37°\hat{j})$$

$$\vec{v}_A = (40\hat{i} + 20\hat{j})\ \text{m/s}$$

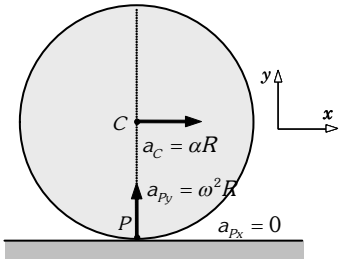


Superposition Method

In rolling $v_C = v_{AC} = \omega R = 25\text{m/s}$. The superposition i.e. vector addition of the terms of equation $\vec{v}_A = \vec{v}_C + \vec{v}_{AC}$ are shown in the following figure. Resolving $v_{AC} = \omega R = 25\text{m/s}$ into its Cartesian components and adding to $\vec{v}_C$, we obtain

$\vec{v}_A = \vec{v}_C + \vec{v}_{A/C} \rightarrow \qquad \vec{v}_A = 25\hat{i} + 15\hat{i} + 20\hat{j} = \left(40\hat{i} + 20\hat{j}\right)$ m/s

Acceleration relations by Analytical Method



The point of contact P does not slide on the surface, therefore component of its acceleration parallel to the surface must be zero. However, it has an acceleration component towards the center. The center always moves parallel to the horizontal surface and does not changes direction of its velocity; therefore, its acceleration can only be parallel to the surface.

Relation between acceleration of acceleration vector of the center C and point of contact P can be obtained with the help of relative motion [15] equation together with the above fact.

$\vec{a}_C = \vec{a}_P + \vec{\alpha} \times \overrightarrow{PC} - \omega^2 \overrightarrow{PC} \rightarrow \qquad a_C\hat{i} = a_{Py}\hat{j} + \left(-\alpha\hat{k}\right) \times R\hat{j} - \omega^2 R\hat{j} = a_{Py}\hat{j} + \alpha R\hat{i} - \omega^2 R\hat{j}$

Equating coefficients of x and y-components on both the sides of the above equation, we have

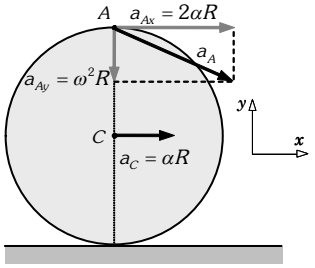
$$\vec{a}_C = \alpha R\hat{i} \qquad [20]$$

$$\vec{a}_P = \omega^2 R\hat{j} \qquad [21]$$

The eq. [20] *is used as condition for rolling without slipping together with eq.* [16]

In the given figure, acceleration vectors the point of contact; center and the top point are shown. Now we will see how these accelerations can be calculated by using relative motion equation.

Once velocity of the center is obtained, we can use relative motion between A and C as well. Now we calculate acceleration of the top point A.



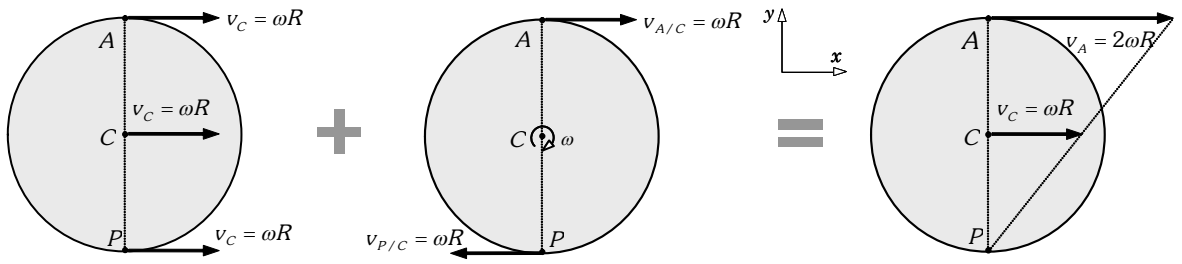
$\vec{a}_A = \vec{a}_C + \vec{\alpha} \times \overrightarrow{CA} - \omega^2 \overrightarrow{CA} \rightarrow \qquad \vec{a}_A = \alpha R\hat{i} + \left(-\alpha\hat{k}\right) \times R\hat{j} - \omega^2 R\hat{j}$

$$\vec{a}_A = 2\alpha R\hat{i} - \omega^2 R\hat{j} \qquad [22]$$

Acceleration vector of point A and its components are shown in the given figure.

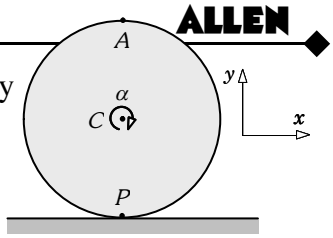
Acceleration relations by Superposition Method

Now we see how acceleration relations are expressed for a rolling wheel by assuming its rolling as superposition of its translation with the velocity of center and simultaneous rotation about the centre.



Pure Translation of the center **Pure rotation about the center** **Rolling**

Ex. A body of round section of radius 10 cm starts rolling on a horizontal stationary surface with uniform angular acceleration 2 rad/s^2.



(a) Find initial acceleration of the center *C* and top point A.

(b) Find expression for acceleration of the top point *A* as function of time.

Sol. Initially when the body starts, it has no angular velocity; therefore, the last term in relative motion equation [15] for acceleration vanishes and for a pair of two points *A* and *B* the equation reduces to

$$\vec{a}_A = \vec{a}_B + \vec{\alpha} \times \overrightarrow{BA}$$

The angular acceleration vector is $\vec{\alpha} = -2\hat{k}$ rad/s^2.

(a) Acceleration of the center *C* is obtained by using condition for rolling without slipping.

$\vec{a}_C = \vec{\alpha} \times \overrightarrow{PC} \rightarrow$ $\vec{a}_C = -2\hat{k} \times 10\hat{j} = 20\hat{i}$ cm/s^2 **Ans.**

Acceleration of the point *A* can be obtained either by analytical method, superposition method or by use of ICR. These methods for calculation of acceleration of the top point are already described; therefore, we use the result.

$\vec{a}_A = 2\alpha R\hat{i} \rightarrow$ $\vec{a}_A = 40\hat{i}$ cm/s^2 **Ans.**

(b) Initially at the instant $t = 0$, when the body starts, its angular velocity is zero. At latter time it acquires angular velocity $\vec{\omega}$, therefore acceleration of any point on the body, other than its center, has an additional component of acceleration. This additional component is accounted for by the last term in the relative motion equation [15].

Angular velocity acquired by the body at time *t* is obtained by eq.[4] used for a body rotating with constant angular acceleration.

$\vec{\omega} = \vec{\omega}_o + \vec{\alpha}t \rightarrow$ Substituting $\omega_o = 0$, we have

$\vec{\omega} = -2t\hat{k}$

Analytical Method

Using the relative motion equation for the pair of points *C* and *A*, we have

$\vec{a}_A = \vec{a}_C + \vec{\alpha} \times \overrightarrow{CA} - \omega^2 \overrightarrow{CA} \rightarrow$ $\vec{a}_A = \alpha R\hat{i} + \left(-\alpha\hat{k}\right) \times R\hat{j} - \omega^2 R\hat{j} = 2\alpha R\hat{i} - \omega^2 R\hat{j}$

Substituting the known values

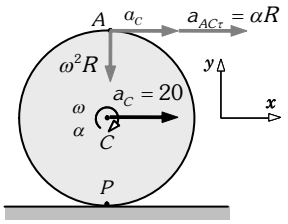
$\vec{\alpha} = -2\hat{k}$ rad/s^2, $\vec{\omega} = -2t\hat{k}$ rad/s and $R = 10$ cm

we have $\vec{a}_A = 40\hat{i} - 40t^2\hat{j}$ cm/s^2 **Ans.**

Superposition Method

We superimpose translation motion of the center and rotation motion about the center. In fact it is vector addition of terms of above equation used in analytical method.

From the above figure, we have



$\vec{a}_A = \left(a_C + \alpha R\right)\hat{i} - \omega^2 R\hat{j}$

Substituting known values $\vec{\alpha} = -2\hat{k}$ rad/s^2, $\vec{\omega} = -2t\hat{k}$ rad/s and R=10 cm,

we have $\vec{a}_A = 40\hat{i} - 40t^2\hat{j}$ cm/s^2 **Ans.**

Methods of Impulse and Momentum

Methods of impulse and momentum describe what happens over a time interval. When motion of a body involves rotation we have to consider angular impulse as well as angular momentum. In this section we discuss concept of angular impulse, angular momentum of rigid body, angular impulse

momentum principle and conservation of angular momentum.

Angular Impulse

Like impulse of a force angular impulse of a constant torque equals to product of the torque and concerned time interval and if the torque is not constant it must be integrated with time over the concerned time interval.

If torque $\vec{\tau}_o$ about an axis passing through O is constant, its angular impulse during a time interval from t_1 to t_2 denoted by $\vec{J}_{o,1\to2}$ is given by the following equation.

$$\vec{J}_{o,1\to2} = \vec{\tau}_o\left(t_2 - t_1\right)$$

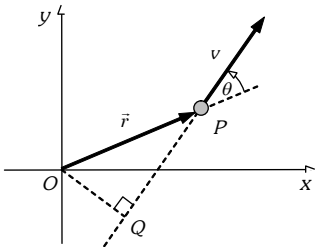
If torque $\vec{\tau}_o$ about an axis passing through O is time varying, its angular impulse during a time interval from t_1 to t_2 denoted by $\vec{J}_{o,1\to2}$ is given by the following equation.

$$\vec{J}_{o,1\to2} = \int_{t_1}^{t_2} \vec{\tau}_o dt$$

Angular momentum of a particle

Angular momentum $\vec{L}_o$ about the origin O of a particle of mass m moving with velocity $\vec{v}$ is defined as the moment of its linear momentum $\vec{p} = m\vec{v}$ about the point O.

$$\vec{L}_o = \vec{r} \times \left(m\vec{v}\right)$$



Angular Momentum of a Rigid Body

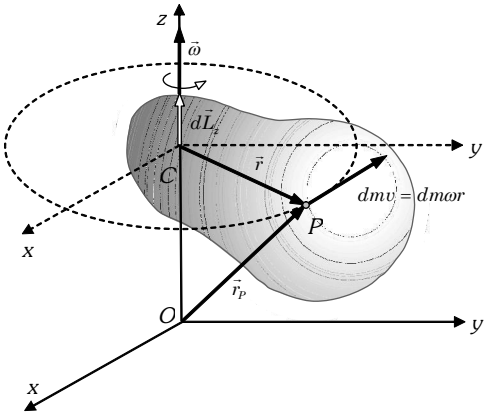
Angular momentum is quantity of rotation motion in a body. The angular momentum of a system of particles is the sum of angular momentum all the particles within the system. A rigid body is an assemblage of large number of particles maintaining their mutual distances intact under all circumstances, therefore angular momentum of a rigid body must be sum of angular momenta of all of its particles.

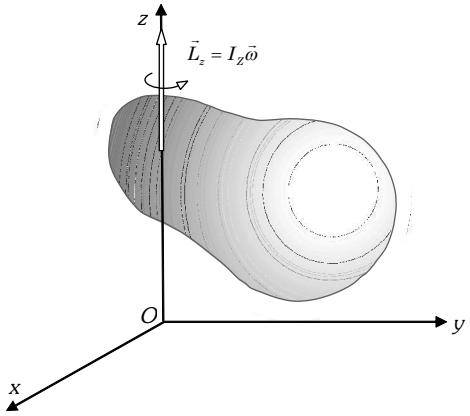
Angular Momentum about a point and about an axis

Angular momentum of a particle is not defined about an axis instead it is defined about a point. Therefore above idea of summing up angular momenta of all the particles about a point gives angular momentum of the rigid body about a point. But while dealing with fixed axis rotation or rotation about axis in translation we need angular momentum about an axis.

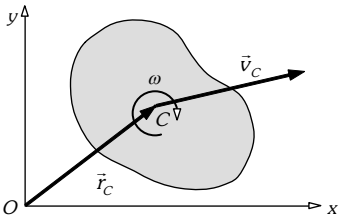
Angular momentum about an axis is calculated similar to torque abut an axis. To calculate angular momentum of a particle of rigid body about an axis we take moment of momentum of the particle about the point where plane of motion of the point of application of the force intersects the axis.

In the following figure is shown angular momentum $d\vec{L}_z = \vec{r} \times (dm\vec{v}) = r^2 dm\omega$ of a particle P of a rigid body rotating about the z-axis. It is along the z-axis i.e. axis of rotation. In the next figure total angular momentum $\vec{L}_z = \int d\vec{L}_z = I_z\vec{\omega}$ about the axis of rotation is shown. It is also along the axis of rotation.





Angular Momentum in general plane motion



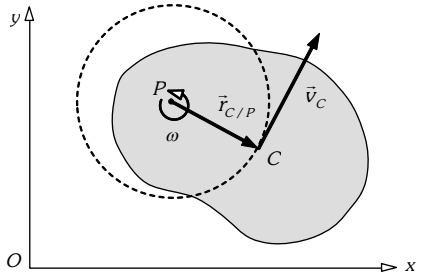
Angular momentum of a body in plane motion can also be written similar to torque equation or kinetic energy as sum of angular momentum about the axis due to translation of mass center and angular momentum of centroidal rotation about centroidal axis parallel to the original axis.

Consider a rigid body of mass M in plane motion. At the instant shown its mass center has velocity $\vec{v}$ and it is rotating with angular velocity $\vec{\omega}$ about an axis perpendicular to the plane of the figure. It angular momentum $\vec{L}_o$ about an axis passing though the origin and parallel to the original is expressed by the following equation.

$$\vec{L}_o = \vec{r}_C \times \left(M\vec{v}_C\right) + I_C\vec{\omega}$$

The first term of the above equation represent angular momentum due to translation of the mass center and the second term represents angular momentum in centroidal rotation.

Angular momentum in rotation about fixed axis

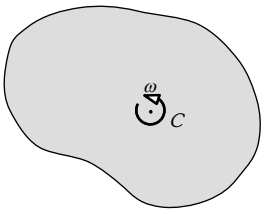


Consider a body of mass M rotating with angular velocity ωabout a fixed axis perpendicular to plane of the figure passing through point P. Making use of the parallel axis theorem $I_P = Mr_{C/P}^2 + I_C$ and equation $\vec{v}_C = \vec{\omega}\times\vec{r}_{C/P}$ we can express the angular momentum $\vec{L}_P$ of the body about the fixed rotational axis.

$$\vec{L}_P = I_P\vec{\omega}$$

The above equation reveals that the angular momentum of a rigid body in plane motion can also be expressed in a single term due to rotation about the instantaneous axis of rotation.

Angular momentum in pure centroidal rotation



In pure centroidal rotation, mass center remains at rest, therefore angular momentum due to translation of the mass center vanishes.

$$\vec{L}_C = I_C\vec{\omega}$$

Rotational Equivalent of the Newton's Laws of Motion

Differentiating terms on both the sides of equation $\vec{L}_o = \vec{r}_C \times \left(M\vec{v}_C\right) + I_C\vec{\omega}$ with respect to time, and making substitution of $\vec{v}_C = d\vec{r}_C/dt$, $\vec{a}_C = d\vec{v}_C/dt$ and $\vec{\alpha} = d\vec{\omega}/dt$ we have

$$\frac{d\vec{L}_o}{dt} = \vec{v}_C \times \left(M\vec{v}_C\right) + \vec{r}_C \times M\vec{a}_C + I_C\vec{\alpha}$$

The first on the right hand side vanishes, so we can write

$$\frac{d\vec{L}_o}{dt} = \vec{r}_C \times M\vec{a}_C + I_C\vec{\alpha}$$

Now comparing the above equation with torque equation $\Sigma\vec{\tau}_o = \vec{r}_C \times M\vec{a}_C + I_C\vec{\alpha}$, we have

$$\sum \vec{\tau}_o = \frac{d\vec{L}_o}{dt}$$

The above equation though developed for plane motion only yet is valid for rotation about an axis in rotation also. It states that the net torque about the origin of an inertial frame equals to the time rate of change in angular momentum about the origin and can be treated as a parallel to Newton's second law which states that net external force on a body equals to time rate of change in its linear momentum.

Angular Impulse Momentum Principle

Rearranging the terms and integrating both the sides obtained form previous equation, we can write

$$\Sigma\int_{t_1}^{t_2} \vec{\tau}_o dt = \vec{L}_{o2} - \vec{L}_{o1}$$

The left hand side of the above equation is the angular impulse of torque of all the external forces in the time interval in the time interval t_1 to t_2.

$$\Sigma\vec{J}_{o,1\to2} = \vec{L}_{o2} - \vec{L}_{o1}$$

The idea expressed by the above equation is known as angular impulse momentum principle and states that increment in the angular momentum of a body about a point in a time interval equals to the net angular impulse of all the external forces acting on it during the concerned time interval.

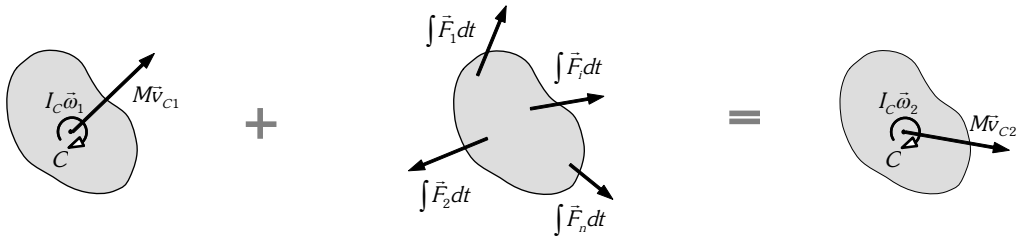
For the ease of application the above equation is rearranged as

$$\vec{L}_{o1} + \sum \vec{J}_{o,1\to2} = \vec{L}_{o2}$$

Like linear impulse momentum principle, the angular impulse momentum principle provides us solution of problems concerned with change in angular velocity in a time interval or change in angular velocity during very short interval interactions.

Method of Impulse Momentum Principle for Plane motion of a Rigid Body

Linear momentum and angular momentum serve as measures of amount of translation and rotation motion respectively. The external forces acting on a rigid body can change its state of translation as well as rotation motion which is reflected by change in linear as well as angular momentum according to the principles of linear impulse and momentum and angular impulse and momentum.



Linear and angular momenta at the instant t_1 | **Impulse of all the forces during time interval t_1 to t_2** | **Linear and angular momenta at the instant t_2**

In the above figure is shown strategy to apply method of impulse and momentum. Consider a rigid body of mass M in plane motion. Its moment of inertial about the centroidal axis perpendicular to plane of motion is I_C. Let $\vec{v}_{C1}$ and $\vec{\omega}_1$ represent velocity of its mass center and its angular velocity at the beginning of a time interval t_1 to t_2. Under the action of several forces $\vec{F}_1$, $\vec{F}_2$ ..... $\vec{F}_i$ ..... $\vec{F}_n$ during the time interval its mass center velocity and angular velocity become $\vec{v}_{C2}$ and $\vec{\omega}_2$ respectively.

The adjacent figure shows strategy representing how to write equations for linear and angular impulse momentum principles.

While applying the principle it becomes simpler to consider translation of the mass center and centroidal rotation separately. Thus in an alternative way we apply linear impulse momentum principle for translation of the mass center and angular impulse momentum principle for centroidal rotation.

Translation of mass center: Linear impulse momentum principle.

$$\vec{p}_1 + \sum \vec{I}_{mp1\to 2} = \vec{p}_2$$

Here $\vec{p}_1 = M\vec{v}_{C1}$ and $\vec{p}_2 = M\vec{v}_{C2}$ represent linear momentums at the beginning and end of the time interval and $\sum \vec{I}_{mp1\to 2}$ stands for impulse of all the external forces during the time interval.

Centroidal rotation: Angular impulse momentum principle.

$$\vec{L}_{C1} + \sum \vec{J}_{C,1\to 2} = \vec{L}_{C2}$$

Here $\vec{L}_{C1} = I_C\vec{\omega}_1$ and $\vec{L}_{C2} = I_C\vec{\omega}_2$ represent angular momentums about the centroidal axis at the beginning and end of the time interval and $\sum \vec{J}_{C,1\to 2}$ stands for angular impulse of all the external forces about the centroidal axis during the time interval.

Conservation of Angular Momentum

If angular impulse of all the external forces about an axis in time interval vanishes, the angular momentum of the system about the same axis in that time interval remain unchanged.

$$\text{If } \Sigma\int_{t_1}^{t_2} \vec{\tau}_o dt = 0 \text{, we have } \vec{L}_{o1} = \vec{L}_{o2}$$

The condition of zero net angular impulse required for conservation of angular momentum can be fulfilled in the following cases.

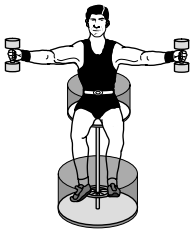
- If no external force acts, the angular impulse about all axes will be zero and hence angular momentum remains conserved about all axes.
- If net torque of all the external forces or torques of each individual force about an axis vanishes the angular momentum about that axes will be conserved.
- If all the external forces are finite in magnitude and the concerned time interval is infinitely small, the angular momentum remain conserved.
- If a system of rigid bodies changes its moment of inertia by changing its configuration due to internal forces only its angular momentum about any axes remains conserved. If we denote the moment of inertias in two configurations by I_1 and I_2 and angular velocities by ω_1 and ω_2, we can write

$$I_1\vec{\omega}_1 = I_2\vec{\omega}_2$$

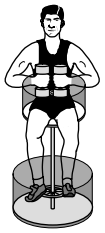
The principle of conservation of angular momentum governs a wide range of physical processes from subatomic to celestial world. The following examples explicate some of these applications.

Student on rotating turntable

The student, the turntable and dumbbells make an isolated system on which no external torque acts, if we ignore friction in the bearing of the turntable and air friction. Initially the student has his arm stretched on rotating turntable. When he pulls dumbbells close to his body, angular velocity increases due to conservation of angular momentum.

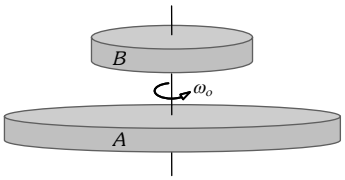


Larger moment of inertia and smaller angular velocity



Smaller moment of inertia and larger angular velocity

Ex. Consider the disk A of moment of inertia I_1 rotating freely in horizontal plane about its axis of symmetry with angular velocity ω_o. Another disk B of moment of inertia I_2 held at rest above the disk A. The axis of symmetry of the disk B coincides with that of the disk A as shown in the figure. The disk B is released to land on the disk A. When sliding stops, what will be the angular velocity of both the disks?



Sol. Both the disks are symmetric about the axis of rotation therefore does not require any external torque to keep the axis stationary. When the disk B lands on A slipping starts. The force of friction provides an internal torque to system of both the disk. It slows down rotation rate of A and increases that of B till both acquire same angular velocity ω.

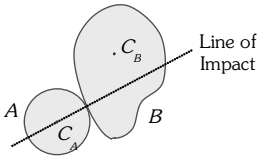
Since there is no external torques on the system of both the disks about the axis of rotation, the total angular momentum of the system remains conserved. The total angular momentum of the system is the sum of angular momentum of both disks. Denoting the angular momentum of the disk A before B lands on it and long after slipping between them stops by symbols $\vec{L}_{A1}$, $\vec{L}_{B1}$, $\vec{L}_{A2}$ and $\vec{L}_{B2}$ respectively, we can express conservation of angular momentum by the following equation.

$$\vec{L}_{A1}+\vec{L}_{B1}=\vec{L}_{A2}+\vec{L}_{B2} \rightarrow \; I_1\omega_o+0=I_1\omega+I_2\omega \Rightarrow \omega=\frac{I_1\omega_o}{I_1+I_2}$$ **Ans.**

Eccentric Impact

In eccentric impact the line of impact which is the common normal drawn at the point of impact does not passes through mass center of at least one of the colliding bodies. It involves change in state of rotation motion of either or both the bodies.

Consider impact of two A and B such that the mass center C_B of B does not lie on the line of impact as shown in figure. If we assume bodies to be frictionless their mutual forces must act along the line of impact. The reaction force of A on B does not passes through the mass center of B as a result state of rotation motion of B changes during the impact.



Problems of Eccentric Impact

Problems of eccentric impact can be divided into two categories. In one category both the bodies under going eccentric impact are free to move. No external force act on either of them. There mutual forces are responsible for change in their momentum and angular momentum. In another category either or both of the bodies are hinged.

Eccentric Impact of bodies free to move

Since no external force acts on the two body system, we can use principle of conservation of linear momentum, principle of conservation of angular momentum about any point and concept of coefficient of restitution.

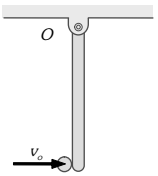
The coefficient of restitution is defined for components of velocities of points of contacts of the bodies along the line of impact.

While applying principle of conservation of angular momentum care must be taken in selecting the point about which we write the equation. The point about which we write angular momentum must be at rest relative to the selected inertial reference frame and as far as possible its location should be selected on line of velocity of the mass center in order to make zero the first term involving moment of momentum of mass center.

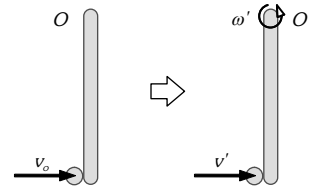
Eccentric Impact of hinged bodies

When either or both of the bodies are hinged the reaction of the hinge during the impact act as external force on the two body system, therefore linear momentum no longer remain conserved and we cannot apply principle of conservation of linear momentum. When both the bodies are hinged we cannot also apply conservation of angular momentum, and we have to use impulse momentum principle on both the bodies separately in addition to making use of coefficient of restitution. But when one of the bodies is hinged and other one is free to move, we can apply conservation of angular momentum about the hinge.

Ex. A uniform rod of mass m and length ℓ is suspended from a fixed support and can rotate freely in the vertical plane. A small ball of mass m moving horizontally with velocity v_o strikes elastically the lower end of the rod as shown in the figure. Find the angular velocity of the rod and velocity of the ball immediately after the impact.



Sol. The rod is hinged and the ball is free to move. External forces acting on the rod ball system are their weights and reaction from the hinge. Weight of the ball as well as the rod are finite and contribute negligible impulse during the impact, but impulse of reaction of the hinge during impact is considerable and cannot be neglected. Obviously linear momentum of the system is not conserved. The angular impulse of the reaction of hinge about the hinge is zero. Therefore angular momentum of the system about the hinge is conserved. Let velocity of the ball after the impact becomes v'_B and angular velocity of the rod becomes ω'.



Before the impact Immediately after the impact

We denote angular momentum of the ball and the rod about the hinge before the impact by L_{B1} and L_{R1} and after the impact by L_{B2} and L_{R2}.

Applying conservation of angular momentum about the hinge, we have

$$\vec{L}_{B1}+\vec{L}_{R1}=\vec{L}_{B2}+\vec{L}_{R2}\rightarrow \quad mv_o\ell+0=mv'_B\ell+I_o\omega'$$

Substituting $\frac{1}{3}M\ell^2$ for I_o, we have

$$3mv'_B+M\ell\omega'=3mv_o \qquad (1)$$

The velocity of the lower end of the rod before the impact was zero and immediately after the impact it becomes $\ell\omega'$ towards right. Employing these facts we can express the coefficient of restitution according to eq.

$$e=\frac{v'_{Qn}-v'_{Pn}}{v_{pn}-v_{Qn}}\rightarrow \qquad \ell\omega'-v'_B=ev_o \qquad (2)$$

From eq. (1) and (2), we have

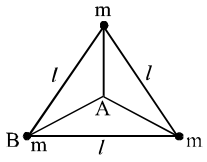
Velocity of the ball immediately after the impact $\quad v'_B=\dfrac{(3m-eM)v_o}{3m+M}$ **Ans.**

Angular velocity of the rod immediately after the impact $\quad \omega'=\dfrac{3(1+e)mv_o}{(3m+M)\ell}$ **Ans.**

EXERCISE (S)

Moment of inertia

1. Three equal masses m are rigidly connected to each other by massless rods of length l forming an equilateral triangle, as shown in the figure. What is the ratio of the moment of inertia of the assembly for an axis through B compared with that for an axis through A (centroid). Both the axis are perpendicular to the plane of triangle.



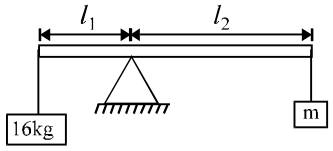
RD0001

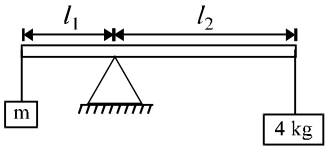
2. Two rods of equal mass m and length l lie along the x axis and y axis with their centres at origin. What is the moment of inertia of the system about the line x=y :

RD0002

Equilibrium

3. In an experiment with a massless beam balance an unknown mass m is balanced by two known masses of 16kg and 4 kg as shown in figure.Find the value of the unknown mass m.

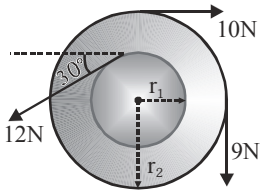




RD0004

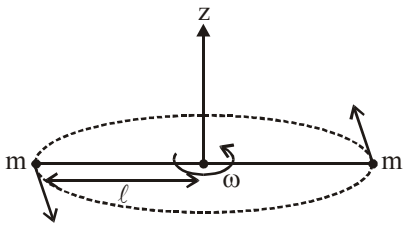
$\tau = I\alpha$ and its calculation

4. In the following figure r_1 and r_2 are 5 cm and 30 cm respectively. If the moment of inertia of the wheel is 5100 kg-m^2 about the axis passing through its centre and perpendicular to the plane of wheel, then what will be its angular acceleration?



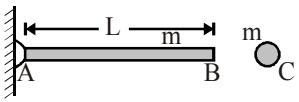
RD0007

5. A dumbbell consists of two identical particles of mass m connected by a rigid light rod of length 2ℓ. The dumbbell is set spinning with angular speed ω_0 on a surface with a small friction coefficient μ_k. If dumbbell stops in time $t = \frac{K\omega_0\ell}{2\mu g}$ where K is a constant, then find the value of K.



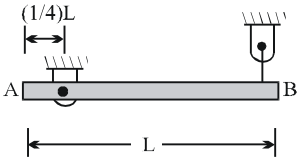
RD0008

6. A uniform bar AB of mass m and a ball of the same mass are released from rest from the same horizontal position. The bar is hinged at end A. There is gravity downwards. What is the distance of the point from point B that has the same acceleration as that of ball, immediately after release?



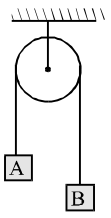
RD0009

7. A uniform beam of length L and mass m is supported as shown. If the cable at B suddenly breaks, determine; (a) the acceleration of end B. (b) the reaction at the pin support.



RD0010

8. In the figure, A & B are two blocks of mass 4 kg & 2 kg respectively attached to the two ends of a light string passing over a disc C of mass 40 kg and radius 0.1 m. The disc is free to rotate about a fixed horizontal axes, coinciding with its own axis. The system is released from rest and the string does not slip over the disc. Find: (i) the linear acceleration of mass B .

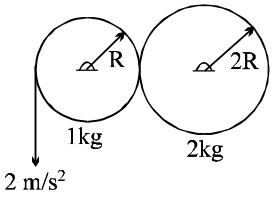


(ii) the number of revolutions made by the disc at the end of 10sec. from the start.

(iii) the tension in the string segment supporting the block A.

RD0011

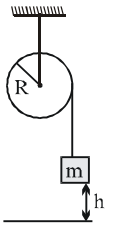
9. Two discs A and B touch each other as shown in figure. A rope tightly wound on A is pulled down at 2 m/s^2. Find the friction force between A and B if slipping is absent



RD0012

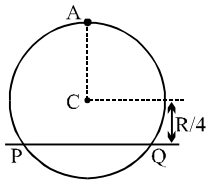
Kinetic energy in pure rotation

10. A mass m is attached to a pulley through a cord as shown in the figure. The pulley is a solid disk with radius R. The cord does not slip on the disk. The mass is released from rest at a height h from the ground and at the instant the mass reaches the ground, the disk is rotating with angular velocity ω. Find the mass of the disk.



RD0013

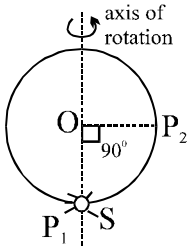
11. A uniform circular disc has radius R and mass m. A particle also of mass m is fixed at a point A on the edge of the disc as in figure. The disc can rotate freely about a fixed horizontal chord PQ that is at a distance R/4 from the centre C of the disc. The line AC is perpendicular to PQ. Initially the disc is held vertical with the point A at its highest position. It is then allowed to fall so that it starts rotating about PQ. Find the linear speed of the particle at it reaches its lowest position.



RD0014

Angular momentum and its conservation

12. A uniform ring is rotating about vertical axis with angular velocity ω initially. A point insect (S) having the same mass as that of the ring starts walking from the lowest point P_1 and finally reaches the point P_2(as shown in figure). What is the final angular velocity of the ring?

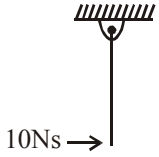


RD0016

13. Two men, each of mass 75 kg , stand on the rim of a horizontal large disc, diametrically opposite to each other. The disc has a mass 450 kg and is free to rotate about its axis. Each man simultaneously start along the rim clockwise with the same speed and reaches their original starting points on the disc. Find the angle turned by the disc with respect to the ground in this duration.

RD0017

14. A thin uniform straight rod of mass 2 kg and length 1 m is free to rotate about its upper end when at rest. It receives an impulsive blow of 10 Ns at its lowest point, normal to its length as shown in figure. Find the kinetic energy of rod just after impact.



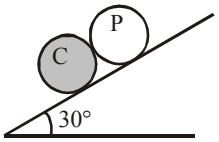
RD0018

Combined rotation and translation

15. A solid uniform disk of mass m rolls down a fixed inclined plane without slipping with an acceleration a. Find the frictional force on the disk due to surface of the plane :

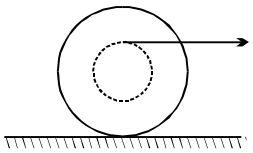
RD0019

16. A solid cylinder C and a hollow pipe P of same diameter are in contact when they are released from rest as shown in the figure on a long incline plane. Cylinder C and pipe P roll without slipping. Determine the clear gap (in m) between them after 6 seconds.



RD0020

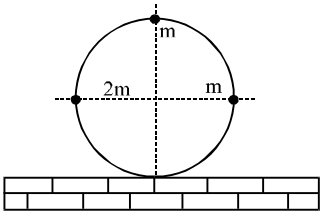
17. A spool of inner radius R and outer radius 3R has a moment of inertia = MR^2 about an axis passing through its geometric centre, where M is the mass of the spool. A thread wound on the inner surface of the spool is pulled horizontally with a constant force = Mg. Find the acceleration of the point on the thread which is being pulled assuming that the spool rolls purely on the floor.



RD0021

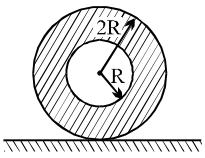
Kinetic energy in rolling

18. A ring of mass m and radius R has three particles attached to the ring as shown in the figure. The centre of the ring has a speed v_0. Find the kinetic energy of the system. (Slipping is absent)



RD0022

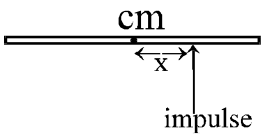
19. A hollow cylinder with inner radius R, outer radius 2R and mass M is rolling with speed of its axis v. Its kinetic energy is :-



RD0023

Angular impulse

20. A uniform rod of length *l* is given an impulse at right angle to its length as shown. Find the distance of instantaneous centre of rotation from the centre of the rod.



RD0024

21. A solid sphere of mass m and radius R is placed on a smooth horizontal surface. A sudden blow is given horizontally to the sphere at a height h = 4R/5 above the centre line. If I is the impulse of the blow then find

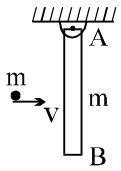
(a) the minimum time after which the highest point B will touch the ground

(b) the displacement of the centre of mass during this internal.

RD0026

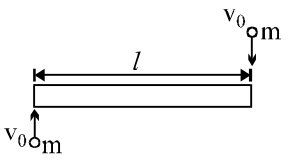
Eccentric collision

22. A uniform rod AB of length L and mass m is suspended freely at A and hangs vertically at rest when a particle of same mass m is fired horizontally with speed v to strike the rod at its mid point. If the particle comes to rest after the impact, then find the impulsive reaction at A.



RD0027

23. On a smooth table two particles of mass m each, travelling with a velocity v_0 in opposite directions, strike the ends of a rigid massless rod of length *l*, kept perpendicular to their velocity. The particles stick to the rod after the collision.Find the tension in rod during subsequent motion.



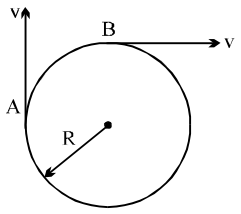
RD0028

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

Conecpt of ω, α, v, a

1. Two points of a rigid body are moving as shown. The angular velocity of the body is:



(A) $\frac{\upsilon}{2R}$ (B) $\frac{\upsilon}{R}$ (C) $\frac{2\upsilon}{R}$ (D) $\frac{2\upsilon}{3R}$

RD0055

Moment of inertia

2. Three bodies have equal mass m. Body A is solid cylinder of radius R, body B is a square lamina of side R, and body C is a solid sphere of radius R. Which body has the smallest moment of inertia about an axis passing through their centre of mass and perpendicular to the plane (in case of lamina)

(A) A (B) B (C) C (D) A and C both

RD0056

3. For the same total mass which of the following will have the largest moment of inertia about an axis passing through its centre of mass and perpendicular to the plane of the body

(A) a disc of radius a (B) a ring of radius a

(C) a square lamina of side 2a (D) four rods forming a square of side 2a

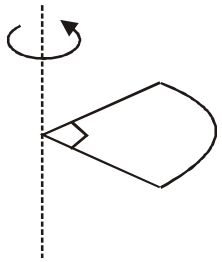
RD0057

4. Let I be the moment of inertia of a uniform square plate about an axis AB that passes through its centre and is parallel to two of its sides. CD is a line in the plane of the plate that passes through the centre of the plate and makes an angle θ with AB. The moment of inertia of the plate about the axis CD is then equal to

(A) I (B) $I\sin^2\theta$ (C) $I\cos^2\theta$ (D) $I\cos^2(\theta/2)$

RD0058

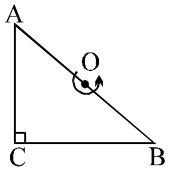
5. One quarter sector is cut from a uniform circular disc of radius R. This sector has mass M. It is made to rotate about a line perpendicular to its plane and passing through the center of the original disc. Its moment of inertia about the axis of rotation is **[IIT-JEE 2000]**



(A) $\frac{1}{2}$ MR^2 (B) $\frac{1}{4}$MR^2 (C) $\frac{1}{8}$ MR^2 (D) $\sqrt{2}$ MR^2

RD0059

6. Find the moment of inertia of a plate cut in shape of a right angled triangle of mass M, about an axis perpendicular to the plane of the plate and passing through the mid point of side AB. (Side AC = BC = a)



(A) $\frac{Ma^2}{12}$ (B) $\frac{Ma^2}{6}$ (C) $\frac{Ma^2}{3}$ (D) $\frac{2Ma^2}{3}$

RD0060

7. A circular disc X of radius R is made from an iron plate of thickness t and another disc Y of radius 4R is made from an iron plate of thickness t/4. Then the relation between the moment of inertia I_X and I_Y is- **[AIEEE - 2003]**

(A) $I_Y = 32\ I_X$ (B) $I_Y = 16 I_X$ (C) $I_Y = I_X$ (D) $I_Y = 64\ I_X$

RD0061

8. A thin uniform rod of mass M and length L has its moment of inertia I_1 about its perpendicular bisector. The rod is bend in the form of a semicircular arc. Now its moment of inertia through the centre of the semi circular arc and perpendicular to its plane is I_2. The ratio of $I_1 : I_2$ will be _____

(A) < 1 (B) > 1 (C) = 1 (D) can't be said

RD0063

9. One solid sphere A and another hollow sphere B are of same mass and same outer radii. Their moment of inertia about their diameters are respectively I_A and I_B such that- **[AIEEE-2004]**

(A) $I_A = I_B$ (B) $I_A > I_B$ (C) $I_A < I_B$ (D) $\frac{I_A}{I_B} = \frac{d_A}{d_B}$

where d_A and d_B are their densities.

RD0064

10. A rigid body can be hinged about any point on the x-axis. When it is hinged such that the hinge is at x, the moment of inertia is given by $I = 2x^2 - 12x + 27$. The x-coordinate of centre of mass is

(A) x = 2 (B) x = 0 (C) x = 1 (D) x = 3

RD0066

Equilibrium

11. A weightless rod is acted on by upward parallel forces of 2N and 4N at ends A and B respectively. The total length of the rod is AB = 3m. To keep the rod in equilibrium a force of 6N should act in the following manner:

(A) Downwards at any point between A and B.

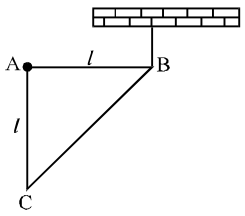
(B) Downwards at mid point of AB.

(C) Downwards at a point C such that AC = 1m.

(D) Downwards at a point D such that BD = 1m.

RD0067

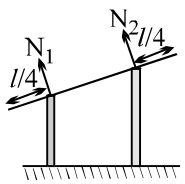
12. A right triangular plate ABC of mass m is free to rotate in the vertical plane about a fixed horizontal axis through A. It is supported by a string such that the side AB is horizontal. The reaction at the support A in equilibrium is:



(A) $\frac{mg}{3}$ (B) $\frac{2mg}{3}$ (C) $\frac{mg}{2}$ (D) mg

RD0068

13. A uniform rod of length l is placed symmetrically on two walls as shown in figure. The rod is in equilibrium. If N_1 and N_2 are the normal forces exerted by the walls on the rod then :-

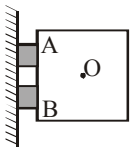


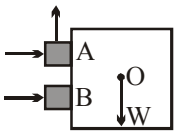
(A) $N_1 > N_2$ (B) $N_1 < N_2$

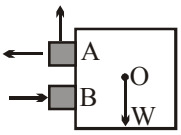
(C) $N_1 = N_2$ (D) N_1 and N_2 would be in the vertical directions

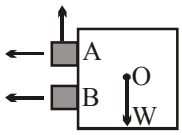
RD0069

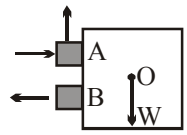
14. A vertical rectangular door with its centre of gravity at O (see figure) is fixed on two hinges A and B along one vertical length side of the door. The entire weight of the door is supported by the hinge A. Then the free body force diagram for the door (the arrows indicate the direction of the forces) is :-



(A) 

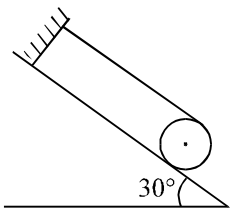
(B) 

(C) 

(D) 

RD0070

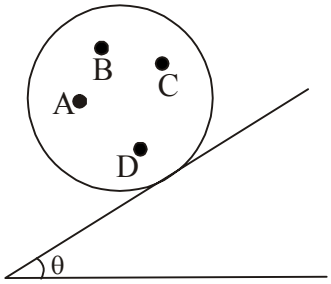
15. A thin hoop of weight 500 N and radius 1 m rests on a rough inclined plane as shown in the figure. The minimum coefficient of friction needed for this configuration to be in equilibrium is:



(A) $\frac{1}{3\sqrt{3}}$ (B) $\frac{1}{\sqrt{3}}$ (C) $\frac{1}{2}$ (D) $\frac{1}{2\sqrt{3}}$

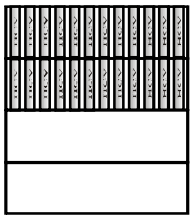
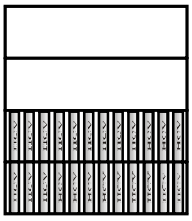
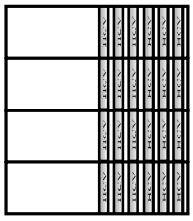
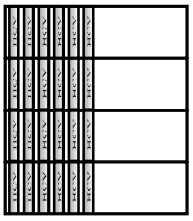
RD0071

16. A non uniform sphere can be kept on a rough inclined plane so that it is in equilibrium. In the figure below the dots represents location of centre of mass. In which one of the positions can sphere be in equilibrium?



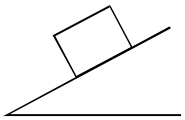
RD0072

17. Same number of books are placed in four book cases as shown. Which bookcase is most likely to topple forward if pulled a little at the top towards right :

(A)  (B)  (C)  (D) 

RD0073

18. A homogeneous cubical brick lies motionless on a rough inclined surface. The half of the brick which applies greater pressure on the plane is :

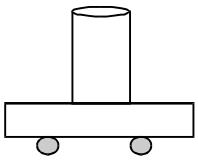


(A) left half (B) right half

(C) both applies equal pressure (D) the answer depend upon coefficient of friction

RD0074

19. A uniform 2 kg cylinder rests on a laboratory cart as shown. The coefficient of static friction between the cylinder and the cart is 0.5. If the cylinder is 4 cm in diameter and 10 cm in height, which of the following is closest to the maximum acceleration of the cart such that cylinder neither slips nor tips over?

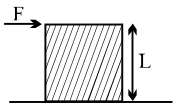


(A) 2 m/s^2 (B) 4 m/s^2 (C) 5 m/s^2 (D) 6 m/s^2

RD0075

20. A cubical block of side L rests on a rough horizontal surface with coefficient of friction μ. A horizontal force F is applied on the block as shown. If the coefficient of friction is sufficiently high so that the block does not slide before toppling, the minimum force required to topple the block is:

[IIT-JEE'(Scr)'2000]

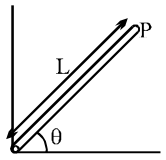


(A) infinitesimal (B) mg/4 (C) mg/2 (D) mg(1-μ)

RD0076

$\tau = I\alpha$

21. A uniform flag pole of length L and mass M is pivoted on the ground with a frictionless hinge. The flag pole makes an angle θ with the horizontal. The moment of inertia of the flag pole about one end is $(1/3)ML^2$. If it starts falling from the position shown in the accompanying figure, the linear acceleration of the free end of the flag pole (labeled P) immediately after it starts falling off would be:



(A) $(2/3)\ g\cos\theta$ (B) $(2/3)\ g$ (C) g (D) $(3/2)\ g\cos\theta$

RD0077

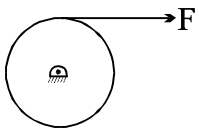
22. A pulley is hinged at the centre and a massless thread is wrapped around it. The thread is pulled with a constant force F starting from rest. As time increases,

(A) its angular velocity increases, but force on hinge remains constant

(B) its angular velocity remains same, but force on hinge increases

(C) its angular velocity increases and force on hinge increases

(D) its angular velocity remains same and force on hinge is constant



RD0078

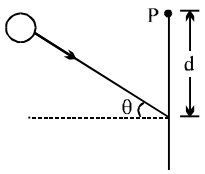
Angular momentum and its conservation

23. A particle of mass 2 kg located at the position $(\hat{i}+\hat{j})$ m has a velocity $2(+\hat{i}-\hat{j}+\hat{k})$ m/s. Its angular momentum about z-axis in kg-m^2/s is:

(A) zero (B) +8 (C) 12 (D) –8

RD0079

24. A ball of mass m moving with velocity v, collide with the wall elastically as shown in the figure. After impact the change in angular momentum about P is:



(A) 2 *mvd* (B) 2 *mvd* $\cos\theta$ (C) 2 *mvd* $\sin\theta$ (D) zero

RD0080

25. Two uniform spheres of mass M have radii R and 2R. Each sphere is rotating about a fixed axis through a diameter. The rotational kinetic energies of the spheres are identical. What is the ratio of the magnitude of the angular momenta of these spheres? That is, $\frac{L_{2R}}{L_R} =$

(A) 4 (B) $2\sqrt{2}$ (C) 2 (D) $\sqrt{2}$

RD0081

26. A spinning ice skater can increase his rate of rotation by bringing his arms and free leg closer to his body. How does this procedure affect the skater's angular momentum and kinetic energy and what is the work done by the skater?

(A) angular momentum remains the same while kinetic energy increases and work done is positive.

(B) angular momentum remains the same while kinetic energy decreases and work done is negative.

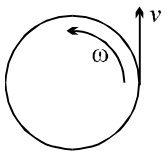
(C) both angular momentum and kinetic energy remain the same and work done is zero.

(D) angular momentum increases while kinetic energy remains the same and work done may be positive or negative.

RD0082

27. A child with mass m is standing at the edge of a disc with moment of inertia I, radius R, and initial angular velocity ω. See figure given below. The child jumps off the edge of the disc with tangential velocity v with respect to the ground. The new angular velocity of the disc is

(A) $\sqrt{\dfrac{I\omega^2 - mv^2}{I}}$ (B) $\sqrt{\dfrac{(I+mR^2)\omega^2 - mv^2}{I}}$



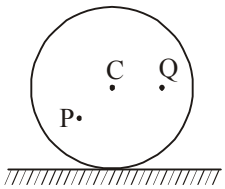
(C) $\dfrac{I\omega - mvR}{I}$ (D) $\dfrac{(I+mR^2)\omega - mvR}{I}$

RD0083

Combined rotation and translation

Kinematics

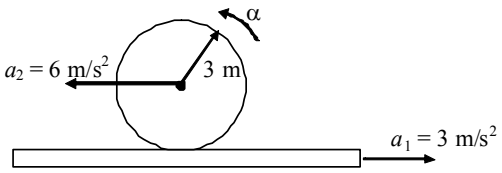
28. A disc is rolling without slipping with angular velocity ω. P and Q are two points equidistant from the centre C. The order of magnitude of velocity is :- **[IIT-JEE 2004]**



(A) $v_Q > v_C > v_P$ (B) $v_P > v_C > v_Q$ (C) $v_P = v_C$, $v_Q = v_C/2$ (D) $v_P < v_C > v_Q$

RD0086

29. In the following figure, a sphere of radius 3 m rolls on a plank. The accelerations of the sphere and the plank are indicated. The value of α is

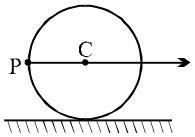


(A) 3 rad/s^2 (B) 6 rad/s^2

(C) 3 rad/s^2 (opposite to the direction shown in figure) (D) 1 rad/s^2

RD0087

30. A disc of radius R is rolling purely on a flat horizontal surface, with a constant angular velocity. The angle between the velocity and acceleration vectors of point P is

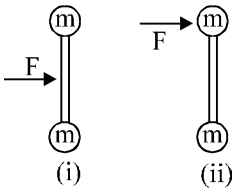


(A) zero (B) 45° (C) 135° (D) $\tan^{-1}(1/2)$

RD0088

Dynamics

31. A force F is applied to a dumbbell for a time interval, t, first as in (i) and then as in (ii). In which case does the dumbbell acquire the greater centre-of-mass speed?



(A) (i) (B) (ii) (C) there is no difference

(D) the answer depends on the rotational inertia of the dumbbell

RD0089

32. A body kept on a smooth horizontal surface is pulled by a constant horizontal force applied at the top point of the body. If the body rolls purely on the surface, its shape can be :

(A) thin pipe (B) uniform cylinder

(C) uniform sphere (D) thin spherical shell

RD0092

33. A solid sphere with a velocity (of centre of mass) v and angular velocity ω is gently placed on a rough horizontal surface. The frictional force on the sphere:

(A) must be forward (in direction of v) (B) must be backward (opposite to v)

(C) cannot be zero (D) none of the above

RD0093

Kinetic energy

34. A hoop and a solid cylinder have the same mass and radius. They both roll, without slipping, on a horizontal surface. If their kinetic energies are equal

(A) the hoop has a greater translational speed than the cylinder

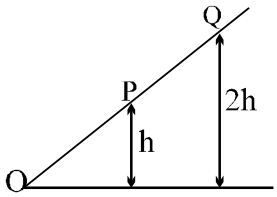
(B) the cylinder has a greater translational speed than the hoop

(C) the hoop and the cylinder have the same translational speed

(D) the hoop has a greater rotational speed than the cylinder

RD0094

35. A ball rolls down an inclined plane as shown in figure. The ball is first released from rest from P and then later from Q. Which of the following statement is/ are correct?



(i) The ball takes twice as much time to roll from Q to O as it does to roll from P to O.
(ii) The acceleration of the ball at Q is twice as large as the acceleration at P.
(iii) The ball has twice as much K.E. at O when rolling from Q as it does when rolling from P.
(A) i, ii only (B) ii, iii only (C) i only (D) iii only

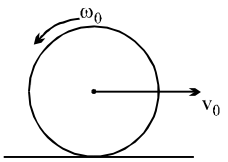
RD0095

36. The moment of inertia of a solid cylinder about its axis is given by $(1/2)MR^2$. If this cylinder rolls without slipping, the ratio of its rotational kinetic energy to its translational kinetic energy is
(A) 1 : 1 (B) 2 : 2 (C) 1 : 2 (D) 1 : 3

RD0096

Angular momentum

37. A uniform circular disc placed on a rough horizontal surface has initially a velocity v_0 and an angular velocity ω_0 as shown in the figure. The disc comes to rest after moving some distance in the direction of motion. Then



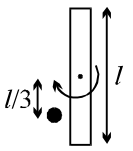
$\frac{v_0}{r\omega_0}$ is

(A) $\frac{1}{2}$ (B) 1 (C) $\frac{3}{2}$ (D) 2

RD0097

Eccentric collision

38. A uniform rod of length l and mass M rotating about a fixed vertical axis on a smooth horizontal table. It elastically strikes a particle placed at a distance $l/3$ from its axis and stops. Mass of the particle is



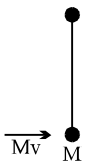
(A) 3M (B) $\frac{3M}{4}$ (C) $\frac{3M}{2}$ (D) $\frac{4M}{3}$

RD0100

39. A mass m is moving at speed v perpendicular to a rod of length d and mass $M = 6m$ which pivots around a frictionless axle running through its centre. It strikes and sticks to the end of the rod. The moment of inertia of the rod about its centre is $Md^2/12$. Then the angular speed of the system right after the collision is
(A) $2v/d$ (B) $2v/(3d)$ (C) v/d (D) $3v/(2d)$

RD0101

40. Two particles each of mass M are connected by a massless rod of length l. The rod is lying on the smooth sufrace. If one of the particle is given an impulse MV as shown in the figure then angular velocity of the rod would be **[IIT-JEE'(Scr)2003]**



(A) v/l (B) 2v/l (C) v/2l (D) None

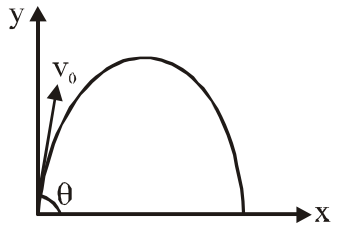
RD0102

EXERCISE (J-M)

Directions : Question number 1 contain Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best discribes the two statements.

1. A small particle of mass m is projected at an angle θ with the x-axis with an initial velocity v_0 in the x-y plane as shown in the figure. At a time $t < \frac{v_0 \sin\theta}{g}$, the angular momentum of the particle is:

Where $\hat{i}$, $\hat{j}$ and $\hat{k}$ are unit vectors along x, y and z-axis respectively. **[AIEEE-2010]**



(1) $\frac{1}{2} mg\, v_0\, t^2 \cos\theta\, \hat{i}$ (2) $-mg\, v_0\, t^2 \cos\theta\, \hat{j}$ (3) $mg\, v_0\, t \cos\theta\, \hat{k}$ (4) $-\frac{1}{2} mg\, v_0\, t^2 \cos\theta\, \hat{k}$

RD0154

2. A pulley of radius 2 m is rotated about its axis by a force $F = (20t - 5t^2)$ newton (where t is measured in seconds) applied tangentially. If the moment of inertia of the pulley about its axis of rotation is 10 kg m^2, the number of rotations made by the pulley before its direction of motion it reversed, is :-

(1) more than 6 but less than 9 (2) more than 9 **[AIEEE-2011]**

(3) less than 3 (4) more than 3 but less than 6

RD0155

3. A thin horizontal circular disc is rotating about a vertical axis passing through its centre. An insect is at rest at a point near the rim of the disc. The insect now moves along a diameter of the disc to reach its other end. During the journey of the insect, then angular speed of the disc :-

[AIEEE-2011]

(1) continuously increases (2) first increases and then decreases

(3) remains unchanged (4) continuously decreses

RD0156

4. A particle of mass 'm' is projected with a velocity v making an angle of 30° with the horizontal. The magnitude of angular momentum of the projectile about the point of projection when the particle is at its maximum height 'h' is :- **[AIEEE-2011]**

(1) $\frac{\sqrt{3}}{2}\frac{mv^2}{g}$ (2) zero (3) $\frac{mv^3}{\sqrt{2}g}$ (4) $\frac{\sqrt{3}}{16}\frac{mv^3}{g}$

RD0157

5. This question has Statement I and Statement II. Of the four choices given after the Statements, choose the one that best describes the two Statements. **[JEE Mains-2013]**

Statement - I : A point particle of mass m moving with speed v collides with stationary point particle of mass M. If the maximum energy loss possible is given as $f\left(\frac{1}{2}mv^2\right)$ then $f = \left(\frac{m}{M+m}\right)$.

Statement - II : Maximum energy loss occurs when the particles get stuck together as a result of the collision. **[JEE Mains- 2013]**

(1) Statement–I is true, Statement–II is true, Statement–II is a correct explanation of Statement–I.

(2) Statement–I is true, Statement–II is true, Statement–II is a not correct explanation of Statement–I.

(3) Statement–I is true, Statement–II is false.

(4) Statement–I is false, Statement–II is true

CM0112

6. A hoop of radius r and mass m rotating with an angular velocity ω_0 is placed on a rough horizontal surface. The initial velocity of the centre of the hoop is zero. What will be the velocity of the centre of the hoop when it ceases to slip? **[JEE Mains-2013]**

(1) $\frac{r\omega_0}{4}$ (2) $\frac{r\omega_0}{3}$ (3) $\frac{r\omega_0}{2}$ (4) $r\omega_0$

RD0158

7. A bob of mass m attached to an inextensible string of length ℓ is suspended from a vertical support. The bob rotates in a horizontal circle with an angular speed ω rad/s about the vertical. About the point of suspension : **[JEE Mains-2014]**

(1) Angular momentum changes in direction but not in magnitude

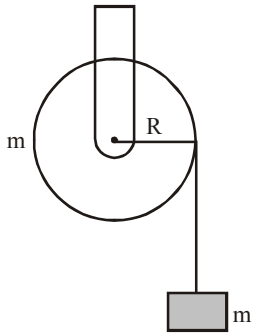
(2) Angular momentum changes both in direction and magnitude

(3) Angular momentum is conserved

(4) Angular momentum changes in magnitude but not in direction.

RD0159

8. A mass 'm' is supported by a massless string wound around a uniform hollow cylinder of mass m and radius R. If the string does not slip on the cylinder, with what acceleration will the mass fall on release? **[JEE Mains-2014]**



(1) $\frac{5g}{6}$ (2) g (3) $\frac{2g}{3}$ (4) $\frac{g}{2}$

RD0160

9. A particle of mass m moving in the x direction with speed 2υ is hit by another particle of mass 2m moving in the y direction with speed υ. If the collisions perfectly inelastic , the percentage loss in the energy during the collision is close to : **[JEE Mains-2015]**

(1) 56 % (2) 62% (3) 44% (4) 50%

CM0113

10. Distance of the centre of mass of a solid uniform cone from its vertex is z_0. If the radius of its base is R and its height is h then z_0 is equal to :- **[JEE Mains-2015]**

(1) $\frac{5h}{8}$ (2) $\frac{3h^2}{8R}$ (3) $\frac{h^2}{4R}$ (4) $\frac{3h}{4}$

CM0114

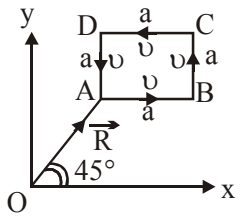
11. From a solid sphere of mass M and radius R a cube of maximum possible volume is cut. Moment of inertia of cube about an axis passing through its centre and perpendicular to one of its faces is:- **[JEE Mains-2015]**

(1) $\frac{4MR^2}{9\sqrt{3}\pi}$ (2) $\frac{4MR^2}{3\sqrt{3}\pi}$ (3) $\frac{MR^2}{32\sqrt{2}\pi}$ (4) $\frac{MR^2}{16\sqrt{2}\pi}$

RD0161

12. A particle of mass m is moving along the side of a square of side 'a', with a uniform speed υ in the x-y plane as shown in the figure : **[JEE Mains-2016]**

Which of the following statement is false for the angular momentum $\vec{L}$ about the origin ?



(1) $\vec{L} = \frac{m\upsilon}{\sqrt{2}} R\hat{k}$ when the particle is moving from D to A

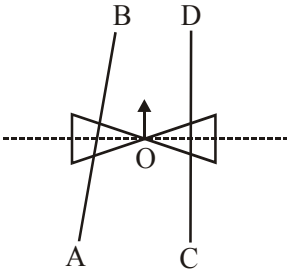
(2) $\vec{L} = -\frac{m\upsilon}{\sqrt{2}} R\hat{k}$ when the particle is moving from A to B

(3) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} - a\right]\hat{k}$ when the particle is moving from C to D

(4) $\vec{L} = m\upsilon\left[\frac{R}{\sqrt{2}} + a\right]\hat{k}$ when the particle is moving from B to C

RD0162

13. A roller is made by joining together two cones at their vertices O. It is kept on two rails AB and CD which are placed asymmetrically (see figure), with its axis perpendicular to CD and its centre O at the centre of line joining AB and CD (see figure). It is given a light push so that it starts rolling with its centre O moving parallel to CD in the direction shown. As it moves, the roller will tend to :- **[JEE Main-2016]**



(1) turn left and right alternately. (2) turn left.

(3) turn right. (4) go straight.

RD0163

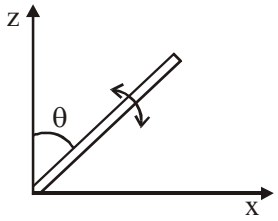
14. The moment of inertia of a uniform cylinder of length ℓ and radius R about its perpendicular bisector is I. What is the ratio ℓ/R such that the moment of inertia is minimum ? **[JEE Main-2017]**

(1) 1 (2) $\frac{3}{\sqrt{2}}$ (3) $\sqrt{\frac{3}{2}}$ (4) $\frac{\sqrt{3}}{2}$

RD0164

15. A slender uniform rod of mass M and length ℓ is pivoted at one end so that it can rotate in a vertical plane (see figure). There is negligible friction at the pivot. The free end is held vertically above the pivot and then released. The angular acceleration of the rod when it makes an angle θ with the vertical is :

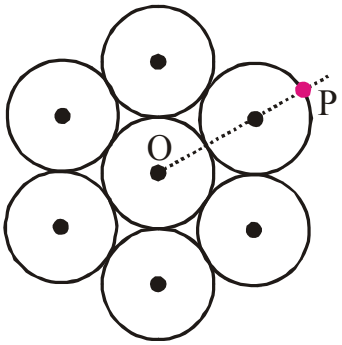
[JEE Main-2017]



(1) $\frac{3g}{2\ell}\cos\theta$ (2) $\frac{2g}{3\ell}\cos\theta$ (3) $\frac{3g}{2\ell}\sin\theta$ (4) $\frac{2g}{3\ell}\sin\theta$

RD0165

16. Seven identical circular planar disks, each of mass M and radius R are welded symmetrically as shown. The moment of inertia of the arrangement about the axis normal to the plane and passing through the point P is : **[JEE Main-2018]**



(1) $\frac{55}{2}$ MR^2 (2) $\frac{73}{2}$ MR^2 (3) $\frac{181}{2}$ MR^2 (4) $\frac{19}{2}$ MR^2

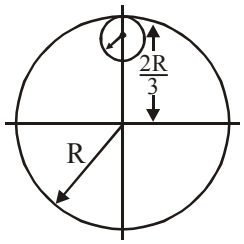
RD0167

17. The mass of a hydrogen molecule is 3.32×10^{-27} kg. If 10^{23} hydrogen molecules strike, per second, a fixed wall of area 2 cm^2 at an angle of 45° to the normal, and rebound elastically with a speed of 10^3 m/s, then the pressure on the wall is nearly : **[JEE Main-2018]**

(1) 4.70×10^3 N/m^2 (2) 2.35×10^2 N/m^2 (3) 4.70×10^2 N/m^2 (4) 2.35×10^3 N/m^2

CM0117

18. From a uniform circular disc of radius R and mass 9M, a small disc of radius $\frac{R}{3}$ is removed as shown in the figure. The moment of inertia of the remaining disc about an axis perpendicular to the plane of the disc and passing through centre of disc is : **[JEE Main-2018]**



(1) $\frac{40}{9}MR^2$ (2) $10\ MR^2$ (3) $\frac{37}{9}MR^2$ (4) $4\ MR^2$

RD0180

19. In a collinear collision, a particle with an initial speed v_0 strikes a stationary particle of the same mass. If the final total kinetic energy is 50 % greater than the original kinetic energy, the magnitude of the relative velocity between the two particles, after collision, is **[JEE Main-2018]**

(1) $\sqrt{2}\,v_0$ (2) $\frac{v_0}{2}$ (3) $\frac{v_0}{\sqrt{2}}$ (4) $\frac{v_0}{4}$

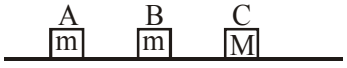
CM0116

20. It is found that if a neutron suffers an elastic collinear collision with deuterium at rest, fractional loss of its energy is p_d ; while for its similar collision with carbon nucleus at rest, fractional loss of energy is p_c. The values of p_d and p_c are respectively : **[JEE Main-2018]**

(1) (·28, ·89) (2) (0, 0) (3) (0, 1) (4) (·89, ·28)

CM0115

21. Three blocks A, B and C are lying on a smooth horizontal surface, as shown in the figure. A and B have equal masses, m while C has mass M. Block A is given an brutal speed v towards B due to which it collides with B perfectly inelastically. The combined mass collides with C, also perfectly inelastically. $\frac{5}{6}$ th of the initial kinetic energy is lost in whole process. What is value of $\frac{M}{m}$? **[JEE Main-2019]**



(1) 4 (2) 5 (3) 3 (4) 2

CM0123

22. A piece of wood of mass 0.03 kg is dropped from the top of a 100 m height building. At the same time, a bullet of mass 0.02 kg is fired vertically upward, with a velocity 100 ms^{-1}, from the ground. The bullet gets embedded in the wood. Then the maximum height to which the combined system reaches above the top of the building before falling below is : ($g = 10ms^{-2}$)

[JEE Main-2019]

(1) 30 m (2) 10 m (3) 40 m (4) 20 m

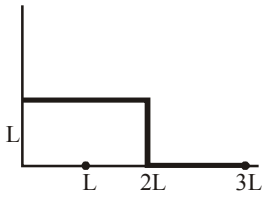
CM0124

23. A simple pendulum, made of a string of length l and a bob of mass m, is released from a small angle θ_0. It strikes a block of mass M, kept on a horizontal surface at its lowest point of oscillations, elastically. It bounces back and goes up to an angle θ_1. Then M is given by : **[JEE Main-2019]**

(1) $\frac{m}{2}\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$ (2) $\frac{m}{2}\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (3) $m\left(\frac{\theta_0+\theta_1}{\theta_0-\theta_1}\right)$ (4) $m\left(\frac{\theta_0-\theta_1}{\theta_0+\theta_1}\right)$

CM0125

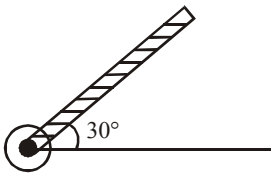
24. The position vector of the centre of mass $\vec{r}_{cm}$ of asymmetric uniform bar of negligible area of cross-section as shown in figure is : **[JEE Main-2019]**



(1) $\vec{r}_{cm}=\frac{13}{8}L\hat{x}+\frac{5}{8}L\hat{y}$ (2) $\vec{r}_{cm}=\frac{11}{8}L\,\hat{x}+\frac{3}{8}L\hat{y}$ (3) $\vec{r}_{cm}=\frac{3}{8}L\,\hat{x}+\frac{11}{8}L\hat{y}$ (4) $\vec{r}_{cm}=\frac{5}{8}L\,\hat{x}+\frac{13}{8}L\hat{y}$

CM0126

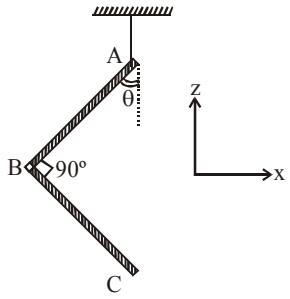
25. A rod of length 50cm is pivoted at one end. It is raised such that if makes an angle of 30° from the horizontal as shown and released from rest. Its angular speed when it passes through the horizontal (in rad s^{-1}) will be ($g = 10ms^{-2}$) **[JEE Main-2019]**



(1) $\sqrt{30}$ (2) $\sqrt{\frac{30}{2}}$ (3) $\frac{\sqrt{30}}{2}$ (4) $\frac{\sqrt{20}}{3}$

RD0197

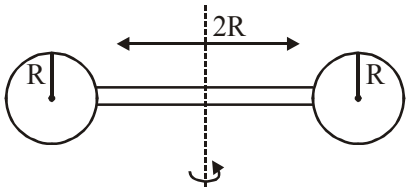
26. An L-shaped object, made of thin rods of uniform mass density, is suspended with a string as shown in figure. If AB = BC, and the angle made by AB with downward vertical is θ, then : **[JEE Main-2019]**



(1) $\tan\theta=\frac{2}{\sqrt{3}}$ (2) $\tan\theta=\frac{1}{3}$ (3) $\tan\theta=\frac{1}{2}$ (4) $\tan\theta=\frac{1}{2\sqrt{3}}$

RD0198

27. Two identical spherical balls of mass M and radius R each are stuck on two ends of a rod of length 2R and mass M (see figure). The moment of inertia of the system about the axis passing perpendicularly through the centre of the rod is : **[JEE Main-2019]**



(1) $\frac{152}{15}MR^2$ (2) $\frac{17}{15}MR^2$ (3) $\frac{137}{15}MR^2$ (4) $\frac{209}{15}MR^2$

RD0199

28. To mop-clean a floor, a cleaning machine presses a circular mop of radius R vertically down with a total force F and rotates it with a constant angular speed about its axis. If the force F is distributed uniformly over the mop and if coefficient of friction between the mop and the floor is μ, the torque, applied by the machine on the mop is : **[JEE Main-2019]**

(1) $\frac{2}{3}\mu FR$ (2) μFR/3 (3) μFR/2 (4) μFR/6

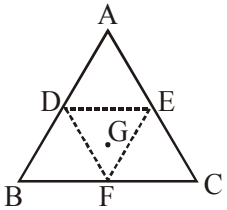
RD0200

29. A homogeneous solid cylindrical roller of radius R and mass M is pulled on a cricket pitch by a horizontal force. Assuming rolling without slipping, angular acceleration of the cylinder is : **[JEE Main-2019]**

(1) $\frac{3F}{2m\,R}$ (2) $\frac{F}{3m\,R}$ (3) $\frac{2F}{3m\,R}$ (4) $\frac{F}{2m\,R}$

RD0201

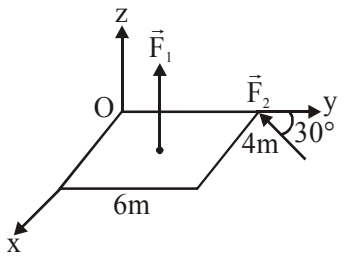
30. An equilateral triangle ABC is cut from a thin solid sheet of wood. (see figure) D, E and F are the mid-points of its sides as shown and G is the centre of the triangle. The moment of inertia of the triangle about an axis passing through G and perpendicular to the plane of the triangle is I_0. It the smaller triangle DEF is removed from ABC, the moment of inertia of the remaining figure about the same axis is I. Then: **[JEE Main-2019]**



(1) $I=\frac{9}{16}I_0$ (2) $I=\frac{3}{4}I_0$ (3) $I=\frac{I_0}{4}$ (4) $I=\frac{15}{16}I_0$

RD0202

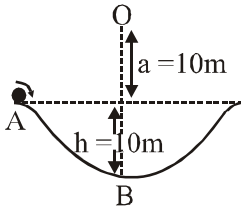
31. A slob is subjected to two forces $\vec{F}_1$ and $\vec{F}_2$ of same magnitude F as shown in the figure. Force $\vec{F}_2$ is in XY-plane while force F_1 acts along z-axis at the point $\left(2\vec{i}+3\vec{j}\right)$. The moment of these forces about point O will be : **[JEE Main-2019]**



(1) $\left(3\hat{i}-2\hat{j}-3\hat{k}\right)F$ (2) $\left(3\hat{i}+2\hat{j}+3\hat{k}\right)F$ (3) $\left(3\hat{i}+2\hat{j}-3\hat{k}\right)F$ (4) $\left(3\hat{i}-2\hat{j}+3\hat{k}\right)F$

RD0203

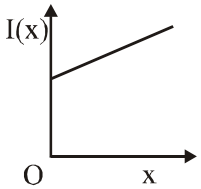
32. A particle of mass 20 g is released with an initial velocity 5 m/s along the curve from the point A, as shown in the figure. The point A is at height h from point B. The particle slides along the frictionless surface. When the particle reaches point B, its angular momentum about O will be : (Take g= 10 m/s^2) **[JEE Main-2019]**

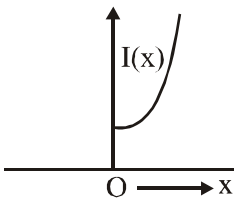


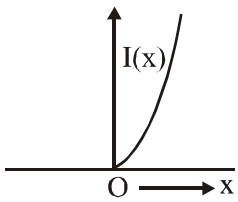
(1) 8kg-m^2/s (2) 6kg-m^2/s (3) 3kg-m^2/s (4) 2kg-m^2/s

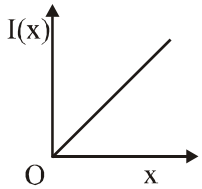
RD0204

33. The moment of inertia of a solid sphere, about an axis parallel to its diameter and at a distance of x from it, is I(x)'. Which one of the graphs represents the variation of I(x) with x correctly? **[JEE Main-2019]**

(1) 

(2) 

(3) 

(4) 

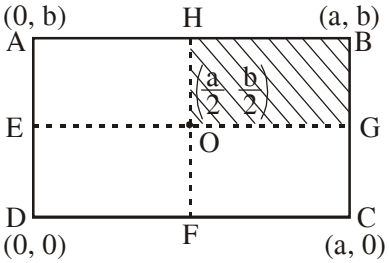
RD0205

34. Let the moment of inertia of a hollow cylinder of length 30 cm (inner radius 10 cm and outer radius 20 cm), about its axis be I. The radius of a thin cylinder of the same mass such that its moment of inertia about its axis is also I, is: **[JEE Main-2019]**

(1) 12 cm (2) 18 cm (3) 16 cm (4) 14 cm

RD0206

35. A uniform rectangular thin sheet ABCD of mass M has length a and breadth b, as shown in the figure. If the shaded portion HBGO is cut-off, the coordinates of the centre of mass of the remaining portion will be :- **[JEE Main-2019]**



(1) $\left(\frac{2a}{3},\frac{2b}{3}\right)$ (2) $\left(\frac{5a}{3},\frac{5b}{3}\right)$ (3) $\left(\frac{3a}{4},\frac{3b}{4}\right)$ (4) $\left(\frac{5a}{12},\frac{5b}{12}\right)$

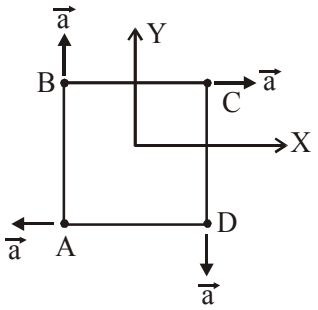
CM0127

36. A body of mass m_1 moving with an unknown velocity of $v_1\hat{i}$, undergoes a collinear collision with a body of mass m_2 moving with a velocity $v_2\hat{i}$. After collision, m_1 and m_2 move with velocities of $v_3\hat{i}$ and $v_4\hat{i}$, respectively. If $m_2 = 0.5\ m_1$ and $v_3 = 0.5\ v_1$, then v_1 is :- **[JEE Main-2019]**

(1) $v_4 - \frac{v_2}{4}$ (2) $v_4 - \frac{v_2}{2}$ (3) $v_4 - v_2$ (4) $v_4 + v_2$

CM0128

37. Four particles A, B, C and D with masses $m_A = m$, $m_B = 2m$, $m_C = 3m$ and $m_D = 4m$ are at the corners of a square. They have accelerations of equal magnitude with directions as shown. The acceleration of the centre of mass of the particles is : **[JEE Main-2019]**



(1) $\frac{a}{5}\left(\hat{i}-\hat{j}\right)$ (2) $\frac{a}{5}\left(\hat{i}+\hat{j}\right)$ (3) Zero (4) $a\left(\hat{i}+\hat{j}\right)$

CM0129

38. A wedge of mass M = 4m lies on a frictionless plane. A particle of mass m approaches the wedge with speed v. There is no friction between the particle and the plane or between the particle and the wedge. The maximum height climbed by the particle on the wedge is given by :-

[JEE Main-2019]

(1) $\frac{2v^2}{7g}$ (2) $\frac{v^2}{g}$ (3) $\frac{2v^2}{5g}$ (4) $\frac{v^2}{2g}$

RD0207

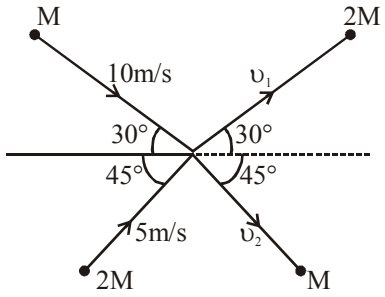
39. A body of mass 2 kg makes an eleastic collision with a second body at rest and continues to move in the original direction but with one fourth of its original speed. What is the mass of the second body ?

[JEE Main-2019]

(1) 1.8 kg (2) 1.2 kg (3) 1.5 kg (4) 1.0 kg

CM0130

40. Two particles, of masses M and 2M, moving, as shown, with speeds of 10 m/s and 5 m/s, collide elastically at the origin. After the collision, they move along the indicated directions with speeds υ_1 and υ_2, respectively. The values of υ_1 and υ_2 are nearly : **[JEE Main-2019]**

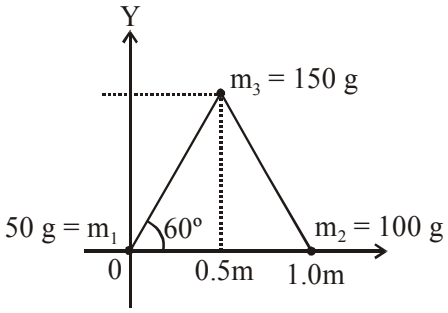


(1) 3.2 m/s and 6.3 m/s (2) 3.2 m/s and 12.6 m/s
(3) 6.5 m/s and 6.3 m/s (4) 6.5 m/s and 3.2 m/s

CM0131

41. Three particles of masses 50 g, 100 g and 150 g are placed at the vertices of an equilateral triangle of side 1 m (as shown in the figure). The (x, y) coordinates of the centre of mass will be :

[JEE Main-2019]



(1) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{8}m\right)$ (2) $\left(\frac{\sqrt{3}}{4}m, \frac{5}{12}m\right)$ (3) $\left(\frac{7}{12}m, \frac{\sqrt{3}}{4}m\right)$ (4) $\left(\frac{\sqrt{3}}{8}m, \frac{7}{12}m\right)$

CM0132

42. A man (mass = 50 kg) and his son (mass = 20 kg) are standing on a frictionless surface facing each other. The man pushes his son so that he starts moving at a speed of 0.70 ms^{-1} with respect to the man. The speed of the man with respect to the surface is : **[JEE Main-2019]**

(1) 0.20 ms^{-1} (2) 0.14 ms^{-1} (3) 0.47 ms^{-1} (4) 0.28 ms^{-1}

CM0133

43. A solid sphere and solid cylinder of identical radii approach an incline with the same linear velocity (see figure). Both roll without slipping all throughout. The two climb maximum heights h_{sph} and h_{cyl} on the incline. The ratio $\frac{h_{sph}}{h_{cyl}}$ is given by :- **[JEE Main-2019]**

(1) $\frac{14}{15}$ (2) $\frac{4}{5}$ (3) 1 (4) $\frac{2}{\sqrt{5}}$

RD0208

44. A thin circular plate of mass M and radius R has its density varying as $\rho(r) = \rho_0 r$ with ρ_0 as constant and r is the distance from its centre. The moment of Inertia of the circular plate about an axis perpendicular to the plate and passing through its edge is $I = aMR^2$. The value of the coefficient a is : **[JEE Main-2019]**

(1) $\frac{3}{2}$ (2) $\frac{1}{2}$ (3) $\frac{3}{5}$ (4) $\frac{8}{5}$

RD0209

45. Moment of inertia of a body about a given axis is 1.5 kg m^2. Initially the body is at rest. In order to produce a rotational kinetic energy of 1200 J, the angular accleration of 20 rad/s^2 must be applied about the axis for a duration of :- **[JEE Main-2019]**

(1) 2 s (2) 5s (3) 2.5 s (4) 3 s

RD0210

46. A thin smooth rod of length L and mass M is rotating freely with angular speed ω_0 about an axis perpendicular to the rod and passing through its center. Two beads of mass m and negligible size are at the center of the rod initially. The beads are free to slide along the rod. The angular speed of the system , when the beads reach the opposite ends of the rod, will be :- **[JEE Main-2019]**

(1) $\frac{M\omega_0}{M+3m}$ (2) $\frac{M\omega_0}{M+m}$ (3) $\frac{M\omega_0}{M+2m}$ (4) $\frac{M\omega_0}{M+6m}$

RD0211

47. A stationary horizontal disc is free to rotate about its axis. When a torque is applied on it, its kinetic energy as a function of θ, where θ is the angle by which it has rotated, is given as $k\theta^2$. If its moment of inertia is I then the angular acceleration of the disc is : **[JEE Main-2019]**

(1) $\frac{k}{2I}\theta$ (2) $\frac{k}{I}\theta$ (3) $\frac{k}{4I}\theta$ (4) $\frac{2k}{I}\theta$

RD0212

48. The time dependence of the position of a particle of mass m = 2 is given by $\vec{r}(t) = 2t\hat{i} - 3t^2\hat{j}$. Its angular momentum, with respect to the origin, at time t = 2 is : **[JEE Main-2019]**

(1) $36\hat{k}$ (2) $-34(\hat{k} - \hat{i})$ (3) $48(\hat{i} + \hat{j})$ (4) $-48\hat{k}$

RD0213

49. A solid sphere of mass M and radius R is divided into two unequal parts. The first part has a mass of $\frac{7M}{8}$ and is converted into a uniform disc of radius 2R. The second part is converted into a uniform solid sphere. Let I_1 be the moment of inertia of the disc about its axis and I_2 be the moment of inertia of the new sphere about its axis. The ratio I_1/I_2 is given by : **[JEE Main-2019]**

(1) 185 (2) 65 (3) 285 (4) 140

RD0214

50. A thin disc of mass M and radius R has mass per unit area $\sigma(r) = kr^2$ where r is the distance from its centre. Its moment of inertia about an axis going through its centre of mass and perpendicular to its plane is : **[JEE Main-2019]**

(1) $\frac{MR^2}{6}$ (2) $\frac{MR^2}{3}$ (3) $\frac{2MR^2}{3}$ (4) $\frac{MR^2}{2}$

RD0215

51. A particle of mass m is moving along a trajectory given by

$x = x_0 + a\cos\omega_1 t$

$y = y_0 + b\sin\omega_2 t$

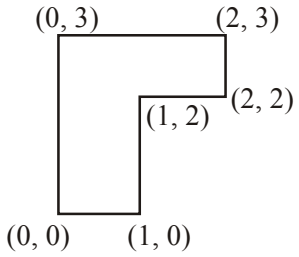
The torque, acting on the particle about the origin, at t = 0 is :

(1) $m(-x_0 b + y_0 a)\omega_1^2\hat{k}$ (2) $+my_0 a\omega_1^2\hat{k}$

(3) $-m(x_0 b\omega_2^2 - y_0 a\omega_1^2)\hat{k}$ (4) Zero

RD0216

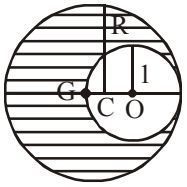
52. The coordinates of centre of mass of a uniform flag shaped lamina (thin flat plate) of mass 4kg. (The coordinates of the same are shown in figure) are : **[JEE Main-2020]**



(1) (1.25m, 1.50m) (2) (1m, 1.75m) (3) (0.75m, 0.75m) (4) (0.75m, 1.75m)

CM0134

53. As shown in figure, when a spherical cavity (centred at O) of radius 1 is cut out of a uniform sphere of radius R (centred at C), the centre of mass of remaining (shaded) part of sphere is at G, i.e, on the surface of the cavity. R can be detemined by the equation : **[JEE Main-2020]**



(1) $(R^2 - R + 1)(2 - R) = 1$ (2) $(R^2 + R - 1)(2 - R) = 1$
(3) $(R^2 + R + 1)(2 - R) = 1$ (4) $(R^2 - R - 1)(2 - R) = 1$

CM0135

54. A particle of mass m is dropped from a height h above the ground. At the same time another particle of the same mass is thrown vertically upwards from the ground with a speed of $\sqrt{2gh}$. If they collide head-on completely inelastically, the time taken for the combined mass to reach the ground, in units of $\sqrt{\frac{h}{g}}$ is : **[JEE Main-2020]**

(1) $\frac{1}{2}$ (2) $\sqrt{\frac{1}{2}}$ (3) $\sqrt{\frac{3}{4}}$ (4) $\sqrt{\frac{3}{2}}$

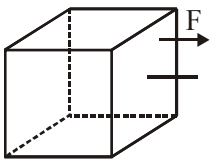
CM0136

55. The radius of gyration of a uniform rod of length l, about an axis passing through a point $\frac{l}{4}$ away from the centre of the rod, and perpendicular to it, is : **[JEE Main-2020]**

(1) $\frac{1}{8}l$ (2) $\sqrt{\frac{7}{48}}l$ (3) $\sqrt{\frac{3}{8}}l$ (4) $\frac{1}{4}l$

RD0217

56.



Consider a uniform cubical box of side a on a rough floor that is to be moved by applying minimum possible force F at a point b above its centre of mass (see figure). If the coefficient of friction is $\mu = 0.4$, the maximum possible value of $100 \times \frac{b}{a}$ for a box not to topple before moving is _____ .

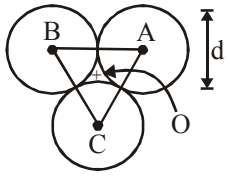
[JEE Main-2020]
RD0218

57. Consider a uniform rod of mass M = 4m and length ℓ pivoted about its centre. A mass m moving with velocity v making angle $\theta = \frac{\pi}{4}$ to the rod's long axis collides with one end of the rod and sticks to it. The angular speed of the rod-mass system just after the collision is : **[JEE Main-2020]**

(1) $\frac{3}{7\sqrt{2}}\frac{v}{\ell}$ (2) $\frac{3\sqrt{2}}{7}\frac{v}{\ell}$ (3) $\frac{4}{7}\frac{v}{\ell}$ (4) $\frac{3}{7}\frac{v}{\ell}$

RD0219

58.

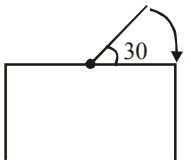


Three solid spheres each of mass m and diameter d are stuck together such that the lines connecting the centres form an equilateral triangle of side of length d. The ratio I_0/I_A of moment of inertia I_0 of the system about an axis passing the centroid and about center of any of the spheres I_A and perpendicular to the plane of the triangle is : **[JEE Main-2020]**

(1) $\frac{13}{23}$ (2) $\frac{15}{13}$ (3) $\frac{23}{13}$ (4) $\frac{13}{15}$

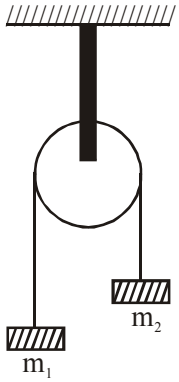
RD0220

59. One end of a straight uniform 1m long bar is pivoted on horizontal table. It is released from rest when it makes an angle 30º from the horizontal (see figure). Its angular speed when it hits the table is given as $\sqrt{n}\,s^{-1}$, where n is an integer. The value of n is _____ . **[JEE Main-2020]**



RD0221

60. A uniformly thick wheel with moment of inertia I and radius R is free to rotate about its centre of mass (see fig). A massless string is wrapped over its rim and two blocks of masses m_1 and m_2 ($m_1 > m_2$) are attached to the ends of the string. The system is released from rest. The angular speed of the wheel when m_1 descents by a distance h is : **[JEE Main-2020]**



(1) $\left[\frac{m_1+m_2}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}} gh$ (2) $\left[\frac{2(m_1-m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$

(3) $\left[\frac{2(m_1+m_2)gh}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}}$ (4) $\left[\frac{(m_1-m_2)}{(m_1+m_2)R^2+I}\right]^{\frac{1}{2}} gh$

RD0222

ANSWER KEYS

01_CENTRE OF MASS, MOMENTUM & COLLISION

EXERCISE (S)

1. Ans. (1/7, 23/14) **2. Ans.** √13 m, $\left(\frac{14}{5},\frac{19}{5}\right)$ **3. Ans.** $L(\sqrt{2}+1)/3$

4. Ans. 4R from O **5. Ans.** x = 6m **6. Ans.** (3 m, 1 m, 8 m)

7. Ans. g/9 downwards **8. Ans.** $\frac{L}{3}$ **9. Ans.** $\vec{v}_C = -\vec{v}_B$

10. Ans. 150 kg **11. Ans.** (i) 4 m/s, (ii) 24 J **12. Ans.** (i) 360 m, (ii) 10800 J

13. Ans. $\frac{\sqrt{13}}{2}v_0$ **14. Ans.** 0.3 **15. Ans.** $m \times \sqrt{u^2 - uv + v^2}$

16. Ans. $\frac{7}{18}$ **17. Ans.** (i) 3 J, (ii) $\frac{12}{5}$ N-s **18. Ans.** (i) $v_0/3$, (ii) $3\sqrt{5gR}$.

19. Ans. 1 N

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (C)	**2. Ans. (D)**	**3. Ans. (C)**	**4. Ans. (B)**	**5. Ans. (C)**	**6. Ans. (A)**
7. Ans. (C)	**8. Ans. (A)**	**9. Ans. (C)**	**10. Ans. (D)**	**11. Ans. (D)**	**12. Ans. (C)**
13. Ans. (A)	**14. Ans. (B)**	**15. Ans. (A)**	**16. Ans. (B)**	**17. Ans. (B)**	**18. Ans. (B)**
19. Ans. (C)	**20. Ans. (D)**	**21. Ans. (D)**	**22. Ans. (C)**	**23. Ans. (D)**	

02_ROTATIONAL DYNAMICS

EXERCISE (S)

1. Ans. 2 **2. Ans.** $\frac{ml^2}{12}$ **3. Ans.** 8 kg **4. Ans.** 10^{-3} rad/s^2 clockwise

5. Ans. 2 **6. Ans.** $\frac{L}{3}$ **7. Ans.** (a) $\frac{9g}{7}\downarrow$ (b) $\frac{4mg}{7}\uparrow$

8. Ans. (i) $10/13 m/s^2$, (ii) $5000/26\pi$,(iii) 480/13N **9. Ans.** 2N

10. Ans. $M = 2m\left(\frac{2gh}{R^2\omega^2} - 1\right)$ **11. Ans.** $\sqrt{5gR}$ **12. Ans.** $\frac{\omega}{3}$

13. Ans. $\frac{4\pi}{5}$ **14. Ans.** 75 J **15. Ans.** 1/2 ma **16. Ans.** 15

17. Ans. 16 m/s² **18. Ans.** $6mv_0^2$ **19. Ans.** $\frac{13}{16}Mv^2$ **20. Ans.** $\frac{l^2}{12x}$

21. Ans. (a) $t = \frac{\pi Rm}{2I}$; (b) $s = \frac{\pi R}{2}$ **22. Ans.** $\frac{mv}{4}$ **23. Ans.** $\frac{2mv_0{}^2}{l}$

EXERCISE (O)

ANSWER KEY

SINGLE CORRECT TYPE QUESTIONS

1. Ans. (B)	**2. Ans. (B)**	**3. Ans. (D)**	**4. Ans. (A)**	**5. Ans. (A)**	**6. Ans. (B)**
7. Ans. (D)	**8. Ans. (A)**	**9. Ans. (C)**	**10. Ans. (D)**	**11. Ans. (D)**	**12. Ans. (B)**
13. Ans. (C)	**14. Ans. (B)**	**15. Ans. (D)**	**16. Ans. (C)**	**17. Ans. (C)**	**18. Ans. (A)**
19. Ans. (B)	**20. Ans. (C)**	**21. Ans. (D)**	**22. Ans. (A)**	**23. Ans. (D)**	**24. Ans. (B)**
25. Ans. (C)	**26. Ans. (A)**	**27. Ans. (D)**	**28. Ans. (A)**	**29. Ans. (A)**	**30. Ans. (B)**
31. Ans. (C)	**32. Ans. (A)**	**33. Ans. (D)**	**34. Ans. (B)**	**35. Ans. (D)**	**36. Ans. (C)**
37. Ans. (A)	**38. Ans. (B)**	**39. Ans. (B)**	**40. Ans. (A)**		

EXERCISE (J-M)

1. Ans. (4)	**2. Ans. (4)**	**3. Ans. (2)**	**4. Ans. (4)**	**5. Ans. (4)**	**6. Ans. (3)**
7. Ans. (1)	**8. Ans. (4)**	**9. Ans. (1)**	**10. Ans. (4)**	**11. Ans. (1)**	
12. Ans. (1 or 3)	**13. Ans. (2)**	**14. Ans. (3)**	**15. Ans. (3)**	**16. Ans. (3)**	**17. Ans. (4)**
18. Ans. (4)	**19. Ans. (1)**	**20. Ans. (4)**	**21. Ans. (1)**	**22. Ans. (3)**	**23. Ans. (3)**
24. Ans. (1)	**25. Ans. (1)**	**26. Ans. (2)**	**27. Ans. (3)**	**28. Ans. (1)**	**29. Ans. (3)**
30. Ans. (4)	**31. Ans. (4)**	**32. Ans. (2)**	**33. Ans. (2)**	**34. Ans. (3)**	**35. Ans. (4)**
36. Ans. (3)	**37. Ans. (1)**	**38. Ans. (3)**	**39. Ans. (2)**	**40. Ans. (3)**	**41. Ans. (3)**
42. Ans. (1)	**43. Ans. (1)**	**44. Ans. (4)**	**45. Ans. (1)**	**46. Ans. (4)**	**47. Ans. (4)**
48. Ans. (4)	**49. Ans. (4)**	**50. Ans. (3)**	**51. Ans. (2)**	**52. Ans. (4)**	**53. Ans. (3)**
54. Ans. (4)	**55. Ans. (2)**	**56. Ans. (75)**	**57. Ans. (2)**	**58. Ans. (1)**	
59. Ans. (15.00)	**60. Ans. (2)**				