NEWTON'S LAWS OF MOTION & FRICTION

KEY CONCEPTS

FORCE

A push or pull that one object exerts on another.

FORCES IN NATURE

There are four fundamental forces in nature:

1. Gravitational force

2. Electromagnetic force

3. Strong nuclear force

4. Weak force

TYPES OF FORCES ON MACROSCOPIC OBJECTS

(a) Field Forces or Range Forces:

These are the forces in which contact between two objects is not necessary.

Ex. (i) Gravitational force between two bodies. (ii) Electrostatic force between two charges.

(b) Contact Forces:

Contact forces exist only as long as the objects are touching each another.

Ex. (i) Normal forces.

(ii) Frictional force

(c) Attachment to Another Body:

Tension (T) in a string and spring force (F = kx) comes in this group.

NEWTON'S FIRST LAW OF MOTION (OR GALILEO'S LAW OF INERTIA)

Every body continues in its state of rest or uniform motion in a straight line unless compelled by an external force to change that state.

Definition of force from Newton's first law of motion "Force is that push or pull which changes or tends to change the state of rest or of uniform motion in a straight line".

Inertia : Inertia is the property of the body due to which body oppose the change of its state. Inertia of a body is measured by mass of the body. Inertia ∞ mass

TYPES OF INERTIA

Inertia of rest: It is the inability of a body to change, its state of rest by itself.

Examples :

- When we shake a branch of a mango tree, the mangoes fall down.
- When a bus or train starts suddenly the passengers sitting inside tends to fall backwards.
- The dust particles in a blanket fall off when it is beaten with a stick.
- When a stone hits a window pane, the glass is broken into a number of pieces whereas if the high speed bullet strikes the pane, it leaves a clean hole.

Inertia of motion : It is the inability of a body to change its state of uniform motion by itself.

Examples:

- When a bus or train stop suddenly, a passenger sitting inside tends to fall forward.
- A person jumping out of a speeding train may fall forward.
- A ball thrown upwards in a running train continues to move along with the train.

- **Examples:**When a car rounds a curve suddenly, the person sitting inside is thrown outwards.
- Rotating wheels of vehicle throw out mud, mudguard over the wheels stop this mud.
- A body released from a balloon rising up, continues to move in the direction of balloon.

NEWTON'S SECOND LAW OF MOTION

Rate of change of momentum of a body is directly proportional to the external force applied on it and the change in momentum takes place in the direction of force

$$\vec{F} \propto \frac{d\vec{p}}{dt} \text{ or } \qquad \vec{F} = \frac{d\vec{p}}{dt}$$

if m = constant then
$$\vec{F} = m \frac{d\vec{v}}{dt} = m\vec{a}$$

- Newton's Second Law Provides the Definition of the Concept of Force.
- **Definition of the 1 Newton (N):**—If an object of mass one kilogram has an acceleration of 1ms⁻² relative to an inertial reference frame, then the net force exerted on the object is one newton.

CONSEQUENCES OF NEWTON'S II LAW OF MOTION

• Concept of inertial mass: From Newton's II law of motion $a = \frac{F}{M}$

i.e., the magnitude of acceleration produced by a given body is inversely proportional to mass i.e. greater the mass, smaller is the acceleration produced in the body. Thus, mass is the measure of inertia of the body. The mass given by above equation is therefore called the inertial mass.

• An accelerated motion is the result of application of the force :

There may be two types of accelerated motion:

- (i) When only the magnitude of velocity of the body changes: In this types of motion the force is applied along the direction of motion or opposite to the direction of motion.
- (ii) When only the direction of motion of the body changes: In this case the force is applied at right angles to the direction of motion of the body, e.g. uniform circular motion.
- **Ex.** A force $\vec{F} = (6\hat{i} 8\hat{j} + 10\hat{k})N$ produces acceleration of 1 ms⁻² in a body. Calculate the mass of the body.

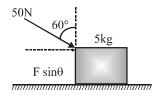
Sol. : Acceleration
$$a = \frac{|\vec{F}|}{m}$$
 : mass $m = \frac{|\vec{F}|}{a} = \frac{\sqrt{6^2 + 8^2 + 10^2}}{1} = 10\sqrt{2} \text{ kg}$

Ex. A force of 50 N acts in the direction as shown in figure. The block of mass 5kg, resting on a smooth horizontal surface. Find out the acceleration of the block.



Sol. Horizontal component of the force = $F\sin\theta = 50\sin 60^\circ = \frac{50\sqrt{3}}{2}$ N

acceleration of the block
$$a = \frac{F \sin \theta}{m} = \frac{50\sqrt{3}}{2} \times \frac{1}{5} = 5\sqrt{3} \text{ m/s}^2$$



NEWTON'S THIRD LAW OF MOTION

The first and second laws are statements about a single object, whereas the third law is a statement about two objects.

- According to this law, every action has equal and opposite reaction. Action and reaction act on different bodies and they are simultaneous. There can be no reaction without action.
- If an object A exerts a force $\vec{\mathbf{F}}$ on an object B, then B exerts an equal and opposite force $(-\vec{\mathbf{F}})$ on A.





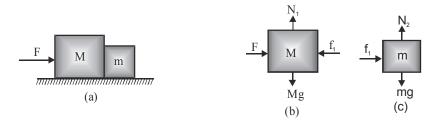
The forces between two objects A and B are equal and opposite, whether they are attractive or repulsive.

- Action and reaction never cancel each other, since they act on different bodies.
- **First law**: If no net force acts on a particle, then it is possible to select a set of reference frames, called inertial reference frames, observed from which the particle moves without any change in velocity.
- Second law: Observed from an inertial reference frame, the net force on a particle is proportional to the time rate of change of its linear momentum: $\frac{d(m\vec{v})}{dt}$
- **Third law**: Whenever a particle A exerts a force on another particle B, B simultaneously exerts a force on A with the same magnitude in the opposite direction.

FREE BODY DIAGRAM

A diagram showing all external forces acting on an object is called "Free Body Diagram" (F.B.D.) In a specific problem, first we are required to choose a body and then we find the number of forces acting on it, and all the forces are drawn on the body, considering it as a point mass. The resulting diagram is known as free body diagram (FBD).

For example, if two bodies of masses m and M are in contact and a force F on M is applied from the left as shown in figure (a), the free body diagrams of M and m will be as shown in figure (b) and (c).



Important Point:

Two forces in Newton's third law never occur in the same free-body diagram. This is because a free-body diagram shows forces acting on a single object, and the action-reaction pair in Newton's third law always act on different objects.

MOTION OF BODIES IN CONTACT

Case I:

When two bodies of masses m₁ and m₂ are kept on the frictionless surface and a force F is applied on one body, then the force with which one body presses the other at the point of contact is called Force of Contact. These two bodies will move with same acceleration a.

When the force F acts on the body with mass m₁ (i) as shown in fig. (1) $F = (m_1 + m_2)$ a. If the force exerted by m₂ on m₁ is f₁ (force of contact) then for body $m_1 : (F - f_1) = m_1 a$

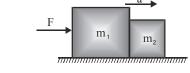
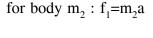


Fig.(1): When the force F acts on mass m₁.



$$\Rightarrow$$
 action of m_1 on m_2 : $f_1 = \frac{m_2 F}{m_1 + m_2}$

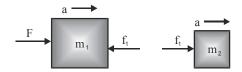


Fig. 1(a): F.B.D. representation of action and reaction forces.

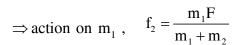
When the force F acts on the body with mass m₂ (ii)as shown in figure 2

$$F = (m_1 + m_2) a$$

for body with mass m₂

$$F - f_2 = m_2 a$$

for body m_1 , $f_2 = m_1 a$



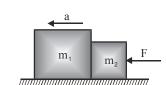
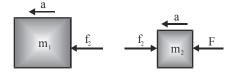


Fig. (2): When the force F acts on mass m2



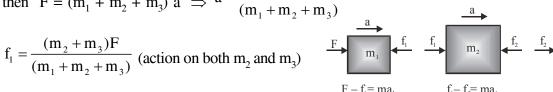
m,

Fig. 2 (a): F.B.D. representation of action and reaction forces.

Case II:

Three bodies of masses m₁, m₂ and m₃ placed one after another and in contact with each other. Suppose a force F is applied horizontally on mass m₁

then
$$F = (m_1 + m_2 + m_3) a \implies a = \frac{F}{(m_1 + m_2 + m_3)}$$

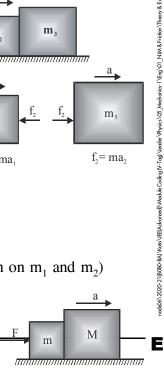


and
$$f_2 = \frac{m_3 F}{(m_1 + m_2 + m_3)}$$
 (action on m_3 alone)

when the force F is applied on m₃, then

$$f_1 = \frac{m_1 F}{(m_1 + m_2 + m_3)}$$
 (action on m_1 alone) and $f_2 = \frac{(m_1 + m_2) F}{(m_1 + m_2 + m_3)}$ (action on m_1 and m_2)

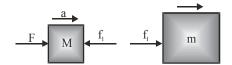
Two blocks of mass m = 2 kg and M = 5 kg are in contact on a



frictionless table. A horizontal force F (= 35 N) is applied to m. Find the force of contact between the block, will the force of contact remain same if F is applied to M?

Sol. As the blocks are rigid under the action of a force F, both will move with same acceleration

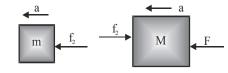
$$a = \frac{F}{m+M} = \frac{35}{2+5} = 5 \text{ m/s}^2$$



force of contact $f_1 = Ma = 5 \times 5 = 25 \text{ N}$

If the force is applied to M then its action on m will be

$$f_2 = ma = 2 \times 5 = 10 \text{ N}.$$

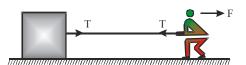


From this problem it is clear that acceleration does not depends on the fact that whether the force is applied to m or M, but force of contact does.

SYSTEM OF MASSES TIED BY STRINGS

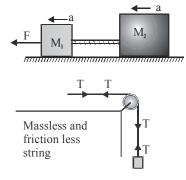
Tension in a String:

It is an intermolecular force between the atoms of a string, which acts or reacts when the string is streched.

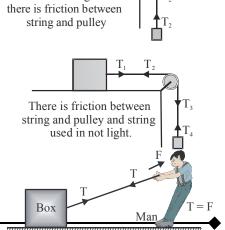


Important points about the tension in a string:

- Force of tension act on a body in the direction away from the point of contact or tied ends of the string.
- String is assumed to be inextensible so that the magnitude of accelerations of any number of masses connected through strings is always same.
- If the string is extensible the acceleration of different masses connected through it will be different until the string can stretch.
- String is massless and frictionless so that tension throughout the string remains same.



- If the string is massless but not frictionless, at every contact tension changes.
- If the string is not light, tension at each point will be different depending on the acceleration of the string.
- If a force is directly applied on a string as say man is pulling

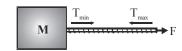


Massless string but

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a tied string from the other end with some force the tension will be equal to the applied force irrespective of the motion of the pulling agent, irrespective of whether the box will move or not, man will move or not.

• String is assumed to be massless unless stated, hence tension in it everywhere remains the same and equal to applied force. However, if a string has a mass, tension at different points will be different being maximum (= applied force) at the end through which force is applied and minimum at the other end connected to a body.



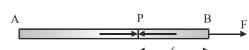
• In order to produce tension in a string two equal and opposite stretching forces must be applied. The tension thus produced is equal in magnitude to either applied force (i.e., T = F) and is directed inwards opposite to F. Here it must be noted that a string can never be compressed like a spring.

$$F \leftarrow T \qquad T \qquad F$$

• If string is cut so that element b is replaced by a spring scale (the rest of the string being undisturbed), the scale reads the tension T.

- Every string can bear a maximum tension, i.e. if the tension in a string is continuously increased it will break if the tension is increased beyond a certain limit. The maximum tension which a string can bear without breaking is called "breaking strength". It is finite for a string and depends on its material and dimensions.
- **Ex.** A uniform rope of length L is pulled by a constant force F. What is the tension in the rope at a distance ℓ from the end where it is applied?
- Sol. Let mass of rope is M and T be tension in the rope at

point P, then. Acceleration of rope, $a = \frac{F}{M}$



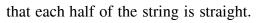
Equation of motion of part PB is F - T = (m) a

$$\Rightarrow T = F - (m) \ a = F - \left(\frac{M}{L}\right) \ \left(\ell\right) \ \left(\frac{F}{M}\right) \ = \left[1 - \frac{\ell}{L}\right] \ F$$



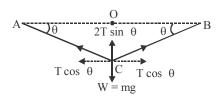
Ex. A bird with mass m perches at the middle of a stretched string

Show that the tension in the string is give by $T = \frac{mg}{2 \sin \theta}$. Assume





Sol. Initial position of wire = AOB. Final position of wire = ACB Let θ be the angle made by wire with horizontal, which is very small. Resolving tension T of string in horizontal and vertical directions, we note that the horizontal components cancel while vertical components add and balance the weight.



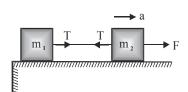
For equilibrium 2T
$$\sin\theta = W = mg$$
 $\Rightarrow T = \frac{W}{2\sin\theta} = mg/2\sin\theta$

MOTION OF BODIES CONNECTED BY STRINGS

Case: I

Two bodies:

Let us consider the case of two bodies of masses m₁ and m₂ connected by a thread and placed on a smooth horizontal surface as shown in figure. A force F is applied on the body of mass m₂ in forward direction as shown. Our aim is to consider the acceleration of the system and the tension T in the thread. The forces acting separately on two bodies are also shown



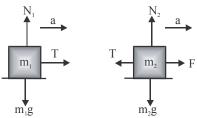
in the figure:

From figure
$$T = m_1 a$$

and
$$F - T = m_2 a$$

$$\Rightarrow$$
 F = $(m_1 + m_2)a$

$$\Rightarrow a = \frac{F}{m_1 + m_2} \& T = \frac{m_1 F}{m_1 + m_2}$$

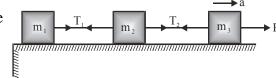


Case II:

Three bodies

In case of three bodies, the situation is shown in figure

Acceleration
$$a = \frac{F}{m_1 + m_2 + m_3}$$
,

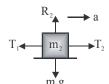


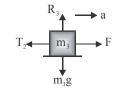
$$T_1 = m_1 a = \frac{m_1 F}{m_1 + m_2 + m_3}$$

∴ for block of mass
$$m_3$$
 $F - T_2 = m_3 a$

∴ $T_2 = F - \frac{m_3 F}{m_1 + m_2 + m_3} = \frac{(m_1 + m_2) F}{m_1 + m_2 + m_3}$
 \vdots

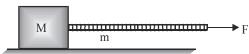






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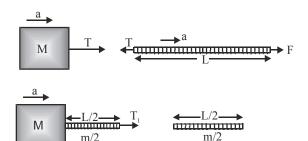
- Ex. A block of mass M is pulled along a horizontal frictionless surface by a rope of mass m as shown in fig. A horizontal force F is applied to one end of the rope. Find (i) The acceleration of the rope and block (ii) The force that the rope exerts on the block. (iii) Tension in the rope at its mid point.
- **Sol.** (i) Accelaration $a = \frac{F}{(m+M)}$



(ii) Force exerted by rope $T = Ma = \frac{M.F}{(m+M)}$

(iii)
$$T_1 = \left(\frac{m}{2} + M\right)$$
 $a = \left(\frac{m + 2M}{2}\right) \left(\frac{F}{m + M}\right)$

Tension in rope at midpiont $T_1 = \frac{(m+2M)F}{2(m+M)}$

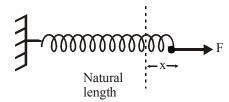


Spring Force (According to Hooke's law):

In equilibrium F = kx

k is spring constant

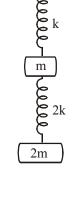
Note: Spring force is non impulsive in nature.



- If the lower spring is cut, find acceleration of the blocks, immediately after cutting the spring. Ex.
- Sol. Intial stretches

$$x_{upper} = \frac{3mg}{k}$$
 & $x_{lower} = \frac{mg}{k}$

On cutting the lower spring, by virture of non-impulsive nature of spring the stretch in upper spring remains same. Thus,



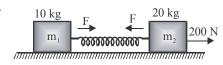
$$2mg = 2ma \Rightarrow a = g$$

Upper block :



$$k \left(\frac{3mg}{k}\right) - mg = ma \Rightarrow a = 2 g$$

Two masses 10 kg and 20 kg respectively are connected by a



ALLEN

massless spring as shown in figure force of 200 N acts on the 20 kg mass. At the instant shown in figure the 10 kg mass has acceleration of 12 m/s², what is the acceleration of 20 kg mass?

Sol. Equation of motion for m_1 is $F = m_1 a_1 = 10 \times 12 = 120 \text{ N}$.

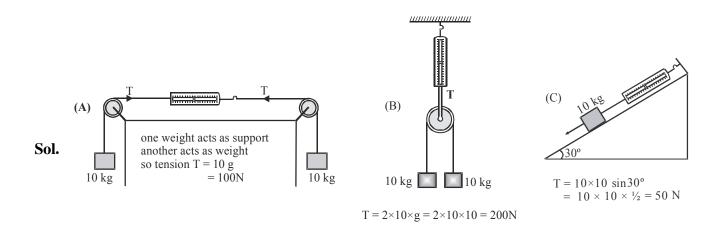
Force on 10 kg-mass is 120 N to the right. As action and reaction are equal and opposite, the reaction force F on 20 kg mass F = 120 N to the left.

 \therefore Equation of motion for m₂ is $200 - F = 20 a_2$

$$\Rightarrow 200 - 120 = 20 \text{ a}_2 \qquad \Rightarrow 20 \text{ a}_2 = 80 \qquad \Rightarrow \text{a}_2 = \frac{80}{20} = 4 \text{ ms}^{-2}$$

SPRING BALANCE: It measures force acting on hook.

Ex. The system shown in figure are in equilibrium. If the spring balance is calibrated in newtons, what does it record in each case? $(g = 10 \text{ m/s}^2)$



FRAME OF REFERENCE

It is a conveniently chosen co-ordinate system which describes the position and motion of a body in space.

INERTIAL AND NON-INERTIAL FRAMES OF REFERENCE

Inertial frames of reference:

A reference frame which is either at rest or in uniform motion along the straight line. A non–accelerating frame of reference is called an inertial frame of reference.

- All the fundamental laws of physics have been formulated in respect of inertial frame of reference.
- All the fundamental laws of physics can be expressed as to have the same mathematical form in all the inertial frames of reference.
- The mechanical and optical experiments performed in an inertial frame in any direction will always yield the same results. It is called isotropic property of the inertial frame of reference.

Examples of inertial frames of reference:

- A frame of reference remaining fixed w.r.t. distant stars is an inertial frame of reference.
- A space–ship moving in outer space without spinning and with its engine cut–off is also inertial frame of reference.
- For practical purposes, a frame of reference fixed to the earth can be considered as an inertial frame. Strictly speaking, such a frame of reference is not an inertial frame of reference, because the motion

of earth around the sun is accelerated motion due to its orbital and rotational motion. However, due to negligibly small effects of rotation and orbital motion, the motion of earth may be assumed to be uniform and hence a frame of reference fixed to it may be regarded as inertial frame of reference.

Non-inertial frame of reference:

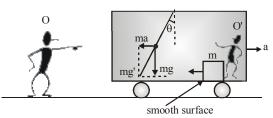
An accelerating frame of reference is called a non-inertial frame of reference.

Newton's laws of motion are not directly applicable in such frames, before application we must add pseudo force.

Note: A rotating frame of references is a non–inertial frame of reference, because it is also an accelerating one due to its centripetal acceleration.

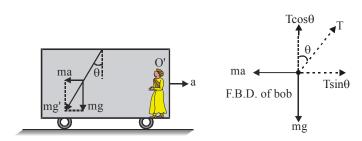
PSEUDO FORCE

The force on a body due to acceleration of non–inertial frame is called fictitious or apparent or pseudo force and is given by $\vec{F} = -m\vec{a}_0$, where \vec{a}_0 is acceleration of non–inertial frame with respect to an inertial frame and m is mass of the particle or body. The direction of pseudo force must be opposite to the direction of acceleration of the non–inertial frame.



For observer O on ground train is moving with acceleration on "a" for observer O' in side the train there is pseudo force in opposite direction shown in figure.

- When we draw the free body diagram of a mass, with respect to an **inertial frame of reference** we apply only the real forces (forces which are actually acting on the mass).
- But when the free body diagram is drawn from a non–inertial frame of reference a pseudo force (in addition to all real forces) has to be applied to make the equation $\vec{F} = m\vec{a}$ to be valid in this frame also.
- Ex. A pendulum of mass m is suspended from the ceiling of a train moving with an acceleration 'a' as shown in figure. Find the angle θ in equilibrium position.



Sol. Non-inertial frame of reference (Train)

F.B.D. of bob w.r.t. train. (real forces + pseudo force) : with respect to train, bob is in equilibrium $\therefore \ \Sigma F_v = 0 \Rightarrow T \cos \theta = mg \qquad \text{and}$

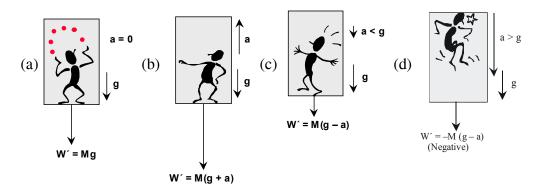
$$\Sigma F_x = 0 \Rightarrow T \sin \theta = ma$$

$$\Rightarrow \tan \theta = \frac{a}{g} \Rightarrow \theta = \tan^{-1} \left(\frac{a}{g}\right)$$

MOTION IN A LIFT

The weight of a body is simply the force exerted by earth on the body. If body is on an accelerated platform, the body experiences fictitious force, so the weight of the body appears changed and this new weight is called apparent weight. Let a man of weight W = Mg be standing in a lift.

We consider the following cases:



Case (a):

If the lift moving with constant velocity v upwards or downwards.

In this case there is no accelerated motion hence no pseudo force experienced by observer inside the lift. So apparent weight W' = Mg (Actual weight).

Case (b):

If the lift is accelerated upward with constant acceleration a.

Then net forces acting on the man are (i) weight W = Mg downward (ii) fictitious force $F_0 = Ma$ downward.

So apparent weight $W' = W + F_0 = Mg + Ma = M(g + a)$

Case (c)

If the lift is accelerated downward with acceleration a < g

Then fictitious force $F_0 = Ma$ acts upward while weight of man W = Mg always acts downward.

So apparent weight $W' = W + F_0 = Mg - Ma = M(g - a)$

Special Case : If a = g then W' = 0 (condition of weightlessness).

Thus, in a freely falling lift the man will experience weightlessness.

Case (d):

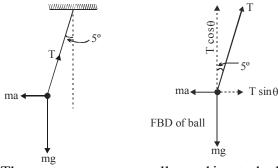
If lift accelerates downward with acceleration a > g:

Then as in Case c. Apparent weight W' = M(g - a) is negative, i.e., the man will be accelerated upward w.r.t. to lift and will stay at the ceiling of the lift.

- **Ex.** A spring weighing machine inside a stationary lift reads 50 kg when a man stands on it. What would happen to the scale reading if the lift is moving upward with (i) constant velocity, and (ii) constant acceleration?
- **Sol.** (i) In the case of constant velocity of lift, there is no fictitious force; therefore the apparent weight = actual weight. Hence the reading of machine is 50 kgwt.
 - (ii) In this case the acceleration is upward, the fictitious force ma acts downward, therefore apparent weight is more than actual weight i.e. W' = m (g + a).

- Hence scale shows a reading = m (g + a) = $\frac{mg\left(1 + \frac{a}{g}\right)}{g} = \left(50 + \frac{50a}{g}\right)$ kg wt.
- **Ex.** Two objects of equal mass rest on the opposite pans of an arm balance. Does the scale remain balanced when it is accelerated up or down in a lift?
- Sol. Yes, since both masses experience equal fictitious forces in magnitude as well as direction.
- **Ex.** A passenger on a large ship sailing in a quiet sea hangs a ball from the ceiling of her cabin by means of a long thread. Whenever the ship accelerates, she notes that the pendulum ball lags behind the point of suspension and so the pendulum no longer hangs vertically. How large is the ship's acceleration when the pendulum stands at an angle of 5° to the vertical?
- **Sol.** The ball is accelerated by the force T $\sin 5^{\circ}$. Therefore T $\sin 5^{\circ} = ma$. Vertical component $\Sigma F = 0$, so T $\cos 5^{\circ} = mg$. By solving $a = g \tan 5^{\circ} = 0.0875 g$

 $= 0.86 \text{ ms}^{-2}$.



- Ex. A 12 kg monkey climbs a light rope as shown in figure. The rope passes over a pulley and is attached to a 16 kg bunch of bananas. Mass and friction in the pulley are negligible so that the pulley's only effect is to reverse the direction of the rope. What is the maximum acceleration the monkey can have without lifting the bananas?
- **Sol.** Effective weight of monkey As per given condition

ey
$$W_{m} = M_{m}(g + a)$$

 $W_{m} = W_{b}$
 $M_{m}(g + a) = M_{b}g$

$$\Rightarrow a = \frac{(M_{b} - M_{m})}{M_{m}} = \left(\frac{16 - 12}{12}\right) \times 9.8$$

$$= \frac{9.8}{3} = 3.26 \text{ m/s}^{2}$$

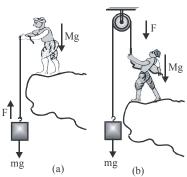


PULLEY SYSTEM

A single fixed pulley changes the direction of force only and in general, assumed to be massless and frictionless.

It is clear from example given below.

- **Ex.** A block of mass 25 kg is raised by a 50 kg man in two different ways as shown in figure. What is the action on the floor by the man in the two cases? If the floor yields to a normal force of 700 N, which mode should be the man adopt to lift the block without the floor yielding?
- **Sol.** Mass of the block, m = 25 kg; mass of the man, M = 50 kgForce applied to lift the block $F = mg = 25 \times 9.8 = 245 \text{ N}$ Weight of the man, $M = 50 \times 9.8 = 490 \text{ N}$





- (a) When the block is raised by the man by applying force F in upward direction, reaction equal and opposite to F will act on the floor in addition to the weight of the man.
 - \therefore action on the floor Mg + F = 490 + 245 = 735 N
- (b) When the block is raised by the mass applying force F over the rope (passed over the pulley) in downward direction, reaction equal and opposite to F will act on the floor,
 - \therefore action on the floor Mg F = 490 425 = 245 N

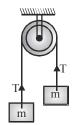
floor yields to a normal force of 700 N, the mode (b) should be adopted by the man to lift block.

Some cases of pulley

I Case

$$m_1 = m_2 = m$$

Tension in the string $T = mg$
Acceleration 'a' = zero

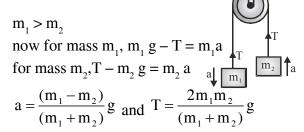


Reaction at the suspension of the pulley

$$R = 2T = 2 \text{ mg}.$$

Acceleration =
$$\frac{\text{net pulling force}}{\text{total mass to be pulled}}$$

II Case



Tension =
$$\frac{2 \times Product of masses}{Sum of two masses} g$$

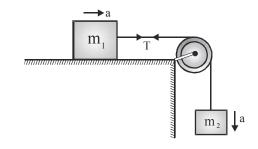
Reaction at the suspension of pulley $R = 2T = \frac{4m_1m_2 g}{(m_1 + m_2)}$

III Case:

For mass m_1 : $T = m_1 a$

For mass m_2 : $m_2 g - T = m_2 a$

acceleration $a = \frac{m_2 g}{(m_1 + m_2)}$ and $T = \frac{m_1 m_2}{(m_1 + m_2)} g$



IV Case:

$$(m_1 > m_2)$$

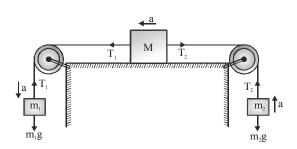
For
$$m_1, m_1g - T_1 = m_1a$$

For
$$m_2$$
, $T_2 - m_2 g = m_2 a$

For $MT_1 - T_2 = Ma$

$$\Rightarrow a = \frac{(m_1 - m_2)}{(m_1 + m_2 + M)} g,$$

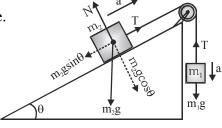
$$T_1 = \frac{(2m_2 + M)m_1g}{m_1 + m_2 + M}, T_2 = \frac{(2m_1 + M)m_2g}{m_1 + m_2 + M}$$



V Case:

Mass suspended over a pulley from another on an inclined plane.

For mass $m_1 : m_1 g - T = m_1 a$



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For mass
$$m_2$$
: $T - m_2 g \sin \theta = m_2 a$

acceleration
$$a = \frac{(m_1 - m_2 \sin \theta)}{(m_1 + m_2)} g$$

$$T = \frac{m_1 m_2 (1 + \sin \theta)}{(m_1 + m_2)} g$$

VI Case:

Masses m_1 and m_2 are connected by a string passing over a pulley $(m_1 > m_2)$

Acceleration
$$a = \frac{(m_1 \sin \alpha - m_2 \sin \beta)}{(m_1 + m_2)} g$$

Tension
$$T = \frac{m_1 m_2 (\sin \alpha + \sin \beta)}{(m_1 + m_2)} g$$



For mass
$$m_1 : T_1 - m_1 g = m_1 a$$

For mass
$$m_1 : m_2g + T_2 - T_1 = m_2a$$

For mass
$$m_3 : m_3 g - T_2 = m_3 a$$

Acceleration
$$a = \frac{(m_2 + m_3 - m_1)}{(m_1 + m_2 + m_3)}g$$

we can calculate tensions T_1 and T_2 from above equations

VIII Case:

From case (iii)

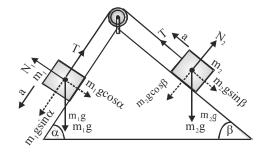
tension
$$T = \frac{m_1 m_2}{(m_1 + m_2)} g$$

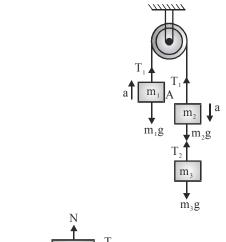
If x is the extension in the spring,

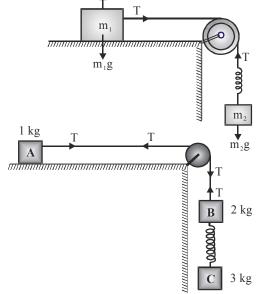
then
$$T = kx$$

$$x = \frac{T}{k} = \frac{m_1 m_2 g}{k(m_1 + m_2)}$$

- **Ex.** In the system shown in figure all surface are smooth, string is massless and inextensible. Find:
 - (a) acceleration of the system
 - (b) tension in the string and
 - (c) extension in the spring if force constant of spring is k = 50 N/m (Take $g = 10 \text{ m/s}^2$)
- **Sol.** (a) In this case net pulling force = $m_c g + m_B g = 50 \text{ N}$ and total mass to be pulled is (1 + 2 + 3) kg = 6 kg.
 - \therefore Acceleration of the system is $a = \frac{50}{6} \text{ ms}^{-2}$
- (b) Free body diagram of 1 kg block gives $T = ma = (1) \left(\frac{50}{6}\right) N = \frac{50}{6} N$









(c) Free body diagram of 3 kg block gives

$$30 - kx = ma$$
 but $ma = 3 \times \frac{50}{6} = 25N$
 $x = \frac{30 - 25}{k} = \frac{5}{50} = 0.1$ $m = 10$ cm

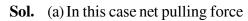


Ex. In the adjacent figure, masses of A, B and C are 1 kg, 3 kg and 2 kg respectively.

Find: (a) the acceleration of the system and

(b) tensions in the string

Neglect friction. $(g = 10 \text{ ms}^{-2})$



$$= m_{A}g \sin 60^{\circ} + m_{B}g \sin 60^{\circ} - m_{C}g \sin 30^{\circ}$$

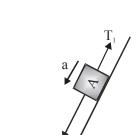
$$= (m_{A} + m_{B}) g \sin 60^{\circ} - m_{C}g \sin 30^{\circ}$$

$$= (1 + 3) \times 10 \times \frac{\sqrt{3}}{2} - 2 \times 10 \times \frac{1}{2}$$

$$= 20\sqrt{3} - 10 = 20 \times 1.732 - 10 = 24.64 \text{ N}$$

Total mass being pulled = 1 + 3 + 2 = 6 kg

$$\therefore$$
 Acceleration of the system $a = \frac{24.64}{6} = 4.1 \text{ m/s}^2$



 $m_{_A}g \ sin 60^\circ$

(b) For the tension T_1 in the string between A and B, $m_A g \sin 60^\circ - T_1 = (m_A)$ (a)

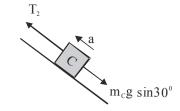
:.
$$T_1 = m_A g \sin 60^\circ - m_A a = m_A (g \sin 60^\circ - a)$$

$$\Rightarrow T_1 = (1) \left(10 \times \frac{\sqrt{3}}{2} - 4.1 \right) = 4.56 \text{ N}$$

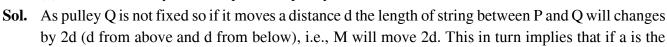
For the tension T₂ in the string between B and C.

$$T_2 - m_C g \sin 30^\circ = m_C a$$

$$\Rightarrow$$
 T₂ = m_C (a + g sin30°) = 2 $\left[4.1+10 \left(\frac{1}{2}\right)\right]$ = 18.2 N



- **Ex.**. Consider the situation shown in figure. Both the pulleys and the strings are light and all the surfaces are frictionless. Calculate
 - (a) the acceleration of mass M,
 - (b) tension in the string PQ and
 - (c) force exerted by the clamp on the pulley P.



acceleration of M the acceleration of Q and of 2M will be $\frac{a}{2}$.

Now if we consider the motion of mass M, it is accelerated downward, so

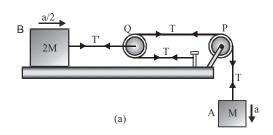
$$T = M(g - a) \dots (i)$$

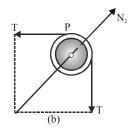
And for the motion of Q



$$2T - T' = 0 \times \frac{a}{2} = 0 \implies T' = 2T$$
(ii)

And for the motion of mass 2M $T' = 2M\left(\frac{a}{2}\right)$, $\Rightarrow T' = Ma$ (iii)





(a) From equation (ii) and (iii) as $T = \frac{1}{2}Ma$, so equation (i) reduces to

$$T = \frac{1}{2} Ma = M(g - a) \implies a = \frac{2}{3} g$$

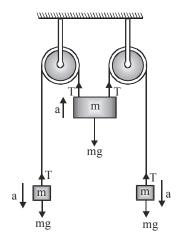
(b) So the acceleration of mass M is $\frac{2}{3}$ g while tension in the string PQ from equation (1) will be

$$T = M(g - \frac{2}{3}g) = \frac{1}{3} Mg$$

(c) Now from figure (b), it is clear that force on pulley by the clamp will be equal and opposite to the resultant of T and T at 90° to each other, i.e.,

$$(N_2) = \sqrt{T^2 + T^2} = \sqrt{2} T = \frac{\sqrt{2}}{3} Mg$$

Ex. Consider the double Atwood's machine as shown in the figure



- (a) What is acceleration of the masses?
- (b) What is the tension in each string?
- **Sol.** (a) Here the system behaves as a rigid system, therefore every part of the system will move with same acceleration. Thus Applying newton's law

$$mg - T = ma$$
 ...(1)

$$2T - mg = ma$$
 ...(2)

Doubling the first equation and adding

$$mg = 3 \text{ ma} \Rightarrow \text{acceleration } a = \frac{1}{3}g$$

(b) Tension in the string
$$T = m (g - a) = m \left(g - \frac{g}{3}\right) = \frac{2}{3} mg$$

Ex. Consider the system of masses and pulleys shown in fig. with massless string and frictionless pulleys.

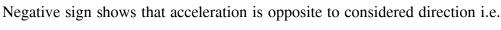
- (a) Give the necessary relation between masses m₁ and m₂ such that system is in equilibrium and does not move.
- (b) If $m_1 = 6$ kg and $m_2 = 8$ kg, calculate the magnitude and direction of the acceleration of m_1 .
- **Sol.** (a) Applying newton's law $m_2g 2T = 0$ (because there is no acceleration) and $T m_1g = 0$ $\Rightarrow (m_2 - 2m_1) \ g = 0 \ \Rightarrow m_2 = 2m_1$
 - (b) If the upwards acceleration of m₁ is a,

then acceleration of m_2 is $\frac{a}{2}$ downwards

for mass
$$m_2$$
: $m_2g - 2T = m_2\left(\frac{a}{2}\right) \Rightarrow 2m_2g - 4T = m_2a$

for mass $m_1 : T - m_1 g = m_1 a$

$$\Rightarrow \qquad a = \left(\frac{2m_2 - 4m_1}{m_2 + 4m_1}\right)g = \frac{2(8 - 12)}{8 + 24}g = -\frac{g}{4}$$



it is downwards for m_1 and upwards for m_2 .

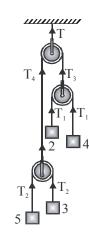
- Ex. In the given figure If $T_1 = 2T_2 = 50$ N then find the value of T.
- Sol. As given in figure,

$$T_3 = 2T_1 = 2(2T_2) = 4T_2$$

and
$$T_4 = 2T_2$$

$$T = T_3 + T_4 = 4T_2 + 2T_2 = 6T_2$$

$$=6\times\frac{50}{2}=150$$
N



CONSTRAINT RELATIONS

These equations establish the relation between accelerations (or velocities) of different masses attached by string(s). Normally number of constraint equations are equal to number of strings in the system under consideration.

Ex. Find the relation between acceleration of 1 and 2.

Sol. At any instant of time let x_1 and x_2 be the displacements of

1 and 2 from a fixed line. Then $x_1 + x_2 = constant$

Differentiating w.r.t. time, $v_1 + v_2 = 0$

Again differentiating w.r.t. time, $a_1 + a_2 = 0 \Rightarrow a_1 = -a_2$

So acceleration of 1 and 2 are equal but in opposite directions.

Ex. At certain moment of time, velocities of 1 and 2 both are 1 ms⁻¹ upwards. Find the velocity of 3 at that moment.

Sol.
$$x_1 + x_4 = \ell_1$$
 (length of first string)
 $x_2 - x_4 + x_3 - x_4 = \ell_2$ (length of second string)

$$\Rightarrow v_1 + v_4 = 0 & v_2 + v_3 - 2v_4 = 0$$

\Rightarrow v_2 + v_3 + 2v_1 = 0

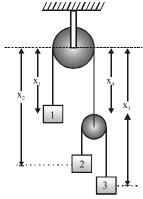
Taking upward direction as positive

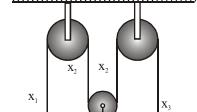
$$v_1 = v_2 = 1$$

so
$$1^{1} + v_{3}^{2} + 2 \times 1 = 0 \implies v_{3} = -3 \text{ ms}^{-1}$$

i.e. velocity of block 3 is 3 ms⁻¹ downwards.

Ex. Find the relation between acceleration of blocks a_1 , a_2 and a_3 .



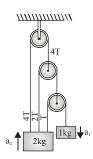


Sol.
$$x_1 + 2 x_2 + x_3 = \ell$$

 $v_1 + 2v_2 + v_3 = 0$

 $a_1 + 2a_2 + a_3 = 0$

Ex. Using contraint equation. Find the relation between
$$a_1$$
 and a_2 .



Sol. For this system $a_1T = a_2(4T+2T+T) \Rightarrow a_1=7a_2$

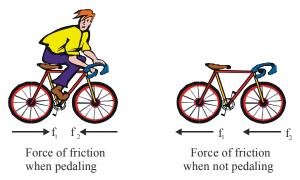
FRICTION

INTRODUCTION

Friction is the force of two surfaces in contact, or the force of a medium acting on a moving object. (i.e. air on aircraft.). Frictional forces may also exist between surfaces when there is no relative motion. Frictional forces arise due to molecular interactions. In some cases friction acts as a supporting force and in some cases it acts as opposing force.



- **Supporting:** Walking process can only take place because there is friction between the shoes and ground.
- **Opposing :** When a block slides over a surface the force of friction acts as an opposing force in the opposite direction of the motion
- Both Supporting and Opposing :
- **Pedaling:** When cyclist pedals the friction force on rear wheel acts as a supporting force and on front wheel as a opposing force.

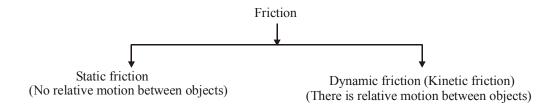


• **Non–Pedaling :** When cyclist not pedals the friction force on rear wheel & front wheel act as a opposing force.

CAUSE OF FRICTION

- Old View: When two bodies are in contact with each other, the irregularities in the surface of one body set interlocked in the irregularities of another surface. This locking opposes the tendency of motion.
- **Modern View:** Friction is arises on account of strong atomic or molecular forces of attraction between the two surfaces at the point of actual contact.

TYPES OF FRICTION



STATIC FRICTION

- It is the frictional force which is effective before motion starts between two planes in contact with each other.
- It's nature is self adjusting.
- Numerical value of static friction is equal to external force parallel to surface which creates the tendency
 of motion of body.
- Maximum value of static friction is called limiting friction. $0 \le f_s \le \mu_s N$, $\vec{f}_s = -\vec{F}_{applied \; (parallel \; to \; surface)}$

LAWS OF LIMITING FRICTION

- The magnitude of the force of limiting friction (F) between any two bodies in contact is directly proportional to the normal reaction (N) between them $F \propto N$
- The direction of the force of limiting friction is always opposite to the direction in which one body is on the verge of moving over the other.
- The force of limiting friction is independent of the apparent contact area, as long as normal reaction between the two bodies in contact remains the same.
- Limiting friction between any two bodies in contact depends on the nature of material of the surfaces in contact and their roughness and smoothness.
- Value of limiting friction is more than to other types of friction force.

DYNAMIC FRICTION

The friction opposing the relative motion between two bodies is called dynamic or kinetic friction $f_{_{k}}=\left(\mu_{_{k}}N\right)$

- This is always slightly less than the limiting friction
- Direction of kinetic friction is in opposite direction of relative velocity of surface w.r.t. other surface in contact.

COEFFICIENT OF FRICTION

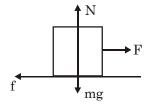
The frictional coefficient is a dimensionless scalar value which describes the ratio of the force of friction between two bodies and the force pressing them together.

- Static friction coefficient $\mu_s = \frac{F}{N}$
- Kinetic friction coefficient $\mu_k = \frac{F_k}{N}$

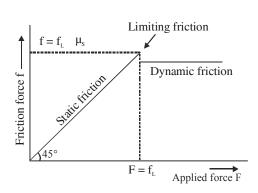
The values of $\mu_{\scriptscriptstyle S}$ and $\mu_{\scriptscriptstyle k}$ depend on the nature of both the surfaces in contact.



GRAPH BETWEEN APPLIED FORCE AND FORCE OF FRICTION



If we slowly increase the force with which we are pulling the box, graph shows that the friction force increases with our force upto a certain critical value, f_L , the box suddenly begins to move, and as soon as it starts moving, a smaller force is required to maintain its motion as in motion friction is reduced. The friction value from 0 to f_L is known as static friction, which balances the external force on the body and prevent it from sliding. The value f_L is the



maximum limit up to which the static friction acts is known as limiting friction, after which body starts sliding and friction reduces to kinetic friction.

- When two highly polished surfaces are pressed hard, then a situation similar to welding occurs. It is called **cold welding.**
- When two copper plates are highly polished and placed in contact with each other, then instead of decreasing, the force of friction increases. This arises due to the fact that for two highly polished surfaces in contact, the number of molecules coming in contact increases and as a result the cohesive/adhesive forces increases. This in turn, increases the force of friction.

Net contact force is the resultant of normal reaction and frictional force.

APPROXIMATE COEFFICIENTS OF FRICTION

	Materials	Coefficient of	Coefficien of
		static friction, $\mu_{_{s}}$	kinetic friction, μ_{k}
	Steel on steel	0.74	0.57
	Aluminum on steel	0.61	0.47
	Copper on steel	0.53	0.36
	Copper on cast iron	1.05	0.29
	Brass on steel	0.51	0.44
1	Teflon on teflon	0.04	0.04
	Rubber on concrete (dry)	1.0	0.8
5	Rubber on concrete (wet)	0.30	0.25

Ex. A block of mass 1 kg is at rest on a rough horizontal surface having coefficient of static friction 0.2 and kinetic friction 0.15, find the frictional forces if a horizontal force,

(a)
$$F = 1N$$

(b)
$$F = 1.96 \text{ N}$$

(c) F = 2.5 N, is applied on a block

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Maximum force of friction $f_{max} = 0.2 \times 1 \times 9.8 \text{ N} = 1.96 \text{ N}$ Sol.

(a)

for $F_{ext} = 1 \text{ N}$, $F_{ext} < f_{max}$ So, body is in rest means static friction is present and hence $f_s = F_{ext} = 1 \text{ N}$

- f = 1.96 N(b)
- for $F_{ext} = 1.96 \text{ N}$, $F_{ext} = f_{max} = 1.96 \text{ N}$ so for $F_{ext} = 2.5 \text{ N}$, so $F_{ext} > f_{max}$. (c) now body is in moving condition

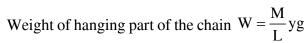
$$\therefore \ f_{\text{max.}} = f_{\text{k}} = \mu_{\text{k}} N = \mu_{\text{k}} \ mg = 0.15 \times \ 1 \times 9.8 = 1.47 \ N$$

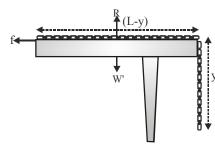
Ex. Length of a chain is L and coefficient of static friction is μ. Calculate the maximum length of the chain which can be hang from the table without sliding.

Sol. Let y be the maximum length of the chain can be hold out side the table without sliding.

Length of chain on the table = (L - y)

Weight of part of the chain on table W' = $\frac{M}{I}$ (L-y)g





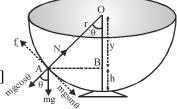
For equlibrium: limiting force of friction = weight of hanging part of the chain

$$\mu R = W \Rightarrow \qquad \mu \; W' = W \qquad \Rightarrow \mu \frac{M}{L} \big(L - y \big) g = \frac{M}{L} y g \; \Rightarrow \mu L - \mu y = y \; \Rightarrow y = \frac{\mu L}{1 + \mu}$$

If the coefficient of friction between an insect and bow ℓ is μ and the radius of the bowl is r, find the maximum height to which the insect can crawl up in the bow ℓ .

The insect will crawl up the bowl till the component of its weight Sol. along the bowl is balanced by limiting frictional force. So, resolving weight perpendicular to the bowl and along the bowl,

 $N = mg \cos \theta$, $f_L = mg \sin \theta \implies \tan \theta = \frac{f_L}{N} \implies \tan \theta = \mu \ [\because f_L = \mu N]$



$$\Rightarrow \frac{\sqrt{(r^2 - y^2)}}{y} = \mu \Rightarrow y = \frac{r}{\sqrt{1 + \mu^2}} \text{ So } h = r - y = r \left[1 - \frac{1}{\sqrt{1 + \mu^2}} \right]$$

- A body of mass M is kept on a rough horizontal surface (friction coefficient = μ). A person is trying to pull the body by applying a horizontal force F, but the body is not moving. What is the force by the surface on A.
- **Sol.** Let f is the force of friction and N is the normal reaction,

then the net force by the surface on the body is $F = \sqrt{N^2 + f^2}$

Let the applied force is F' (varying), applied horizontally then $f \le \mu_s N$ (adjustable with f = F'). Now if F' is zero, f = 0 and $F_{min} = N = Mg$

and when F' is increased to maximum value permissible for no motion. $f = \mu_s N$,

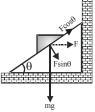
giving
$$F_{max} = \sqrt{N^2 + \mu_s^2 N^2} = Mg \sqrt{1 + \mu_s^2}$$

 $Mg \le F \le Mg\sqrt{1+\mu_s^2}$ therefore we can write

Ex. A block rest on a rough inclined plane as shown in fig. A horizontal force F is applied to it (a) Find out the force of reaction, (b) Can the force of friction be zero if yes when? and (c) Assuming that friction is not zero find magnitude and direction of its limiting value.



- **Sol.** (a) $N = mg \cos \theta + F \sin \theta$
 - (b) Yes, if mg sin $\theta = F \cos \theta$
 - (c) $f = \mu N = \mu$ (mg cos $\theta + F \sin \theta$); up the plane if the body has tendency to slide down and down the plane if the body has tendency to move up.



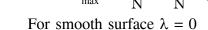
Applied

ANGLE OF FRICTION

The angle of friction is the angle which the resultant of friction force f_S and normal reaction N makes with the normal reaction.

It is represented by $\lambda \tan \lambda = \frac{f_S}{N}$

$$tan \ \lambda_{max} \, = \, \frac{f_{s \, max}}{N} = \frac{\mu N}{N} = \mu$$



ANGLE OF REPOSE (θ)

If a body is placed on an inclined plane and if its angle of inclination is gradually increased, then at some angle of inclination θ the body will just on the point to slide down. The angle is called angle of repose (θ) .

$$F_s = \text{mg sin}\theta \text{ and } N = \text{mg cos}\theta$$

so
$$\frac{F_s}{N} = \tan \theta \implies \mu = \tan \theta$$

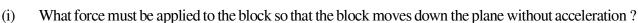


 $tan \ \lambda = \mu \ and \ \mu = tan \ \theta, \ hence \ tan \ \lambda = tan \ \theta \Rightarrow \theta = \lambda$

Thus, angle of repose = angle of friction

Ex. A block of mass 2 kg slides down an inclined plane which makes an angle of 30° with the horizontal.

The coefficient of friction between the block and the surface is $\frac{\sqrt{3}}{2}$.



(ii) What force should be applied to the block so that it can move up without any acceleration?

Sol. Make a 'free-body' diagram of the block. Take the force of friction opposite to the direction of motion.

(i) Project forces along and perpendicular to the plane

perpendicular to plane

 $N = mg \cos \theta$

along the plane

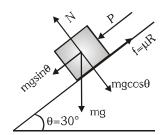
 $F + mg \sin \theta - f = 0$

(: there is no acceleration along the plane)

 $F + mg \sin \theta - \mu N = 0 \Rightarrow F + mg \sin \theta = \mu mg \cos \theta$

F = mg (μ cos θ – sin θ) = 2 × 9.8 (
$$\frac{\sqrt{3}}{2}$$
 cos 30° – sin 30°)

= 19.6
$$(\frac{\sqrt{3}}{2} \times \frac{\sqrt{3}}{2} - \frac{1}{2})$$
 = 19.6 $(\frac{3}{4} - \frac{1}{2})$ = 4.9 N



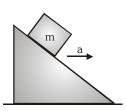
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(ii) This time the direction of F is reversed and that of the frictional force is also reversed.

$$\therefore N = mg \cos \theta ; F = mg \sin \theta + f$$

$$\Rightarrow$$
 F = mg (μcos θ + sin θ) = 19.6 ($\frac{3}{4} + \frac{1}{2}$) = 24.5 N

Ex. A block of mass 1 kg rest on an incline as shown in figure.

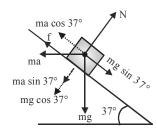


- (a) What must be the frictional force between block and incline if the block is not to slide along the incline when the incline is accelerating to the right at 3 m/s²?
- (b) What is the least value μ_s can have for this to happen?
- **Sol.** N = m (g cos37° + a sin37°) = 1(9.8 × 0.8 + 3 × 0.6) = 9.64 N mg sin 37° = ma cos 37° + f

(a)
$$f = 1(9.8 \times 0.6 - 3 \times 0.8) = 3.48$$

(b)
$$\cdot$$
 f = μ N

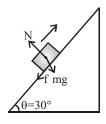
$$\therefore \mu = \frac{f}{N} = \frac{3.48}{9.64} = 0.36$$



- **Ex.** A body of mass 5×10^{-3} kg is launched up on a rough inclined plane making an angle of 30° with the horizontal. Obtain the coefficient of friction between the body and the plane if the time of ascent is half of the time of descent.
- **Sol.** For upward motion: upward retardation $a_1 = \frac{\mu N + mg \sin \theta}{m}$

$$a_1 = \mu g \cos 30^\circ + g \sin 30^\circ = (\sqrt{3} \mu + 1) \frac{g}{2}$$

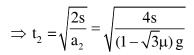
$$\ \, : \ \, s \, = \, \frac{1}{2} \, a_1 t_1^{\ 2} \quad \therefore \ \, t_1 \ \, = \, \sqrt{\frac{2s}{a_1}} \, = \sqrt{\frac{4s}{(\sqrt{3}\mu + 1)\,g}}$$

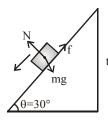


length of plane = stime of ascent = t_1

For downward motion: downward acceleration $a_2 = \frac{mg \sin \theta - \mu N}{m}$

$$a_2 = g \sin 30^\circ - g \cos 30^\circ = (1 - \sqrt{3} \mu) \frac{g}{2}$$





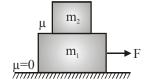
time of descent = t_2

Now according to question $2t_1 = t_2$

$$\Rightarrow 2\sqrt{\frac{4s}{(\sqrt{3}\mu+1)g}} = \sqrt{\frac{4s}{(1-\sqrt{3}\mu)g}}$$

$$\Rightarrow \frac{1 - \sqrt{3} \,\mu}{1 + \sqrt{3} \mu} = \frac{1}{4} \Rightarrow \mu = \frac{\sqrt{3}}{5}$$

Ex. When force F applied on m_1 and there is no friction between m_1 and surface and the coefficient of friction between m_1 and m_2 is μ . What should be the maximum value of F so that there is no relative motion between m_1 and m_2

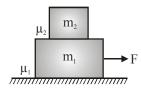


Sol. For m_1 $f \leftarrow N_1 \downarrow N_2$ $\downarrow N_2$ $\downarrow N_2$ $\downarrow N_2$ $\downarrow N_2$ $\downarrow N_2$ $\downarrow N_2$

For system acceleration $a = \frac{F}{m_1 + m_2}$

For
$$m_2$$
 $f = m_2 a \Rightarrow \mu m_2 g = m_2 \left(\frac{F}{m_1 + m_2}\right) \Rightarrow F_{max} = \mu (m_1 + m_2) g$

Ex. When force F applied on m_1 and the coefficient of friction between m_1 and surface. is μ_1 and the coefficient of friction between m_1 and m_2 is μ_2 . What should be the maximum value of F so that there is no relative motion between m_1 and m_2 .

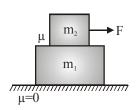


Sol. For m_1 $\mu_2 N_2$ $\mu_2 N_2$ $\mu_1 N_1$ $\mu_2 N_2$ $\mu_2 N_2$ $\mu_2 N_3$

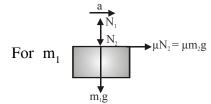
For system
$$a = \frac{F - \mu_1(m_1 + m_2) g}{m_1 + m_2}$$

For
$$m_2$$
, $\mu_2 (m_2 g) = m_2 a = m_2 \left(\frac{F - \mu_1 (m_1 + m_2) g}{m_1 + m_2} \right)$
 $\Rightarrow F_{max} = (m_1 + m_2) (\mu_1 + \mu_2) g$

Ex. When force F applied on m_2 and there is no friction between m_1 and surface and the coefficient of friction between m_1 and m_2 is μ . What should be the maximum value of F so that there is no relative motion between m_1 and m_2



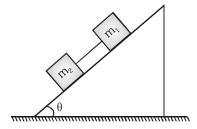
Sol. For $m_2 \mu N_2$ $m_2 g$



for system: acceleration = $\frac{F}{m_1 + m_2}$

for
$$m_1$$
: $\mu m_2 g = m_1 a = m_1 \left(\frac{F}{m_1 m_2}\right)$, $F_{max} = (m_1 + m_2) \left(\frac{\mu m_2 g}{m_1}\right)$

Ex. Two blocks with masses $m_1 = 1$ kg and $m_2 = 2$ kg are connected by a string and slide down a plane inclined at an angle $\theta = 45^0$ with the horizontal. The coefficient of sliding friction between m_1 and plane is $\mu_1 = 0.4$, and that between m_2 and plane is $\mu_2 = 0.2$. Calculate the common acceleration of the two blocks and the tension in the string.



Sol. As $\mu_2 < \mu_1$, block m_2 has greater acceleration than m_1 if we separately consider the motion of blocks. But they are connected so they move together as a system with common acceleration. So acceleration of the blocks:

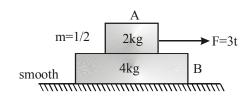
$$a = \frac{\left(m_1 + m_2\right)g\sin\theta - \mu_1m_1g\cos\theta - \mu_2m_2g\cos\theta}{m_1 + m_2}$$

$$= \frac{\left(1 + 2\right)\left(10\right)\left(\frac{1}{\sqrt{2}}\right) - 0.4 \times 1 \times 10 \times \frac{1}{\sqrt{2}} - 0.2 \times 2 \times 10 \times \frac{1}{\sqrt{2}}}{1 + 2} = \frac{22}{3\sqrt{2}} \text{ ms}^{-2}$$

For block $\mathbf{m_2}$: $m_2 g sin\theta - \mu_2 m_2 g cos\theta - T = m_2 a \Rightarrow T = m_2 g sin\theta - \mu_2 m_2 g cos\theta - m_2 a$

$$= 2 \times 10 \times \frac{1}{\sqrt{2}} - 0.2 \times 2 \times 10 \times \frac{1}{\sqrt{2}} - 2 \times \frac{22}{3\sqrt{2}} = \frac{2}{3\sqrt{2}}$$
N

Ex. For shown situation draw a graph showing accelerations of A and B on y-axis and time on x-axis. $(g=10ms^{-2})$



Sol. Limiting friction between A & B, $f_L = \mu m_A g = \left(\frac{1}{2}\right) (2) (10) = 10 \text{ N}$

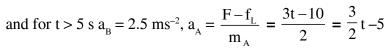
Block B moves due to friction only. So maximum acceleration of B,

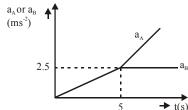
$$a_{max} = \frac{f_L}{m_B} = \frac{10}{4} = 2.5 \text{ ms}^{-2}.$$

So both the blocks move together till the common acceleration becomes 2.5 ms⁻², after that acceleations of B will become constant while that of A will go on increasing. Slipping will starts between A & B at 2.5 ms⁻²

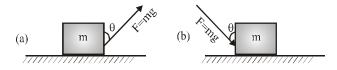
$$\Rightarrow 2.5 = \frac{F}{m_A + m_B} = \frac{3t}{6} \Rightarrow t = 5 \text{ s}$$

Hence for $t \le 5s$, $a_A = a_B = \frac{F}{m_A + m_B} = \frac{3t}{6} = \frac{t}{2}$





Ex. A block of mass m rests on a rough horizontal surface as shown in figure (a) and (b). Coefficient of friction between block and surface is μ . A force F = mg acting at an angle θ with the vertical side of the block. Find the condition for which block will move along the surface.



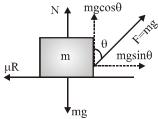
Sol. For (a): normal reaction $N = mg - mg \cos \theta$, frictional force $= \mu N = \mu (mg - mg \cos \theta)$ Now block can be pulled when: Horizontal component of force \geq frictional force i.e. $mg \sin \theta \geq \mu (mg - mg \cos \theta)$

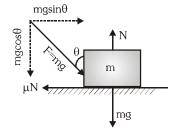
or
$$2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \ge \mu(1-\cos\theta)$$

or
$$2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \ge 2\mu\sin^2\frac{\theta}{2}$$
 or $\cot\frac{\theta}{2} \ge \mu$

For (b): Normal reaction $N = mg + mg \cos \theta = mg (1 + \cos \theta)$ Hence, block can be pushed along the horizontal surface when horizontal component of force \geq frictional force i.e. $mg \sin \theta \geq \mu \ mg(1 + \cos \theta)$

or
$$2\sin\frac{\theta}{2}\cos\frac{\theta}{2} \ge \mu \times 2\cos^2\frac{\theta}{2} \implies \tan\frac{\theta}{2} \ge \mu$$



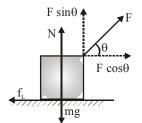


node06/2020-21(B0B0-BA) Vicota VEE(Advanced) Vivodule Coding (V-Tag) Neoder-Physics \05_AMedranics-1 \Eng\01_NUM & Friction \Theory & Ex

- 28
- Ex. A body of mass m rests on a horizontal floor with which it has a coefficient of static friction μ . It is desired to make the body move by applying the minimum possible force F. Find the magnitude of F and the direction in which it has to be applied.
- **Sol.** Let the force F be applied at an angle θ with the horizontal as shown in figure. For vertical equilibrium,

$$N + F \sin \theta = mg \implies N = mg - F \sin \theta$$

for horizontal motion $F \cos \theta \ge f_L \Rightarrow F \cos \theta \ge \mu N$ [as $f_L = \mu N$] substituting value of N from equation (i) in (ii),



(i)

(ii)

$$F\cos\theta \ge \mu(mg - F\sin\theta) \implies F \ge \frac{\mu mg}{(\cos\theta + \mu\sin\theta)}$$
 (iii)

For the force F to be minimum ($\cos \theta + \mu \sin \theta$) must be maximum,

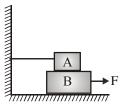
maximum value of
$$\cos\theta + \mu \sin\theta$$
 is $\sqrt{1 + \mu^2}$ so that $F_{min} = \frac{\mu \, mg}{\sqrt{1 + \mu^2}}$ with $\theta = \tan^{-1}(\mu)$

- Ex. A book of 1 kg is held against a wall by applying a perpendicular force F. If $\mu_S = 0.2$ then what is the minimum value of F?
- **Sol.** The situation is shown in fig. The forces acting on the book are-For book to be at rest it is essential that $Mg = f_s$

But
$$f_{s \text{ max}} = \mu_S N$$
 and $N = F$

$$\therefore Mg = \mu_S F \implies F = \frac{Mg}{\mu_S} = \frac{1 \times 9.8}{0.2} = 49 \text{ N}$$

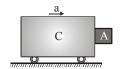
Ex. A is a 100 kg block and B is a 200 kg block. As shown in fig., the block A is attached to a string tied to a wall. The coefficient of friction between A and B is 0.2 and the coefficient of friction between B and floor is 0.3. Then calculate the minimum force required to move the block B. (take $g = 10 \text{ m/s}^2$).



Sol. When B is tied to move, by applying a force F, then the frictional forces acting on the block B are f_1 and f_2 with limiting values, $f_1 = (\mu_S)_A m_A g$ and $f_2 = (\mu_S)_B (m_A + m_B) g$ then minimum value of F should be (for just tending to move),

$$F = f_1 + f_2 = 0.2 \times 100 \text{ g} + 0.3 \times 300 \text{ g} = 110 \text{ g} = 1100 \text{ N}$$

Ex. Consider the figure shown here of a moving cart C. If the coefficient of friction between the block A and the cart is μ , then calculate the minimum acceleration a of the cart C so that the block A does not fall.

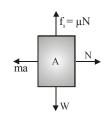




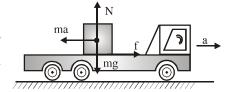
Sol. The forces acting on the block A (in block A's frame (i.e. non inertial frame) are :

For A to be at rest in block A's frame i.e. no fall,

we require
$$W = f_s$$
 $\Rightarrow mg = \mu(ma)$ Thus $a = \frac{g}{\mu}$



Ex. A block of mass 1kg lies on a horizontal surface in a truck, the coefficient of static friction between the block and the surface is 0.6, What is the force of friction on the block. If the acceleration of the truck is 5 m/s².



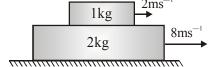
Sol. Fictitious force on the block $F = ma = 1 \times 5 = 5N$

While the limiting friction force

$$F = \mu_s N = \mu_s mg = 0.6 \times 1 \times 9.8 = 5.88 N$$

As required force F lesser than limiting friction force. The block will remain at rest in the truck and the force of friction will be equal to 5N and in the direction of acceleration of the truck.

- Ex. Coefficient of friction between two blocks shown in figure is $\mu = 0.4$ The blocks are given velocities of 2ms⁻¹ and 8 ms⁻¹ in the directions shown in figure. Find
 - (i) The time when relative motion between them will stop.
 - (ii) The common velocities of blocks upto that instant.
 - (iii) Displacement of blocks upto that instant ($g = 10 \text{ ms}^{-2}$)



Sol. (i) Frictional force between two blocks will oppose the relative motion. For 1 kg block friction support the motion & for 2 kg friction oppose the motion. Let common velocity be v then

for 1 kg
$$v = 2 + a_1 t$$
 where $a_1 = \frac{\mu(1g)}{1} = \frac{0.4 \times 10}{1} = 4 \text{ ms}^{-2}$

for 2 kg v = 8 -
$$a_2$$
 t where $a_2 = \frac{\mu(1g)}{2} = \frac{0.4 \times 10}{2} = 2 \text{ ms}^{-2} \Rightarrow 2 + 4t = 8 - 2 t \Rightarrow 6t = 6 \Rightarrow t = 1s$

- (ii) $v = 2 + 4 t = 2 + 4 \times 1 = 6 \text{ ms}^{-1}$
- (iii) Displacement of 1 kg block from rest

$$s = ut + \frac{1}{2}at^2 \implies s_1 = 2 \times 1 + \frac{1}{2} \times 4 \times 1^2 = 2 + 2 = 4m$$

Displacement of 2 kg block from rest

$$s = ut + \frac{1}{2}at^2 \implies s_2 = 8 \times 1 - \frac{1}{2} \times 2 \times 1^2 = 8 - 1 = 7 m$$

JEE-Physics ALLEN

Friction is a Necessary Evil:

Friction is a necessary evil. It means it has advantage as well as disadvantages. In other words, friction is not desirable but without friction, we cannot think of survival.

Disadvantages:

- (i) A significant amount of energy of a moving object is wasted in the form of heat energy to overcome the force of friction.
- (ii) The force of friction restricts the speed of moving vehicles like buses, trains, aeroplanes, rockets etc.
- (iii) The efficiency of machines decreases due to the presence of force of friction.
- (iv) The force of friction causes lot of wear and tear in the moving parts of a machine.
- (v) Sometimes, a machine gets burnt due to the friction force between different moving parts.

Advantages:

- (i) The force of friction helps us to move on the surface of earth. In the absence of friction, we cannot think of walking on the surface. That is why, we fall down while moving on a smooth surface.
- (ii) The force of friction between the tip of a pen and the surface of paper helps us to write on the paper. It is not possible to write on the glazed paper as there is not force of friction.
- (iii) The force of friction between the tyres of a vehicle and the road helps the vehicle to stop when brake is applied. In the absence of friction, the vehicle skid off the road when brake is applied.
- (iv) moving belts remain on the rim of a wheel because of friction.
- (v) The force of friction between a chalk and the black board helps us to write on the board. Thus, we observe that inspect of various disadvantages of the friction, it is very difficult to part with it. So, friction is a necessary evil.

METHODS OF REDUCING FRICTION

As friction causes the wastage of energy so it becomes necessary to reduce the friction. Friction can be reduced by the following methods.

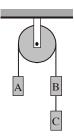
- (i) Polishing the surface. We know, friction between rough surface is much more than between the polished surfaces. So we polish the surface to reduce the friction. The irregularities on the surface are filled with polish and hence the friction decreases.
- (ii) Lubrication. To reduce friction, lubricants like oil or greese are used. When the oil or greese is put in between the two surfaces, the irregularities remain apart and do not interlock tightly. Thus, the surface can move over each other with less friction between them.
- (iii) By providing the streamlined shape. When a body (e.g. bus, train, aeroplane etc.) moves with high speed, air resistance (friction) opposes its motion. The effect of air resistance on the motion of the objects (stated above) is decreased by providing them a streamlined shape.

EXERCISE (S)

- 1. A force F applied to an object of mass m_1 produces an acceleration of 3.00 m/s². The same force applied to a second object of mass m_2 produces an acceleration of 1.00 m/s².
 - (i) What is the value of the ratio m_1/m_2 ?
 - (ii) If m_1 and m_2 are combined, find their acceleration under the action of the force F.

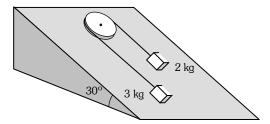
NL0001

2. In the system shown, the blocks A, B and C are of weight 4W, W and W respectively. The system set free. The tension in the string connecting the blocks B and C is



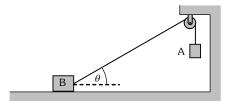
NL0002

3. Two blocks of masses 2.0 kg and 3.0 kg are connected by light inextensible string. The string passes over an ideal pulley pivoted to a fixed axel on a smooth incline plane as shown in the figure. When the blocks are released, find magnitude of their accelerations.

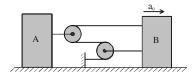


NL0003

4. The block A is moving downward with constant velocity v_0 . Find the velocity of the block B, when the string makes an angle θ with the horizontal

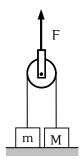


5. Calculate the relative acceleration of A w.r.t. B if B is moving with acceleration a_0 towards right.



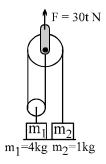
NL0015

6. In the system shown, pulley and strings are ideal. The vertically upward pull *F* is being increased gradually, find magnitude of *F* and acceleration of the 5 kg block at the moment the 10 kg block leaves the floor.



NL0004

7. Force F is applied on upper pulley. If F = 30t N where t is time in second. Find the time when m_1 loses contact with floor.

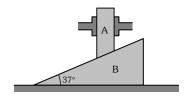


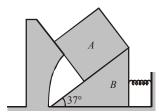
NL0005

8. A 40 kg boy climbs a rope that passes over an ideal pulley. The other end of the rope is attached to a 60 kg weight placed on the ground. What is the maximum upward acceleration the boy can have without lifting the weight? If he climbs the rope with upward acceleration 2 g, with what acceleration the weight will rise up?

NL0006

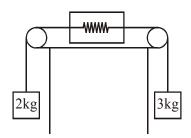
9. Rod A can slide in vertical direction pushing the triangular wedge B towards left. The wedge is moving toward right with uniform acceleration a_B. Find acceleration of the rod A.





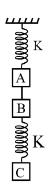
NL0008

11. Find the reading of the massless spring balance in the given condition



NL0009

12. The system shown adjacent is in equilibrium. Find the acceleration of the blocks *A*, *B* & *C* all of equal masses *m* at the instant when (Assume springs to be ideal)

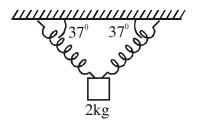


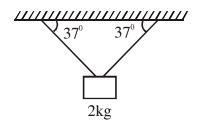
- (i) The spring between ceiling & A is cut.
- (ii) The string (inextensible) between A & B is cut.
- (iii) The spring between *B* & *C* is cut.

Also find the tension in the string when the system is at rest and in the above 3 cases.

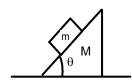
NL0010

13. The blocks are of mass 2 kg shown is in equilibrium. At t = 0 right spring in figure (i) and right string in figure (ii) breaks. Find the ratio of instantaneous acceleration of blocks?





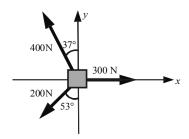
14. A block of mass m lies on wedge of mass M as shown in figure.



With what minimum acceleration must the wedge be moved towards right horizontally so that block m falls freely.

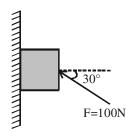
NL0011

15. A block is placed on a rough horizontal plane. Three horizontal forces are applied on the block as shown in the figure. If the block is in equilibrium, find the friction force acting on the block.



NL0016

16. A force of 100 N is applied on a block of mass 3 kg as shown in figure. The coefficient of friction between the wall and the surface of the block is 1/4. Calculate frictional force acting on the block.

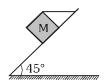


NL0017

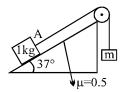
17. A thin rod of length 1 m is fixed in a vertical position inside a train, which is moving horizontally with constant acceleration 4 m/s². A bead can slide on the rod, and friction coefficient between them is 1/2. If the bead is released from rest at the top of the rod, find the time when it will reach at the bottom. $[g = 10 \text{ m/s}^2]$

NL0019

18. A block of mass 15 kg is resting on a rough inclined plane as shown in figure. The block is tied up by a horizontal string which has a tension of 50 N. Calculate the minimum coefficient of friction between the block and inclined plane.

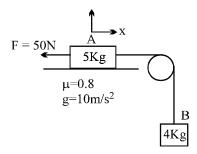


19. In the figure, what should be mass m so that block A slides up with a constant velocity?



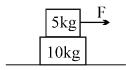
NL0023

20. Find the acceleration of the blocks and magnitude & direction of frictional force between block *A* and table, if block *A* is pulled towards left with a force of 50 N.



NL0024

21. Coefficient of friction between 5 kg and 10 kg block is 0.5. If friction between them is 20 N. What is the value of force being applied on 5 kg. The floor is frictionless.

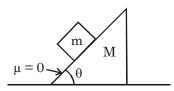


NL0026

22. A block of mass 1 kg is horizontally thrown with a velocity of 10 m/s on a stationary long plank of mass 2 kg whose surface has $\mu = 0.5$. Plank rests on frictionless surface. Find the time when block comes to rest w.r.t. plank.

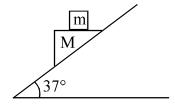
NL0020

23. A block of mass *m* lies on wedge of mass *M* as shown in figure. Find the minimum friction coefficient required between wedge *M* and ground so that it does not move while block *m* slips down on it.



NL0021

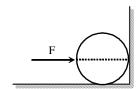
24. Block M slides down on frictionless incline as shown. Find the minimum friction coefficient so that m does not slide with respect to M.

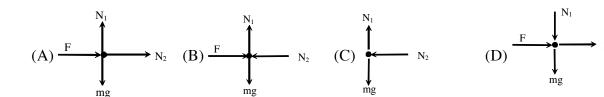


EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

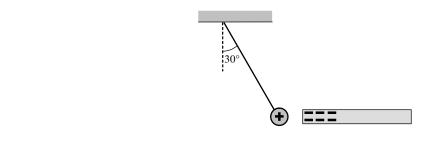
1. A ball of mass m kept at the corner as shown in the figure, is acted by a horizontal force F. The correct free body diagram of ball is

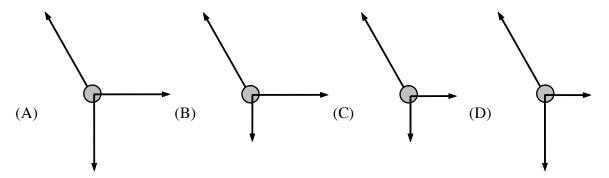




NL0036

2. A small electrically charged sphere is suspended vertically from a thread. An oppositely charged rod is brought close to the sphere such that the sphere is in equilibrium displaced from the vertical by an angle of 30°. Which one of the following best represents the free body diagram for the sphere?





NL0037



- **3.** Under what condition(s) will an object be in equilibrium?
 - (A) Only if it is at rest
 - (B) Only if it is moving with constant velocity
 - (C) Only if it is moving with constant acceleration
 - (D) If it is either at rest or moving with constant velocity

NL0038

4. Four blocks of same mass connected by cords are pulled by force F on a smooth horizontal surface, as in figure. The tension T_1 , T_2 and T_3 will be

(A)
$$T_1 = F/4$$
, $T_2 = 3F/2$, $T_3 = F/4$

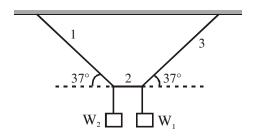
(B)
$$T_1 = F/4$$
, $T_2 = F/2$, $T_3 = F/2$

(C)
$$T_1 = 3F/4$$
, $T_2 = F/2$, $T_3 = F/4$

(D)
$$T_1 = 3F/4$$
, $T_2 = F/2$, $T_3 = F/2$

NL0039

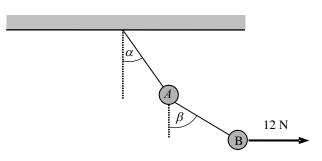
5. In a given figure system is in equilibrium. If $W_1 = 300 \text{ N}$. Then W_2 is approximately equal to



- (A) 500 N
- (B) 400 N
- (C) 670 N
- (D) 300 N

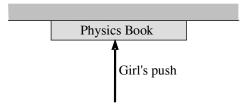
NL0040

6. Two balls A and B weighing 7 N and 9 N are connected by a light cord. The system is suspended from a fixed support by connecting the ball A with another light cord. The ball B is pulled aside by a horizontal force 12 N and equilibrium is established. Angles α and β respectively are



- (A) 30° and 60°
- (B) 60° and 30°
- (C) 37° and 53°
- (D) 53° and 37°

7. A girl pushes her physics book up against the horizontal ceiling of her room as shown in the figure. The book weighs 20 N and she pushes upwards with a force of 25 N. The choices below list the magnitudes of the contact force F_{CR} between the ceiling and the book, and F_{RH} between the book and her hand. Select the correct pair.



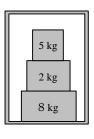
- (A) $F_{CB} = 20 \text{ N}$ and $F_{BH} = 25 \text{ N}$ (C) $F_{CB} = 5 \text{ N}$ and $F_{BH} = 25 \text{ N}$
- (B) $F_{CB} = 25 \text{ N}$ and $F_{BH} = 45 \text{ N}$ (D) $F_{CB} = 5 \text{ N}$ and $F_{BH} = 45 \text{ N}$

NL0042

- 8. Two astronauts A and B connected with a rope stay stationary in free space relative to their spaceship. Mass of A is more than that of B and the rope is straight. Astronaut A starts pulling the rope but astronaut B does not. If you were the third astronaut in the spaceship, what do you observe?
 - (A) Astronaut B accelerates towards A and A remains stationery.
 - (B) Both accelerate towards each other with equal accelerations of equal modulus.
 - (C) Both accelerate towards each other but acceleration of B is greater than that of A.
 - (D) Both accelerate towards each other but acceleration of B is smaller than that of A.

NL0043

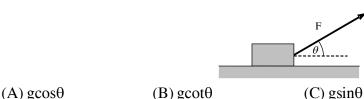
9. Three boxes are placed in a lift. When acceleration of the lift is 4 m/s², the net force on the 8 kg box is closest to



- (A) 80 N
- (B) 48 N
- (C) 40 N
- (D) 32 N

- A man is standing on a weighing machine with a block in his hand. The machine records w. When he **10.** takes the block upwards with some acceleration the machine records w₁. When he takes the block down with some acceleration, the machine records w₂. Then choose correct option
 - (A) $W_1 = W = W_2$
- (B) $w_1 < w < w_2$
- (C) $w_2 < w < w_1$
- (D) $w_2 = w_1 > w$

A block is being pulled by a force F on a long frictionless level floor. Magnitude of the force is 11. gradually increases from zero until the block lifts off the floor. Immediately before the block leaves the floor, its acceleration is

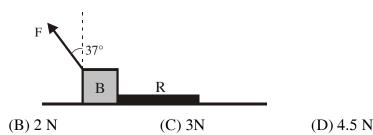


(D) g tan θ

(A) 1.5 N

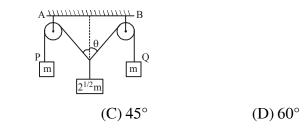
 $(A) 0^{\circ}$

12. A block B is tied to one end of a uniform rope R as shown. The mass of block is 2 kg and that of rope is 1 kg. A force F = 15 N is applied at angle 37° with vertical. The tension at the mid-point of rope is



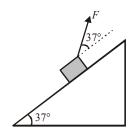
NL0047

13. The pulleys and strings shown in the figure are smooth and of negligible mass. For the system to remain in equilibrium, the angle θ should be [JEE (Scr) 2001]



NL0048

14. A block resting on a smooth inclined plane is acted upon by a force F as shown. If mass of block is 2 kg and F = 20 N and sin $37^{\circ} = 3/5$, the acceleration of block is



(A) 2 m/s^2

(B) 6 m/s^2

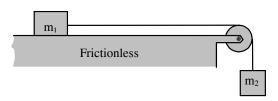
 $(B) 30^{\circ}$

(C) 8 m/s^2

(D) zero

NL0049

15. In the arrangement shown, the blocks of unequal masses are held at rest. When released, acceleration of the blocks is



(A) g/2.

(B) g.

(C) a value between zero and g.

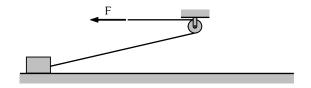
(D) a value that could be greater than g.

- 16. A monkey weighing 10 kg is climbing up a light rope and frictionless pulley attached to 15 kg mass at other end as in figure. In order to raise the 15 kg mass off the ground the monkey in ust climb-up
 - (A) with constant acceleration g/3.
 - (B) with an acceleration greater than g/2.
 - (C) with an acceleration greater than g/4.
 - (D) It is not possible because weight of monkey is lesser than the block.



NL0051

17. A heavy cart is pulled by a constant force F along a horizontal track with the help of a rope that passes over a fixed pulley, as shown in the figure. Assume the tension in the rope and the frictional forces on the cart remain constant and consider motion of the cart until it reaches vertically below the pulley.

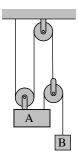


As the cart moves to the right, its acceleration

- (A) decreases.
- (B) increases.
- (C) remains constant.
- (D) is zero

NL0052

18. In arrangement shown the block A of mass 15 kg is supported in equilibrium by the block B. Mass of the block B is closest to



(A) 2 kg

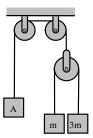
(B) 3 kg

(C) 4 kg

(D) 5 kg

NL0053

19. In the given figure, find mass of the block A, if it remains at rest, when the system is released from rest. Pulleys and strings are massless. $[g = 10 \text{ m/s}^2]$



(A) m

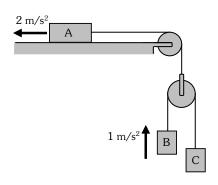
(B) 2m

(C) 2.5m

(D) 3m



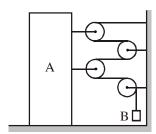
In the setup shown, find acceleration of the block C.



- (A) $3 \text{ m/s}^2 \uparrow$
- (B) $3 \text{ m/s}^2 \downarrow$
- (C) 5 m/s 2 \uparrow
- (D) 5 m/s² \downarrow

NL0063

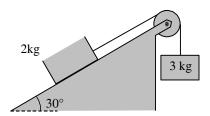
21. Block A is moving away from the wall at a speed v and acceleration a.



- (A) Velocity of B is v with resect to A.
- (B) Acceleration of B is a with respect to A.
- (C) Acceleration of B is 4a with respect to A. (D) Acceleration of B is $\sqrt{17}$ a with respect to A.

NL0062

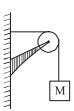
In the arrangement shown, the 2 kg block is held to keep the system at rest. The string and pulley are ideal. 22. When the 2 kg block is set free, by what amount the tension in the string changes? [$g = 10 \text{ m/s}^2$]



- (A) Increase of 12 N
- (B) Decrease of 12 N (C) Increase of 18 N
- (D) Decrease of 18 N

NL0055

A string of negligible mass going over a clamped pulley of mass m supports a block of mass M as 23. shown in the figure. The force on the pulley by the clamp is given [JEE (Scr) 2001]



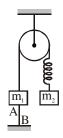
(A)
$$\sqrt{2} Mg$$

(B)
$$\sqrt{2} mg$$

(C)
$$\sqrt{(M+m)^2 + m^2}$$
 g (D) $\sqrt{(M+m)^2 + M^2}$ g

(D)
$$\sqrt{(M+m)^2 + M^2}$$
 g

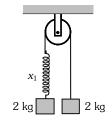
24. In a given figure two masses $m_1 \& m_2 (m_2 > m_1)$ are at rest in equilibrium position. Find the tension in string AB

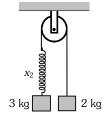


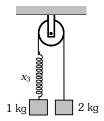
- $(A) m_1 g$
- $(B) m_{\gamma}g$
- $(C)(m_1+m_2)g$
- (D) $(m_2 m_1)g$

NL0057

25. Same spring is attached with 2 kg, 3 kg and 1 kg blocks in three different cases as shown. If x_1 , x_2 and x_3 be the extensions in the spring in these three cases, when acceleration of both the blocks have same magnitude, then



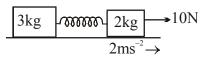




- (A) $x_2 > x_3 > x_1$
- (B) $x_2 > x_1 > x_3$
- (C) $x_3 > x_1 > x_2$
- (D) $x_1 > x_2 > x$

NL0058

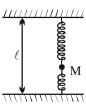
26. Find the acceleration of 3 kg mass when acceleration of 2 kg mass is 2 ms⁻² as shown in figure.



- (A) 3 ms^{-2}
- (B) 2 ms^{-2}
- (C) 0.5 ms⁻²
- (D) zero

NL0059

27. A small ball of mass M is held in equilibrium with two identical springs as shown in the figure. Force constant of each spring is k and relaxed length of each spring is $\ell/2$. What is distance between the ball and roof?



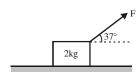
- (A) $\frac{\ell}{2} + \frac{Mg}{k}$
- (B) $\frac{\ell}{2} \frac{Mg}{k}$
- (C) $\frac{\ell}{2} + \frac{Mg}{2k}$
- (D) $\frac{\ell}{2} \frac{Mg}{2k}$

- An elastic spring of relaxed length ℓ_0 and force constant k is cut into two parts of lengths ℓ_1 and ℓ_2 . 28. The force constants of these parts are respectively

- (A) $\frac{k\ell_o}{\ell_o}$ and $\frac{k\ell_o}{\ell_o}$ (B) $\frac{k\ell_1}{\ell_o}$ and $\frac{k\ell_2}{\ell_o}$ (C) $\frac{k\ell_0}{\ell_2}$ and $\frac{k\ell_0}{\ell_1}$ (D) $\frac{k\ell_2}{\ell_0}$ and $\frac{k\ell_1}{\ell_0}$

NL0061

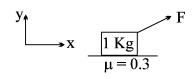
A block of mass 2 kg is kept on a rough horizontal floor and pulled with a force F. If the coefficient 29. of friction is 0.5. then the minimum force required to move the block is :-



- (A) 10 N
- (B) $\frac{100}{11}$ N
- (C) $\frac{100}{8}$ N
- (D) 20 N

NL0064

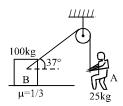
A force $\vec{F} = \hat{i} + 4\hat{j}$ acts on block shown. The force of friction acting on the block is : **30.**



- $(A) = \hat{i}$
- (C) $-2.4\,\hat{i}$
- (D) $-3\hat{i}$

NL0066

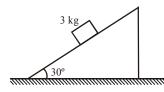
31. Block B of mass 100 kg rests on a rough surface of friction coefficient $\mu = 1/3$. A rope is tied to block B as shown in figure. The maximum acceleration with which boy A of 25 kg can climbs on rope without making block move is:



- (D) $\frac{3g}{4}$

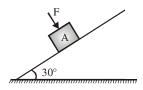
NL0067

32. A block of mass 3 kg is at rest on a rough inclined plane as shown in the figure. The magnitude of net force exerted by the surface on the block will be $(g = 10 \text{ m/s}^2)$



- (A) 26 N
- (B) 19.5 N
- (C) 10 N
- (D) 30 N

33. A block of mass m = 2 kg is resting on a rough inclined plane of inclination 30° as shown in figure. The coefficient of friction between the block and the plane is $\mu = 0.5$. What minimum force F should be applied perpendicular to the plane on the block, so that block does not slip on the plane (g=10m/s²)



(A) zero

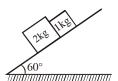
(B) 6.24 N

(C) 2.68 N

(D) 4.34 N

NL0069

- 34. In the figure shown if friction coefficient of block 1kg and 2kg with inclined plane is μ_1 =0.5 and μ_2 = 0.4 respectively, then
 - (A) both block will move together.
 - (B) both block will move separately.
 - (C) there is a non zero contact force between two blocks.
 - (D) none of these

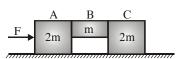


NL0070

- 35. A block is pushed with some velocity up a rough inclined plane. It stops after ascending few meters and then reverses its direction and returns back to point from where it started. If angle of inclination is 37° and the time to climb up is half of the time to return back then coefficient of friction is
 - (A) $\frac{9}{20}$
- (B) $\frac{7}{5}$
- (C) $\frac{7}{12}$
- (D) $\frac{5}{7}$

NL0071

36. The system is pushed by a force F as shown in figure. All surfaces are smooth except between B and C. Friction coefficient between B and C is μ . Minimum value of F to prevent block B from downward slipping is:-



(A) $\left(\frac{3}{2u}\right) mg$

(B) $\left(\frac{5}{2\mu}\right)mg$

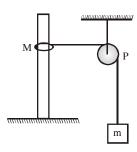
(C) $\left(\frac{5}{2}\right) \mu mg$

(D) $\left(\frac{3}{2}\right)\mu mg$

NL0072

- 37. The rear side of a truck is open and a box of mass 20 kg is placed on the truck 4 m away from the open end, $\mu = 0.15$ and g = 10 m/s². The truck starts from rest with an acceleration of 2m/s² on a straight road. The distance moved by the truck when box starts fall down is :-
 - (A) 4 m
- (B) 8 m
- (C) 16 m
- (D) $32 \, \text{m}$

38. In the figure shown a ring of mass M and a block of mass m are in equilibrium. The string is light and pulley P does not offer any friction and coefficient of friction between pole and M is μ . The frictional force offered by the pole on M is



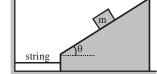
- (A) Mg directed up
- (C) (M-m)g directed down

- (B) µmg directed up
- (D) µmg directed down

EXERCISE (O-2)

MULTIPLE CORRECT TYPE QUESTIONS

- Refer the system shown in the figure. Block is sliding down the wedge. All surfaces are frictionless. Find correct statement(s)
 - (A) Acceleration of block is $g\sin\theta$
 - (B) Acceleration block is $g\cos\theta$
 - (C) Tension in the string is $mgcos^2\theta$
 - (D) Tension in the string is $mgsin\theta.cos\theta$



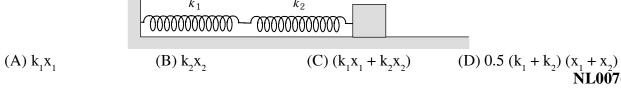
- 2. A carpenter of mass 50 kg is standing on a weighing machine placed in a lift of mass 20 kg. A light string is attached to the lift. The string passes over a smooth pulley and the other end is held by the carpenter as shown. When carpenter keeps the lift moving upward with constant velocity:- $(g = 10 \text{ m/s}^2)$
 - (A) the reading of weighing machine is 15 kg
 - (B) the man applies a force of 350 N on the string
 - (C) net force on the man is 150 N
 - (D) Net force on the weighing machine is 150 N



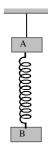
NL0073

NL0077

3. The mass in the figure can slide on a frictionless surface. When the mass is pulled out, spring 1 is stretched a distance x_1 and spring 2 is stretched a distance x_2 . The spring constants are k_1 and k_2 respectively. Magnitude of spring force pulling back on the mass is



4. In the given figure both the blocks have equal mass. When the thread is cut, which of the following statements give correct description immediately after the thread is cut?



- (A) Relative to the block A, acceleration of block B is 2g upwards.
- (B) Relative to the block B, acceleration of block A is 2g downwards.
- (C) Relative to the ground, accelerations of the blocks A and B are both g downwards.
- (D) Relative to the ground, accelerations of the blocks A and B are 2g downwards and zero respectively.



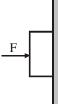
- A block is kept on a rough horizontal surface as shown. Its mass is 2 kg and coefficient of friction 5. between block and surface (μ) = 0.5. A horizontal force F is acting on the block. When
 - (A) F = 4 N, acceleration is zero.
 - (B) F = 4 N, friction is 10 N and acceleration is 3 m/s².



- (C) F = 14 N, acceleration is 2 m/s².
- (D) F = 14 N, friction is 14 N.

NL0075

- **6.** A block of mass 1 kg is held at rest against a rough vertical surface by pushing by a force F horizontally. The coefficient of friction is 0.5. When
 - (A) F = 40 N, friction on the block is 20 N.
 - (B) F = 30 N, friction on the block is 10 N.
 - (C) F = 20 N, friction on the block is 10 N.
 - (D) Minimum value of force F to keep block at rest is 20 N.



NL0074

COMPREHENSION TYPE QUESTIONS

Paragraph for Question No. 07 to 10

A uniform rope of mass (m) and length (L) placed on frictionless horizontal ground is being pulled by two forces F_A and F_B at its ends as shown in the figure. As a result, the rope accelerates toward the right.



- 7. Acceleration (A) of the rope is
 - (A) zero

- (B) $a = \frac{F_A + F_B}{m}$ (C) $a = \frac{F_A F_B}{m}$ (D) $a = \frac{F_B F_A}{m}$

NL0079

- 8. Tension (T) at the mid point of the rope is
- (A) $T = F_B F_A$ (B) $T = F_A + F_B$ (C) $T = \frac{1}{2} (F_B F_A)$ (D) $T = \frac{1}{2} (F_A + F_B)$

NL0079

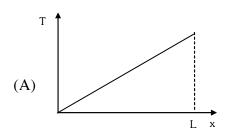
- 9. Expression (T_x) of tension at a point at distance x from the end A is
 - (A) $T_x = \left(\frac{F_B F_A}{I}\right) x + F_A$

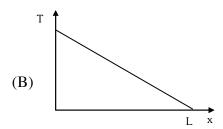
(B) $T_x = \left(\frac{F_B - F_A}{I}\right) x - F_A$

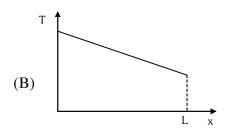
(C) $T_x = \left(\frac{F_B - F_A}{I}\right) x + F_B$

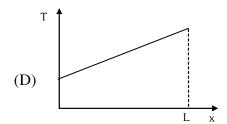
(D) $T_x = \left(\frac{F_B - F_A}{I}\right) x - F_B$

10. Which of the following graph best represents variation in tension at a point on the rope with distance *x* of the point from the end *A*?





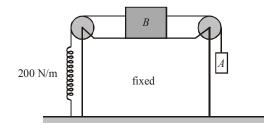




NL0079

Paragraph for Question No. 11 to 13

The figure shown blocks A and B are of mass 2 kg and 8 kg and they are connected through strings to a spring connected to ground. The blocks are in equilibrium. $(g = 10 \text{m/s}^2)$



- 11. The elongation of the spring is
 - (A) 1 cm
- (B) 10 cm
- (C) 0.1 cm
- (D) 1m

NL0080

- **12.** Now the block A is pulled downwards by a force gradually increasing to 20 N. The new elongation of spring is:-
 - (A) 2 cm
- (B) 4 cm
- (C) 20 cm
- (D) 40 cm

NL0080

- 13. Now the force on A is suddenly removed. The acceleration of block B becomes:
 - (A) 1.0 m/s
- (B) 2.0 m/s^2
- (C) 3.0 m/s^2
- (D) 4.0 m/s^2

Paragraph for Question No. 14 to 16

A block of mass m is placed on a smooth horizontal floor is attached to one end of spring. The other end of the spring is attached to fixed support. When spring is vertical it is relaxed. Now the block is pulled towards right by a force F, which is being increased gradually. When the spring makes angle 53° with the vertical, block leaves the floor.



- 14. When blocks leaves the table, the normal force on it from table is
 - (A) mg
- (B) zero
- (C) $\frac{4mg}{3}$
- (D) $\frac{3mg}{4}$

NL0106

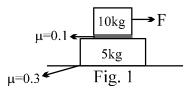
- **15.** Force constant of the spring is :-
 - (A) $\frac{5mg}{2\ell}$
- (B) $\frac{15mg}{8\ell}$
- (C) $\frac{5mg}{3\ell}$
- (D) $\frac{5mg}{4\ell}$

NL0106

- **16.** When the block leaves the table, the force F is :-
 - (A) $\frac{3mg}{4}$
- (B) $\frac{4mg}{3}$
- (C) $\frac{3mg}{5}$
- (D) $\frac{4mg}{5}$

NL0106

Paragraph for Question No. 17 to 21



- 17. When F = 2N, the frictional force between 5 kg block and ground is
 - (A) 2N
- (B)0

- (C) 8 N
- (D) 10 N

NL0107

- 18. When F = 2N, the frictional force between 10 kg block and 5 kg block is
 - (A) 2N
- (B) 15 N
- (C) 10 N
- (D) None

NL0107

- 19. The maximum F which will not cause motion of any of the blocks is
 - (A) 10 N
- (B) 15 N
- (C) data insufficient
- (D) None

NL0107

- **20.** The maximum acceleration of 5 kg block is :-
 - (A) 1 m/s^2
- (B) 3 m/s^2
- (C)0

(D) None

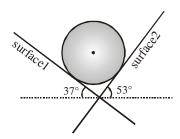
NL0107

- **21.** The acceleration of 10 kg block when F = 30 N is
 - (A) 2 m/s^2
- (B) 3 m/s^2
- (C) 1 m/s^2
- (D) None

NL0107

MATRIX MATCH TYPE QUESTION

22. A sphere of mass 10 kg is placed in equilibrium in a V shaped groove plane made of two smooth surfaces 1 and 2 as shown in figure. $(g = 10 \text{ ms}^{-2})$



Column I

Column II

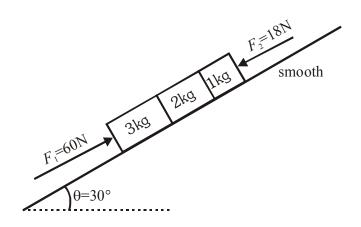
- (A) Normal reaction by Surface 1
- (P) Zero
- (B) Normal reaction by surface 2
- (Q) 60 N
- (C) Force on sphere by Earth
- (R) 80 N

(D) Net force on sphere

- (S) 100 N
- (T) 120 N

NL0084

23. In the diagram shown in figure $(g = 10 \text{ m/s}^2)$



Column I

(A) Acceleration of 2 kg block in m/s²

(B) Net force on 3 kg block in newton

(C) Normal reaction between 2 kg and 1 kg in newton

(D) Normal reaction between 3 kg and 2 kg in newton

Column II

(P) 8

(Q) 25

(R) 2

(S) 45

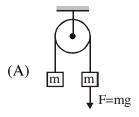
(T) None

NL0082

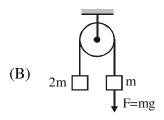
24. Match the situations in column I to the accelerations of blocks in the column II (acceleration due to gravity is g and F is an additional force applied to one of the blocks?

Column I

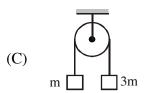
Column II



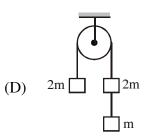
(P) $\frac{g}{5}$



 $(Q) = \frac{g}{2}$



(R) $\frac{g}{2}$



(S) $\frac{2g}{3}$

(T) zero

NL0083

Velocity of three particles A, B and C varies with time t as, $\vec{v}_A = (2t\hat{i} + 6\hat{j})$ m/s; $\vec{v}_B = (3\hat{i} + 4\hat{j})$ m/s and **25.**

 $\vec{v}_C = (6\hat{i} - 4t\hat{j})$ m/s. Regarding the pseudo force match the following table

Column I

- (A) On A as observed by B
- (B) On B as observed by C
- On A as observed by C (C)
- On C as observed by A (D)

Column II

- (P) Along positive x-direction
- (Q) Along negative x-direction
- Along positive y-direction (R)
- **(S)** Along negative y-direction
- (T) Zero

NL0109

CIRCULAR MOTION

KEY CONCEPT

Cross product (Vector Product) of two vectors:

Vector product of two vectors \vec{A} and \vec{B} , also called the cross product, is denoted by $\vec{A} \times \vec{B}$. As the name suggests, the vector product is itself a vector.

Vector product of two vectors \vec{A} and \vec{B} which are at an angle ϕ is defined as

$$\vec{C} = \vec{A} \times \vec{B},$$

then

 $C = AB \sin \phi \hat{n}$

where \hat{n} is an unit vector perpendicular to plane containing vector \vec{A} and \vec{B} . Direction of \hat{n} is given by right hand rule as described below.

Note: the vector product of two parallel or antiparallel vectors is always zero. In particular, the vector product of any vector with itself is zero.

 $\vec{A} \times \vec{B}$ is perpendicular to the plane containing the vectors \vec{A} and \vec{B} (direction of \hat{n})

Its direction is determined by the right-hand rule. \vec{B} .

Place the vectors tail to tail They define a plane.

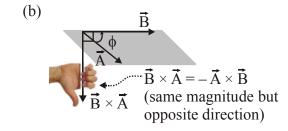
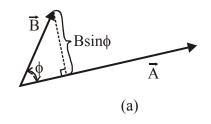


Figure : (a) The vector product $\vec{A} \times \vec{B}$. determined by the right-hand rule. (b) $\vec{B} \times \vec{A} = -\vec{A} \times \vec{B}$ Note: The vector product is not commutative! In fact, for any two vectors \vec{A} and \vec{B} ,

$$\vec{A} \times \vec{B} = -\vec{B} \times \vec{A}$$

(Magnitude of $A \times \vec{B}$) equals $A (B \sin \phi)$.

(Magnitude of \vec{A}) times (Component of \vec{B} perpendicular to \vec{A})



(Magnitude of $\vec{A} \times \vec{B}$) also equals $\vec{B}(A \sin \phi)$.

(Magnitude of \vec{B}) times (Component of \vec{A} perpendicular to \vec{B})

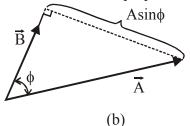


Figure: Calculating the magnitude AB $\sin \phi$ of the vector product of two vector, $\vec{A} \times \vec{B}$. Calculating the Vector Product Using Components

$$\hat{\mathbf{i}} \times \hat{\mathbf{i}} = \hat{\mathbf{j}} \times \hat{\mathbf{j}} = \hat{\mathbf{k}} \times \hat{\mathbf{k}} = 0$$

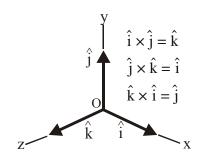
Using the right-hand rule, we find

$$\hat{i} \times \hat{j} = -\hat{j} \times \hat{i} = \hat{k}$$
;

$$\hat{\mathbf{k}} \times \hat{\mathbf{i}} = -\hat{\mathbf{i}} \times \hat{\mathbf{k}} = \hat{\mathbf{i}}$$

$$\hat{j} \times \hat{k} = -\hat{k} \times \hat{j} = \hat{i}$$
;

(a) A right-handed coordinate system



(b) A left-handed coordinate system; we will not use these

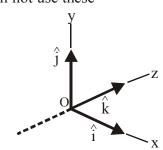


Fig.:

- (a) We will always use a right handed coordinate system, like this one.
- (b) We will never use a left handed coordinate system (in which $\hat{i} \times \hat{j} = -\hat{k}$, and so on.)

$$\begin{split} \vec{A} \times \vec{B} &= \left(A_x \hat{i} + A_y \hat{j} + A_z \hat{k} \right) \times \left(B_x \hat{i} + B_y \hat{j} + B_z \hat{k} \right) \\ &= \left(A_y B_z - A_z B_y \right) \hat{i} + \left(A_z B_x - A_x B_z \right) \hat{j} + \left(A_x B_y - A_y B_z \right) \hat{k} \end{split}$$

Thus the components of $\vec{C} = \vec{A} \times \vec{B}$ are given by

$$C_x = A_y B_z - A_z B_y$$

$$Cy = A_z B_x - A_x B_z$$
 $C_z = A_x B_y - A_y B_x$

$$C_z = A_x B_y - A_y B_x$$

 $C_x = A_y B_z - A_z B_y$ $Cy = A_z B_x - A_x B_z$ $C_z =$ Find the area of the triangle formed by the points having position vectors Ex.

$$\vec{A} = \hat{i} - \hat{j} - 3\hat{k}$$
, $\vec{B} = 4\hat{i} - 3\hat{j} + \hat{k}$ and $\vec{C} = 3\hat{i} - \hat{j} + 2\hat{k}$

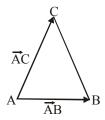
Sol. Let ABC is the triangle formed by the points A, B and C. Then

$$\overrightarrow{AB} = \overrightarrow{B} - \overrightarrow{A} = (4\hat{i} - 3\hat{j} + \hat{k}) - (\hat{i} - \hat{j} - 3\hat{k}) = 3\hat{i} - 2\hat{j} + 4\hat{k}$$

$$\overrightarrow{AC} = \overrightarrow{C} - \overrightarrow{A} = \left(3\hat{i} - \hat{j} + 2\hat{k}\right) - \left(\hat{i} - \hat{j} - 3\hat{k}\right) = 2\hat{i} + 5\hat{k}$$

Now,
$$\overrightarrow{AB} \times \overrightarrow{AC} = (3\hat{i} - 2\hat{j} + 4\hat{k}) \times (2\hat{i} + 5\hat{k})$$

$$= \hat{i}(-10-0) + \hat{j}(8-15) + \hat{k}(0+4) = -10\hat{i} - 7\hat{j} + 4\hat{k}$$



Ex. Find unit vector perpendicular to the plane determined by the points P(1, -1, 2), Q(2, 0, -1) and R(0, 2, 1).

Sol. $\overrightarrow{P}Q = (Position \ vector \ of \ Q) - (Position \ vector \ of \ P)$

$$= (2\hat{i} - \hat{k}) - (\hat{i} - \hat{j} + 2\hat{k}) = \hat{i} + \hat{j} - 3\hat{k}$$

Similarly,
$$\vec{P}R = (2\hat{j} + \hat{k}) - (\hat{i} - \hat{j} + 2\hat{k}) = -\hat{i} + 3\hat{j} - \hat{k}$$

$$|\vec{P}Q \times \vec{P}R| = \sqrt{8^2 + 4^2 + 4^2} = 4\sqrt{6}$$

:. Required unit vectors

$$= \pm \frac{1}{4\sqrt{6}} \left(8\hat{i} + 4\hat{j} + 4\hat{k} \right) = \pm \frac{1}{\sqrt{6}} \left(2\hat{i} + \hat{j} + 9\hat{k} \right)$$

Definition of Circular Motion

When a particle moves in a plane such that its distance from a fixed (or moving) point remains constant then its motion is called as circular motion with respect to that fixed point.

That fixed point is called centre and the distance is called radius of circular path.

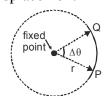
Radius Vector

The vector joining the centre of the circle and the center of the particle performing circular motion is called radius vector. It has constant magnitude and variable direction.

KINEMATICS OF CIRCULAR MOTION

Angular Displacement

• Angle traced by position vector of a particle moving w.r.t. some fixed point is called angular displacement.



 $\Delta\theta$ = angular displacement

Angle =
$$\frac{Arc}{Radius}$$
 or $\Delta\theta = \frac{Arc}{r}$

- Small Angular displacement $d\vec{\theta}$ is a vector quantity, but large angular displacement θ is scalar quantity. $d\vec{\theta}_1 + d\vec{\theta}_2 = d\vec{\theta}_2 + d\vec{\theta}_1 \text{ But } \vec{\theta}_1 + \vec{\theta}_2 \neq \vec{\theta}_2 + \vec{\theta}_1$
- Its direction is perpendicular to plane of rotation and given by right hand screw rule.
- It is dimensionless and has S.I. unit "Radian" while other units are degree or revolution.

$$2\pi$$
 radian = 360° =1 revolution

Frequency (n): Number of revolutions described by particle per second is its frequency. Its unit is revolutions per second (r.p.s.) or revolutions per minute (r.p.m.)

Time Period (T): It is time taken by particle to complete one revolution. $T = \frac{1}{n}$

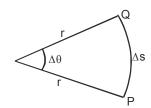
Angular Velocity (ω) : It is defined as the rate of change of angular displacement of a moving particle

$$\omega = \frac{\text{Angle traced}}{\text{Time taken}} = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$$

Relation between linear and Angular velocity

Angle =
$$\frac{Arc}{Radius}$$
 or $\frac{\Delta s}{r}$ $\Delta \theta = \frac{\Delta s}{r}$ or $\Delta s = r\Delta \theta$

$$\Delta\theta = \frac{\Delta s}{r}$$
 or $\Delta s = r\Delta\theta$



$$\therefore \frac{\Delta s}{\Delta t} = \frac{r\Delta \theta}{\Delta t} \text{ if } \Delta t \to 0 \text{ then } \frac{ds}{dt} = r\frac{d\theta}{dt}$$
 v = \omega r

$$\vec{v} = \vec{\omega} \times \vec{r}$$
 (direction of \vec{v} is according to right hand thumb rule)

Average Angular Velocity ($\omega_{\alpha \nu}$)

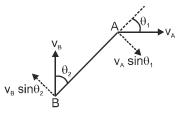
$$\omega_{av} = \frac{\text{total angle of rotation}}{\text{total time taken}} = \frac{\theta_2 - \theta_1}{t_2 - t_1} = \frac{\Delta \theta}{\Delta t} = \frac{2\pi}{T} = 2\pi n$$

where θ_1 and θ_2 are angular position of the particle at instant t_1 and t_2 . **Instantaneous Angular Velocity**

The angular velocity at some particular instant $\omega = \lim_{\Delta t \to 0} \frac{\Delta \theta}{\Delta t} = \frac{d\theta}{dt}$ or $\vec{\omega} = \frac{d\theta}{dt}$

Relative Angular Velocity

Relative angular velocity of a particle 'A' w.r.t. other moving particle B is the angular velocity of the position vector of A w.r.t. B. That means it is the rate at which position vector of 'A' w.r.t. B rotates at that instant



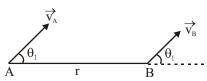
$$\omega_{AB} = \frac{\left(v_{AB}\right)_{\perp}}{r_{AB}} = \frac{\text{Relative velocity of A w.r.t. B perpendicular to line AB}}{\text{seperation between A and B}}$$

here
$$(v_{AB})_{\perp} = v_A \sin \theta_1 + v_B \sin \theta_2$$

$$\therefore \omega_{AB} = \frac{v_A \sin \theta_1 + v_B \sin \theta_2}{r}$$

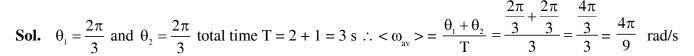
- It is an axial vector quantity.
- Its direction is same as that of angular displacement i.e. perpendicular to the plane of rotation and along the axis according to right hand screw rule.
- Its unit is radian/second.

If particles A and B are moving with a velocity \vec{v}_A and \vec{v}_B

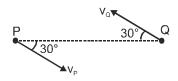


and separated by a distance r at a given instant then
(i)
$$\frac{dr}{dt} = v_B \cos\theta_2 - v_A \cos\theta_1$$
 (ii) $\frac{d\theta_{AB}}{dt} = \omega_{AB} = \frac{v_B \sin\theta_2 - v_A \sin\theta_1}{r}$

Ex. A particle revolving in a circular path completes first one third of circumference in 2 s, while next one third in 1 s. Calculate the average angular velocity of particle.

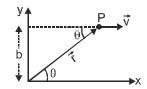


Ex. Two moving particles P and Q are 10 m apart at any instant. Velocity of P is 8 m/s at 30°, from line joining the P and Q and velocity of Q is 6m/s at 30°. Calculate the angular velocity of P w.r.t. Q

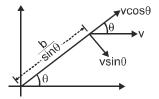


Sol.
$$\omega_{PQ} = \frac{8 \sin 30^{\circ} - (-6 \sin 30^{\circ})}{10} = 0.7 \text{ rad/s}.$$

Ex. A particle moving parallel to x-axis as shown in fig. such that at all instant the y-axis component of its position vector is constant and is equal to 'b'. Find the angular velocity of the particle about the origin when its radius vector makes angle θ from the x-axis.



Sol.
$$\therefore \omega_{PO} = \frac{v \sin \theta}{\frac{b}{\sin \theta}} = \frac{v}{b} \sin^2 \theta$$



Ex. The angular velocity of a particle is given by $\omega = 1.5 \text{ t} - 3t^2 + 2$, Find the time when its angular acceleration becomes zero.

Sol.
$$\alpha = \frac{d\omega}{dt} = 1.5 - 6 t = 0 \implies t = 0.25 \text{ s.}$$

Ex. A disc starts from rest and on the application of a torque, it gains an angular acceleration given by $\alpha = 3t - t^2$. Calculate the angular velocity after 2 s.

Sol.
$$\frac{d\omega}{dt} = 3t - t^2$$
 $\Rightarrow \int_0^{\omega} d\omega = \int_0^t (3t - t^2) dt \Rightarrow \omega = \frac{3t^2}{2} - \frac{t^3}{3} \Rightarrow \text{ at } t = 2 \text{ s}, \qquad \omega = \frac{10}{3} \text{ rad/s}$

Angular Acceleration (α)

- Rate of change of angular velocity is called angular acceleration. $\alpha = \lim_{\Delta t \to 0} \frac{\Delta \omega}{\Delta t} = \frac{d\omega}{dt}$ or $\vec{\alpha} = \frac{d\vec{\omega}}{dt}$
- Its an axial vector quantity. It direction is along the axis according to right hand screw rule.
- Unit \rightarrow rad/s²

58

Relation between Angular and Linear Acceleration

 $\vec{v} = \vec{\omega} \times \vec{r}$ (\vec{v} is a tangential vector, $\vec{\omega}$ is a axial vector and \vec{r} is a radial vector.)

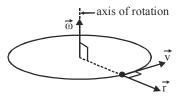
These three vectors are mutually perpendicular. but $\vec{a} = \frac{d\vec{v}}{dt} = \frac{d}{dt}(\vec{\omega} \times \vec{r}) = \frac{d\vec{\omega}}{dt} \times \vec{r} + \vec{\omega} \times \frac{d\vec{r}}{dt}$

or
$$\vec{a} = \vec{\alpha} \times \vec{r} + \vec{\omega} \times \vec{v}$$
 $(\frac{d\vec{\omega}}{dt} = \vec{\alpha} \text{ and } \frac{d\vec{r}}{dt} = \vec{v}) \Rightarrow \vec{a} = \vec{a}_T + \vec{a}_C$

 $(\vec{a}_T = \vec{\alpha} \times \vec{r} \text{ is tangential acceleration } \&$

 $\vec{a}_C = \vec{\omega} \times \vec{v}$ is centripetal acceleration)

 $\vec{a} = \vec{a}_T + \vec{a}_C$ (\vec{a}_T and \vec{a}_C are two component of net linear acceleration)



Tangential Acceleration

 $\vec{a}_T = \vec{\alpha} \times \vec{r}$, its direction is parallel to velocity. $\vec{v} = \vec{\omega} \times \vec{r}$ and $\vec{a}_T = \vec{\alpha} \times \vec{r}$ as $\vec{\omega}$ and $\vec{\alpha}$ both are parallel and along the axis so that \vec{v} and \vec{a}_T are also parallel and along the tangential direction. Magnitude of tangential acceleration in case of circular motion.

$$\mathbf{a}_{r} = \alpha \, r \sin 90^{\circ} = \alpha r \, (\vec{\alpha} \text{ is axial, } \vec{r} \text{ is radial so that } \vec{\alpha} \perp \vec{r})$$

As \vec{a}_T is along the direction of motion (in the direction of \vec{v}) so that \vec{a}_T is responsible for change in speed of the particle. Its magnitude is rate of change of speed of the particle. If particle is moving on a circular path with constant speed then tangential acceleration is zero.

Centripetal acceleration

$$\vec{a}_C = \vec{\omega} \times \vec{v} \Rightarrow \vec{a}_C = \vec{\omega} \times (\vec{\omega} \times \vec{r}) \ (\because \vec{v} = \vec{\omega} \times \vec{r})$$

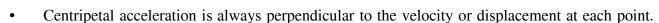
Let \vec{r} is in \hat{i} direction and $\vec{\omega}$ is in \hat{j} direction then direction

of
$$\vec{a}_C$$
 is along

$$\hat{j} \times (\hat{j} \times \hat{i}) = \hat{j} \times (-\hat{k}) = -\hat{i}$$

opposite direction of \vec{r} i.e., from P to O and it is

centripetal direction. Magnitude of centripetal acceleration, $a_C = \omega v = \frac{v^2}{r} = \omega^2 r$, $\vec{a}_C = \frac{v^2}{r}(-\hat{r})$



Net Linear Acceleration:

$$\vec{a} = \vec{a}_{\mathrm{T}} + \vec{a}_{\mathrm{C}}$$
 and $\vec{a}_{\mathrm{T}} \perp \vec{a}_{\mathrm{C}}$ so that $|\vec{a}| = \sqrt{a_{\mathrm{T}}^2 + a_{\mathrm{C}}^2}$

About uniform circular motion :-

- Position vector (\vec{r}) is always perpendicular to the velocity vector (\vec{v}) i.e. $\vec{r} \cdot \vec{v} = 0$
- velocity vector is always perpendicular to the acceleration. $\vec{v} \cdot \vec{a} = 0$
- $|\vec{v}| = \text{constant}$ so tangential acc. a = 0 \therefore f = 0
- Important difference between the projectile motion and uniform circular motion:
 In projectile motion, both the magnitude and the direction of acceleration (g) remain constant, while in uniform circular motion the magnitude remains constant but the direction continuously changes.

DYNAMICS OF CIRCULAR MOTION

If a particle moves with constant speed in a circle, motion is called uniform circular. In uniform circular motion a resultant non-zero force acts on the particle. The acceleration is due to the change in direction of the velocity vector. In uniform circular motion tangential acceleration (a_r)

is zero. The acceleration of the particle is towards the centre and its magnitude is $\frac{v^2}{r}$. Here, v is the speed of the particle and r the radius of the circle. The direction of the resultant force F is mv^2

therefore towards centre and its magnitude is $F = \frac{mv^2}{r} = mr\omega^2$ (as $v = r\omega$)

Here, $_{\omega}$ is the angular speed of the particle. This force F is called the centripetal force. Thus, a centripetal force of magnitude $\frac{mv^2}{r}$ is needed to keep the particle moving in a circle with constant speed. This force is provided by some external source such as friction, magnetic force, coulomb force, gravitation, tension, etc.

Circular Turning of Roads

When vehicles go through turnings, they travel along a nearly circular arc. There must be some force which will produce the required centripetal acceleration. If the vehicles travel in a horizontal circular path, this resultant force is also horizontal. The necessary centripetal force is being provided to the vehicles by following three ways:

By friction only.
By banking of roads only.
By friction and banking of roads both.

In real life the necessary centripetal force is provided by friction and banking of roads both.

• By Friction only

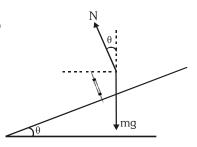
Suppose a car of mass m is moving at a speed v in a horizontal circular arc of radius r. In this case, the necessary centripetal force to the car will be provided by force of friction f acting towards centre.

Thus,
$$f = \frac{mv^2}{r}$$
 $\therefore f_{max} = \mu N = \mu mg$

Therefore, for a safe turn without sliding $\frac{mv^2}{r} \le f_{max} \Rightarrow \frac{mv^2}{r} \le \mu mg \Rightarrow v \le \sqrt{\mu rg}$

By Banking of Roads only

Friction is not always reliable at circular turns if high speeds and sharp turns are involved. To avoid dependence on friction, the roads are banked at the turn so that the outer part of the road is some what lifted compared to the inner part.

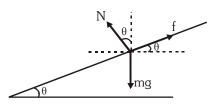


60

$$N \sin \theta = \frac{mv^2}{r}$$
 and $N \cos \theta = mg \implies \tan \theta = \frac{v^2}{rg} \implies v = \sqrt{rg \tan \theta}$

Friction and Banking of Road both

If a vehicle is moving on a circular road which is rough and banked also, then three forces may act on the vehicle, of these the first force, i.e., weight (mg) is fixed both in magnitude and direction. The direction of second force, i.e., normal reaction N is also fixed



(perpendicular to road) while the direction of the third force, i.e., friction f can be either inwards or outwards while its magnitude can be varied upto a maximum limit ($f_{max} = \mu N$). So, the magnitude of normal reaction N and direction plus magnitude of friction f are so adjusted that the resultant

of the three forces mentioned above is $\frac{mv^2}{r}$ towards the centre.

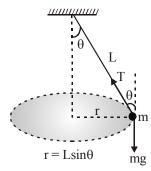
Conical Pendulum

If a small particle of mass m tied to a string is whirled in a horizontal circle, as shown in figure. The arrangement is called the 'conical pendulum'. In case of conical pendulum the vertical component of tension balances the weight while its horizontal component provides the necessary centripetal force. Thus,,

$$T \sin \theta = \frac{mv^2}{r}$$
 and $T \cos \theta = mg \implies v = \sqrt{rg \tan \theta}$

$$\therefore \text{ Angular speed } \omega = \frac{v}{r} = \sqrt{\frac{g \tan \theta}{r}}$$

So, the time period of pendulum is
$$T = \frac{2\pi}{\omega} = 2\pi \sqrt{\frac{r}{g \tan \theta}} = 2\pi \sqrt{\frac{L \cos \theta}{g}}$$

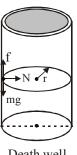


'Death Well' or Rotor

In case of 'death well' a person drives a motorcycle on a vertical surface of a large wooden well while in case of a rotor at a certain angular speed of rotor a person hangs resting against the wall without any support from the bottom. In death well walls are at rest and person revolves while in case of rotor person is at rest and the walls rotates.

In both cases friction balances the weight of person while reaction provides the centripetal force for circular motion,

i.e.,
$$f = mg$$
 and $N = \frac{mv^2}{r} = mr\omega^2$



Death well



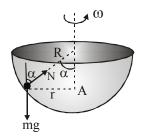
- Ex. Find the maximum speed at which a car can turn round a curve of 30 m radius on a level road if the coefficient of friction between the tyres and the road is 0.4 [acceleration due to gravity = 10 m/s²]
- Sol. Here centripetal force is provided by friction so

$$\frac{mv^2}{r} \le \mu mg \implies v_{max} = \sqrt{\mu rg} = \sqrt{120} \approx 11 \text{ ms}^{-1}$$

- Ex. For traffic moving at 60 km/hr, if the radius of the curve is 0.1 km, what is the correct angle of banking of the road ? $(g = 10 \text{ m/s}^2)$
- **Sol.** In case of banking $\tan \theta = \frac{v^2}{rg}$ Here $v = 60 \text{ km/hr} = 60 \times \frac{5}{18} \text{ ms}^{-1} = \frac{50}{3} \text{ ms}^{-1} \text{ r} = 0.1 \text{ km} = 100 \text{ m}$

So
$$\tan \theta = \frac{50/3 \times 50/3}{100 \times 10} = \frac{5}{18} \Rightarrow \theta = \tan^{-1} \left(\frac{5}{18}\right)$$

Ex. A hemispherical bowl of radius R is rotating about its axis of symmetry which is kept vertical. A small ball kept in the bowl rotates with the bowl without slipping on its surface. If the surface of the bowl is smooth and the angle made by the radius through the ball with the vertical is α . Find the angular speed at which the bowl is rotating.

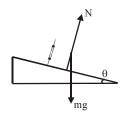


Sol. Ncos α =mg and Nsin α = mr ω ² but r =R sin α

$$\Rightarrow$$
 Nsin α = mRsin $\alpha\omega^2 \Rightarrow$ N=mR ω^2

$$\Rightarrow (mR\omega^2)\cos\alpha = mg \Rightarrow \omega = \sqrt{\frac{g}{R\cos\alpha}}$$

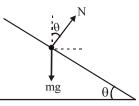
Ex. A car is moving along a banked road laid out as a circle of radius r. (a) What should be the banking angle $_{\theta}$ so that the car travelling at speed v needs no frictional force from the tyres to negotiate the turn? (b) The coefficients of friction between tyres and road are $\mu_s = 0.9$ and $\mu_k = 0.8$. At what maximum speed can a car enter the curve without sliding towards the top edge of the banked turn?



Sol. (a)
$$N \sin \theta = \frac{mv^2}{r}$$
 and $N \cos \theta = mg \implies \tan \theta = \frac{v^2}{rg}$

Note: In above case friction does not play any role in negotiating the turn.

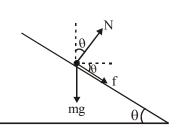
(b) If the driver moves faster than the speed mentioned above, a friction



force must act parallel to the road, inward towards centre of the turn.

$$\Rightarrow$$
 fcosθ + Nsinθ = $\frac{mv^2}{r}$ and Ncosθ = mg + fsinθ

For maximum speed $f = \mu N$



$$\Rightarrow N(\mu\cos\theta + \sin\theta) = \frac{mv^2}{r} \text{ and } N(\cos\theta - \mu\sin\theta) = mg$$

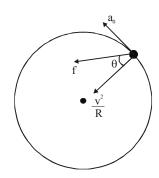
$$\Rightarrow \frac{v^2}{rg} = \frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta} \Rightarrow v = \sqrt{\left(\frac{\sin\theta + \mu\cos\theta}{\cos\theta - \mu\sin\theta}\right)rg}$$

- Ex. A car starts from rest with a consant tangential acceleration a_0 in a circular path of radius r. At time t, the car skids, find the value of coefficient of friction.
- **Sol.** The tangential and centripetal acceleration is provided only by the frictional force.

Thus,
$$f \sin\theta = ma_0$$
, $f \cos\theta = \frac{mv^2}{r} = \frac{m(a_0t)^2}{r}$

$$\Rightarrow f = m \ \sqrt{a_0^2 + \frac{\left(a_0 t\right)^4}{r^2}} = m a_0 \ \sqrt{1 + \frac{a_0^2 t^4}{r^2}} \ = f_{max}$$

$$\mu mg = ma_0 \sqrt{1 + \frac{a_0^2 t^4}{r^2}} \ \Rightarrow \ \mu = \frac{a_0}{g} \sqrt{1 + \frac{a_0^2 t^4}{r^2}}$$



Centrifugal force:-

Centrifugal force is a pseudo force which an observer needs to consider while making observations in a rotating frame. This force is non physical and arise from kinematics and not due to physical interactions. Centrifugal force is directed away from axis of rotation of rotating frame and its value is $m\omega^2 r$, where ω is angular speed of rotating frame where observer has kept himself fixed and r is distance of object of mass m from axis of rotation.

WORK, POWER & ENERGY

KEY CONCEPT

WORK

- Whenever a force acting on a body, displaces it in its direction, work is said to be done by the force.
- Work done by a force is equal to scalar product of force applied and displacement of the point of application, $W = \vec{F} \cdot \vec{d}$
- Work is a scalar quantity.

Work done by a constant force:

If the direction and magnitude of a force applied on a body is constant, the force is said to be constant. Work done by a constant force, $W = Force \times component$ of displacement along force

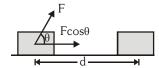
= displacement × component of force along displacement.

The work done will, be $W = (F \cos \theta) d$

$$= F (d \cos \theta)$$

In vector form,

$$W = \vec{F} \cdot \vec{d}$$



Note: The force of gravity is the example of constant force, hence work done by it is the example of work done by a constant force.

Work done by a variable force

If the force applying on a body is changing its direction or magnitude or both, the force is said to be variable. Suppose a variable force causes displacement in a body from position P_1 to position P_2 . To calculate the work done by the force the path from P_1 to P_2 can be divided into infinitesimal element, each element is so small that during displacement of body through it, the force is supposed to be constant. If $d\vec{r}$ be small displacement of point of application and \vec{F} be the force applied on the body, the work done by force is $dW = \vec{F} \cdot d\vec{r}$

The total work done in displacing body from P_1 to P_2 is given by $\int dW = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r} \implies W = \int_{P_1}^{P_2} \vec{F} \cdot d\vec{r}$

If \vec{r}_1 and \vec{r}_2 be the position vectors of the points P_1 and P_2 respectively, the total work done $W = \int_{r_1}^{r_2} \vec{F} . d\vec{r}$

Note: When we consider a block attached to a spring, the force on the block is k times the elongation of the spring, where k is spring constant. As the elongation changes with the motion of the block, therefore the force is variable. This is an example of work done by variable force.

Here:
$$W_s = \int_{x_i}^{x_f} -k \ x \ dx = \frac{1}{2} k \left(x_i^2 - x_f^2\right)$$

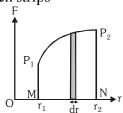
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Calculation of work done from force displacement graph:

Suppose a body, whose initial position is r_1 , is acted upon by a variable force \vec{F} and consequently the body acquires its final position r_2 . From position r to r + dr or for small displacement dr, the work done will be $\vec{F} \cdot d\vec{r}$ whose value will be the area of the shaded strip of width dr. The work done on the body in displacing it from position r₁ to r₂ will be equal to the sum of areas of all such strips

Thus, total work done,
$$W = \sum_{r_1}^{r_2} dW = \sum_{r_1}^{r_2} F.dr = Area of P_1 P_2 NM$$

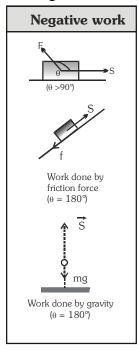
The area of the graph between curve and displacement axis is equal to the work done.

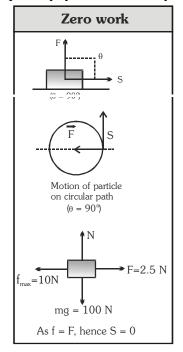


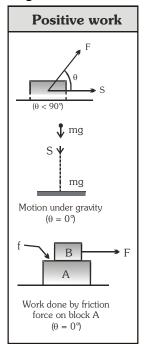
Note: To calculate the work done by graphical method, for the sake of simplicity, here we have assumed the direction of force and displacement as same, but if they are not in same direction, the graph must be plotted between F $\cos \theta$ and r.

Nature of work done

Although work done is a scalar quantity, yet its value may be positive, negative or even zero







UNITS:

SI Unit: joule (J).

One joule of work is said to be done when a force of one newton displaces a ioule body by one meter in the direction of force.

1 joule = 1 newton \times 1 meter = 1 kgm²s⁻²

One erg of work is said to be done when a force of one dyne displaces a body by erg one cm. in the direction of force.

Other Units: (a) 1 joule = 10^7 erg

(b) 1 erg =
$$10^{-7}$$
 joule

(c) 1 eV =
$$1.6 \times 10^{-19}$$
 joule
(e) 1 MeV = 1.6×10^{-13} J

(d) 1 joule =
$$6.25 \times 10^{18} \text{ eV}$$

(e) 1 MeV =
$$1.6 \times 10^{-13}$$
 .

(f)
$$1 \text{ J} = 6.25 \times 10^{12} \text{ MeV}$$

(g) 1 kilo watt hour (kWh) =
$$3.6 \times 10^6$$
 joule

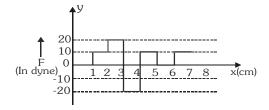
DIMENSIONS:

 $[Work] = [Force] [Displacement] = [MLT^{-2}][L] = [ML^2T^{-2}]$

Ex. A position dependent force $F = 7 - 2x + 3x^2$ acts on a small body of mass 2kg and displaces it from x = 0 to x = 5 m. Calculate the work done in joule.

Sol. W =
$$\int_{x_1}^{x_2} F dx = \int_{0}^{5} (7 - 2x + 3x^2) dx = \left[7x - \frac{2x^2}{2} + \frac{3x^3}{3} \right]_{0}^{5} = 135J$$

Ex. For the force displacement diagram shown in adjoining diagram. Calculate the work done by the force in displacing the body from x = 1 cm to x = 5 cm.



- **Sol.** Work = Area under the curve and displacement axis = 10 + 20 20 + 10 = 20 erg
- **Ex.** A uniform chain of mass M and length L is lying on a frictionless table in such a way that its 1/3 part is hanging vertically down. Calculate the work done in pulling the chain up the table.
- **Sol.** If length x of the chain is pulled up on the table, then the length of hanging part of the chain would be

$$\left(\frac{L}{3} - x\right)$$
 and its weight would be $\frac{M}{L} \left(\frac{L}{3} - x\right) g$

If it is pulled up further by a distance dx, the work done in pulling up.

$$\Rightarrow dW = \frac{M}{L} \left(\frac{L}{3} - x \right) g dx \implies W = \int_{0}^{L/3} \frac{M}{L} \left(\frac{L}{3} - x \right) g dx = \frac{MgL}{18}$$

- Ex. Calculate work done to move a body of mass 10 kg along a smooth inclined plane ($\theta = 30^{\circ}$) with constant velocity through a distance of 10 m.
- **Sol.** Here the motion is not accelerated, the resultant force parallel to the plane must be zero. So $F Mg \sin 30^{\circ}=0 \Rightarrow F = Mg \sin 30^{\circ} \& d=10m$

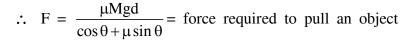
W = Fdcos
$$\theta$$
 = (Mgsin30°)d cos0° = 10 × 10 × $\frac{1}{2}$ × 10 × 1 = 500 J

- Ex. Calculate work done in pulling an object with a constant force F as shown in figure. (given that the ground is rough with coefficient of friction μ)
- **Sol.** From the figure $F \sin \theta + N = Mg$

∴ N=Mg–F
$$\sin\theta$$

$$F \cos\theta = f = \mu N = \mu [Mg - F \sin\theta]$$

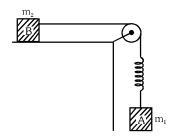
$$F (cosθ + μsinθ) = μMg$$



Work done in pulling an object
$$W = Fd = \frac{\mu Mgd}{\cos\theta + \mu \sin\theta}$$



- 66
- **Ex.** If the two blocks moves with a constant uniform speed then find coefficient of friction between the surface of the block B and the table. The spring is massless and the pulley is frictionless.

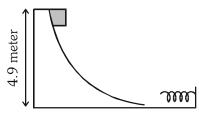


Sol. For block $B : T = f = \mu m_2 g$ and For block $A : T = m_1 g$

By solving above equations $\mu = \frac{m_1}{m_2}$

Ex. Figure shows a smooth curved track terminating in a smooth horizontal part, A spring of force constant 400 N/m is attached at one end to a wedge fixed rigidly with a horizontal part.

A 40 g mass is released from rest at a height of 4.9 m on the curved track. Find the maximum compression of the spring.



Sol. From the law of conservation of mechanical energy \Rightarrow mgh = $\frac{1}{2}$ kx²

$$\Rightarrow \quad x = \sqrt{\frac{2mgh}{k}} = \sqrt{\frac{2 \times (0.04) \times 9.8 \times 4.9}{400}} = 9.8 \text{ cm}.$$

ENERGY

- The energy of a body is defined as the capacity of doing work.
- There are various form of energy
 - (i) mechanical energy
- (ii) chemical energy
- (iii) electrical energy (iv) sound energy

- (v) light energy etc
- (vi) magnetic energy (vii) nuclear energy
- Energy of system always remain constant it can neither be created nor it can be destroyed however it may be converted from one form to another

Examples

Electrical energy Mechanical energy Mechanical energy Electrical energy Light energy Photocell Electrical energy Electrical energy Heat energy Heater Electrical energy Sound energy Nuclear energy Nuclear Reactor Electrical energy Cell Chemical energy Electrical energy Electrical energy Chemical energy

- Energy is a scalar quantity
- Unit : Its unit is same as that of work or torque. In MKS : joule, watt second ; In CGS : erg **Note** : $1 \text{ eV} = 1.6 \times 10^{-19} \text{ joule}$; $1 \text{ kWh} = 3.6 \times 10^{6} \text{ joule}$; $10^{7} \text{ erg} = 1 \text{ joule}$.
- Dimension [M¹L²T⁻²]
- According to Einstein's mass energy equivalence principle mass and energy are inter convertible i.e. they can be changed into each other. Energy equivalent of mass m is, $E = mc^2$ where, m: mass of the particle, c: velocity of light, E: equivalent energy corresponding to mass m.
- In mechanics we are concerned with mechanical energy only which is of two type :
 - (i) kinetic energy (ii) potential energy

Kinetic energy

- The energy possessed by a body by virtue of its motion is called kinetic energy.
- If a body of mass m is moving with velocity v, its kinetic energy $KE = 1/2 \text{ mv}^2$.
- Kinetic energy is always positive.
- If linear momentum of body is p, the kinetic energy for translatory motion is $KE = \frac{p^2}{2m} = \frac{1}{2}mv^2$.
- The kinetic energy of a moving body is measured by the amount of work which has been done in bringing the body from the rest position to its present moving position.
- The kinetic energy of a moving body is measured by the amount of work which the body can do against the external forces before it comes to rest.
- **Ex.** In a ballistics demonstration, a police officer fires a bullet of mass 50.0 g with speed 200 ms⁻¹ on soft plywood of thickness 2.00 cm. The bullet emerges with only 10% of its initial kinetic energy. What is the emergent speed of the bullet?

Sol. Initial kinetic energy,
$$K_i = \frac{1}{2} \times \frac{50}{1000} \times 200 \times 200 J = 1000 J$$

Final kinetic energy,
$$K_f = \frac{10}{100} \times 1000 J = 100 J$$

If v_f is emergent speed of the bullet, then $\frac{1}{2} \times \frac{50}{1000} \times v_f^2 = 100$

$$\Rightarrow$$
 $v_f^2 = 4000 \Rightarrow v_f = 63.2 \text{ ms}^{-1}.$

Note that the speed is reduced by approximately 68% and not 90%.

Work Energy Theorem

Work done by all the forces (conservative or non conservative, external or internal) acting on a particle or an object is equal to the change in it's kinetic energy. So work done by all the forces = change in kinetic energy

$$W = \Delta KE = \frac{1}{2} m v_f^2 - \frac{1}{2} m v_i^2$$

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Ex. A particle of mass m moves with velocity $v = a\sqrt{x}$ where a is a constant. Find the total work done by all the forces during a displacement from x = 0 to x = d.

Sol. Work done by all forces = W =
$$\Delta KE = \frac{1}{2} mv_2^2 - \frac{1}{2} mv_1^2$$

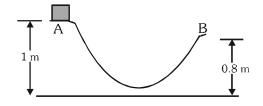
Here
$$v_1 = a\sqrt{0} = 0$$
, $v_2 = a\sqrt{d}$, So $W = \frac{1}{2} ma^2 d - 0 = \frac{1}{2} ma^2 d$

Ex. The displacement x of a body of mass 1 kg on horizontal smooth surface as a function of time t is given by $x = \frac{t^3}{3}$. Find the work done by the external agent for the first one second.

Sol.
$$\therefore$$
 $x = \frac{t^3}{3}$ \therefore $v = \frac{dx}{dt} = t^2$, Velocity at t=0, u=0 and at t=1s v=1 m/s

Using work energy theorem :
$$W = \frac{1}{2} mv^2 - \frac{1}{2} mu^2 = \frac{1}{2} 1(1)^2 = 0.5 J$$

Ex. A block of mass 1kg is placed at the point A of a rough track shown in figure. If it slightly pushed towards right it stops at the point B of the track. Calculate the workdone by the frictional force on the block during its transit from A to B.



Sol.
$$W_c + W_{nc} + W_{ext} = \Delta K$$

 $mg (1 - 0.8) + W_{nc} + 0 = 0 \Rightarrow W_{nc} = -1 \times 9.8 \times 0.2 = -1.96 J$

POWER

When we purchase a car or jeep we are interested in the horsepower of its engine. We know that usually an engine with large horsepower is most effective in accelerating the automobile. In many cases it is useful to know not just the total amount of work being done, but how fast the work is done. We define power as the rate at which work is being done.

Average Power =
$$\frac{\text{Work done}}{\text{Time taken to do work}}$$

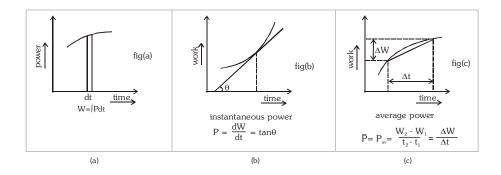
If
$$\Delta W$$
 is the amount of work done in the time interval Δt . Then $P = \frac{\Delta W}{\Delta t} = \frac{W_2 - W_1}{t_2 - t_1}$

When work is measured in joules and t is in seconds, the unit for power is the joule per second, which is called watt. For motors and engines, power is usually measured in horsepower, where horsepower is 1 hp = 746 W. The definition of power is applicable to all types of work like mechanical, electrical, thermal.

Instanteneous power
$$P = \frac{dW}{dt} = \frac{\vec{F} \cdot d\vec{r}}{dt} = \vec{F} \cdot \vec{v}$$

Where v is the instantaneous velocity of the particle and dot product is used as only that component of force will contribute to power which is acting in the direction of instantaneous velocity.

- Power is a scalar quantity with dimension M¹L²T⁻³
- SI unit of power is J/s or watt
- 1 horsepower = 746 watt = 550 ft-lb/sec.



- Area under power-time graph gives the work done. $W = \int Pdt$ (See Fig. a)
- The slope of tangent at a point on work time graph gives instantaneous power (See Fig. b)
- The slope of a straight line joining two points on work time graph gives average power between two points (See Fig. c)
- Ex. A boy is able to chew 20g of ice in 5 minute. Calculate the power of the boy in horse power.

Sol. Power,
$$P = \frac{W}{t} = \frac{JML}{t} = \frac{4.2 \text{ J/cal} \times 20\text{g} \times 80\text{cal/g}}{5 \times 60 \text{ sec}} = 22.4 \text{ watt};$$

$$P = \frac{22.4}{746} = 0.03 \text{ hp}$$

- **Ex.** A pump is used to deliver water at a certain rate from a given pipe. To obtain n times water from same pipe in the same time by what amount should power of motor be increased?
- **Sol.** Amount of water /second $\frac{dm}{dt} = Av\rho$

v = velocity of flow, A is area of cross-section $\rho = density$ of liquid

To get n times water in same time $\left(\frac{dm}{dt}\right)' = n\frac{dm}{dt} \Rightarrow Av'\rho = nAv\rho \Rightarrow v'=nv$

$$\therefore P = v^2 \frac{dm}{dt} \quad \text{So } \frac{P'}{P} = \frac{v'^2 (dm/dt)'}{v^2 (dm/dt)} = \frac{n^2 v^2 n \ dm/dt}{v^2 \ dm/dt} = n^3 \quad \Rightarrow P' = n^3 P$$

Thus to get n times water the power must be increased n³ times.

CONSERVATIVE FORCE

- A force is said to be conservative if the work done by or against the force is independent of path and depends only on initial and final positions
- It does not depend on the nature of path followed between the initial and final positions.

Examples of Conservative Force

All central forces are conservative like gravitational, electrostatic, elastic force, restoring force due to spring etc.

- In presence of conservative forces mechanical energy remains constant.
- Work done along a closed path or in a cyclic process is zero. i.e. $\oint \vec{F} . d\vec{r} = 0$
- If \vec{F} is a conservative force then $\vec{\nabla} \times \vec{F} = \vec{0}$

CENTRAL FORCE

The force whose line of action always passes through a fixed point (which is known as centre of force) and magnitude of force depends only on the distance from this point is known as **central force.**

$$\vec{F} = F(r)\hat{r}$$

All forces following inverse square law are called central forces.

 $\vec{F} = \frac{k}{r^2}\hat{r}$ is central force like Gravitational force and Coulomb force.

- All central forces are conservative forces
- Central forces are function of position

NON CONSERVATIVE FORCE

A force is said to be non-conservative if work done by or against the force in moving a body depends upon the path between the initial and final positions.

Work done in a closed path is not zero in a non-conservative force field.

The frictional forces are non-conservative forces. This is because the work done against friction depends on the length of the path along which a body is moved. It does not depend on the initial and final positions. The work done by frictional force in a round trip is not zero.

Examples of non-conservative force

The velocity-dependent forces such as air resistance, viscous force etc. are non-conservative forces.

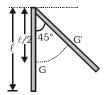
Conservative Forces		Non-conservative Forces	
•	Work done does not depend upon path.	•	Work done depends upon path.
•	Work done in a round trip is zero.	•	Work done in a round trip is not zero.
•	Central forces, spring forces etc. are	•	Forces are retarding in
	conservative forces		nature e.g. friction, viscous force etc.
•	When only a conservative force acts	•	Work done against a non-conservative force
	within a system, the kinetic energy		may be dissipated as heat energy.
	and potential energy can change.		
	However, their sum, the mechanical		
	energy of the system, doesnot change.		
•	Work done is completely recoverable.	•	Work done is not completely recoverable.

Potential energy

- The energy which a body has by virtue of its position or configuration in a conservative force field.
- Potential energy is a relative quantity.
- Potential energy is defined only for conservative force field.
- Potential energy of a body at any position in a conservative force field is defined as the workdone by an external agent against the action of conservative force in order to shift it from reference point. (PE = 0) to the present position.
- Potential energy of a body in a conservative force field is equal to the work done by the body in moving from its present position to reference position.
- At reference position, the potential energy of the body is zero or the body has lost the capacity of doing work.
- Relationship between conservative force field and potential energy:

$$\vec{F} = -\nabla U = -grad(U) = -\frac{\partial U}{\partial x}\hat{i} - \frac{\partial U}{\partial y}\hat{j} - \frac{\partial U}{\partial z}\hat{k}$$

- If force varies only with one dimension (along x-axis) then $F = -\frac{dU}{dx} \Rightarrow U = -\int_{x_1}^{x_2} F dx$
- **Ex.** A meter scale of mass m initially vertical is displaced at 45° keeping the upper end fixed. Find out the change in potential energy.

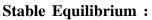


Sol.
$$\Delta U = \text{mg } \Delta h_{\text{cm}} = \text{mg} \frac{\ell}{2} (1 - \cos \theta) = \text{mg} \times \frac{\ell}{2} (1 - \cos 45^{\circ}) = \frac{\text{mg}}{2} \left(1 - \frac{1}{\sqrt{2}} \right)$$

- **Ex.** A uniform rod of length 4m and mass 20kg is lying horizontal on the ground. Calculate the work done in keeping it vertical with one of its ends touching the ground.
- **Sol.** As the rod is kept in vertical position the shift in the centre of gravity is equal to the half the length $= \ell/2$ Work done W = mgh = mg $\frac{\ell}{2} = (20)(9.8) \left(\frac{4}{2}\right) = 392$ J

It is a curve which shows change in potential energy

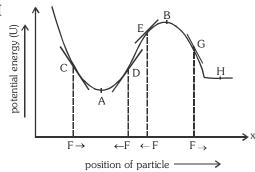
with postion of a particle.



When a particle is slightly displaced from equilibrium position and it tends to come back towards equilibrium

then it is said to be in stable equilibrium

At point C: slope $\frac{dU}{dx}$ is negative so F is positive



At point A: it is the point of stable equilibrium.

At
$$\mathbf{A} \ \mathbf{U} = \mathbf{U}_{min}$$
, $\frac{d\mathbf{U}}{d\mathbf{x}} = 0$ and $\frac{d^2\mathbf{U}}{d\mathbf{x}^2} = positive$

Unstable equilibrium:

When a particle is slightly displaced from equilbrium and it tends to move away from equilibrium position then it is said to be in unstable equilbrium

At point
$${\bf E}$$
 : slope $\frac{dU}{dx}$ is positive so F is negative

At point
$$\mathbf{G}$$
 : slope $\frac{dU}{dx}$ is negative so F is postive

At point B: it is the point of unstable equilibrium.

At **B**
$$U = U_{max}$$
, $\frac{dU}{dx} = 0$ and $\frac{d^2U}{dx^2} = negative$

Neutral equilibrium:

When a particle is slightly displaced from equilibrium position and no force acts on it then equilibrium is said to be neutral equilibrium

Point **H** is at neutral equilibrium
$$\Rightarrow$$
 U = constant; $\frac{dU}{dx} = 0$, $\frac{d^2U}{dx^2} = 0$

Ex. The potential energy for a conservative force system is given by $U = ax^2 - bx$. Where a and b are constants find out

- (a) The expression of force
- (b) Equilibrium position
- (c) Potential energy at equilibrium.

Sol. (a) For conservative force
$$F = -\frac{dU}{dx} = -(2ax - b) = -2ax + b$$

(b) At equilibrium
$$F = 0 \implies -2ax + b = 0 \implies x = \frac{b}{2a}$$
 (c) $U = a\left(\frac{b}{2a}\right)^2 - b\left(\frac{b}{2a}\right) = \frac{b^2}{4a} - \frac{b^2}{2a} = -\frac{b^2}{4a}$

Law of conservation of Mechanical energy

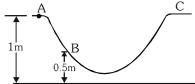
Total mechanical (kinetic + potential) energy of a system remains constant if only conservative forces are acting on the system of particles and the work done by all other forces is zero.

From work energy theorem $W = \Delta KE$

For internal conservative forces $W_{int} = -\Delta U$

So
$$W = W_{ext} + W_{int} = 0 + W_{int} = -\Delta U \Rightarrow -\Delta U = \Delta KE \Rightarrow (KE + U) = 0 \Rightarrow KE + U = (constant)$$

Ex. A particle is placed at the point A of a frictionless track ABC. It is pushed slightly towards right. Find its speed when it reaches the point B. [Take $g=10 \text{ m/s}^2$]



- **Sol.** $mg(1 0.5) = \frac{1}{2} \times m \times v^2 \implies v^2 = (2) (10) (0.5) \implies v = \sqrt{10} \text{ m/s}$
- Ex. Determine the average force necessary to stop a bullet of mass 20 g and speed 250 ms⁻¹ as it penetrates wood to a distance of 12 cm.
- **Sol.** If F newton be the retarding force, then the work done by force W = F × s=F × 0.12 joule Loss of kinetic energy = $\frac{1}{2} \times \frac{20}{1000} \times (250)^2 = 625$ joule

(This kinetic energy is consumed in stopping the bullet and is converted into heat energy)

Applying work-energy theorem,
$$F \times 0.12 = 625 \Rightarrow F = \frac{625}{0.12}$$
 $N = 5.2 \times 10^3$ N

It is interesting to note that the retarding force is nearly 30,000 times the weight of the bullet.

- **Ex.** On passing through a wooden sheet a bullet loses 1/20 of its initial velocity. Calculate the minimum number of sheets required to completely stop the bullet.
- **Sol.** For a sheet of thickness s v = (19/20)u

Use
$$v^2 = u^2 + 2as \Rightarrow \left(\frac{19}{20}u\right)^2 = u^2 + 2as \Rightarrow 2as = (361/400)u^2 - u^2 \Rightarrow a = -\left(\frac{39}{400}\right)\frac{u^2}{2s}$$

Let minimum number of steets is n then $0^2 = u^2 + 2a$ (ns) \Rightarrow n = $-\frac{u^2}{2as} = \frac{u^2}{2\left(\frac{39}{400}\right)\frac{u^2}{2s} \times s} \approx 11$

- Ex. A body of mass 8 kg moves under the influence of a force. The position of the body and time are related as $x = \frac{1}{2}t^2$ where x is in meter and t in sec. Find the work done by the force in first two seconds.
- **Sol.** Work done = change in kinetic energy = $\frac{1}{2}$ mv² = $\frac{1}{2}$ m $\left(\frac{dx}{dt}\right)^2 = \frac{1}{2}$ m $\left(\frac{2t}{2}\right)^2 = \frac{1}{2} \times 8 \times \left[\frac{2 \times 2}{2}\right]^2 = 16$ J
- **Ex.** A body falls on the surface of the earth from a height of 20 cm. If after colliding with the earth, its mechanical energy is lost by 75%, then determine height upto the body would reach.
- **Sol.** $\frac{1}{4}$ mgh = mgh' : h' = $\frac{h}{4} = \frac{1}{4} \times 20 = 5$ cm



- Ex. Calculate the stopping distance for a vehicle of mass m moving with speed v along level road. (μ is the coefficient of friction between tyres and the road)
- **Sol.** When the vehicle of mass m is moving with velocity v, the kinetic energy of the vehicle $K = \frac{1}{2} \text{ mv}^2$ and if S is the stopping distance, work done by the friction $W = FS \cos \theta = \mu mgS \cos 180^\circ = -\mu mgS$

So by Work-Energy theorem,
$$W = \Delta K = K_f - K_i \Rightarrow -\mu \text{ mgS} = 0 - 1/2 \text{ mv}^2 \Rightarrow S = \frac{v^2}{2\mu g}$$

- **Ex.** A particle of mass m is moving in a horizontal circle of radius r, under a centripetal force equal to $(-k/r^2)$, where k is constant. Calculate the total energy of the particle.
- **Sol.** As the particle is moving in a circle, so $\frac{mv^2}{r} = \frac{k}{r^2}$. Now K.E = $\frac{1}{2}$ mv² = $\frac{k}{2r}$

As
$$U = -\int_{\infty}^{r} F \cdot dr = \int_{\infty}^{r} + \left(\frac{k}{r^2}\right) dr = -\frac{k}{r}$$
. So total energy = U + K.E. $= -\frac{k}{r} + \frac{k}{2r} = -\frac{k}{2r}$

Negative energy means that particle is in bound state.

- **Ex.** A man throws the bricks to the height of 12 m where they reach with a speed of 12 m/sec. If he throws the bricks such that they just reach this height, what percentage of energy will he save?
- **Sol.** In first case, $W_1 = \frac{1}{2} m(v_1)^2 + mgh = \frac{1}{2} m(12)^2 + m \times 10 \times 12 = 72 m + 120 m = 192m$ and in second case, $W_2 = mgh = 120 m$

The percentage of energy saved =
$$\frac{192\text{m} - 120\text{m}}{192\text{m}} \times 100 = 38\%$$

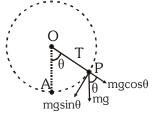
CIRCULAR MOTION IN VERTICAL PLANE

Suppose a particle of mass m is attached to an inextensible light string of length R. The particle is moving in a vertical circle of radius R about a fixed point O. It is imparted a velocity u in horizontal direction at lowest point A. Let v be its velocity at point P of the circle as shown in figure. Here, $h = R (1 - \cos\theta)$...(i)

From conservation of mechanical energy

$$\frac{1}{2}$$
 m(u²- v²)=mgh \Rightarrow v² = u²-2gh(ii)

The necessary centripetal force is provided by the resultant of tension



T and mg
$$\cos\theta$$
 T - mg $\cos\theta = \frac{mv^2}{R}$...(iii)

Since speed of the particle decreases with height, hence tension is maximum at the bottom, where $\cos\theta = 1$ (as $\theta = 0^{\circ}$)

$$\Rightarrow$$
 $T_{max} = \frac{mu^2}{R} + mg$; $T_{min} = \frac{mv'^2}{R} - mg$ at the top. Here, $v' = speed$ of the particle at the top.

E

Condition of Looping the Loop $(u \ge \sqrt{5gR})$

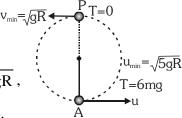
The particle will complete the circle if the string does not slack even at the highest point $(\theta = \pi)$. Thus, tension in the string should be greater than or equal to zero $(T \ge 0)$ at $\theta = \pi$. In critical case substituting

T = 0 and
$$\theta = \pi$$
 in Eq. (iii), we get mg = $\frac{mv_{min}^2}{R} \Rightarrow v_{min} = \sqrt{gR}$ (at highest point)

$$u_{min}^2 = v_{min}^2 + 2gh = gR + 2g(2R) = 5gR \implies u_{min} = \sqrt{5gR}$$

Substituting $\theta=\pi$ in Eq. (i), Therefore, from Eq. (ii) $v_{\text{min}}=\sqrt{gR} \qquad P = 0$ $u_{\text{min}}^2=v_{\text{min}}^2+2gh=gR+2g(2R)=5gR \implies u_{\text{min}}=\sqrt{5gR}$ Thus, if $u \geq \sqrt{5gR}$, the particle will complete the circle. At $u=\sqrt{5gR}$, T=6mg

velocity at highest point is $v = \sqrt{gR}$ and tension in the string is zero.



Substituting $\theta = 0^{\circ}$ and $v = \sqrt{5gR}$ in Eq. (iii), we get T = 6 mg or in the critical condition tension in the string at lowest position is 6 mg. This is shown in figure. If $u < \sqrt{5gR}$, following two cases are possible.

Condition of Leaving the Circle $(\sqrt{2gR} < u < \sqrt{5gR})$

If $u < \sqrt{5gR}$, the tension in the string will become zero before reaching the highest point. From Eq.

(iii), tension in the string becomes zero (T = 0) where,
$$\cos \theta = \frac{-v^2}{Rg} \implies \cos \theta = \frac{2gh - u^2}{Rg}$$

Substituting, this value of $\cos \theta$ in Eq. (i), we get $\frac{2gh - u^2}{R\sigma} = 1 - \frac{h}{R} \Rightarrow h = \frac{u^2 + Rg}{3\sigma} = h_1$ (say)

or we can say that at height h_1 tension in the string becomes zero. Further, if $u < \sqrt{5gR}$, velocity

of the particle becomes zero when $0 = u^2 - 2gh \Rightarrow h = \frac{u^2}{2g} = h_2(say)...(v)$ i.e., at height h_2 velocity of particle becomes zero.

Now, the particle will leave the circle if tension in the string becomes zero but velocity is not zero. or T = 0 but $v \neq 0$. This is possible only when $h_1 < h_2$

$$\Rightarrow \frac{u^2 + Rg}{3g} < \frac{u^2}{2g} \Rightarrow 2u^2 + 2Rg < 3u^2 \Rightarrow u^2 > 2Rg \Rightarrow u > \sqrt{2Rg}$$

Therefore, if $\sqrt{2gR}$ < u < $\sqrt{5gR}$, the particle leaves the circle.

From Eq. (iv), we can see that h > R if $u^2 > 2gR$. Thus, the particle, will leave the circle when h > R or 90° < θ < 180° . This situation is shown in the figure

$$\sqrt{2gR} < u < \sqrt{5gR} \text{ or } 90^{\circ} < \theta < 180^{\circ}$$

Note: That after leaving the circle, the particle will follow a parabolic path.

Condition of Oscillation $(0 < u \le \sqrt{2gR})$

The particle will oscillate if velocity of the particle becomes zero but tension in the string is not zero or v=0, but $T\neq 0$. This is possible when $h_2 < h_1$

$$\Rightarrow \frac{u^2}{2g} < \frac{u^2 + Rg}{3g} \Rightarrow 3u^2 < 2u^2 + 2Rg \Rightarrow u^2 < 2Rg \ \Rightarrow u < \sqrt{2Rg}$$

comes zero v = 0 $T \neq 0$ $h \leq R$

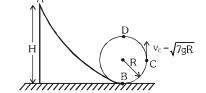
Moreover, if $h_1 = h_2$, $u = \sqrt{2Rg}$ and tension and velocity both becomes zero simultaneously. Further, from Eq. (iv), we can see that $h \le R$ if $u \le \sqrt{2Rg}$.

Thus, for $0 < u \le \sqrt{2gR}$, particle oscillates in lower half of the circle $(0^o < \theta \le 90^\circ)$

This situation is shown in the figure. $0 < u \le \sqrt{2gR}$ or $0^{\circ} < \theta \le 90^{\circ}$

- Ex. Calculate following for shown situation:-
 - (a) Speed at D (b) Normal reaction at D (c) Height H

Sol. (a)
$$v_D^2 = v_C^2 - 2gR = 5gR \implies v_D = \sqrt{5gR}$$



(b)
$$mg + N_D = \frac{mv_D^2}{R} \implies N_D = \frac{m(5gR)}{R} - mg = 4mg$$

(c) by energy conservation between point A & C

$$mgH = \frac{1}{2}mv_{C}^{2} + mgR = \frac{1}{2}m(5gR) + mg2R = \frac{9}{2}mgR \implies H = \frac{9}{2}R$$

Ex. A stone of mass 1 kg tied to a light string of length $\ell = \frac{10}{3}$ m is whirling in a circular path in vertical plane. If the ratio of the maximum to minimum tension in the string is 4, find the speed of the stone at the lowest and highest points

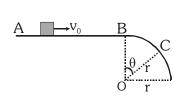
Sol.
$$\frac{T_{\text{max}}}{T_{\text{min}}} = 4 : \frac{\frac{mv_{\ell}^2}{\ell} + mg}{\frac{mv_{p}^2}{\ell} - mg} = 4 \Rightarrow \frac{v_{\ell}^2 + g\ell}{v_{p}^2 - g\ell} = 4$$



We know
$$v_{\ell}^2 = v_p^2 + 4g\ell \implies \frac{v_p^2 + 5g\ell}{v_p^2 - g\ell} = 4 \implies 3v_p^2 = 9g\ell$$

$$\Rightarrow v_{_{P}}{^{2}} = \sqrt{3g\ell} = \sqrt{3\times10\times\frac{10}{3}} = 10 \text{ ms}^{_{-1}} \qquad \Rightarrow v_{_{\ell}} = \sqrt{7g\ell} = \sqrt{7\times10\times\frac{10}{3}} = 15.2 \text{ ms}^{_{-1}}$$

Ex. A small block slides with velocity $0.5\sqrt{gr}$ on the horizontal frictionless $A \longrightarrow V_0$ surface as shown in the figure. The block leaves the surface at point C. Calculate angle θ in the figure.

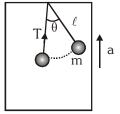


As block leaves the surface at C so at C, normal reaction = $0 \Rightarrow \text{mgcos}_{\theta} = \frac{\text{mv}_{\text{C}}^2}{\text{...}}$ Sol.

By energy conservation at point B & C $\frac{1}{2} \text{mv}_c^2 - \frac{1}{2} \text{mv}_0^2 = \text{mgr} (1-\cos\theta)$

$$\Rightarrow \frac{1}{2} \, m \, \left(rg cos \theta \right) - \frac{1}{2} \, m \, \left(0.5 \sqrt{gr} \right)^2 = mgr \, \left(1 - cos \, \theta \right) \\ \Rightarrow cos \, \theta = \frac{3}{4} \\ \Rightarrow \theta = cos^{-1} \left(\frac{3}{4} \right)$$

A particle of mass m is attached to the ceiling of a cabin with an inextensible light string of length ℓ . The cabin is moving upward with an acceleration 'a'. The particle is taken to a position such that the string makes an angle θ with vertical. When string becomes vertical, find the tension in the string.

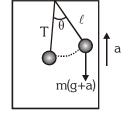


Sol. In a frame associated with cabin work done on the particle when it comes in the vertical position = $mg\ell(1-\cos\theta) + ma(1-\cos\theta)$ By work energy theorem,

$$\frac{mv^{2}}{2} = \left(mg\ell + ma\ell\right)\left(1 - \cos\theta\right) \Rightarrow \frac{v^{2}}{2} = (g+a) \cdot 1 \cdot (1 - \cos\theta)$$

At vertical position, $T = (mg + ma) = \frac{mv^2}{\rho}$

$$\Rightarrow T = (mg + ma) + 2m(g+a) (1-\cos\theta) = mg (g+a) (3-2\cos\theta)$$



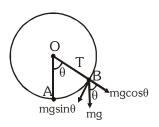
- A heavy particle hanging from a fixed point by a light inextensible string of length ℓ , is projected horizontally with speed $\sqrt{(g\ell)}$. Find the speed of the particle and the inclination of the string to the vertical at the instant of the motion when the tension in the string is equal to the weight of the particle.
- Let tension in the string becomes equal to the weight of the particle when particle reaches the point B and deflection of the string from vertical is θ . Resolving mg along the string and perpendicular to the string, we get net radial force on the particle at B i.e.

$$F_R = T - mg \cos \theta$$
(i)

 $F_{R} = T - mg \cos \theta$ If v_{B} be the speed of the particle at B, then

$$F_{R} = \frac{mv_{B}^{2}}{\ell} \qquad(ii)$$

From (i) and (ii), we get, $T - mg \cos \theta = \frac{mv_B^2}{\ell}$ (iii)



Since at B, T = mg
$$\Rightarrow$$
 mg $(1-\cos\theta) = \frac{mv_B^2}{\ell} \Rightarrow v_B^2 = g\ell(1-\cos\theta)$...(iv)

Applying conservation of mechanical energy of the particle at point A and B, we have

$$\frac{1}{2} m v_A^2 = mg\ell \ (1 - \cos \theta) + \frac{1}{2} m v_B^2; \text{ where } v_A = \sqrt{g\ell} \text{ and } v_B = \sqrt{g\ell(1 - \cos \theta)}$$

$$\Rightarrow g\ell = 2g\ell \left(1 - \cos \theta\right) + g\ell \ (1 - \cos \theta) \Rightarrow \cos \theta = \frac{2}{3} \Rightarrow \theta = \cos^{-1}\left(\frac{2}{3}\right)$$

Putting the value of $\cos \theta$ in equation (iv), we get : $v = \sqrt{\frac{g\ell}{3}}$

ALLEN

EXERCISE (S)

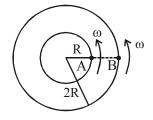
- 1. A particle moves in a circle of radius 1.0 cm at a speed given by v = 2.0 t where v is in cm/s and t in seconds.
 - (a) Find the radial acceleration of the particle at t = 1s.
 - (b) Find the tangential acceleration at t = 1s.
 - (c) Find the magnitude of the acceleration at t = 1 s.

CR0003

- 2. A particle is travelling in a circular path of radius 4m. At a certain instant the particle is moving at 20m/s and its acceleration is at an angle of 37° from the direction to the centre of the circle as seen from the particle
 - (i) At what rate is the speed of the particle increasing?
 - (ii) What is the magnitude of the acceleration?

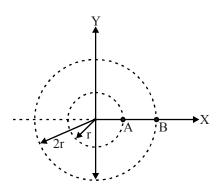
CR0004

3. Two particles A & B are moving on two concentric circles of radius R & 2R respectively with constant angular velocity ω as shown in figure. Find the angular velocity of B wrt A.

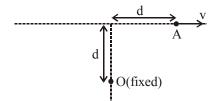


CR0079

Two particles A and B are moving in a horizontal plane anticlockwise on two different concentric circles with different constant angular velocities 2ω and ω respectively. Find the relative velocity (in m/s) of B w.r.t. A after time $t = \pi/\omega$. They both start at the position as shown in figure (Take $\omega = 3\text{rad/sec}$, r = 2m)



5. Find angular velocity of A with respect to O at the instant shown in the figure.



CR0007

6. A stone is thrown horizontally with the velocity 15m/s. Determine the tangential and normal accelerations of the stone in 1 second after it begins to move.

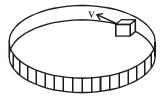
CR0008

7. A body is projected with a velocity 10 ms⁻¹ at an inclination 45° to the horizontal. Minimum radius of curvature of the trajectory described by the particle is ______.

CR0080

DYNAMICS OF CIRCULAR MOTION

8. A block of mass m moves with speed v against a smooth, fixed vertical circular groove of radius r kept on smooth horizontal surface.



Find:

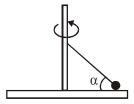
- (i) normal reaction of the floor on the block.
- (ii) normal reaction of the vertical wall on the block.

CR0011

9. A cyclist speeding at 18 km/h on a level road takes a sharp circular turn of radius 3 m without reducing the speed. The co~efficient of static-friction between the tyres and the road is 0.1. Will the cyclist slip while taking the turn?

CR0012

10. A circular platform rotates around a vertical axis with angular velocity $\omega = 10$ rad/s. On the platform is a ball of mass 1 kg, attached to the long axis of the platform by a thin rod of length 10 cm ($\alpha = 30^{\circ}$). Find normal force exerted by the ball on the platform (in newton). Friction is absent.



CR0016

11. A circular racetrack of radius 300 m is banked at an angle of 15°. If the coefficient of friction between the wheels of a race-car and the road is 0.2, what is the (a) optimum speed of the race- car to avoid wear and tear on its tyres, and [0) maximum permissible speed to avoid slipping?

80

- 12. A block of mass m = 20 kg is kept at a distance R = 1m from central axis of rotation of a round turn table (A table whose surface can rotate about central axis). Table starts from rest and rotates with constant angular acceleration, $\alpha = 3$ rad/sec². The friction coefficient between block and table is $\mu = 1m$
 - 0.5. At time $t = \frac{x}{3}$ sec from starting of motion (i.e. t = 0 sec) the block is just about to slip. Find the value of x.

CR0018

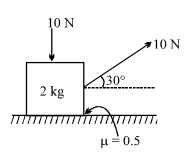
13. A stone is fastened to one end of a string and is whirled in a vertical circle of radius R. Find the minimum speed the stone can have at the highest point of the circle.

WE0106

- 14. The sign of work done by a force on a body is important to understand. State carefully if the following quantities are positive or negative: (NCERT)
 - (a) work done by a man in lifting a bucket out of a well by means of a rope tied to the bucket.
 - (b) work done by gravitational force in the above case,
 - (c) work done by friction on a body sliding down an inclined plane,
 - (d) work done by an applied force on a body moving on a rough horizontal plane with uniform velocity,
 - (e) work done by the resistive force of air on a vibrating pendulum in bringing it to rest.

WE0001

- 15. A particle of mass 2kg moves on a smooth horizontal plane under the action of a single force $\vec{F} = (3\hat{i} + 4\hat{j})$ newton. Under this force it is displaced from (0,0) to (-1m, 1m). Find the work done by F (in Joule) **WE0107**
- **16.** A body of mass 2 kg is kept on a rough horizontal surface as shown in the figure. Find the work done by frictional force in the time interval t = 0 to t = 5 sec.



WE0108

17. The net fore exerted on a particle acts in the +x direction. Its magnitude increases linearly from zero at x = 0, to 24.0 N at x = 3.0 m. It remains constant at 24.0 N from x = 3.0m to x = 8.0m and then decreases linearly to zero at x = 13.0m. Determine the work done to move the particle from x = 0 to x = 13.0.

WORK ENERGY THEOREM

18. A point mass of 0.5 kg is moving along x-axis as $x = t^2 + 2t$, where, x is in meters and t is in seconds. Find the work done (in J) by all the forces acting on the body during the time interval [0, 2s].

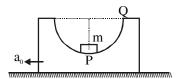
WE0003

19. A block is released from rest from top of a rough curved track as shown in figure. It comes to rest at some point on the horizontal part. If its mass is 200 gm, calculate negative of work by friction is joules.



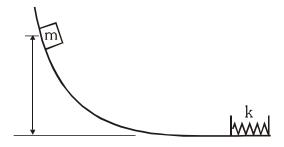
WE0011

20. A small block of mass m is lying at rest at point P of a wedge having a smooth semi circular track of radius R. The minimum value of horizontal acceleration a₀ of wedge so that mass can just reach the point Q, is



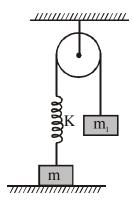
WE0013

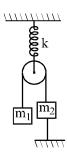
21. A mass m slides from rest at height h down a smooth curved surface which becomes horizontal at zero height (see figure). A spring is fixed horizontally on the level part of the surface. The spring constant is k N/m. When the mass encounters the spring it compresses it by an amount x = h/10. If m = 1 kg, h = 5 m then find k /100.



WE0012

22. For what minimum value of m_1 the block of mass m will just leave the contact with surface? Initially srping is in natural length and block of mass m_1 is released from rest.



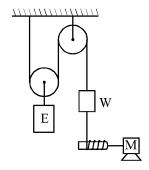


WE0017

24. The potential function for a conservative force is given by U = k(x + y). Find the work done by the conservative force in moving a particle from the point A(1, 1) to point B(2, 3).

WE0110

25. The elevator E has a mass of 3000 kg when fully loaded and is connected as shown to a counterweight W of mass 1000 kg. Determine the power in kilowatts delivered by the motor



- (a) when the elevator is moving down at a constant speed of 3 m/s,
- (b) when it has an upward velocity of 3 m/s and a deceleration of 0.5 m/s².

WE0025

26. The potential energy (in joules) function of a particle in a region of space is given as :

$$U = (2x^2 + 3y^3 + 2z)$$

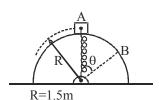
Here x, y and z are in metres. Find the magnitude of x component of force (in newton) acting on the particle at point P (1m, 2m, 3m).

WE0018

27. The P.E. of a particle oscillating on x-axis is given as $U = 20 + (x - 2)^2$ here U is in Joules & x is in meters. Total mechanical energy of particle is 36 J

WE0019

- (i) Find the mean position (ii) Find the max. K.E. of the particle
- 28. A 2 kg block is gently pushed from rest at A and it slides down along the fixed smooth circular surface as shown in figure. If the attached spring has a force constant k = 20 N/m. What is unstretched length of spring (in m) so that it does not allow the block to leave the surface until angle with the vertical is $\theta = 60^{\circ}$.



EXERCISE (O-1)

- 1. The second's hand of a watch has length 6 cm. Speed of end point and magnitude of difference of velocities at two perpendicular positions will be
 - (A) $2\pi \& 0 \text{ mm/s}$

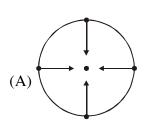
(B) $2\sqrt{2} \pi \& 4.44 \text{ mm/s}$

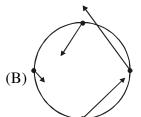
(C) $2\sqrt{2} \pi \& 2\pi \text{ mm/s}$

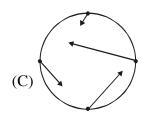
(D) $2\pi \& 2\sqrt{2} \pi \text{ mm/s}$

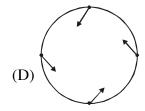
CR0033

2. A car speeds up with constant magnitude of tangential acceleration in circular path moving in anticlockwise direction. Which of the following figure represents acceleration of the car?







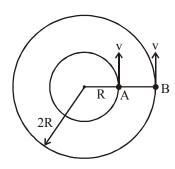


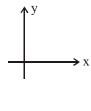
CR0082

- **3.** A particle is moving in a circle:
 - (A) the resultant force on the particle must be towards the centre
 - (B) the cross product of the tangential acceleration and the angular velocity will be zero
 - (C) the direction of the angular acceleration and the angular velocity must be the same
 - (D) the resultant force may be towards the centre

CR0037

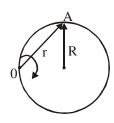
4. A particle is moving on a circle of radius R with velocity v. Another particle is moving on a concentric circle of radius 2R with some velocity v. Then, choose the correct statement(s)





- (A) Direction of angular velocity of B relative to A will be in +z direction
- (B) Direction of angular velocity of B relative to A will be in –z direction
- (C) Angular velocity of B relative to A is zero always.
- (D) Angular velocity of B relative to A is zero for the situation shown.

5. A particle A moves along a circle of radius R=50 cm so that its radius vector r relative to the point O (figure) rotates with the constant angular velocity ω =0.40 rad/s. Then modulus of the velocity of the particle, and the modulus of its total acceleration will be



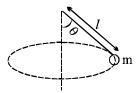
(A) v = 0.4 m/s, $a = 0.4 \text{ m/s}^2$

(B) $v = 0.32 \text{ m/s}, a = 0.32 \text{ m/s}^2$

- (C) $v = 0.32 \text{ m/s}, a = 0.4 \text{ m/s}^2$
- (D) v = 0.4 m/s, $a = 0.32 \text{ m/s}^2$

CR0057

6. A conical pendulum is moving in a circle with angular velocity ω as shown. If tension in the string is T, which of following equations are correct?



- (A) $T = m\omega^2 l$
- (B) T $\sin\theta = m\omega^2 l$
- (C) $T = mg \cos\theta$
- (D) $T = m\omega^2 l \sin\theta$

CR0040

- 7. A ring of radius r and mass per unit length m rotates with an angular velocity ω in free space. The tension in the ring is :
 - (A) zero
- (B) $\frac{1}{2}m\omega^2r^2$
- (C) $m\omega^2 r^2$
- (D) $mr\omega^2$

CR0051

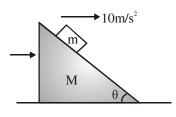
- **8**. A rope is used to lower vertically a block of mass M by a distance x with a constant downward acceleration g/2. The work done by the rope on the block is:
 - (A) Mgx
- (B) ½ Mgx²
- $(C) \frac{1}{2} Mgx$
- (D) Mgx²

WE0040

- 9. A bicyclist comes to a skidding stop in 10 m. During this process, the force on the bicycle due to the road is 200N and is directly opposed to the motion. The work done by the cycle on the road is
 - (A) + 2000J
- (B) 200J
- (C) zero
- (D) 20,000J

WE0112

10. In the figure shown all the surfaces are frictionless, and mass of the block, m = 1 kg. The block and wedge are held initially at rest. Now wedge is given a horizontal acceleration of 10 m/s² by applying a force on the wedge, so that the block does not slip on the wedge. Then work done by the normal force in ground frame on the block in $\sqrt{3}$ seconds is:

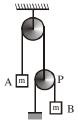


- (A) 30J
- (B) 60 J
- (C) 150 J
- (D) $100 \sqrt{3} \text{ J}$

- In a shotput event an athlete throws the shotput of mass 10 kg with an initial speed of 1m/s at 45° from 11. a height 1.5 m above ground. Assuming air resistance to be negligible and acceleration due to gravity to be 10 m/s², the kinetic energy of the shotput when it just reaches the ground will be
 - (A) 2.5 J
- (B) 5.0 J
- (C) 52.5 J
- (D) 155.0 J

WE0048

In the figure shown, the system is released from rest. Find the velocity of block A when block B has **12.** fallen a distance '\mathcal{l}'. Assume all pulleys to be massless and frictionless.



- (A) $\sqrt{\frac{g\ell}{5}}$
- (B) $\sqrt{g\ell}$
- (D) None of these

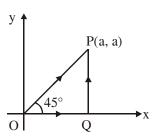
WE0050

- The work done in joules in increasing the extension of a spring of stiffness 10 N/cm from 4 cm to 6 **13.** cm is:
 - (A) 1

- (B) 10
- (C) 50
- (D) 100

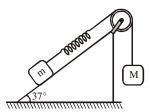
WE0113

A particle is moved from (0,0) to (a,a) under a force $\vec{F} = (3\hat{i} + 4\hat{j})$ from two paths. Path 1 is OP and **14.** path 2 is OQP. Let \mathbf{W}_1 and \mathbf{W}_2 be the work done by this force in these two paths. Then :



- (A) $W_1 = W_2$ (B) $W_1 = 2W_2$
- (C) $W_2 = 2W_1$ (D) $W_2 = 4W_1$

15. A block of mass m is attached with a massless spring of force constant k. The block is placed over a rough inclined surface for which the coefficient of friction is $\mu = \frac{3}{4}$. The minimum value of M required to move the block up the plane is: Initially spring is in natural length and block is released from rest. (Neglect mass of string and pulley and friction in pulley)



- (A) $\frac{3}{5}$ m
- (B) $\frac{4}{5}$ m
- (C) 2 m
- (D) $\frac{3}{2}$ m

WE0052

- **16.** When a conservative force does positive work on a system
 - (A) the potential energy increases
- (B) the potential energy decreases

(C) total energy increases

(D) total energy decreases

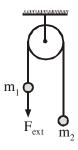
WE0060

- 17. A body is moving unidirectionally under the influence of a source of constant power. Its displacement in time t is proportional to (NCERT)
 - (A) $t^{1/2}$
- (B) t

- (C) $t^{3/2}$
- (D) t^2

WE0114

18. Two bodies of mass m_1 and m_2 ($m_2 > m_1$) are connected by a light inextensible string which passes through a smooth fixed pulley. The instantaneous power delivered by an external agent to pull m_1 with constant velocity v is:



- $(A) (m_2 m_1) g/v$
- (B) $(m_2 m_1) v/g$
- $(C) (m_2 m_1) gv$
- (D) $(m_1 m_2)$ gv

WE0115

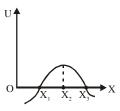
- **19.** Assume the aerodynamic drag force on a car is proportional to its speed. If the power output from the engine is doubled, then the maximum speed of the car.
 - (A) is unchanged

(B) increases by a factor of $\sqrt{2}$

(C) is also doubled

(D) increases by a factor of four.

20. In the figure shown the potential energy (U) of a particle is plotted against its position 'x' from origin. Then which of the following statement is correct. A particle at :



(A) x_1 is in stable equilibrium

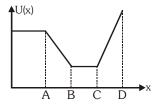
(B) x, is in stable equilibrium

(C) x_3 is in stable equilibrium

(D) None of these

WE0063

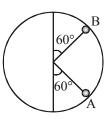
21. As a particle moves along the x-axis it is acted upon by a conservative force. The potential energy is shown below as a function of the coordinate x of the particle. Rank the labelled regions according to the magnitude of the force, least to greatest.



- (A) AB, BC, CD
- (B) AB, CD, BC
- (C) BC, CD, AB
- (D) BC, AB, CD

WE0064

22. A particle is attached to one end of string the other end of which is fixed. The particle moves in vertical plane. The velocity at the lowest point is such that the particle can just complete the circle. The ratio of tension in string at A & B is:-



(A) 2

(B)3

(C)4

(D) $\sqrt{3}$

WE0116

23. A block of mass m is hung vertically from an elastic thread of force constant $\frac{2mg}{a}$. Initially the thread

was at its natural length and the block is allowed to fall freely. The work done on block by earth when it passes through the equilibrium position will be:

- (A) zero
- (B) $\frac{\text{mga}}{2}$
- (C) mga
- (D) $\frac{\text{mga}}{4}$

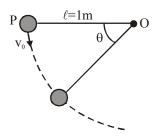
- 24. A ball is thrown vertically upwards with a velocity v and an initial kinetic energy E. When half way to the top of its flight, it can have velocity and kinetic energy respectively of:
 - $(A) \frac{v}{2}, \frac{E}{2}$
- (B) $\frac{v}{\sqrt{2}}, \frac{E}{2}$ (C) $\frac{v}{4}, \frac{E}{2}$
- (D) $\frac{v}{4}, \frac{E}{\sqrt{2}}$

WE0117

- 25. A spring of spring constant 5×10^3 N/m is stretched initially by 10 cm from the unstretched position. Then the work required to stretch it further by another 10cm is-
 - (A) 12.50 N-m
- (B) 18.75 N-m
- (C) 75.00 N-m
- (D) 62.5 N-m

WE0118

The sphere at P is given a downward velocity v_0 and swings in a vertical plane at the end of a rope of **26.** $\ell = 1$ m attached to a support at O. The rope breaks at angle 30° from horizontal, knowing that it can with stand a maximum tension equal to three times the weight of the sphere. Then the value of \boldsymbol{v}_0 will be : $(g = \pi^2 \text{ m/s}^2)$



- $(A) \frac{g}{2} \text{ m/s}$
- (B) $\frac{2g}{3}$ m/s
- (C) $\sqrt{\frac{3g}{2}}$ m/s
- (D) $\frac{g}{3}$ m/s

WE0119

An object of weight 4N swings freely on the end of a string. At the bottom of the swing, the tension 27. in the string is 6N. If g is the magnitude of the acceleration due to gravity, the magnitude of the centripetal acceleration at the bottom of the swing is



(A) 0

- (C)g
- (D) $\frac{3g}{2}$

WE0120

- Force acting on a particle moving in a straight line varies with the velocity of the particle as $F = \frac{K}{v}$ 28. Here K is a constant. The work done by this force in time t is:
 - $(A)\frac{K}{v^2}.t$
- (B) 2Kt
- (C) Kt
- (D) $\frac{2Kt}{v^2}$

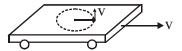
WE0121

E

ALLEN

EXERCISE (O-2)

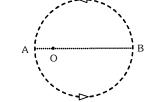
- 1. On a train moving along east with a constant speed v, a boy revolves a bob with string of length ℓ on smooth surface of a train, with equal constant speed v relative to train. Mark the correct option(s).
 - (A) Maximum speed of bob is 2 v in ground frame.
 - (B) Tension in string connecting bob is $\frac{4mv^2}{\ell}$ at an instant.



- (C) Tension in string is $\frac{mv^2}{\ell}$ at all the moments.
- (D) Minimum speed of bob is zero in ground frame.

CR0059

- 2. An object moves counter–clockwise along the circular path shown. As it moves along the path, its acceleration vector continuously points towards point O. In the figure, line AB is a diameter.
 - (A) The object speeds up from A to B and slows down from B to A.
 - (B) The object slows down from A to B and speeds up from B to A.

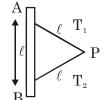


- (C) The object has maximum speed at A and minimum speed at B.
- (D) The object has minimum speed at A and maximum speed at B.

CR0084

- 3. An ant travels along a long rod with a constant velocity \vec{u} relative to the rod starting from the origin. The rod is kept initially along the positive x-axis. At t = 0, the rod also starts rotating with an angular velocity ω (anticlockwise) in x-y plane about origin. Then
 - (A) the position of the ant at any time t is $\vec{r} = ut[\cos \omega t\hat{i} + \sin \omega t\hat{j}]$
 - (B) the speed of the ant at any time t is $u\sqrt{1+\omega^2t^2}$
 - (C) the magnitude of the tangential acceleration of the ant at any time t is $\frac{\omega^2 tu}{\sqrt{1+\omega^2 t^2}}$
 - (D) the speed of the ant at any time t is $\sqrt{1+2\omega^2t^2u}$

4.

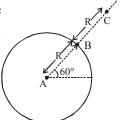


- are T_1 and T_2
 - (A) $T_1 = T_2$ (B) $T_1 + T_2 = m\omega^2 l$
 - (C) $T_1 T_2 = 2mg$
 - (D) BP will remain taut only if $\omega \ge \sqrt{\frac{2g}{\ell}}$

- Diagram shows three particles A, B & C. Particle B is moving in a circle **5.** of radius R with constant angular velocity w clockwise A & C are at rest. Choose the correct statement(s)



- (B) Angular velocity of B wrt A is ω anticlockwise
- (C) Angular velocity of B wrt C is constant & equal to ω
- (D) Angular velocity of B wrt C is not constant and is $\leq \omega$



CR0085

CR0067

A particle is projected from ground at an angle θ . At a certain instant the velocity vector \vec{v} of particle 6. makes an angle α with horizontal.

A particle P of mass m is attached to a vertical axis by two strings AP and BP of \$length l each. The

separation AB=l. P rotates around the axis with an angular velocity ω. The tensions in the two strings

(A)
$$\frac{d\vec{v}}{dt} = \vec{g}$$

(B) Modulus of
$$\frac{d|\vec{v}|}{dt} = g \sin \alpha$$

- (C) Radius of curvature is $\frac{v^2 \cos^2 \alpha}{g \sin \alpha}$
- (D) Magnitude of radial acceleration is g cosα
 - CR0086

7. A car is moving with constant speed on a rough banked road.

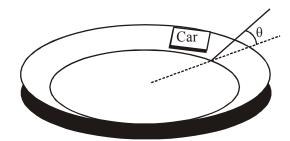
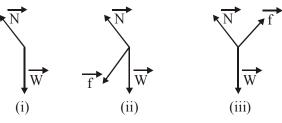


Figure (i), (ii) and (iii) show the free body diagram of car A, B & C respectively:-



- (A) Car A has more speed than car C
- (B) Car A has less speed than car B
- (C) FBD for car A is not possible
- (D) If $\mu > \tan\theta$ the FBD for car C is not possible



- **8.** A heavy particle is tied to the end A of a string of length 1.6 m. Its other end O is fixed. It revolves as a conical pendulum with the string making 60° with the vertical. Then
 - (A) its period of revolution is $\frac{4\pi}{7}$ sec.
 - (B) the tension in the string is double the weight of the particle
 - (C) the velocity of the particle = $2.8\sqrt{3}$ m/s
 - (D) the centripetal acceleration of the particle is $9.8\sqrt{3}$ m/s².

CR0048

9. In the shown figure inside a fixed hollow cylinder with vertical axis a pendulum is moving conically with its axis same as that of the cylinder with uniform angular velocity. Radius of cylinder is 30 cm, length of string is 50 cm and mass of bob is 400 gm. The bob makes contact with the inner frictionless wall of the cylinder while moving:-



- (A) The minimum value of angular velocity of the bob so that it does not leave contact is 5 rad/s
- (B) Tension in the string is 5N for all values of angular velocity
- (C) For angular velocity of 10 rad/s the bob pushes the cylinder with a force of 9N
- (D) For angular velocity of 10 rad/s, tension in the string is 20N

CR0049

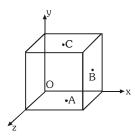
- **10.** A particle of mass 2 kg is projected with an initial speed u = 10 m/sec at an angle $\theta = 30^{\circ}$ with the horizontal
 - (A) The magnitude of work done on the particle during the first half of the total time of flight of the particle is 25J.
 - (B) The magnitude of work done on the particle during the total time of flight of the particle is 0J.
 - (C) The magnitude of average power delivered to the particle during the first half of the flight is 50watt.
 - (D) The radius of curvature of the trajectory of the particle at the highest point of the projectile is 7.5m.

WE0077

E



A particle is shifted from A to B and then from B to C where A, B and C are the midpoints of the 11. corresponding faces of a cube of side 2m. If a force $\vec{F} = (3\hat{i} + 4\hat{j} - 5\hat{k})N$ is continuously acting on the particle, then select correct alternative is



- (A) work done from A to B is 7 J
- (B) work done from B to C is 1 J

(C) work done A to C is 8 J

(D) force \vec{F} , is conservative force

WE0078

12. Which of the following is/are conservative force(s)?

(A)
$$\vec{F} = 2r^3\hat{r}$$

$$(B) \vec{F} = -\frac{5}{r} \hat{r}$$

(C)
$$\vec{F} = \frac{3(xi+y\hat{j})}{(x^2+y^2)^{3/2}}$$

(B)
$$\vec{F} = -\frac{5}{r}\hat{r}$$
 (C) $\vec{F} = \frac{3(xi+y\hat{j})}{(x^2+y^2)^{3/2}}$ (D) $\vec{F} = \frac{3(yi+x\hat{j})}{(x^2+y^2)^{3/2}}$

WE0079

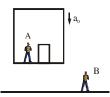
- **13.** If one of the forces acting on a particle is conservative then:
 - (A) Its work is zero when the particle moves exactly once around any closed path.
 - (B) Its work equals the change in the kinetic energy of the particle.
 - (C) It obeys Newton's second law.
 - (D) Its work depends on the end points of the motion, not on the path between.

WE0080

COMPREHENSION TYPE QUESTIONS

Comprehension for Q. 14 to 18

A block of mass m is kept in an elevator which starts moving downward with an acceleration a₀ as shown in figure. The block is observed by two observers A and B for a time interval t₀.



The observer B finds that the work done by gravity is 14.

(A)
$$\frac{1}{2} \text{mg}^2 t_0^2$$

(B)
$$-\frac{1}{2} \text{ mg}^2 t_0^2$$

(C)
$$\frac{1}{2} \text{mga}_0 t_0^2$$

(A)
$$\frac{1}{2} \text{mg}^2 t_0^2$$
 (B) $-\frac{1}{2} \text{mg}^2 t_0^2$ (C) $\frac{1}{2} \text{mga}_0 t_0^2$ (D) $-\frac{1}{2} \text{mga}_0 t_0^2$



- The observer B finds that work done by normal reaction N is **15.**
 - (A) zero
- (B) $-Na_0t_0^2$
- (C) $\frac{Na_0t_0^2}{2}$
- (D) $-\frac{Na_0t_0^2}{2}$

WE0083

- **16.** The observer B finds that work done by pseudo force is
 - (A) zero
- $(B) ma_0^2 t_0$
- $(C) + ma_0^2 t_0$
- $(D) mga_0t_0$

WE0083

- **17.** According to observer B, the net work done on the block is
 - (A) $-\frac{1}{2} \operatorname{ma_0}^2 t_0^2$ (B) $\frac{1}{2} \operatorname{ma_0}^2 t_0^2$ (C) $\frac{1}{2} \operatorname{mga_0} t_0^2$ (D) $-\frac{1}{2} \operatorname{mga_0} t_0^2$

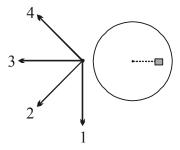
WE0083

- According to the observer A 18.
 - (A) the work done by gravity is zero
- (B) the work done by normal reaction is zero
- (C) the work done by pseudo force is zero
- (D) all the above

WE0083

MATRIX MATCH TYPE QUESTIONS

A block is placed on a horizontal table which can rotate about its axis. The block is placed at a certain **19.** distance from centre as shown in figure. Table rotates such that particle does not slide. Select possible direction of net acceleration of block at the instant shown in figure.

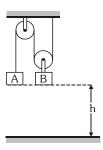


Column-I

Column-II

- When rotation is clockwise with constant ω (A)
- (P) 1
- When rotation is clock wise with decreasing ω
- (Q) 2
- (C) When rotation is clockwise with increasing ω
- 3 (R)
- (D) Just after clockwise rotation begins from rest
- (S)4

20. In the figure shown are two blocks A and B of same mass connected with pulley and string to each other. Initially both of them are at a height of h = 0.5 m from ground. After they are released they move in either direction and one of them strike the ground. For, the interval from releasing to when one of them strike, some physical quantities are in column I and their modulus values in SI units are in column II.



Column II Column II

- (A) Velocity of A immediately before any one of them strike ground. (P) 1
- (B) Velocity of B immediately before any one of them strike ground. (Q) 2
- (C) Ratio of work done by gravity on A to workdone by gravity on B. (R) 3
- (D) Acceleration of block A before any one of them strike ground. (S)
 - (T) 5

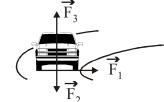
WE0071

21. Column-I shows certain situations and column-2 shows information about forces.

Column - I

Column - II

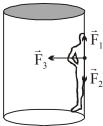
(A)



Situation

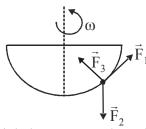
- (P) $\vec{F}_1 + \vec{F}_2 + \vec{F}_3$ is centripetal force.
- Front view of a car rounding a curve with constant speed.

E

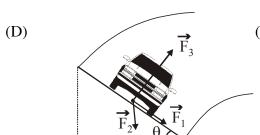


(Q) \vec{F}_1 is static friction.

Passengers in a rotor not sliding relative to rotor wall cylindrical rotor is rotating with constant angular velocity about its symmetry axis.



- (C) Particle kept on rough surface of a bowl, no relative motion of particle in bowl, bowl has constant angular velocity
- (R) \vec{F}_1 can be in direction opposite to that shown in figure.



Car moving on a banked road with constant speed, no sideways skidding

(S)
$$\vec{F}_1 + \vec{F}_2 = \vec{0}$$

(T)
$$\vec{F}_1 + \vec{F}_2 + \vec{F}_3 = \vec{0}$$

CR0068

22. A block of mass m is released from top of a smooth track as shown in the figure. The end part of the track is a circle in vertical plane of radius R. N is normal reaction of the track at any point of the track. Match the entries of column I with entries of column–II.

Column I

Column II

$$(A) \quad h = \frac{5}{2} R$$

(P) Net force on the block at C is mg

(B)
$$h = \frac{9}{2}R$$

$$(Q) N_A - N_B = 6 mg$$

(C) h = R

(D) h = 2R

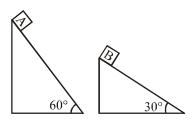
- (S) Block will keep contact with the track is region between A & B.
- (T) $N_C > mg$



EXERCISE (JM)

1. Two fixed frictionless inclined planes making an angle 30° and 60° with the vertical are shown in the figure. Two blocks A and B are placed on the two planes. What is the relative vertical acceleration of A with respect to B?

[AIEEE - 2010]



- (1) 4.9 ms⁻² in vertical direction.
- (2) 4.9 ms⁻² in horizontal direction
- (3) 9.8 ms⁻² in vertical direction
- (4) Zero

NL0110

- 2. At time t = 0s particle starts moving along the x-axis. If its kinetic energy increases uniformly with time 't', the net force acting on it must be proportional to :- [AIEEE 2011]
 - $(1) \sqrt{t}$
- (2) constant
- (3) t

 $(4) \frac{1}{\sqrt{t}}$

WE0088

3. The minimum force required to start pushing a body up a rough (frictional coefficient μ) inclined plane is F_1 while the minimum force needed to prevent it from sliding down is F_2 . If the inclined plane

makes an angle θ from the horizontal such that $tan\theta$ = 2μ then the ratio $\frac{F_1}{F_2}$ is :- [AIEEE - 2011]

(1) 4

(2) 1

(3)2

(4) 3

NL0111

4. Two cars of masses m_1 and m_2 are moving in circles of radii r_1 and r_2 , respectively. Their speeds are such that they make complete circles in the same time t. The ratio of their centripetal acceleration is:

[AIEEE - 2012]

- (1) 1 : 1
- (2) $m_1 r_1 : m_2 r_2$
- $(3) m_1 : m_2$
- $(4) r_1 : r_2$

If two springs S₁ and S₂ of force constants k₁ and k₂, respectively, are stretched by the same force, it is found that more work is done on spring S_1 than on spring S_2 .

Statement-1: If stretched by the same amount, work done on S_1 , will be more than that on S_2 **Statement-2:** $k_1 < k_2$

- (1) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of Statement-1.
- (2) Statement-1 is false, Statement-2 is true
- (3) Statement-1 is true, Statement-2 is false
- (4) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of statement-1.

- A block of mass m is placed on a surface with a vertical cross section given by $y = \frac{x^3}{6}$. If the coefficient **6.** of friction is 0.5, the maximum height above the ground at which the block can be placed without slipping is:-[**JEE-MAIN-2014**]
 - (1) $\frac{1}{3}$ m
- (2) $\frac{1}{2}$ m
- (3) $\frac{1}{6}$ m
- $(4) \frac{2}{3} m$

NL0112

- 7. When a rubber-band is stretched by a distance x, it exerts a restoring force of magnitude $F = ax + bx^2$ where a and b are constants. The work done in stretching the unstretched rubber-band by L is:-[JEE Main - 2014]

 - $(1) \frac{aL^2}{2} + \frac{bL^3}{3} \qquad (2) \frac{1}{2} \left(\frac{aL^2}{2} + \frac{bL^3}{3} \right) \qquad (3) aL^2 + bL^3 \qquad (4) \frac{1}{2} (aL^2 + bL^3)$

WE0090

- 8. A rocket is fired vertically from the earth with an acceleration of 2g, where g is the gravitational acceleration. On an inclined plane inside the rocket, making an angle θ with the horizontal, a point object of mass m is kept. The minimum coefficient of friction μ_{min} between the mass and the inclined surface such that the mass does not move is: [JEE-Main online-2016]
 - (1) $2 \tan \theta$
- (2) $3 \tan \theta$
- (3) $\tan \theta$
- (4) $\tan 2\theta$

NL0114

9. A person trying to lose weight by burning fat lifts a mass of 10 kg upto a height of 1 m 1000 times. Assume that the potential energy lost each time he lowers the mass is dissipated. How much fat will he use up considering the work done only when the weight is lifted up? Fat supplies 3.8×10^7 J of energy per kg which is converted to mechanical energy with a 20% efficiency rate. Take $g = 9.8 \text{ ms}^{-2}$:

[JEE Main - 2016]

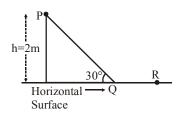
- (1) 12.89×10^{-3} kg (2) 2.45×10^{-3} kg (3) 6.45×10^{-3} kg
- $(4) 9.89 \times 10^{-3} \text{ kg}$

WE0091

E

10. A point particle of mass, moves along the uniformly rough track PQR as shown in the figure. The coefficient of friction, between the particle and the rough track equals μ . The particle is released, from rest, from the point P and it comes to rest at a point R. The energies, lost by the ball, over the parts, PQ and QR, of the track, are equal to each other, and no energy is lost when particle changes direction from PQ to QR. The values of the coefficient of friction μ and the distance x(=QR) are, respectively close to:

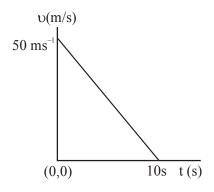
[JEE Main - 2016]



- (1) 0.29 and 6.5 m
- (2) 0.2 and 6.5 m
- (3) 0.2 and 3.5 m
- (4) 0.29 and 3.5 m

WE0092

11. Velocity-time graph for a body of mass 10 kg is shown in figure. Work-done on the body in first two seconds of the motion is :-[JEE Main (Online)-2016]



- (1) 9300 J
- (2) 12000 J
- (3) 12000 J
- (4) 4500 J

CR0087

- **12.** A particle of mass M is moving in a circle of fixed radius R in such a way that its centripetal acceleration at time t is given by n²R t² where n is a constant. The power delivered to the particle by the force acting on it, is: [JEE Main (Online)-2016]
 - (1) $M n R^2 t^2$
- (2) Mn R²t
- (3) $\frac{1}{2}$ M n²R²t²

CR0088

A body of mass $m = 10^{-2}$ kg is moving in a medium and experiences a frictional force $F = -kv^2$. Its intial **13.**

speed is $v_0 = 10 \text{ ms}^{-1}$. If, after 10 s, its energy is $\frac{1}{8} \text{mv}_0^2$, the value of k will be:-[JEE Main - 2017]

- (1) 10^{-4} kg m⁻¹
- (2) 10^{-1} kg m⁻¹ s⁻¹ (3) 10^{-3} kg m⁻¹
- $(4) 10^{-3} \text{ kg s}^{-1}$

WE0093

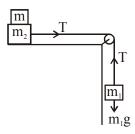
- 14. A time dependent force F = 6t acts on a particle of mass 1 kg. If the particle starts from rest, the work done by the force during the first 1 sec. will be: [JEE Main - 2017]
 - (1) 9 J
- (2) 18 J
- (3) 4.5 J
- (4) 22 J

WE0094

E

Two masses $m_1 = 5$ kg and $m_2 = 10$ kg, connected by an inextensible string over a frictionless pulley, **15.** are moving as shown in the figure. The coefficient of friction of horizontal surface is 0.15. The minimum weight m that should be put on top of m₂ to stop the motion is:-

[JEE Main - 2018]



- (1) 27.3 kg
- (2) 43.3 kg
- (3) 10.3 kg
- (4) 18.3 kg

NL0115

- A particle is moving in a circular path of radius a under the action of an attractive potential 16. $U = -\frac{k}{2r^2}$. Its total energy is :-[JEE Main - 2018]
 - $(1) \ \frac{k}{2a^2}$
- (2) Zero
- $(3) -\frac{3}{2} \frac{k}{a^2} \qquad (4) -\frac{k}{4a^2}$

WE0095

17. A particle is moving with a uniform speed in a circular orbit of radius R in a central force inversely proportional to the nth power of R. If the period of rotation of the particle is T, then,

[JEE Main - 2018]

- (1) $T \propto R^{\frac{n}{2}+1}$
- (2) $T \propto R^{(n+1)/2}$ (3) $T \propto R^{n/2}$
- (4) T \propto R^{3/2} for any n

CR0073

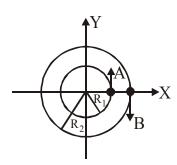
- 18. A body is projected at t = 0 with a velocity 10 ms^{-1} at an angle of 60° with the horizontal. The radius of curvature of its trajectory at t = 1s is R. Neglecting air resistance and taking acceleration due to gravity $g = 10 \text{ ms}^{-2}$, the value of R is: [JEE Main - 2019]
 - (1) 2.5 m
- (2) 10.3 m
- (3) 2.8 m
- (4) 5.1 m

CR0089

- **19.** A particle is moving along a circular path with a constant speed of 10 ms⁻¹. What is the magnitude of the change is velocity of the particle, when it moves through an angle of 60° around the centre of the circle? [JEE Main - 2019]
 - (1) zero
- (2) 10 m/s
- (3) $10\sqrt{3}$ m/s
- (4) $10\sqrt{2}$ m/s

Two particles A, B are moving on two concentric circles of radii R₁ and R₂ with equal angular speed 20. ω . At t = 0, their positions and direction of motion are shown in the figure :

[JEE Main - 2019]



The relative velocity $\vec{v}_{A} - \vec{v}_{B}$ at $t = \frac{\pi}{2\omega}$ is given by :

- (1) $-\omega(R_1 + R_2)\hat{i}$ (2) $\omega(R_1 + R_2)\hat{i}$ (3) $\omega(R_1 R_2)\hat{i}$ (4) $\omega(R_2 R_1)\hat{i}$

CR0091

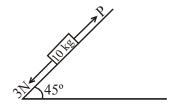
21. A mass of 10 kg is suspended vertically by a rope from the roof. When a horizontal force is applied on the rope at some point, the rope deviated at an angle of 45° at the roof point. If the suspended mass is at equilibrium, the magnitude of the force applied is $(g = 10 \text{ ms}^{-2})$

[JEE Main - 2019]

- (1) 200 N
- (2) 100 N
- (3) 140 N
- (4) 70 N

WE0122

22. A block of mass 10 kg is kept on a rough inclined plane as shown in the figure. A force of 3 N is applied on the block. The coefficient of static friction between the plane and the block is 0.6. What should be the minimum value of force P, such that the block doesnot move downward? [JEE Main - 2019]

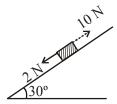


 $(take g = 10 ms^{-2})$

- (1) 32 N
- (2) 25 N
- (3) 23 N
- (4) 18 N

- A particle of mass m is moving in a straight line with momentum p. Starting at time t = 0, a force F = kt acts in the same direction on the moving particle during time interval T so that its momentum changes from p to 3p. Here k is a constant. The value of T is :-[JEE Main - 2019]
- (2) $\sqrt{\frac{2p}{k}}$
- $(3) \sqrt{\frac{2k}{p}}$
- $(4) 2\sqrt{\frac{k}{n}}$

A block kept on a rough inclined plane, as shown in the figure, remains at rest upto a maximum force 2 N down the inclined plane. The maximum external force up the inclined plane that does not move the block is 10 N. The coefficient of static friction between the block and the plane is: [Take g = 10 m/s^2 [JEE Main - 2019]



 $(1) \frac{2}{3}$

(2) $\frac{\sqrt{3}}{2}$

(3) $\frac{\sqrt{3}}{4}$

 $(4) \frac{1}{2}$

WE0125

A force acts on a 2 kg object so that its position is given as a function of time as $x = 3t^2 + 5$. What is 25. the work done by this force in first 5 seconds? [JEE Main - 2019]

(1)850J

(2) 900 J

(3) 950 J

(4)875J

WE0126

A particle which is experiencing a force, given by $\vec{F} = 3\vec{i} - 12\vec{j}$, undergoes a displacement of $\vec{d} = 4\vec{i}$. 26. If the particle had a kinetic energy of 3 J at the beginning of the displacement, what is its kinetic energy at the end of the displacement? [JEE Main - 2019]

(1) 15 J

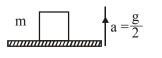
(2) 10 J

(3) 12 J

(4) 9 J

WE0127

27. A block of mass m is kept on a platform which starts from rest with constant acceleration g/2 upward, as shown in fig. Work done by normal reaction on block in time t is:[JEE Main - 2019]



(1)0

(2) $\frac{3mg^2t^2}{8}$ (3) $-\frac{mg^2t^2}{8}$ (4) $\frac{mg^2t^2}{8}$

WE0128

28. A particle of mass m is fixed to one end of a light spring having force constant k and unstretched length ℓ . The other end is fixed. The system is given an angular speed ω about the fixed end of the spring such that it rotates in a circle in gravity free space. Then the stretch in the spring is:

[**JEE-MAIN-2020**]

 $(1) \frac{m\ell\omega^2}{k+m\omega^2}$

 $(2) \frac{m\ell\omega^2}{k - m\omega^2} \qquad (3) \frac{m\ell\omega^2}{k - \omega m}$

 $(4) \frac{m\ell\omega^2}{k+m\omega}$

CR0092

29. A spring mass system (mass m, spring constant k and natural length l) rest in equilibrium on a horizontal disc. The free end of the spring is fixed at the centre of the disc. If the disc together with spring mass system, rotates about it's axis with an angular velocity ω , $(k \gg m\omega^2)$ the relative change in the length of the spring is best given by the option: [**JEE-MAIN-2020**]

 $(1) \frac{2m\omega^2}{k}$

 $(2) \frac{m\omega^2}{3k}$

 $(3) \sqrt{\frac{2}{3}} \left(\frac{m\omega^2}{k} \right) \qquad (4) \frac{m\omega^2}{k}$

30. A mass of 10 kg is suspended by a rope of length 4 m, from the ceiling. A force F is applied horizontally at the mid-point of the rope such that the top half of the rope makes an angle of 45° with the vertical. Then F equals: (Take $g = 10 \text{ ms}^{-2}$ and the rope to be massless)

[**JEE-MAIN-2020**]

(1) 100 N

(2) 90 N

(3) 75 N

(4) 70 N

WE0129

A 60 HP electric motor lifts an elevator having a maximum total load capacity of 2000 kg. If the 31. frictional force on the elevator is 4000 N, the speed of the elevator at full load is close to: $(1 \text{ HP} = 746 \text{ W}, \text{ g} = 10 \text{ ms}^{-2})$ [JEE-MAIN-2020]

 $(1) 1.7 \text{ ms}^{-1}$

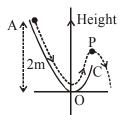
 $(2) 2.0 \text{ ms}^{-1}$

 $(3) 1.9 \text{ ms}^{-1}$

 $(4) 1.5 \text{ ms}^{-1}$

WE0130

A particle (m = 1 kg) slides down a frictionless track (AOC) starting from rest at a point **32.** A (height 2 m). After reaching C, the particle continues to move freely in air as a projectile. When it reaching its highest point P (height 1 m), the kinetic energy of the particle (in J) is: (Figure drawn is schematic and not to scale; take g=10 ms⁻²) [**JEE-MAIN-2020**]



WE0131

33. An elevator in a building can carry a maximum of 10 persons, with the average mass of each person being 68 kg. The mass of the elevator itself is 920 kg and it moves with a constant speed 3 m/s. The frictional force opposing the motion is 6000 N. If the elevator is moving up with its full capacity, the power delivered by the motor to the elevator $(g = 10 \text{ m/s}^2)$ must be at least: [**JEE-MAIN-2020**]

(1) 56300 W

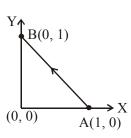
(2) 48000 W

(3) 66000 W

(4) 62360 W

WE0132

Consider a force $\vec{F} = -x\hat{i} + y\hat{j}$. The work done by this force in moving a particle from point A(1, 0) 34. [**JEE-MAIN-2020**] to B(0, 1) along the line segment is :(all quantities are in SI units)



 $(1) \frac{3}{2}$

(2) 1

(3)2

 $(4) \frac{1}{2}$

EXERCISE (JA)

Two particles of mass m each are tied at the ends of a light string of length 2a. The whole system is 1. kept on a frictionless horizontal surface with the string held tight so that each mass is at a distance 'a' from the center P (as shown in the figure). Now, the mid-point of the string is pulled vertically upwards with a small but constant force F. As a result, the particles move towards each other on the surfaces. The magnitude of acceleration, when the separation between them becomes 2x, is

[IIT-JEE 2007]



(A)
$$\frac{F}{2m} \frac{a}{\sqrt{a^2 - x^2}}$$
 (B) $\frac{F}{2m} \frac{x}{\sqrt{a^2 - x^2}}$ (C) $\frac{F}{2m} \frac{x}{a}$

(B)
$$\frac{F}{2m} \frac{x}{\sqrt{a^2 - x^2}}$$

(C)
$$\frac{F}{2m}\frac{x}{a}$$

(D)
$$\frac{F}{2m} \frac{\sqrt{a^2 - x^2}}{x}$$

WE0134

2. **Statement-1**

A block of mass m starts moving on a rough horizontal surface with a velocity v. It stops due to friction between the block and the surface after moving through a certain distance. The surface is now tilted to an angle of 30° with the horizontal and the same block is made to go up on the surface with the same initial velocity v. The decrease in the mechanical energy in the second situation is smaller than that in the first situation. [IIT-JEE 2007]

because

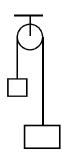
Statement-2

The coefficient of friction between the block and the surface decreases with the increase in the angle of inclination.

- (A) Statement-1 is True, Statement-2 is True; Statement-2 is a correct explanation for Statement-1
- (B) Statement-1 is True, Statement-2 is True; Statement-2 is NOT a correct explanation for Statement-1
- (C) Statement-1 is True, Statement-2 is False
- (D) Statement-1 is False, Statement-2 is True

3. A light inextensible string that goes over a smooth fixed pulley as shown in the figure connects two blocks of masses 0.36 kg and 0.72 kg. Taking $g = 10 \text{ m/s}^2$, find the work done (in joules) by the string on the block of mass 0.36 kg during the first second after the system is released from rest.

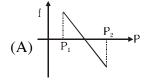
[IIT-JEE-2009]

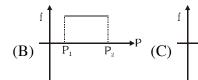


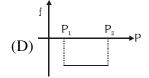
WE0096

4. A block of mass m is on an inclined plane of angle θ . The coefficient of friction between the block and the plane is μ and $\tan\theta > \mu$. The block is held stationary by applying a force P parallel to the plane. The direction of force pointing up the plane is taken to be positive. As P is varied from $P_1 = mg(\sin\theta - \mu\cos\theta)$ to $P_2 = mg(\sin\theta + \mu\cos\theta)$, the frictional force f versus P graph will look like [IIT-JEE-2010]



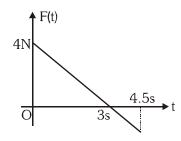






NL0117

5. A block of mass 2 kg is free to move along the x-axis. It is at rest and from t=0 onwards it is subjected to a time-dependent force F(t) in the x-direction. The force F(t) varies with t as shown in the figure. The kinetic energy of the block after 4.5 second is [IIT-JEE-2010]



(A) 4.50 J

(B) 7.50 J

(C) 5.06 J

(D) 14.06 J

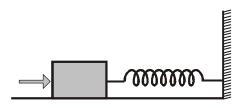
CM0106

A block is moving on an inclined plane making an angle 45° with the horizontal and the coefficient of friction is μ . The force required to just push it up the inclined plane is 3 times the force required to just prevent it from sliding down. If we define $N = 10\mu$, then N is [IIT-JEE-2011]

NL0118

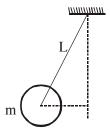
7. A block of mass 0.18 kg is attached to a spring of force-constant 2 N/m. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is un-stretched. An impulse is given to the block as shown in the figure. The block slides a distance of 0.06 m and comes to rest for the first time. The initial velocity of the block in m/s is V = N/10. Then N is

[IIT-JEE-2011]



WE0097

8. A ball of mass (m) 0.5 kg is attached to the end of a string having length (L) 0.5 m. The ball is rotated on a horizontal circular path about vertical axis. The maximum tension that the string can bear is 324 N. The maximum possible value of angular velocity of ball (in radian/s) is [IIT-JEE-2011]



(A)9

(B) 18

(C) 27

(D)36

CR0074

The work done on a particle of mass m by a force, $K\left[\frac{x}{\left(x^2+y^2\right)^{3/2}}\hat{i} + \frac{y}{\left(x^2+y^2\right)^{3/2}}\hat{j}\right]$ (K being a constant 9.

of appropriate dimensions), when the particle is taken from the point (a, 0) to the point (0, a) along a circular path of radius a about the origin in the x-y plane is :-[IIT-JEE-2013]

(A) $\frac{2K\pi}{a}$

(C) $\frac{K\pi}{2a}$

(D)0

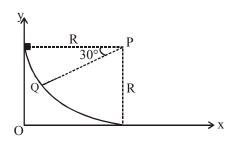
WE0098

10. A particle of mass 0.2 kg is moving in one dimension under a force that delivers a constant power 0.5W to the particle. If the initial speed (in ms⁻¹) of the particle is zero, the speed (in ms⁻¹) after 5 s is. [IIT-JEE-2013]

ALLEN

Paragraph for Questions 11 and 12

A small block of mass 1 kg is released from rest at the top of a rough track. The track is a circular arc of radius 40 m. The block slides along the track without toppling and a frictional force acts on it in the direction opposite to the instantaneous velocity. The work done in overcoming the friction up to the point Q, as shown in the figure below, is 150 J. (Take the acceleration due to gravity, $g = 10 \text{ m s}^{-2}$) [IIT-JEE-2013]



- 11. The magnitude of the normal reaction that acts on the block at the point Q is
 - (A) 7.5 N
- (B) 8.6 N
- (C) 11.5 N
- (D) 22.5 N

WE0100

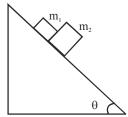
- **12.** The speed of the block when it reaches the point Q is
 - (A) 5 ms⁻¹
- (B) 10 ms⁻¹
- (C) $10\sqrt{3}$ ms⁻¹
- (D) 20 ms⁻¹

WE0100

13. A block of mass $m_1 = 1$ kg another mass $m_2 = 2$ kg, are placed together (see figure) on an inclined plane with angle of inclination θ . Various values of θ are given in List I. The coefficient of friction between the block m_1 and the plane is always zero. The coefficient of static and dynamic friction between the block m_2 and the plane are equal to $\mu = 0.3$. In List II expressions for the friction on block m_2 are given. Match the correct expression of the friction in List II with the angles given in List I, and choose the correct option. The acceleration due to gravity is denoted by g.

[**useful information :** $\tan (5.5^{\circ}) \approx 0.1$; $\tan (11.5^{\circ}) \approx 0.2$; $\tan (16.5^{\circ}) \approx 0.3$]

[IIT-JEE-2014]



List-I

- (P) $\theta = 5^{\circ}$
- (Q) $\theta = 10^{\circ}$
- (R) $\theta = 15^{\circ}$
- (S) $\theta = 20^{\circ}$

Code:

- (A) P-1, Q-1, R-1, S-3
- (C) P-2,. Q-2, R-2, S-4

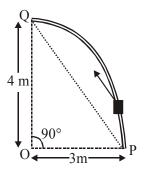
List-II

- (1) $m_2 g \sin \theta$
- (2) $(m_1 + m_2)g \sin \theta$
- (3) μ m, $g\cos\theta$
- (4) $\mu(m_1 + m_2)g \cos \theta$
- (B) P-2. Q-2, R-2, S-3
- (D) P-2, Q-2, R-3, S-3

NL0119

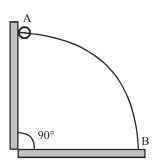
14. Consider an elliptically shaped rail PQ in the vertical plane with OP = 3 m and OQ = 4m. A block of mass 1 kg is pulled along the rail from P to Q with a force of 18 N, which is always parallel to line PQ (see the figure given). Assuming no frictional losses, the kinetic energy of the block when it reaches Q is (n × 10) Joules. The value of n is (take acceleration due to gravity = 10 ms⁻²)

[JEE-Advance-2014]



WE0101

15. A wire, which passes through the hole in a small bead, is bent in the form of quarter of a circle. The wire is fixed vertically on ground as shown in the figure. The bead is released from near the top of the wire and it slides along the wire without friction. As the bead moves from A to B, the force it applies on the wire is:-



- (A) Always radially outwards
- (B) Always radially inwards
- (C) Radially outwards initially and radially inwards later.
- (D) Radially inwards initially and radially outwards later.

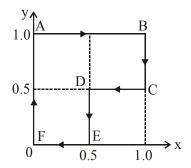
16. A particle of mass m is initially at rest at the origin. It is subjected to a force and starts moving along the x-axis. Its kinetic energy K changes with time as $dK/dt = \gamma t$, where γ is a positive constant of appropriate dimensions. Which of the following statements is (are) true ?

[JEE-Advance-2018]

- (A) The force applied on the particle is constant
- (B) The speed of the particle is proportional to time
- (C) The distance of the particle from the origin increses linerarly with time
- (D) The force is conservative

WE0103

17. A particle is moved along a path AB-BC-CD-DE-EF-FA, as shown in figure, in presence of a force $\vec{F} = (\alpha y \hat{i} + 2\alpha x \hat{j}) N$, where x and y are in meter and $\alpha = -1 \text{ N/m}^{-1}$. The work done on the particle by this force \vec{F} will be _____ Joule. [JEE-Advance-2019]



ANSWER KEY

01 NLM & FRICTION

EXERCISE (S)

1. Ans. (i)
$$\frac{m_1}{m_2} = \frac{1}{3}$$
 (ii) $a = 3/4$ m/s² **2. Ans.** $\frac{4}{3}W$ **3. Ans.** $\frac{g}{10}m/s^2$ **4. Ans.** $v_0/\cos\theta$

5. Ans.
$$\frac{a_0}{2}$$
 6. Ans. 200 N, 10 m/s² **7.** Ans. 2 sec **8.** Ans. 0.5g, g **9.** Ans. 3a_B /4

12. Ans. (i)
$$a_A = \frac{3g \downarrow}{2} = a_B$$
; $a_C = 0$; $T = mg/2$; (ii) $a_A = 2g \uparrow$, $a_B = 2g \downarrow$, $a_C = 0$, $T = 0$;

(iii)
$$a_A = a_B = g/2\uparrow$$
, $a_c = g\downarrow$, $T = \frac{3mg}{2}$;

13. Ans.
$$\frac{25}{24}$$
 14. Ans. $a = g \cot \theta$ **15.** Ans. $(100\hat{i} - 200\hat{j})$ N

4. Ans. (A.B.D) 5. Ans. (A.C)

20. Ans.
$$10,\hat{i}$$
 21. Ans. 30 N **22.** Ans. $4/3 \text{ s}$ **23.** Ans. $\mu_{\min} = \frac{m \sin \theta \cos \theta}{m \cos^2 \theta + M}$

24. Ans. 3/4

1. Ans. (A.D)

EXERCISE (0-1)

1. Alls. (D)	2. Alls. (D)	3. Alls. (D)	4. Alls. (C)	3. Alls. (D)	0. Ans. (C)
7. Ans. (C)	8. Ans. (C)	9. Ans. (D)	10. Ans. (C)	11. Ans. (B)	12. Ans. (A)
13. Ans. (C)	14. Ans. (A)	15. Ans. (C)	16. Ans. (B)	17. Ans. (A)	18. Ans. (B)
19. Ans. (D)	20. Ans. (A)	21. Ans. (D)	22. Ans. (B)	23. Ans. (D)	24. Ans. (D)
25. Ans. (B)	26. Ans. (B)	27. Ans. (C)	28. Ans. (A)	29. Ans. (B)	30. Ans. (A)
31. Ans. (B)	32. Ans. (D)	33. Ans. (C)	34. Ans. (B)	35. Ans. (A)	36. Ans. (B)
37. Ans. (C)	38. Ans. (A)				

2. Ans. (A.B)

EXERCISE (O-2)

MULTIPLE CORRECT TYPE QUESTIONS

- () /	- () /	() /	- () /	() - /	
6. Ans. (B,C,D)	7. Ans. (D)	8. Ans. (D)	9. Ans. (A)	10. Ans. (D)	11. Ans. (B)
12. Ans. (C)	13. Ans. (B)	14. Ans. (B)	15. Ans. (A)	16. Ans. (B)	17. Ans. (A)

MATRIX MATCH TYPE QUESTION

3. Ans. (A.B)

110

02 CIRCULAR MOTION, WORK, POWER & ENERGY

EXERCISE (S)

- **1.** Ans. (a) 4 cm/s² (b) 2 cm/s² (c) $\sqrt{20}$ cm/s²
- 2. Ans. (i) 75m/s^2 , (ii) 125m/s^2

- 3. Ans. ω
- 4. Ans. 024
- 5. Ans. $\frac{v}{2d}$ 6. Ans. $a_t = \frac{2g}{\sqrt{13}}$, $a_n = \frac{3g}{\sqrt{13}}$

- 7. Ans. 5m
- 8. Ans. (i) $N_F = mg$ (ii) $N_W = \frac{mv^2}{r}$.
- 9. Ans. Yes 10. Ans. 5

- 11. Ans. (a) 28.3 m/s (b) $v_{max} = 38.4 \text{ ms}^{-1}$.
- 12. Ans. 2
- 13. Ans. \sqrt{Rg}

- 14. Ans. (a) +ve (b) -ve (c) -ve (d) +ve (e) -ve
- 15. Ans. 1
- 16. Ans. zero

- 17. Ans. 216 J 18. Ans. 8
- 19. Ans. 4
- 20. Ans. g
- 21. Ans. 4

- 22. Ans. $m_1 = m/2$
- 23. Ans. $\frac{2m_1^2g^2}{k}$ 24. Ans. -3k
- 25. Ans. (a) -30 kW, 19.5 kW

- 26. Ans. 4
- 27. Ans. (i) x = 2, (ii) 16 J
- 28. Ans. 1

EXERCISE (O-1)

- 1. Ans. (D)
- 2. Ans. (C)
- 3. Ans. (D)
- 4. Ans. (D)
- **5.** Ans. (D)
- 6. Ans. (A)

- 7. Ans. (C)
- 8. Ans. (C)
- 9. Ans.(C)
- 10. Ans. (C)
- 11. Ans. (D)
- 12. Ans. (A)

- 13. Ans. (A) 19. Ans. (B)
- 14. Ans. (A)
- 15. Ans. (A)
- 16. Ans. (B) 22. Ans. (B)
- 17. Ans. (C) 23. Ans. (B)
- 18. Ans. (C) 24. Ans. (B)

- 25. Ans. (C)
- 20. Ans. (D) 26. Ans. (C)
- 21. Ans. (D) 27. Ans. (B)
- 28. Ans. (C)

EXERCISE (O-2)

- 1. Ans. (A, C, D)
- 2. Ans. (B,C)
- 3. Ans. (A, B, C)

11. Ans. (A,B,C,D)

4. Ans. (B, C, D)

- 5. Ans. (A, D)
- 6. Ans. (A,B,D)
- 7. Ans. (A, B)
- **8.** Ans. (A, B, C, D)

(A,B,C)

- 9. Ans. (A, B, C) 13. Ans. (A,C,D)
- 10. Ans. (A,B,C,D)

- 14. Ans. (C)
- 15. Ans. (D)
- 16. Ans. (A)

12. Ans.

- 17. Ans. (B)
- 18. Ans. (D)
- 19. Ans. (A)-R; (B)-S; (C)-Q; (D)-P
- 20. Ans. (A) \rightarrow (Q); (B) \rightarrow (P); (C) \rightarrow (Q); (D) \rightarrow (S) 21. Ans. (A) P,Q (B) P,Q,S (C) P,Q,R (D) P,Q,R 22. Ans. (A) \rightarrow (Q,S,T); (B) \rightarrow (Q,S,T); (C) \rightarrow (P,R); (D) \rightarrow (R,T)

EXERCISE (JM)

- 1. Ans. (1)
- 2. Ans. (4)
- 3. Ans. (4)
- 4. Ans. (4)
- 5. Ans. (2)
- 6. Ans. (3)

- 7. Ans. (1)
- 8. Ans. (3)
- 9. Ans. (1)
- 11. Ans. (4)
- 12. Ans. (4)

- 13. Ans. (1)
- 14. Ans. (3)
- 15. Ans. (1)
- **10.** Ans. (4) 16. Ans. (2)
- 17. Ans. (2)
- 18. Ans. (3)

- 19. Ans. (2)
- 20. Ans. (4)
- 21. Ans. (2)
- 22. Ans. (1)
- 23. Ans. (1)
- 24. Ans. (2)

- 25. Ans. (2)
- 26. Ans. (1)
- 27. Ans. (2)
- 28. Ans. (2)
- 29. Ans. (4)
- 30. Ans. (1)

- 31. Ans. (3)
- 32. Ans. 10
- 33. Ans. (3)
- 34. Ans. (2)
- **EXERCISE (JA)**
 - 4. Ans. (A)
- 5. Ans. (C)
- 6. Ans. 5

- 13. Ans. (D)
- 2. Ans. (C) 8. Ans. (D)
- 3. Ans. 8 9. Ans. (D)
- 10. Ans. 5
- 11. Ans. (A)
- 12. Ans. (B)

7. Ans. 4

1. Ans. (B)

- 14. Ans. 5
- 15. Ans. (D)
- 16. Ans.(A,B,D) 17. Ans. (0.75)

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