MAGNETIC EFFECT OF CURRENT

KEY CONCEPT

The branch of physics which deals with the magnetism due to electric current or moving charge (i.e. electric current is equivalent to the charges or electrons in motion) is called electromagnetism.

Biot-Savart Law

Currents which arise due to the motion of charges are the source of magnetic fields. When charges move in a conducting wire and produce a current *I*, the magnetic field at any point *P* due to the current can be calculated by adding up the magnetic field contributions, $d\vec{B}$, from small segments of the wire $d\vec{s}$ (Figure).

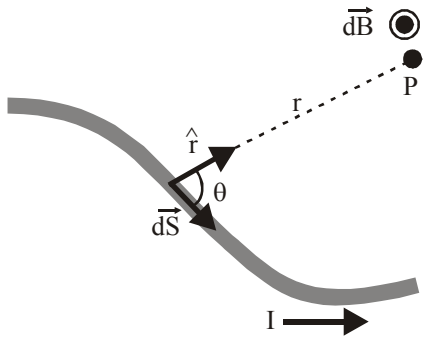


Figure : Magnetic field $d\vec{B}$ at point *P* due to a current-carrying element $Id\vec{s}$

These segments can be thought of as a vector quantity having a magnitude of the length of the segment and pointing in the direction of the current flow. The infinitesimal current source can then be written as $Id\vec{s}$.

Let *r* denote as the distance form the current source to the field point *P* and $\hat{r}$ the corresponding unit vector. The Biot-Savart law gives an expression for the magnetic field contribution, $d\vec{B}$, from the current source, $Id\vec{s}$,

$$d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{s}\times\hat{r}}{r^2}$$

where μ_0 is a constant called the permeability of free space:

$\mu_0 = 4\pi \times 10^{-7}$ T.m/A here Tesla (T) is SI unit of $\vec{B}$

Adding up these contributions to find the magnetic field at the point *P* requires integrating over the current source,

$$\vec{B} = \int d\vec{B} = \frac{\mu_0 I}{4\pi}\int\frac{d\vec{s}\times\hat{r}}{r^2}$$

- According to $d\vec{B} = \frac{\mu_0}{4\pi}\frac{Id\vec{\ell}\times\vec{r}}{r^3}$, direction of magnetic field vector $d\vec{B}$ is always perpendicular to the plane of vectors $\left(Id\vec{\ell}\right)$ and $(\vec{r})$, where plane of $\left(Id\vec{\ell}\right)$ and $(\vec{r})$ is the plane of wire.
- Magnetic field on the axis of current carrying conductor is always zero ($\theta = 0°$ or $\theta = 180°$)

RIGHT HAND THUMB RULE

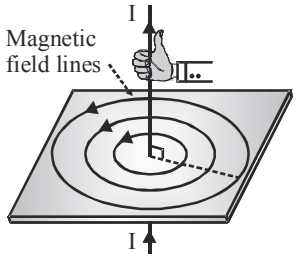
This rule gives the pattern of magnetic field lines due to current carrying wire.

(i) Straight current

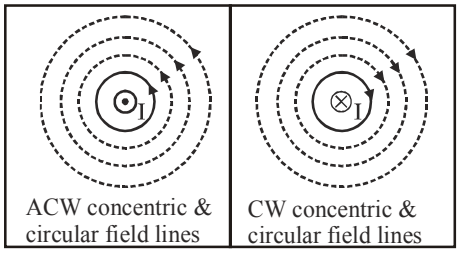
Thumb → In the direction of current

Curling fingers → Gives field line pattern

Case I : wire in the plane of the paper



Case II : Wire is ⊥ to the plane of the paper.

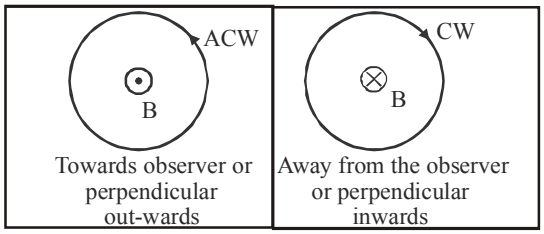


(ii) Circular current

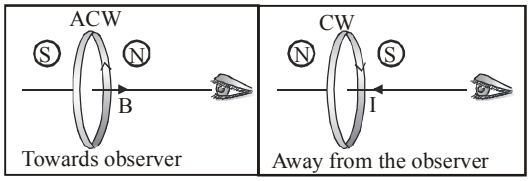
Curling fingers → In the direction of current,

Thumb → Gives field line pattern

Case I : wire in the plane of the paper



Case II : Wire is ⊥ to the plane of the paper



APPLICATION OF BIOT-SAVART LAW :

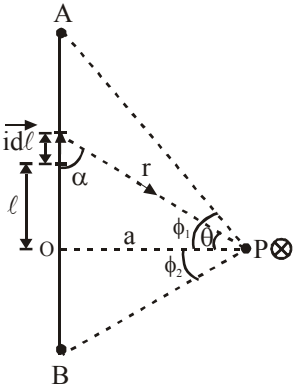
- **Magnetic field surrounding a thin straight current carrying conductor**

AB is a straight conductor carrying current i from B to A.

At a point P, whose perpendicular distance from AB is OP = a, the direction of field is perpendicular to the plane of paper, inwards (represented by a cross)

$\ell = a\tan\theta \Rightarrow dl = a\sec^2\theta d\theta$...(i)

$\alpha = 90° - \theta$ & $r = a\sec\theta$



- **By Biot-Savart's law**

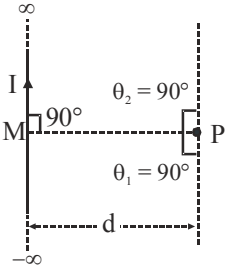
$$\overrightarrow{dB} = \frac{\mu_0}{4\pi}\frac{id\ell\sin\alpha}{r^2}\otimes \text{ (due to a current element } id\ell \text{ at point P)}$$

$$\Rightarrow B = \int dB = \int \frac{\mu_0}{4\pi}\frac{id\ell\sin\alpha}{r^2} \text{ (due to wire AB) } \therefore B = \frac{\mu_0 i}{4\pi}\int \cos\theta d\theta$$

Taking limits of integration as $-\phi_2$ to ϕ_1

$$B = \frac{\mu_0 i}{4\pi a}\int_{-\phi_2}^{\phi_1}\cos\theta d\theta = \frac{\mu_0 i}{4\pi a}\left[\sin\theta\right]_{-\phi_2}^{\phi_1} = \frac{\mu_0 i}{4\pi a}\left[\sin\phi_1 + \sin\phi_2\right] \text{ (inwards)}$$

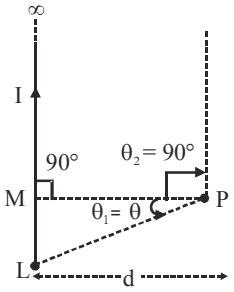
Ex. Magnetic field due to infinite length wire at point 'P'



Sol. $B_P = \frac{\mu_0 I}{4\pi d}[\sin 90° + \sin 90°]$

$$B_P = \frac{\mu_0 I}{2\pi d}$$

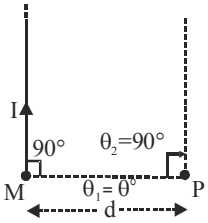
Ex. Magnetic field due to semi infinite length wire at point 'P'



Sol. $B_P = \frac{\mu_0 I}{4\pi d}[\sin\theta + \sin 90°]$

$B_P = \frac{\mu_0 I}{4\pi d}[\sin\theta + 1]$

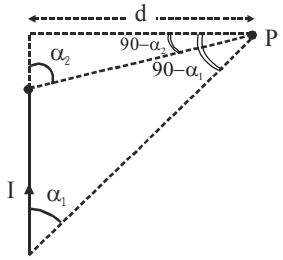
Ex. Magnetic field due to special semi infinite length wire at point 'P'



Sol. $B_P = \frac{\mu_0 I}{4\pi d}[\sin 0° + \sin 90°]$

$B_P = \frac{\mu_0 I}{4\pi d}$

Ex. If point 'P' lies out side the line of wire then magnetic field at point 'P' :



Sol. $B_P = \frac{\mu_0 I}{4\pi d}$

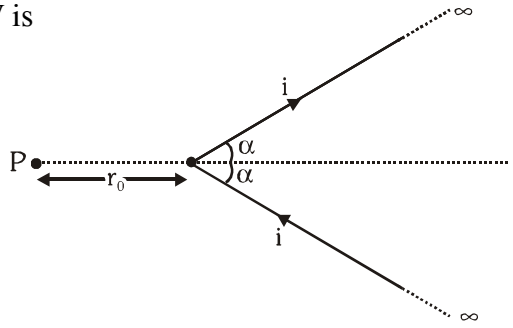
$\left[\sin(90-\alpha_1) - \sin(90-\alpha_2)\right]$

$= \frac{\mu_0 I}{4\pi d}(\cos\alpha_1 - \cos\alpha_2)$

Ex. A current carrying wire in the form of 'V' alphabet is kept as shown in the figure. Magnetic field intensity at point P which lies on the angular bisector of V is

(A) $\frac{\mu_0 i}{4\pi r_0}[1-\cos\alpha]$ (B) $\frac{\mu_0 i}{2\pi r_0}[1-\cos\alpha]$

(C) $\frac{\mu_0 i}{4\pi r_0}\frac{[1-\cos\alpha]}{\sin\alpha}$ (D) $\frac{\mu_0 i}{2\pi r_0}\frac{[1-\cos\alpha]}{\sin\alpha}$



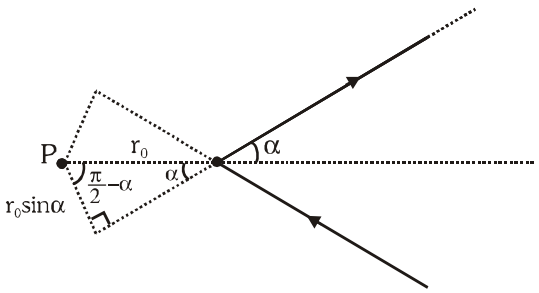
Ans. (D)

Sol. By using formula for stragith wire i.e.

$B = \frac{\mu_0 I}{4\pi d}(\sin\theta_1 + \sin\theta_2)$

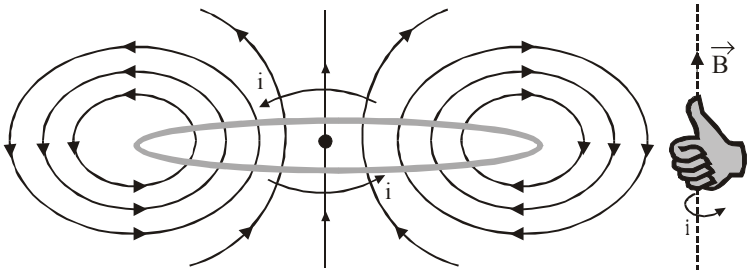
$B = 2\left[\frac{\mu_0 i}{4\pi r_0 \sin\alpha}(\sin 90° - \sin(90°-\alpha))\right]$

$= \frac{\mu_0 i}{2\pi r_0 \sin\alpha}[1-\cos\alpha]$



- **Magnetic field due to a loop of current**

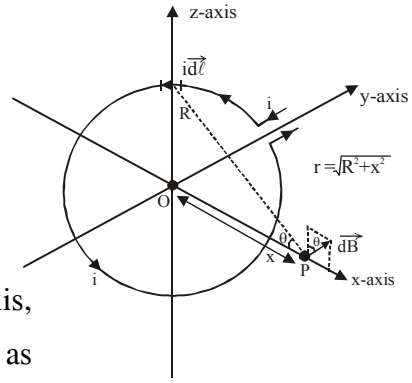
Magnetic field lines due to a loop of wire are shown in the figure



The direction of magnetic field on the axis of current loop can be determined by right hand thumb rule. If fingers of right hand are curled in the direction of current, the stretched thumb is in the direction of magnetic field.

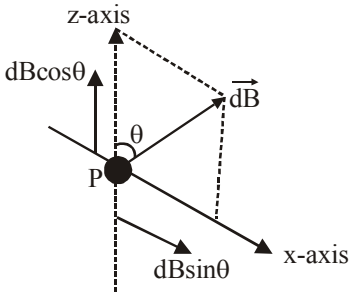
- **Calculation of magnetic field**

Consider a current loop placed in y-z plane carrying current i in anticlockwise sense as seen from positive x-axis. Due to a small current element $id\vec{\ell}$ shown in the figure, the magnetic field at P is given by $dB = \frac{\mu_0}{4\pi}\frac{id\ell \sin 90^0}{r^2}$.



The angle between $id\vec{\ell}$ and $\vec{r}$ is 90^0 because $id\vec{\ell}$ is along y-axis, while $\vec{r}$ lies in x-z plane. The direction of $\overrightarrow{dB}$ is perpendicular to $\vec{r}$ as shown. The vector $\overrightarrow{dB}$ can be resolved into two components, dBcosθ along z-axis and dB sinθ along x-axis.

For any two diametrically opposite current elements, the components along x-axis add up, while the other two components cancel out. Therefore, the field at P is due to x-component of field only. Hence, we have



$$\therefore\ B = \frac{\mu_0}{4\pi}\frac{i\times 2\pi R^2}{\left(R^2+x^2\right)^{3/2}} \quad \left(\because r = \sqrt{R^2+x^2}\right)$$

(a) At the centre, x = 0, $B_{centre} = \frac{\mu_0 i}{2R}$

(b) At points very close to centre, $x << R \Rightarrow B = \frac{\mu_0 i}{2}\left(1+\frac{x^2}{R^2}\right)^{-3/2} = \frac{\mu_0 i}{2}\left(1-\frac{3x^2}{2R^2}\right)$

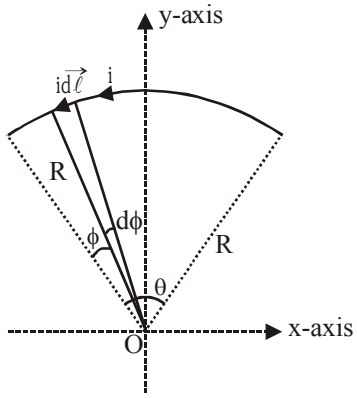
(c) At points far off from the centre, $x >> R \Rightarrow B = \frac{\mu_0 \ell}{4\pi}\frac{2\pi R^2}{x^3}$

(d) The result in point (c) is also expressed as $B = \frac{\mu_0}{4\pi}\frac{2M}{x^3}$

where $M = \ell \times \pi R^2$, is called magnetic dipole moment.

Ex. Find the magnetic field at the centre of a current carrying conductor bent in the form of an arc subtending angle θ at its centre. Radius of the arc is R.

Sol. Let the arc lie in x-y plane with its centre at the origin.



Consider a small current element $id\vec{\ell}$ as shown.

The field due to this element at the centre is

$$dB=\frac{\mu_0}{4\pi}\frac{id\ell\sin 90^0}{R^2}\quad\left(\because id\vec{\ell}\text{ and R are perpendicular}\right)$$

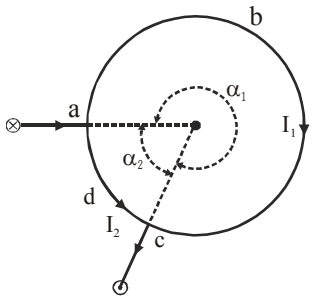
$$\text{Now } d\ell = Rd\phi \ \therefore dB=\frac{\mu_0}{4\pi}\frac{iRd\phi}{R^2}\Rightarrow dB=\frac{\mu_0}{4\pi}\frac{i}{R}d\phi$$

The direction of field is outward perpendicular to plane of paper

$$\text{Total magnetic field } B=\int dB\ \therefore B=\frac{\mu_0 i}{4\pi R}\int_0^{\theta}d\phi=\frac{\mu_0 i}{4\pi R}[\phi]_0^{\theta}\ \therefore B=\frac{\mu_0 i}{4\pi R}\theta$$

Ex. Find the magnetic field at the centre of a current carrying conductor bent in the form of an arc subtending angle α_1 and α_2 at the centre.

Sol. Magnetic field at the centre of arc abc and adc wire of circuit loop



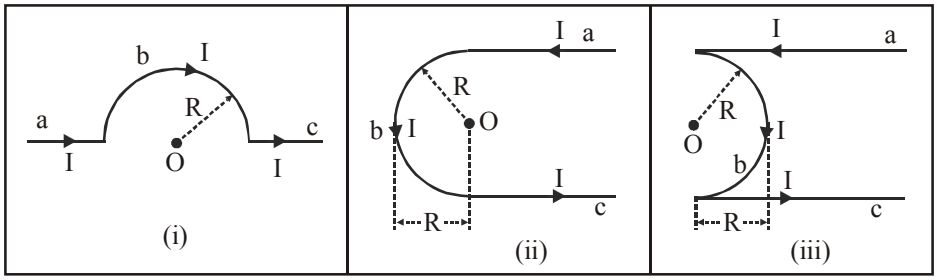
$$B_{abc}=\frac{\mu_0 I_1\alpha_1}{4\pi r}\ \text{ and }\ B_{adc}=\frac{\mu_0 I_2\alpha_2}{4\pi r}\Rightarrow\frac{B_{abc}}{B_{adc}}=\frac{I_1\alpha_1}{I_2\alpha_2}$$

$$\because\ \text{angle}=\frac{\text{arc length}}{\text{radius}}\Rightarrow\frac{\alpha_1}{\alpha_2}=\frac{\ell_1}{\ell_2}$$

$$\therefore V=I_1R_1=I_2R_2\Rightarrow\frac{I_1}{I_2}=\frac{R_2}{R_1}\Rightarrow\frac{I_1}{I_2}=\frac{\ell_2}{\ell_1}\left(\because R=\frac{\rho\ell}{A}\Rightarrow R\propto\ell\right)$$

$$\therefore\frac{B_{abc}}{B_{adc}}=\left(\frac{\ell_2}{\ell_1}\right)\left(\frac{\ell_1}{\ell_2}\right)\Rightarrow\frac{B_{\alpha_1}}{B_{\alpha_2}}=\frac{1}{1}$$

Ex. Calculate the field at the centre of a semi-circular wire of radius R in situations depicted in figure (i), (ii) and (iii) if the straight wire is of infinite length.



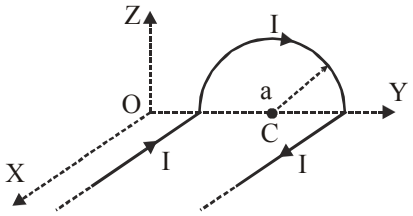
Sol. The magnetic field due to a straight current carring wire of infinite length, for a point at a distance R from one of its ends is zero if the point is along its length and $\frac{\mu_0 I}{4\pi R}$ if the point is on a line perpendicular to its length while at the centre of a semicircular coil is $\frac{\mu_0 I}{4R}$ so net magnetic field at the centre of semicircular wire is $\vec{B}_R=\vec{B}_a+\vec{B}_b+\vec{B}_c$

(i) $\vec{B}_R = 0 + \frac{\mu_0}{4}\frac{I}{R}\otimes + 0 = \frac{\mu_0 I}{4R}\otimes$(into the page)

(ii) $\vec{B}_R = \frac{\mu_0}{4\pi}\frac{I}{R}\odot + \frac{\mu_0}{4}\frac{I}{R}\odot + \frac{\mu_0}{4\pi}\frac{I}{R}\odot = \frac{\mu_0}{4\pi}\frac{I}{R}[\pi + 2]\odot$ (out of the page)

(iii) $\vec{B}_R = \frac{\mu_0}{4\pi}\frac{I}{R}\odot + \frac{\mu_0}{4}\frac{I}{R}\otimes + \frac{\mu_0}{4\pi}\frac{I}{R}\odot = \frac{\mu_0}{4\pi}\frac{I}{R}[\pi - 2]\otimes$ (into the page)

Ex. A long wire bent as shown in the figure carries current I. If the radius of the semi-circular portion is "a" then find the magnetic induction at the centre C.



Sol. Due to semi circular part

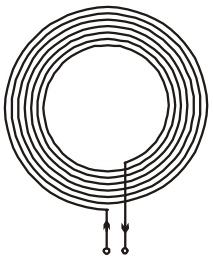
$$\vec{B}_1 = \frac{\mu_0 I}{4a}\left(-\hat{i}\right)$$

due to parallel parts of currents

$$\vec{B}_2 = 2\times\frac{\mu_0 I}{4\pi a}(-\hat{k}),\ B_{net} = B_C = \vec{B}_1 + \vec{B}_2 = \frac{\mu_0 I}{4a}(-\hat{i}) + \frac{\mu_0 I}{2\pi a}(-\hat{k})$$

magnitude of resultant field $B = \sqrt{B_1^2 + B_2^2} = \frac{\mu_0 I}{4\pi a}\sqrt{\pi^2 + 4}$

Ex. A thin insulated wire forms a plane spiral of N = 100 tight turns carrying a current I = 8 mA. The radii of inside and outside turns (Fig.) are equal to a = 50 mm and b = 100 mm.
Find the magnetic induction at the centre of the spiral;



Ans. $B = \frac{\mu_0\, IN\, ln\,(b/a)}{2(b-a)} = 7\ \mu T$;

Sol. From Biot-Savart's law, the magnetic induction due to a circular current carrying wire loop at its centre is given by,

$$B_r = \frac{\mu_0}{2r}i$$

The plane spiral is made up of concentric circular loops, having different radii, varying from a to b. Therefore, the total magnetic induction at the centre,

$$B_0 = \int\frac{\mu_0}{2r}dN$$

where $\frac{\mu_0}{2r}i$ is the contribution of one turn of radius r and dN is the number of turns in the interval (r, r + dr).

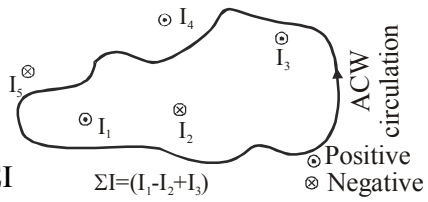
i.e. $dN = \frac{N}{b-a}dr$

Substituting in equation (1) and integrating the result over r between a and b, we obtain,

$$B_0 = \int_a^b \frac{\mu_0 i}{2r}\frac{N}{(b-a)}dr = \frac{\mu_0 iN}{2(b-a)}\ln\frac{b}{a}$$

AMPERE'S CIRCUITAL LAW

Ampere's circuital law state that line integral of the magnetic field around any closed path in free space or vacuum is equal to μ_0 times of net current or total current which crossing through the area bounded by the closed path. Mathematically $\oint \vec{B} \cdot d\vec{\ell} = \mu_0 \Sigma I$



This law independent of size and shape of the closed path.

Any current outside the closed path is not included in writing the right hand side of law

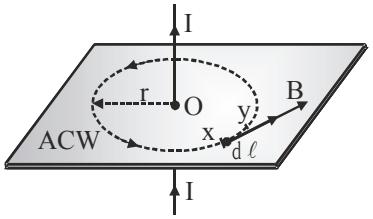
Note :

- This law suitable for infinite long and symmetrical current distribution.
- Radius of cross section of thick cylinderical conductor and current density must be given to apply this law.

APPLICATION OF AMPERE'S CIRCUITAL LAW

- **Magnetic field due to infinite long thin current carrying straight conductor**

Consider a circle of radius 'r'. Let XY be the small element of length $d\ell$. $\vec{B}$ and $d\vec{\ell}$ are in the same direction because direction of along the tangent of the circle. By A.C.L.

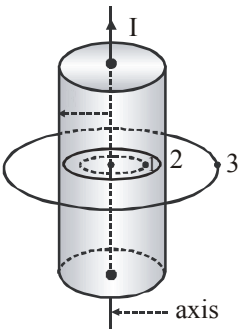


$\oint \vec{B} . d\vec{\ell} = \mu_0 \Sigma I$, $\oint B\, d\ell \cos\theta = \mu_0 I$ (where $\theta = 0°$)

$\oint B\, d\ell \cos 0° = \mu_0 I \Rightarrow B \oint d\ell = \mu_0 I$ (where $\oint d\ell = 2\pi r$)

$B\,(2\pi r) = \mu_0 I \Rightarrow B = \dfrac{\mu_0 I}{2\pi r}$

- **Magnetic field due to infinite long solid cylinderical conductor**
- For a point inside the cylinder r < R, Current from area πR^2 is = I

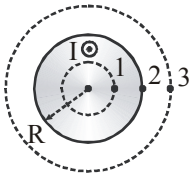


so current from area πr^2 is = $\dfrac{I}{\pi R^2}(\pi r^2) = \dfrac{I r^2}{R^2}$

By Ampere circuital law for circular path 1 of radius r

$B_{in}\,(2\pi r) = \mu_0 I' = \mu_0 \dfrac{I r^2}{R^2} \Rightarrow B_{in} = \dfrac{\mu_0 I r}{2\pi R^2} \Rightarrow B_{in} \propto r$

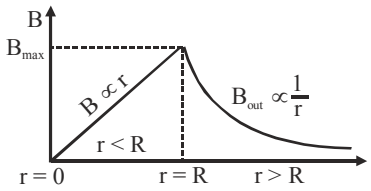
- For a point on the axis of the cylinder (r = 0); $B_{axis} = 0$
- For a point on the surface of cylinder (r = R)



By Ampere circuital law for circular path 2 of radius R

$B_s\,(2\,\pi\,R) = \mu_0 I \Rightarrow B_s = \dfrac{\mu_0 I}{2\pi R}$ (it is maximum)

- For a point outside the cylinder (r > R) :-



By Ampere circuital law for circular path 3 of radius r

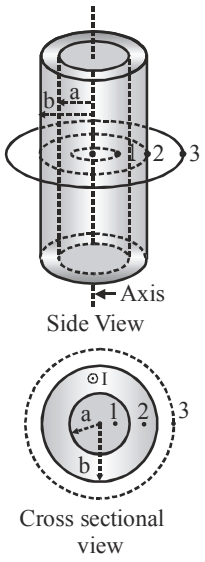
$B_{out}\,(2\,\pi\,r) = \mu_0 I \Rightarrow B_{out} = \dfrac{\mu_0 I}{2\pi r} \Rightarrow B_{out} \propto \dfrac{1}{r}$

Magnetic field outside the cylinderical conductor does not depend upon nature (thick/thin or solid/hollow) of the conductor as well as its radius of cross section.

- **Magnetic field due to infinite long hollow cylinderical conductor**
- For a point at a distance r such that $r < a < b$ $\quad B_1 = 0$
- For a point at a distance r such that $a < r < b$

$$B_2(2\pi r) = \mu_0 I' \Rightarrow B_2(2\pi r) = \mu_0 I\left(\frac{r^2 - a^2}{b^2 - a^2}\right)$$

$$B_2 = \frac{\mu_0 I}{2\pi r}\left(\frac{r^2 - a^2}{b^2 - a^2}\right) \begin{cases} \rightarrow r = a \text{ (inner surface)} \Rightarrow B_{IS} = 0 \\ \rightarrow r = b \text{ (outer surface)} \Rightarrow B_{OS} = \frac{\mu_0 I}{2\pi b} \text{ (maximum)} \end{cases}$$



- For a point at a distance r such that $r > b > a$, $B_3(2\pi r) = \mu_0 I \Rightarrow B_3 = \frac{\mu_0 I}{2\pi r}$
- For a point at the axis of cylinder $r = 0$ $\quad B_{axis} = 0$

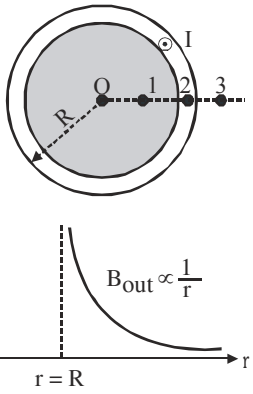
Magnetic field at specific positions for thin hollow cylinderical conductor

At point 1 $B_1 = 0$

At point 2 $B_2 = \frac{\mu_0 I}{2\pi R}$ (maximum) [outer surface] and

$B_2 = 0$ (minimum) [inner surface]

At point 3 $B_3 = \frac{\mu_0 I}{2\pi r}$ (for the point on axis $B_{axis} = 0$)



Ex. **Non-Uniform Current Density :** Consider an infinitely long, cylindrical conductor of radius *R* carrying a current *I* with a non-uniform current density $J = \alpha r$ where α is a constant. Find the magnetic field everywhere.

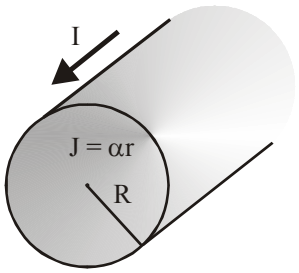


Figure : Non-uniform current density

Solution :

The problem can be solved by using the Ampere's law:

$$\oint \vec{B}.d\vec{s} = \mu_0 I_{enc}$$

Where the enclosed current I_{enc} is given by

$$I_{enc} = \int \vec{J} \cdot d\vec{A} = \int (\alpha r')(2\pi r' dr')$$

(a) For r < R, the enclosed current is $I_{enc} = \int_0^r 2\pi\alpha r'^2\, dr' = \frac{2\pi\alpha r^3}{3}$

Applying Ampere's law, the magnetic field at P_1 is given by

$$B_1(2\pi r) = \frac{2\mu_0\pi\alpha r^3}{3} \text{ or } B_1 = \frac{\alpha\mu_0}{3} r^2$$

The direction of the magnetic field $\vec{B}_1$ is tangential to the Amperian loop which encloses the currect.

(b) For r > R, the enclosed current is : $I_{enc} = \int_0^R 2\pi\alpha r'^2 dr' = \frac{2\pi\alpha R^3}{3}$

which yields $B_2(2\pi r) = \frac{2\mu_0\pi\alpha R^3}{3}$

Thus, the magnetic field at a point P_2 outside the conductor is ;

$$B_2 = \frac{\alpha\mu_0 R^3}{3r}$$

A plot of B as a function of r is shown in figure.

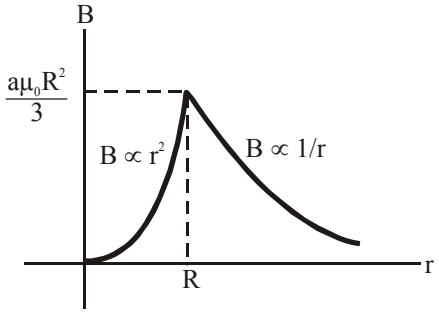
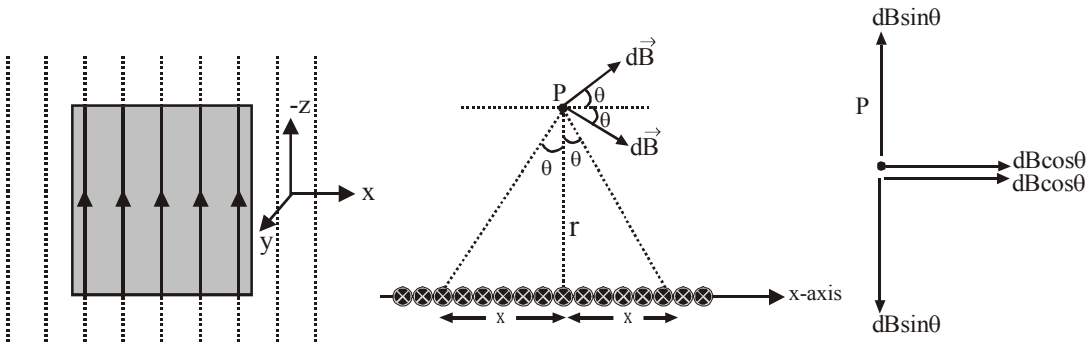


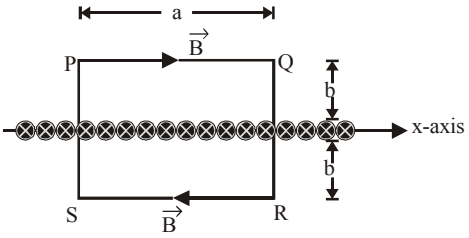
Figure : The magnetic field as a function of distance away from the conductor

Magnetic field due to an infinite plane sheet of current



An infinite sheet of current lies in x-z plane, carrying current along-z axis. The field at any point P on y is along a line parallel to x-z plane. We can take a rectangular amperian loop as shown. If you traverse the loop in clockwise direction, inward current will be positive.

By Ampere circuital law, $\oint_{PQRS} \vec{B}.d\vec{\ell} = \mu_0 \ell_{enclosed}$ ....(i)



Let λ represents current per unit length.

The current enclosed is given by $\ell_{enclosed}=\lambda a$

Now, $\oint_{PQRS} \vec{B}.d\vec{\ell} = \int_{PQ} \vec{B}.d\vec{\ell} + \int_{QR} \vec{B}.d\vec{\ell} + \int_{RS} \vec{B}.d\vec{\ell} + \int_{SR} \vec{B}.d\vec{\ell}$

Now, $\int_{QR} \vec{B}.d\vec{\ell} = \int_{SP} \vec{B}.d\vec{\ell} = 0$ as $\vec{B} \perp d\vec{\ell}$

Also, $\int_{PQ} \vec{B}.d\vec{\ell} + \int_{RS} \vec{B}.d\vec{\ell} = 2\times B\times a \left(\text{as } \vec{B} \| d\vec{\ell}\right) \therefore 2B\times a = \mu_0 \lambda a \Rightarrow B = \frac{\mu_0 \lambda}{2}$

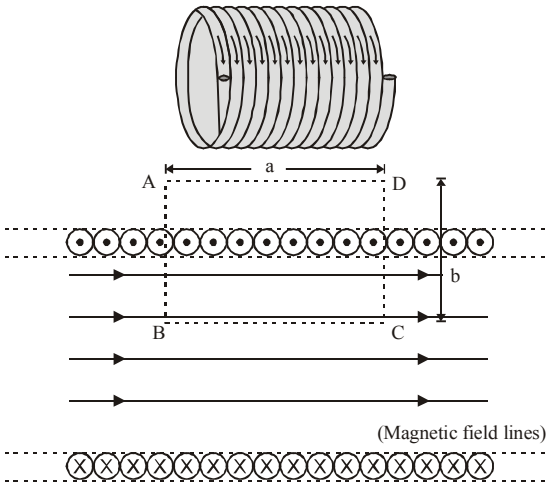
MAGNETIC FIELD DUE TO SOLENOID

It is a coil which has length and used to produce uniform magnetic field of long range. It consists a conducting wire which is tightly wound over a cylinderical frame in the form of helix. All the adjacent turns are electrically insulated to each other. The magnetic field at a point on the axis of a solenoid can be obtained by superposition of field due to large number of identical circular turns having their centres on the axis of solenoid.

Magnetic field due to a long solenoid

A solenoid is a tightly wound helical coil of wire. If length of solenoid is large, as compared to its radius, then in the central region of the solenoid, a reasonably uniform magnetic field is present. Figure shows a part of long solenoid with number of turns/length n.We can find the field by using Ampere circuital law.

Consider a rectangular loop ABCD. For this loop $\oint_{ABCD} \vec{B}.d\vec{\ell} = \mu_0 i_{enc}$



Now

$$\oint_{ABCD} \vec{B}.d\vec{\ell} = \oint_{AB} \vec{B}.d\vec{\ell} + \oint_{BC} \vec{B}.d\vec{\ell} + \oint_{CD} \vec{B}.d\vec{\ell} + \oint_{DA} \vec{B}.d\vec{\ell} = B\times a$$

This is because $\oint_{AB} \vec{B}.d\vec{\ell} = \oint_{CD} \vec{B}.d\vec{\ell} = 0,\ \vec{B} \perp d\vec{\ell}$.

And, $\oint_{DA} \vec{B}.d\vec{\ell} = 0$

($\because$ $\vec{B}$ outside the solenoid is negligible

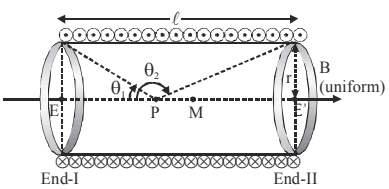
Now, $i_{enc} = (n\times a)\times i$ s $\Rightarrow B\times a = \mu_0 (n\times a\times i)$

$\Rightarrow B = \mu_0 ni$

Finite length solenoid :

Its length and diameter are comparable. By the concept of BSL magnetic field at the axial point 'P' obtained as :

$$B_P = \frac{\mu_0 nI}{2}(\cos\theta_1 - \cos\theta_2)$$



Angle θ_1 and θ_2 both measured in same sense from the axis of the solenoid to end vectors.

Infinite length solenoid :

Its length very large as compared to its diameter i.e. ends of solenoid tends to infinity.

(a) Magnetic field at axial point which is well inside the solenoid

$$\theta_1 \simeq 0^\circ \text{ and } \theta_2 \simeq 180^\circ \Rightarrow B \simeq \frac{\mu_0 nI}{2}[\cos 0^\circ - \cos 180^\circ] \simeq \frac{\mu_0 nI}{2}[(1)-(-1)] \simeq \mu_0 nI$$

(b) Magnetic field at both axial end points of solenoid

$$\theta_1 = 90^\circ \text{ and } \theta_2 \simeq 180^\circ \Rightarrow B \simeq \frac{\mu_0 nI}{2}[\cos 90^\circ - \cos 180^\circ] \simeq \frac{\mu_0 nI}{2}[(0)-(-1)] \simeq \frac{\mu_0 nI}{2}$$

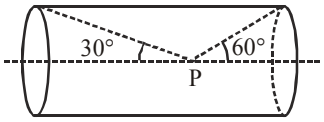
Ex. The length of solenoid is 0.1m. and its diameter is very small. A wire is wound over it in two layers. The number of turns in inner layer is 50 and that of outer layer is 40. The strength of current flowing in two layers in opposite direction is 3A. Then find magnetic induction at the middle of the solenoid.

Sol. Direction of magnetic field due to both layers is opposite, as direction of current is opposite so

$$B_{net} = B_1 - B_2 = \mu_0 n_1 I_1 - \mu_0 n_2 I_2 = \mu_0 \frac{N_1}{\ell} I - \mu_0 \frac{N_2}{\ell} I \quad (\because I_1 = I_2 = I)$$

$$= \frac{\mu_0 I}{\ell}(N_1 - N_2) = \frac{4\pi \times 10^{-7} \times 3}{0\cdot 1}(50-40) = 12\pi \times 10^{-5}\ T$$

Ex. Find out magnetic field at axial point 'P' of solenoid shown in figure (where turn density 'n' and current through it is I)



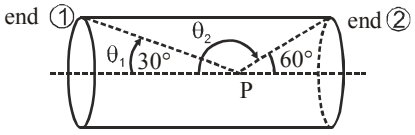
Sol. Magnetic field at point 'P' due to finite length solenoid

$$B_P = \frac{\mu_0 nI}{2}[\cos\theta_1 - \cos\theta_2],$$

where $\theta_1 = 30^\circ$ (CW),

$$\theta_2 = (180^\circ - 60^\circ) = 120^\circ \text{ (CW)} = \frac{\mu_0 nI}{2}[\cos 30^\circ - \cos 120^\circ]$$

$$= \frac{\mu_0 nI}{2}\left[\frac{\sqrt{3}}{2} - \left(-\frac{1}{2}\right)\right] = \frac{\mu_0 nI}{4}(\sqrt{3}+1)$$

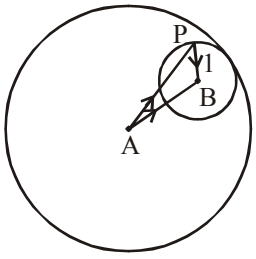


Ex. Inside a long straight uniform wire of round cross-section there is a long round cylindrical cavity whose axis is parallel to the axis of the wire and displaced from the latter by a distance I. A direct current of density j flows along the wire. Find the magnetic induction inside the cavity. Consider, in particular, the case I = 0.

Sol. We can think of the given current which will be assumed uniform, as arising due to a negative current, flowing in the cavity, superimposed on the true current, everywhere including the cavity. Then from the previous problem, by superposition

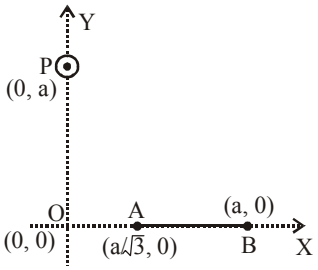
$$\vec{B} = \frac{1}{2}\mu_0 \vec{j} \times (\vec{AP} - \vec{BP}) = \frac{1}{2}\mu_0 \vec{j} \times \vec{\ell}$$

If $\vec{\ell}$ vanishes so that the cavity is concentric with the conductor, there is no magnetic field in the cavity.



Ex. An infinite current carrying conductor, parallel to z-axis is situated at point P as shown in the figure.

Value of $\int_A^B \vec{B}\cdot\vec{d\ell}$ is given by $\alpha\frac{\mu_0 i}{96}$, then fill the value of α in OMR sheet ?

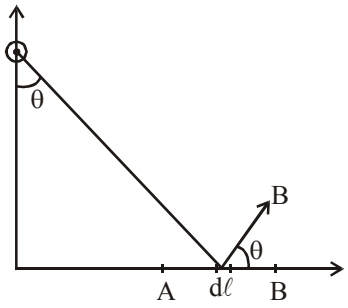


Ans. 4

Sol. $d\ell = d(a\tan\theta) = a\sec^2\theta d\theta.$

$$B = \frac{\mu_0 i}{2\pi a}\cos\theta$$

$$\therefore \int B.d\ell = \int_{\frac{\pi}{6}}^{\frac{\pi}{4}} \frac{\mu_0 i}{2\pi a}\cos\theta\, a\sec^2\theta d\theta.\cos\theta = \frac{\mu_0 i}{2\pi}\int_{\pi/6}^{\pi/4} d\theta = \frac{\mu_0 i}{2\pi}\cdot\frac{\pi}{12} = \frac{\mu_0 i}{24}.$$



MOTION OF A CHARGED PARTICLE IN A MAGNETIC FIELD

Motion of a charged particle when it is moving collinear with the field magnetic field is not affected by the field (i.e. if motion is just along or opposite to magnetic field) ($\because$ F = 0). The following two cases are possible :

- **Case I :**

When the charged particle is moving perpendicular to the field.

The angle between $\vec{B}$ and $\vec{v}$ is $\theta=90°$. So the force will be maximum (= qvB) and always perpendicular to motion (and also field); Hence the charged particle will move along a circular path (with its plane perpendicular to the field). Centripetal force is provided by the force qvB, So $\frac{mv^2}{r} = qvB \Rightarrow r = \frac{mv}{qB}$

Angular frequency of circular motion, called cyclotron or gyro-frequency. $\omega = \frac{v}{r} = \frac{qB}{m}$

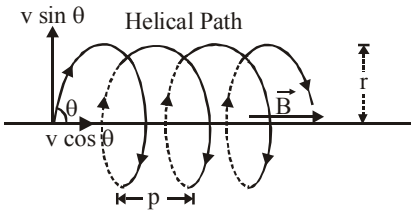
and the time period, $T = \frac{2\pi}{\omega} = 2\pi\frac{m}{qB}$ i.e., time period (or frequency) is independent of speed of particle and radius of the orbit. Time period depends only on the field B and the nature of the particle, i.e., specific charge (q/m) of the particle.

This principle has been used in a large number of devices such as cyclotron (a particle accelerator), bubble-chamber (a particle detector) or mass-spectrometer etc.

Note that $\vec{F}_B$ is always perpendicular to $\vec{v}$ and $\vec{B}$, and cannot change the particle's speed *v* (and thus the kinetic energy). In other words, magnetic force cannot speed up or slow down a charged particle. Consequently, $\vec{F}_B$ can do no work on the particle :

$$dW = \vec{F}_B\cdot d\vec{s} = q(\vec{v}\times\vec{B})\cdot\vec{v}dt = q(\vec{v}\times\vec{v})\cdot\vec{B}dt = 0$$

- **Case II :**



The charged particle is moving at an angle θ to the field :
(θ ≠ 0°, 90° or 180°)
Resolving the velocity of the particle along and perpendicular to the field.The particle moves with constant velocity v cos θ along the field
(∵ no force acts on a charged particle when it moves parallel to the field).
And at the same time it is also moving with velocity v sinθ perpendicular to the field due to which it will describe a circle (in a plane perpendicular to the field)

Radius of the circular path $r = \frac{m(v\sin\theta)}{qB}$ and Time period $T = \frac{2\pi r}{v\sin\theta} = \frac{2\pi m}{qB}$

So the resultant path will be a helix with its axis parallel to the field $\vec{B}$ as shown in fig.

The pitch p of the helix = linear distance travelled in one rotation $p = T(v\cos\theta) = \frac{2\pi m}{qB}(v\cos\theta)$

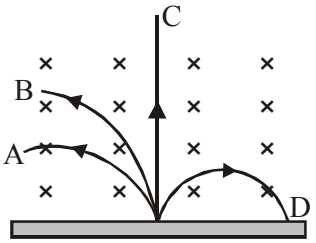
Ex. An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV enters a region with uniform magnetic field of 0.15 T. Determine the radius of the trajectory of the electron if the field is –
(a) Transverse to its initial velocity (b) Makes an angle of 30° with the initial velocity
[Given : $m_e = 9 \times 10^{-31}$ kg]

Sol. $\frac{1}{2}mv^2 = eV \Rightarrow v = \sqrt{\frac{2eV}{m}} = \sqrt{\frac{2\times1.6\times10^{-19}\times2\times10^{3}}{9\times10^{-31}}} = \frac{8}{3}\times10^{7}$ m/s

(a) Radius $r_1 = \frac{mv}{qB} = \frac{9\times10^{-31}\times(8/3)\times10^{7}}{1.6\times10^{-19}\times0.15} = 10^{-3}\,m = 1mm$

(b) Radius $r_2 = \frac{mv\sin\theta}{qB} = r_1\sin\theta = 1\times\sin30° = 1\times\frac{1}{2} = 0.5$ mm

Ex. A neutron, a proton, an electron an α-particle enter a region of constant magnetic field with equal velocities. The magnetic field is along the inwards normal to the plane of the paper. The tracks of the particles are shown in fig. Relate the tracks to the particles.



Sol. Force on a charged particle in magnetic field $\vec{F} = q(\vec{v}\times\vec{B})$

For neutron q = 0, F = 0 hence it will pass undeflected.
i.e., tracks C corresponds to neutron.

If the particle is negatively charged, i.e. electron. $\vec{F} = -e(\vec{v}\times\vec{B})$

It will experience a force to the right; so track D corresponds to electron.
If the charge on particle is positive. It will experience a force to the left; so both tracks A and B corresponds to positively charged particles (i.e., protons and α-particles). When motion of charged particle perpendicular to the magnetic field the path is a circle with radius

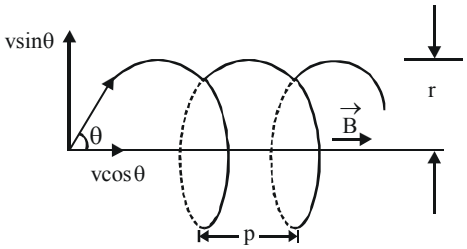
$r = \frac{mv}{qB}$ i.e. $r \propto \frac{m}{q}$ and as $\left(\frac{m}{q}\right)_\alpha = \left(\frac{4m}{2e}\right)$ while $\left(\frac{m}{q}\right)_p = \frac{m}{e} \Rightarrow \left(\frac{m}{q}\right)_\alpha > \left(\frac{m}{q}\right)_p$

So $r_\alpha > r_p \Rightarrow$ track B to α-particle and A corresponds to proton.

Ex. A beam of protons with velocity 4×10^5 m/s enters a uniform magnetic field of 0.3 tesla at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of helix. Mass of proton=1.67×10^{-27}kg.

Sol. Radius of helix $r = \frac{mv\sin\theta}{qB}$ ($\therefore$ component of velocity $\perp$ to field is $v\sin\theta$)

$$= \frac{(1.67\times10^{-27})(4\times10^5)\sqrt{\frac{3}{2}}}{(1.6\times10^{-19})0.3} = \frac{2}{\sqrt{3}} \times 10^{-2}\text{m} = 1.2\text{cm}$$



Again, pitch $p = v\cos\theta \times T$ (where $T = \frac{2\pi r}{v\sin\theta}$)

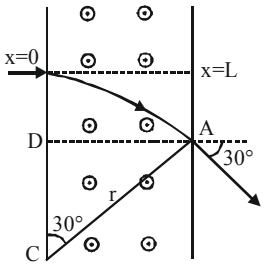
$$\therefore\ p = \frac{v\cos\theta\times 2\pi r}{v\sin\theta} = \frac{\cos 60^\circ\times 2\pi\times(1.2\times10^{-2})}{\sin 60^\circ} = 4.35\times 10^{-2}\text{m} = 4.35\text{cm}$$

Ex. The region betwen x = 0 and x = L is filled with uniform, steady magnetic field $B_0\hat{k}$. A particle of mass m, positive charge q and velocity $v_0\hat{i}$ travels along X-axis and enters the region of magnetic field. Neglect the gravity throughout the question.

(a) Find the value of L if the particle emerges from the region of magnetic field with its final velocity at an angle 30° to its initial velocity.

(b) Find the final velocity of the particle and the time spent by it in the magnetic field, if the magnetic field now extends upto 2.1 L.

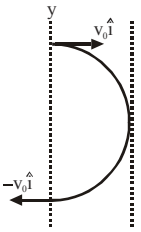
Sol. (a) The particle is moving with velocity $v_0\hat{i}$, perpendicular to magnetic field $B_0\hat{k}$. Hence the particle will move along a circular arc OA of radius $r = \frac{mv_0}{qB_0}$



Let the particle leave the magnetic field at A.

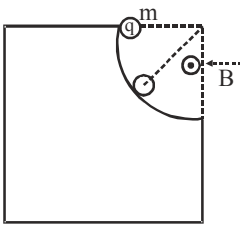
From ΔCDA, $\sin 60^\circ = \frac{AD}{CA} = \frac{L}{r} \Rightarrow L = r\sin 30^\circ = \frac{r}{2}$ $\therefore L = \frac{mv_0}{2qB_0}$

(b) As the magnetic field extends upto 2.1 L i.e., L > 2r, so the particle completes half cycle before leaving the magnetic field, as shown in figure.
The magnetic field is always perpendicular to velocity vector, therefore the magnitude of velocity will remain the same.



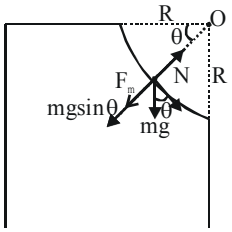
$\therefore$ Final velocity $= v_0(-\hat{i}) = -v_0\hat{i}$ Time spent in magnetic field $= \frac{\pi r}{v_0} = \frac{\pi m}{qB_0}$

Ex. A charged sphere of mass m and charge q starts sliding from rest on a vertical fixed circular track of radius R from the position as shown in figure. There exists a uniform and constant horizontal magnetic field of induction B. Find the maximum force exerted by the track on the sphere.



Sol. Magnetic force on sphere $F_m = qvB$ (directed radially outward)

$\because N - mg\sin\theta - qvB = \frac{mv^2}{R}$



$\Rightarrow N = \frac{mv^2}{R} + mg\sin\theta + qvB$

Hence, at $\theta = \pi/2$ we get $N_{max} = \frac{2mgR}{R} + mg + qB\sqrt{2gR} = 3mg + qB\sqrt{2gR}$

Ex. A particle of charge q and mass m starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{i}$ with velocity $\vec{v} = v_0\hat{j}$. The speed of the particle will become $2v_0$ after a time :-

(A) $t = \frac{2mv_0}{qE}$ (B) $t = \frac{2Bq}{mv_0}$ (C) $t = \frac{\sqrt{3}\,Bq}{mv_0}$ (D) $t = \frac{\sqrt{3}\,mv_0}{qE}$

Ans. (D)

Sol. Charged particle will move in a Helical path.

$\vec{F} = q\vec{E} + q\vec{v}\times\vec{B}$

$\vec{a} = \frac{qE\hat{i}}{m}$ this part will increase x component of velocity

+

$\frac{qv_0\times B}{m}$ (in y-z plane) this term will provide centripetal acceleration.

$v_x = \frac{qE}{m}\cdot t$

$v = \sqrt{v_x^2 + v_0^2}$

$2v_0 = \sqrt{\left(\frac{qE}{m}t\right)^2 + v_0^2}$

$\sqrt{3}v_0 = \frac{qE}{m}t$

$t = \frac{\sqrt{3}mv_0}{qE}$

(1) Velocity Selector :

In the presence of both electric field $\vec{E}$ and magnetic field $\vec{B}$, the total force on a charged particle is

$$\vec{F} = q(\vec{E} + \vec{v} \times \vec{B})$$

This is known as the Lorentz force. By combining the two fields, particles which move with a certain velocity can be selected. This was the principle used by J. J. Thomson to measure the charge-to-mass ratio of the electrons. In Figure the schematic diagram of Thomson's apparatus is depicted.

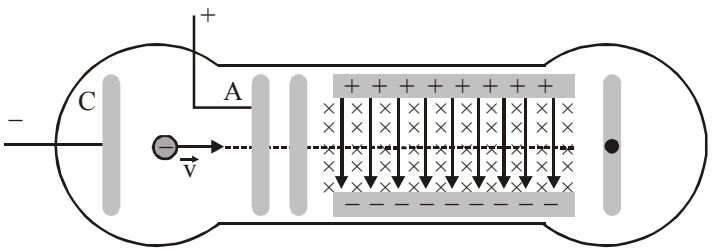


Figure : Thomson's apparatus

The electrons with charge $q = -e$ and mass m are emitted from the cathode C and then accelerated toward slit A. Let the potential difference between A and C be $V_A - V_C = \Delta V$. The change in potential energy is equal to the external work done in accelerating the electrons: $\Delta U = W_{ext} = q\Delta V = -e\Delta V$. By energy conservation, the kinetic energy gained is $\Delta K = -\Delta U = mv^2/2$. Thus, the speed of the electrons is given by

$$v = \sqrt{\frac{2e\Delta V}{m}}$$

If the electrons further pass through a region where there exists a downward uniform electric field, the electrons, being negatively charged, will be deflected upward. However, if in addition to the electric field, a magnetic field directed into the page is also applied, then the electrons will experience an additional downward magnetic force $-e\vec{v} \times \vec{B}$. When the two forces exactly cancel, the electrons will move in a straight path. From Eq., we see that when the condition for the cancellation of the two forces is given by $eE = evB$. which implies

$$v = \frac{E}{B}$$

In other words, only those particles with speed $v = E / B$ will be able to move in a straight line. Combining the two equations, we obtain

$$\frac{e}{m} = \frac{E^2}{2(\Delta V)B^2}$$

By measuring E, ΔV and B, the charge-to-mass ratio can be readily determined. The most precise measurement to date is $e/m = 1.758820174(71) \times 10^{11}$ C/kg.

(2) Mass Spectrometer :

Various methods can be used to measure the mass of an atom. One possibility is through the use of a mass spectrometer. The basic feature of a *Bainbridge* mass spectrometer is illustrated in Figure. A particle carrying a charge $+q$ is first sent through a velocity selector.

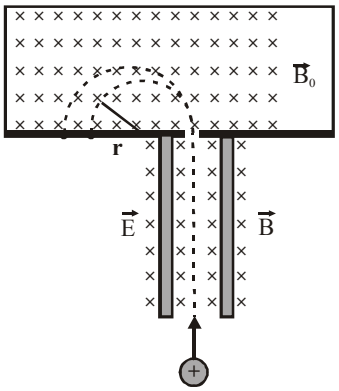


Figure : A Bainbridge mass spectrometer

The applied electric and magnetic fields satisfy the relation $E = vB$ so that the trajectory of the particle is a straight line. Upon entering a region where a second magnetic field $\vec{B}_0$ pointing into the page has been applied, the particle will move in a circular path with radius r and eventually strike the photographic plate. Using Eq., we have

$$r = \frac{mv}{qB_0}$$

Since v = E/B, the mass of the particle can be written as

$$m = \frac{qB_0 r}{v} = \frac{qB_0 Br}{E}$$

Ex. Particle A with charge q and mass m_A and particle B with charge $2q$ and mass m_B, are accelerated from rest by a potential difference ΔV, and subsequently deflected by a uniform magnetic field into semicircular paths. The radii of the trajectories by particle A and B are R and $2R$, respectively. The direction of the magnetic field is perpendicular to the velocity of the particle. What is their mass ratio?

Sol. The kinetic energy gained by the charges is equal to

$$\frac{1}{2}mv^2 = q\Delta V$$

which yields $v = \sqrt{\frac{2q\Delta V}{m}}$

The charges move in semicircles, since the magnetic force points radially inward and provides the source of the centripetal force :

$$\frac{mv^2}{r} = qvB$$

The radius of the circle can be readily obtained as :

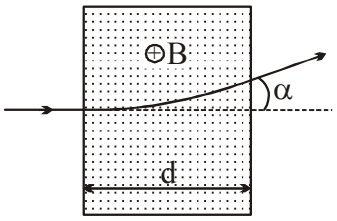
$$r = \frac{mv}{qB} = \frac{m}{qB}\sqrt{\frac{2q\Delta V}{m}} = \frac{1}{B}\sqrt{\frac{2m\Delta V}{q}}$$

which shows that r is proportional to $(m/q)^{1/2}$. The mass ratio can then be obtained from

$$\frac{r_A}{r_B} = \frac{(m_A/q_A)^{1/2}}{(m_B/q_B)^{1/2}} \Rightarrow \frac{R}{2R} = \frac{(m_A/q)^{1/2}}{(m_B/2q)^{1/2}}$$

which gives $\frac{m_A}{m_B} = \frac{1}{8}$

Ex. A proton accelerated by a potential difference V = 500 kV flies through a uniform transverse magnetic field with induction B = 0.51 T. The field occupies a region of space d = 10 cm in thickness (Fig.). Find the angle a through which the proton deviates from the initial direction of its motion.



Ans. $\alpha = \arcsin\left(dB\sqrt{\frac{q}{2mV}}\right) = 30°$

Sol. From the figure,

$$\sin\alpha = \frac{d}{R} = \frac{dqB}{mv}$$

As radius of the arc $R = \frac{mv}{qB}$, where v is the velocity of the particle, when it enters into the field.

From initial condition of the problem,

$$qV = \frac{1}{2}mv^2 \qquad \text{or, } v = \sqrt{\frac{2qV}{m}}$$

Hence, $\sin\alpha = \frac{dqB}{m\sqrt{2qV/m}} = dB\sqrt{\frac{q}{2mV}}$

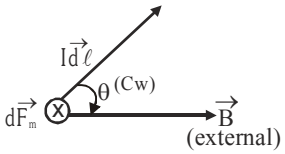
and $a = \sin^{-1}\left(dB\sqrt{\frac{q}{2mV}}\right) = 30°$, on putting the values.

CURRENT CARRYING CONDUCTOR IN MAGNETIC FIELD

When a current carrying conductor placed in magnetic field, a magnetic force exerts on each free electron which are present inside the conductor. The resultant of these forces on all the free electrons is called magnetic force on conductor.

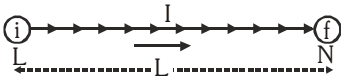
- **Magnetic force on current element**

Through experiments Ampere established that when current element $I\vec{d\ell}$ is placed in magnetic field $\vec{B}$, it experiences a magnetic force $\vec{dF_m} = I(\vec{d\ell} \times \vec{B})$



- Current element in a magnetic field does not experience any force if the current in it is parallel or anti–parallel with the field $\theta = 0°$ or $180°$
  $dF_m = 0$ (min.)
- Current element in a magnetic field experiences maximum force if the current in it is perpendicular with the field $\theta = 90°$
  $dF_m = BId\ell$ (max.)
- Magnetic force on current element is always perpendicular to the current element vector and magnetic field vector. $\vec{dF_m} \perp I\vec{d\ell}$ and $\vec{dF_m} \perp \vec{B}$ (always)
- Total magnetic force on straight current carrying conductor in uniform magnetic field given as

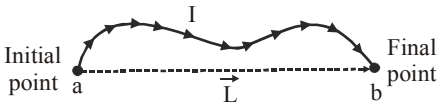
$$\vec{F}_m = \int_i^f \vec{dF}_m \left[\int_i^f \vec{d\ell}\right] = I \times \vec{B},\ F_m = I(\vec{L} \times \vec{B})$$



Where $\vec{L} = \int_i^f d\vec{\ell}$, vector sum of all length elements from initial to final point, which is in accordance with the law of vector addition and $|\vec{L}|$ = length of the condutor.

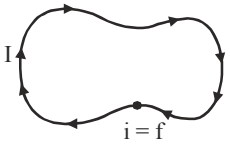
- Total magnetic force on arbitrary shape current carrying conductor in uniform magnetic field $\vec{B}$ is

$$\int_i^f d\vec{F}_m = I\left[\int_i^f d\vec{\ell}\right] \times \vec{B},\ \vec{F}_m = I(\vec{L} \times \vec{B})\ (L = ab)$$



Where $\vec{L} = \int_i^f d\vec{\ell}$, vector sum of all length elements from initial to final point or displacement between free ends of an arbitrary conducter from initial to final point.

- A current carrying closed loop (or coil) of any shape placed in uniform magnetic field then no net magnetic force act on it (Torque may or may not be zero)



$$\vec{L} = \int_i^f d\vec{\ell} = 0 \text{ or } \oint d\vec{\ell} = 0$$

So net magnetic force acting on a current carrying closed loop $\vec{F}_m = 0$ (always)

- When a current carrying closed loop (or coil) of any shape placed in non uniform magnetic field then net magnetic force is always acts on it (Torque may or may not be zero)

Ex. : Magnetic Force on a Semi-Circular Loop

Consider a closed semi-circular loop lying in the *xy* plane carrying a current *I* in the counterclockwise direction, as shown in Figure.

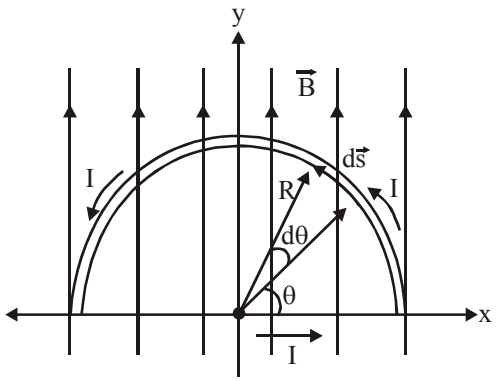


Figure 8.3.6 Semi-circular loop carrying a current *I*

A uniform magnetic field pointing in the +*y* direction is applied. Find the magnetic force acting on the straight segment and the semicircular arc.

Solution : Let $\vec{B} = B\hat{j}$ and $\vec{F}_1$ and $\vec{F}_2$ the forces acting on the straight segment and the semicircular parts, respectively. Using Eq. and noting that the length of the straight segment is 2*R*, the magnetic force is

$$\vec{F}_1 = I(2R\hat{i}) \times (B\hat{j}) = 2IRB\hat{k}$$

where $\hat{k}$ is directed out of the page.

To evaluate $\vec{F}_2$, we first note that the differential length element $d\vec{s}$ on the semicircle can be written as $d\vec{s} = ds\hat{\theta} = Rd\theta(-\sin\theta\hat{i} + \cos\theta\hat{j})$. The force acting on the length element is $d\vec{s}$ is :

$$d\vec{s} = Id\vec{s} \times \vec{B} = IRd\theta(-\sin\theta\hat{i} + \cos\theta\hat{j}) \times (B\hat{j}) = -IBR\sin\theta d\theta\hat{k}$$

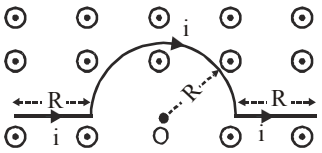
Here we see that $d\vec{F}_2$ points into the page. Integrating over the entire semi-circular arc, we have

Thus, the net force acting on the semi-circular wire is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 = \vec{0}$$

This is consistent from our previous claim that the net magnetic force acting on a closed current-carrying loop must be zero.

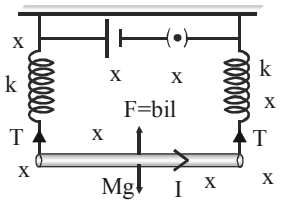
Ex. A wire bent as shown in fig carries a current i and is placed in a uniform field of magnetic induction $\vec{B}$ that emerges from the plane of the figure. Calculate the force acting on the wire.



Sol. The total force on the whole wire is

$$F_m = I|\vec{L}|B = I(R + 2R + R)B = 4RIB$$

Ex. A metal rod of mass 10 gm and length 25 cm is suspended on two springs as shown in figure. The springs are extended by 4 cm. When a 20 ampere current passes through the rod it rises by 1 cm. Determine the magnetic field assuming acceleration due to gravity to be 10 m/s^2.



Sol. Let tension in each spring is = T_0

Initially the rod will be in equilibrium if $2T_0 = Mg$

then $T_0 = kx_0$ ...(i)

Now when the current I is passed through the rod it will experience a force F = BIL vertically up; so in this situation for its equilibrium,

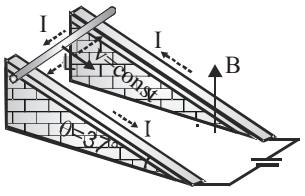
$2T + BIL = Mg$ with $T = kx$ ...(ii) $(x = 4 - 1 = 3cm)$

So from eq. (i) and eq.(ii) $\frac{T}{T_0} = \frac{Mg - BIL}{Mg}$

$$\Rightarrow \frac{x}{x_0} = 1 - \frac{BIL}{Mg}$$

$$\Rightarrow B = \frac{Mg(x_0 - x)}{ILx_0} = \frac{10\times10^{-3}\times10\times3\times10^{-2}}{20\times25\times10^{-2}\times4\times10^{-2}} = 1.5\times10^{-2}T$$

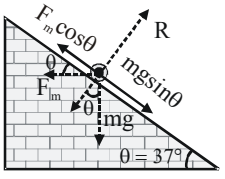
Ex. Two conducting rails are connected to a source of e.m.f. and form an incline as shown in fig. A bar of mass 50 g slides without friction down the incline through a vertical magnetic field B. If the length of the bar is 50 cm and a current of 2.5 A is provided by the battery, for what value of B will the bar slide at a constant velocity ? [g = 10 m/s^2]



Sol. Force on current carrying wire F = BIL

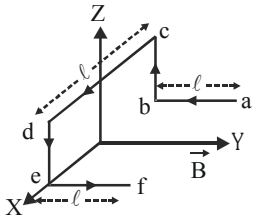
The rod will move down the plane with constant velocity only if

$F\cos\theta = mg\sin\theta \Rightarrow BIL\cos\theta = mg\sin\theta$



or, $B = \frac{mg}{IL}\tan\theta = \frac{50\times10^{-3}\times10}{2.5\times50\times10^{-2}}\times\frac{3}{4} = 0.3\ T$

Ex. A wire abcdef with each side of length 'ℓ' bent as shown in figure and carrying a current I is placed in a uniform magnetic field B parallel to +y direction.What is the force experienced by the wire.

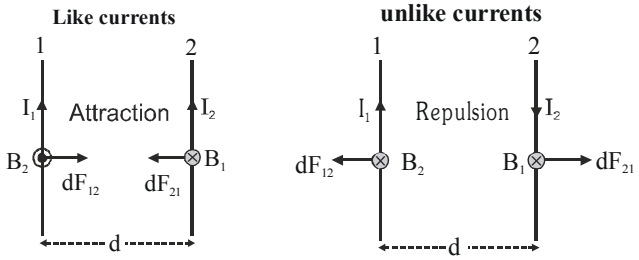


Sol. Magnetic force on wire abcdef in uniform magnetic field is $\vec{F}_m = I(\vec{L}\times\vec{B})$,

$\vec{L}$ is displacement between free ends of the conductor from initial to final point. $\vec{L} = (\ell)\,\hat{i}$ and $\vec{B} = (B)\,\hat{j}$

$F_m = I(\vec{L}\times\vec{B}) = BIL\ (\hat{i}\times\hat{j}) = BI\ell(\hat{k}) = BI\ell$, along +z direction

MAGNETIC FORCE BETWEEN TWO PARALLEL CURRENT CARRYING CONDUCTORS



The net magnetic force acts on a current carrying conductor due to its own field is zero. So consider two infinite long parallel conductors separated by distence 'd' carrying currents I_1 and I_2.

Magnetic field at each point on conductor (ii) due to current I_1 is $B_1 = \dfrac{\mu_0 I_1}{2\pi d}$

[uniform field for conductor (2)]

Magnetic field at each point on conductor (i) due to curent I_2 is $B_2 = \dfrac{\mu_0 I_2}{2\pi d}$

[Uniform field for conductor (1)]
consider a small element of length 'dℓ' on each conductor. These elements are right angle to the external magnetic field, so magnetic force experienced by elements of each conductor given as

$$dF_{12} = B_2 I_1 d\ell = \left(\frac{\mu_0 I_2}{2\pi d}\right) I_1\, d\ell \qquad ...(i) \qquad (\text{Where } I_1\, d\ell \perp B_2)$$

$$dF_{21} = B_1 I_2 d\ell = \left(\frac{\mu_0 I_1}{2\pi d}\right) I_2\, d\ell \qquad ... (ii) \qquad (\text{Where } I_2\, d\ell \perp B_1)$$

Where dF_{12} is magnetic force on element of conductor (i), due field of conductor (i) and dF_{21} is magnetic force on element of conductor (ii), due to field of conductor (i).

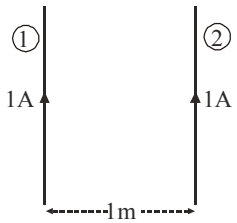
Magnetic force per unit length of each conductor is $\dfrac{dF_{12}}{d\ell} = \dfrac{dF_{21}}{d\ell} = \dfrac{\mu_0 I_1 I_2}{2\pi d}$

$f = \dfrac{\mu_0 I_1 I_2}{2\pi d}$ N/m (in S.I.) $\qquad$ $f = \dfrac{2 I_1 I_2}{d}$ dyne/cm (In C.G.S.)

Definition of ampere :

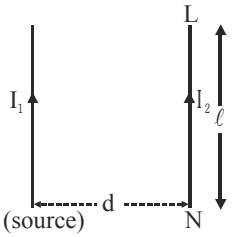
Magnetic force/unit length for both infinite length conductor gives as

$$f = \frac{\mu_0 I_1 I_2}{2\pi d} = \frac{(4\pi\times10^{-7})(1)(1)}{2\pi(1)} = 2\times10^{-7}\ \text{N/m}$$



'Ampere' is the current which, when passed through each of two parallel infinite long straight conductors placed in free space at a distance of 1 m from each other, produces between them a force of 2×10^{-7} N/m

- Force scale $f = \frac{\mu_0 I_1 I_2}{2\pi d}$ is applicable when at least one conductor must be of infinite length so it behaves like source of uniform magnetic field for other conductor.



Magnetic force on conductor 'LN' is $F_{LN} = f \times \ell \Rightarrow F_{LN} = \left(\frac{\mu_0 I_1 I_2}{2\pi d}\right)\ell$

Torque on a Current Loop :

What happens when we place a rectangular loop carrying a current *I* in the *xy* plane and switch on a uniform magnetic field $\vec{B} = B\hat{i}$ which runs parallel to the plane of the loop, as shown in Figure (a)?

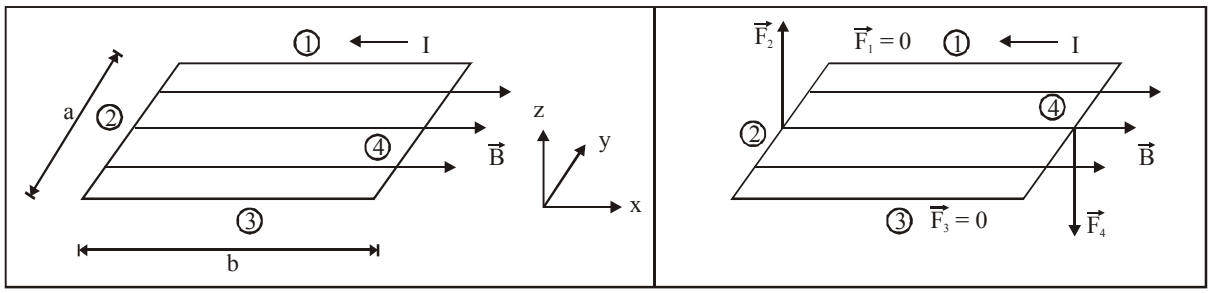


Figure : (a) A rectangular current loop placed in a uniform magnetic field.
(b) The magnetic forces acting on sides 2 and 4.

From Eq., we see the magnetic forces acting on sides 1 and 3 vanish because the length vectors $\vec{\ell}_1 = -b\hat{i}$ and $\vec{\ell}_3 = b\hat{i}$ are parallel and anti-parallel to $\vec{B}$ and their cross products vanish. On the other hand, the magnetic forces acting on segments 2 and 4 are non-vanishing:

$$\begin{cases}\vec{F}_2 = I(-a\hat{j})\times(B\hat{i}) = IaB\hat{k}\\ \vec{F}_4 = I(a\hat{j})\times(B\hat{i}) = -IaB\hat{k}\end{cases}$$

with $\vec{F}_2$ pointing out of the page and $\vec{F}_4$ into the page. Thus, the net force on the rectangular loop is

$$\vec{F}_{net} = \vec{F}_1 + \vec{F}_2 + \vec{F}_3 + \vec{F}_4 = \vec{0}$$

as expected. Even though the net force on the loop vanishes, the forces $\vec{F}_2$ and $\vec{F}_4$ will produce a torque which causes the loop to rotate about the *y*-axis (Figure). The torque with respect to the center of the loop is

$$\vec{\tau} = \left(-\frac{b}{2}\hat{i}\right)\times\vec{F}_2 + \left(\frac{b}{2}\hat{i}\right)\times\left(IaB\hat{k} + \left(\frac{b}{2}\hat{i}\right)\times\left(-IaB\hat{k}\right)\right)$$

$$= \left(\frac{IabB}{2} + \frac{IabB}{2}\right)\hat{j} = IabB\hat{j} = IAB\hat{j}$$

where A = ab represents the area of the loop and the positive sign indicates that the rotation is clockwise about the *y*-axis. It is convenient to introduce the area vector $\vec{A} = A\hat{n}$ where $\hat{n}$ is a unit vector in the direction normal to the plane of the loop. The direction of the positive sense of $\hat{n}$ is set by the conventional right-hand rule. In our case, we have $\hat{n} = +\hat{k}$. The above expression for torque can then be rewritten as

$$\vec{\tau} = I\vec{A}\times\vec{B}$$

Notice that the magnitude of the torque is at a maximum when $\vec{B}$ is parallel to the plane of the loop (or perpendicular to).

Consider now the more general situation where the loop (or the area vector $\vec{A}$) makes an angle θ with respect to the magnetic field.

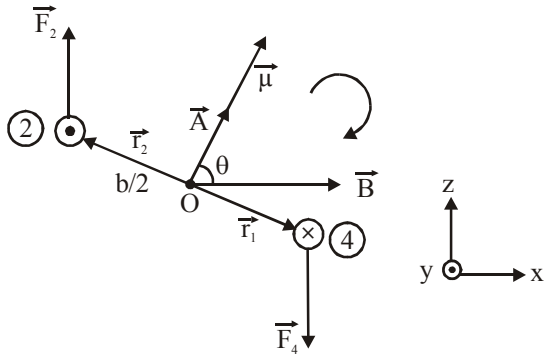


Figure : Rotation of a rectangular current loop

From Figure, the lever arms and can be expressed as:

$$\vec{r}_2 = \frac{b}{2}\left(-\sin\theta\hat{i} + \cos\theta\hat{k}\right) = -\vec{r}_4$$

and the net torque becomes

$$\vec{\tau} = \vec{r}_2\times\vec{F}_2 + \vec{r}_4 = 2\vec{r}_2\times\vec{F}_2 = 2.\frac{b}{2}\left(-\sin\theta\hat{i} + \cos\theta\hat{k}\right)\times\left(IaB\hat{k}\right)$$

$$jIabB\sin\theta\,\hat{j} = I\vec{A}\times\vec{B}$$

For a loop consisting of *N* turns, the magnitude of the toque is

$$\tau = NIAB\sin\theta$$

The quantity $NI\vec{A}$ is called the magnetic dipole moment $\vec{\mu}$

$$\vec{\mu} = NI\vec{A}$$

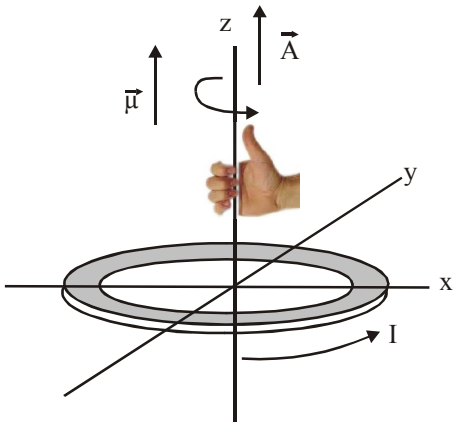


Figure : Right-hand rule for determining the direction of **μ**

The direction of $\vec{\mu}$ is the same as the area vector $\vec{A}$ (perpendicular to the plane of the loop) and is determined by the right-hand rule (Figure). The SI unit for the magnetic dipole moment is ampere-meter^2 (A•m^2).. Using the expression for $\vec{\mu}$, the torque exerted on a current-carrying loop can be rewritten as

$$\vec{\tau} = \vec{\mu} \times \vec{B}$$

The above equation is analogous to $\vec{\tau} = \vec{p} \times \vec{E}$ in Eq., the torque exerted on an electric dipole moment $\vec{p}$ in the presence of an electric field $\vec{E}$.

Configuratoin energy of current loop in uniform magnetic field.

Recalling that the potential energy for an electric dipole is $U = -\vec{p} \cdot \vec{E}$ [see Eq.], a similar form is expected for the magnetic case. The work done by an external agent to rotate the magnetic dipole from an angle θ_0 to θ is given by

$$W_{ext} = \int_{\theta_0}^{\theta} (\mu B \sin\theta')d\theta' = \mu B(\cos\theta_0 - \cos\theta)$$

$$= \Delta U = U - U_0$$

Once again, $W_{ext} = -W$, where W is the work done by the magnetic field. Choosing $U_0 = 0$ at $\theta_0 = \pi/2$, the dipole in the presence of an external field then has a potential energy os

$$U = -\mu B \cos\theta = -\vec{\mu} \cdot \vec{B}$$

The configuration is at a stable equilibrium when $\vec{\mu}$ is aligned parallel to $\vec{B}$, making U a minimum with $U_{min} = -\mu B$. On the other hand, when $\vec{\mu}$ and $\vec{B}$ are anti-parallel, $U_{max} = +\mu B$ is a maximum and the system is unstable.

Ex. A non-conducting thin disc of radius R charged uniformly over one side with surface density σ rotates about its axis with an angular velocity ω. Find:
(a) the magnetic induction at the centre of the disc;
(b) the magnetic moment of the disc.

Ans. (a) $B = 1/2\ \mu_0\sigma\omega R$; (b) $p_m = 1/4\ \pi\sigma\omega R^4$

Sol. (a) Let us take a ring element of radius r and thickness dr, then charge on the ring element.,

$$dq = \sigma 2\pi\ r\ dr$$

and current, due to this element, $di = \frac{(\sigma 2\pi r\, dr)\omega}{2\pi} = \sigma\omega r\, dr$

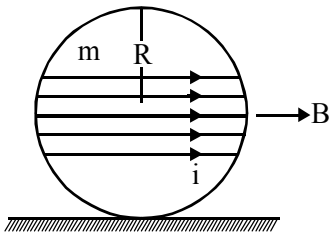
So, magnetic induction at the centre, due to this element : $dB = \frac{\mu_0}{2}\frac{di}{r}$

and hence, from symmetry : $B = \int dB = \int_0^R \frac{\mu_0\sigma\omega\, r\, dr}{r} = \frac{\mu_0}{2}\sigma\omega R$

(b) Magnetic moment of the element, considered,
$dp_m = (di)\,\pi r^2 = \sigma\omega dr\pi\, r^2 = \sigma\pi\omega r^3\, dr$
Hence, the sought magnetic moment,

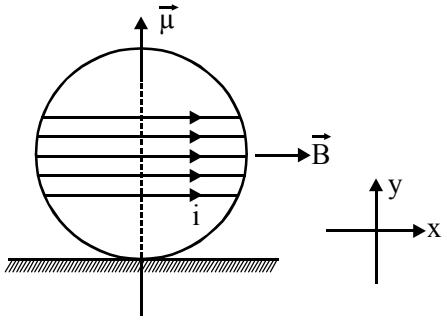
$$p_m = \int dp_m = \int_0^R \sigma\pi\omega r^3\, dr = \sigma\omega\pi\frac{R^4}{4}$$

Ex. A wire is wrapped N = 10 times over a solid sphere of mass m = 5kg, current I = 2A, which is placed on a smooth horizontal surface. A horizontal magnetic field of induction $|\vec{B}| = 10T$ is present. Find the angular acceleration experienced by the sphere. Assume that the mass of the wire is negligible compared to the mass of the sphere. If answer is $20n\pi$. Write value of n.



Ans. 5

Sol. (a) The net torque acting on the sphere is



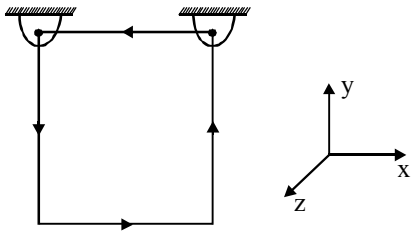
$\vec{\tau} = \vec{\mu}\times\vec{B} = \left(NiA\hat{J}\right)\times\left(B\hat{i}\right) = -NiAB\,\hat{k}$, where $A = \pi R^2$

or $\vec{\tau} = -N\pi R^2 iB\hat{k}$

(b) $\vec{\alpha} = \frac{\vec{\tau}}{I_C}$ ($\because$ the sphere is free to rotate, it must rotate about the centroidal axis)

$$= -\frac{N\pi R^2 iB}{\frac{2}{5}mR^2}\hat{k} \qquad \left(\because I_C = \frac{2}{5}mR^2\right) \qquad = \frac{5N\pi iB}{2m}\hat{k}$$

Ex. A current carrying uniform square frame is suspended from hinged supports as shown in the figure such that it can freely rotate about its upper side. The length and mass of each side of the frame is 2m and 4kg respectively. A uniform magnetic field $\vec{B}=\left(3\hat{i}+4\hat{j}\right)$ is applied. When the wire frame is rotated to 45° from vertical and released it remains in equilibrium. If the magnitude of current (in A) in the wire frame is I then find $\left(\frac{3}{5}\right)I$.



Ans. 6

Sol. $\vec{\mu}$ (Magnetic moment of loop) when it is lifted by 45° = $i\ell^2\left(\frac{\hat{j}+\hat{k}}{\sqrt{2}}\right)$

$\therefore$ $\vec{\tau}$ due to magnetic field = $\vec{\mu}\times\vec{B}=\frac{i\ell^2}{\sqrt{2}}\left[\left(\hat{j}+\hat{k}\right)\times\left(3\hat{i}+4\hat{j}\right)\right]$

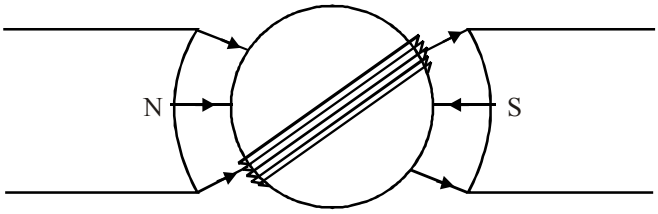
$\vec{\tau}$ due to mg (about top edge) = $4mg\frac{\ell}{2}\cos 45°\hat{i}$

$\therefore$ For equilibrium net torque along X-axis = 0

$\therefore$ $\frac{4mg\ell}{2\sqrt{2}}=\frac{4i\ell^2}{\sqrt{2}}\Rightarrow i=\frac{mg}{2\ell}=10A$

MOVING COIL GALVANOMETER :

The main parts of a moving-coil galvanometer are shown in figure.



The current to be measured is passed through the galvanometer. As the coil is in the magnetic field $\vec{B}$ of the permanent magnet, a torque $\vec{\Gamma}=ni\vec{A}\times\vec{B}$ acts on the coil. Here n = number of turns, i = current in the coil $\vec{A}$ = area-vector of the coil and $\vec{B}$ = magnetic field at the site of the coil. This torque deflects the coil from its equilibrium position.

The pole pieces are made cylindrical. As a result, the magnetic field at the arms of the coil remains parallel to the plane of the coil everywhere even as the coil rotates. The deflecting torque is then $\Gamma = niAB$. As the upper end of the suspension strip W is fixed, the strip gets twisted when the coil rotates. This produces a restoring torque acting on the coil. If the deflection of the coil is θ and the torsional constant of the suspension strip is k, the restoring torque is kθ. The coil will stay at a deflection θ where

$$niAB = k\theta$$

or, $$i=\frac{k}{nAB}\theta$$

Hence, the current is proportional to the deflection. The constant $\frac{k}{nAB}$ is called the galvanometer constant.

We define the **current sensitivity** of the galvanometer as the deflection per unit current. From Eq. this current sensitivity is.

$$\frac{\phi}{I}=\frac{NAB}{k}$$

A convenient way for the manufacturer to increase the sensitivity is to increase the number of turns N. We choose galvanometers having sensitivities of value, required by our experiment.

We define the **voltage sensitivity** as the deflection per unit volt of applied potential difference

$$\frac{\phi}{I}=\left(\frac{NAB}{k}\right)\frac{I}{V}=\left(\frac{NAB}{k}\right)\frac{1}{R}$$

An interesting point to note is that increasing the current sensitivity may not necessarily increase the voltage sensitivity. If $N \rightarrow 2N$, i.e., we double the number of turns, then

$$\frac{\phi}{I}\rightarrow 2\frac{\phi}{I}$$

Thus, the current sensitivity doubles. However, the resistance of the galvanometer is also likely to double, since it is proportional to the length of the wire. In eq. $N \rightarrow 2N$, and $R \rightarrow 2R$, thus the voltage sensitivity,

$$\frac{\phi}{V}\rightarrow\frac{\phi}{V}$$

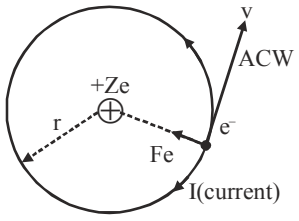
remains unchanged.

ATOMIC MAGNETISM

An atomic orbital electron, which doing bounded uniform circular motion around nucleus. A current constitues with this orbital motion and hence orbit behaves like current carrying loop. Due to this magnetism produces at nucleus position. This phenomenon called as 'atomic magnetism.

Bohr's postulates :

(i) $\frac{mv^2}{r}=\frac{kze^2}{r^2}$ (ii) $L = mvr = n\left(\frac{h}{2\pi}\right)$, where n = 1, 2,3 .....



Basic elements of atomic magnetism :

(a) **Orbital current :-** $I = ef = \frac{e}{T} = \frac{ev}{2\pi r} = \frac{e\omega}{2\pi}$

(b) **Magnetic induction at nucleus position :-** As circular orbit behaves like current carrying loop, so magnetic induction at nucleus position $B_N=\frac{\mu_0 I}{2r}$

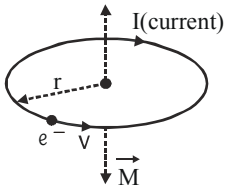
$$B_N=\frac{\mu_0 ef}{2r}=\frac{\mu_0 e}{2Tr}=\frac{\mu_0 ev}{4\pi r^2}=\frac{\mu_0 e\omega}{4\pi r}$$

(c) **Magnetic moment of circular orbit :-** Magnetic dipole moment of circular orbit

M = IA where A is area of circular orbit. $M = ef\left(\pi r^2\right) = \frac{\pi er^2}{T} = \frac{evr}{2} = \frac{e\omega r^2}{2}$

- **Relation between magnetic moment and angular momentum of orbital electron**

Magnetic moment $M = \frac{evr}{2} \times \frac{m}{m} = \frac{eL}{2m}$ ($\because$ angular momentum L = mvr)



Vector form $\vec{M} = \frac{-e\vec{L}}{2m}$

For orbital electron its $\vec{M}$ and $\vec{L}$ both are antiparallel axial vectors.

A NONCONDUCTING CHARGED BODY IS ROTATED WITH SOME ANGULAR SPEED.

In this case the ratio of magnetic moment and angular momentum is constant which is equal to $\frac{q}{2m}$

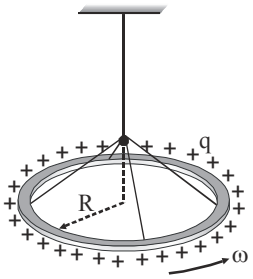
here q = charge and m = the mass of the body.

Ex. :- In case of a ring, of mass m, radius R and charge q distributed on it circumference.

Angular momentum $L = I\omega = (mR^2)(\omega)$ ... (i)

Magnetic moment $M = iA = (qf)(\pi R^2)$

$$M = (q)\left(\frac{\omega}{2\pi}\right)(\pi R^2) = q\frac{\omega R^2}{2} \text{ ...(ii)}$$



$\because f = \frac{\omega}{2\pi}$ From Eqs. (i) and (ii) $\frac{M}{L} = \frac{q}{2m}$

Although this expression is derived for simple case of a ring, it holds good for other bodies also. For example, for a disc or a sphere. $M = \frac{qL}{2m} \Rightarrow M = \frac{q(I\omega)}{2m}$, where $L = I\omega$

Rigid body	**Ring**	**Disc**	**Solid sphere**	**Spherical shell**
Moment of inertia (I)	mR^2	$\frac{mR^2}{2}$	$\frac{2}{5}mR^2$	$\frac{2}{3}mR^2$
Magnetic moment= $\frac{qI\omega}{2m}$	$\frac{q\omega R^2}{2}$	$\frac{q\omega R^2}{4}$	$\frac{q\omega R^2}{5}$	$\frac{q\omega R^2}{3}$

EXERCISE (S)

Biot savart law

1. Two long, straight wires, each carrying a current of 5 A, are placed along the X- and Y-axes respectively. The currents point along the positive directions of the axes. Find the magnetic field at the points (a) (1 m, 1 m), (b) (–1 m, 1 m), (c) (– 1 m, – 1 m) and (d) (1 m, – 1 m).

MG0001

2. A circular loop of radius 4.0 cm is placed in a horizontal plane and carries an electric current of 5.0 A in the clockwise direction as seen from above. Find the magnetic field
(a) At a point 3.0 cm above the centre of the loop.
(b) At a point 3.0 cm below the centre of the loop.

MG0002

3. Two concentric circular coils X and Y of radii 16 cm and 10 cm, respectively, lie in the same vertical plane containing the north to south direction. Coil X has 20 turns and carries a current of 16A ; coil Y has 25 turns and carries a current of 18 A. The sense of the current in X is anticlockwise, and clockwise in Y, for an observer looking at the coils facing west. Give the magnitude and direction of the net magnetic field due to the coils at their centre. ***(NCERT)***

MG0003

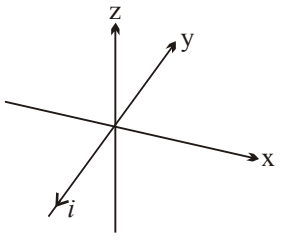
4. A current element $\Delta\vec{\ell} = \Delta x\,\hat{i} - \Delta y\,\hat{j}$ carries 10 A current. It is placed at origin. Calculate magnetic field at point 'P' which is at position vector $\vec{r} = (\hat{i} + \hat{j})$ m with respect to origin. (where $\Delta x = \Delta y = 1$mm)

MG0004

5. A circular loop of radius r carries a current i. How should a long, straight wire carrying a current 4i be placed in the plane of the circle so that the magnetic field at the centre becomes zero ?

MG0005

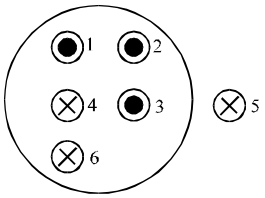
6. A long straight wire carries a current of 10 A directed along the negative y-axis as shown in figure. A uniform magnetic field B_0 of magnitude 10^{-6} T is directed parallel to the x-axis. What is the resultant magnetic field at the following points?
(a) x = 0, z = 2 m; (b) x = 2 m, z = 0; (c) x = 0, z = – 0.5 m



MG0006

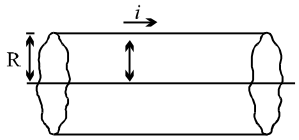
Ampere's law

7. Six wires of current $I_1 = 1A, I_2 = 2A, I_3 = 3A, I_4 = 1A, I_5 = 5A$ and $I_6 = 4A$ cut the page perpendicularly at the points 1, 2, 3, 4, 5 and 6 respectively as shown in the figure. Find the value of the integral $\oint \vec{B}.d\vec{\ell}$ around the closed path.



MG0007

8. A cylindrical conductor of radius R carries a current along its length. The current density J, however, it is not uniform over the cross section of the conductor but is a function of the radius according to J = br, where b is a constant. Find an expression for the magnetic field B.
(a) at $r_1 < R$ & (b) at distance $r_2 > R$, measured from the axis



MG0017

Motion of charged particle

9. A charged particle (charge q, mass m) has velocity v_0 at origin in +x direction. In space there is a uniform magnetic field B in – z direction. Find the y coordinate of particle when is crosses y axis.

MG0008

10. A beam of protons with a velocity 4×10^5 m/s enters a uniform magnetic field of 0.3 T at an angle of 60° to the magnetic field. Find the radius of the helical path taken by the proton beam. Also find the pitch of the helix (which is the distance travelled by a proton in the beam parallel to the magnetic field during one period of rotation).

MG0009

11. An electron emitted by a heated cathode and accelerated through a potential difference of 2.0 kV, enters a region with uniform magnetic field of 0.15 T. Determine the trajectory of the electron if the field (a) is transverse to its initial velocity, (b) makes an angle of 30°with the initial velocity.***(NCERT)***

MG0010

12. A particle of mass 1×10^{-26} kg and charge $+1.6 \times 10^{-19}$ C travelling with a velocity 1.28×10^6 m/s in the +x direction enters a region in which a uniform electric field E and a uniform magnetic field of induction B are present such that $E_x = E_y = 0$, $E_z = -102.4$ kV/m and $B_x = B_z = 0$, $B_y = 8 \times 10^{-2}$ weber/m^2. The particle enters this region at the origin at time t = 0. Determine the location (x, y and z coordinates) of the particle at $t = 5 \times 10^{-6}$ s. If the electric field is switched off at this instant (with the magnetic field still present), what will be the position of the particle at $t = 7.45 \times 10^{-6}$ s ?

MG0019

Ampere force & torque

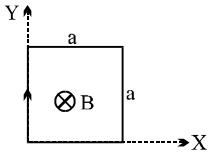
13. A straight horizontal conducting rod of length 0.45 m and mass 60g is suspended by two vertical wires at its ends. A current of 5.0 A is set up in the rod through the wires. ***(NCERT)***

(a) What magnetic field should be set up normal to the conductor in order that the tension in the wires is zero ?

(b) What will be the total tension in the wires if the direction of current is reversed keeping the magnetic field same as before ? [Ignore the mass of the wires). $g = 9.8\ ms^{-2}$.

MG0011

14. A rectangular loop of wire is oriented with the left corner at the origin, one edge along X-axis and the other edge along Y-axis as shown in the figure. A magnetic field is into the page and has a magnitude that is given by $\beta = \alpha y$ where α is contant. Find the total magnetic force on the loop if it carries current i.



MG0032

15. A circular coil of 20 turns and radius 10 cm is placed in a uniform magnetic field of 0.10 T normal to the plane of the coil. If the current in the coil is 5.0 A, what is the ***(NCERT)***

(a) total, torque on the coil,

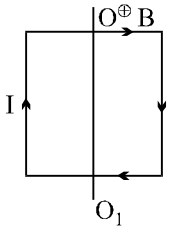
(b) total force on the coil,

(c) average force on each electron in the coil due to the magnetic field ?

(The coil is made of copper wire of cross-sectional area $10^{-5}\ m^2$, and the free electron density in copper is given to be about $10^{29}\ m^{-3}$.)

MG0014

16. A square current carrying loop made of thin wire and having a mass m =10g can rotate without friction with respect to the vertical axis OO_1, passing through the centre of the loop at right angles to two opposite sides of the loop. The loop is placed in a homogeneous magnetic field with an induction $B = 10^{-1}$ T directed at right angles to the plane of the drawing. A current I = 2A is flowing in the loop. Find the period of small oscillations that the loop performs about its position of stable equilibrium.



MG0015

17. Two moving coil meters. M_1 and M_2 have the following particulars : ***(NCERT)***

$R_1 = 10\ \Omega$, $N_1 = 30$,

$A_1 = 3.6 \times 10^{-3}\ m^2$. $B_1 = 0.25$ T

$R_2 = 14\ \Omega$, $N_2 = 42$ $A_2 = 1.8 \times 10^{-3}\ m^2$, $B_2 = 0.50$ T

(The spring constants are identical for the two meters). Determine the ratio of (a) current sensitivity and (b) voltage sensitivity of M_2 and M_1.

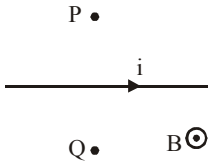
MG0016

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

Biot savart law

1. A long, straight wire carrying a current of 1.0 A is placed horizontally in a uniform magnetic field $B = 1.0 \times 10^{-5}$ T pointing vertically upward (figure). The magnitude of the resultant magnetic field at the points P and Q, both situated at a distance of 2.0 cm from the wire in the same horizontal plane are respectively



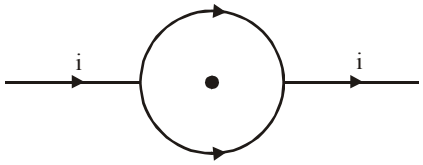
(A) zero, 20 μT (B) 20 μT, zero (C) zero, zero (D) 20 μT, 20 μT

MG0036

2. Two parallel wires carry equal currents of 10 A along the same direction and are separated by a distance of 2.0 cm. Find the magnetic field at a point which is 2.0 cm away from each of these wires.
(A) 3.4×10^{-4} T in a direction parallel to the plane of the wires and perpendicular to the wires
(B) 1.7×10^{-4} T in a direction parallel to the plane of the wires and parallel to the wires
(C) 1.7×10^{-4} T in a direction parallel to the plane of the wires and perpendicular to the wires
(D) 3.4×10^{-4} T in a direction parallel to the plane of the wires and parallel to the wires

MG0038

3. A conducting circular loop of radius a is connected to two long, straight wires. The straight wires carry a current i as shown in figure. Find the magnetic field B at the centre of the loop.



(A) zero (B) $\frac{\mu_0 I}{2a}$ (C) $\frac{\mu_0 I}{a}$ (D) $\frac{\mu_0 I}{a} + \frac{\mu_0 I}{2\pi a}$

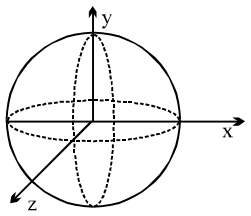
MG0039

4. A piece of wire carrying a current of 6.00 A is bent in the form of a circular arc of radius 10.0 cm, and it subtends an angle of 120° at the centre. Find the magnetic field B due to this piece of wire at the centre.

(A) zero (B) 1.26×10^{-5} T (C) 5×10^{-5} T (D) 7.2×10^{-5} T

MG0040

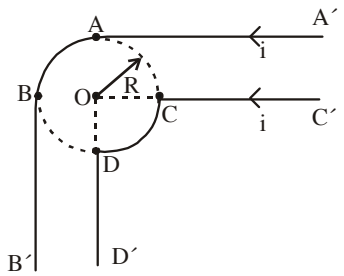
5. Three rings, each having equal radius R, are placed mutually perpendicular to each other and each having its centre at the origin of co-ordinate system. If current I is flowing through each ring then the magnitude of the magnetic field at the common centre is



(A) $\sqrt{3}\frac{\mu_0 I}{2R}$ (B) zero (C) $\left(\sqrt{2}-1\right)\frac{\mu_0 I}{2R}$ (D) $\left(\sqrt{3}-\sqrt{2}\right)\frac{\mu_0 I}{2R}$

MG0088

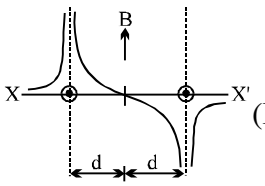
6. All straight wires are very long. Both AB and CD arc area of the same circle, both subtending right angles at the centre O. Then the magnetic field at O is

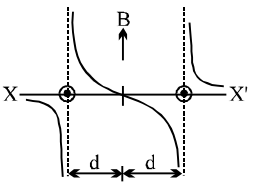


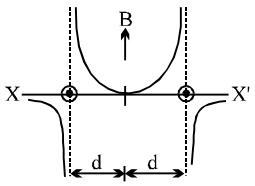
(A) $\frac{\mu_0 i}{4\pi R}$ (B) $\frac{\mu_0 i}{4\pi R}\sqrt{2}$ (C) $\frac{\mu_0 i}{2\pi R}$ (D) $\frac{\mu_0 i}{2\pi R}(\pi+1)$

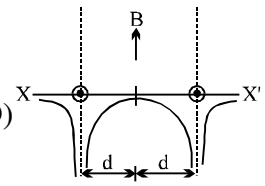
MG0041

7. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper, as shown. The variation of the magnetic field B along the XX' is given by

(A) 

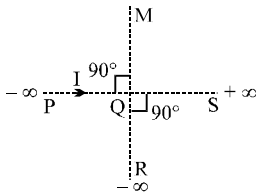
(B) 

(C) 

(D) 

MG0044

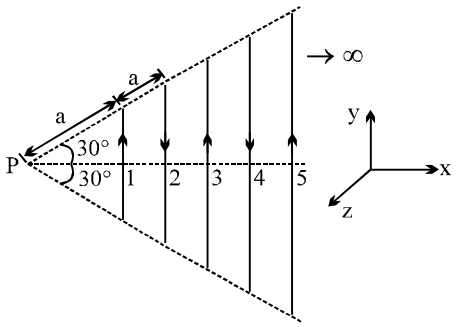
8. An infinitely long conductor PQR is bent to form a right angle as shown. A current I flows through PQR. The magnetic field due to this current at the point M is H_1. Now, another infinitely long straight conductor QS is connected at Q so that the current in PQ remaining unchanged. The magnetic field at M is now H_2. The ratio H_1/H_2 is given by :-



(A) 1/2 (B) 1 (C) 2/3 (D) 2

MG0143

9. Infinite number of straight wires each carrying current I are equally placed as shown in the figure. Adjacent wires have current in opposite direction. Net magnetic field at point P is :-

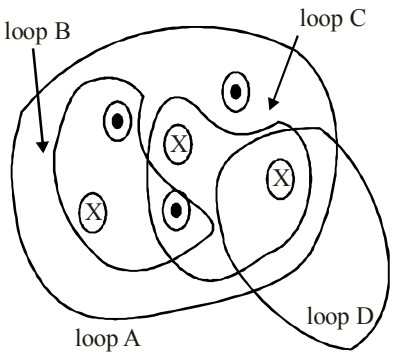


(A) $\frac{\mu_0 I}{4\pi}\frac{l\text{n}\,2}{\sqrt{3}\,a}\hat{k}$ (B) $\frac{\mu_0 I}{4\pi}\frac{l\text{n}\,4}{\sqrt{3}\,a}\hat{k}$ (C) $\frac{\mu_0 I}{4\pi}\frac{l\text{n}\,4}{\sqrt{3}\,a}(-\hat{k})$ (D) Zero

MG0146

Ampere's law

10. Consider six wires coming into or out of the page, all with the same current. Rank the line integral of the magnetic field (from most positive to most negative) taken counterclockwise around each loop shown.



(A) B > C > D > A (B) B > C = D > A (C) B > A > C = D (D) C > B = D > A

MG0047

11. **Statement-1 :** Ampere law can be used to find magnetic field due to finite length of a straight current carrying wire.
Statement-2 : The magnetic field due to finite length of a straight current carrying wire is symmetric about the wire.
(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

MG0048

12. A long, cylindrical wire of radius b carries a current i distributed uniformly over its cross-section. Find the magnitude of the magnetic field at a point inside the wire at a distance a from the axis.

(A) zero (B) $\frac{\mu_0 ib}{2\pi a^2}$ (C) $\frac{\mu_0 ia^2}{2\pi b^3}$ (D) $\frac{\mu_0 ia}{2\pi b^2}$

MG0049

13. A copper wire of diameter 1.6 mm carries a current of 20 A. Find the maximum magnitude of the magnetic field $\vec{B}$ due to this current.
(A) 5.0 mT (B) 10 mT (C) 15 mT (D) 15.5 mT

MG0050

14. A closely wound solenoid 80 cm long has 5 layers of windings of 400 turns each. The diameter of the solenoid is 1.8 cm. If the current carried is 8.0 A. estimate the magnitude of B inside the solenoid near its centre. ***(NCERT)***
(A) zero (B) $8\pi \times 10^{-3}$ T (C) $15\pi \times 10^{-3}$ T (D) $\pi \times 10^{-3}$ T

MG0051

15. A hollow cylinder having infinite length and carrying uniform current per unit length λ along the circumference as shown. Magnetic field inside the cylinder is :-

(A) $\frac{\mu_0 \lambda}{2}$ (B) $\mu_0 \lambda$ (C) $2\mu_0 \lambda$ (D) none

MG0150

Motion of charge particle

16. A charged particle enters a non-uniform uni-directional field such that initial velocity is parallel to magnetic field, then the radius of curvature of its path is (in standard notation):
(A) mV/qB (B) 0 (C) ∞ (D) qB/mV

MG0059

17. A charge particle moves in a uniform magnetic field such that initial velocity is perpendicular to the magnetic field . No other force acts on the particle .
(A) the motion is uniform rectilinear
(B) the motion can be non uniform circular motion
(C) the motion can be uniform circular motion
(D) the motion must be uniform circular motion .

MG0060

18. A tightly-wound, long solenoid carries a current of 2.00 A. An electron is found to execute a uniform circular motion inside the solenoid with a frequency of 1.00×10^8 rev/s. Find the number of turns per metre in the solenoid.
(A) 500 Turns/m (B) 1020 Turns/m
(C) 1232 Turns/m (D) 1420 Turns/m

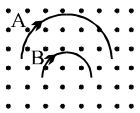
MG0061

19. An electron having kinetic energy T is moving in a circular orbit of radius R perpendicular to a uniform magnetic induction $\vec{B}$. If kinetic energy is doubled and magnetic induction tripled, the radius will become

(A) $\frac{3R}{2}$ (B) $\sqrt{\frac{3}{2}}\,R$ (C) $\sqrt{\frac{2}{9}}\,R$ (D) $\sqrt{\frac{4}{3}}\,R$

MG0064

20. Two particles A and B of masses m_A and m_B respectively and having the same charge are moving in a plane. A uniform magnetic field exists perpendicular to this plane. The speeds of the particles are v_A and v_B respectively and the trajectories are as shown in the figure. Then **[JEE, 2001 (Scr)]**



(A) $m_A v_A < m_B v_B$ (B) $m_A v_A > m_B v_B$
(C) $m_A < m_B$ and $v_A < v_B$ (D) $m_A = m_B$ and $v_A = v_B$

MG0065

21. A charged particle moves in a magnetic field $\vec{B} = 10\hat{i}$ with initial velocity $\vec{u} = 5\hat{i} + 4\hat{j}$. The path of the particle will be
(A) straight line (B) circle (C) helical (D) none

MG0066

22. An electron makes 3×10^5 revolutions per second in a circle of radius 0.5 angstrom. Find the magnetic field B at the centre of the circle.
(A) 6×10^{-10} T (B) 12×10^{-10} T (C) 18×10^{-10} T (D) 24×10^{-10} T

MG0067

23. Electrons moving with different speeds enter a uniform magnetic field in a direction perpendicular to the field. They will move along circular paths.
(A) of same radius
(B) with larger radii for the faster electrons
(C) with smaller radii for the faster electrons
(D) either (B) or (C) depending on the magnitude of the magnetic field

MG0068

24. In the previous question, time periods of rotation will be :
(A) same for all electrons
(B) greater for the faster electrons
(C) smaller for the faster electrons
(D) either (B) or (C) depending on the magnitude of the magnetic field

MG0069

25. A particle of mass m and charge q moves with a constant velocity v along the positive x direction. It enters a region containing a uniform magnetic field B directed along the negative z direction, extending from x = a to x = b. The minimum value of v required so that the particle can just enter the region x > b is :- **JEE 2002 (screening)]**

(A) q b B/m (B) q(b – a) B/m (C) q a B/m (D) q(b + a) B/2m

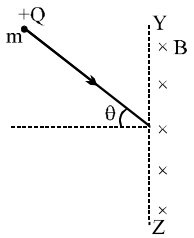
MG0070

26. A particle having charge of 1 C, mass 1 kg and speed 1 m/s enters a uniform magnetic field, having magnetic induction of 1 T, at an angle $\theta = 30^\circ$ between velocity vector and magnetic induction. The pitch of its helical path is (in meters)

(A) $\frac{\sqrt{3}\pi}{2}$ (B) $\sqrt{3}\pi$ (C) $\frac{\pi}{2}$ (D) π

MG0073

27. A particle with charge +Q and mass m enters a magnetic field of magnitude B, existing only to the right of the boundary YZ. The direction of the motion of the particle is perpendicular to the direction of B. Let $T = 2\pi\frac{m}{QB}$. The time spent by the particle in the field will be



(A) Tθ (B) 2Tθ (C) $T\left(\frac{\pi+2\theta}{2\pi}\right)$ (D) $T\left(\frac{\pi-2\theta}{2\pi}\right)$

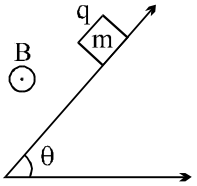
MG0074

28. In the previous question, if the particle has –Q charge, the time spend by the particle in the field will be :-

(A) Tθ (B) 2Tθ (C) $T\left(\frac{\pi+2\theta}{2\pi}\right)$ (D) $T\left(\frac{\pi-2\theta}{2\pi}\right)$

MG0075

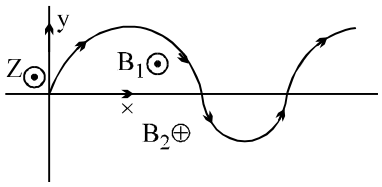
29. A block of mass m & charge q is released on a long smooth inclined plane magnetic field B is constant, uniform, horizontal and parallel to surface as shown. Find the time from start when block loses contact with the surface.



(A) $\frac{m\cos\theta}{qB}$ (B) $\frac{m\,\text{cosec}\,\theta}{qB}$ (C) $\frac{m\cot\theta}{qB}$ (D) none

MG0076

30. At t = 0 a charge q is at the origin and moving in the y-direction with velocity $\vec{v} = v\hat{j}$. The charge moves in a magnetic field that is for y > 0 out of page and given by $B_1\hat{z}$ and for y < 0 into the page and given $-B_2\hat{z}$. The charge's subsequent trajectory is shown in the sketch. From this information, we can deduce that



(A) q > 0 and $|B_1| < |B_2|$ (B) q < 0 and $|B_1| < |B_2|$
(C) q > 0 and $|B_1| > |B_2|$ (D) q < 0 and $|B_1| > |B_2|$

MG0153

31. A particle of specific charge (charge/mass) α starts moving from the origin under the action of an electric field $\vec{E} = E_0\hat{i}$ and magnetic field $\vec{B} = B_0\hat{k}$. Its velocity at $(x_0, y_0, 0)$ is $(4\hat{i} - 3\hat{j})$. The value of x_0 is:

(A) $\frac{13}{2}\frac{\alpha E_0}{B_0}$ (B) $\frac{16\alpha B_0}{E_0}$ (C) $\frac{25}{2\alpha E_0}$ (D) $\frac{5\alpha}{2B_0}$

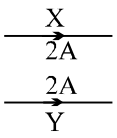
MG0154

32. In a cyclotron, a charged particle
(A) undergoes acceleration all the time.
(B) speeds up between the dees because of the magnetic field.
(C) speeds up in a dee.
(D) slows down within a dee and speeds up between dees.

MG0077

Ampere force & torque

33. In given figure, X and Y are two long straight parallel conductors each carrying a current of 2 A. The force on each conductor is F newtons. When the current in each is changed to 1 A and reversed in direction, the force on each is now



(A) F/4 and unchanged in direction (B) F/2 and reversed in direction

(C) F/2 and unchanged in direction (D) F/4 and reversed in direction

MG0078

34. A wire of mass 100 g carrying a current of 2A towards increasing x is in the form of $y = x^2$ ($-2m \le x \le +2m$). This wire is placed in a magnetic field $B = -0.02\hat{k}$ Tesla & gravity free region. The acceleration of the wire (in m/s^2) is:-

(A) $-1.6\hat{j}$ (B) $-3.2\hat{j}$ (C) $1.6\hat{j}$ (D) $2.4\hat{j}$

MG0081

35. A very long wire carrying current I is fixed along x-axis. Another parallel finite wire carrying a current in the opposite direction is kept at a distance d above the wire in xy plane. The second wire is free to move parallel to itself. The options available for its small displacements are in

(i) +ve x direction (ii) +ve y direction (iii) +ve z direction

Taking gravity in negative y direction, the nature of equilibrium of second wire is

(A) stable for movement in x direction, unstable for movement in y direction, neutral for movement in z direction

(B) stable for movement in y direction, unstable for movement in z direction, neutral for movement in x direction

(C) stable for movement in z direction, unstable for movement in y direction, neutral for movement in x direction

(D) stable for movement in y direction, unstable for movement in x direction, neutral for movement in z direction

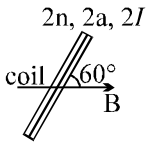
MG0091

36. A circular loop of radius R carries a current I. Another circular loop of radius r(<<R) carries a current i and is placed at the centre of the larger loop. The planes of the two circles are at right angle to each other. Find the torque acting on the smaller loop.

(A) zero (B) $\frac{\mu_0 \pi i I r^2}{4R}$ (C) $\frac{\mu_0 \pi i I r^2}{2R}$ (D) $\frac{\mu_0 \pi i I r^2}{R}$

MG0082

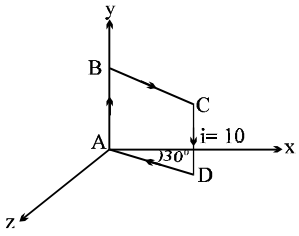
37. A rectangular coil PQ has 2n turns, an area 2a and carries a current 2*I*, (refer figure). The plane of the coil is at 60° to a horizontal uniform magnetic field of flux density B. The torque on the coil due to magnetic force is :-



(A) Bna*I* sin60° (B) 8Bna*I* cos60° (C) 4na*I* Bsin60° (D) none

MG0084

38. Figure shows a square current carrying loop ABCD of side 10 cm and current i = 10A. The magnetic moment $\vec{M}$ of the loop is

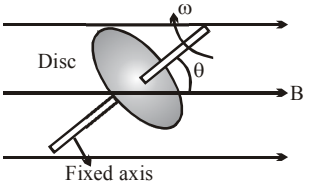


(A) (0.05) $\left(\hat{i}-\sqrt{3}\hat{k}\right)A-m^2$ (B) (0.05) $\left(\hat{j}+\hat{k}\right)A-m^2$

(C) (0.05) $\left(\sqrt{3}\hat{i}+\hat{k}\right)A-m^2$ (D) $\left(\hat{i}+\hat{k}\right)A-m^2$

MG0086

39. A disc of radius r and carrying positive charge q is rotating with an angular speed ω in a uniform magnetic field B about a fixed axis as shown in figure, such that angle made by axis of disc with magnetic field is θ. Torque applied by axis on the disc is

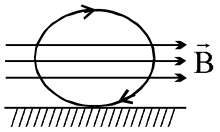


(A) $\frac{q\omega r^2 B\sin\theta}{2}$, clockwise (B) $\frac{q\omega r^2 B\sin\theta}{4}$, anticlockwise

(C) $\frac{q\omega r^2 B\sin\theta}{2}$, anticlockwise (D) $\frac{q\omega r^2 B\sin\theta}{4}$, clockwise

MG0087

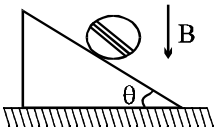
40. A conducting ring of mass 2 kg and radius 0.5 m is placed on a smooth horizontal plane. The ring carries a current i = 4A. A horizontal magnetic field B = 10T is switched on at time t = 0 as shown in figure. The initial angular acceleration of the ring will be :-



(A) 40 π rad/s^2 (B) 20 π rad/s^2 (C) 5 π rad/s^2 (D) 15 π rad/s^2

MG0159

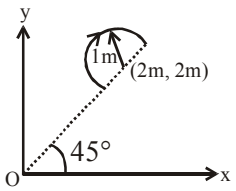
41. In the figure shown a coil of single turn is wound on a sphere of radius R and mass m. The plane of the coil is parallel to the plane and lies in the equatorial plane of the sphere. Current in the coil is i. The value of B if the sphere is in equilibrium is :-



(A) $\frac{mg\cos\theta}{\pi iR}$ (B) $\frac{mg}{\pi iR}$ (C) $\frac{mg\tan\theta}{\pi iR}$ (D) $\frac{mg\sin\theta}{\pi iR}$

MG0160

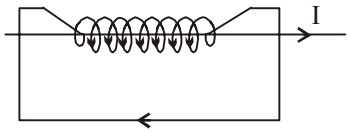
42. A uniform magnetic field $\vec{B}=\left(3\hat{i}+4\hat{j}+\hat{k}\right)$ Tesla exist in a region of space. A semicircular wire of radius 1 m carrying a current of 1A having its centre at (2,2,0) m is placed on the X-Y plane as shown. The force on the semicircular wire will be



(A) $\frac{1}{\sqrt{2}}\left(\hat{i}-\hat{j}+\hat{k}\right)$N (B) $\sqrt{2}\left(\hat{i}-\hat{j}+\hat{k}\right)$N (C) $\frac{1}{\sqrt{2}}\left(\hat{i}+\hat{j}-\hat{k}\right)$N (D) $\sqrt{2}\left(\hat{i}+\hat{j}-\hat{k}\right)$N

MG0090

43. In the diagram shown, a wire carries current I. What is the value of the $\oint\vec{B}\cdot d\vec{s}$ (as in Ampere's law) on the helical loop shown in the figure? The integration in done in the sense shown. The loop has N turns and part of helical loop on which arrows are drawn is outside the plane of paper.



(A) $-\mu_0(NI)$ (B) $\mu_0(I)$ (C) $\mu_0(NI)$ (D) Zero

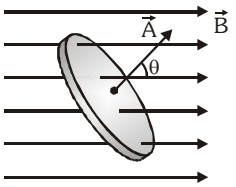
MG0147

ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

KEY CONCEPT

MAGNETIC FLUX

The magnetic flux (ϕ) linked with a surface held in a magnetic field (B) is defined as the number of magnetic lines of force crossing that area (A). If θ is the angle between the direction of the field and normal to the area, (area vector) then $\phi = \vec{B}.\vec{A} = BA\cos\theta$



FLUX LINKAGE

If a coil has more than one turn, then the flux through the whole coil is the sum of the fluxes through the individual turns. If the magnetic field is uniform, the flux through one turn is $\phi = BA\cos\theta$ If the coil has N turns, the total flux linkage $\phi = NBA\cos\theta$

- Magnetic lines of force are imaginary, magnetic flux is a real scalar physical quantity with dimensions

$$[\phi] = B \times \text{area} = \left[\frac{F}{IL}\right][L^2] \qquad \because\ B = \frac{F}{IL\sin\theta} \qquad [\because\ F = B\,I\,L\sin\theta]$$

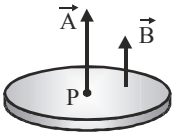
$$\therefore \qquad [\phi] = \left[\frac{M\,L\,T^2}{A\,L}\right][L^2] = [M\,L^2\,T^{-2}\,A^{-1}]$$

SI UNIT of magnetic flux :

$\because$ $[M\,L^2T^{-2}]$ corresponds to energy

$$\frac{\text{joule}}{\text{ampere}} = \frac{\text{joule} \times \text{second}}{\text{coulomb}} = \text{weber (Wb)}$$

or $T\text{–}m^2$ (as tesla = Wb/m^2) $\left[\text{ampere} = \frac{\text{coulomb}}{\text{sec ond}}\right]$

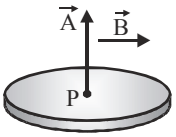


$\phi_{max} = B\,A$

- For a given area flux will be maximum :

when magnetic field $\vec{B}$ is normal to the area

$\theta = 0^\circ \quad \Rightarrow \cos\theta = \text{maximum} = 1 \quad \phi_{max} = B\,A$



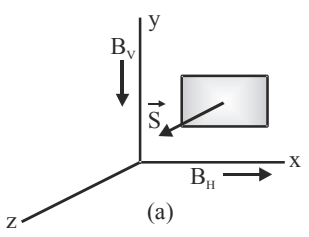
For a given area flux will be minimum :

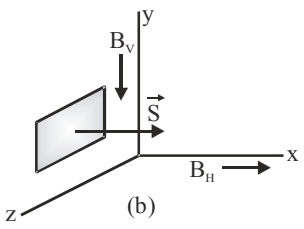
when magnetic field $\vec{B}$ is parallel to the area

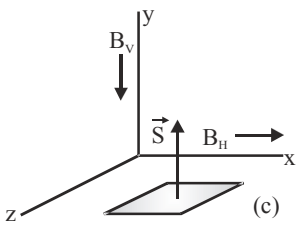
$\theta = 90^\circ \Rightarrow \cos\theta = \text{minimum} = 0 \qquad \phi_{min} = 0$

$\phi_{min} = 0$

Ex. At a given plane, horizontal and vertical components of earth's magnetic field B_H and B_V are along x and y axes respectively as shown in figure. What is the total flux of earth's magnetic field associated with an area S, if the area S is in (a) x-y plane (b) y-z plane and (c) z-x plane ?







Sol. $\vec{B} = \hat{i}B_H - \hat{j}B_V = \text{constant, so } \phi = \vec{B}.\vec{S}$ [$\vec{B}$ = constant]

(a) For area in x-y plane $\vec{S} = S\hat{k}$, $\phi_{xy} = (\hat{i}B_H - \hat{j}B_V).(\hat{k}S) = 0$

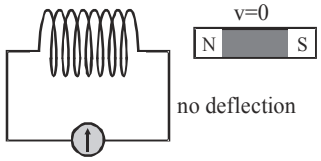
(b) For area S in y-z plane $\vec{S} = S\hat{i}$, $\phi_{yz} = (\hat{i}B_H - \hat{j}B_V).(\hat{i}S) = B_H S$

(c) For area S in z-x plane $\vec{S} = S\hat{j}$, $\phi_{zx} = (\hat{i}B_H - \hat{j}B_V).(\hat{j}S) = -B_V S$
Negative sign implies that flux is directed vertically downwards.

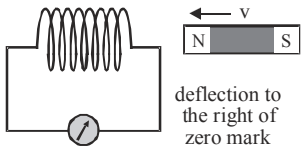
FARADAY'S EXPERIMENT

Faraday performed various experiments to discover and understand the phenomenon of electromagnetic induction. Some of them are :

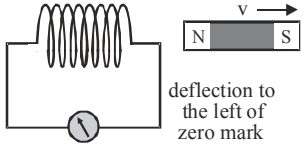
- When the magnet is held stationary anywhere near or inside the coil, the galvanometer does not show any deflection.



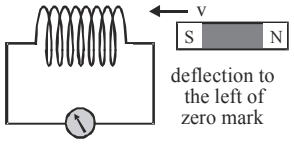
- When the N-pole of a strong bar magnet is moved towards the coil, the galvanometer shows a deflection right to the zero mark.



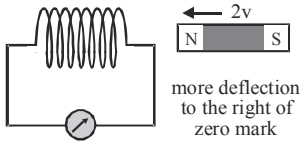
- When the N-pole of a strong bar magnet is moved away from the coil, the galvanometer shows a deflection left to the zero mark.



- If the above experiments are repeated by bringing the S-pole of the, magnet towards or away from the coil, the direction of current in the coil is opposite to that obtained in the case of N-pole.



- The deflection in galvanometer is more when the magnet moves faster and less when the magnet moves slow.



CONCLUSIONS

Whenever there is a relative motion between the source of magnetic field (magnet) and the coil, an emf is induced in the coil. When the magnet and coil move towards each other then the flux linked with the coil increases and emf is induced. When the magnet and coil move away from each other the magnetic flux linked with the coil decreases, again an emf is induced. This emf lasts so long the flux is changing. Due to this emf an electric current start to flow and the galvanometer shows deflection.
The deflection in galvanometer last as long the relative motion between the magnet and coil continues. Whenever relative motion between coil and magnet takes place an induced emf produced in coil. If coil is in closed circuit then current and charge is also induced in the circuit. This phenomenon is called electro magnetic induction.

FARADAY'S LAWS OF ELECTROMAGNETIC INDUCTION

Faraday's law of induction may be stated as follows:
The induced emf ε in a coil is proportional to the negative of the rate of change of magnetic flux:

$$\varepsilon = -\frac{d\Phi_B}{dt}$$

For a coil that consists of N loops, the total induced emf would be N times as large:

$$\varepsilon = -N\frac{d\Phi_B}{dt}$$

Thus, we see that an emf may be induced in the following ways:

(i) by varying the magnitude of $\vec{B}$ with time (illustrated in Figure)

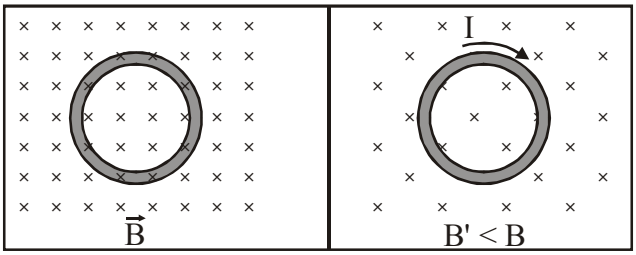


Figure : Inducing emf by varying the magnetic field strength

(ii) by varying the magnitude of $\vec{A}$, i.e., the area enclosed by the loop with time (illustrated in Figure)

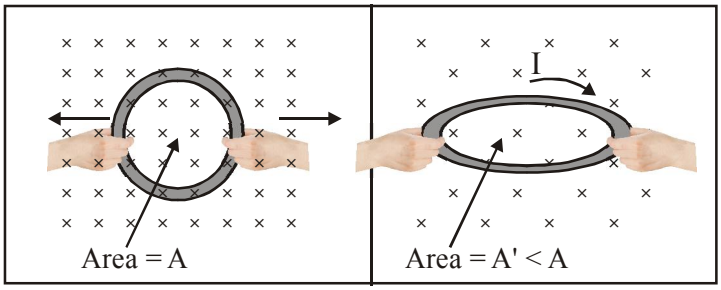


Figure : Inducing emf by changing the area of the loop

(iii) varying the angle between $\vec{B}$ and the area vector $\vec{A}$ with time (illustrated in Figure)

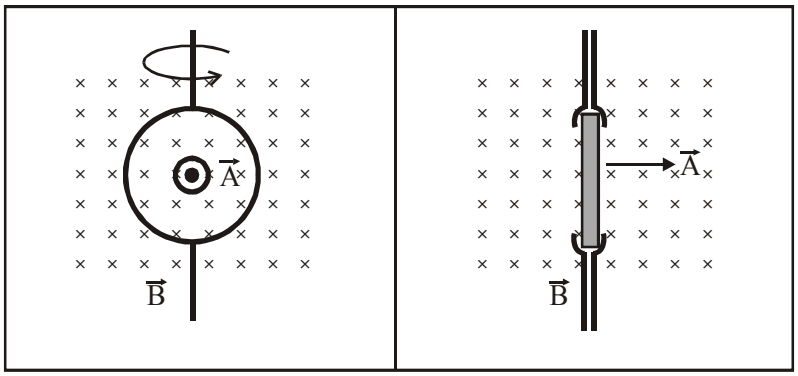


Figure : Inducing emf by varying the angle between **B** and **A**

LENZ'S LAW

Lenz's Law :

The direction of the induced current is determined by Lenz's law:

The induced current produces magnetic fields which tend to oppose the change in magnetic flux that induces such currents.

To illustrate how Lenz's law works, let's consider a conducting loop placed in a magnetic field. We follow the procedure below:

1. Define a positive direction for the area vector $\vec{A}$.

2. Assuming that $\vec{B}$ is uniform, take the dot product of $\vec{B}$ and $\vec{A}$. This allows for the determination of the sign of the magnetic flux Φ_B.

3. Obtain the rate of flux change $d\Phi_B / dt$ by differentiation. There are three possibilities:

$$\frac{d\Phi_B}{dt} \begin{cases} > 0 \Rightarrow \text{induced emf } \varepsilon < 0 \\ < 0 \Rightarrow \text{induced emf } \varepsilon > 0 \\ = 0 \Rightarrow \text{induced emf } \varepsilon = 0 \end{cases}$$

4. Determine the direction of the induced current using the right-hand rule. With your thumb pointing in the direction of $\vec{A}$, curl the fingers around the closed loop. The induced current flows in the same direction as the way your fingers curl if $\varepsilon > 0$, and the opposite direction if $\varepsilon < 0$, as shown in Figure.

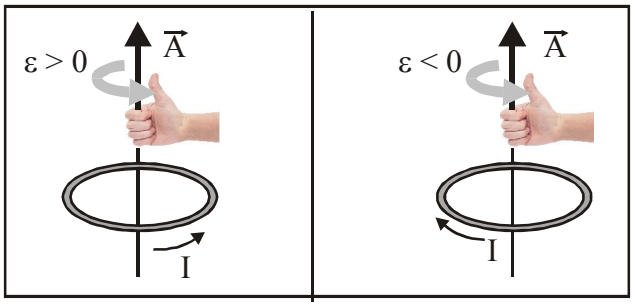


Figure : Determination of the direction of induced current by the right-hand rule

In Figure we illustrate the four possible scenarios of time-varying magnetic flux and show how Lenz's law is used to determine the direction of the induced current *I*.

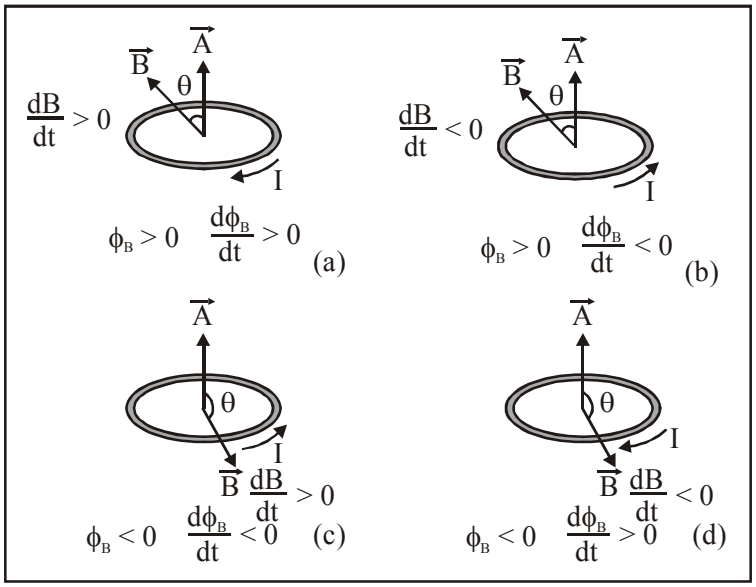


Figure : Direction of the induced current using Lenz's law

The above situations can be summarized with the following sign convention:

Φ_B	$d\Phi_B/dt$	ε	I
+	+	−	−
	−	+	+
−	+	−	−
	−	+	+

The positive and negative signs of *I* correspond to a counter clockwise and clockwise currents, respectively.

Ex. The radius of a coil decreases steadily at the rate of 10^{-2} m/s. A constant and uniform magnetic field of induction 10^{-3} Wb/m^2 acts perpendicular to the plane of the coil. What will be the radius of the coil when the induced e.m.f. in the 1μV

Sol. Induced emf $e = \frac{d(BA)}{dt} = \frac{Bd(\pi r^2)}{dt} = 2B\pi r\frac{dr}{dt}$ radius of coil $r = \frac{e}{2B\pi\left(\frac{dr}{dt}\right)} = \frac{1\times10^{-6}}{2\times10^{-3}\times\pi\times10^{-2}} = \frac{5}{\pi}$ cm

Ex. The ends of a search coil having 20 turns, area of cross-section 1 cm^2 and resistance 2 ohms are connected to a ballistic galvanometer of resistance 40 ohms. If the plane of search coil is inclined at 30° to the direction of a magnetic field of intensity 1.5 Wb/m^2, coil is quickly pulled out of the field to a region of zero magnetic field, calculate the charge passed through the galvanometer.

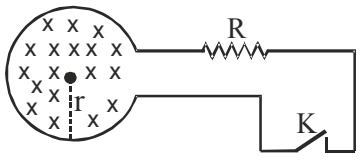
Sol. The total flux linked with the coil having turns N and area A is

$$\phi_1 = N(\vec{B}.\vec{A}) = NBA\cos\theta = NBA\cos(90°-30°) = \frac{NBA}{2}$$

when the coil is pulled out, the flux becomes zero, $\phi_2 = 0$ so change in flux is $\Delta\phi = \frac{NBA}{2}$

the charge flowed through the circuit is $q = \frac{\Delta\phi}{R} = \frac{NBA}{2R} = \frac{20\times1.5\times10^{-4}}{2\times42} = 0.357\times10^{-4}$ C

Ex. Shown in the figure is a circular loop of radius r and resistance R. A variable magnetic field of induction $B = B_0 e^{-t}$ is established inside the coil. If the key (K) is closed. Then calculate the electrical power developed right after closing the key.



Sol. Induced emf $e = \frac{d\phi}{dt} = \frac{d}{dt}(BA) = A\frac{dB}{dt} = \pi r^2 B_0\frac{d}{dt}(e^{-t}) = -\pi r^2 B_0 e^{-t}$

At t = 0, $e_0 = B_0 e^{-0}\cdot\pi r^2 = B_0\pi r^2$

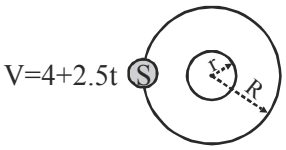
The electric power developed in the resistor R just at the instant of closing the key is $P = \frac{e_0^2}{R} = \frac{B_0^2\pi^2 r^4}{R}$

Ex. Two concentric coplanar circular loops made of wire, resistance per unit length 10^{-4} Ωm^{-1}, have diameters 0.2 m and 2 m. A time-varying potential difference (4 + 2.5 t) volt is applied to the larger loop. Calculate the current in the smaller loop.

Sol. The magnetic field at the centre O due to the current in the larger loop is $B = \frac{\mu_0 I}{2R}$

If ρ is the resistance per unit length, then

$$I = \frac{\text{potential difference}}{\text{resistance}} = \frac{4+2.5t}{2\pi R\cdot\rho}$$



$\therefore \quad B = \frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}$

$\because$ r << R, so the field B can be taken almost constant over the entire area of the smaller loop.

$\therefore$ the flux linked with the smaller loop is $\phi = B\times\pi r^2 = \frac{\mu_0}{2R}\cdot\frac{4+2.5t}{2\pi R\rho}\cdot\pi r^2$

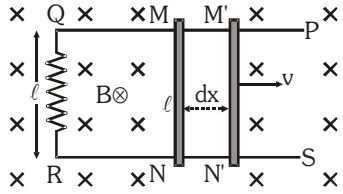
$$\text{Induced emf } e = \frac{d\phi}{dt} = \frac{\mu_0 r^2}{4R^2\rho}\times 2.5$$

The corresponding current in the smaller loop is I' then

$$I' = \frac{e}{R} = \frac{\mu_0 r^2}{4R^2\rho}\times 2.5\times\frac{1}{2\pi r\rho} = \frac{2.5\mu_0 r}{8\pi R^2\rho^2} = \frac{2.5\times 4\pi\times 10^{-7}\times 0.1}{8\pi\times(1)^2\times(10^{-4})^2} = 1.25A$$

Induced emf by changing the area of the coil

A U shaped frame of wire, PQRS is placed in a uniform magnetic field B perpendicular to the plane and vertically inward. A wire MN of length ℓ is placed on this frame. The wire MN moves with a speed v in the direction shown. After time dt the wire reaches to the position M'N' and distance covered = dx. The change in area ΔA = Length × area = ℓdx



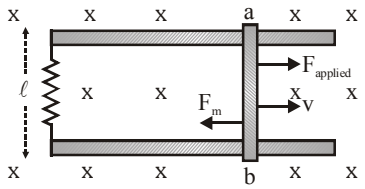
Change in the magnetic flux linked with the loop in the dt is $d\phi = B\times\Delta A = B\times\ell dx$

$$\text{induced emf} \qquad e = \frac{d\phi}{dt} = B\ell\frac{dx}{dt} = B\ell v \quad \because \left[v = \frac{dx}{dt}\right]$$

If the resistance of circuit is R and the circuit is closed then the current through the circuit

$$I = \frac{e}{R} \qquad \Rightarrow I = \frac{Bv\ell}{R}$$

A magnetic force acts on the conductor in opposite direction of velocity is



$$F_m = i\ell B = \frac{B^2\ell^2 v}{R}\ .$$

So, to move the conductor with a constant velocity v an equal and opposite force F has to be applied in the conductor.

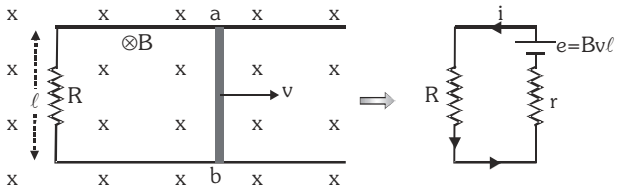
$$F = F_m = \frac{B^2\ell^2 v}{R}$$

The rate at which work is done by the applied force is, $P_{applied} = Fv = \frac{B^2\ell^2 v^2}{R}$

and the rate at which energy is dissipated in the circuit is, $P_{dissipated} = i^2 R = \left[\frac{Bv\ell}{R}\right]^2 R = \frac{B^2\ell^2 v^2}{R}$

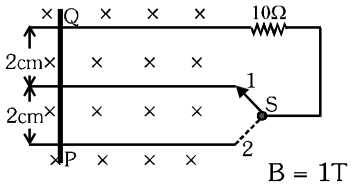
This is just equal to the rate at which work is done by the applied force.

- In the figure shown, we can replaced the moving rod ab by a battery of emf $B v \ell$ with the positive terminal at a and the negative terminal at b. The resistance r of the rod ab may be treated as the internal resistance of the battery.



Hence, the current in the ciruit is $i = \frac{e}{R+r} = \frac{B v \ell}{R+r}$

Ex. Wire PQ with negligible resistance slides on the three rails with 5 cm/sec. Calculate current in 10 Ω resistance when switch S is connected to (a)position 1 (b)position 2



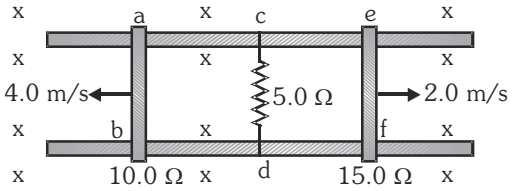
Sol. (a) For position 1

$$\text{Induced current } I = \frac{e}{R} = \frac{B v \ell}{R} = \frac{1 \times 5 \times 10^{-2} \times 2 \times 10^{-2}}{10} = 0.1 \text{ mA}$$

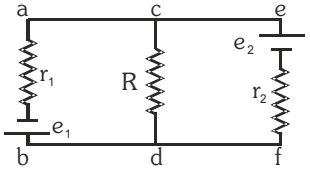
(b) For position 2

$$\text{Induced current } I = \frac{e}{R} = \frac{B v (2\ell)}{R} = \frac{1 \times 5 \times 10^{-2} \times 4 \times 10^{-2}}{10} = 0.2 \text{ mA}$$

Ex. Two parallel rails with negligible resistance are 10.0 cm apart. They are connected by a 5.0 Ω resistor. The circuit also contains two metal rods having resistance of 10.0Ω and 15.0 Ω along the rails (fig). The rods are pulled away from the resistor at constantspeeds 4.00 m/s and 2.00 m/s respectively. A uniform magnetic field of magnitude 0.01 T is applied perpendicular to the plane of the rails. Determine the current in the 5.0 Ω resistor.



Sol. Two conductors are moving in uniform magnetic field, so motional emf will induced across them.
The rod ab will act as a source of emf $e_1 = B v \ell = (0.01)(4.0)(0.1) = 4 \times 10^{-3}$ V
and internal resistance $r_1 = 10.0\ \Omega$
Similarly, rod ef will also act as source of emf $e_2 = (0.01)(2.0)(0.1) = 2 \times 10^{-3}$ V
and internal resistance $r_2 = 15.0\ \Omega$
From right hand rule : $V_b > V_a$ and $V_e > V_f$ Also $R = 5.0\ \Omega$,

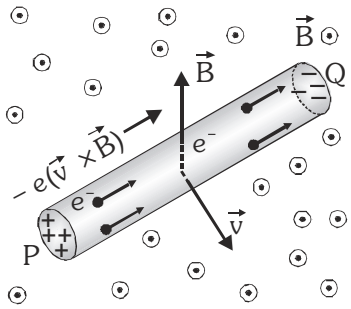


$$E_{eq} = \frac{e_1 r_2 - e_2 r_1}{r_1 + r_2} = \frac{6 \times 10^{-3} - 20 \times 10^{-3}}{15+10} = \frac{40}{25} \times 10^{-3} = 1.6 \times 10^{-3} \text{ volt}$$

$$r_{eq} = \frac{15 \times 10}{15+10} = 6\,\Omega \quad \text{and}$$

$$I = \frac{E_{eq}}{r_{eq} + R} = \frac{1.6 \times 10^{-3}}{6+6} = \frac{1.6}{11} \times 10^{-3} = \frac{8}{55} \times 10^{-3} \text{ amp from d to c}$$

MOTIONAL EMF FROM LORENTZ FORCE



A conductor PQ is placed in a uniform magnetic field B, directed normal to the plane of paper outwards. PQ is moved with a velocity v, the free electrons of PQ also move with the same velocity. The electrons experience a magnetic Lorentz force, $\vec{F}_m = (\vec{v} \times \vec{B})$. According to Fleming's left hand rule, this force acts in the direction PQ and hence the free electrons will move towards Q. A negative chagre accumulates at Q and a positive charge at P. An electric field E is setup in the conductor from P to Q. Force exerted by electric field on the free electrons is, $\vec{F}_e = e\vec{E}$

The accumulation of charge at the two ends continues till these two forces balance each other.

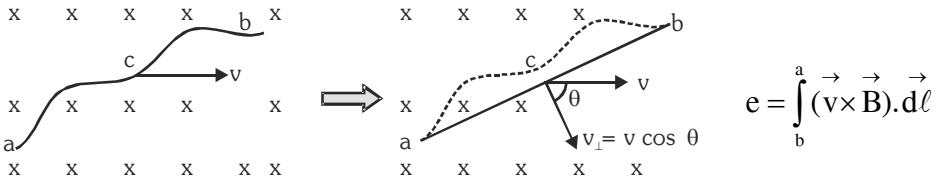
so $\vec{F}_m = -\vec{F}_e \Rightarrow \; e(\vec{v} \times \vec{B}) = -e\vec{E} \Rightarrow \; \vec{E} = -(\vec{v} \times \vec{B})$

The potential difference between the ends P and Q is $V = \vec{E}.\vec{\ell} = (\vec{v} \times \vec{B}).\vec{\ell}$ .It is the magnetic force on the moving free electrons that maintains the potential difference and produces the emf $\mathcal{E} = B\,\ell\,v$ (for $\vec{B} \perp \vec{v} \perp \vec{\ell}$)

As this emf is produced due to the motion of a conductor, so it is called a motional emf.

The concept of motional emf for a conductor can be generalized for any shape moving in any magnetic field uniform or not. For an element $\vec{d\ell}$ of conductor the contribution de to the emf is the magnitude $d\ell$ multiplied by the component of $\vec{v} \times \vec{B}$ parallel to $\vec{d\ell}$, that is $de = (\vec{v} \times \vec{B}).\vec{d\ell}$

For any two points a and b the motional emf in the direction from b to a is,

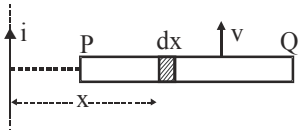


$$e = \int_b^a (\vec{v} \times \vec{B}).\vec{d\ell}$$

Motional emf in wire acb in a uniform magnetic field is the motional emf in an imaginary wire ab. Thus, $e_{acb} = e_{ab}$ = (length of ab) ($v_\perp$) (B), $v_\perp$ = the component of velocity perpendicular to both $\vec{B}$ and ab. From right hand rule : b is at higher potential and a at lower potential. Hence, $V_{ba} = V_b - V_a$ = (ab) (v cosθ) (B)

Ex. A rod PQ of length L moves with a uniform velocity v parallel to a long straight wire carrying a current i, the end P remaining at a distance r from the wire. Calculate the emf induced across the rod. Take v = 5.0 m/s, i = 100 amp, r = 1.0 cm and L = 19 cm.

Sol. The rod PQ is moving in the magnetic field produced by the current-carrying long wire. The field is not uniform throughout the length of the rod (changing with distance). Let us consider a small element of length dx at distance x from wire. if magnetic field at the position of dx is B then emf induced



$$d\mathcal{E} = B\,v\,dx = \frac{\mu_0}{2\pi}\frac{i}{x}v\,dx$$

$\therefore$ emf $\mathcal{E}$ is induced in the entire length of the rod PQ is $\mathcal{E} = \int d\mathcal{E} = \int_P^Q \frac{\mu_0}{2\pi}\frac{i}{x}v\,dx$

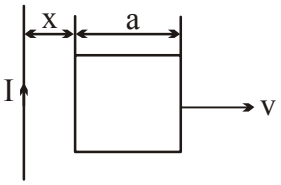
Now x = r at P, and x = r + L at Q. hence

$$\mathcal{E} = \frac{\mu_0 i v}{2\pi} = \int_r^{r+L} \frac{dx}{x} = \frac{\mu_0 i v}{2\pi}\left[\log_e x\right]_r^{r+L} = \frac{\mu_0 i v}{2\pi}[\log_e (r+L) - \log_e r] = \frac{\mu_0 i v}{2\pi} \log \frac{r+L}{r}$$

Putting the given values :

$$\mathcal{E} = (2 \times 10^{-7})\,(100)\,(5.0) \log_e \frac{1.0+19}{1.0} = 10^{-4} \log_e 20 \text{ Wb/s} = 3 \times 10^{-4} \text{ volt}$$

Ex. A square frame with side a and a long straight wire carrying a current I are located in the same plane as shown in Fig. The frame translates to the right with a constant velocity v. Find the emf induced in the frame as a function of distance x.



Ans. $\xi_i = \dfrac{\mu_0}{4\pi} \dfrac{2Ia^2v}{x\ (x\ +\ a)}$

Sol. Field, due to the current carrying wire, at a perpendicular distance x from it is given by,

$$B(x) = \frac{\mu_0}{2\pi} \frac{i}{x}$$

Motional emf is given by $\left|\int -(\vec{v} \times \vec{B})\ .\ d\vec{\ell}\right|$

There will be no induced emf in the segments (2) and (4) as, $\vec{v} \uparrow\uparrow d\vec{\ell}$ and magnitude of emf induced 1 and 3, will be

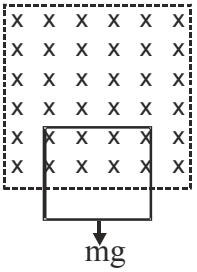
$$\xi_1 = v\left(\frac{\mu_0}{2\pi}\frac{i}{x}\right) a \text{ and } \xi_2 = v\left(\frac{\mu_0}{2\pi}\frac{i}{(a+x)}\right)a,$$

respectively, and their sense will be in the direction of $(\vec{v} \times \vec{B})$. So, emf induced in the network

$= \xi_1 - \xi_2$ [as $\xi_1 > \xi_2$]

$$= \frac{av\mu_0 i}{2\pi}\left[\frac{1}{x} - \frac{1}{a+x}\right] = \frac{va^2\mu_0 i}{2\pi x(a+x)}$$

Ex. A horizontal magnetic field B is produced across a narrow gap between square iron pole-pieces as shown. A closed square wire loop of side ℓ, mass m and resistance R is allowed of fall with the top of the loop in the field. Show that the loop attains a terminal velocity given by $v = \dfrac{Rmg}{B^2\ell^2}$ while it is between the poles of the magnet.

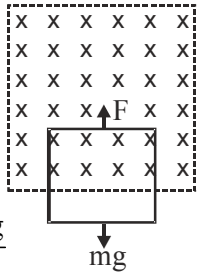


Sol. As the loop falls under gravity, the flux passing through it decreases and so an induced emf is set up in it. Then a force F which opposes its fall. When this force becomes equal to the gravity force mg, the loop attains a terminal velocity v.

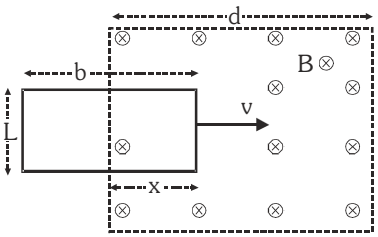
The induced emf e = B v ℓ, and the induced current is $i = \dfrac{e}{R} = \dfrac{Bv\ell}{R}$

The force experienced by the loop due to this current is $F = B\ell i = \dfrac{B^2 v \ell^2}{R}$

When v is the terminal (constant) velocity F = mg or $\dfrac{B^2 v \ell^2}{R} = mg$ or $v = \dfrac{R\,mg}{B^2\ell^2}$



Ex. Figure shows a rectangular conducting loop of resistance R, width L, and length b being pulled at constant speed v through a region of width d in which a uniform magnetic field B is set up by an electromagnet.Let L = 40 mm, b = 10 cm, d = 15 cm, R = 1.6 Ω, B = 2.0 T and v = 1.0 m/s



(i) Plot the flux ϕ through the loop as a function of the position x of the right side of the loop.
(ii) Plot the induced emf as a function of the positioin of the loop.
(iii) Plot the rate of production of thermal energy in the loop as a function of the position of the loop.

Sol. (i) When the loop is not in the field :
The flux linked with the loop $\phi = 0$
When the loop is entirely in the field :
Magnitic flux linked with the loop
$\phi = B\ L\ b$
$= 2 \times 40 \times 10^{-3} \times 10^{-1} = 8$ mWb
When the loop is entering the field :
The flux linked with the loop $\phi = B\ L\ x$
When the loop is leaving the field :
The flux $\phi = B\ L\ [b - (x - d)]$

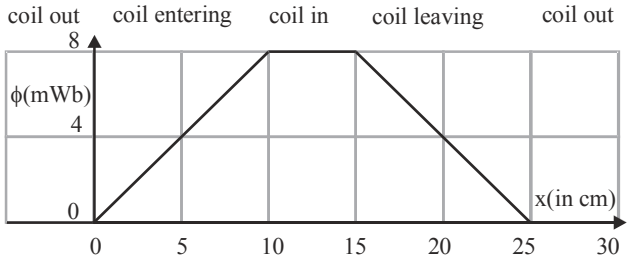


fig. (i)

(ii) Induced emf is $e = -\frac{d\phi}{dt} = -\frac{d\phi}{dx}\frac{dx}{dt} = -\frac{d\phi}{dx}v$
= – slope of the curve of figure (i) × v
The emf for 0 to 10 cm :

$$e = -\frac{(8-0)\times10^{-3}}{(10-0)\times10^{-2}}\times1 = -80\,mV$$

The emf for 10 to 15 cm : $e = 0 \times 1 = 0$
The emf for 15 to 25 cm :

$$e = -\frac{(0-8)\times10^{-3}}{(25-15)\times10^{-2}}\times1 = +80\,mV$$

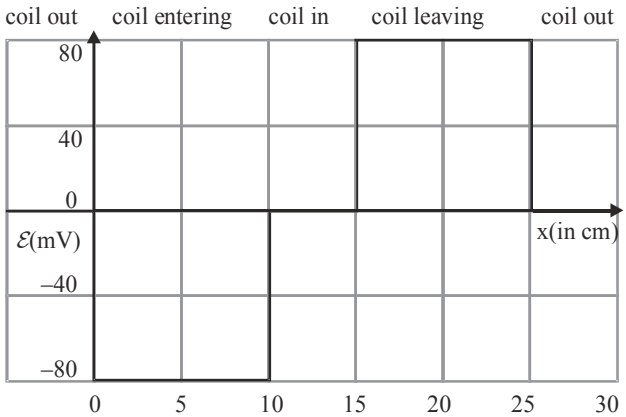


fig. (ii)

(iii) The rate of thermal energy production is $P = \frac{e^2}{R}$

for 0 to 10 cm : $P = \frac{(80\times10^{-3})^2}{1.6} = 4\,mW$

for 10 to 15 cm : $P = 0$

for 15 to 25 cm : $P = \frac{(80\times10^{-3})^2}{1.6} = 4\,mW$

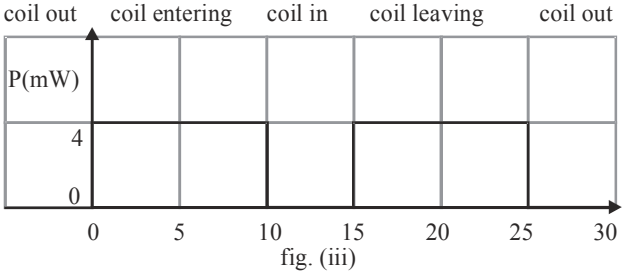
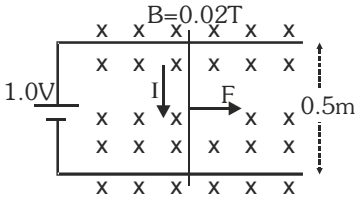


fig. (iii)

Ex. Two long parallel wires of zero resistance are connected to each other by a battery of 1.0 V. The separation between the wires is 0.5 m. A metallic bar, which is perpendicular to the wires and of resistance 10 Ω, moves on these wires. When a magneticfield of 0.02 testa is acting perpendicular to the plane containing the bar and the wires. Find the steady-state veclocity of the bar. If the mass of the bar is 0.002 kg then find its velocity as a function of time.

Sol. The current in the 10Ω bar is $I = \frac{1.0\,V}{10\,\Omega} = 0.1\,A$



The current carrying bar is placed in the magnetic field $\vec{B}$ (0.2 T) perpendicular to the plane of paper and directed downwards.
The magnetic force of the bar is $F = B\,I\,\ell = 0.02 \times 0.5 \times 0.1\,0 = 1 \times 10^{-3}$ N

The moving bar cuts the lines of force of $\vec{B}$. If v be the instantaneous velocity of the bar, then the emf induced in the bar is $\mathcal{E} = B\ell v = 0.02 \times 0.5 \times v = 0.01\,v$ volt. By Lenz's law, $\mathcal{E}$ will oppose the motion of the bar which will ultimately attain a steady velocity. In this state, the induced emf $\mathcal{E}$ will be equal to the applied emf (1.0 volt). $\therefore$ 0.01 v = 1.0 or $v = \frac{1.0}{0.01} = 100$ ms^{-1}

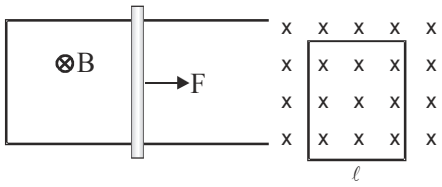
Again, a magnetic force F acts on the bar. If m be the mass of the bar, the acceleration of the rod is

$$\frac{dv}{dt} = \frac{F}{m} \Rightarrow dv = \frac{F}{m}.dt \qquad \text{Integrating, } \int dv = \int \frac{F}{m} dt \Rightarrow v = \frac{F}{m}t + C \text{ (constant)}$$

If at t = 0, v = 0 then C = 0. $\therefore$ $v = \frac{F}{m}t$ But $F = 1 \times 10^{-3}$ N, m = 0.002 kg

$$\therefore \quad v = \frac{1\times10^{-3}}{0.002}t = 0.5\,t$$

Ex. In figure, a rod closing the circuit moves along a U-shaped wire at a constant speed v under the action of the force F. The circuit is in a uniform magnetic field perpendicualr to its plane. Calculate F if the rate generation of heat is P.

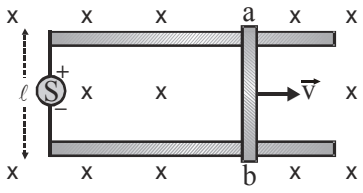


Sol. The emf induced across the ends of the rod, $\mathcal{E} = B\ell v$

Current in the circuit, $I = \frac{\mathcal{E}}{R} = \frac{B\ell v}{R}$ Magnetic force on the conductor, F' = IℓB, towards left

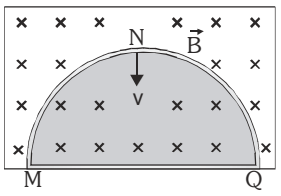
$\because$ acceleration is zero F' = F $\Rightarrow$ $BI\ell = F$ or $I = \frac{F}{B\ell}$ $\because$ $P = \mathcal{E}I = B\,\ell\,v \times \frac{F}{B\ell} = Fv$ $\therefore$ $F = \frac{P}{v}$

Ex. The diagram shows a wire ab of length ℓ and resistance R sliding on a smooth pair of rails with a velocity v towards right. A uniform magnetic field of induction B acts normal to the plane containing the rails and the wire inwards. S is a current source providing a constant I in the circuit. Determine the potential difference between a and b.

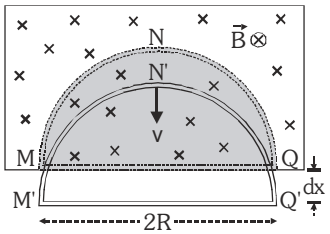


Sol. The wire ab which is moving with a velocity v is equivalent to an emf source of value B v ℓ with its positive terminal towards a.
$\therefore$ Potential difference $V_a - V_b = Bv\ell - IR$

Ex. A thin semicircular conduting ring of radius R is falling with its plane vertical in a horizontal magnetic induction $\vec{B}$ (fig.). At the position MNQ, the speed of the ring is v. What is the potential difference developed across the ring at the position MNQ ?



Sol. Let semiconductor ring falls through an infinitesimally small distance dx from its initial position MNQ to M'Q'N' in time dt (fig).



decrease in area of the ring inside the magnetic field,

$dA = -MQQ'M' = -M'Q' \times QQ' = -2R\,dx$

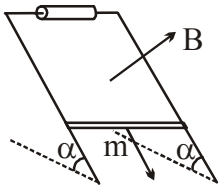
$\therefore$ change in magnetic flux linked with the ring,

$d\phi = B \times dA = B \times (-R\,dx) = -2BR\,dx$

The potential difference developed across the ring, $e = -\frac{d\phi}{dt} = -\left[-2BR\frac{dx}{dt}\right] = 2BRv$

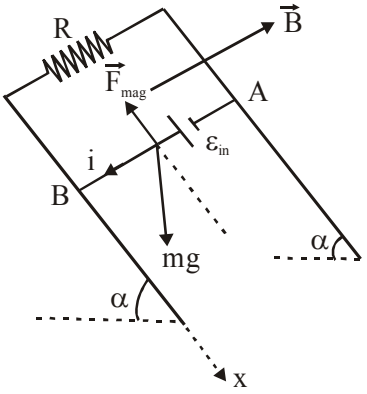
the speed with which the ring is falling $v = \frac{dx}{dt}$

Ex. A copper connector of mass m slides down two smooth copper bars, set at an angle α to the horizontal, due to gravity (Fig.). At the top the bars are interconnected through a resistance R. The separation between the bars is equal to l. The system is located in a uniform magnetic field of induction B, perpendicular to the plane in which the connector slides. The resistances of the bars, the connector and the sliding contacts, as well as the self-inductance of the loop, are assumed to be negligible. Find the steady-state velocity of the connector.



Ans. $v = \frac{mgR\sin\alpha}{B^2l^2}$

Sol. From Lenz's law, the current through the connector is directed from A to B. Here $\xi_{in} = vB\ell$ between A and B.



where v is the velocity of the rod at any moment.

For the rod, from $F_x = mw_x$

or, $mg\sin\alpha - i\ell B = mw$

For steady state, acceleration of the rod must be equal to zero.

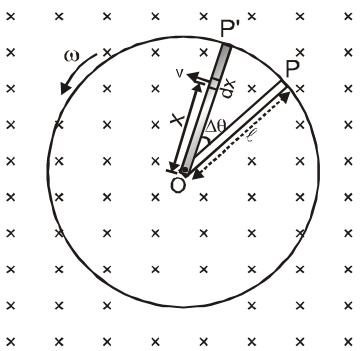
Hence, $mg\sin\alpha = i\ell B$ ..... (1)

But, $i = \frac{\xi_{in}}{R} = \frac{vB\ell}{R}$

from (1) and (2) $v = \frac{mg\sin\alpha R}{B^2\ell^2}$

INDUCED E.M.F. DUE TO ROTATION OF A CONDUCTOR ROD IN A UNIFORM MAGNETIC FIELD

Let a conducting rod is rotating in a magnetic field around an axis passing through its one end, normal to its plane.



Consider an small element dx at a distance x from axis of rotation.

Suppose velocity of this small element = v

So, according to Lorent's formula induced e.m.f. across this small element

$d\varepsilon = B\,v.\,dx$

$\because$ This small element dx is at distance x from O (axis of rotation)

$\therefore$ Linear velocity of this element dx is $v = \omega x$

substitute of value of v in eq^n (i) $d\varepsilon = B\,\omega\,x\,dx$

Every element of conducting rod is normal to magnetic field and moving in perpendicular direction to the field

So, net induced e.m.f. across conducting rod $\varepsilon = \int d\varepsilon = \int_0^{\ell} B\omega\ x\ dx = \omega\ B\left(\frac{x^2}{2}\right)_0^{\ell}$

or $\varepsilon = \frac{1}{2}B\omega\ell^2$ $\varepsilon = \frac{1}{2}B\times 2\pi f\times \ell^2$ [f = frequency of rotation]

$= B\ f\ (\pi\ell^2)$ area traversed by the rod $A = \pi\ \ell^2$ or $\varepsilon = BAf$

Ex. A wheel with 10 metallic spokes each 0.5 m long is rotated with angular speed of 120 revolutions per minute in a plane normal to the earth's mangetic field. If the earth's magnetic field at the given place is 0.4 gauss, find the e.m.f. induced between the axle and the rim of the wheel.

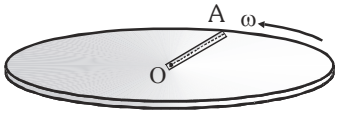
Sol. $\omega = 2\pi n = 2\pi\times\frac{120}{60} = 4\pi$, $B = 0.4\ G = 4\times\ 10^{-5}\ T$, length of each spoke = 0.5 m

induced emf $e = \frac{1}{2}B\omega\ell^2 = \frac{1}{2}\times 4\times 10^{-5}\times 4\pi\times(0.5)^2 = 6.28\times 10^{-5}$ volt

As all the ten spokes are connected with their one end at the axle and the other end at the rim, so they are connected in parallel and hence emf across each spoke is same. The number of spokes is immaterial.

Ex. A horizontal copper disc of diameter 20 cm, makes 10 revolutions/sec about a vertical axis passing through its centre. A uniform magnetic field of 100 gauss acts perpendicular to the plane of the disc. Calculate the potential difference its centre and rim in volts.

Sol. $B = 100$ gauss $= 100\times\ 10^{-4}\ Wb/m^2 = 10^{-2}$,
$r = 10$ cm $= 0.10$ m, frequency of rotaion = 10 rot/sec



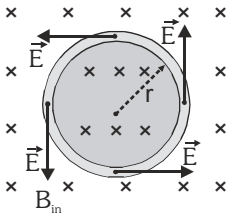
The emf induced between centre and rim $\mathcal{E} = \frac{1}{2}B\omega\ell^2 = \frac{1}{2}B\omega r^2\ (\because r = \ell)$

$\omega = 2\pi f = 2\times\ 3.14\times\ 10 = 62.8\ s^{-1}$

$\therefore\ \mathcal{E} = \frac{1}{2}\times 10\times 62.8\times(0.1)^2 = 3.14\times\ 10^{-3}\ V = 3.14\ mV.$

INDUCED ELECTRIC FIELD

When the magnetic field changes with time (let it increases with time) there is an induced electric field in the conductor caused by the changing magnetic flux.



Important properties of induced electric field :

(i) It is non conservative in nature. The line integral of $\vec{E}$ around a closed path is not zero. When a charge q goes once around the loop, the total work done on it by the electric field is equal to q times the emf.

Hence $\oint \vec{E}.d\vec{\ell} = e = -\frac{d\phi}{dt}$ ...(i)

This equation is valid only if the path around which we integrate is stationary.

(ii) Due to of symmetry, the electric field $\vec{E}$ has the same magnitude at every point on the circle and it is tangential at each point (figure).

(iii) Being nonconservative field, so the concept of potential has no meaning for such a field.

(iv) This field is different from the conservative electrostatic field produced by stationary charges.

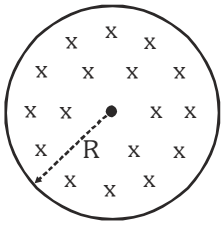
(v) The relation $\vec{F} = q\vec{E}$ is still valid for this field. (vi) This field can vary with time.

- For symmetrical situations $E\ell = \left|\frac{d\phi}{dt}\right| = A\left|\frac{dB}{dt}\right|$

ℓ = the length of closed loop in which electric field is to be calculated
A = the area in which magnetic field is changing.

Direction of induced electric field is the same as the direction of included current.

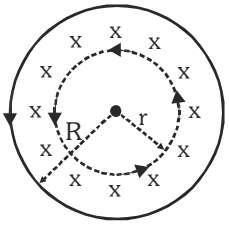
Ex. The magnetic field at all points within the cylindrical region whose cross-section is indicated in the figure start increasing at a constant rate $\alpha \frac{\text{tesla}}{\text{sec ond}}$. Find the magnitude of electric field as a function of r, the distance from the geomatric centreof the region.



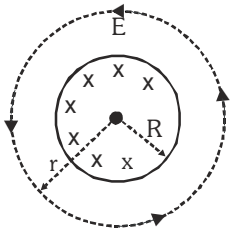
Sol. **For r ≤ R :**

$\because \quad E\ell = A\left|\frac{dB}{dt}\right|$

$\therefore \quad E(2\pi r) = (\pi r^2)\alpha \Rightarrow E = \frac{r\alpha}{2} \Rightarrow E \propto r$



E-r graph is straight line passing through origin.

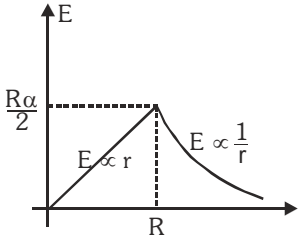


At $r = R, \quad E = \frac{R\alpha}{2}$

For r ≥ R :

$\because \quad E\ell = A\left|\frac{dB}{dt}\right|$

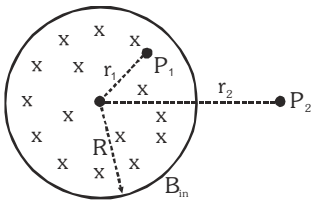
$\therefore \quad E(2\pi r) = (\pi R^2)\alpha \quad \Rightarrow E = \frac{\alpha R^2}{2r} \Rightarrow E \propto \frac{1}{r}$



Ex. For the situation described in figure the magnetic field changes with time according to,
$B = (2.00\ t^3 - 4.00\ t^2 + 0.8)$ T and $r_2 = 2R = 5.0$ cm

(a) Calculate the force on an electron located at P_2 at t = 2.00 s

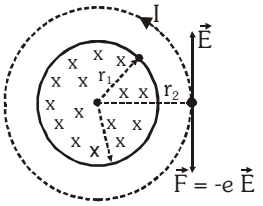
(b) What are the magnetude and direction of the electric field at P_1 when t = 3.00 s and $r_1 = 0.02$m.



Sol. $E\ell = A\left|\frac{dB}{dt}\right| \Rightarrow E = \frac{\pi R^2}{2\pi r_2}\frac{d}{dt}(2t^3 - 4t^2 + 0.8) = \frac{R^2}{2r_2}(6t^2 - 8t)$

(a) Force on electron at P_2 is F = eE

$$\therefore \text{ at t = 2 s } F = \frac{1.6\times10^{-19}\times(2.5\times10^{-2})^2}{2\times5\times10^{-2}}\times[6(2)^2 - 8(2)]$$



$$= \frac{1.6}{4}\times2.5\times10^{-21}\times(24-16) = 8\times10^{-21}\text{ N at t = 2s,}$$

$\frac{dB}{dt}$ is positive so it is increasing.

$\therefore$ direction of induced current and E are as shown in figure and hence force of electron having charge –e is right perpendicular to r_2 downwards

(b) For $r_1 = 0.02$ m and at t = 3s, $E = \frac{\pi r_1^2}{2\pi r_1}(6t^2 - 8t) = \frac{0.02}{2}\times[6(3)^2 - 8(3)]$

$= 0.3$ V/m at t = 3sec, $\frac{dB}{dt}$

is positive so B is increasing and hence direction of E is same as in case (a) and it is left perpendicular to r_1 upwards.

Generators :

One of the most important applications of Faraday's law of induction is to generators and motors. A generator converts mechanical energy into electric energy, while a motor converts electrical energy into mechanical energy.

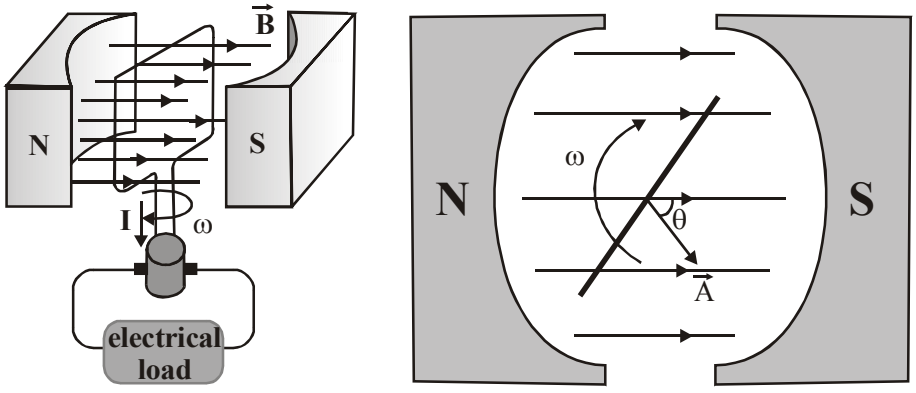


Figure : (a) A simple generator. (b) The rotating loop as seen from above.

Figure (a) is a simple illustration of a generator. It consists of an *N*-turn loop rotating in a magnetic field which is assumed to be uniform. The magnetic flux varies with time, thereby inducing an emf. From Figure (b), we see that the magnetic flux through the loop may be written as

$$\Phi_B = \vec{B}\ .\ \vec{A} = BA\cos\theta = BA\cos\omega t$$

The rate of change of magnetic flux is

$$\frac{d\Phi_B}{dt} = -BA\,\omega \sin \omega t$$

Since there are N turns in the loop, the total induced emf across the two ends of the loop is

$$\varepsilon = -N\frac{d\Phi_B}{dt} = NB\,A\omega \sin \omega t$$

If we connect the generator to a circuit which has a resistance R, then the current generated in the circuit is given by $I = \frac{|\varepsilon|}{R} = \frac{NBA\omega}{R} \sin \omega t$

The current is an alternating current which oscillates in sign and has an amplitude $I_0 = NBA\omega / R$. The power delivered to this circuit is

$$P = I|\varepsilon| = \frac{(NBA\omega)^2}{R} \sin^2 \omega t$$

On the other hand, the torque exerted on the loop is

$$\tau = \mu B \sin \theta = \mu B \sin \omega t$$

Thus, the mechanical power supplied to rotate the loop is

$$P_m = \tau\omega = \mu B\,\omega \sin \omega t$$

Since the dipole moment for the N-turn current loop is

$$\mu = NIA = \frac{N^2A^2B\omega}{R} \sin \omega t$$

the above expression becomes

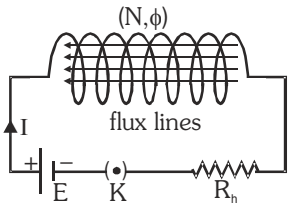
$$P_m = \left(\frac{N^2A^2B\omega}{R}\sin \omega t\right) B\omega \sin \omega t = \frac{(NAB\omega)^2}{R}\sin^2 \omega t$$

As expected, the mechanical power put in is equal to the electric power output.

SELF INDUCTION

When the current through the coil changes, the magnetic flux linked with the coil also changes. Due to this change of flux a current induced in the coil itself according to lenz concept it opposes the change in magnetic flux. This phenomenon is called self induction and a factor by virtue of coil shows opposition for change in magnetic flux called cofficient of self inductance of coil.

Considering this coil circuit in two cases.



Case - I Current through the coil is constant

If $I \to B \to \phi$ (constant) $\Rightarrow$ No EMI

total flux of coil ($N\phi$) $\propto$ current through the coil

$$N\phi \propto I \Rightarrow N\phi = LI \qquad L = \frac{N\phi}{I} = \frac{NBA}{I} = \frac{\phi_{total}}{I}$$

where L = coefficient of self inductance of coil

S I unit of L : $1\ \frac{Wb}{amp} = 1$ Henry $= 1\frac{N.m}{A^2} = 1\frac{J}{A^2}$ **Dimensions :** $[M^1L^2T^{-2}\,A^{-2}]$

Note : L is constant of coil it **does not depends on current** flow through the coil.

Case - II Current through the coil changes w.r.t. time

$$\text{If } \frac{dI}{dt} \rightarrow \frac{dB}{dt} \rightarrow \frac{d\phi}{dt} \Rightarrow \text{Static EMI} \quad \Rightarrow N\phi = LI$$

$$-N\frac{d\phi}{dt} = -L\frac{dI}{dt}, \qquad \left(-N\frac{d\phi}{dt}\right) \text{ called total self induced emf of coil '}e_s\text{'}$$

$$e_s = -L\frac{dI}{dt} \qquad \textbf{S.I. unit of L} \rightarrow \frac{V.s}{A}$$

SELF-INDUCTANCE OF A PLANE COIL

Total magnetic flux linked with N turns,

$$\phi = NBA = N\left(\frac{\mu_0 NI}{2r}\right)A = \frac{\mu_0 N^2 I}{2r}\ A = \frac{\mu_0 N^2 I}{2r} \times \pi r^2 = \frac{\mu_0 \pi N^2 r}{2} I \text{ But } \phi = L\,I \therefore L = \frac{\mu_0 \pi N^2 r}{2}$$

Ex. Self-Inductance of a Solenoid :

Compute the self-inductance of a solenoid with turns, length ℓ, and radius *NR* with a current *I* flowing through each turn, as shown in Figure.

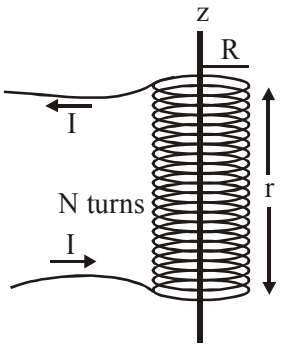


Figure : Solenoid

Solution:

Ignoring edge effects and applying Ampere's law, the magnetic field inside a solenoid is given by Eq. :

$$\vec{B} = \frac{\mu_0 NI}{\ell}\hat{k} = \mu_0 nI \quad \hat{k}$$

where $n = N / \ell$ is the number of turns per unit length. The magnetic flux through each turn is

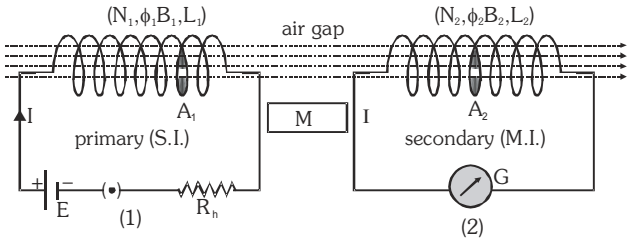
$$\Phi_B = BA = \mu_0 nI \,.\, (\pi R_2) = \mu_0 nI\pi R^2$$

Thus, the self-inductance is $L = \frac{N\Phi_B}{I} = \mu_0 n^2 \pi R^2 \ell$

We see that *L* depends only on the geometrical factors (*n*, *R* and ℓ) and is independent of the current *I*.

MUTUAL INDUCTION

Whenever the current passing through primary coil or circuit change then magnetic flux neighbouring secondary coil or circuit will also change. Acc. to Lenz for opposition of flux change, so an emf induced in the neighbouring coil or circuit.



This phenomenon called as 'Mutual induction'. In case of mutual inductance for two coils situated close to each other, flux linked with the secondary due to current in primary.

Due to Air gap always $\phi_2<\phi_1$ and $\phi_2 = B_1A_2$ $(\theta=0°)$.

Case - I When current through primary is constant

Total flux of secondary is directly proportional to current flow through the primary coil

$N_2\,\phi_2 \propto I_1 \Rightarrow N_2\,\phi_2 = MI_1,\ M = \frac{N_2\phi_2}{I_1} = \frac{N_2B_1A_2}{I_1} = \frac{(\phi_T)_s}{I_p}$ where M : is coefficient of mutual induction.

Case - II When current through primary changes with respect to time

If $\frac{dI_1}{dt} \to \frac{dB_1}{dt} \to \frac{d\phi_1}{dt} \to \frac{d\phi_2}{dt}$

$\Rightarrow$ Static EMI $N_2\phi_2 = MI_1 - N_2\frac{d\phi_2}{dt} = -M\frac{dI_1}{dt}, \left[-N_2\frac{d\phi}{dt}\right]$

$e_m = -M\left(\frac{dI_1}{dt}\right)$

Secondary ← → Primary

called total mutual induced emf of secondary coil e_m.

- The units and dimension of M are same as 'L'.
- The mutual inductance does not depends upon current through the primary and it is constant for circuit system.

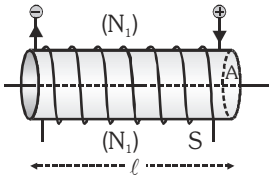
'M' depends on :

- Number of turns (N_1, N_2).
- Area of cross section.
- Distance between two coils (As d↓ = M↑).
- **Coupling factor** 'K' between two coils.
- Cofficient of self inductance (L_1, L_2).
- Magnetic permeability of medium (μ_r).
- Orientation between two coils.

DIFFERENT COEFFICIENT OF MUTUAL INDUCTANCE

- In terms of their number of turns
- In terms of their coefficient of self inductances
- **In terms of their nos of turns (N_1, N_2)**

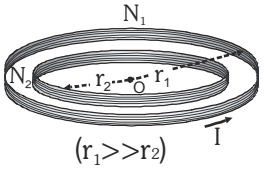
(a) Two co–axial solenoids :- $(M_{S_1S_2})$



Coefficient of mutual inductance between two solenoids

$M_{s_1s_2} = \frac{N_2B_1A}{I_1} = \frac{N_2}{I_1}\left[\frac{\mu_0N_1I_1}{\ell}\right]A \qquad \Rightarrow M_{s_1s_2} = \left[\frac{\mu_0N_1N_2A}{\ell}\right]$

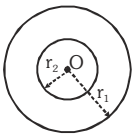
(b) Two plane concentric coils ($M_{C_1C_2}$)

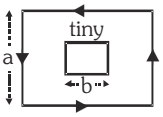


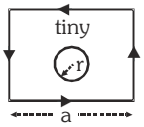
$M_{c_1c_2} = \frac{N_2B_1A_2}{I_1}$ where $B_1 = \frac{\mu_0N_1I_1}{2r_1}$, $A_2 = \pi r_2^2$

$M_{c_1c_2} = \frac{N_2}{I_1}\left[\frac{\mu_0N_1I_1}{2r_1}\right](\pi r_2^2) \Rightarrow M_{c_1c_2} = \frac{\mu_0N_1N_2\pi r_2^2}{2r_1}$

Two concentric loop : **Two concentric square loops :** **A square and a circular loop**

$M \propto \frac{r_2^2}{r_1}(r_1 >> r_2)$ 

$M \propto \frac{b^2}{a}$ 

$M \propto \frac{r^2}{a}$ 

INDUCTANCE IN SERIES AND PARALLEL

Two coil are connected in series : Coils are lying close together (M)

If $M = 0$, $L = L_1 + L_2$

If $M \neq 0$ $L = L_1 + L_2 + 2M$

(a) When current in both is in the same direction Then $L = (L_1 + M) + (L_2 + M)$

(b) When current flow in two coils are mutually in opposite directions.

$L = L_1 + L_2 - 2M$

Two coils are connected in parallel :

(a) If $M = 0$ then $\frac{1}{L} = \frac{1}{L_1} + \frac{1}{L_2}$ or $L = \frac{L_1 L_2}{L_1 + L_2}$

(b) If $M \neq 0$ then $\frac{1}{L} = \frac{1}{(L_1 + M)} + \frac{1}{(L_2 + M)}$

Ex. A coil is wound on an iron core and looped back on itself so that the core has two sets of closely would wires in series carrying current in the opposite sense. What do you expect about its self-inductance ? Will it be larger or small ?

Sol. As the two sets of wire carry currents in opposite directions, their induced emf's also act in opposite directions. These induced emf's tend to cancel each other, making the self-inductance of the coil very small.

This situation is similar to two coils connected in series and producing fluxes in opposite directions. Therefore, their equivalent inductance must be $L_{eq} = L + L - 2M = L + L - 2L = 0$

Ex. A solenoid has 2000 turns wound over a length of 0.3 m. The area of cross-section is 1.2×10^{-3} m^2. Around its central section a coil of 300 turns is closely would. If an initial current of 2A is reversed in 0.25 s, find the emf induced in the coil.

Sol. $M = \frac{\mu_0 N_1 N_2 A}{\ell} = \frac{4\pi \times 10^{-7} \times 2000 \times 300 \times 1.2 \times 10^{-3}}{0.3} = 3 \times 10^{-3}\,H$

$$\mathcal{E} = -M\frac{dI}{dt} = -3 \times 10^{-3}\left[\frac{-2-2}{0.25}\right] = 48 \times 10^{-3}\,V = 48\ mV$$

ENERGY STORED IN INDUCTOR

The energy of a capacitor is stored in the electric field between its plates. Similary, an inductor has the capability of storing energy in its magnetic field.An increasing current in an inductor causes an emf between its terminals.

Power P = The work done per unit time $= \frac{dW}{dt} = -ei = -\left[L\frac{di}{dt}\right]i = -L\,i\frac{di}{dt}$

here i = instanteneous current and L = inductance of the coil

From dW = – dU (energy stored) so $\frac{dW}{dt} = -\frac{dU}{dt}$ $\therefore$ $\frac{dU}{dt} = Li\frac{di}{dt} \Rightarrow dU = Li\,di$

The total energy U supplied while the current increases from zero to final value i is,

$$U = L\int_0^I i\,di = \frac{1}{2}L(i^2)_0^I \quad \therefore U = \frac{1}{2}L\,I^2$$

the energy stored in the magnetic field of an inductor when a current I is $= \frac{1}{2}L\,I^2$.

The source of this energy is the external source of emf that supplies the current.

- After the current has reached its final steady state value I, $\frac{di}{dt} = 0$ and no more energy is input to the inductor.
- When the current decreases from i to zero, the inductor acts as a source that supplies a total amount of energy $\frac{1}{2}Li^2$ to the external circuit. If we interrupt the circuit suddenly by opening a switch the current decreases very rapidly, the induced emf is very large and the energy may be dissipated in an arc the switch.

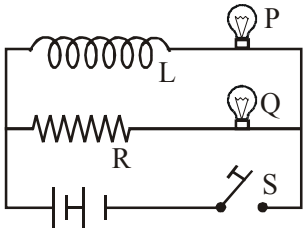
MAGNETIC ENERGY PER UNIT VOLUME OR ENERGY DENSITY

- The energy is an inductor is actually stored in the magnetic field within the coil. For a long solenoid its magnetic field can be assumed completely within the solenoid.
  The energy U stored in the solenoid when a current I is,

$$U = \frac{1}{2}L\,I^2 = \frac{1}{2}(\mu_0\, n^2\, V)\,I^2 \qquad (L = \mu_0\, n^2\, V) \qquad (V = \text{Volume} = A\ell)$$

The energy per unit volume $u = \frac{U}{V} = \frac{1}{2}\mu_0\, n^2\, I^2 = \frac{(\mu_0\, n\, I)^2}{2\mu_0} = \frac{B^2}{2\mu_0}$ $(B = \mu_0\, n\, I)$ $\therefore$ $u = \frac{1}{2}\frac{B^2}{\mu_0}$

Ex. Figure shows an inductor L a resistor R connected in paralled to a battery through a switch. The resistance of R is same as that of the coil that makes L. Two identical bulb are put in each arm of the circuit.



(a) Which of two bulbs lights up earlier when S is closed?
(b) Will the bulbs be equally bright after some time?

Sol. (i) When switch is closed induced e.m.f. in inductor i.e. back e.m.f. delays the glowing of lamp P so lamp Q light up earlier.
(ii) Yes. At steady state inductive effect becomes meaningless so both lamps become equally bright after some time.

Ex. A very small circular loop of area 5×10^{-4} m^2, resistance 2 ohm and negligible inductance is initially coplanar and concentric with a much larger fixed circular loop of radius 0.1 m. A constant current of 1 ampere is passed in the bigger loop and the smaller loop is rotated with angular velocity ω rad/s about a diameter. Calculate (a) the flux linked with the smaller loop (b) induced emf and induced current in the smaller loop as a function of time.

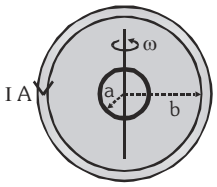
Sol. (a) The field at the centre of larger loop $B_1 = \frac{\mu_0 I}{2R} = \frac{2\pi \times 10^{-7}}{0.1} = 2\pi \times 10^{-6}$ Wb/m^2 is initially along the normal to the area of smaller loop. Now as the smaller loop (and hence normal to its plane) is rotating at angular velocity ω, with respect to $\vec{B}$ so the flux linked with the smaller loop at time t is, $\phi_2 = B_1 A_2 \cos\theta = (2\pi \times 10^{-6})(5 \times 10^{-4}) \cos\omega t$

i.e., $\phi_2 = \pi \times 10^{-9} \cos\omega t$ Wb

(b) The induced emf in the smaller loop

$$e_2 = -\frac{d\phi_2}{dt} = -\frac{d}{dt}(\pi \times 10^{-9} \cos\omega t)$$

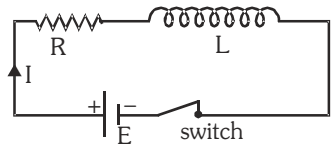
$$= \pi \times 10^{-9}\, \omega \sin\omega t \text{ volt}$$



(c) The induced current in the smaller loop is, $I_2 = \frac{e_2}{R} = \frac{1}{2}\pi\omega \times 10^{-9} \sin\omega t$ ampere

R-L DC CIRCUIT

Current Growth



(i) emf equation $E = IR + L\frac{dI}{dt}$

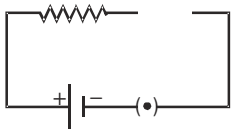
(ii) Current at any instant

When key is closed the current in circuit increases exponentially with respect to time. The current in circuit at any instant 't' given by $I = I_0\left[1 - e^{\frac{-t}{\lambda}}\right]$

t = 0 (just after the closing of key) $\Rightarrow$ I = 0

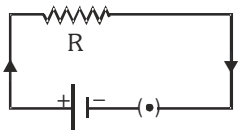
t = ∞ (some time after closing of key) $\Rightarrow$ $I \to I_0$

(iii) Just after the closing of the key inductance behaves like open circuit and current in circuit is zero.



Open circuit, t = 0, I = 0 Inductor provide infinite resistence

(iv) Some time after closing of the key inductance behaves like simple connecting wire (short circuit) and current in circuit is constant.



Short circuit, $t \to \infty, I \to I_0$, Inductor provide zero resistence $I_0 = \frac{E}{R}$

(Final, steady, maximum or peak value of current) or ultimate current

Note : Peak value of current in circuit does not depends on self inductance of coil.

(v) Time constant of circuit (λ)

$\lambda = \frac{L}{R_{sec.}}$ It is a time in which current increases up to 63% or 0.63 times of peak current value.

(vi) Half life (T)

It is a time in which current increases upto 50% or 0.50 times of peak current value.

$$I = I_0(1 - e^{-t/\lambda}),\ t = T,\ I = \frac{I_0}{2} \Rightarrow \frac{I_0}{2} = I_0(1 - e^{-T/\lambda}) \Rightarrow e^{-T/\lambda} = \frac{1}{2} \Rightarrow e^{T/\lambda} = 2$$

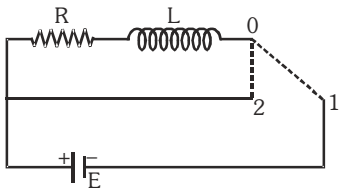
$$\frac{T}{\lambda}\log_e e = \log_e 2 \quad \Rightarrow T = 0.693\lambda \quad \Rightarrow \quad T = 0.693\frac{L}{R_{sec.}}$$

(vii) Rate of growth of current at any instant :–

$$\left[\frac{dI}{dt}\right] = \frac{E}{L}(e^{-t/\lambda}) \Rightarrow \quad t = 0 \ \Rightarrow \quad \left[\frac{dI}{dt}\right]_{max} = \frac{E}{L} \quad t = \infty \ \Rightarrow \quad \left[\frac{dI}{dt}\right] \to 0$$

Note : Maximum or initial value of rate of growth of current does not depends upon resistance of coil.

Current Decay



(i) Emf equation $IR + L\frac{dI}{dt} = 0$

(ii) Current at any instant

Once current acquires its final max steady value, if suddenly switch is put off then current start decreasing exponentially wrt to time. At switch put off condition $t = 0$, $I = I_0$, source emf E is cut off from circuit $I = I_0(e^{-t/\lambda})$

Just after opening of key $t = 0$ $\Rightarrow I = I_0 = \frac{E}{R}$

Some time after opening of key $t \to \infty$ $\Rightarrow I \to 0$

(iii) Time constant (λ)

It is a time in which current decreases up to 37% or 0.37 times of peak current value.

(iv) Half life (T)

It is a time in which current decreases upto 50% or 0.50 times of peak current value.

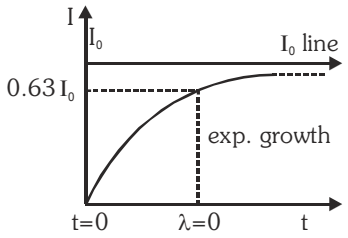
(v) Rate of decay of current at any instant

$$\left[-\frac{dI}{dt}\right] = \left[\frac{E}{L}\right]e^{-t/\lambda} \qquad t = 0\ ,\ \left[-\frac{dI}{dt}\right]_{max.} = \frac{E}{L}\ t \to \infty \qquad \Rightarrow \quad \left[-\frac{dI}{dt}\right] \to 0$$

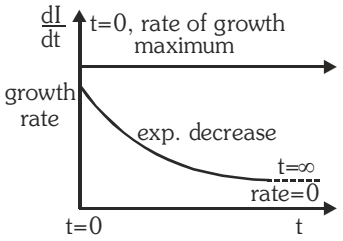
Graph for R–L circuit :–

Current Growth :–

(a)

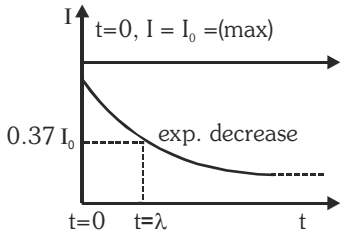


(b)

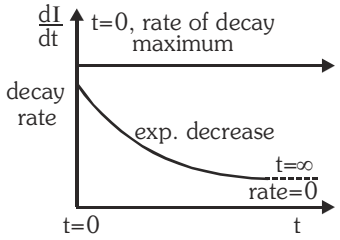


Current decay :–

(a)



(b)



LC Oscillations :

Consider an *LC* circuit in which a capacitor is connected to an inductor, as shown in Figure.

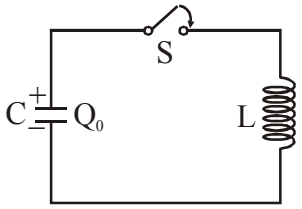


Figure *LC* Circuit

Suppose the capacitor initially has charge Q_0. When the switch is closed, the capacitor begins to discharge and the electric energy is decreased. On the other hand, the current created from the discharging process generates magnetic energy which then gets stored in the inductor. In the absence of resistance, the total energy is transformed back and forth between the electric energy in the capacitor and the magnetic energy in the inductor. This phenomenon is called electromagnetic oscillation.

The total energy in the *LC* circuit at some instant after closing the switch is

$$U = U_C + U_L = \frac{1}{2}\frac{Q^2}{C} + \frac{1}{2}LI^2$$

The fact that U remains constant implies that

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}\frac{Q^2}{C}+\frac{1}{2}LI^2\right) = \frac{Q}{C}\frac{dQ}{dt} + LI\frac{dI}{dt} = 0$$

$$\frac{Q}{C} + L\frac{d^2Q}{dt^2} = 0$$

where $I = -dQ / dt$ (and $dI / dt = -d^2Q/dt^2$). Notice the sign convention we have adopted here. The negative sign implies that the current I is equal to the rate of decrease of change in the capacitor plate immediately after the switch has been closed. The same equation can be obtained by applying the modified Kirchhoff's loop rule clockwise.

$$\frac{Q}{C} - L\frac{dI}{dt} = 0$$

followed by our definition of current.

The general solution to equation is $Q(t) = Q_0 \cos(\omega_0 t + \phi)n$

where Q_0 is the amplitude of the charge and ϕ is the phase. The angular frequency ω_0 is given by

$$\omega_0 = \frac{1}{\sqrt{LC}}$$

The corresponding current in the inductor is

$$I(t) = -\frac{dQ}{dt} = \omega_0 Q_0 \sin(\omega_0 t + \phi) = I_0 \sin(\omega_0 t + \phi)$$

where $I_0 = \omega_0 Q_0$. From the initial conditions $Q(t = 0) = Q_0$ and $I(t = 0) = 0$, the phase ϕ can be determined to $\phi = 0$. Thus, the solutions for the charge and the current in our LC circuit are

$$Q(t) = Q_0 \cos \omega_0 t$$

and $I(t) = I_0 \sin w_0 t$

The time dependence of Q(t) and I(t) are depicted in figure.

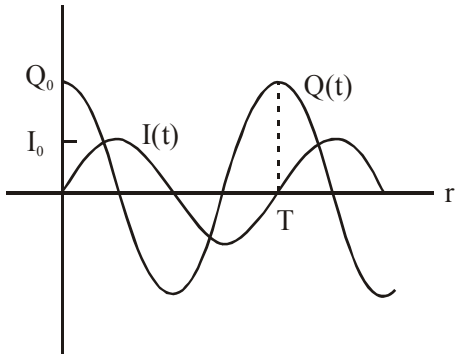


Figure : Charge and current in the *LC* circuit as a function of time

Using Eqs., we see that at any instant of time, the electric energy and the magnetic energies are given by

$$U_E = U_E = \frac{Q^2(t)}{2C} = \left(\frac{Q_0^2}{2C}\right)\cos^2 \omega_0 t$$

and $$U_B = \frac{1}{2}LI^2(t) = \frac{LI_0^2}{2}\sin^2 \omega t = \frac{L(-\omega_0 Q_0)^2}{2}\sin^2 \omega_0 t = \left(\frac{Q_0^2}{2C}\right)\sin^2 \omega_0 t = \frac{Q_0^2}{2C}$$

The electric and magnetic energy oscillation is illustrated in figure.

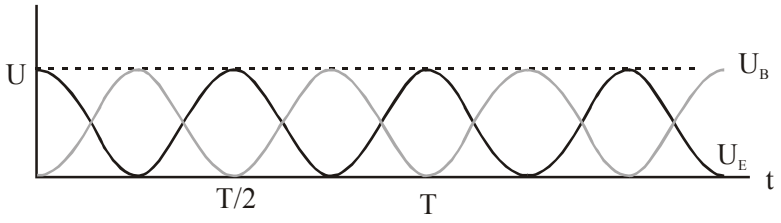


Figure : Electric and magnetic energy oscillations

The mechanical analog of the *LC* oscillations is the mass-spring system, shown in Figure.

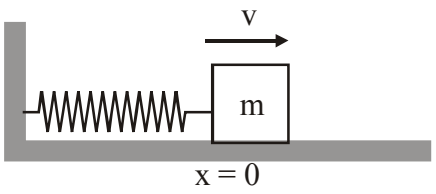


Figure : Mass-spring oscillations

If the mass is moving with a speed *v* and the spring having a spring constant *k* is displaced from its equilibrium by *x*, then the total energy of this mechanical system is

$$U = K + U_{sp} = \frac{1}{2}mv^2 + \frac{1}{2}kx^2$$

where K and U_{sp} are the kinetic energy of the mass and the potential energy of the spring, respectively. In the absence of friction, U is conserved and we obtain

$$\frac{dU}{dt} = \frac{d}{dt}\left(\frac{1}{2}mv^2 + \frac{1}{2}kx^2\right) = mv\,\frac{dv}{dt} + kx\,\frac{dx}{dt} = 0$$

Using $v = dx/dt$ and $dv/dt = d^2x/dt^2$, the above equation may be rewritten as

$$m\frac{d^2x}{dt^2} + kx = 0$$

The general solution for the displacement is

$$x(t) = x_0 \cos(\omega_0 t + \phi)$$

where $\omega_0 = \sqrt{\frac{k}{m}}$ is the angular frequency and x_0 is the amplitude of the oscillations. Thus, at any instant in time, the energy of the system may be written as

$$U = \frac{1}{2} m x_0^2 \omega_0^2 \sin^2(\omega_0 t + \phi) + \frac{1}{2} k x_0^2 \cos^2(\omega_0 t + \phi)$$

$$= \frac{1}{2} k x_0^2 [\sin^2(\omega_0 t + \phi) + \cos^2(\omega_0 t + \phi) = \frac{1}{2} k x_0^2$$

In figure we illustrate the energy oscillations in the LC circuit and the mass spring system (harmonic oscillator).

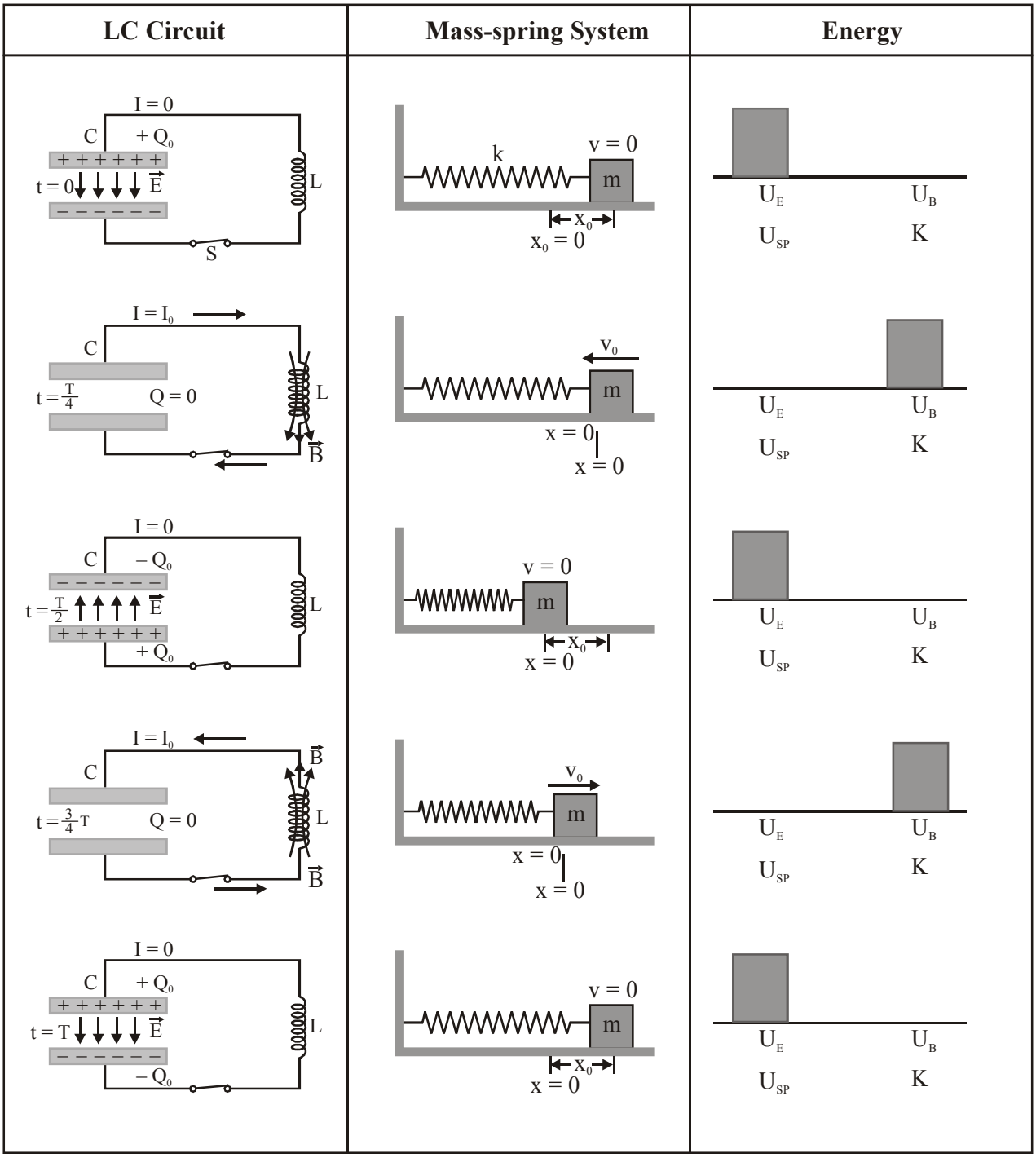


Figure : Energy oscillations in the *LC* Circuit and the mass-spring system

LC Circuit :

Ex. Consider the circuit shown in Figure. Suppose the switch which has been connected to point *a* for a long time is suddenly thrown to *b* at $t = 0$.

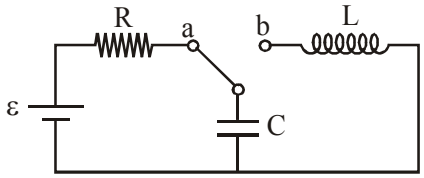


Figure : *LC* circuit

Find the following quantities :

(a) the frequency of oscillation of the *LC* circuit.

(b) the maximum charge that appears on the capacitor.

(c) the maximum current in the inductor.

(d) the total energy the circuit possesses at any time *t*.

Solution :

(a) The (angular) frequency of oscillation of the *LC* circuit is given by $\omega = 2\pi f = 1/\sqrt{LC}$. Therefore, the frequency is :

$$f = \frac{1}{2\pi\sqrt{LC}}$$

(b) The maximum charge stored in the capacitor before the switch is thrown to b is

$$Q = C\varepsilon$$

(c) The energy stored in the capacitor before the switch is thrown is :

$$U_E = \frac{1}{2}C\varepsilon^2$$

On the other hand, the magnetic energy stored in the inductor is :

$$U_B = \frac{1}{2}LI^2$$

Thus, when the current is at its maximum, all the energy originally stored in the capacitor is now in the inductor :

$$\frac{1}{2}C\varepsilon^2 = \frac{1}{2}LI_0^2$$

This implies a maximum current

$$I_0 = \varepsilon\sqrt{\frac{C}{L}}$$

(d) At any time, the total energy in the circuit would be equal to the initial energy that the capacitance stored, that is

$$U = U_E + U_B = \frac{1}{2}C\varepsilon^2$$

ALTERNATING CURRENT

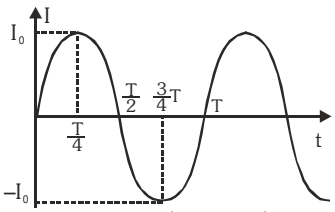
ALTERNATING CURRENT AND VOLTAGE

Voltage or current is said to be alternating if it is change continously in magnitude and perodically in direction with time. It can be represented by a sine curve or cosine curve

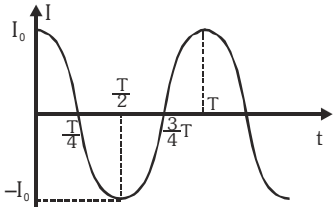
$I = I_0 \sin \omega t$ or $I = I_0 \cos \omega t$

where I = Instantaneous value of current at time t, I_0 = Amplitude or peak value

ω = Angular frequency $\omega = \frac{2\pi}{T} = 2\pi f$ T = time period f = frequency



I as a sine function of t



I as a cosine function of t

AMPLITUDE OF AC

The maximum value of current in either direction is called peak value or the amplitude of current. It is represented by I_0. Peak to peak value = $2I_0$

PERIODIC TIME

The time taken by alternating current to complete one cycle of variation is called periodic time or time period of the current.

FREQUENCY

The number of cycle completed by an alternating current in one second is called the frequency of the current.

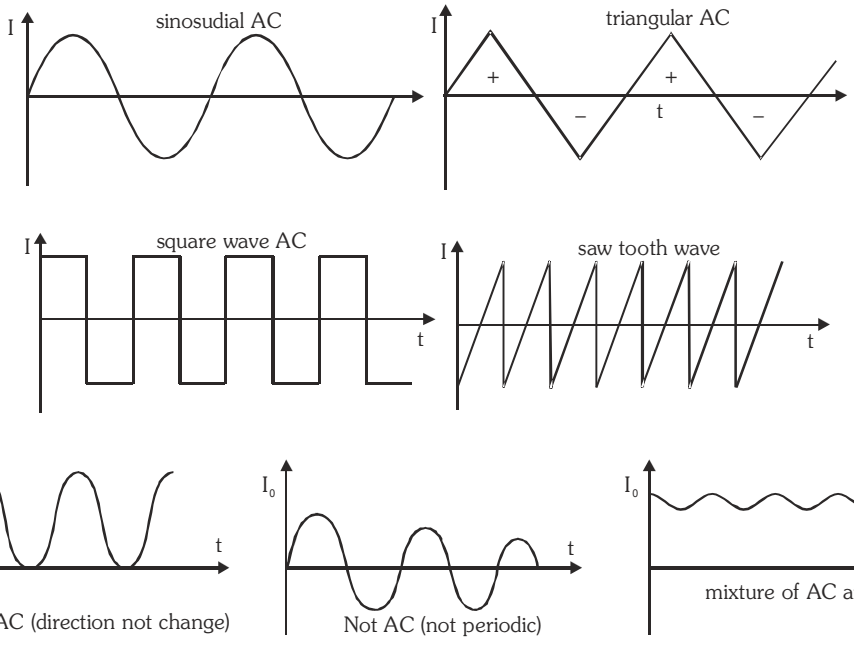
UNIT : cycle/s ; (Hz)

In India : f = 50 Hz , supply voltage = 220 volt **In USA :** f = 60 Hz ,supply voltage = 110 volt

CONDITION REQUIRED FOR CURRENT/ VOLTAGE TO BE ALTERNATING

- Amplitude is constant
- Alternate half cycle is positive and half negative

The alternating current continuously varies in magnitude and periodically reverses its direction.



AVERAGE VALUE OR MEAN VALUE

The mean value of A.C over any half cycle (either positive or negative) is that value of DC which would send same amount of charge through a circuit as is sent by the AC through same circuit in the same time.

$$\text{average value of current for half cycle } \langle I \rangle = \frac{\int_0^{T/2} I\,dt}{\int_0^{T/2} dt}$$

Average value of $I = I_0 \sin \omega t$ over the positive half cycle :

$$I_{av} = \frac{\int_0^{\frac{T}{2}} I_0 \sin \omega t\, dt}{\int_0^{\frac{T}{2}} dt} = \frac{2I_0}{\omega T}\left[-\cos \omega t\right]_0^{\frac{T}{2}} = \frac{2I_0}{\pi}$$

$\langle \sin\theta \rangle = \langle \sin 2\theta \rangle = 0$
$\langle \cos\theta \rangle = \langle \cos 2\theta \rangle = 0$
$\langle \sin\theta \cos\theta \rangle = 0$
$\langle \sin^2\theta \rangle = \langle \cos^2\theta \rangle = \frac{1}{2}$

- For symmetric AC, average value over full cycle = 0,
  Average value of sinusoidal AC

Full cycle	(+ve) half cycle	(–ve) half cycle
0	$\frac{2I_0}{\pi}$	$\frac{-2I_0}{\pi}$

As the average value of AC over a complete cycle is zero, it is always defined over a half cycle which must be either positive or negative

MAXIMUM VALUE

- $I = a \sin\theta \Rightarrow I_{Max.} = a$
- $I = a + b \sin\theta \Rightarrow I_{Max.} = a + b$ (if a and b > 0)
- $I = a \sin\theta + b \cos\theta \Rightarrow I_{Max.} = \sqrt{a^2 + b^2}$
- $I = a \sin^2\theta \Rightarrow I_{Max.} = a$ (a > 0)

ROOT MEAN SQUARE (rms) VALUE

It is value of DC which would produce same heat in given resistance in given time as is done by the alternating current when passed through the same resistance for the same time.

$$I_{rms} = \sqrt{\frac{\int_0^T I^2 dt}{\int_0^T dt}}$$ rms value = Virtual value = Apparent value

rms value of $I = I_0 \sin \omega t$:

$$I_{rms} = \sqrt{\frac{\int_0^T (I_0 \sin \omega t)^2 dt}{\int_0^T dt}} = \sqrt{\frac{I_0^2}{T}\int_0^T \sin^2 \omega t\, dt}$$

$$= I_0 \sqrt{\frac{1}{T}\int_0^T \left[\frac{1-\cos 2\omega t}{2}\right] dt} = I_0 \sqrt{\frac{1}{T}\left[\frac{t}{2} - \frac{\sin 2\omega t}{2\times 2\omega}\right]_0^T} = \frac{I_0}{\sqrt{2}}$$

- If nothing is mentioned then values printed in a.c circuit on electrical appliances, any given or unknown values, reading of AC meters are assumed to be RMS.

Current	Average	Peak	RMS	Angular fequency
$I_1 = I_0 \sin \omega t$	0	I_0	$\frac{I_0}{\sqrt{2}}$	ω
$I_2 = I_0 \sin \omega t \cos \omega t = \frac{I_0}{2} \sin 2\omega t$	0	$\frac{I_0}{2}$	$\frac{I_0}{2\sqrt{2}}$	2ω
$I_3 = I_0 \sin\omega t + I_0 \cos\omega t$	0	$\sqrt{2}\, I_0$	I_0	ω

- For above varieties of current rms = $\frac{\text{Peak value}}{\sqrt{2}}$

Ex. If $I = 2\sqrt{t}$ ampere then calculate average and rms values over t = 2 to 4 s

Sol. $< I > = \frac{\int_2^4 2\sqrt{t}.dt}{\int_2^4 dt} = \frac{4}{3}\frac{(t^{\frac{3}{2}})_2^4}{(t)_2^4} = \frac{2}{3}\left[8 - 2\sqrt{2}\right]$ and $I_{rms} = \sqrt{\frac{\int_2^4 (2\sqrt{t})^2 dt}{\int_2^4 dt}} = \sqrt{\frac{\int_2^4 4t\, dt}{2}} = \sqrt{2\left[\frac{t^2}{2}\right]_2^4} = 2\sqrt{3}$ A

Ex. Find the time required for 50Hz alternating current to change its value from zero to rms value.

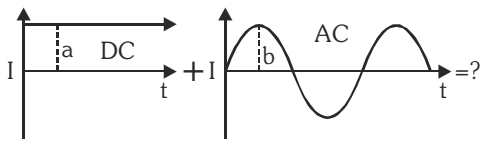
Sol. $\because I = I_0 \sin \omega t \ \therefore \frac{I_0}{\sqrt{2}} = I_0 \sin \omega t \Rightarrow \sin \omega t = \frac{1}{\sqrt{2}}\ \omega t = \frac{\pi}{4}$

$\Rightarrow \left(\frac{2\pi}{T}\right) t = \frac{\pi}{4} \Rightarrow t = \frac{T}{8} = \frac{1}{8 \times 50} = 2.5\,\text{ms}$

Ex. If $E = 20 \sin (100\pi t)$ volt then calculate value of E at $t = \frac{1}{600}$ s

Sol. At $t = \frac{1}{600}$ s $E = 20 \text{ Sin } \left[100\pi \times \frac{1}{600}\right] = 20 \sin \left[\frac{\pi}{6}\right] = 20 \times \frac{1}{2} = 10$V

Ex. If a direct current of value a ampere is superimposed on an alternating current $I = b \sin\omega t$ flowing through a wire, what is the effective value of the resulting current in the circuit ?



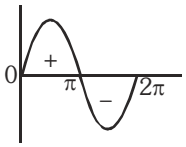
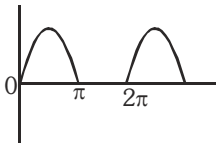
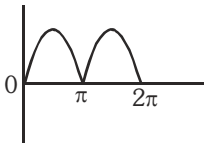
Sol. As current at any instant in the circuit will be,

$I = I_{DC} + I_{AC} = a + b \sin\omega t$

$\therefore \quad I_{eff} = \sqrt{\frac{1}{T}\int_0^T I^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a + b\sin\omega t)^2 dt} = \sqrt{\frac{1}{T}\int_0^T (a^2 + 2ab\sin\omega t + b^2 \sin^2 \omega t) dt}$

but as $\frac{1}{T}\int_0^T \sin\omega t dt = 0$ and $\frac{1}{T}\int_0^T \sin^2 \omega t dt = \frac{1}{2}$ $\quad \therefore \quad I_{eff} = \sqrt{a^2 + \frac{1}{2}b^2}$

SOME IMPORTANT WAVE FORMS AND THEIR RMS AND AVERAGE VALUE

Nature of wave form	Wave–form	RMS Value	Average or mean Value
Sinusoidal		$\frac{I_0}{\sqrt{2}}$ $= 0.707\ I_0$	$\frac{2I_0}{\pi}$ $= 0.637\ I_0$
Half wave rectifired		$\frac{I_0}{2}$ $= 0.5\ I_0$	$\frac{I_0}{\pi}$ $= 0.318\ I_0$
Full wave rectifired		$\frac{I_0}{\sqrt{2}}$ $= 0.707\ I_0$	$\frac{2I_0}{\pi}$ $0.637\ I_0$

PHASE AND PHASE DIFFERENCE

(a) Phase

$I = I_0 \sin(\omega t \pm \phi)$

Initial phase = ϕ (it does not change with time)

Instantaneous phase = $\omega t \pm \phi$ (it changes with time)

- Phase decides both value and sign. ◎ **UNIT:** radian

(b) Phase difference

Voltage $V = V_0 \sin(\omega t + \phi_1)$ Current $I = I_0 \sin(\omega t + \phi_2)$

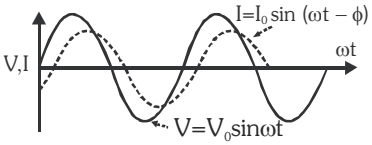
- Phase difference of I w.r.t. V $\phi = \phi_2 - \phi_1$
- Phase difference of V w.r.t. I $\phi = \phi_1 - \phi_2$

LAGGING AND LEADING CONCEPT

(a) V leads I or I lags V → It means, V reach maximum before I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t - \phi)$

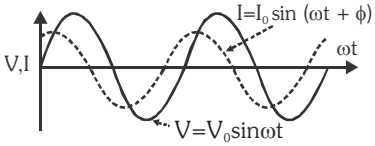
and if $V = V_0 \sin(\omega t + \phi)$ then $I = I_0 \sin \omega t$



(b) V lags I or I leads V → It means V reach maximum after I

Let if $V = V_0 \sin \omega t$ then $I = I_0 \sin(\omega t + \phi)$

and if $V = V_0 \sin(\omega t - \phi)$ then $I = I_0 \sin \omega t$



PHASOR AND PHASOR DIAGRAM

A diagram representing alternating current and voltage (of same frequency) as vectors (phasor) with the phase angle between them is called phasor diagram.

Let $V = V_0 \sin \omega t$ and $I = I_0 \sin(\omega t + \phi)$

In figure (a) two arrows represents phasors. The length of phasors represents the maximum value of quantity. The projection of a phasor on y-axis represents the instantaneous value of quantity

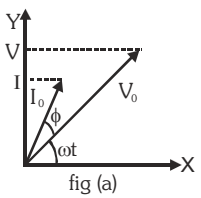


fig (a)

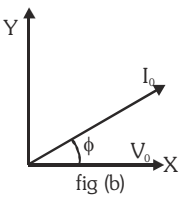


fig (b)

Ex. The Equation of current in AC circuit is $I = 4\sin\left[100\pi t + \frac{\pi}{3}\right]$ A. Calculate.

(i) RMS Value (ii) Peak Value (iii) Frequency (iv) Initial phase (v) Current at t = 0

Sol. (i) $I_{rms} = \frac{I_0}{\sqrt{2}} = \frac{4}{\sqrt{2}} = 2\sqrt{2}$ A (ii) Peak value $I_0 = 4$A

(iii) $\because$ $\omega = 100\ \pi$ rad/s $\therefore$ frequency $f = \frac{100\pi}{2\pi} = 50$ Hz

(iv) Initial phase $= \frac{\pi}{3}$ (v) At $t = 0$, $I = 4\sin\left[100\pi \times 0 + \frac{\pi}{3}\right] = 4 \times \frac{\sqrt{3}}{2} = 2\sqrt{3}$ A

Ex. If $I = I_0 \sin\omega t$, $E = E_0 \cos\left[\omega t + \frac{\pi}{3}\right]$. Calculate phase difference between E and I

Sol. $I = I_0 \sin\omega t$ and $E = E_0 \sin\left[\frac{\pi}{2} + \omega t + \frac{\pi}{3}\right]$ $\therefore$ phase difference $= \frac{\pi}{2} + \frac{\pi}{3} = \frac{5\pi}{6}$

Ex. If $E = 500 \sin(100\ \pi t)$ volt then calculate time taken to reach from zero to maximum.

Sol. $\because \omega = 100\ \pi \Rightarrow T = \frac{2\pi}{100\pi} = \frac{1}{50}$ s, time taken to reach from zero to maximum $= \frac{T}{4} = \frac{1}{200}$ s

Ex. Show that average heat produced during a cycle of AC is same as produced by DC with $I = I_{rms}$.

Sol. For AC, $I = I_0 \sin\omega t$, the instantaneous value of heat produced (per second) in a resistance R is,

$H = I^2R = I_0^2\sin^2\omega t \times R$ the average value of heat produced during a cycle is :

$$H_{av} = \frac{\int_0^T H\,dt}{\int_0^T dt} = \frac{\int_0^T (I_0^2 \sin^2\omega t \times R)dt}{\int_0^T dt} = \frac{1}{2} I_0^2 R \quad \left[\because \int_0^T I_0^2 \sin^2\omega t\,dt = \frac{1}{2} I_0^2 T\right]$$

$$\Rightarrow H_{av} = \left(\frac{I_0}{\sqrt{2}}\right)^2 R = I_{rms}^2 R \ldots..(i)$$

However, in case of DC, $H_{DC} = I^2 R$...(ii) $\because$ $I = I_{rms}$ so from equation (i) and (ii) $H_{DC} = H_{av}$

AC produces same heating effects as DC of value $I = I_{rms}$. This is also why AC instruments which are based on heating effect of current give rms value.

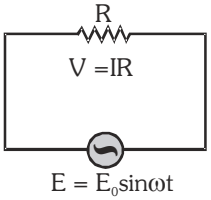
DIFFERENT TYPES OF AC CIRCUITS

In order to study the behaviour of A.C. circuits we classify them into two categories :

(a) Simple circuits containing only one basic element i.e. resistor (R) or inductor (L) or capacitor (C) only.

(b) Complicated circuit containing any two of the three circuit elements R, L and C or all of three elements.

AC CIRCUIT CONTAINING PURE RESISTANCE



Let at any instant t the current in the circuit = I.

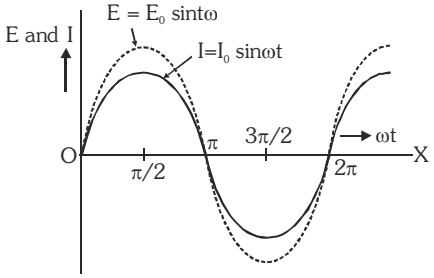
Potential difference across the resistance = I R.

with the help of kirchoff's circuital law $E - I R = 0$

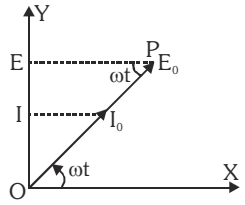
$\Rightarrow E_0 \sin\omega t = I R$

$\Rightarrow I = \frac{E_0}{R}\sin\omega t = I_0 \sin\omega t$ ($I_0 = \frac{E_0}{R}$ = peak or maximum value of current)

Alternating current developed in a pure resistance is also of sinusoidal nature. In an a.c. circuits containing pure resistance, the voltage and current are in the same phase. The vector or phasor diagram which represents the phase relationship between alternating current and alternating e.m.f. as shown in figure.



In the a.c. circuit having R only, as current and voltage are in the same phase, hence in fig. both phasors E_0 and I_0 are in the same direction, making an angle ωt with OX. Their projections on Y-axis represent the instantaneous values of alternating current and voltage.



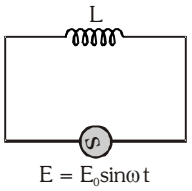
i.e. $I = I_0 \sin\omega t$ and $E = E_0 \sin\omega t$.

Since $I_0 = \frac{E_0}{R}$, hence $\frac{I_0}{\sqrt{2}} = \frac{E_0}{R\sqrt{2}} \Rightarrow I_{rms} = \frac{E_{rms}}{R}$

AC CIRCUIT CONTAINING PURE INDUCTANCE

A circuit containing a pure inductance L (having zero ohmic resistance) connected with a source of alternating emf.

Let the alternating e.m.f. $E = E_0 \sin\omega t$



When a.c. flows through the circuit, emf induced across inductance $= -L\frac{dI}{dt}$

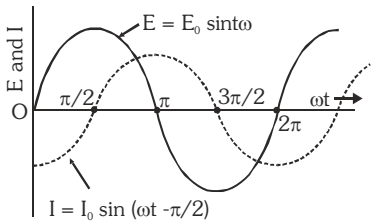
Negative sign indicates that induced emf acts in opposite direction to that of applied emf.

Because there is no other circuit element present in the circuit other then inductance so with the help of

Kirchoff's circuital law $E + \left(-L\frac{dI}{dt}\right) = 0 \Rightarrow E = L\frac{dI}{dt}$ so we get $I = \frac{E_0}{\omega L}\sin\left(\omega t - \frac{\pi}{2}\right)$

Maximum current $I_0 = \frac{E_0}{\omega L} \times 1 = \frac{E_0}{\omega L}$, Hence, $I = I_0 \sin\left(\omega t - \frac{\pi}{2}\right)$

In a pure inductive circuit current always lags behind the emf by $\frac{\pi}{2}$.

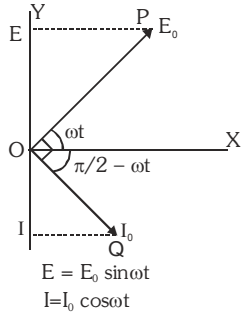


or alternating emf leads the a. c. by a phase angle of $\frac{\pi}{2}$.

Expression $I_0 = \frac{E_0}{\omega L}$ resembles the expression $\frac{E}{I} = R$.

This non-resistive opposition to the flow of A.C. in a circuit is called the inductive reactance (X_L) of the circiut.

$X_L = \omega L = 2\pi f L$ where f = frequency of A.C.



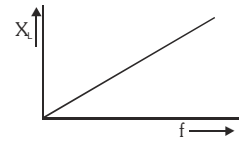
Unit of X_L : ohm

(ωL) = Unit of L × Unit of ω = henry × sec^{-1}

$$= \frac{\text{Volt}}{\text{Ampere/sec}} \times \sec^{-1} = \frac{\text{Volt}}{\text{Ampere}} = \text{ohm}$$

Inductive reactance $X_L \propto f$

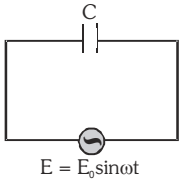
Higher the frequency of A.C. higher is the inductive reactance offered by an inductor in an A.C. circuit.



For d.c. circuit, f = 0 $\therefore X_L = \omega L = 2\pi f L = 0$

Hence, inductor offers no opposition to the flow of d.c. whereas a resistive path to a.c.

AC CIRCUIT CONTAINING PURE CAPACITANCE

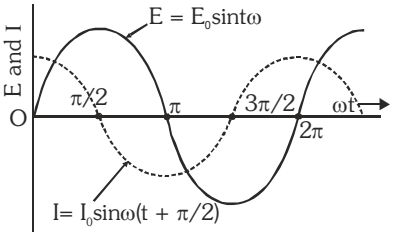


A circuit containing an ideal capacitor of capacitance C connected with a source of alternating emf as shown in fig. The alternating e.m.f. in the circuit $E = E_0 \sin \omega t$

When alternating e.m.f. is applied across the capacitor a similarly varying alternating current flows in the circuit.

The two plates of the capacitor become alternately positively and negatively charged and the magnitude of the charge on the plates of the capacitor varies sinusoidally with time. Also the electric field between the plates of the capacitor varies sinusoidally with time.Let at any instant t charge on the capacitor = q

Instantaneous potential difference across the capacitor $E = \frac{q}{C}$

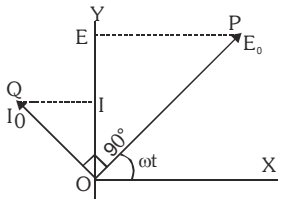


$\Rightarrow \quad q = C\,E \quad \Rightarrow q = CE_0 \sin \omega t$

The instantaneous value of current

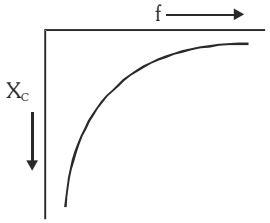
$$I = \frac{dq}{dt} = \frac{d}{dt}\left(CE_0 \sin \omega t\right) = CE_0 \omega \cos \omega t$$

$$\Rightarrow \quad I = \frac{E_0}{(1/\omega C)} \sin\left(\omega t + \frac{\pi}{2}\right) = I_0 \sin\left(\omega t + \frac{\pi}{2}\right) \text{ where } I_0 = \omega C V_0$$



In a pure capacitive circuit, the current always leads the e.m.f. by a phase angle of $\pi/2$. The alternating emf lags behinds the alternating current by a phase angle of $\pi/2$.

IMPORTANT POINTS



$\frac{E}{I}$ is the resistance R when both E and I are in phase, in present case they differ in phase by $\frac{\pi}{2}$, hence $\frac{1}{\omega C}$ is not the resistance of the capacitor, the capacitor offer opposition to the flow of A.C. This non-resistive opposition to the flow of A.C. in a pure capacitive circuit is known as capacitive reactance X_C. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi fC}$

Unit of X_C : ohm

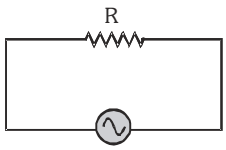
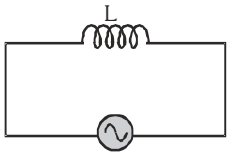
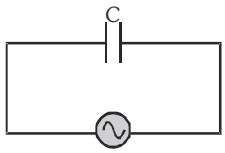
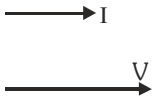
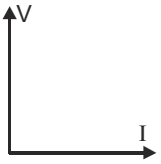
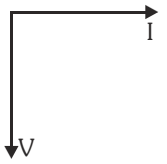
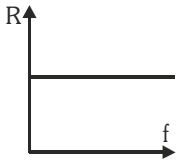
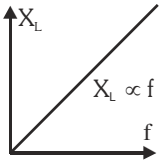
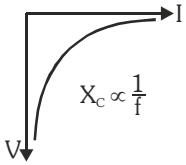
Capacitive reactance X_C is inversely proportional to frequence of A.C. X_C decreases as the frequency increases.

This is because with an increase in frequency, the capacitor charges and discharges rapidly following the flow of current.

For d.c. circuit f = 0 $\therefore X_C = \frac{1}{2\pi fC} = \infty$ but has a very small value for a.c.

This shows that capacitor blocks the flow of d.c. but provides an easy path for a.c.

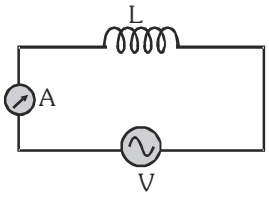
INDIVIDUAL COMPONENTS (R or L or C)

TERM	R	L	C
Circuit			
Supply Voltage	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$	$V = V_0 \sin \omega t$
Current	$I = I_0 \sin \omega t$	$I = I_0 \sin (\omega t - \frac{\pi}{2})$	$I = I_0 \sin (\omega t + \frac{\pi}{2})$
Peak Current	$I_0 = \frac{V_0}{R}$	$I_0 = \frac{V_0}{\omega L}$	$I_0 = \frac{V_0}{1/\omega C} = V_0 \omega C$
Impedance (Ω)	$\frac{V_0}{I_0} = R$	$\frac{V_0}{I_0} = \omega L = X_L$	$\frac{V_0}{I_0} = \frac{1}{\omega C} = X_C$
$Z = \frac{V_0}{I_0} = \frac{V_{rms}}{I_{rms}}$	R = Resistance	X_L=Inductive reactance.	X_C=Capacitive reactance.
Phase difference	zero (in same phase)	$+\frac{\pi}{2}$ (V leads I)	$-\frac{\pi}{2}$ (V lags I)
Phasor diagram			
Variation of Z with *f*			
	R does not depend on *f*		
G,S_L,S_C (mho, seiman)	G=1/R = conductance.	$S_L = 1/X_L$ Inductive susceptance	$S_C = 1/X_C$ Capacitive susceptance
Behaviour of device in D.C. and A.C	Same in A C and D C	L passes DC easily (because $X_L = 0$) while gives a high impedance for the A.C. of high frequency ($X_L \propto f$)	C - blocks DC (because $X_C = \infty$) while provides an easy path for the A.C. of high frequency $\left[X_C \propto \frac{1}{f}\right]$
Ohm's law	$V_R = IR$	$V_L = IX_L$	$V_C = IX_C$

Ex. A capacitor of 50 pF is connected to an a.c. source of frequency 1kHz Calculate its reactance.

Sol. $X_C = \frac{1}{\omega C} = \frac{1}{2\pi \times 10^3 \times 50 \times 10^{-12}} = \frac{10^7}{\pi}\ \Omega$

Ex. In given circuit applied voltage $V = 50\sqrt{2}$ sin 100πt volt and ammeter reading is 2A then calculate value of L



Sol. $V_{rms} = I_{rms} X_L$ $\because$ Reading of ammeter = I_{rms}

$$X_L = \frac{V_{rms}}{I_{rms}} = \frac{V_0}{\sqrt{2}\, I_{rms}} = \frac{50\sqrt{2}}{\sqrt{2}\times 2} = 25\ \Omega \Rightarrow L = \frac{X_L}{\omega} = \frac{25}{100\pi} = \frac{1}{4\pi}\ H$$

Ex. A 50 W, 100 V lamp is to be connected to an AC mains of 200 V, 50 Hz. What capacitance is essential to be put in series with the lamp ?

Sol. $\because$ resistance of the lamp $R = \frac{V_s^2}{W} = \frac{(100)^2}{50} = 200\ \Omega$ and the maximum curent $I = \frac{V}{R} = \frac{100}{200} = \frac{1}{2}A$

$\therefore$ when the lamp is put in series with a capacitance and run at 200 V AC, from V = IZ

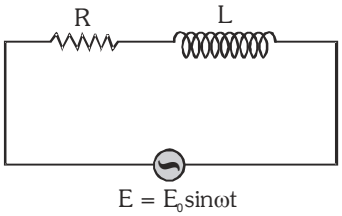
$Z = \frac{V}{I} = \frac{200}{\frac{1}{2}} = 400\Omega$ Now as in case of C–R circuit $Z = \sqrt{R^2 + \frac{1}{(\omega C)^2}}$,

$$\Rightarrow R^2 + \frac{1}{(\omega C)^2} = (400)^2 \qquad \Rightarrow \frac{1}{(\omega C)^2} = 16\times 10^4 - (200)^2 = 12\times 10^4 \qquad \Rightarrow \frac{1}{\omega C} = \sqrt{12}\times 10^2$$

$$\Rightarrow C = \frac{1}{100\pi \times \sqrt{12}\times 10^2} F = \frac{100}{\pi\sqrt{12}}\mu F = 9.2\mu F$$

RESISTANCE AND INDUCTANCE IN SERIES (R-L CIRCUIT)

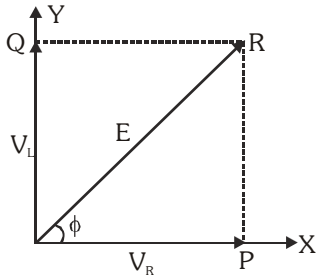
A circuit containing a series combination of a resistance R and an inductance L, connected with a source of alternating e.m.f. E as shown in figure.



PHASOR DIAGRAM FOR L-R CIRCUIT

Let in a L-R series circuit, applied alternating emf is $E = E_0 \sin\omega t$. As R and L are joined in series, hence current flowing through both will be same at each instant. Let I be the current in the circuit at any instant and V_L and V_R the potential differences across L and R respectively at that instant.

Then $V_L = IX_L$ and $V_R = IR$



Now, V_R is in phase with the current while V_L leads the current by $\frac{\pi}{2}$.

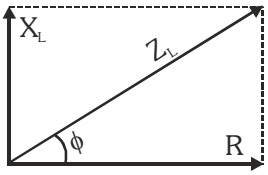
So V_R and V_L are mutually perpendicular (Note : $E \neq V_R + V_L$)

The vector OP represents V_R (which is in phase with I), while OQ represents V_L (which leads I by 90°).

The resultant of V_R and V_L = the magnitude of vector OR $E = \sqrt{V_R^2 + V_L^2}$

Thus $E^2 = V_R^2 + V_L^2 = I^2 (R^2 + X_L^2) \Rightarrow I = \frac{E}{\sqrt{R^2 + X_L^2}}$

The phasor diagram shown in fig. also shows that in L-R circuit the applied emf E leads the current I or conversely the current I lags behind



the e.m.f. E. by a phase angle ϕ $\tan\phi = \frac{V_L}{V_R} = \frac{IX_L}{IR} = \frac{X_L}{R} = \frac{\omega L}{R}$

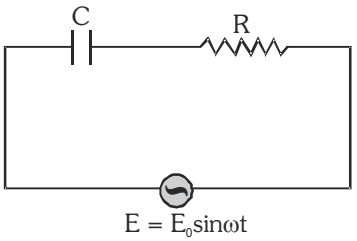
$\Rightarrow \phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

Inductive Impedance Z_L :

In L-R circuit the maximum value of current $I_0 = \frac{E_0}{\sqrt{R^2+\omega^2L^2}}$ Here $\sqrt{R^2+\omega^2L^2}$ represents the effective opposition offered by L-R circuit to the flow of a.c. through it. It is known as impedance of L-R circuit and is represented by Z_L. $Z_L = \sqrt{R^2+\omega^2L^2} = \sqrt{R^2+(2\pi fL)^2}$ The reciprocal of impedance is called admittance $Y_L = \frac{1}{Z_L} = \frac{1}{\sqrt{R^2+\omega^2L^2}}$

RESISTANCE AND CAPACITOR IN SERIES (R-C CIRCUIT)

A circuit containing a series combination of a resistance R and a capacitor C, connected with a source of e.m.f. of peak value E_0 as shown in fig.



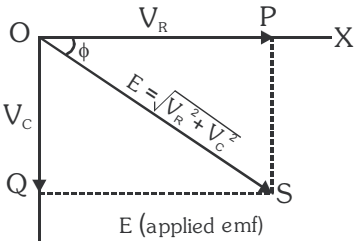
PHASOR DIAGRAM FOR R-C CIRCUIT

Current through both the resistance and capacitor will be same at every instant and the instantaneous potential differences across C and R are

$V_C = I\,X_C$ and $V_R = I\,R$

where X_C = capacitive reactance and I = instantaneous current.

Now, V_R is in phase with I, while V_C lags behind I by 90°.



The phasor diagram is shown in fig.

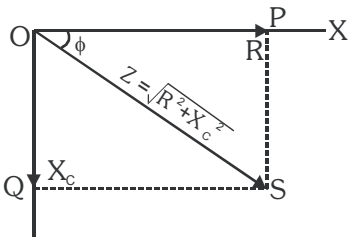
The vector OP represents V_R (which is in phase with I)

and the vector OQ represents V_C (which lags behind I by $\frac{\pi}{2}$).

The vector OS represents the resultant of V_R and V_C = the applied e.m.f. E.

Hence $V_R^2 + V_C^2 = E^2 \Rightarrow E = \sqrt{V_R^2+V_C^2}$

$\Rightarrow E^2 = I^2(R^2 + X_C^2) \Rightarrow I = \frac{E}{\sqrt{R^2+X_C^2}}$



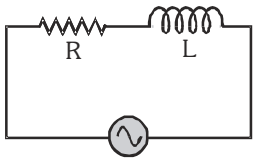
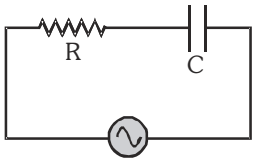
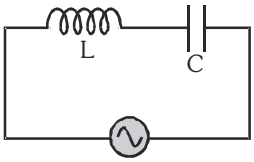
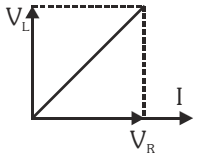
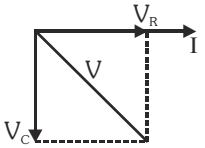
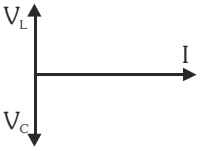
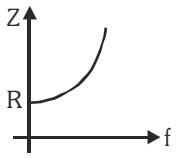
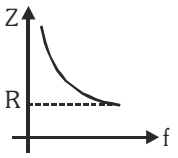
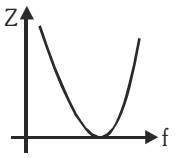
The term $\sqrt{(R^2 + X_C^2)}$ represents the effective resistance of the R-C circuit and called the capacitive impedance Z_C of the circuit. Hence, in C-R circuit $Z_C = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \left(\frac{1}{\omega C}\right)^2}$

Capacitive Impedance Z_C :

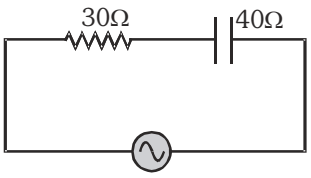
In R-C circuit the term $\sqrt{R^2 + X_C^2}$ effective opposition offered by R-C circuit to the flow of a.c. through it. It is known as impedance of R-C circuit and is represented by Z_C
The phasor diagram also shows that in R-C circuit the applied e.m.f. lags behind the current I (or the current I leads the emf E) by a phase angle ϕ given by

$$\tan\phi = \frac{V_C}{V_R} = \frac{X_C}{R} = \frac{1/\omega C}{R} = \frac{1}{\omega CR}, \quad \tan\phi = \frac{X_C}{R} = \frac{1}{\omega CR} \quad \Rightarrow \phi = \tan^{-1}\left(\frac{1}{\omega CR}\right)$$

COMBINATION OF COMPONENTS (R-L or R-C or L-C)

TERM	R-L	R-C	L-C
Circuit			
	I is same in R & L	I is same in R & C	I is same in L & C
Phasor diagram			
	$V^2 = V_R^2 + V_L^2$	$V^2 = V_R^2 + V_C^2$	$V = V_L - V_C (V_L > V_C)$ $V = V_C - V_L (V_C > V_L)$
Phase difference	V leads I ($\phi = 0$ to $\frac{\pi}{2}$)	V lags I ($\phi = -\frac{\pi}{2}$ to 0)	V lags I ($\phi = -\frac{\pi}{2}$, if $X_C > X_L$)
in between V and I		V leads I($\phi = +\frac{\pi}{2}$, if $X_L > X_C$)	Impedance
$Z = \sqrt{R^2 + X_L^2}$	$Z = \sqrt{R^2 + (X_C)^2}$	$Z = \lvert X_L - X_c \rvert$	
Variation of Z	as f↑,Z ↑	as f↑,Z↓	as f↑, Z first ↓ then↑
with f			
At very low f	$Z \simeq R$ ($X_L \to 0$)	$Z \simeq X_C$	$Z \simeq X_C$
At very high f	$Z \simeq X_L$	$Z \simeq R$ ($X_C \to 0$)	$Z \simeq X_L$

Ex. Calculate the impedance of the circuit shown in the figure.

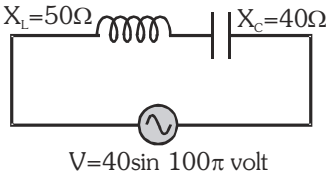


Sol. $Z = \sqrt{R^2 + (X_c)^2} = \sqrt{(30)^2 + (40)^2} = \sqrt{2500} = 50\ \Omega$

Ex. If $X_L = 50\ \Omega$ and $X_C = 40\ \Omega$ Calculate effective value of current in given circuit.

Sol. $Z = X_L - X_C = 10\ \Omega$

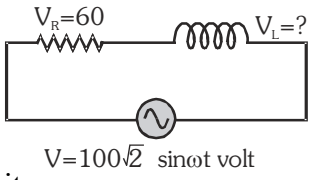
$$I_0 = \frac{V_0}{Z} = \frac{40}{10} = 4A \Rightarrow I_{rms} = \frac{4}{\sqrt{2}} = 2\sqrt{2}\ A$$



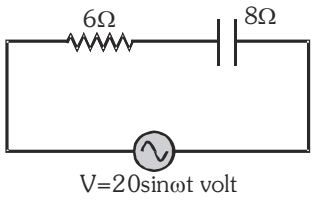
Ex. In given circuit calculate, voltage across inductor

Sol. $\because V^2 = V_R^2 + V_L^2 \quad \therefore \quad V_L^2 = V^2 - V_R^2$

$$V_L \sqrt{V^2 - V_R^2} = \sqrt{(100)^2 - (60)^2} = \sqrt{6400} = 80\ V$$



Ex. In given circuit find out (i) impedance of circuit (ii) current in circuit



Sol. (i) $Z = \sqrt{R^2 + X_C^2} = \sqrt{(6)^2 + (8)^2} = 10\ \Omega$

(ii) $V = IZ \Rightarrow I = \frac{V_0}{Z} = \frac{20}{10} = 2A$ so $I_{rms} = \frac{2}{\sqrt{2}} = \sqrt{2}\ A$

Ex. When 10V, DC is applied across a coil current through it is 2.5 A, if 10V, 50 Hz A.C. is applied current reduces to 2 A. Calculate reactance of the coil.

Sol. For 10 V D.C. $\because V = IR \quad \therefore$ Resistance of coil $R = \frac{10}{2.5} = 4\Omega$ For 10 V A.C. $\leftarrow V = I Z$

$$\Rightarrow \quad Z = \frac{V}{I} = \frac{20}{10} = 5\Omega$$

$$\because \ Z = \sqrt{R^2 + X_L^2} = 5 \ \Rightarrow R^2 + X_L^2 = 25 \ \Rightarrow \ X_L^2 = 5^2 - 4^2 \qquad \Rightarrow X_L = 3\ \Omega$$

Ex. When an alternating voltage of 220V is applied across a device X, a current of 0.5 A flows through the circuit and is in phase with the applied voltage. When the same voltage is applied across another device Y, the same current again flows through the circuit but it leads the applied voltage by $\pi/2$ radians.

(a) Name the devices X and Y.

(b) Calculate the current flowing in the circuit when same voltage is applied across the series combination of X and Y.

Sol. (a) X is resistor and Y is a capacitor

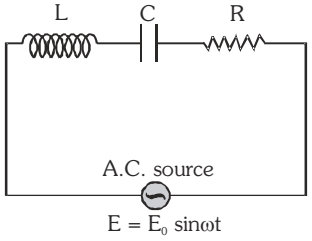
(b) Since the current in the two devices is the same (0.5A at 220 volt)

When R and C are in series across the same voltage then

$$R = X_C = \frac{220}{0.5} = 440\ \Omega \Rightarrow I_{rms} = \frac{V_{rms}}{\sqrt{R^2 + X_C^2}} = \frac{220}{\sqrt{(440)^2 + (440)^2}} = \frac{220}{440\sqrt{2}} = 0.35A$$

INDUCTANCE, CAPACITANCE AND RESISTANCE IN SERIES (L-C-R SERIES CIRCUIT)

A circuit containing a series combination of an resistance R, a coil of inductance L and a capacitor of capacitance C, connected with a source of alternating e.m.f. of peak value of E_0, as shown in fig.



PHASOR DIAGRAM FOR SERIES L-C-R CIRCUIT

Let in series LCR circuit applied alternating emf is $E = E_0 \sin \omega t$. As L,C and R are joined in series, therefore, current at any instant through the three elements has the same amplitude and phase.

However voltage across each element bears a different phase relationship with the current.

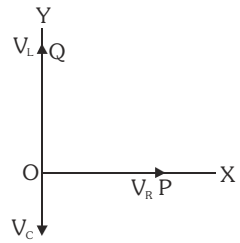
Let at any instant of time t the current in the circuit is I

Let at this time t the potential differences across L, C, and R

$V_L = I X_L$, $V_C = I X_C$ and $V_R = I R$

Now, V_R is in phase with current I but V_L leads I by 90°

While V_C legs behind I by 90°.



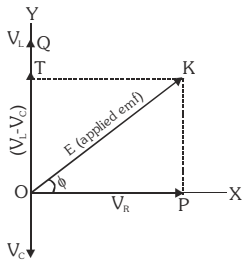
The vector OP represents V_R (which is in phase with I) the vector OQ represent VL (which leads I by 90°)

and the vector OS represents V_C (which legs behind I by 90°)

V_L and V_C are opposite to each other.

If $V_L > V_C$ (as shown in figure) the their resultant will be $(V_L - V_C)$ which is represented by OT.

Finally, the vector OK represents the resultant of V_R and $(V_L - V_C)$, that is, the resultant of all the three = applied e.m.f.

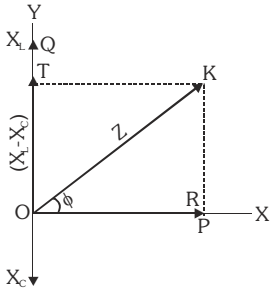


Thus $E = \sqrt{V_R^2 + (V_L - V_C)^2} = I\sqrt{R^2 + (X_L - X_C)^2} \Rightarrow I = \dfrac{E}{\sqrt{R^2 + (X_L - X_C)^2}}$

Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{R^2 + \left(\omega L - \dfrac{1}{\omega C}\right)^2}$

The phasor diagram also shown that in LCR circuit the applied e.m.f.

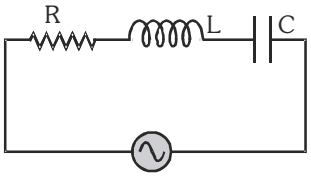
leads the current I by a phase angle ϕ $\quad \tan\phi = \dfrac{X_L - X_C}{R}$



SERIES LCR AND PARALLEL LCR COMBINATION

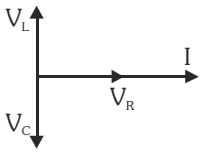
SERIES L-C-R CIRCUIT

1. Circuit diagram

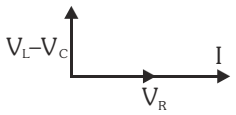


I same for R, L & C

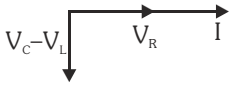
2. Phasor diagram



(i) If $V_L > V_C$ then



(ii) If $V_C > V_L$ then

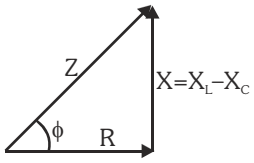


(iii) $V = \sqrt{V_R^2 + (V_L - V_C)^2}$

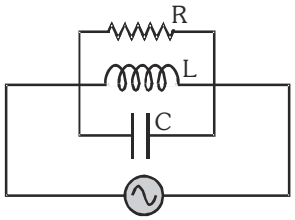
Impedance $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$\tan\phi = \frac{X_L - X_C}{R} = \frac{V_L - V_C}{V_R}$

(iv) Impedance triangle

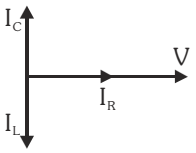


PARALLEL L-C-R CIRCUIT

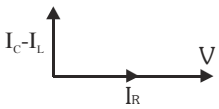


V same for R,L and C

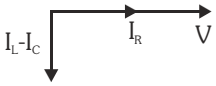
V same for R, L & C



(i) if $I_C > I_L$ then



(ii) if $I_L > I_C$ then

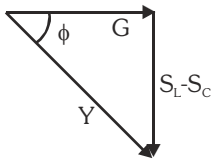


(iii) $I = \sqrt{I_R^2 + (I_L - I_C)^2}$

Admittance $Y = \sqrt{G^2 + (S_L - S_C)^2}$

$\tan\phi = \frac{S_L - S_C}{G} = \frac{I_L - I_C}{I_R}$

(iv) Admittance triangle



GOLDEN KEY POINTS

Series

(a) if $X_L > X_C$ then V leads I,ϕ (positive) circuit nature inductive

(b) if $X_C > X_L$ then V lags I,ϕ (negative) circuit nature capacitive

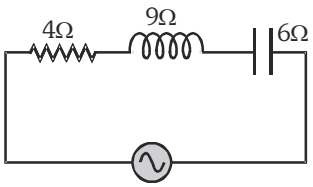
Parallel

(a) if $S_L > S_C$ ($X_L < X_C$) then V leads I,ϕ (positive) circuit nature inductive

(b) if $S_C > S_L$ ($X_C < X_L$) then V lags I, ϕ (negative) circuit nature capacitive

- In A.C. circuit voltage for L or C may be greater than source voltage or current but it happens only when circuit contains L and C both and on R it never greater than source voltage or current.
- In parallel A.C.circuit phase difference between I_L and I_C is π

Ex. Find out the impedance of given circuit.

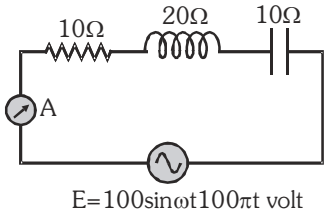


Sol. $Z = \sqrt{R^2 + (X_L - X_C)^2} = \sqrt{4^2 + (9-6)^2} = \sqrt{4^2 + 3^2} = \sqrt{25} = 5\Omega$

($\because X_L > X_C \therefore$ Inductive)

Ex. Find out reading of A C ammeter and also calculate the potential difference across, resistance and capacitor.

Sol. $Z = \sqrt{R^2 + (X_L - X_C)^2} = 10\sqrt{2}\ \Omega \Rightarrow I_0 = \frac{V_0}{Z} = \frac{100}{10\sqrt{2}} = \frac{10}{\sqrt{2}} A$



$\because$ ammeter reads RMS value, so its reading = $\frac{10}{\sqrt{2}\sqrt{2}} = 5A$

so $V_R = 5 \times 10 = 50$ V and $V_C = 5 \times 10 = 50$ V

Ex. In LCR circuit with an AC source R = 300 Ω , C = 20 μF, L = 1.0 H, E_{rms} = 50V and f = 50/π Hz. Find RMS current in the circuit.

Sol. $I_{rms} = \frac{E_{rms}}{Z} = \frac{E_{rms}}{\sqrt{R^2 + \left[\omega L - \frac{1}{\omega C}\right]^2}} = \frac{50}{\sqrt{300^2 + \left[2\pi \times \frac{50}{\pi} \times 1 - \frac{1}{20 \times 10^{-6} \times 2\pi \times \frac{50}{\pi}}\right]^2}}$

$\Rightarrow I_{rms} = \frac{50}{\sqrt{(300)^2 + \left[100 - \frac{10^3}{2}\right]^2}} = \frac{50}{100\sqrt{9+16}} = \frac{1}{10} = 0.1A$

RESONANCE

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

There are two types of resonance : (i) Series Resonance (ii) Parallel Resonance

SERIES RESONANCE

(a) At Resonance

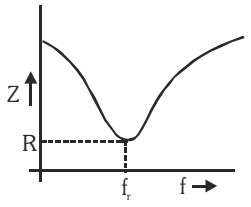
(i) $X_L = X_C$ (ii) $V_L = V_C$ (iii) $\phi = 0$ (V and I in same phase)

(iv) $Z_{min} = R$ (impedance minimum) (v) $I_{max} = \frac{V}{R}$ (current maximum)

(b) Resonance frequency

$$\because \quad X_L = X_C \quad \Rightarrow \quad \omega_r L = \frac{1}{\omega_r C} \quad \Rightarrow \quad \omega_r^2 = \frac{1}{LC} \quad \Rightarrow \quad \omega_r = \frac{1}{\sqrt{LC}} \Rightarrow \quad f_r = \frac{1}{2\pi\sqrt{LC}}$$

(c) Variation of Z with f



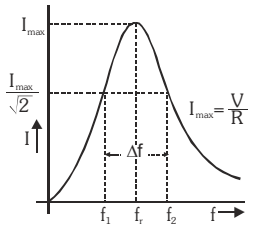
(i) If $f < f_r$ then $X_L < X_C$ circuit nature capacitive, ϕ (negative)

(ii) At $f = f_r$ then $X_L = X_C$ circuit nature, Resistive, ϕ = zero

(iii) If $f > f_r$ then $X_L > X_C$ circuit nature is inductive,ϕ (positive)

Variation of I with f as f increase, Z first decreases then increase

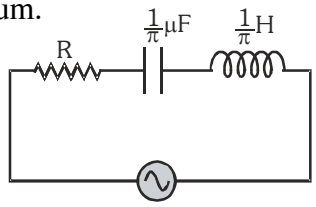
(d)



as f increase, I first increase then decreases

- At resonance impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

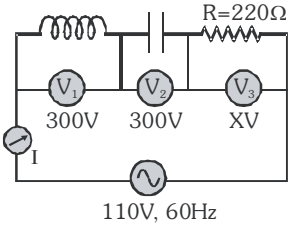
Ex. For what frequency the voltage across the resistance R will be maximum.



Sol. It happens at resonance

$$f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{\frac{1}{\pi}\times 10^{-6}\times\frac{1}{\pi}}} = 500 \text{ Hz}$$

Ex. A capacitor, a resistor and a 40 mH inductor are connected in series to an AC source of frequency 60Hz, calculate the capacitance of the capacitor, if the current is in phase with the voltage. Also calculate the value of X and I.



Sol. At resonance

$$\omega L = \frac{1}{\omega C}, \; C = \frac{1}{\omega^2 L} = \frac{1}{4\pi^2 f^2 L} = \frac{1}{4\pi^2\times(60)^2\times 40\times 10^{-3}} = 176\mu F$$

$$V = V_R \quad \Rightarrow \quad X = 110 \text{ V} \text{ and } I = \frac{V}{R} = \frac{110}{220} = 0.5 \text{ A}$$

Ex. A coil, a capacitor and an A.C. source of rms voltage 24 V are connected in series, By varying the frequency of the source, a maximum rms current 6 A is observed, If this coil is connected to a bettery of emf 12 V, and internal resistance 4Ω , then calculate the current through the coil.

Sol. At resonance current is maximum. $I = \frac{V}{R} \Rightarrow$ Resistance of coil $R = \frac{V}{I} = \frac{24}{6} = 4\ \Omega$

When coil is connected to battery, suppose I current flow through it then

$$I = \frac{E}{R+r} = \frac{12}{4+4} = 1.5\ A$$

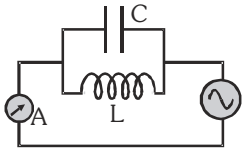
Ex. Radio receiver recives a message at 300m band, If the available inductance is 1 mH, then calculate required capacitance

Sol. Radio recives EM waves. (velocity of EM waves c = 3 x 10^8 m/s)

$$\therefore \quad c = f\lambda \Rightarrow f = \frac{3\times10^8}{300} = 10^6\ Hz$$

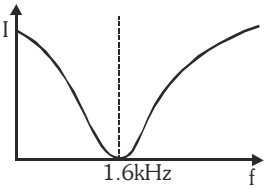
$$\text{Now } f = \frac{1}{2\pi\sqrt{LC}} = 1 \times 10^6 \Rightarrow C = \frac{1}{4\pi^2 L \times 10^{12}} = 25\ pF$$

Ex. In a L–C circuit parallel combination of inductance of 0.01 H and a capacitor of 1 μF is connected to a variable frequency alternating current source as shown in figure. Draw a rough sketch of the current variation as the frequency is changed from 1kHz to 3kHz.



Sol. L and C are connected in parallel to the AC source,

so resonance frequency $f = \frac{1}{2\pi\sqrt{LC}} = \frac{1}{2\pi\sqrt{0.01\times10^{-6}}} = \frac{10^4}{2\pi} \simeq 1.6kHz$



In case of parallel resonance, current in L–C circuit at resonance is zero, so the I-f curve will be as shown in figure.

POWER IN AC CIRCUIT

The average power dissipation in LCR AC circuit

Let $V = V_0 \sin\omega t$ and $I = I_0 \sin(\omega t - \phi)$

Instantaneous power $P = (V_0 \sin\omega t)(I_0 \sin(\omega t - \phi) = V_0 I_0 \sin\omega t (\sin\omega t\cos\phi - \sin\phi\cos\omega t)$

$$\text{Average power } \langle P \rangle = \frac{1}{T}\int_0^T (V_0 I_0 \sin^2\omega t\cos\phi - V_0 I_0 \sin\omega t\cos\omega t\sin\phi)dt$$

$$= V_0 I_0 \left[\frac{1}{T}\int_0^T \sin^2\omega t\cos\phi dt - \frac{1}{T}\int_0^T \sin\omega t\cos\omega t\sin\phi dt\right] = V_0 I_0 \left[\frac{1}{2}\cos\phi - 0\times\sin\phi\right]$$

$$\Rightarrow \quad <P> = \frac{V_0 I_0 \cos\phi}{2} = V_{rms}\, I_{rm,s} \cos\phi$$

Instantaneous power	Average power/actual power/Virtual power/ apparent dissipated power/power loss	Power/rms	Peak power Power
P = VI	$P = V_{rms}\, I_{rms} \cos\phi$	$P = V_{rms}\, I_{rms}$	$P = V_0\, I_0$

- $I_{rms} \cos\phi$ is known as active part of current or wattfull current, workfull current. It is in phase with voltage.
- $I_{rms} \sin\phi$ is known as inactive part of current, wattless current, workless current. It is in quadrature (90°) with voltage.

Power factor :

Average power $\overline{P} = E_{rms}\, I_{rms} \cos\phi = \text{r m s power} \times \cos\phi$

Power factor $(\cos\phi) = \frac{\text{Average power}}{\text{r m s Power}}$ and $\cos\phi = \frac{R}{Z}$

Power factor : (i) is leading if I leads V (ii) is lagging if I lags V

GOLDEN KEY POINTS

- $P_{av} \le P_{rms}$.
- Power factor varies from 0 to 1

• Pure/Ideal	ϕ	V	Power factor = $\cos\phi$	Average power
R	0	V, I same Phase	1 (maximum)	$V_{rms}.\, I_{rms}$
L	$+\frac{\pi}{2}$	V leads I	0	0
C	$-\frac{\pi}{2}$	V lags I	0	0
Choke coil	$+\frac{\pi}{2}$	V leads I	0	0

- At resonance power factor is maximum ($\phi = 0$ so $\cos\phi = 1$) and $P_{av} = V_{rms}\, I_{rms}$

Ex. A voltage of 10 V and frequency 10^3 Hz is applied to $\frac{1}{\pi}$ μF capacitor in series with a resistor of 500Ω. Find the power factor of the circuit and the power dissipated.

Sol. $\because X_C = \frac{1}{2\pi f C} = \frac{1}{2\pi \times 10^3 \times \frac{10^{-6}}{\pi}} = 500\Omega \quad \therefore Z = \sqrt{R^2 + X_C^2} = \sqrt{(500)^2 + (500)^2} = 500\sqrt{2}\,\Omega$

Power factor $\cos\phi = \frac{R}{Z} = \frac{500}{500\sqrt{2}} = \frac{1}{\sqrt{2}}$,

Power dissipated = $V_{rms}\, I_{rms} \cos\phi = \frac{V_{rms}^2}{Z}\cos\phi = \frac{(10)^2}{500\sqrt{2}} \times \frac{1}{\sqrt{2}} = \frac{1}{10}$ W

Ex. If V = 100 sin 100 t volt and I = 100 sin (100 t + $\frac{\pi}{3}$) mA for an A.C. circuit then find out

(a) phase difference between V and I (b) total impedance, reactance, resistance

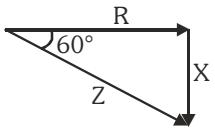
(c) power factor and power dissipated (d) components contains by circuits

Sol. (a) Phase difference $\phi = -\frac{\pi}{3}$ (I leads V)

(b) Total impedance $Z = \frac{V_0}{I_0} = \frac{100}{100 \times 10^{-3}} = 1k\Omega$

Now resistance $R = Z\cos 60° = 1000 \times \frac{1}{2} = 500\Omega$

reactance $X = Z\sin 60° = 1000 \times \frac{\sqrt{3}}{2} = \frac{500}{\sqrt{3}}\Omega$



(c) $\phi = -60° \Rightarrow$ Power factor = $\cos\phi = \cos(-60°) = 0.5$ (leading)

Power dissipated $P = V_{rms}\, I_{rms} \cos\phi = \frac{100}{\sqrt{2}} \times \frac{0.1}{\sqrt{2}} \times \frac{1}{2} = 2.5$ W

(d) Circuit must contains R as $\phi \neq \frac{\pi}{2}$ and as ϕ is negative

so C must be their, (L may exist but $X_C > X_L$)

Ex. If power factor of a R-L series circuit is $\frac{1}{2}$ when applied voltage is V = 100 sin 100πt volt and resistance of circuit is 200Ω then calculate the inductance of the circuit.

Sol. $\cos\phi = \frac{R}{Z} \Rightarrow \frac{1}{2} = \frac{R}{Z} \Rightarrow Z = 2R \Rightarrow \sqrt{R^2 + X_L^2} = 2R \Rightarrow X_L = \sqrt{3}R$

$\omega L = \sqrt{3}R \Rightarrow L = \frac{\sqrt{3}R}{\omega} = \frac{\sqrt{3} \times 200}{100\pi} = \frac{2\sqrt{3}}{\pi}$ H

Ex. A circuit consisting of an inductance and a resistacne joined to a 200 volt supply (A.C.). It draws a current of 10 ampere. If the power used in the circuit is 1500 watt. Calculate the wattless current.

Sol. Apparent power = 200 × 10 = 2000 W

$\therefore$ Power factor $\cos\phi = \frac{\text{True power}}{\text{Apparent power}} = \frac{1500}{2000} = \frac{3}{4}$

Wattless current = $I_{rms} \sin \phi = 10 \sqrt{1-\left(\frac{3}{4}\right)^2} = \frac{10\sqrt{7}}{4} A$

Ex. A coil has a power factor of 0.866 at 60 Hz. What will be power factor at 180 Hz.

Sol. Given that $\cos \phi = 0.866$, $\omega = 2\pi f = 2\pi \times 60 = 120\pi$ rad/s,
$\omega' = 2\pi f' = 2\pi \times 180 = 360\pi$ rad/s
Now, $\cos \phi = R/Z \Rightarrow R = Z \cos \phi = 0.866\ Z$

But $Z = \sqrt{R^2 + (\omega L)^2} \Rightarrow \omega L = \sqrt{Z^2 - R^2} = \sqrt{Z^2 - (0.866\ Z)^2} = 0.5\ Z$

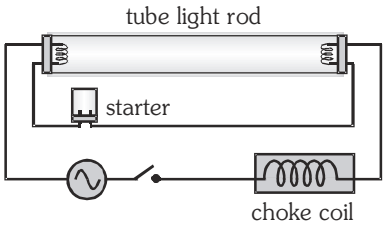
$\therefore L = \frac{0.5Z}{\omega} = \frac{0.5Z}{120\pi}$

When the frequency is changed to $\omega' = 2\pi \times 180 = 3 \times 120\pi = 300$ rad/s, then inductive reactance $\omega' L = 3\ \omega L = 3 \times 0.5\ Z = 1.5\ Z$

$\therefore$ New impedence $Z' = \sqrt{[R'+(\omega' L)^2]} = \sqrt{(0.866\ Z)^2 + (1.5\ Z)^2} = Z\sqrt{[(0.866)^2 + (1.5)^2]}$
$= 1.732Z$

$\therefore$ New power factor $= \frac{R}{Z'} = \frac{0.866\ Z}{1.732\ Z} = 0.5$

CHOKE COIL



In a direct current circuit, current is reduced with the help of a resistance. Hence there is a loss of electrical energy $I^2 R$ per sec in the form of heat in the resistance. But in an AC circuit the current can be reduced by choke coil which involves very small amount of loss of energy. Choke coil is a copper coil wound over a soft iron laminated core. This coil is put in series with the circuit in which current is to be reduced. It also known as ballast.

Circuit with a choke coil is a series L-R circuit. If resistance of choke coil = r (very small)

The current in the circuit $I = \frac{E}{Z}$ with $Z = \sqrt{(R+r)^2 + (\omega L)^2}$ So due to large inductance L of the coil, the current in the circuit is decreased appreciably. However, due to small resistance of the coil r,

The power loss in the choke $P_{av} = V_{rms}\ I_{rms} \cos \phi \to 0$ $\because \cos\phi = \frac{r}{Z} = \frac{r}{\sqrt{r^2 + \omega^2 L^2}} \approx \frac{r}{\omega L} \to 0$

Ex. A choke coil and a resistance are connected in series in an a.c circuit and a potential of 130 volt is applied to the circuit. If the potential across the resistance is 50 V. What would be the potential difference across the choke coil.

Sol. $V = \sqrt{V_R^2 + V_L^2} \Rightarrow V_L = \sqrt{V^2 - V_R^2} = \sqrt{(130)^2 - (50)^2} = 120$ V

Ex. An electric lamp which runs at 80V DC consumes 10 A current. The lamp is connected to 100 V – 50 Hz ac source compute the inductance of the choke required.

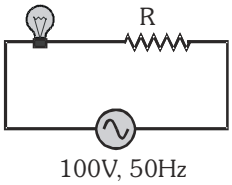
Sol. Resistance of lamp $R = \frac{V}{I} = \frac{80}{10} = 8\Omega$

Let Z be the impedance which would maintain a current of 10 A through the Lamp when it is run on 100 Volt a.c. then. $Z = \frac{V}{I} = \frac{100}{10} = 10\ \Omega$ but $Z = \sqrt{R^2 + (\omega L)^2}$

$\Rightarrow \quad (\omega L)^2 = Z^2 - R^2 = (10)^2 - (8)^2 = 36$

$\Rightarrow \quad \omega L = 6 \quad \Rightarrow \; L = \frac{6}{\omega} = \frac{6}{2\pi \times 50} = 0.02H$

Ex. Calculate the resistance or inductance required to operate a lamp (60V, 10W) from a source of (100 V, 50 Hz)

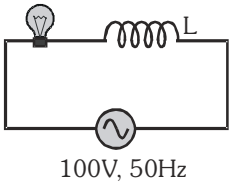


Sol. (a) Maximum voltage across lamp = 60V

$\because \quad V_{Lamp} + V_R = 100 \qquad \therefore \quad V_R = 40V$

Now current throught Lamp is $= \frac{\text{Wattage}}{\text{voltage}} = \frac{10}{60} = \frac{1}{6}A$

But $V_R = IR \quad \Rightarrow \quad 40 = \frac{1}{6}(R) \quad \Rightarrow \; R = 240\ \Omega$



(b) Now in this case $(V_{Lamp})^2 + (V_L)^2 = (V)^2$

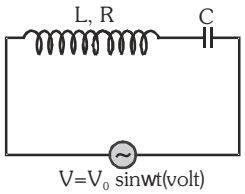
$(60)^2 + (V_L)^2 = (100)^2 \Rightarrow V_L = 80\ V$

Also $V_L = IX_L = \frac{1}{6}X_L$ so $\quad X_L = 80 \times 6 = 480\ \Omega = L(2\pi f) \Rightarrow \; L = 1.5\ H$

A capacitor of suitable capacitance replace a choke coil in an AC circuit, the average power consumed in a capacitor is also zero. Hence, like a choke coil, a capacitor can reduce current in AC circuit without power dissipation.

Cost of capacitor is much more than the cost of inductance of same reactance that's why choke coil is used.

Ex. A choke coil of resistance R and inductance L is connected in series with a capacitor C and complete combination is connected to a.c. voltage, Circuit resonates when angular frequency of supply is $\omega = \omega_0$.



(a) Find out relation betwen ω_0, L and C

(b) What is phase difference between V and I at resonance, is it changes when resistance of choke coil is zero.

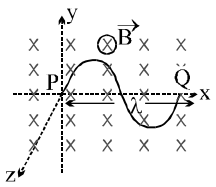
Sol. (a) At resonance condition $X_L = X_C \Rightarrow \omega_0 L = \frac{1}{\omega_0 C} \quad \Rightarrow \quad \omega_0 = \frac{1}{\sqrt{LC}}$

(b) $\because \quad \cos\phi = \frac{R}{Z} = \frac{R}{R} = 1 \quad \therefore \; \phi = 0° \quad$ No, It is always zero.

EXERCISE (S)

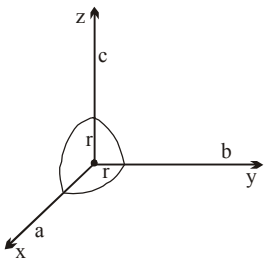
Faraday's law & Motional emf.

1. A wire forming one cycle of sine curve is moved in x-y plane with velocity $\vec{V} = V_x\hat{i} + V_y\hat{j}$. There exist a magnetic field $\vec{B} = -B_0\hat{k}$. Find the motional emf develop across the ends PQ of wire.



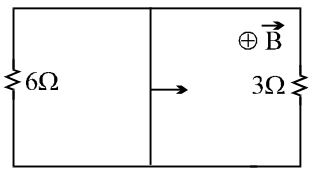
EM0001

2. A wire is bent into 3 circular segments of radius r = 10 cm as shown in figure. Each segment is a quadrant of a circle, ab lying in the xy plane, bc lying in the yz plane & ca lying in the zx plane.
(i) if a magnetic field B points in the positive x direction, what is the magnitude of the emf developed in the wire, when B increases at the rate of 3 mT/s ?
(ii) what is the direction of the current in the segment bc.



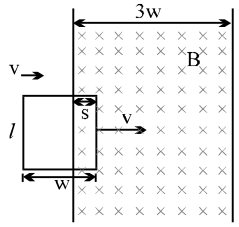
EM0002

3. A rectangular loop with a sliding connector of length l = 1.0 m is situated in a uniform magnetic field B = 2T perpendicular to the plane of loop. Resistance of connector is r = 2Ω. Two resistances of 6Ω and 3Ω are connected as shown in figure. Find the external force required to keep the connector moving with a constant velocity v = 2m/s.



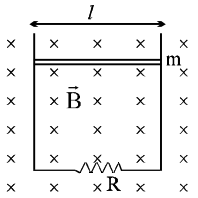
EM0003

4. A rectangular loop of dimensions l & w and resistance R moves with constant velocity V to the right as shown in the figure. It continues to move with same speed through a region containing a uniform magnetic field B directed into the plane of the paper & extending a distance 3 w. Sketch the flux, induced emf & external force acting on the loop as a function of the distance.



EM0004

5. A horizontal wire is free to slide on the vertical rails of a conducting frame as shown in figure. The wire has a mass m and length *l* and the resistance of the circuit is R. If a uniform magnetic field B is directed perpendicular to the frame, then find the terminal speed of the wire as it falls under the force of gravity.



EM0005

6. It is desired to measure the magnitude of field between the poles of a powerful loud speaker magnet. A small flat search coil of area 2 cm^2 with 25 closely wound turns, is positioned normal to the field direction, and then quickly snatched out of the field region. Equivalently, one can give it a quick 90° turn to bring its plane parallel to the field direction. The total charge flown In the coil (measured by a ballistic galvanometer connected to coil) is 7.5 mC. The combined resistance of the coil and the galvanometer is 0.50 Ω. Estimate the field strength of magnet. **(NCERT)**

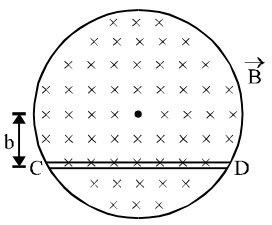
EM0006

Induced electric field

7. There exists a uniform cylindrically symmetric magnetic field directed along the axis of a cylinder but varying with time as B = kt. If an electron is released from rest in this field at a distance of 'r' from the axis of cylinder, its acceleration, just after it is released would be (e and m are the electronic charge and mass respectively)

EM0007

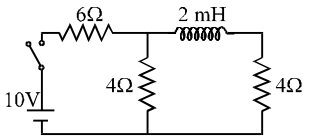
8. A uniform magnetic field $\vec{B}$ fills a cylindrical volumes of radius R. A metal rod CD of length l is placed inside the cylinder along a chord of the circular cross-section as shown in the figure. If the magnitude of magnetic field increases in the direction of field at a constant rate dB/dt, find the magnitude and direction of the EMF induced in the rod.



EM0009

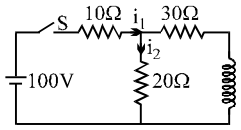
Inductance

9. In the given circuit, find the ratio of i_1 to i_2 where i_1 is the initial (at t = 0) current and i_2 is steady state (at t = ∞) current through the battery.



EM0010

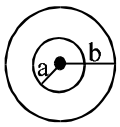
10. Find the values of i_1 and i_2



(i) immediately after the switch S is closed.
(ii) long time later, with S closed.
(iii) immediately after S is open.
(iv) long time after S is opened.

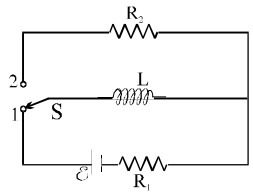
EM0039

11. Two concentric and coplanar circular coils have radii a and b(>>a)as shown in figure. Resistance of the inner coil is R. Current in the outer coil is increased from 0 to i, then find the total charge circulating the inner coil.



EM0011

12. In the circuit shown, initially the switch is in position 1 for a long time. Then the switch is shifted to position 2 for a long time. Find the total heat produced in R_2.

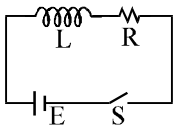


EM0012

13. An emf of 15 volt is applied in a circuit containing 5 H inductance and 10 Ω resistance. Find the ratio of the currents at time t = ∞ and t = 1 second.

EM0013

14. In the circuit shown in figure switch S is closed at time t = 0. Find the charge which passes through the battery in one time constant.



EM0014

15. An inductor of inductance 2.0mH,is connected across a charged capacitor of capacitance 5.0μF and the resulting LC circuit is set oscillating at its natural frequency. Let Q denote the instantaneous charge on the capacitor, and I the current in the circuit .It is found that the maximum value of Q is 200μC.

(a) when Q = 100μC,what is the value of $|dI/dt|$?

(b) when Q = 200 μC ,what is the value of I ?

(c) Find the maximum value of I.

(d) when I is equal to one half its maximum value, what is the value of $|Q|$

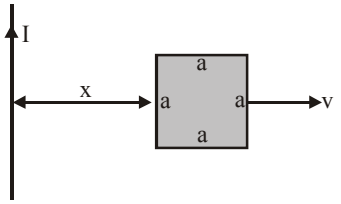
EM0017

16. A pair of adjacent coils has a mutual inductance of 1.5 H. If the current in one coil changes from 0 to 20 A in 0.5 s, what is the change of flux linkage with the other coil ? **(NCERT)**

EM0018

17. (a) Obtain an expression for the mutual inductance between a long straight wire and a square loop of side a as shown In figure.

(b) Now assume that the straight wire carries a current of 50 A and the loop is moved to the right with a constant velocity, v = 10 m/s.Calculate the induced emf in the loop at the instant when x = 0.2m. Take a = 0.1 m and assume that the loop has a large resistance. **(NCERT)**



EM0019

Alternating current

18. A current of 4 A flows in a coil when connected to a 12 V dc source. If the same coil is connected to a 12V, 50 rad/s ac source a current of 2.4 A flows in the circuit. Determine the inductance of the coil. Also find the power developed in the circuit if a 2500 μF capacitor is connected in series with the coil.

EM0021

19. An LCR series circuit with 100Ω resistance is connected to an ac source of 200 V and angular frequency 300 rad/s. When only the capacitance is removed, the current lags behind the voltage by 60°. When only the inductance is removed, the current leads the voltage by 60°. Calculate the current and the power dissipated in the LCR circuit.

EM0022

20. A series LCR circuit containing a resistance of 120Ω has angular resonance frequency 4×10^5 rad s^{-1}. At resonance the voltages across resistance and inductance are 60 V and 40 V respectively. Find the values of L and C. At what frequency the current in the circuit lags the voltage by 45°?

EM0023

21. Find the value of an inductance which should be connected in series with a capacitor of 5μF, a resistance of 10Ω and an ac source of 50 Hz so that the power factor of the circuit is unity.

EM0024

22. In an L-R series A.C circuit the potential difference across an inductance and resistance joined in series are respectively 12 V and 16V. Find the total potential difference across the circuit.

EM0025

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

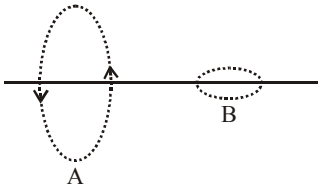
Faraday's law & motional emf.

1. A square of side *2* meters lies in the *x-y* plane in a region, where the magnetic field is given by $\vec{B} = B_0\left(2\hat{i} + 3\hat{j} + 4\hat{k}\right)T$, where B_o is constant. The magnitude of flux passing through the square is:-

(A) 8 B_o Wb. (B) 12 B_o Wb. (C) 16 B_o Wb. (D) $\sqrt{4\times 29}B_0 Wb$

EM0049

2. In the given figure the centre of a small conducting circular loop B lies on the axis of bigger circular loop A and their axis are mutually perpendicular. An anticlockwise (when viewed from the side of B) current in the loop A start increasing then :-



(A) current induced in the loop B is in clockwise direction (when viewed from above the B)
(B) current induced in the loop B is in anti-clockwise direction (when viewed from above the B)
(C) current must induced in the loop B but its direction can not be predicted
(D) no current is induced in the loop B

EM0051

3. A vertical bar magnet is dropped from position on the axis of a fixed metallic coil as shown in fig - I. In fig.II the magnet is fixed and horizontal coil is dropped. The acceleration of the magnet and coil are a_1 and a_2 respectively then :-

Fig. (I)

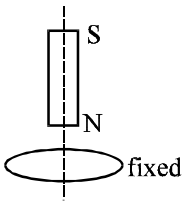
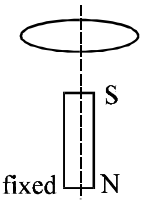


Fig. (II)



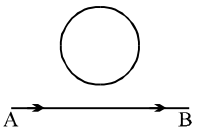
(A) $a_1 > g$, $a_2 > g$ (B) $a_1 > g$, $a_2 < g$ (C) $a_1 < g$, $a_2 < g$ (D) $a_1 < g$, $a_2 > g$

EM0052

4. Two identical coaxial circular loops carry a current *i* each circulating in the same direction. If the loops approach each other
(A) the current in each will decrease
(B) the current in each will increase
(C) the current in each will remain the same
(D) the current in one will increase and in other will decrease

EM0053

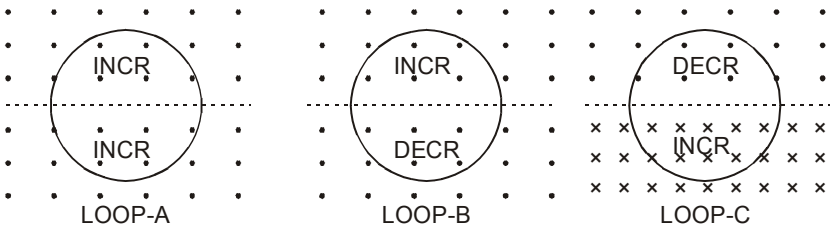
5. In the arrangement shown in given figure current from A to B is increasing in magnitude. Induced current in the loop will



(A) have clockwise direction
(B) have anticlockwise direction
(C) be zero
(D) oscillate between clockwise and anticlockwise

EM0054

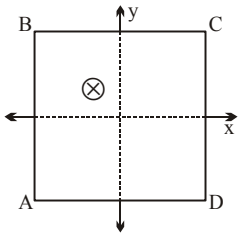
6. Three identical conducting circular loops are placed in uniform magnetic fields. Inside each loop, there are two magnetic field regions, separated by dashed line that coincides with a diameter, as shown. Magnetic fields may either be increasing (marked as INCR) or decreasing (marked as DECR) in magnitude at the same rates. If I_A, I_B and I_C are the magnitudes of the induced currents in the loops A, B and C respectively then choose the **CORRECT** relation :-



(A) $I_A > I_B = I_C$ (B) $I_A = I_C > I_B$ (C) $I_A = I_B = I_C$ (D) $I_C > I_A > I_B$

EM0055

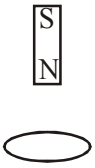
7. A square coil ABCD is placed in x-y plane with its centre at origin. A long straight wire, passing through origin, carries a current in negative z-direction. Current in this wire increases with time. The induced current in the coil is :



(A) clockwise (B) anticlockwise (C) zero (D) alternating

EM0056

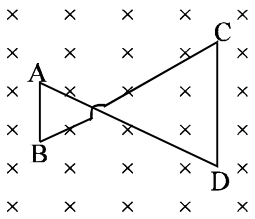
8. A short circuited coil is kept on the ground and a magnet is dropped on it as shown. The coil shows (when viewed from top)



(A) anticlockwise current that increases in magnitude
(B) anticlockwise current that remains constant
(C) clockwise current that remains constant
(D) clockwise current that increases in magnitude

EM0057

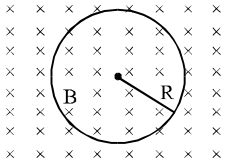
9. A conducting wire frame is placed in a magnetic field which is directed into the paper. The magnetic field is increasing at a constant rate. The directions of induced currents in wires AB and CD are



(A) B to A and D to C
(B) A to B and C to D
(C) A to B and D to C
(D) B to A and C to D

EM0059

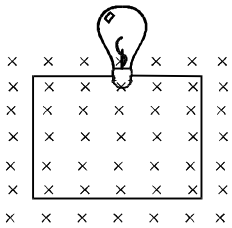
10. A conducting loop of radius R is present in a uniform magnetic field B perpendicular to the plane of the ring. If radius R varies as a function of time 't', as $R = R_0 + t$. The e.m.f induced in the loop is



(A) $2\pi(R_0 + t)B$ clockwise
(B) $\pi(R_0 + t)B$ clockwise
(C) $2\pi(R_0 + t)B$ anticlockwise
(D) zero

EM0060

11. A square wire loop of 10.0 cm side lies at right angles to a uniform magnetic field of 20T. A 10 V light bulb is in a series with the loop as shown in the fig. The magnetic field is decreasing steadily to zero over a time interval Δt. The bulb will shine with full brightness if Δt is equal to :-



(A) 20 ms
(B) 0.02 ms
(C) 2 ms
(D) 0.2 ms

EM0144

12. A thin wire of length 2m is perpendicular to the xy plane. It is moved with velocity $\vec{v} = (2\hat{i} + 3\hat{j} + \hat{k})\,m/s$ through a region of magnetic induction $\vec{B} = (\hat{i} + 2\hat{j})\ Wb/m^2$. Then potential difference induced between the ends of the wire :

(A) 2 volts
(B) 4 volts
(C) 0 volts
(D) none of these

EM0039

13. A square loop of side a and resistance R is moved in the region of uniform magnetic field B (loop remaining completely inside field) ,with a velocity v through a distance x . The work done is :

(A) $\frac{B\ell^2vx}{R}$ (B) $\frac{2B^2\ell^2vx}{R}$ (C) $\frac{4B^2\ell^2vx}{R}$ (D) 0

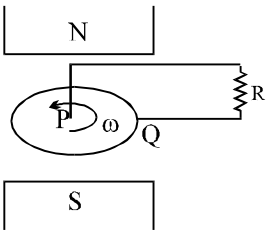
EM0040

14. The magnetic field in a region is given by $\vec{B} = B_0\left(1+\frac{x}{a}\right)\hat{k}$. A square loop of edge - length d is placed with its edge along x & y axis. The loop is moved with constant velocity $\vec{V} = V_0\hat{i}$. The emf induced in the loop is

(A) $\frac{V_0B_0d^2}{a}$ (B) $\frac{V_0B_0d^2}{2a}$ (C) $\frac{V_0B_0a^2}{d}$ (D) None

EM0155

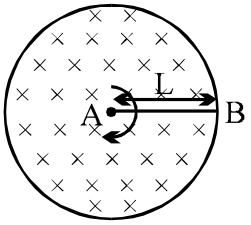
15. A metal disc rotates freely, between the poles of a magnet in the direction indicated. Brushes P and Q make contact with the edge of the disc and the metal axle. What current, if any, flows through R?



(A) a current from P to Q
(B) a current from Q to P
(C) no current, because the emf in the disc is opposed by the back emf
(D) no current, because the emf induced in one side of the disc is opposed by the emf induced in the other side.
(E) no current, because no radial emf is induced in the disc

EM0065

16. A copper rod AB of length L, pivoted at one end A, rotates at constant angular velocity ω, at right angles to a uniform magnetic field of induction B. The e.m.f developed between the mid point C of the rod and end B is



(A) $\frac{B\omega L^2}{4}$ (B) $\frac{B\omega L^2}{2}$ (C) $\frac{3B\omega L^2}{4}$ (D) $\frac{3B\omega L^2}{8}$

EM0066

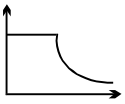
Induced electric field

17. A ring of resistance 10Ω, radius 10cm and 100 turns is rotated at a rate 100 revolutions per second about its diameter is perpendicular to a uniform magnetic field of induction 10mT. The amplitude of the current in the loop will be nearly (Take : $\pi^2 = 10$)

(A) 200A (B) 2A (C) 0.002A (D) none of these

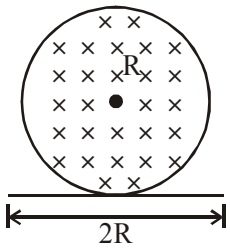
EM0068

18. A uniform but time variant magnetic field exists in a cylindrical region directed along the axis of cylinder of radius R. The graph of induced electric field at a given time v/s. r is (r = distance from axis)

(A)  (B)  (C)  (D) 

EM0064

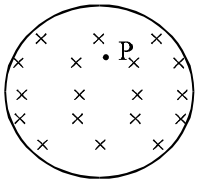
19. A uniform but time varying magnetic field is present in a circular region of radius R. The magnetic field is perpendicular and into the plane of the loop and the magnitude of field is increasing at a constant rate α. There is a straight conducting rod of length 2R placed as shown in figure. The magnitude of induced emf across the rod is



(A) $\pi R^2\alpha$ (B) $\frac{\pi R^2\alpha}{2}$ (C) $\frac{R^2\alpha}{\sqrt{2}}$ (D) $\frac{\pi R^2\alpha}{4}$

EM0066

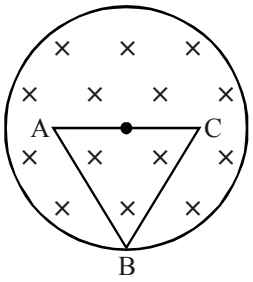
20. Figure shows a uniform magnetic field B confined to a cylindrical volume and is increasing at a constant rate. The instantaneous acceleration experienced by an electron placed at P is



(A) zero (B) towards right (C) towards left (D) upwards

EM0070

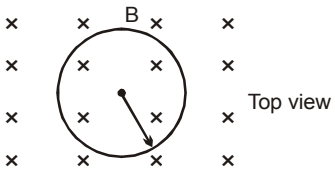
21. An equilateral triangle ABC of side a is placed in the magnetic field with side AC and its centre coinciding with the centre of the magnetic field. The magnetic field varies with time as B = ct. The emf induced across side AB is :-



(A) $\frac{\sqrt{3}}{4}a^2c$ (B) Zero (C) $\frac{\sqrt{3}}{8}a^2c$ (D) $\frac{(\sqrt{2}-1)}{2}a^2c$

EM0154

22. A non conducting ring (of mass m, radius r, having charge Q) is placed on a rough horizontal surface (in a region with transverse magnetic field). The field is increasing with time at the rate R and coefficient of friction between the surface and the ring is μ. For ring to remain in equilibrium μ should be greater than :-



(A) $\frac{QrR}{mg}$ (B) $\frac{QrR}{2mg}$ (C) $\frac{QrR}{3mg}$ (D) $\frac{2QrR}{mg}$

EM0143

23. **Statement–1 :** For a charged particle moving from point P to point Q the net work done by an induced electric field on the particle is independent of the path connecting point P to point Q.

Statement–2 : The net work done by a conservative force on an object moving along closed loop is zero.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1

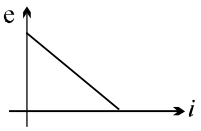
(C) Statement-1 is true, statement-2 is false.

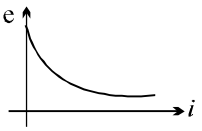
(D) Statement-1 is false, statement-2 is true.

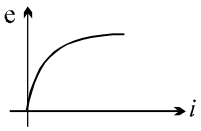
EM0071

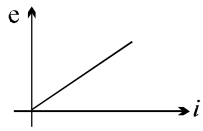
Inductance

24. In an L-R circuit connected to a battery of constant e.m.f. E switch S is closed at time t = 0. If e denotes the magnitude of induced e.m.f. across inductor and *i* the current in the circuit at any time t. Then which of the following graphs shows the variation of e with *i* ?

(A) 

(B) 

(C) 

(D) 

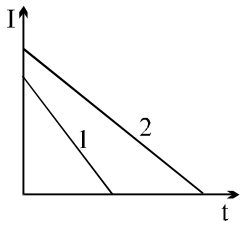
EM0072

25. A current of 2A is increasing at a rate of 4 A/s through a coil of inductance 2H. The energy stored in the inductor per unit time is :-

(A) 2 J/s (B) 1 J/s (C) 16 J/s (D) 4 J/s

EM0073

26. Two identical inductance carry currents that vary with time according to linear laws (as shown in figure). In which of two inductance is the self induction emf greater?

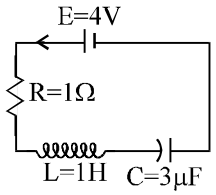


(A) 1 (B) 2

(C) same (D) data are insufficient to decide

EM0074

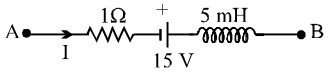
27. The current in the given circuit is increasing with a rate a = 4 amp/s. The charge on the capacitor at an instant when the current in the circuit is 2 amp will be :



(A) 4μC (B) 5μC (C) 6μC (D) none of these

EM0075

28. The network shown in the figure is part of a complete circuit. If at a certain instant, the current I is 5A and it is decreasing at a rate of 10^3 As^{-1} then $V_B - V_A$ equals



(A) 20 V (B) 15 V (C) 10 V (D) 5 V

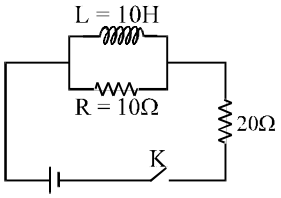
EM0077

29. In **Problem 28**, if I is reversed in direction, then $V_B - V_A$ equals

(A) 5 V (B) 10 V (C) 15 V (D) 20 V

EM0078

30. Two resistors of 10Ω and 20Ω and an ideal inductor of 10H are connected to a 2V battery as shown. The key K is shorted at time t = 0. Find the initial (t = 0) and final ($t \to \infty$) currents through battery.



(A) $\frac{1}{15}A, \frac{1}{10}A$ (B) $\frac{1}{10}A, \frac{1}{15}A$ (C) $\frac{2}{15}A, \frac{1}{10}A$ (D) $\frac{1}{15}A, \frac{2}{25}A$

EM0079

31. An inductor coil stores U energy when i current is passed through it and dissipates energy at the rate of P. The time constant of the circuit, when this coil is connected across a battery of zero internal resistance is :-

(A) $\frac{4U}{P}$ (B) $\frac{U}{P}$ (C) $\frac{2U}{P}$ (D) $\frac{2P}{U}$

EM0080

32. A small coil of radius r is placed at the centre of a large coil of radius R, where R >> r. The coils are coplanar. The coefficient of mutual inductance between the coils is :-

(A) $\frac{\mu_0 \pi r}{2R}$ (B) $\frac{\mu_0 \pi r^2}{2R}$ (C) $\frac{\mu_0 \pi r^2}{2R^2}$ (D) $\frac{\mu_0 \pi r}{2R^2}$

EM0081

33. A long straight wire is placed along the axis of a circular ring of radius R. The mutual inductance of this system is :-

(A) $\frac{\mu_0 R}{2}$ (B) $\frac{\mu_0 \pi R}{2}$ (C) $\frac{\mu_0}{2}$ (D) 0

EM0082

34. Two coils are placed close to each other. The mutual inductance of the pair of coils depends upon-

(A) the rates at which currents are changing in the two coils **[AIEEE - 2003]**

(B) relative position and orientation of the two coils

(C) the materials of the wires of the coils

(D) the currents in the two coils

EM0083

35. L, C and R represent physical quantities inductance, capacitance and resistance. The combination which has the dimensions of frequency is

(A) $\frac{1}{RC}$ and $\frac{R}{L}$ (B) $\frac{1}{\sqrt{RC}}$ and $\sqrt{\frac{R}{L}}$ (C) $\sqrt{LC}$ (D) $\frac{C}{L}$

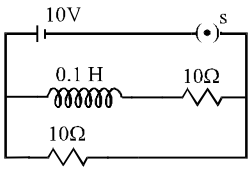
EM0085

36. A coil of inductance 5H is joined to a cell of emf 6V through a resistance 10Ω at time t = 0. The emf across the coil at time t = ln $\sqrt{2}$ s is:

(A) 3V (B) 1.5 V (C) 0.75 V (D) 4.5 V

EM0087

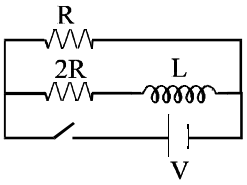
37. In the adjoining circuit, initially the switch S is open. The switch 'S' is closed at t = 0. The difference between the maximum and minimum current that can flow in the circuit is



(A) 2 Amp (B) 3 Amp
(C) 1 Amp (D) nothing can be concluded

EM0089

38. Find the ratio of time constant in build up and decay in the circuit as shown in figure :-



(A) 1 : 1 (B) 3 : 2 (C) 2 : 3 (D) 1 : 3

EM0090

39. In a L–R decay circuit, the initial current at t = 0 is I. The total charge that has flown through the resistor till the energy in the inductor has reduced to one–fourth its initial value, is

(A) LI/R (B) $LI/2R$ (C) $LI\sqrt{2}/R$ (D) None

EM0092

40. The inductor in a L–C oscillation has a maximum potential difference of 16 V and maximum energy of 640 μJ. Find the value of capacitor in μF in L–C circuit.

(A) 5 (B) 4 (C) 3 (D) 2

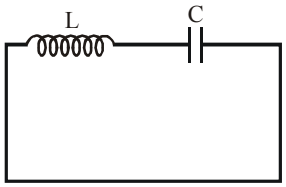
EM0093

41. A condenser of capacity 6 μF is fully charged using a 6-volt battery. The battery is removed and a resistanceless 0.2 mH inductor is connected across the condenser. The current which is flowing through the inductor when one-third of the total energy is in the magnetic field of the inductor is :-

(A) 0.1 A (B) 0.2 A (C) 0.4 A (D) 0.6 A

EM0094

42. In an LC circuit the capacitor has maximum charge q_0. The value of $\left(\frac{dI}{dt}\right)_{max}$ is :-



(A) $\frac{q_0}{LC}$ (B) $\frac{q_0}{\sqrt{LC}}$ (C) $\frac{q_0}{2LC}$ (D) $\frac{2q_0}{LC}$

EM0095

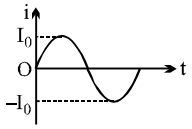
Alternating current

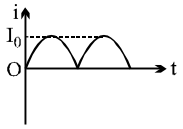
43. When 100 V DC is applied across a solenoid a current of 1 A flows in it. When 100 V AC is applied across the same coil, the current drops to 0.5 A. If the frequency of the AC source is 50 Hz, the impedance and inductance of the solenoid are:

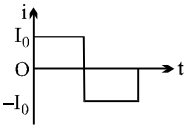
(A) 100Ω, 0.93 H (B) 200Ω, 1.0 H (C) 10Ω, 0.86H (D) 200Ω, 0.55 H

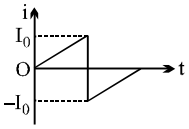
EM0096

44. If I_1, I_2, I_3 and I_4 are the respective r.m.s. values of the time varying currents as shown in the four cases I, II, III and IV. Then identify the correct relations.









(A) $I_1 = I_2 = I_3 = I_4$ (B) $I_3 > I_1 = I_2 > I_4$ (C) $I_3 > I_4 > I_2 = I_1$ (D) $I_3 > I_2 > I_1 > I_4$

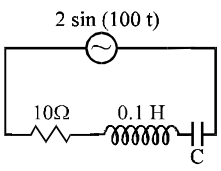
EM0097

45. An current is given by $I = I_0 + I_1 \sin \omega t$ then its rms value will be

(A) $\sqrt{I_0^2 + 0.5 I_1^2}$ (B) $\sqrt{I_0^2 + 0.5 I_0^2}$ (C) 0 (D) $I_0/\sqrt{2}$

EM0163

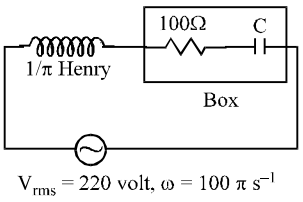
46. The power factor of the circuit is $1/\sqrt{2}$. The capacitance of the circuit is equal to



(A) 400 μF (B) 300 μF (C) 500 μF (D) 200 μF

EM0098

47. In the circuit, as shown in the figure, if the value of R.M.S current is 2.2 ampere, the power factor of the box is



(A) $\frac{1}{\sqrt{2}}$ (B) 1 (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{1}{2}$

EM0099

48. Power factor of an L-R series circuit is 0.6 and that of a C–R series circuit is 0.5. If the element (L, C, and R) of the two circuits are joined in series the power factor of this circuit is found to be 1. The ratio of the resistance in the L-R circuit to the resistance in the C–R circuit is

(A) 6/5 (B) 5/6 (C) $\frac{4}{3\sqrt{3}}$ (D) $\frac{3\sqrt{3}}{4}$

EM0164

49. In ac circuit when ac ammeter is connected it reads i current. If a student uses dc ammeter in place of ac ammeter the reading in the dc ammeter will be:

(A) $\frac{i}{\sqrt{2}}$ (B) $\sqrt{2}$ i (C) 0.637 i (D) zero

EM0101

50. The effective value of current i = 2 sin 100 π t + 2 sin(100 π t + 30°) is :

(A) $\sqrt{2}$ Amp (B) $2\sqrt{2+\sqrt{3}}$ Amp (C) 4 Amp (D) None

EM0103

51. In series LR circuit X_L = 3R. Now a capacitor with X_C = R is added in series. Ratio of new to old power factor is

(A) 1 (B) 2 (C) $\frac{1}{\sqrt{2}}$ (D) $\sqrt{2}$

EM0104

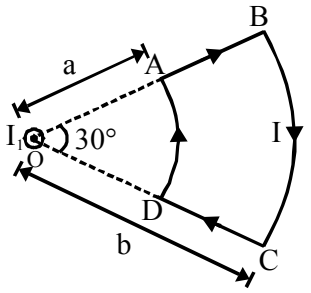
52. In a series R-L-C circuit, the frequency of the source is half of the resonance frequency. The nature of the circuit will be

(A) capacitive (B) inductive (C) purely resistive (D) data insufficient

EM0106

EXERCISE-JM

Direction :- Question number 1 and 2 are based on the following paragraph.
A current loop ABCD is held fixed on the plane of the paper as shown in the figure. The arcs BC (radius = b) and DA (radius = a) of the loop are joined by two straight wires AB and CD. A steady current I is flowing in the loop. Angle made by AB and CD at the origin O is 30°. Another straight thin wire with steady current I_1 flowing out of the plane of the paper is kept at the origin.



1. The magnitude of the magnetic field (B) due to the loop ABCD at the origin (O) is :- **[AIEEE - 2009]**

(1) $\frac{\mu_0 I}{4\pi}\left[\frac{b-a}{ab}\right]$

(2) $\frac{\mu_0 I}{4\pi}\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

(3) Zero

(4) $\frac{\mu_0 I(b-a)}{24ab}$

MG0204

2. Due to the presence of the current I_1 at the origin:- **[AIEEE - 2009]**

(1) The magnitude of the net force on the loop is given by $\frac{I_1 I}{4\pi}\mu_0\left[2(b-a)+\frac{\pi}{3}(a+b)\right]$

(2) The magnitude of the net force on the loop is given by $\frac{\mu_0 I\, I_1}{24ab}(b-a)$

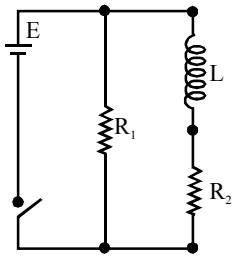
(3) The forces on AB and DC are zero

(4) The forces on AD and BC are zero

MG0205

3. An inductor of inductance L = 400 mH and resistors of resistances $R_1 = 2\Omega$ and $R_2 = 2\Omega$ are connected to a battery of emf 12V as shown in the figure. The internal resistance of the battery is negligible. The switch S is closed at t = 0. The potential drop across L as a function of time is:-

[AIEEE - 2009]

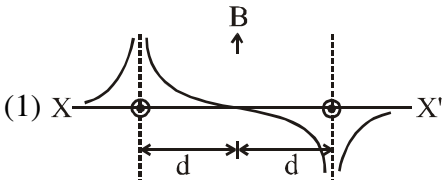


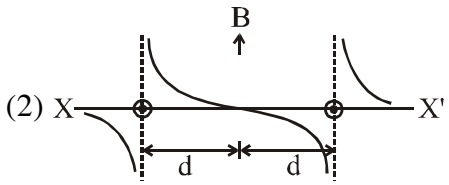
(1) $6(1 - e^{-t/0.2})V$ (2) $12e^{-5t}$ V (3) $6e^{-5t}$ V (4) $\frac{12}{t}e^{-3t}V$

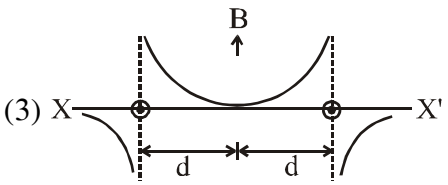
EM0218

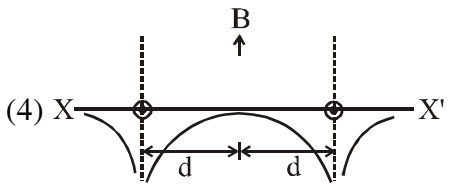
4. Two long parallel wires are at a distance 2d apart. They carry steady equal currents flowing out of the plane of the paper as shown. The variation of the magnetic field B along the line XX' is given by:-

[AIEEE - 2010]

(1)


(2)


(3)


(4)


MG0166

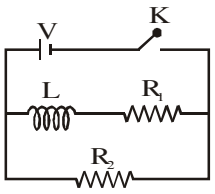
5. An electric charge + q moves with velocity $\vec{V} = 3\hat{i} + 4\hat{j} + \hat{k}$, in an electromagnetic field given by :- $\vec{E} = 3\hat{i} + \hat{j} + 2\hat{k}$ $\vec{B} = \hat{i} + \hat{j} - 3\hat{k}$. The y-component of the force experienced by + q is :-

[AIEEE - 2010]

(1) 2 q (2) 11 q (3) 5 q (4) 3q

MG0168

6. In the circuit show below, the key K is closed at t = 0. The current through the battery is : **[AIEEE - 2010]**



(1) $\frac{V(R_1+R_2)}{R_1R_2}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

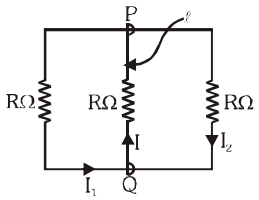
(2) $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = 0 and $\frac{V}{R_2}$ at t = ∞

(3) $\frac{V}{R_2}$ at t = 0 and $\frac{V(R_1+R_2)}{R_1R_2}$ at t = ∞

(4) $\frac{V}{R_2}$ at t = 0 and $\frac{VR_1R_2}{\sqrt{R_1^2+R_2^2}}$ at t = ∞

EM0175

7. A rectangular loop has a sliding connector PQ of length ℓ and resistance RΩ and it is moving with a speed v as shown. The set-up is placed in a uniform magnetic field going into the plane of the paper. The three currents I_1, I_2 and I are :- **[AIEEE - 2010]**



(1) $I_1 = I_2 = \frac{B\ell v}{6R}$, $I = \frac{B\ell v}{3R}$

(2) $I_1 = -I_2 = \frac{B\ell v}{R}$, $I = \frac{2B\ell v}{R}$

(3) $I_1 = I_2 = \frac{B\ell v}{3R}$, $I = \frac{2B\ell v}{3R}$

(4) $I_1 = I_2 = I = \frac{B\ell v}{R}$

EM0176

8. In a series LCR circuit R = 200Ω and the voltage and the frequency of the main supply is 220 V and 50 Hz respectively. On taking out the capacitance from the circuit the current lages behind the voltage by 30°. On taking out the inductor from the circuit the current leads the voltage by 30°. The power dissipated in the LCR circuit is : **[AIEEE - 2010]**

(1) 242 W (2) 305 W (3) 210 W (4) 0 W

EM0177

9. A thin circular disk of radius R is uniformly charged with density σ > 0 per unit area. The disk rotates about its axis with a uniform angular speed ω. The magnetic moment of the disk is :- **[AIEEE - 2011]**

(1) $2\pi R^4\sigma\omega$ (2) $\pi R^4\sigma\omega$ (3) $\frac{\pi R^4}{2}\sigma\omega$ (4) $\frac{\pi R^4}{4}\sigma\omega$

MG0169

10. A boat is moving due east in a region where the earth's magnetic field is $5.0\times10^{-5}NA^{-1}m^{-1}$ due north and horizontal. The boat carries a vertical aerial 2m long. If the speed of the boat is 1.50 ms^{-1}, the magnitude of the indueced emf in the wire of aerial is :- **[AIEEE - 2011]**

(1) 0.50 mV (2) 0.15 mV (3) 1 mV (4) 0.75 mV

EM0178

11. A horizontal straight wire 20 m long extending from east to west is falling with a speed of 5.0 m/s, at right angles to the horizontal component of the earth's magnetic field 0.30×10^{-4} Wb/m^2. The instantaneous value of the e.m.f. induced in the wire will be :- **[AIEEE - 2011]**

(1) 6.0 mV (2) 3 mV (3) 4.5 mV (4) 1.5 mV

EM0179

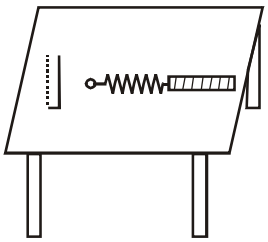
12. A fully charged capacitor C with intial charge q_0 is connected to a coil of self inductance L at t = 0. The time at which the energy is stored equally between the electric and the magnetic fields is :-

[AIEEE - 2011]

(1) $2\pi\sqrt{LC}$ (2) $\sqrt{LC}$ (3) $\pi\sqrt{LC}$ (4) $\frac{\pi}{4}\sqrt{LC}$

EM0180

13. A metallic rod of length 'l' is tied to a string of length 2l and made to rotate with angular speed ω on a horizontal table with one end of the string fixed. If there is a vertical magnetic field 'B' in the region, the e.m.f. induced across the ends of the rod is : **[JEE Main-2013]**



(1) $\frac{2B\omega l^2}{2}$ (2) $\frac{3B\omega l^2}{2}$ (3) $\frac{4B\omega l^2}{2}$ (4) $\frac{5B\omega l^2}{2}$

EM0184

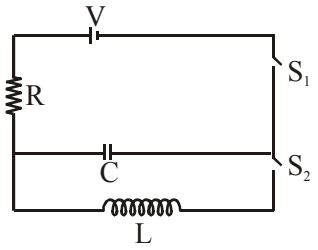
14. A circular loop of radius 0.3 cm lies parallel to a much bigger circular loop of radius 20 cm. The centre of the small loop is on the axis of the bigger loop. The distance between their centres is 15 cm. If a current of 2.0 A flows through the smaller loop, then the flux linked with bigger loop is :-

[JEE Main-2013]

(1) 9.1×10^{-11} weber (2) 6×10^{-11} weber (3) 3.3×10^{-11} weber (4) 6.6×10^{-9} weber

EM0185

15. In an LCR circuit as shown below both switches are open initially. Now switch S_1 is closed, S_2 kept open, (q is charge on the capacitor and $\tau = RC$ is Capacitive time constant). Which of the following statement is correct? **[JEE Main-2013]**



(1) Work done by the battery is half of the energy dissipated in the resistor

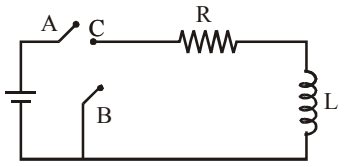
(2) At $t = \tau$, $q = CV/2$

(3) At $t = 2\pi$, $q = CV(1-e^{-2})$

(4) At $t = \frac{\tau}{2}$, $q = CV(1-e^{-1})$

EM0186

16. In the circuit shown here, the point 'C' is kept connected to point 'A' till the current flowing through the circuit becomes constant. Afterward, suddenly, point 'C' is disconnected from point 'A' and connected to point 'B' at time t = 0. Ratio of the voltage across resistance and the inductor at t = L/R will be equal to : **[JEE Main-2014]**

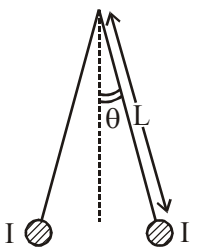


(1) -1 (2) $\frac{1-e}{e}$ (3) $\frac{e}{1-e}$ (4) 1

EM0187

17. Two long current carrying thin wires, both with current I, are held by the insulating threads of length L and are in equilibrium as shown in the figure, with threads making an angle 'θ' with the vertical. If wires have mass λ per unit length then the value of I is :- (g = gravitational acceleration)

[JEE(Mains) - 2015]

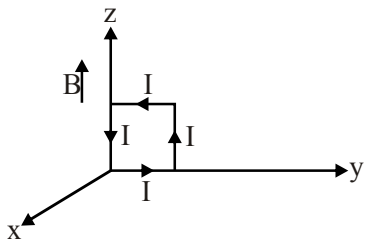


(1) $2\sqrt{\frac{\pi gL}{\mu_0}\tan\theta}$ (2) $\sqrt{\frac{\pi\lambda gL}{\mu_0}\tan\theta}$ (3) $\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$ (4) $2\sin\theta\sqrt{\frac{\pi\lambda gL}{\mu_0\cos\theta}}$

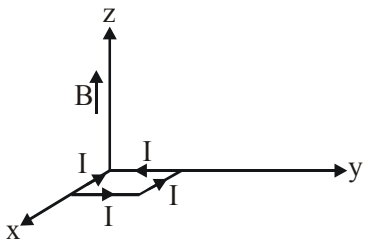
MG0174

18. A rectangular loop of sides 10 cm and 5 cm carrying a current I of 12 A is place in different orientations as shown in the figures below : **[JEE(Mains) - 2015]**

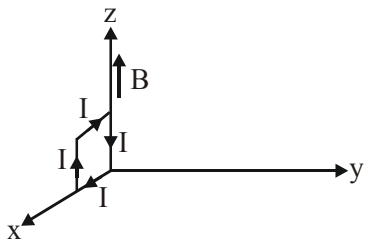
(a)



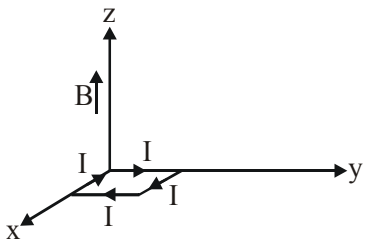
(b)



(c)



(d)



If there is a uniform magnetic field of 0.3 T in the positive z direction, in which orientations the loop would be in (i) stable equilibrium and (ii) unstable equilibrium ?

(1) (b) and (d), respectively
(2) (b) and (c), respectively
(3) (a) and (b), respectively
(4) (a) and (c), respectively

MG0175

19. Two coaxial solenoids of different radii carry current I in the same direction. Let $\vec{F_1}$ be the magnetic force on the inner solenoid due to the outer one and $\vec{F_2}$ be the magnetic force on the outer solenoid due to the inner one. Then : **[JEE Main-2015]**

(1) $\vec{F_1}$ is radially inwards and $\vec{F_2} = 0$

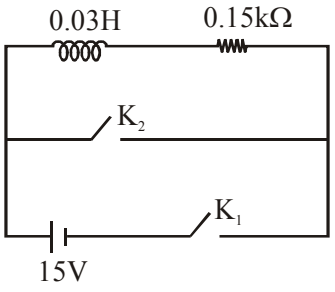
(2) $\vec{F_1}$ is radially outwards and $\vec{F_2} = 0$

(3) $\vec{F_1} = \vec{F_2} = 0$

(4) $\vec{F_1}$ is radially inwards and $\vec{F_2}$ is radially outwards.

MG0176

20. An inductor (L = 0.03 H) and a resistor (R = 0.15 kΩ) are connected in series to a battery of 15V EMF in a circuit shown below. The key K_1 has been kept closed for a long time. Then at t = 0, K_1 is opened and key K_2 is closed simultaneously. At t = 1ms, the current in the circuit will be ($e^5 \cong 150$):- **[JEE Main-2015]**



(1) 6.7 mA (2) 0.67 mA (3) 100 mA (4) 67 mA

EM0188

21. Two identical wires A and B, each of length '*l*', carry the same current I. Wire A is bent into a circle of radius R and wire B is bent to form a square of side 'a'. If B_A and B_B are the values of magnetic field at the centres of the circle and square respectively, then the ratio $\frac{B_A}{B_B}$ is : **[JEE Main-2016]**

(1) $\frac{\pi^2}{8\sqrt{2}}$ (2) $\frac{\pi^2}{8}$ (3) $\frac{\pi^2}{16\sqrt{2}}$ (4) $\frac{\pi^2}{16}$

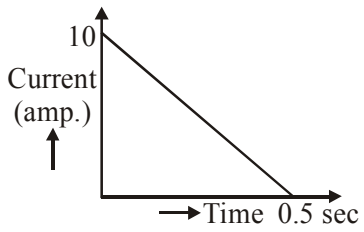
MG0178

22. An arc lamp requires a direct current of 10A at 80V to function. If it is connected to a 220V (rms), 50Hz AC supply, the series inductor needed for it to work is close to :- **[JEE Main-2016]**

(1) 0.065 H (2) 80 H (3) 0.08 H (4) 0.044 H

EM0190

23. In a coil of resistance 100 Ω, a current is induced by changing the magnetic flux through it as shown in the figure. The magnitude of change in flux through the coil is :- **[JEE Main-2017]**



(1) 250 Wb (2) 275 Wb (3) 200 Wb (4) 225 Wb

EM0191

24. An electron, a proton and an alpha particle having the same kinetic energy are moving in circular orbits of radii r_e, r_p, r_α respectively in a uniform magnetic field B. The relation between r_e, r_p, r_α is:- **[JEE Main-2018]**

(1) $r_e < r_p = r_\alpha$ (2) $r_e < r_p < r_\alpha$ (3) $r_e < r_\alpha < r_p$ (4) $r_e > r_p = r_\alpha$

MG0180

25. The dipole moment of a circular loop carrying a current I, is m and the magnetic field at the centre of the loop is B_1. When the dipole moment is doubled by keeping the current constant, the magnetic field at the centre of the loop is B_2. The ratio $\frac{B_1}{B_2}$ is : **[JEE Main-2018]**

(1) $\sqrt{3}$ (2) $\sqrt{2}$ (3) $\frac{1}{\sqrt{2}}$ (4) 2

MG0181

26. In an a. c. circuit, the instantaneous e.m.f. and current are given by

$e = 100 \sin 30\, t$

$i = 20 \sin\left(30t - \frac{\pi}{4}\right)$

In one cycle of a.c., the average power consumed by the circuit and the wattless current are, respectively.

[JEE Main-2018]

(1) $\frac{1000}{\sqrt{2}}$, 10 (2) $\frac{50}{\sqrt{2}}$, 0 (3) 50, 0 (4) 50, 10

EM0193

27. A series AC circuit containing an inductor (20 mH), a capacitor (120 μF) and a resistor (60Ω) is driven by an AC source of 24V/50Hz. The energy dissipated in the circuit in 60 s is :

[JEE Main-2019_Jan]

(1) 2.26×10^3 J (2) 3.39×10^3 J (3) 5.65×10^2 J (4) 5.17×10^2 J

EM0219

28. A power transmission line feeds input power at 2300 V to a step down transformer with its primary windings having 4000 turns. The output power is delivered at 230 V bv the transformer. If the current in the primary of the transformer is 5A and its efficiency is 90%, the output current would be :

[JEE Main-2019_Jan]

(1) 25 A (2) 50 A (3) 35 A (4) 45 A

EM0220

29. The self induced emf of a coil is 25 volts. When the current in it is changed at uniform rate from 10 A to 25 A in 1s, the change in the energy of the inductance is : **[JEE Main-2019_Jan]**

(1) 437.5 J (2) 637.5 J (3) 740 J (4) 540 J

EM0221

30. A solid metal cube of edge length 2 cm is moving in a positive y direction at a constant speed of 6 m/s. There is a uniform magnetic field of 0.1 T in the positive z-direction. The potential difference between the two faces of the cube perpendicular to the x-axis, is : **[JEE Main-2019_Jan]**

(1) 6 mV (2) 1 mV (3) 12 mV (4) 2 mV

MG0206

31. A copper wire is wound on a wooden frame, whose shape is that of an equilateral triangle. If the linear dimension of each side of the frame is increased by a factor of 3, keeping the number of turns of the coil per unit length of the frame the same, then the self inductance of the coil :

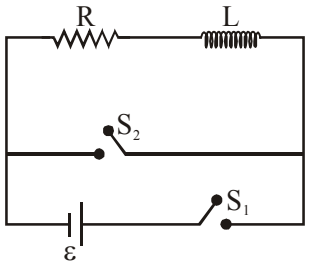
[JEE Main-2019_Jan]

(1) Decreases by a factor of $9\sqrt{3}$ (2) Increases by a factor of 3

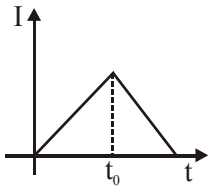
(3) Decreases by a factor of 9 (4) Increases by a factor of 27

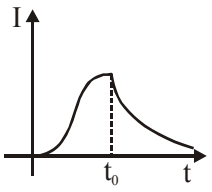
EM0222

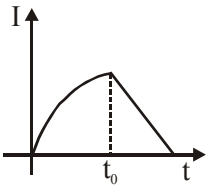
32. In the circuit shown, **[JEE Main-2019_Jan]**

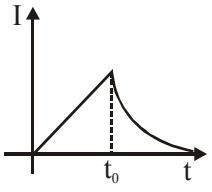


the switch S_1 is closed at time t = 0 and the switch S_2 is kept open. At some later time (t_0), the switch S_1 is opened and S_2 is closed. The behavious of the current I as a function of time 't' is given by :

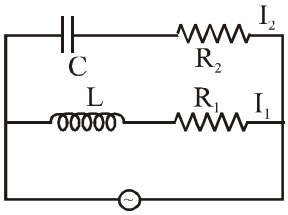
(1) 

(2) 

(3) 

(4) 

EM0223

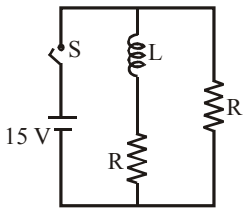
33. 

In the above circuit, $C = \frac{\sqrt{3}}{2}\mu F$, $R_2 = 20\Omega$, $L = \frac{\sqrt{3}}{10}$ H and $R_1 = 10\Omega$. Current in L-R_1 path is I_1 and in C-R_2 path it is I_2. The voltage of A.C source is given by $V = 200\sqrt{2}\sin(100t)$ volts. The phase difference between I_1 and I_2 is : **[JEE Main-2019_Jan]**

(1) 30° (2) 0° (3) 90° (4) 60°

EM0224

34. In the figure shown, a circuit contains two identical resistors with resistance R = 5Ω and an inductance with L = 2mH. An ideal battery of 15 V is connected in the circuit. What will be the current through the battery long after the switch is closed? **[JEE Main-2019_Jan]**



(1) 6A (2) 7.5A (3) 5.5A (4) 3A

EM0225

35. One of the two identical conducting wires of length L is bent in the form of a circular loop and the other one into a circular coil of N identical turns. If the same current is passed in both, the ratio of the magnetic field at the central of the loop (B_L) to that at the centre of the coil (B_C), i.e. R $\frac{B_L}{B_C}$ will be :

[JEE Main-2019_Jan]

(1) $\frac{1}{N}$ (2) N^2 (3) $\frac{1}{N^2}$ (4) N

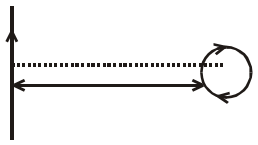
MG0207

36. A particle having the same charge as of electron moves in a circular path of radius 0.5 cm under the influence of a magnetic field of 0.5 T. If an electric field of 100 V/m makes it to move in a straight path, then the mass of the particle is (Given charge of electron $=1.6 \times 10^{-19}$C) **[JEE Main-2019_Jan]**

(1) 2.0×10^{-24} kg (2) 1.6×10^{-19} kg (3) 1.6×10^{-27} kg (4) 9.1×10^{-31} kg

MG0208

37. An infinitely long current carrying wire and a small current carrying loop are in the plane of the paper as shown. The radius of the loop is a and distance of its centre from the wire is d (d»a). If the loop applies a force F on the wire then : **[JEE Main-2019_Jan]**



(1) $F \propto \left(\frac{a^2}{d^3}\right)$ (2) $F \propto \left(\frac{a}{d}\right)$ (3) $F \propto \left(\frac{a}{d}\right)^2$ (4) F = 0

MG0209

38. A hoop and a solid cylinder of same mass and radius are made of a permanent magnetic material with their magnetic moment parallel to their respective axes. But the magnetic moment of hoop is twice of solid cylinder. They are placed in a uniform magnetic field in such a manner that their magnetic moments make a small angle with the field. If the oscillation periods of hoop and cylinder are T_h and T_c respectively, then : **[JEE Main-2019_Jan]**

(1) $T_h = 0.5\ T_c$ (2) $T_h = 2\ T_c$ (3) $T_h = 1.5\ T_c$ (4) $T_h = T_c$

MG0210

39. An insulating thin rod of length ℓ has a x linear charge density $p(x) = \rho_0 \frac{x}{\ell}$ on it. The rod is rotated about an axis passing through the origin (x = 0) and perpendicular to the rod. If the rod makes n rotations per second, then the time averaged magnetic moment of the rod is : **[JEE Main-2019_Jan]**

(1) $\frac{\pi}{4} n\rho\ell^3$ (2) $n\rho\ell^3$ (3) $\pi n\rho\ell^3$ (4) $\frac{\pi}{3} n\rho\ell^3$

MG0211

40. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is :- **[JEE Main-2019_Jan]**

(1) 80 Ω (2) 120 Ω (3) 125 Ω (4) 100 Ω

MG0212

41. The region between y = 0 and y = d contains a magnetic field $\vec{B} = B\hat{z}$. A particle of mass m and charge q enters the region with a velocity $\vec{v} = v\hat{i}$. If $d = \frac{mv}{2qB}$, the acceleration of the charged particle at the point of its emergence at the other side is :- **[JEE Main-2019_Jan]**

(1) $\frac{qvB}{m}\left(\frac{\hat{i}+\hat{j}}{\sqrt{2}}\right)$ (2) $\frac{qvB}{m}\left(\frac{1}{2}\hat{i}-\frac{\sqrt{3}}{\sqrt{2}}\hat{j}\right)$ (3) $\frac{qvB}{m}\left(\frac{-\hat{j}+\hat{i}}{\sqrt{2}}\right)$ (4) $\frac{qvB}{m}\left(\frac{\sqrt{3}}{2}\hat{i}+\frac{1}{2}\hat{j}\right)$

MG0213

42. A particle of mass m and charge q is in an electric and magnetic field given by

$\vec{E} = 2\hat{i}+3\hat{j}$; $\vec{B} = 4\hat{j}+6\hat{k}$.

The charged particle is shifted from the origin to the point P(x = 1 ; y = 1) along a straight path. The magnitude of the total work done is :- **[JEE Main-2019_Jan]**

(1) (0.35)q (2) (0.15)q (3) (2.5)q (4) 5q

MG0214

43. There are two long co-axial solenoids of same length *l*. the inner and outer coils have radii r_1 and r_2 and number of turns per unit length n_1 and n_2 respectively. The rate of mutual inductance to the self-inductance of the inner-coil is : **[JEE Main-2019_Jan]**

(1) $\frac{n_2}{n_1}\cdot\frac{r_2^2}{r_1^2}$ (2) $\frac{n_2}{n_1}\cdot\frac{r_1}{r_2}$ (3) $\frac{n_1}{n_2}$ (4) $\frac{n_2}{n_1}$

EM0226

44. In an experiment electrons are accelerated, from rest, by applying a voltage of 500 V. Calculate the radius of the path if a magnetic field 100 mT is then applied.
[Charge of the electron = 1.6×10^{-19} C, Mass of the electron = 9.1×10^{-31} kg] **[JEE Main-2019_Jan]**

(1) 7.5×10^{-4} m (2) 7.5×10^{-3} m (3) 7.5 m (4) 7.5×10^{-2} m

MG0215

45. A 10 m long horizontal wire extends from North East to South West. It is falling with a speed of $5.0 ms^{-1}$, at right angles to the horizontal component of the earth's magnetic field, of $0.3 \times 10^{-4} Wb/m^2$. The value of the induced emf in wire is : **[JEE Main-2019_Jan]**

(1) 2.5×10^{-3}V (2) 1.1×10^{-3}V (3) 0.3×10^{-3}V (4) 1.5×10^{-3}V

EM0227

46. A proton and an α-particle (with their masses in the ratio of 1:4 and charges in the ratio of 1:2) are accelerated from rest through a potential difference V. If a uniform magnetic field (B) is set up perpendicular to their velocities, the ratio of the radii $r_p : r_\alpha$ of the circular paths described by them will be : **[JEE Main-2019_Jan]**

(1) $1:\sqrt{2}$ (2) 1 : 2 (3) 1 : 3 (4) $1:\sqrt{3}$

MG0216

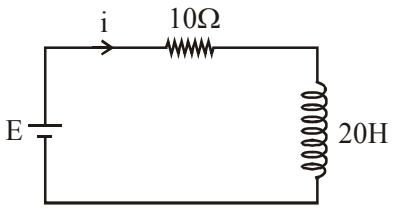
47. A circuit connected to an ac source of emf $e = e_0\sin(100t)$ with t in seconds, gives a phase difference of $\frac{\pi}{4}$ between the emf e and current i. Which of the following circuits will exhibit this ?

[JEE Main-2019_April]

(1) RC circuit with R = 1 kΩ and C = 1μF (2) RL circuit with R = 1kΩ and L = 1mH
(3) RL circuit with R = 1 kΩ and L = 10 mH (4) RC circuit with R = 1kΩ and C = 10 μF

EM0228

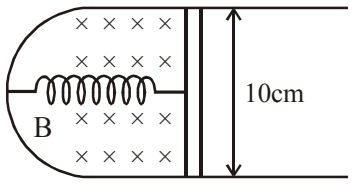
48. A 20 Henry inductor coil is connected to a 10 ohm resistance in series as shown in figure. The time at which rate of dissipation of energy (joule's heat) across resistance is equal to the rate at which magnetic energy is stored in the inductor is : **[JEE Main-2019_April]**



(1) $\frac{2}{\ell n2}$ (2) $\ell n2$ (3) $2\ell n2$ (4) $\frac{1}{2}\ell n2$

EM0229

49. A thin strip 10 cm long is on a U shaped wire of negligible resistance and it is connected to a spring of spring constant 0.5 Nm^{-1} (see figure). The assembly is kept in a uniform magnetic field of 0.1 T. If the strip is pulled from its equilibrium position and released, the number of oscillation it performs before its amplitude decreases by a factor of e is N. If the mass of the strip is 50 grams, its resistance 10Ω and air drag negligible, N will be close to : **[JEE Main-2019_April]**



(1) 50000 (2) 5000 (3) 10000 (4) 1000

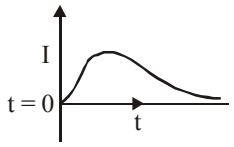
EM0230

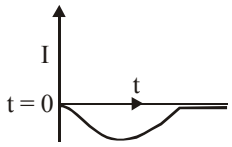
50. An alternating voltage v(t) = 220 sin 100 πt volt is applied to a purely resistance load of 50 Ω. The time taken for the current to rise from half of the peak value to the peak value is : **[JEE Main-2019_April]**

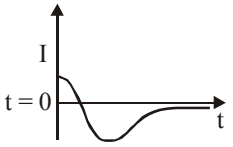
(1) 2.2 ms (2) 5 ms (3) 3.3 ms (4) 7.2 ms

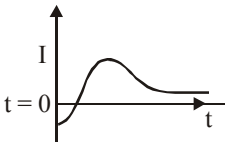
EM0231

51. A very long solenoid of radius R is carrying current $I(t) = kte^{-\alpha t}(k > 0)$, as a function of time ($t \geq 0$). counter clockwise current is taken to be positive. A circular conducting coil of radius 2R is placed in the equatorial plane of the solenoid and concentric with the solenoid. The current induced in the outer coil is correctly depicted, as a function of time, by :- **[JEE Main-2019_April]**

(1) 

(2) 

(3) 

(4) 

EM0232

52. The total number of turns and cross-section area in a solenoid is fixed. However, its length L is varied by adjusting the separation between windings. The inductance of solenoid will be proportional to : **[JEE Main-2019_April]**

(1) $1/L^2$ (2) $1/L$ (3) L (4) L^2

EM0233

53. A coil of self inductance 10 mH and resistance 0.1 Ω is connected through a switch to a battery of internal resistance 0.9 Ω. After the switch is closed, the time taken for the current to attain 80% of the saturation value is : (Take $\ln 5 = 1.6$) **[JEE Main-2019_April]**

(1) 0.103 s (2) 0.016 s (3) 0.002 s (4) 0.324 s

EM0234

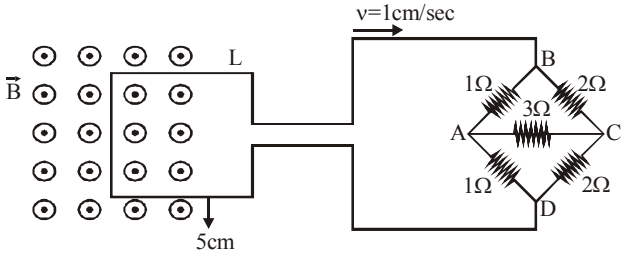
54. A transformer consisting of 300 turns in the primary and 150 turns in the secondary gives output power of 2.2 kW. If the current in the secondary coil is 10A, then the input voltage and current in the primary coil are : **[JEE Main-2019_April]**

(1) 220 V and 10A (2) 440 V and 5A

(3) 440 V and 20 A (4) 220 V and 20 A

EM0235

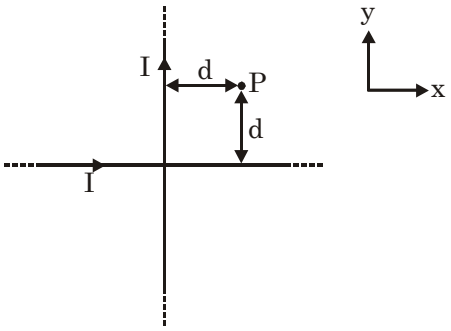
55. The figure shows a square loop L of side 5 cm which is connected to a network of resistances. The whole setup is moving towards right with a constant speed of 1 cms^{-1}. At some instant, a part of L is in a uniform magnetic field of 1T, perpendicular to the plane of the loop. If the resistance of L is 1.7Ω, the current in the loop at that instant will be close to : **[JEE Main-2019_April]**



(1) 115 μA (2) 170 μA (3) 60 μA (4) 150 μA

EM0236

56. Two very long, straight, and insulated wires are kept at 90° angle from each other in xy-plane as shown in the figure. These wires carry currents of equal magnitude I, whose directions are shown in the figure. The net magnetic field at point P will be : **[JEE Main-2019_April]**



(1) Zero / शून्य (2) $\frac{+\mu_0 I}{\pi d}(\hat{z})$ (3) $-\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$ (4) $\frac{\mu_0 I}{2\pi d}(\hat{x}+\hat{y})$

MG0217

57. A circular coil having N turns and radius r carries a current I. It is held in the XZ plane in a magnetic field $B\hat{i}$. The torque on the coil due to the magnetic field is : **[JEE Main-2019_April]**

(1) $B\pi r^2 IN$ (2) $\frac{Br^2 I}{\pi N}$ (3) Zero (4) $\frac{B\pi r^2 I}{N}$

MG0218

58. Two coils 'P' and 'Q' are separated by some distance. When a current of 3 A flows through coil 'P', a magnetic flux of 10^{-3} Wb passes through 'Q'. No current is passed through 'Q'. When no current passes through 'P' and a current of 2 A passes through 'Q', the flux through 'P' is :- **[JEE Main-2019_April]**

(1) 6.67×10^{-3} Wb (2) 6.67×10^{-4} Wb (3) 3.67×10^{-4} Wb (4) 3.67×10^{-3} Wb

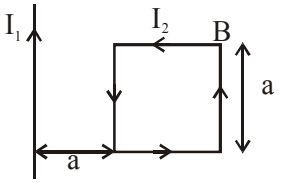
EM0237

59. A moving coil galvanometer has a coil with 175 turns and area 1 cm^2. It uses a torsion band of torsion constant 10^{-6} N-m/rad. The coil is placed in a maganetic field B parallel to its plane. The coil deflects by 1° for a current of 1 mA. The value of B (in Tesla) is approximately :- **[JEE Main-2019_April]**

(1) 10^{-3} (2) 10^{-1} (3) 10^{-4} (4) 10^{-2}

MG0219

60. A rigid square loop of side 'a' and carrying current I_2 is lying on a horizontal surface near a long current I_1 carrying wire in the same plane as shown in figure. The net force on the loop due to wire will be : **[JEE Main-2019_April]**



(1) Attractive and equal to $\frac{\mu_0 I_1 I_2}{3\pi}$ (2) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{4\pi}$

(3) Repulsive and equal to $\frac{\mu_0 I_1 I_2}{2\pi}$ (4) Zero

MG0220

61. A rectangular coil (Dimension 5 cm × 2.5 cm) with 100 turns, carrying a current of 3 A in the clock-wise direction is kept centered at the origin and in the X-Z plane. A magnetic field of 1 T is applied along X-axis. If the coil is tilted through 45° about Z-axis, then the torque on the coil is :

[JEE Main-2019_April]

(1) 0.55 Nm (2) 0.27 Nm (3) 0.38 Nm (4) 0.42 Nm

MG0221

62. The magnitude of the magnetic field at the center of an equilateral triangular loop of side 1m which is carrying a current of 10 A is : [Take $\mu_0 = 4\pi \times 10^{-7}$ NA^{-2}] **[JEE Main-2019_April]**

(1) 18 μT (2) 3 μT (3) 1 μT (4) 9 μT

MG0222

63. A square loop is carrying a steady current I and the magnitude of its magnetic dipole moment is m. If this square loop is changed to a circular loop and it carries the same current, the magnitude of the magnetic dipole moment of circular loop will be : **[JEE Main-2019_April]**

(1) $\frac{3m}{\pi}$ (2) $\frac{4m}{\pi}$ (3) $\frac{2m}{\pi}$ (4) $\frac{m}{\pi}$

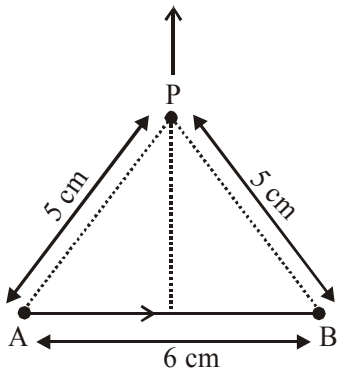
MG0223

64. In the formula $X = 5YZ^2$, X and Z have dimensions of capacitance and magnetic field, respectively. What are the dimensions of Y in SI units ? **[JEE Main-2019_April]**

(1) [M^{-2} L^{-2} T^6 A^3] (2) [M^{-1} L^{-2} T^4 A^2] (3) [M^{-3} L^{-2} T^8 A^4] (4) [M^{-2} L^0 T^{-4} A^{-2}]

MG0224

65. Find the magnetic field at point P due to a straight line segment AB of length 6 cm carrying a current of 5 A. (See figure) ($\mu_0 = 4\pi \times 10^{-7}$ N-A^{-2}) **[JEE Main-2019_April]**

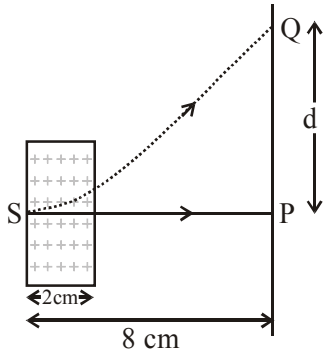


(1) 3.0×10^{-5} T (2) 2.5×10^{-5} T (3) 2.0×10^{-5} T (4) 1.5×10^{-5} T

MG0225

66. An electron, moving along the x-axis with an initial energy of 100 eV, enters a region of magnetic field $\vec{B} = (1.5 \times 10^{-3}\,T)\hat{k}$ at S (See figure). The field extends between x = 0 and x = 2 cm. The electron is detected at the point Q on a screen placed 8 cm away from the point S. The distance d between P and Q (on the screen) is : (electron's charge = 1.6×10^{-19} C, mass of electron = 9.1×10^{-31} kg)

[JEE Main-2019_April]



(1) 12.87 cm (2) 1.22 cm (3) 11.65 cm (4) 2.25 cm

MG0226

67. A thin ring of 10 cm radius carries a uniformly distributed charge. The ring rotates at a constant angular speed of 40 π rad s^{-1} about its axis, perpendicular to its plane. If the magnetic field at its centre is 3.8×10^{-9} T, then the charge carried by the ring is close to ($\mu_0 = 4\pi \times 10^{-7}$ N/A^2) :

[JEE Main-2019_April]

(1) 2×10^{-6} C (2) 3×10^{-5} C (3) 4×10^{-5} C (4) 7×10^{-6} C

EM0238

ANSWER KEY

01_MAGNETIC EFFECT OF CURRENT

EXERCISE (S)

1. Ans. (a) zero (b) 2μT along the Z-axis (c) zero and (d) 2μT along the negative Z-axis

2. Ans. 4.0×10^{-5} T, downwards in both the cases

3. Ans. $5\pi \times 10^{-4}$ T = 1.6×10^{-3} T towards west.

4. Ans. $7.07\times 10^{-10}\ \hat{k}$T

5. Ans. At a distance of $\frac{4r}{\pi}$ from the centre in such a way that the direction of the current in it is opposite to that in the nearest part of the circular wire.

6. Ans. (a) 0 (b) 1.41×10^{-6} T , 45° in xz-plane, **(c)** 5×10^{-6} T , +x-direction]

7. Ans. μ_0 weber.m^{-1} **8. Ans.** $B_1 = \frac{\mu_0 b r_1^2}{3}$, $B_2 = \frac{\mu_0 b R^3}{3r_2}$ **9. Ans.** $\frac{2mv_0}{qB}$

10. Ans. 1.2×10^{-2}m, 4.37×10^{-2}m

11. Ans. (a) Circular trajectory of radius 1.0 mm normal to B.
(b) Helical trajectory of radius 0.5 mm with velocity component 2.3×10^7 ms^{-1} along B.

12. Ans. (6.4 m, 0,0) (6.4m, 0, 2m)

13. Ans. (a) A horizontal magnetic field of magnitude 0.26 T normal to the conductor in such a direction that Fleming a left-hand rule gives a magnetic force upward.
(b) 1.176 N.

14. Ans. $F = \alpha a^2 i\ \hat{j}$

15. Ans. (a) Zero, (b) zero, (c) force on each electron is evB = IB/(nA) = 5×10^{-25}N. Note : Answer (c) denotes only the magnetic force.

16. Ans. $T_0 = 2\pi\sqrt{\frac{m}{6IB}} = 0.57$ s **17. Ans.** (a) 1.4, (b) 1

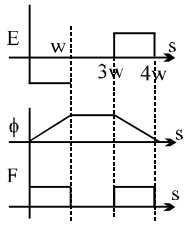
EXERCISE (O)

1. Ans. (B)	**2. Ans. (C)**	**3. Ans. (A)**	**4. Ans. (B)**	**5. Ans. (A)**	**6. Ans. (C)**
7. Ans. (B)	**8. Ans. (C)**	**9. Ans. (B)**	**10. Ans. (C)**	**11. Ans. (D)**	**12. Ans. (D)**
13. Ans. (A)	**14. Ans. (B)**	**15. Ans. (B)**	**16. Ans. (C)**	**17. Ans. (D)**	**18. Ans. (D)**
19. Ans. (C)	**20. Ans. (B)**	**21. Ans. (C)**	**22. Ans. (A)**	**23. Ans. (B)**	**24. Ans. (A)**
25. Ans. (B)	**26. Ans. (B)**	**27. Ans. (C)**	**28. Ans. (D)**	**29. Ans. (C)**	**30. Ans. (A)**
31. Ans. (C)	**32. Ans. (A)**	**33. Ans. (A)**	**34. Ans. (C)**	**35. Ans. (B)**	**36. Ans. (C)**
37. Ans. (B)	**38. Ans. (A)**	**39. Ans. (D)**	**40. Ans. (A)**	**41. Ans. (B)**	**42. Ans. (B)**
43. Ans. (C)					

02_ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT

EXERCISE (S)

1. Ans. $\lambda V_y B_0$ **2. Ans.** (i) 2.4×10^{-5} V (ii) from c to b **3. Ans.** 2 N

4. Ans. 

5. Ans. $\frac{mgR}{B^2\ell^2}$ **6. Ans.** 0.75 T

7. Ans. $\frac{erk}{2m}$ directed along tangent to the circle of radius r, whose centre lies on the axis of cylinder.

8. Ans. $\frac{l}{2}\frac{dB}{dt}\sqrt{R^2-\frac{l^2}{4}}$ **9. Ans.** 0.8

10. Ans. (i) $i_1 = i_2 = 10/3$ A, (ii) $i_1 = 50/11$ A ; $i_2 = 30/11$ A, (iii) $i_1 = 0$, $i_2 = 20/11$ A, (iv) $i_1 = i_2 = 0$

11. Ans. $\frac{\mu_0 i a^2 \pi}{2Rb}$ **12. Ans.** $\frac{LE^2}{2R_1^2}$ **13. Ans.** $\frac{e^2}{e^2-1}$ **14. Ans.** $\frac{EL}{eR^2}$

15. Ans. (a)10^4A/s (b) 0 (c) 2A (d) $100\sqrt{3}$ μC **16. Ans.** 30 Wb. **17. Ans.** $\varepsilon = 1.7 \times 10^{-5}$V

18. Ans. 0.08 H, 17.28 W **19. Ans.** 2A, 400W **20. Ans.** 0.2 mH, $\frac{1}{32}$ μF, 8×10^5 rad/s

21. Ans. $\frac{20}{\pi^2} \cong 2$H **22. Ans.** 20 V

EXERCISE (O)

1. Ans. (C)	**2. Ans. (D)**	**3. Ans. (C)**	**4. Ans. (A)**	**5. Ans. (A)**	**6. Ans. (B)**
7. Ans. (C)	**8. Ans. (A)**	**9. Ans. (A)**	**10. Ans. (C)**	**11. Ans. (A)**	**12. Ans. (A)**
13. Ans. (D)	**14. Ans. (A)**	**15. Ans. (A)**	**16. Ans. (D)**	**17. Ans. (B)**	**18. Ans. (A)**
19. Ans. (D)	**20. Ans. (B)**	**21. Ans. (C)**	**22. Ans. (B)**	**23. Ans. (D)**	**24. Ans. (A)**
25. Ans. (C)	**26. Ans. (A)**	**27. Ans. (C)**	**28. Ans. (B)**	**29. Ans. (C)**	**30. Ans. (A)**
31. Ans. (C)	**32. Ans. (B)**	**33. Ans. (D)**	**34. Ans. (B)**	**35. Ans. (A)**	**36. Ans. (A)**
37. Ans. (C)	**38. Ans. (B)**	**39. Ans. (B)**	**40. Ans. (A)**	**41. Ans. (D)**	**42. Ans. (A)**
43. Ans. (D)	**44. Ans. (B)**	**45. Ans. (A)**	**46. Ans. (C)**	**47. Ans. (A)**	**48. Ans. (D)**
49. Ans. (D)	**50. Ans. (D)**	**51. Ans. (D)**	**52. Ans. (A)**		

EXERCISE-JM

1. Ans. (4)	**2. Ans. (4)**	**3. Ans. (2)**	**4. Ans. (2)**	**5. Ans. (2)**	**6. Ans. (3)**
7. Ans. (3)	**8. Ans. (1)**	**9. Ans. (4)**	**10. Ans. (2)**	**11. Ans. (2)**	**12. Ans. (4)**
13. Ans. (4)	**14. Ans. (1)**	**15. Ans. (3)**	**16. Ans. (1)**	**17. Ans. (4)**	**18. Ans. (1)**
19. Ans. (2)	**20. Ans. (2)**	**21. Ans. (1)**	**22. Ans. (1)**	**23. Ans. (1)**	**24. Ans. (1)**
25. Ans. (2)	**26. Ans. (1)**	**27. Ans. (4)**	**28. Ans. (4)**	**29. Ans. (1)**	**30. Ans. (3)**
31. Ans. (2)	**32. Ans. (1 or 3 or 4)**		**33. Ans. (Bonus)**		**34. Ans. (1)**
35. Ans. (3)	**36. Ans. (1)**	**37. Ans. (3)**	**38. Ans. (4)**	**39. Ans. (1)**	**40. Ans. (1)**
41. Ans. (Bonus)		**42. Ans. (4)**	**43. Ans. (4)**	**44. Ans. (1)**	**45. Ans. (2)**
46. Ans. (1)	**47. Ans. (4)**	**48. Ans. (3)**	**49. Ans. (2)**	**50. Ans. (3)**	
51. Ans. (2) or (4)		**52. Ans. (2)**	**53. Ans. (2)**	**54. Ans. (2)**	**55. Ans. (2)**
56. Ans. (1)	**57. Ans. (1)**	**58. Ans. (2)**	**59. Ans. (1)**	**60. Ans. (2)**	**61. Ans. (2)**
62. Ans. (1)	**63. Ans. (2)**	**64. Ans. (3)**	**65. Ans. (4)**	**66. Ans. (1)**	**67. Ans. (2)**