KINEMATICS

KEY CONCEPT

Kinematics

Study of motion of objects without taking into account the factor which cause the motion (i.e. nature of force).

Motion

If a body changes its position with time, it is said to be moving else it is at rest. Motion is always relative to the observer.

Motion is a combined property of the object under study and the observer. There is no meaning of rest or motion without the viewer. In other words absolute motion or rest is meaningless.

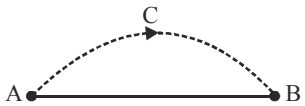
- To locate the position of a particle we need a reference frame. A commonly used reference frame is Cartesian coordinate system or simply coordinate system.

  The coordinates (x, y, z) of a particle specify the position of the particle with respect to origin of that frame. If all the three coordinates of the particle remain unchanged as time passes it means the particle is at rest w.r.t. this frame.
- The reference frame is chosen according to the problems.
- If the frame is not mentioned, then ground is taken as the reference frame.

DISTANCE AND DISPLACEMENT

Total length of path covered by the particle, in definite time interval is called distance.

Let a body moves from A to B via C. The length of path ACB is called the distance travelled by the body.

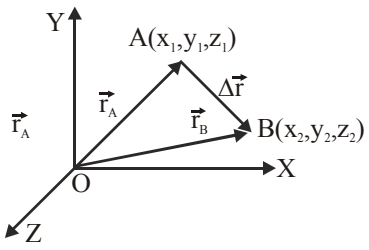


But overall, body is displaced from A to B. A vector from A to B, i.e. $\overrightarrow{AB}$ is its displacement vector or displacement that is the minimum distance and directed from initial position to final position.

- Distance is a scalar while displacement is a vector.
- Distance depends on path while displacement is independent of path but depends only on final and initial position.
- For a moving body, distance can't have zero or negative values but displacement may be +ive, –ive or zero.
- For a moving/stationary object distance can't be decreasing.
- If motion is in straight line without change in direction then
  distance = |displacement| i.e. magnitude of displacement.
- Magnitude of displacement may be equal or less than distance but never greater than distance.
  distance $\geq$ |displacement|

Displacement in terms of position vector

Let a body is displaced from A (x_1, y_1, z_1) to B (x_2, y_2, z_2) then its displacement is given by vector $\vec{AB}$.



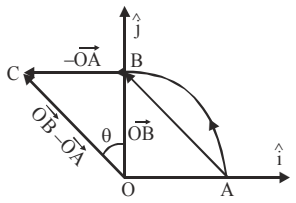
From ΔOAB $\vec{r}_A + \Delta\vec{r} = \vec{r}_B \Rightarrow \Delta\vec{r} = \vec{r}_B - \vec{r}_A$

$\vec{r}_B = x_2\hat{i} + y_2\hat{j} + z_2\hat{k}$ and $\vec{r}_A = x_1\hat{i} + y_1\hat{j} + z_1\hat{k}$

$\Delta\vec{r} = (x_2 - x_1)\hat{i} + (y_2 - y_1)\hat{j} + (z_2 - z_1)\hat{k} \Rightarrow \Delta\vec{r} = \Delta x\hat{i} + \Delta y\hat{j} + \Delta z\hat{k}$

Ex. A particle goes along a quadrant from A to B of a circle radius 10m as shown in fig. Find the direction and magnitude of displacement and distance along path AB.

Sol. Displacement $\vec{AB} = \vec{OB} - \vec{OA} = 10\hat{j} - 10\hat{i}$



$|\vec{AB}| = \sqrt{10^2 + 10^2} = 10\sqrt{2}$ m

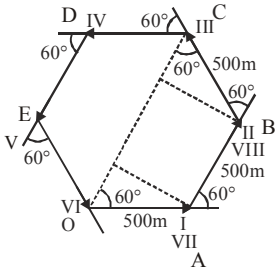
From ΔOBC $\tan\theta = \frac{OA}{OB} = \frac{10}{10} = 1 \Rightarrow \theta = 45°$

Angle between displacement vector $\vec{OC}$ and x–axis = 90° + 45° = 135°

Distance of path AB = $\frac{1}{4}$ (circumference) = $\frac{1}{4}(2\pi R)$ m = (5π) m

Ex. On an open ground a motorist follows a track that turns to his left by an angle of 60° after every 500 m. Starting from a given turn, specify the displacement of the motorist at the third, sixth and eighth turn. Compare the magnitude of displacement with the total path length covered by the motorist in each case.

Sol. **At III turn**



Displacement = $\overrightarrow{OA} + \overrightarrow{AB} + \overrightarrow{BC} = \overrightarrow{OC}$

= 500 cos 60° + 500 + 500 cos 60°

= $500 \times \frac{1}{2} + 500 + 500 \times \frac{1}{2}$ = 1000 m from O to C

Distance = 500 + 500 + 500 = 1500 m. So $\frac{\text{Displacement}}{\text{Distance}} = \frac{1000}{1500} = \frac{2}{3}$

At VI turn

$\because$ initial and final positions are same so displacement = 0 and distance = 500 × 6 = 3000 m

$\therefore \frac{\text{Displacement}}{\text{Distance}} = \frac{0}{3000} = 0$

At VIII turn

Displacement = $2(500)\cos\left(\frac{60°}{2}\right) = 1000 \times \cos 30° = 1000 \times \frac{\sqrt{3}}{2} = 500\sqrt{3}$ m

Distance = 500 × 8 = 4000 m

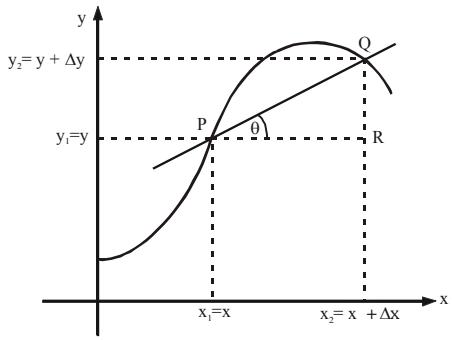
$\therefore \frac{\text{Displacement}}{\text{Distance}} = \frac{500\sqrt{3}}{4000} = \frac{\sqrt{3}}{8}$

DERIVATIVE OF A FUNCTION

Average Rate of Change

Let a function y=f(x) be plotted by an arbitrary graph as shown in the figure. Average rate of change in y with respect to x in an interval $[x_1, x_2]$ is defined as

$$\text{Average rate of change} = \frac{\text{Change in y}}{\text{Change in x}} = \frac{y_2 - y_1}{x_2 - x_1} = \frac{\Delta y}{\Delta x}$$



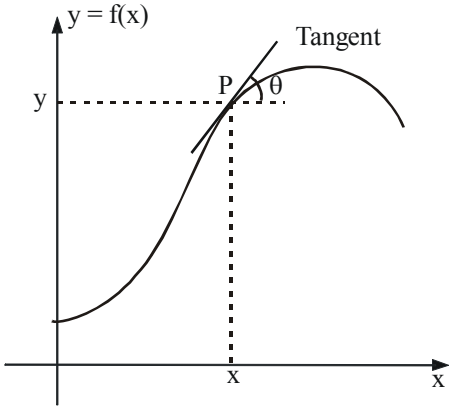
= slope of chord

If both the axes have equal scale

Average rate of change = slope of chord PQ = tanθ

Instantaneous Rate of Change : First derivative

It is defined as the rate of change in y with x at a particular value of x. It is measured graphically by the slope of the tangent drawn to the y-x graph at the point (x, y) and algebraically by the first derivative of the function y=f(x).



$$\text{Instantaneous rate of change} = \frac{dy}{dx} = \text{Slope of the tangent}$$

If both the axes have equal scale then $\frac{dy}{dx} = \tan\theta$

$$\text{Instantaneous rate of change} = \frac{dy}{dx} = \lim_{\Delta x \to 0} \frac{\Delta y}{\Delta x}$$

$$= \lim_{\Delta x \to 0} \frac{f(x + \Delta x) - f(x)}{\Delta x}$$

VELOCITY

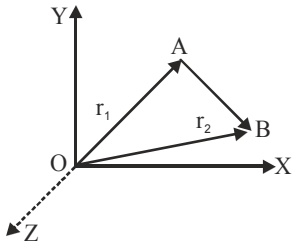
When a particle is moving in space then its motion can be broken up in three co-ordinate axes (x, y& z). The motion in these three directions is governed only by velocity & acceleration in that particular direction and is totally independent of the velocities and acceleration in other directions.

Lets say a particle is moving in space $\vec{r} = x\hat{i} + y\hat{j} + z\hat{k}$, Gives position of particle in space.

Average Velocity

The average velocity of a particle in a time interval t_1 to t_2 is a defined as its displacement divided by the time interval. Let a particle is at a point A at time t_1 and B at time t_2. Position vectors of A and B are $\vec{r}_1$ and $\vec{r}_2$. The displacement in this time interval is the vector $\vec{AB} = (\vec{r}_2 - \vec{r}_1)$.

The average velocity in this time interval is, $\vec{v}_{av} = \dfrac{\text{displacement vector}}{\text{time interval}}$



$$\vec{v}_{av} = \frac{\vec{AB}}{t_2 - t_1} = \frac{\vec{r}_2 - \vec{r}_1}{t_2 - t_1} = \frac{\Delta\vec{r}}{\Delta t}$$

here $\vec{AB} = \vec{OB} - \vec{OA} = \vec{r}_2 - \vec{r}_1$ = change in position vector.

Instantaneous velocity

The velocity of the object at a given instant of time or at a given position during motion is called instantaneous velocity $\vec{v} = \underset{\Delta t \to 0}{\text{Lim}} \dfrac{\Delta\vec{r}}{\Delta t} = \dfrac{d\vec{r}}{dt}$

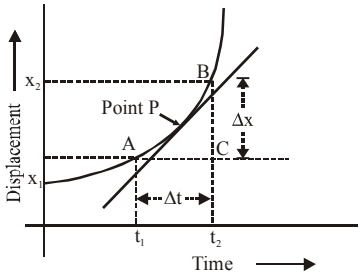
$$\vec{v} = \frac{d\vec{r}}{dt} = \frac{d}{dt}\left(x\hat{i} + y\hat{j} + z\hat{k}\right) = \left(\frac{dx}{dt}\right)\hat{i} + \left(\frac{dy}{dt}\right)\hat{j} + \left(\frac{dz}{dt}\right)\hat{k}$$

where the scalar components of $\vec{v}$ are

$$v_x = \frac{dx}{dt},\ v_y = \frac{dy}{dt},\ v_z = \frac{dz}{dt}$$

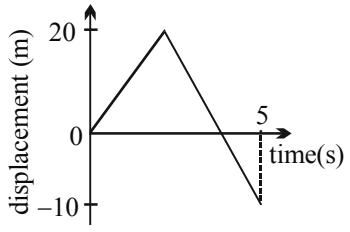
Differentiating $\vec{r}$ w.r.t. time gives us velocity vector of particle at that time.

Note:- The direction of the instantaneous velocity $\vec{v}$ of a particle is always tangent to the particle's path at the particle position. $\vec{V} = \underset{\Delta t \to 0}{\lim} \dfrac{\Delta\vec{r}}{\Delta t}$, $\Delta\vec{r}$ will be along tangent

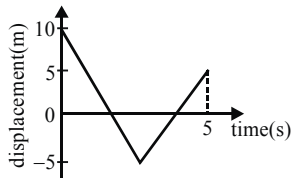


SOLVED EXAMPLE :

Ex. (a) The diagram shows the displacement-time graph for a particle moving in a straight line. Find the average velocity for the interval from t = 0 to t = 5.

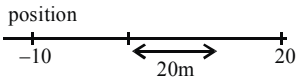


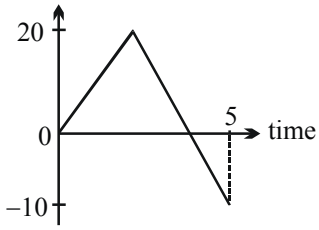
(b) The diagram shows the displacement-time graph for a particle moving in a straight line. Find the average speed for the interval from t = 0 to t = 5.



Ans. (a) – 2 ms^{-1} (b) 5 m/s

Sol. (a)





Average velocity = $\frac{-10}{5} = -2\text{m/s}$

(b) Total distance = 10 + 5 + 5 + 5 = 25

$S_{avg} = \frac{25}{5} = 5$ m/s

ACCELRATION

Similarly, if we differentiate $\vec{V}$ w.r.t. time we get acceleration of particle $\vec{a} = \frac{d\vec{V}}{dt}$

$$\vec{a} = \frac{d\vec{v}}{dt} = \frac{dv_x}{dt}\hat{i} + \frac{dv_y}{dt}\hat{j} + \frac{dv_z}{dt}\hat{k}$$

where the scalar components of $\vec{a}$ are

$$a_x = \frac{dv_x}{dt},\ a_y = \frac{dv_y}{dt},\ a_z = \frac{dv_z}{dt}$$

Now, collecting equations of motion relating to x & y axes separately

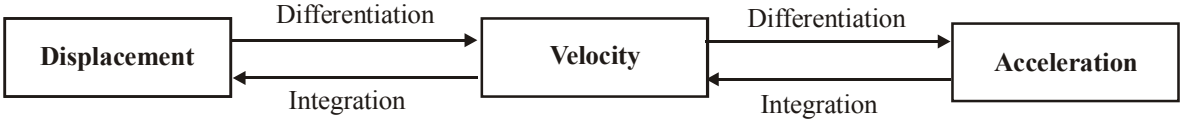
x-axis	y-axis
$V_x = \frac{dx}{dt}$	$V_y = \frac{dy}{dt}$
$a_x = \frac{dV_x}{dt}$	$a_y = \frac{dV_y}{dt}$

Thus we can see that motion in plane is composed of two straight line motions. **These motions are completely independent of each other**. Only thing connecting them is fact that they are occuring simultaneously.

Use of Mathematical Tools in Solving Problems of One–Dimensional Motion

If displacement–time equation is given, we can get velocity–time equation with the help of differentiation. Again, we can get acceleration–time equation with the help of differentiation.

If acceleration–time equation is given, we can get velocity–time equation by integration. From velocity equation, we can get displacement–time equation by integration.



Ex. The velocity of any particle is related with its displacement as; $x = \sqrt{v+1}$, Calculate acceleration at x = 5 m.

Sol. $\therefore\ x = \sqrt{v+1}\ \ \therefore\ x^2 = v + 1 \Rightarrow v = (x^2 - 1)$

Therefore $a = \frac{dv}{dt} = \frac{d}{dt}\left(x^2 - 1\right) = 2x\frac{dx}{dt} = 2x\ \ v = 2x\left(x^2 - 1\right)$

At x = 5 m, $a = 2 \times 5\ (25 - 1) = 240\ m/s^2$

Ex. The velocity of a particle moving in the positive direction of x–axis varies as $v = \alpha\sqrt{x}$ where α is positive constant. Assuming that at the moment t = 0, the particle was located at x = 0 find, (i) the time dependance of the velocity and the acceleration of the particle and (ii) the average velocity of the particle averaged over the time that the particle takes to cover first s metres of the path.

Sol. (i) Given that $v = \alpha\sqrt{x}$

$$\Rightarrow \frac{dx}{dt} = \alpha\sqrt{x}\ \ \therefore \frac{dx}{\sqrt{x}} = \alpha\, dt \ \Rightarrow \int_0^x \frac{dx}{\sqrt{x}} = \int_0^t \alpha\, dt\ \ 2\sqrt{x} = \alpha t \Rightarrow x = \left(\alpha^2 t^2 / 4\right)$$

Velocity $\frac{dx}{dt} = \frac{1}{2}\alpha^2 t$ and Acceleration $\frac{d^2x}{dt^2} = \frac{1}{2}\alpha^2$

(ii) Time taken to cover first s metres $s = \frac{\alpha^2 t^2}{4} \Rightarrow t^2 = \frac{4s}{\alpha^2} \Rightarrow t = \frac{2\sqrt{s}}{\alpha}$;

$$\text{average velocity} = \frac{\text{total displacement}}{\text{total time}} = \frac{s\alpha}{2\sqrt{s}} = \frac{1}{2}\sqrt{s}\alpha$$

Ex. A particle moves in the x–y plane according to the law x = at; y = at(1–αt) where a and α are positive constants and t is time. Find the velocity and acceleration vector. The moment t_0 at which the velocity vector forms angle of 90° with acceleration vector.

Sol. $V_x = a;\ V_y = a - 2a\alpha t \Rightarrow \vec{V} = a\hat{i} + (a - 2a\alpha t)\hat{j}$

$a_x = 0;\ a_y = -2a\alpha \Rightarrow \vec{a} = -2a\alpha\hat{j}$

for 90°, $\vec{V}\cdot\vec{a} = 0$

$-2a\alpha(a - 2a\alpha t) = 0$

$1 - 2\alpha t = 0 \Rightarrow t = 1/(2\alpha)$ sec.

Equations of motion (motion with constant acceleration)

If a particle moves with acceleration $\vec{a}$, then by definition $\vec{a} = \frac{d\vec{v}}{dt} \Rightarrow d\vec{v} = \vec{a}dt$. Let at starting (t = 0)

initial velocity of the particle $\vec{u}$ and at time t its final velocity = $\vec{v}$ then $\int_{\vec{u}}^{\vec{v}} d\vec{v} = \int_0^t \vec{a}dt$

If acceleration is constant

$$\int_{\vec{u}}^{\vec{v}} d\vec{v} = \vec{a}\int_0^t dt \Rightarrow \left[\vec{v}\right]_{\vec{u}}^{\vec{v}} = \vec{a}\left[t\right]_0^t \Rightarrow \vec{v} - \vec{u} = \vec{a}\,t \Rightarrow \vec{v} = \vec{u} + \vec{a}t \quad \text{.....(1)}$$

Now by definition of velocity, equation (1) reduces to

$$\vec{v} = \frac{d\vec{s}}{dt} = \vec{u} + \vec{a}t \Rightarrow \int_0^{\vec{s}} d\vec{s} = \int_0^t \left(\vec{u} + \vec{a}t\right)dt \Rightarrow \vec{s} = \left[\vec{u}t + \frac{1}{2}\vec{a}t^2\right]_0^t \Rightarrow \vec{s} = \vec{u}t + \frac{1}{2}\vec{a}t^2 \quad \text{.....(2)}$$

Now substituting the value of t from equation (1) to equation (2)

$$s = u\,\frac{(v-u)}{a} + \frac{1}{2}a\left[\frac{v-u}{a}\right]^2 \Rightarrow 2as = 2uv - 2u^2 + v^2 + u^2 - 2uv \Rightarrow v^2 = u^2 + 2as \text{.....(iii)}$$

vector form of equation (iii) $\boxed{v^2 = u^2 + 2\vec{a}.\vec{s}}$ .....(3)

These three equation are called equations of motion and are applicable **only and only when acceleration is constant.**

Distance travalled by the body in n^{th} second

$$s_{n^{th}} = s_n - s_{n-1} = un + \frac{1}{2}an^2 - u(n-1) - \frac{1}{2}a(n-1)^2 = un + \frac{1}{2}an^2 - un + u - \frac{1}{2}an^2 + an - \frac{a}{2}$$

vector form of equation (iv)

$$\boxed{s_{n^{th}} = u + \frac{a}{2}(2n-1)} \quad \text{...(4)}$$

Ex. A passenger is standing d distance away from a bus. The bus begins to move with constant acceleration a. To catch the bus, the passenger runs at a constant speed u towards the bus. What must be the minimum speed of the passenger so that he may catch the bus?

Sol. Let the passenger catch the bus after time t.

The distance travelled by the bus, $s_1 = 0 + \frac{1}{2}at^2$ ...(1)

and the distance travelled by the passenger $s_2 = ut + 0$ ...(2)

Now the passenger will catch the bus if $d + s_1 = s_2$ ...(3)

$$\Rightarrow d + \frac{1}{2}at^2 = ut \Rightarrow \frac{1}{2}at^2 - ut + d = 0 \Rightarrow t = \frac{[u \pm \sqrt{u^2 - 2ad}]}{a}$$

So the passenger will catch the bus if t is real, i.e., $u^2 \geq 2ad \Rightarrow u \geq \sqrt{2ad}$

So the minimum speed of passenger for catching the bus is $\sqrt{2ad}$.

Vertical motion under gravity (Example of constant acceleration)

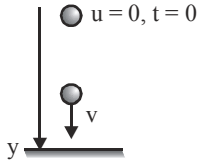
If air resistance is neglected and a body is freely moving along vertical line near the earth surface then an acceleration downward which is 9.8m/s^2 or 980 cm/s^2 or 32 ft/s^2 is experienced by the body.

Freely falling bodies from a height h above the ground

Taking initial position as origin and direction of motion (i.e. downward direction) positive y axis, as body is just released/dropped u = 0

acceleration along +Y axis a = g

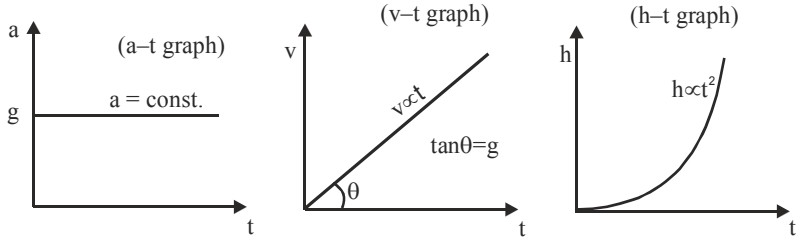
Use equations of motion to describe the motion, i.e.



$$v = u + at,\ y = ut + \frac{1}{2}at^2,\ v^2 = u^2 + 2ay$$

Let the body acquires velocity v(downward) after falling a distance h in time t, then

$$v = gt \Rightarrow t = v/g \because h = \frac{1}{2}gt^2 \Rightarrow t = \sqrt{\frac{2h}{g}},\ v^2 = 2gh \Rightarrow v = \sqrt{2gh}$$



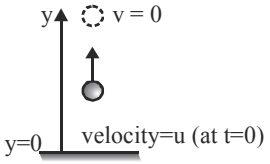
Body is projected vertically upward : With velocity u take initial position as origin and direction of motion

(i.e. vertically upward) as positive y–axis.

v = 0 at maximum height, at t = T,

a = – g (because directed downward)

Put the values in equation of motion



$$v = u + at \Rightarrow 0 = u - gT \Rightarrow u = gT$$

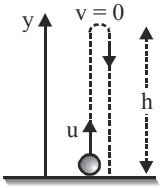
$$s = ut + \frac{1}{2}at^2 \Rightarrow h = ut - \frac{1}{2}gt^2$$

$$\Rightarrow\ h_{max} = uT - \frac{1}{2}gT^2 \Rightarrow h_{max} = (gT)T - \frac{1}{2}gT^2 = \frac{1}{2}gT^2$$

$$v^2 = u^2 + 2as \Rightarrow v^2 = u^2 - 2gh \Rightarrow 0 = u^2 - 2g\,h_{max} \Rightarrow u^2 = 2g\,h_{max} \Rightarrow u = \sqrt{2g\,h_{max}}$$

After attaining maximum height body turns and come back at ground. During complete flight acceleration is constant,

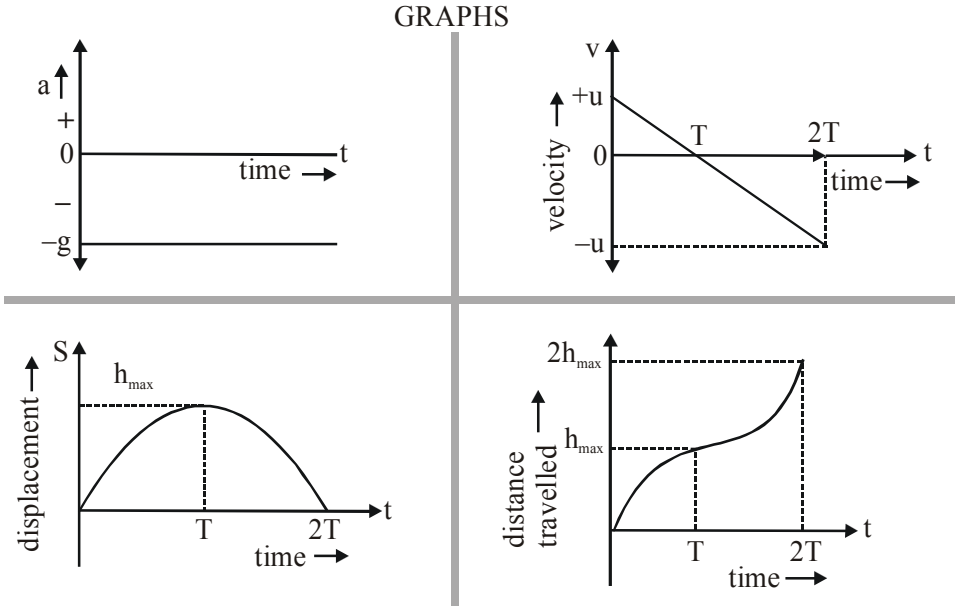
Time taken during up flight and down flight are equal



Time for one side $T = \frac{u}{g}$ and total flight time $= 2T = \frac{2u}{g}$

At each equal height from ground speed of body will be same either going up or coming down.

SOME RELATED GRAPHS FOR ABOVE MOTION'S



Ex. At the height of 500m, a particle A is thrown up with v = 75 ms^{-1} and particle B is released from rest. Draw, accelearation –time, velocity–time, speed–time and displacement–time graph of each particle.

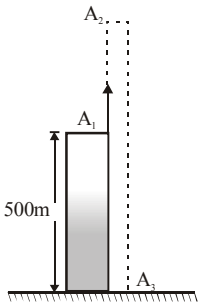
For particle A :

Time of flight

$$-500 = +75\,t - \frac{1}{2} \times 10t^2$$

$\Rightarrow t^2 - 15t - 100 = 0$

$\Rightarrow t = 20$ s



Time taken for A_1A_2

$= 75 - 10t \Rightarrow t = 7.5$ s

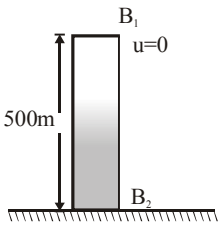
Velocity at A_3, $v = 75 - 10 \times 20 = -125$ ms^{-1}

Height $A_2A_1 = \frac{1}{2}(10)(7.5)^2 = 281.25$ m

For Particle B

Time of flight

$$500 = \frac{1}{2}(10)t^2 \Rightarrow t = 10 \text{ s}$$



Velocity at B_2

$v = 0 - (10)(10) = -100$ ms^{-1}

Ex. A rocket is fired vertically up from the ground with a resultant vertical acceleration of $10m/s^2$. The fuel is finished in 1 minute and it continues to move up.

(a) What is the maximum height reached?

(b) After finishing fuel, calculate the time for which it continues its upwards motion. (Take g = 10 m/s^2)

Sol. (a) The distance travelled by the rocket during burning interval (1minute= 60s) in which resultant acceleration 10 m/s^2 is vertically upwards will be $h_1 = 0 \times 60 + (1/2) \times 10 \times 60^2 = 18000$ m = 18 km and velocity acquired by it will be v = 0 + 10 × 60 = 600 m/s

Now after 1 minute the rocket moves vertically up with initial velocity of 600 m/s and acceleration due to gravity opposes its motion. So, it will go to a height h_2 from this point, till its velocity becomes zero such that

$0 = (600)^2 - 2gh_2 \Rightarrow h_2 = 18000$ m = 18 km [g = 10 ms^{-2}]

So the maximum height reached by the rocket from the ground, $H = h_1 + h_2 = 18 + 18 = 36$ km

(b) As after burning of fuel the initial velocity 600m/s and gravity opposes the motion of rocket, so from 1st equation of motion time taken by it till it velocity v =0

$0 = 600 - gt \Rightarrow t = 60$ s

Ex. If a body travels half its total path in the last second of its fall from rest, find :

(a) The time and

(b) height of its fall. Explain the physically unacceptable solution of the quadratic time equation. (g = 9.8 m/s^2)

Sol. If the body falls a height h in time t, then

$$h = \frac{1}{2} gt^2 \text{ [u = 0 as the body starts from rest]} \quad \text{... (1)}$$

Now, as the distance covered in (t – 1) second is $h' = \frac{1}{2} g(t-1)^2$ ... (2)

So from Equations (1) and (2) distance travelled in the last second.

$$h - h' = \frac{1}{2} gt^2 - \frac{1}{2} g(t-1)^2 \text{ i.e., } h - h' = \frac{1}{2} g\,(2t - 1)$$

But according to given problem as $(h - h') = \frac{h}{2}$

$$\text{i.e., } \left(\frac{1}{2}\right)h = \left(\frac{1}{2}\right)g(2t-1) \text{ or } \left(\frac{1}{2}\right)gt^2 = g(2t-1) \quad \text{[as from equation (1) } h = \left(\frac{1}{2}\right)gt^2\text{]}$$

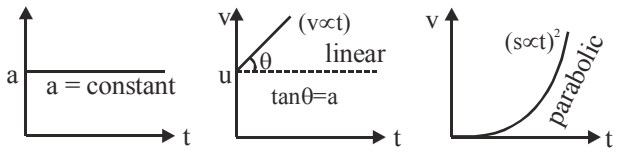
or $t^2 - 4t + 2 = 0$ or $t = [4 \pm \sqrt{(4^2 - 4 \times 2)}/2$ or $t = 2 \pm \sqrt{2} \Rightarrow t = 0.59$ s or 3.41 s

0.59 s is physically unacceptable as it gives the total time t taken by the body to reach ground lesser than one sec while according to the given problem time of motion must be greater than 1s.

so t = 3.41s and $h = \frac{1}{2} \times (9.8) \times (3.41)^2 = 57$ m

Graphs based on 1–D

For constant acceleration, a/t, v/t and s/t curve from equations of motion are –



In case of constant acceleration motion in a straight line, scalar form of equations of motion can be applied and problem becomes fairly simple.

As $d\vec{v} = \vec{a}dt$ or $\left[\vec{v}\right]_{\vec{u}}^{\vec{v}} = \vec{v} - \vec{u} = \int_{t_1}^{t_2} \vec{a}dt$ = Area between curve and time axis from t_1 to t_2.

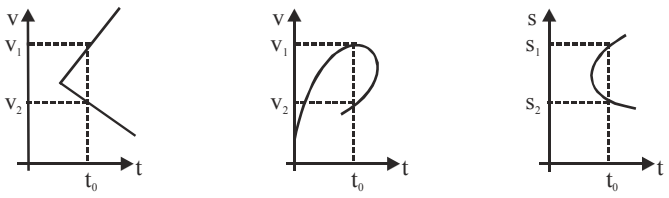
Area under the curve of a – t graph always gives the change in velocity.

Similarly $d\vec{s} = \int \vec{v}dt$ or $\vec{s} = \int_{t_1}^{t_2} \vec{v}dt$ = Area between curve and time axis from t_1 to t_2.

Here $\vec{s}$ is the displacement of particle in time interval t_1 to t_2, i.e. area under the curve of v/t graph always gives the displacement. If only magnitude of area is taken into account then sum of all area is the total distance travelled by the particle.

- Slopes of v–t or s–t graphs can never be infinite at any point, because infinite slope of v–t graph means infinite acceleration. Similarly, infinite slope of s–t graph means infinite velocity. Hence, the following graphs are not possible.

- At one time, two values of velocity or displacement are not possible Hence, the following graphs are not acceptable.



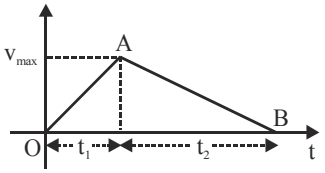
- The slope of velocity–time graph of uniform motion is zero.
- When a body is having uniform motion along a straight line in a given direction, the magnitude of the displacement of body is equal to the actual distance travelled by the body in the given time.
- The average and instantaneous velocity in a uniform motion are equal in magnitude.
- In a uniform motion along a straight line, the slope of position–time graph gives the velocity of the body.
- The position–time graph of a body moving along a straight line can never be a straight line parallel to position axis because it will indicate infinite velocity.
- The speed of a body can never be negative
- Medium effects the motion of a body falling freely under gravity due to thrust and viscous drag.

Ex. A car accelerates from rest at a constant rate α for some time, after which it decelerates at a constant rate β, to come to rest. If the total time elapsed is t evaluate (a) the maximum velocity attained and (b) the total distance travelled.

Sol. (a) Let the car accelerates for time t_1 and decelerates for time t_2 then

$$t = t_1 + t_2 \qquad ...(i)$$

and corresponding velocity–time graph will be as shown in. fig.



From the graph α = slope of line AB = $\frac{v_{max}}{t_1} \Rightarrow t_1 = \frac{v_{max}}{\alpha}$

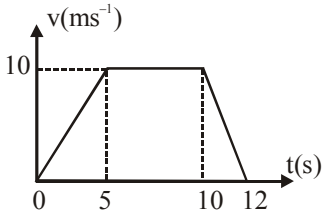
and β = – slope of line OB = $\frac{v_{max}}{t_2} \Rightarrow t_2 = \frac{v_{max}}{\beta}$

$$\Rightarrow \frac{v_{max}}{\alpha} + \frac{v_{max}}{\beta} = t \Rightarrow v_{max}\left(\frac{\alpha+\beta}{\alpha\beta}\right) = t \Rightarrow v_{max} = \frac{\alpha\beta t}{\alpha+\beta}$$

(b) Total distance = area under v–t graph = $\frac{1}{2} \times t \times v_{max} = \frac{1}{2} \times t \times \frac{\alpha\beta t}{\alpha+\beta} = \frac{1}{2}\left(\frac{\alpha\beta t^2}{\alpha+\beta}\right)$

Note: This problem can also be solved by using equations of motion (v = u + at, etc.).

Ex. Draw displacement time and acceleration – time graph for the given velocity–time graph



Sol. For $0 \le t \le 5 v \propto t \Rightarrow s \propto t^2$ and a_1 = constant $\frac{10}{5} = 2\ ms^{-2}$

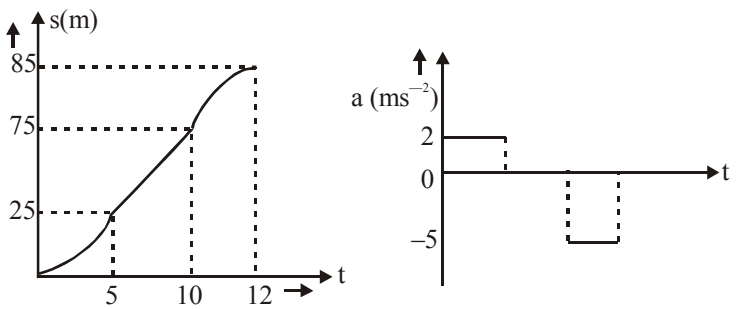
for whole interval s_1 = Area under the curve = $\frac{1}{2} \times 5 \times 10 = 25$ m

For $5 \le t \le 10$ $\quad v = 10 ms^{-1}$ $\quad \Rightarrow a = 0$

for whole interval s_2 = Area under the curve = $\frac{1}{2} \times 5 \times 10 = 50$ m

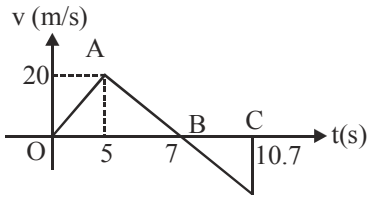
For $10 \le t \le 12$ v linearly decreases with time $\Rightarrow a_3 = -\frac{10}{2} = -5\ ms^{-1}$

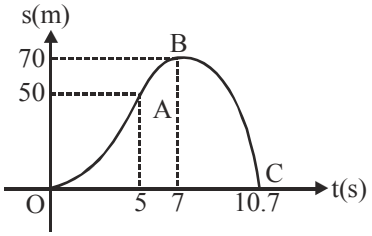
for whole interval s_3 = Area under the curve = $\frac{1}{2} \times 2 \times 10 = 10$ m



Ex. A rocket is fired upwards vertically with a net acceleration of 4 m/s^2 and initial velocity zero. After 5 seconds its fuel is finished and it decelerates with g. At the highest point its velocity becomes zero. Then it accelerates downwards with acceleration g and return back to ground. Plot velocity–time and displacement–time graphs for the complete journey. Take g = 10 m/s^2.

Sol.





In the graphs, $v_A = at_{OA} = (4)(5) = 20$ m/s $\quad v_B = 0 = v_A - gt_{AB}$

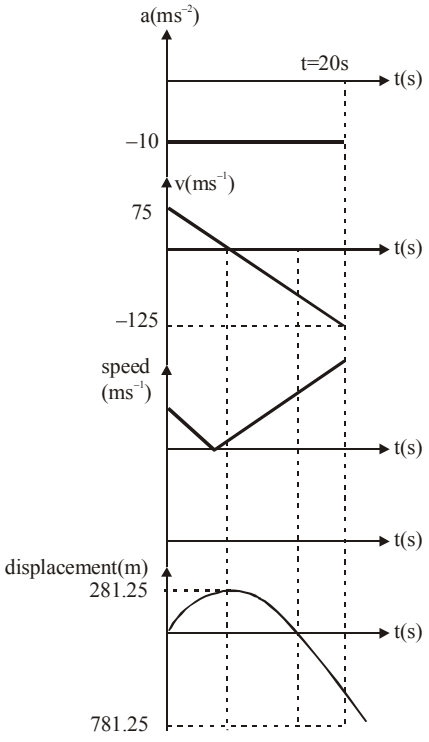
$\therefore t_{AB} = \frac{v_A}{g} = \frac{20}{10} = 2s$ $\quad \therefore t_{OAB} = (5 + 2)s = 7s$

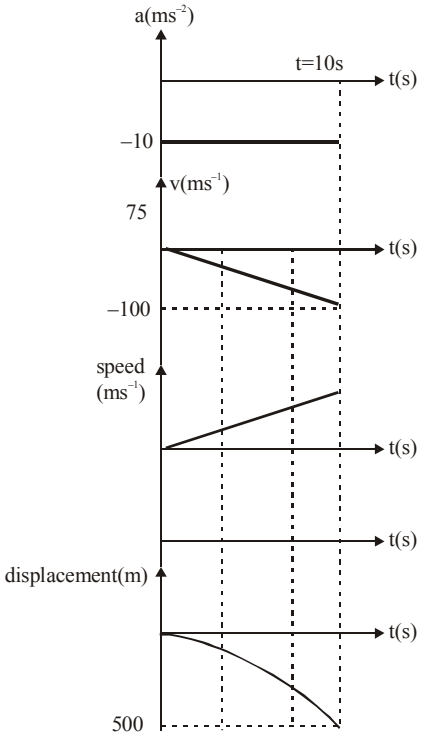
Now, s_{OAB} = area under v–t graph between 0 to 7 s = $\frac{1}{2}(7)(20) = 70$ m

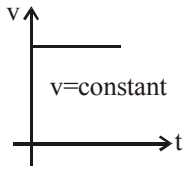
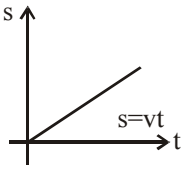
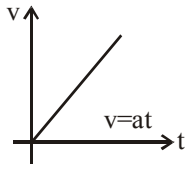
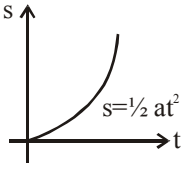
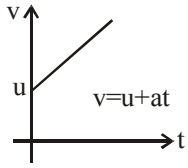
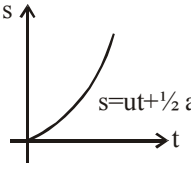
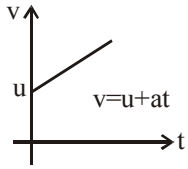
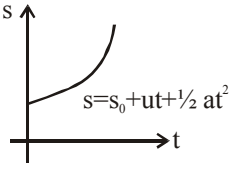
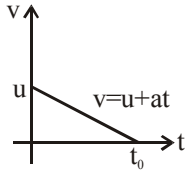
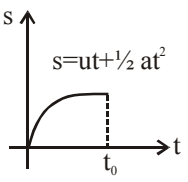
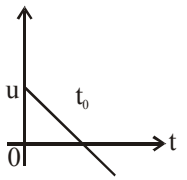
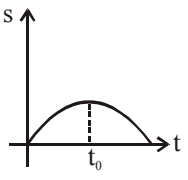
Now, $s_{OAB} = s_{BC} = \frac{1}{2} gt^2_{BC}$ $\quad \therefore 70 = \frac{1}{2}(10)\, t^2_{BC}$

$\therefore \quad t_{BC} = \sqrt{14} = 3.7s$ $\quad \therefore t_{OAB} = 7 + 3.7 = 10.7s$

Also s_{OA} = area under v–t graph between OA = $\frac{1}{2}(5)(20) = 50$ m





S.N.	Different Cases	v–t graph	s–t graph	Important Points
1.	Uniform motion			(i) Slope of s–t graph = v = constant (ii) In s–t graph s = 0 at t = 0
2.	Uniformly accelerated motion with u = 0 at t = 0			(i) u = 0, i.e. v = 0 at t = 0 (ii) u = 0, i.e., slope of s–t graph at t = u, should be zero (iii) a or slope of v – t graph is constant
3.	Uniformly accelerated with u ≠ 0 at t = 0			(i) u ≠ 0, i.e., v or slope motion of s – t graph at t = 0 is not zero (ii) v or slope of s – t graph gradually goes on increasing.
4.	Uniformly accelerated motion with u ≠ 0 and s = s_0 at t = 0			(i) s = s_0 at t = 0
5.	Uniformly retarded motion till velocity becomes zero			(i) Slope of s – t graph at t = 0 gives u (ii) Slope of s – t graph at t = t_0 becomes zero (iii) In this case u can't be zero.
6.	Uniformly retarded then accelerated in opposite direction			(i) At time t = t_o, v = 0 or slope of s – t graph is zero (ii) In s – t graph slope or velocity first decreases then increases with opposite sign.

PROJECTILE MOTION

We next consider a special case of two-dimensional motion: A particle moves in a vertical plane with some initial velocity $\vec{v}_0$ but its acceleration is always the freefall acceleration $\vec{g}$, which is downward. Such a particle is called a projectile (meaning that it is projected or launched) and its motion is called **projectile motion**.

Assumptions:–

Particle remains close to earth's surface, so acceleration due to gravity remains constant.

Air resistance is neglected.

Distance that projectile travels is small so that earth can be treated as plane surface.

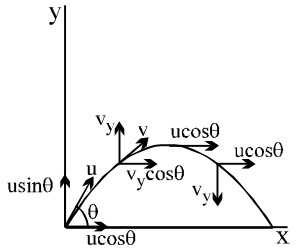
Two straight line motions:–

Our goal here is to analyse projectile motion using the tools for two dimensional motion. This feature allows us to break up a problem involving two dimensional motion into two separate and easier one-dimensional problems,

(a) The horizontal motion is motion with uniform velocity (no effect of gravity)

(b) The vertical motion is motion of uniform acceleration, or freely falling bodies.

Note: In projectile motion, the horizontal motion and the vertical motion are independent of each other, that is either motion does not affects the other.



Treating as two straight line motions:–

The horizontal Motion(x axis):

Because there is no accelration in the horizontal direction, the horizontal component v_x of the projectile's velocity remains unchanged from its initial value v_{0x} throughout the motion,

The vertical motion(y axis):

The vertical motion is the motion we discussed for a particle in free fall.

As is illustrated in figure and equation (1.3), the vertical component behaves just as for a ball thrown vertically upward. It is directed upward initially and its magnitude steadily decreasing to zero, which marks the maximum height of the path. The vertical velocity component then reverses direction, and its magnitude becomes larger with time.

x-axis	**y-axis**
Initial velocity(u_x) = ucosθ	Initial velocity(u_y) = usinθ
acceleration(a_x) = 0	acceleration(a_y) = –g
Thus, velocity after time t	Thus, velocity after time t
v_x = ucosθ	v_y = usinθ – gt
Displacement after time t	Displacement after time t
x = ucosθt	y = usinθt – $gt^2/2$

Resultant velocity

$$(\vec{V}_R) = (u\cos\theta)\hat{i} + (u\sin\theta - gt)\hat{j}$$

$$|\vec{V}_R| = \sqrt{u^2\cos^2\theta + (u\sin\theta - gt)^2}$$

& $\tan\alpha = \dfrac{u\sin\theta - gt}{u\cos\theta}$

where α is angle that velocity vector makes with horizontal. Also known as direction or angle of motion

Time of flight(T)

$$T = \frac{2u\sin\theta}{g}$$

Considering vertical motion

$s_y = 0; u_y = v\sin\theta; a_y = -g$

$0 = u\sin\theta T - gT^2/2 \Rightarrow T = \dfrac{2u\sin\theta}{g}$

Maximum Height(H)

$$H = \frac{u^2\sin^2\theta}{2g}$$

Vertical velocity at maximum height $v_y = 0$

$0 = u^2\sin^2\theta - 2gH \Rightarrow H = \dfrac{u^2\sin^2\theta}{2g}$

Horizontal Range(R)

$$R = \frac{u^2\sin 2\theta}{g} = \frac{2u_x u_y}{g}$$

$$\text{Total time } T = \frac{2u\sin\theta}{g}$$

Velocity in horizontal direction $u_x = u\cos\theta$

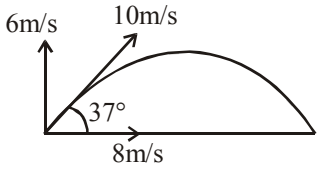
Total displacement in horizontal direction $R = u\cos\theta T$

$$R = \frac{u^2\sin 2\theta}{g}$$

Ex. A body is thrown with initial velocity 10m/sec. at an angle 37° from horizontal. Find
(i) Time of flight (ii) Maximum height.
(iii) Range (iv) Position vector after t = 1 sec.

Ans. (i) 1.2 sec, (ii) 1.8 m, (iii) 9.6 m, (iv) $(16\hat{i} - 8\hat{j}) - (8\hat{i} + \hat{j})$

Sol. (i) Time of flight $T = \frac{2u\sin\theta}{g} = \frac{2\times10\times\frac{3}{5}}{10} = \frac{6}{5} = 1.2$ sec



(ii) Maximum height $H = \frac{u^2\sin^2\theta}{2g} = \frac{100\left(\frac{9}{25}\right)}{2\times10} = \frac{9}{5} = 1.8$ m

(iii) $R = \frac{u^2\sin2\theta}{g} = \frac{100\times2\times\frac{3}{5}\times\frac{4}{5}}{10} = \frac{240}{25} = 9.6$m

(iv) $x = 8\times1 = 8$ m

$y = 6\times1 - \frac{1}{2}\times10\times(1)^2 = 1$m

$\vec{r} = 8\hat{i} + \hat{j}$

Caution: This equation does not given the horizontal distance travelled by a projectile when the final height is not the launch height.

Maximum Range

$R = \frac{u^2\sin2\theta}{g}$

for $\theta = 45°$, R is maximum

$R_{max} = \frac{u^2}{g}$

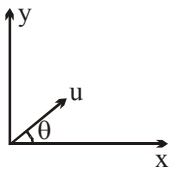
Note:– *For complementry angles i.e. $\theta + \alpha = 90°$, the range is same for same projection speed but maximum height and time of flight are different.*

EQUATION OF TRAJECTORY

Lets say point of projection is our origin and horizontal direction is x-axis and vertically upwards is positive y-axis.

positive y-axis.

We know $x = u\cos\theta t$



$\therefore \quad t = \frac{x}{u\cos\theta}$ .....(1)

also $y = u\sin\theta t - \frac{1}{2}gt^2$ .....(2)

Putting value of 't' from eq. (1) in eq. (2), we get

$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

Ex. A particle is projected with a velocity 10 m/s at an angle 37° to the horizontal. Find the location at which the particle is at a height 1m from point of projection.

Ans. 1.6 m, 8 m.

Sol. $y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$

for $y = 1$; $\theta = 37°$; $u = 10$ m/s

$$1 = \frac{3}{4}x - \frac{10x^2}{2\times100\times\left(\frac{16}{25}\right)}$$

$$1 = \frac{3}{4}x - \frac{5}{64}x^2$$

$5x^2 - 48x + 64 = 0$

$5x^2 - 40x - 8x + 64 = 0$

$x = 8m, 1.6m$

Ex. We have a hose pipe which disposes water at the speed of 10 ms^{-1}. The safe distance from a building on fire, on ground is 5 m. How high can this water go? (take : $g = 10\ ms^{-2}$)

Sol. Here we must understand that taking range of projectile as 10m and making projectile hit the building when it is at maximum height is wrong. By doing this we are not acheiving maximum y for given x = 5m. This just makes highest pt. of path to like on x = 5, But there may be other path for which y will be maximum for given x. This problem will be solved by using equation of trajectory by putting x = 5m and maximising y by varying θ.

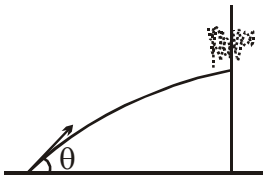
$$y = x\tan\theta - \frac{gx^2}{2u^2\cos^2\theta}$$

Putting we get x = 5m

$$y = 5\tan\theta - \frac{10\times25\sec^2\theta}{2\times100}$$

$5\tan^2\theta - 20\tan\theta + (4y + 5) = 0$

for real roots discriminant must be positive.



$400 - 4 \times 5\,(4y + 5) > 0$

Solving $3.75 \geq y$

hence maximum y = 3.75 m

If we have taken range as 10 m then angle of projection will be $\theta = 45°$ corsponding maximum hight H = 2.5m which is smaller than our answer.

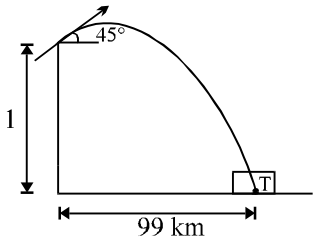
Ex. Prithvi missile is fired to destroy an enemy military base situated on same horizontal level, situated 99 km away. The missile rises vertically for 1 km & then for remainder of flight, it followes parbolic path like a free body under earth's gravity, at an angle of 45°. Calculate its velocity at beginning of parabolic path. ($g = 10\ ms^{-2}$)

Sol. for horizontal motion time t

$$t = \frac{99\times10^3}{u\cos45°}$$

for vertical

$$-1\times10^3 = u\sin45^\circ\, t - \frac{1}{2}\times10\times t^2$$



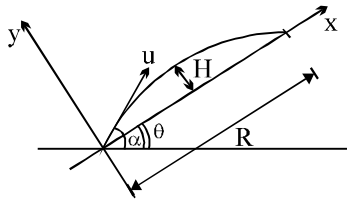
$$1\times10^3 + \frac{u\sin45^\circ}{u\sin45^\circ}\times99\times10^3 = \frac{10}{2}\times\frac{(99\times10^3)^2\times2}{u^2}$$

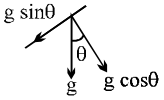
$$u^2 = \frac{(99\times10^3)^2\times10}{100\times10^3}$$

$$u = 99\times10^3\sqrt{\frac{1}{10^4}} = 990\ ms^{-1}$$

PROJECTION ON INCLINED PLANE

There is an inclined plane making an angle θ with horizontal. A particle is projected at an angle α from horizontal.





x-axis

$u_x = u\cos(\alpha-\theta)$

$a_x = -g\sin\theta$

vel. at any time t

$v_x = u\cos(\alpha-\theta) - g\sin\theta\, t$

y-axis

$u_y = u\sin(\alpha-\theta)$

$a_y = -g\cos\theta$

vel. at any time t

$v_y = u\sin(\alpha-\theta) - g\cos\theta\, t$

Time of flight

Displacement in y direction $s_y = 0$

$$0 = u\sin(a-\theta)\,T - \frac{1}{2}g\cos\theta\,T^2$$

$$T = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

Maximum distance of particle from inclined plane

Pt. where $v_y = 0$ is max. height

$(0)^2 = u^2\sin^2(\alpha-\theta) - 2g\cos\theta\, H$

$$H = \frac{u^2\sin^2(\alpha-\theta)}{2g\cos\theta}$$

Range along the inclined plane

$$s_x = u_xT + \frac{1}{2}a_xT^2$$

$$R = \frac{u\cos(\alpha-\theta)\times2u\sin(\alpha-\theta)}{g\cos\theta} - \frac{2\sin\theta\times2\times2u^2\sin^2(\alpha-\theta)}{2g^2\cos^2\theta}$$

$$= \frac{2u^2 \sin(\alpha-\theta)[\cos(\alpha-\theta)\cos\theta - \sin\theta\sin(\alpha-\theta)]}{g\cos^2\theta}$$

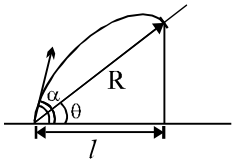
$$R = \frac{2u^2 \sin(\alpha-\theta)\cos\alpha}{g\cos^2\theta}$$

$$R = \frac{u^2[\sin(2\alpha-\theta)-\sin\theta]}{g\cos^2\theta}$$

Alternate Method

$l = u\cos\alpha\, T$

$$R = \frac{l}{\cos\theta}$$



$$R = \frac{u\cos\alpha}{\cos\theta} \times \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

$$R = \frac{2u^2 \sin(\alpha-\theta)\cos\alpha}{g\cos^2\theta}$$

Note : Presence of incline plane does not affect the path of projectile in any way.

Maximum Range :

$$R = \frac{u^2[\sin(2\alpha-\theta)-\sin\theta]}{g\cos^2\theta}$$

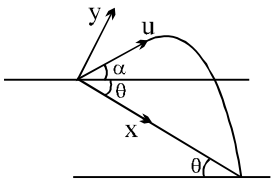
For max. range $2\alpha - \theta = \frac{\pi}{2} \Rightarrow \alpha = \frac{\pi}{4} + \frac{\theta}{2}$

so $R_{max} = \frac{u^2}{g(1+\sin\theta)}$

Projection from top of incline plane:

Incline plane is at an angle θ with horizontal and a particle is projected at an angle α from horizontal.

In all formulae replace θ with –θ



$$\mu = \frac{u^2 \sin^2(\alpha+\theta)}{2g\cos\theta}$$

$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\theta}$$

$$R = \frac{2u^2 \sin(\alpha+\theta)\cos\alpha}{g\cos^2\theta}$$

$$R_{max} = \frac{u^2}{g(1-\sin\theta)} \text{ and } \alpha = \frac{\pi}{4} - \frac{\theta}{2}$$

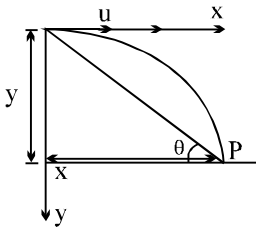
Note : If a particle strikes the incline plane ⊥ then its comp. of velocity along incline must be zero.

Ex. A particle is projected horizontally with a speed u from the top of a plane inclined at an angle θ with the horizontal. How far along the plane, from the point of projection will particle strike the plane?

Sol.

x-axis	**y-axis**
$u_x = u$	$u_x = 0$
$a_x = 0$	$a_x = g$
$x = ut$	$y = \frac{gt^2}{2}$

$$\Rightarrow \quad y = \frac{g x^2}{2u^2}$$



$$\text{also } \frac{y}{x} = \tan\theta \Rightarrow x\tan\theta = \frac{g x^2}{2u^2}$$

$$x = 0, \ \frac{2u^2\tan\theta}{g}$$

$$x = \frac{2u^2\tan\theta}{g} \Rightarrow y = \frac{2u^2\tan^2\theta}{g}$$

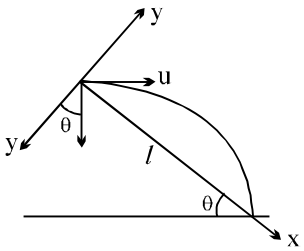
$$\text{dist. } l = \sqrt{x^2 + y^2}$$

$$l = \frac{2u^2\tan\theta\sec\theta}{g}$$

Alternate Method.

$$R = \frac{2u^2\sin(\alpha+\theta)\cos\alpha}{g\cos^2\theta}$$

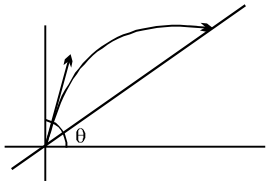
$$R = 2u^2\tan\theta\sec\theta$$



Ex. A particle is projected up an inclined plane. Plane is inclined at an angle θ with horizontal and particle is projected at an angle α with horizontal. If particle strikes the plane horizontally prove that $\tan\alpha = 2\tan\theta$

Sol. We know time of flight

$$T = \frac{2u\sin(\alpha+\theta)}{g\cos\theta}$$



considering vertical motion

$u = v\sin\alpha$

$a = -g$

$v = 0$

$$\therefore \ T = \frac{u\sin\alpha}{g} = \frac{2u\sin(\alpha-\theta)}{g\cos\theta}$$

$\sin\alpha\cos\theta = 2\sin\alpha\cos\theta - 2\cos\alpha\sin\theta$

$2\cos\alpha\sin\theta = \sin\alpha\cos\theta$

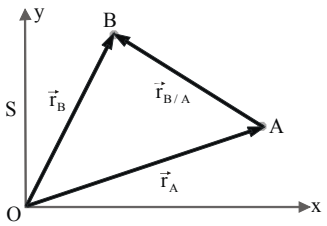
$2\tan\theta = \tan\alpha$

Relative Motion

Motion of a body can only be observed, when it changes its position with respect to some other body. In this sense, motion is a relative concept. To analyze motion of a body say A, therefore we have to fix our reference frame to some other body say B. The result obtained is motion of body A relative to body B.

Relative position, Relative Velocity and Relative Acceleration

Let two bodies represented by particles A and B at positions defined by position vectors $\vec{r}_A$ and $\vec{r}_B$, moving with velocities $\vec{v}_A$ and $\vec{v}_B$ and accelerations $\vec{a}_A$ and $\vec{a}_B$ with respect to a reference frame S. For analyzing motion of terrestrial bodies the reference frame S is fixed with the ground.



The vectors $\vec{r}_{B/A}$ denotes position vector of B relative to A. Following triangle law of vector addition, we have

$$\vec{r}_B = \vec{r}_A + \vec{r}_{B/A} \qquad ...(i)$$

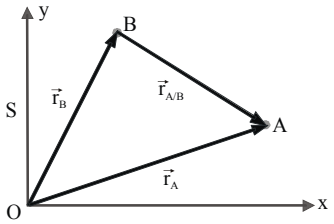
First derivatives of $\vec{r}_A$ and $\vec{r}_B$ with respect to time equals to velocity of particle A and velocity of particle B relative to frame S and first derivative of $\vec{r}_{B/A}$ with respect to time defines velocity of B relative to A.

$$\vec{v}_B = \vec{v}_A + \vec{v}_{B/A} \qquad ...(ii)$$

Second derivatives of $\vec{r}_A$ and $\vec{r}_B$ with respect to time equals to acceleration of particle A and acceleration of particle B relative to frame S and second derivative of $\vec{r}_{B/A}$ with respect to time defines acceleration of B relative to A.

$$\vec{a}_B = \vec{a}_A + \vec{a}_{B/A} \qquad ...(iii)$$

In similar fashion motion of particle A relative to particle B can be analyzed with the help of adjoining figure. You can observe in the figure that position vector of A relative to B is directed from B to A and therefore



$$\vec{r}_{B/A} = -\vec{r}_{A/B},\ \vec{v}_{B/A} = -\vec{v}_{A/B} \text{ and } \vec{a}_{B/A} = -\vec{a}_{A/B}.$$

The above equations elucidate that how a body A appears moving to another body B is opposite to how body B appears moving to body A.

SOLVED EXAMPLE BASED ON RELATIVE MOTION

Ex. Man A sitting in a car moving at 54 km/hr observes a man B in front of the car crossing perpendicularly the road of width 15 m in three seconds. Then the velocity of man B will be

(A) $5\sqrt{10}$ towards the car (B) $5\sqrt{10}$ away from the car

(C) 5 m/s perpendicular to the road (D) None

Ans. (B)

Sol. $D = vt$

$$\frac{15}{3} = v$$

$$5 = v$$

$v_{B/A} = 5$

$v_{A/g} = 15$

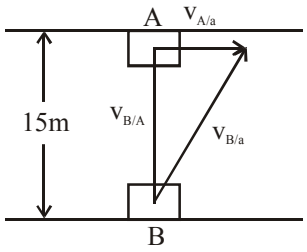
$(v_{B/g})^2 = (v_{B/A})^2 + (V_{A/g})^2$

$= \sqrt{(15)^2 + 5^2}$

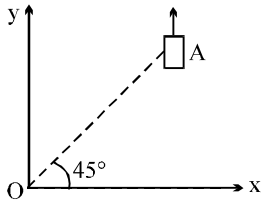
$= \sqrt{225+25}$

$= \sqrt{250}$

$= 5\sqrt{10}$ away from car



Ex. On a frictionless horizontal surface, assumed to be the x-y plane, a small trolley. A is moving along a straight line parallel to the y-axis (see figure) with a constant velocity of $(\sqrt{3}-1)$ m/s. At a particular instant, when the line OA makes an angle of 45° with the x-axis, a ball is thrown along the surface from the origin O. Its velocity makes an angle ϕ with the x-axis and it hits the trolley.



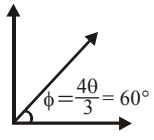
(a) The motion of the ball is observed from the frame of trolley. Calculate the angle θ made by the velocity vector of the ball with the x-axis in this frame.

(b) Find the speed of the ball with respect to the surface, if $\phi = \frac{4\theta}{3}$.

Ans. (a) 45°, (b) 2 m/sec

Sol. From the frame of trolley ball will appear to move along the line joining origin and trolley

$\therefore \quad \theta = 45°$



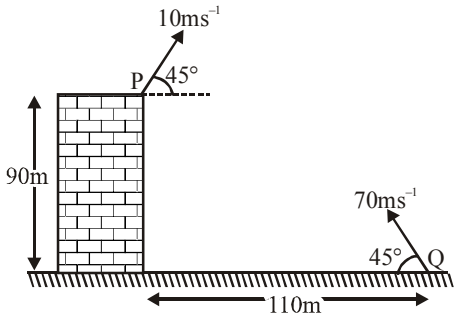
$u_x = v \cos 60$

$u_y = v \sin 60$

$(u_y - v_T)\, t = v_x t$

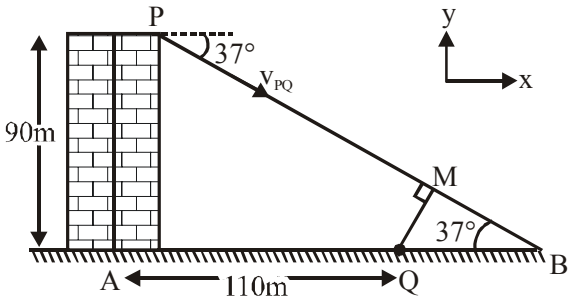
$u_y - v_x = u_T$

Ex. Two particles P and Q are launched simultaneously as shown in figure. Find the minimum distance between particles in meters.



Ans. 6

Sol.



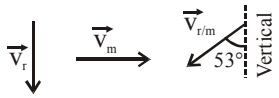
$$\vec{v}_{PQ} = \vec{v}_P - \vec{v}_Q = \left(\frac{10}{\sqrt{2}}\hat{i} + \frac{10}{\sqrt{2}}\hat{j}\right) - \left(-\frac{70}{\sqrt{2}}\hat{i} + \frac{70}{\sqrt{2}}\hat{j}\right) = 40\sqrt{2}\,\hat{i} - 30\sqrt{2}\,\hat{j}$$

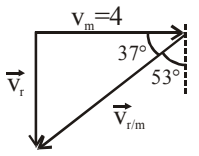
$$AB = 90 \times \frac{4}{3} = 120\text{m} \Rightarrow QB = 10\text{m} \Rightarrow QM = 10\sin 37° = 10 \times \frac{3}{5} = 6\text{m}$$

Ex. A man when standstill observes the rain falling vertically and when he walks at 4 km/h he has to hold his umbrella at an angle of 53° from the vertical. Find velocity of the raindrops.

Sol. Assigning usual symbols $\vec{v}_m$, $\vec{v}_r$ and $\vec{v}_{r/m}$ to velocity of man, velocity of rain and velocity of rain relative to man, we can express their relationship by the following eq. $\vec{v}_r = \vec{v}_m + \vec{v}_{r/m}$

The above equation suggests that a standstill man observes velocity $\vec{v}_r$ of rain relative to the ground and while he is moving with velocity $\vec{v}_m$, he observes velocity of rain relative to himself $\vec{v}_{r/m}$. It is a common intuitive fact that umbrella must be held against $\vec{v}_{r/m}$ for optimum protection from rain. According to these facts, directions of the velocity vectors are shown in the adjoining figure.



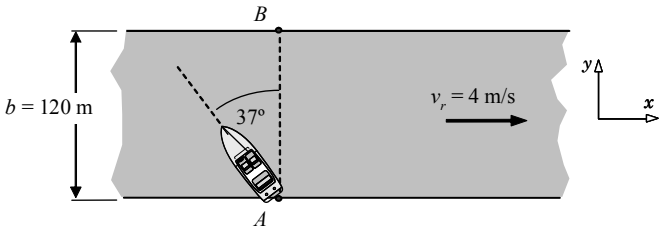


The addition of velocity vectors is represented according to the above equation is also represented. From the figure we have

$v_r = v_m \tan 37° = 3$ km/h **Ans.**

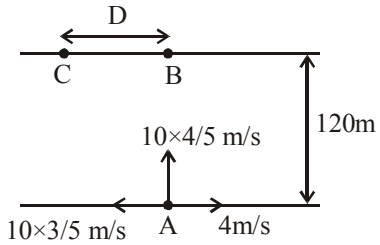
SOLVED EXAMPLE BASED ON RIVER BOAT

Ex. Velocity of the boat with respect to river is 10 m/s. From point A it is steered in the direction shown. Where will it reach on the opposite bank?



Ans. 30 m upstream

Sol.



$$t = \frac{120}{8} = 15 \text{ sec}$$

$\Delta = (6 - 4) \times 15 = 30\text{m}$

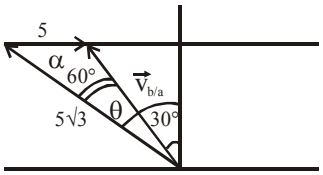
Ex. A motor boat is to reach at a point 30° upstream (w.r.t. normal) on other side of a river flowing with velocity 5m/s. The angle 30° is measured from a direction perpendicular to river flow. Velocity of motorboat with respect to water is $5\sqrt{3}$ m/s. The driver should steer the boat at an angle
(A) 120° with respect to stream direction.
(B) 30° with respect to the perpendicular to the bank.
(C) 30° with respect to the line of destination from starting point.
(D) None of these.

Ans. (C)

Sol. $\vec{v}_{r/g} = 5\hat{i}$

$$\vec{v}_{m/r} = -5\sqrt{3}\sin\theta\hat{i} + 5\sqrt{3}\cos\theta\hat{j}$$

$$\vec{v}_{b/g} = \left(-5 + 5\sqrt{3}\sin\theta\right)\left(-\hat{i}\right) + \left(5\sqrt{3}\cos\theta\right)\hat{j}$$



$$\tan 30° = \frac{-5 + 5\sqrt{3}\sin\theta}{5\sqrt{3}\cos\theta}$$

$$\frac{1}{\sqrt{3}} = \frac{\sqrt{3}\sin\theta - 1}{\sqrt{3}\cos\theta}$$

$$\cos\theta = \sqrt{3}\sin\theta - 1$$

$$\frac{\sqrt{3}}{2}\sin\theta - \frac{1}{2}\cos\theta = \frac{1}{2}$$

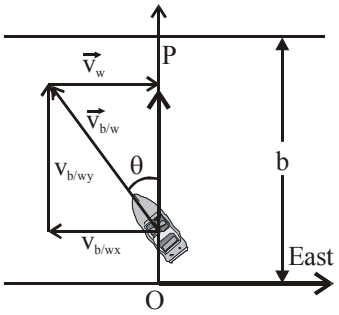
$\sin(\theta - 30°) = \sin 30°$

$\theta = 60°$ $\alpha = 30°$

(c) 30° with respect to line of destination from starting point.

Ex. A boat can be rowed at 5 m/s on still water. It is used to cross a 200 m wide river from south bank to the north bank. The river current has uniform velocity of 3 m/s due east.
(a) In which direction must it be steered to cross the river perpendicular to current?
(b) How long will it take to cross the river in a direction perpendicular to the river flow?
(c) In which direction must the boat be steered to cross the river in minimum time? How far will it drift?

Sol. (a) Velocity of a boat on still water is its capacity to move on water surface and equals to its velocity relative to water.

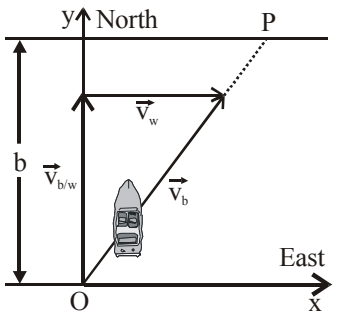


$\vec{v}_{b/w}$ = Velocity of boat relative to water = Velocity of boat on still water

On flowing water, the water carries the boat along with it. Thus velocity $\vec{v}_b$ of the boat relative to the ground equals to vector sum of $\vec{v}_{b/w}$ and $\vec{v}_w$. The boat crosses the river with the velocity $\vec{v}_b$.

$$\vec{v}_b = \vec{v}_{b/w} + \vec{v}_w$$

(b) To cross the river perpendicular to current the boat must be steered in a direction so that one of the components of its velocity ($\vec{v}_{b/w}$) relative to water becomes equal and opposite to water flow velocity $\vec{v}_w$ to neutralize its effect. It is possible only when velocity of boat relative to water is grater than water flow velocity. In the adjoining figure it is shown that the boat starts from the point O and moves along the line OP (y-axis) due north relative to ground with velocity $\vec{v}_b$. To achieve this it is steered at an angle θ with the y-axis.



$$v_{b/w}\sin\theta = v_w \rightarrow 5\sin\theta = 3 \Rightarrow \theta = 37^\circ \textbf{ Ans.}$$

(c) The boat will cover river width b with velocity

$v_b = v_{b/wy} = v_{b/w}\sin 37^\circ = 4$ m/s in time t, which is given by

$$t = b/v_b \rightarrow t = 50\text{s} \textbf{ Ans.}$$

(d) To cross the river in minimum time, the component perpendicular to current of its velocity relative to ground must be kept to maximum value. It is achieved by steering the boat always perpendicular to current as shown in the adjoining figure. The boat starts from O at the south bank and reaches point P on the north bank. Time t taken by the boat is given by

$$t = b/v_{b/w} \rightarrow t = 40\text{s} \textbf{ Ans.}$$

Drift is the displacement along the river current measured from the starting point. Thus, it is given by the following equation. We denote it by x_d.

$$x_d = v_{bx}t$$

Substituting $v_{bx} = v_w = 3$ m/s, from the figure, we have

$$x_d = 120 \text{ m} \textbf{ Ans.}$$

EXERCISE (S)

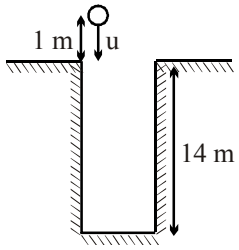
1. A particle moving in one-dimension with constant acceleration of 10 m/s^2 is observed to cover a distance of 100 m during a 4s interval. How far will the particle move in the next 4s?

KM0004

2. A stone is dropped from the top of a tall cliff, and 1s later a second stone is thrown vertically downward with a velocity of 20 ms^{-1}. How far below the top of the cliff will the second stone overtake the first?

KM0010

3. A boy throws a ball with speed u in a well of depth 14 m as shown. On bounce with bottom of the well the speed of the ball gets halved. What should be the minimum value of u (in m/s) such that the ball may be able to reach his hand again? It is given that his hands are at 1 m height from top of the well while throwing and catching.



KM0013

4. A balloon rises from rest on the ground with constant acceleration $\frac{g}{3}$. A stone is dropped when the balloon has rises to a height 60 metre. The time taken by the stone to reach the ground is.

KM0016

5. The position x of a particle w.r.t. time t along x–axis is given by $x = 9t^2 - t^3$ where x is in metre and t in second. Find

(a) Maximum speed along +x direction

(b) Position of turning point

(c) Displacement in first ten seconds

(d) Distance travelled in first ten seconds

KM0017

6. The acceleration of a particle starting from rest vary with respect to time is given by a = (2t – 6), where t is in seconds. Find the time (in seconds) at which velocity of particle in negative direction is maximum.

KM0028

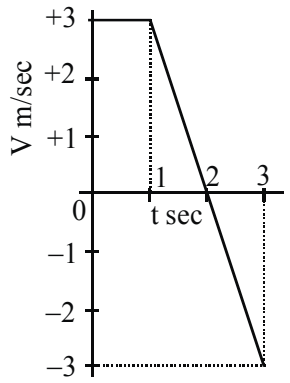
7. A particle moves along a straight line, x. At time t = 0, its position is at x = 0. The velocity, V, of the object changes as a function of time t, as indicated in the figure; t is in seconds, V in m/sec and x in meters.

(a) What is x at t = 3 sec?

(b) What is the instantaneous acceleration (in m/sec^2) at t = 2 sec?

(c) What is the average velocity (in m/sec) between t = 0 and t = 3 sec?

(d) What is the average speed (in m/sec) between t = 1 and t = 3 sec?



KM0033

8. The vertical height y and horizontal distance x of a projectile on a certain planet are given by x = (3t)m, y = (4t – 6t^2) m where t is in seconds. Find the speed of projection (in m/s).

KM0108

9. A particle is projected in the *x*-*y* plane with *y*-axis along vertical. Two second after projection the velocity of the particle makes an angle 45° with the *X*-axis. Four second after projection, it moves horizontally. Find the velocity of projection.

KM0118

10. A particle is projected upwards with a velocity of 100 m/s at an angle of 60° with the vertical. Find the time when the particle will move perpendicular to its initial direction, taking g =10 m/s^2.

KM0115

11. A particle is projected in *x*-*y* plane with *y*-axis along vertical, the point of projection is origin. The equation of a path is $y = \sqrt{3}x - \frac{gx^2}{2}$. Find angle of projection and speed of projection.

KM0116

12. A Bomber flying upward at an angle of 53° with the vertical releases a bomb at an altitude of 800 m. The bomb strikes the ground 20 s after its release. Find: [Given sin 53°=0.8; g =10 m/s^2]

(i) The velocity of the bomber at the time of release of the bomb .

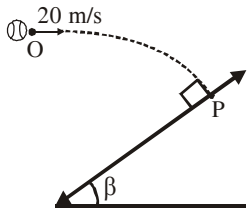
(ii) The maximum height attained by the bomb .

(iii) The horizontal distance travelled by the bomb before it strikes the ground

(iv) The velocity (magnitude & direction) of the bomb just when it strikes the ground .

KM0120

13. A ball is thrown horizontally from a point O with speed 20 m/s as shown. Ball strikes the incline plane along the normal to it after two seconds. Find value of x, if $\beta = \pi/x$ (where β is the angle of incline in degree).

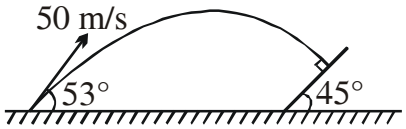


KM0126

14. A projectile is thrown with velocity of 50 m/s towards an inclined plane from ground such that it strikes the inclined plane perpendicularly. The angle of projection of the projectile is 53° with the horizontal and the inclined plane is inclined at an angle of 45° to the horizontal.

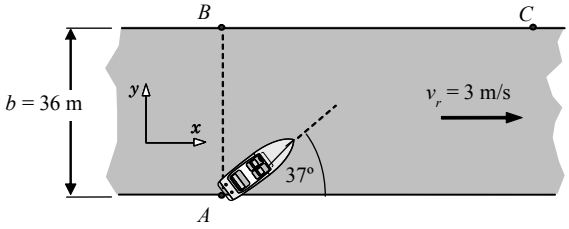
(i) Find the time of flight.

(ii) Find the distance between the point of projection and the foot of inclined plane.



KM0141

15. Velocity of the boat with respect to river is 10 m/s. From point A it is steered in the direction shown to reach point C. Find the time of the trip and distance between B and C.



KM0135

16. A man crosses a river by a boat. If he crosses the river in minimum time he takes 10 minutes with a drift 120 m. If he crosses the river taking shortest path, he takes 12.5 minutes. Assuming $v_{b/r} > v_r$, find

(i) width of the river,

(ii) velocity of the boat with respect to water ($v_{b/r}$)

(iii) speed of the current (v_r)

KM0131

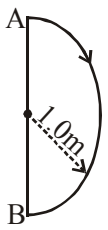
17. Rain is falling vertically with a speed of 20 m/s relative to air. A person is running in the rain with a velocity of 5 m/s and a wind is also blowing with a speed of 15 m/s (both towards east). Find the angle with the vertical at which the person should hold his umbrella for best protection from rain.

KM0132

EXERCISE (O-1)

SINGLE CORRECT TYPE QUESTIONS

1. In 1.0 sec. a particle goes from point A to point B moving in a semicircle of radius 1.0 m. The magnitude of average velocity is : **[JEE '99]**



(A) 3.14 m/sec (B) 2.0 m/sec (C) 1.0 m/sec (D) zero

KM0048

2. A body starts from rest and is uniformly accelerated for 30 s. The distance travelled in the first 10 s is x_1, next 10 s is x_2 and the last 10 s is x_3. Then $x_1 : x_2 : x_3$ is the same as :-

(A) 1 : 2 : 4 (B) 1 : 2 : 5 (C) 1 : 3 : 5 (D) 1 : 3 : 9

KM0050

3. A particle travels 10m in first 5 sec and 10m in next 3 sec. Assuming constant acceleration what is the distance travelled in next 2 sec.

(A) 8.3 m (B) 9.3 m (C) 10.3 m (D) None of above

KM0052

4. A particle is thrown upwards from ground. It experiences a constant resistance force which can produce retardation 2 m/s^2. The ratio of time of ascent to the time of descent is [g=10 m/s^2]

(A) 1 : 1 (B) $\sqrt{\frac{2}{3}}$ (C) $\frac{2}{3}$ (D) $\sqrt{\frac{3}{2}}$

KM0060

5. A ball is thrown vertically upward with initial velocity 30 m/sec. What will be its position vector at time t = 5 sec, taking origin at 45 m above the point of projection, vertical up as positive y-axis and horizontal as x-axis :-

(A) (0, –25) (B) (0, –20) (C) (0, –45) (D) (0, –5)

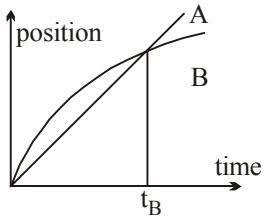
KM0062

6. A point mass moves with velocity $v = (5t–t^2)\ ms^{-1}$ in a straight line. Find the distance travelled (i.e. $\int vdt$) in fourth second.

(A) $\frac{31}{6}$ m (B) $\frac{29}{6}$ m (C) $\frac{37}{6}$ m (D) None of these

KM0065

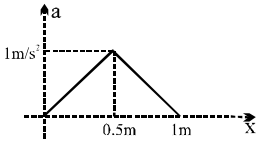
7. The graph shows position as a function of time for two trains running on parallel tracks. Which statement is true?



(A) At time t_B, both trains have the same velocity.
(B) Both trains have the same velocity at some time after t_B
(C) Both trains have the same velocity at some time before t_B.
(D) Somewhere on the graph, both trains have the same acceleration.

KM0073

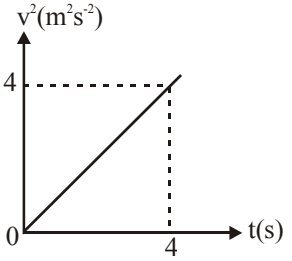
8. A body initially at rest, starts moving along x-axis in such a way so that its acceleration vs displacement plot is as shown in figure. The maximum velocity of particle is :-



(A) 1 m/s (B) 6 m/s (C) 2 m/s (D) $\frac{1}{2}$ m/s

KM0076

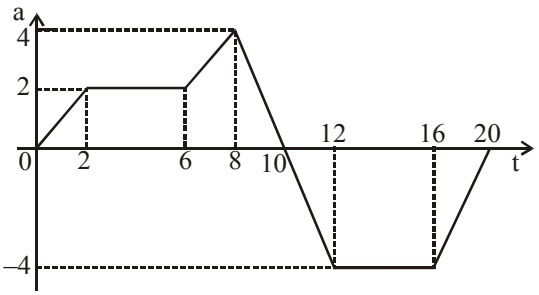
9. A particle is moving along a straight line such that square of its velocity varies with time as shown in the figure. What is the acceleration of the particle at t = 4 s?



(A) 4 m/s² (B) 1/4 m/s² (C) 1/2 m/s² (D) 1

KM0077

10. If initial velocity of particle is 2 m/s, the maximum velocity of particle from t = 0 to t = 20 sec is :



(A) 20 m/s (B) 18 m/s (C) 22 m/s (D) 24 m/s

KM0084

11. The position vector of a particle is determined by $\vec{r} = 3t^2\hat{i} + 4t^2\hat{j} + 7\hat{k}$. The distance travelled in first 10 sec is :-

(A) 100 m (B) 150 m (C) 500 m (D) 300 m

KM0186

12. A particle leaves the origin with an initial velocity $\vec{v} = \left(3\hat{i} + 4\hat{j}\right) ms^{-1}$ and a constant acceleration $\vec{a} = \left(-\hat{i} - 0.5\hat{j}\right) ms^{-2}$. When the particle reaches its maximum x–coordinate, what is the y–coordinate?

(A) $\frac{27}{4}$ m (B) $\frac{37}{4}$ m (C) $\frac{29}{4}$ m (D) $\frac{39}{4}$ m

KM0185

13. A particle is projected from a horizontal plane (*x*-*z* plane) such that its velocity vector at time t is given by $\vec{V} = a\hat{i} + (b - ct)\hat{j}$. Its range on the horizontal plane is given by

(A) $\frac{ba}{c}$ (B) $\frac{2ba}{c}$ (C) $\frac{3ba}{c}$ (D) None

KM0153

14. A particle is projected from the ground with velocity *u* at angle θ with horizontal. The horizontal range, maximum height and time of flight are *R*, *H* and *T* respectively. They are given by

$R = \frac{u^2 \sin 2\theta}{g}$ $H = \frac{u^2 \sin^2 \theta}{2g}$ and $T = \frac{2u \sin \theta}{g}$

Now keeping *u* fixed, θ is varied from 30° to 60°, then

(A) *R* will first increase then decrease, *H* will increase and *T* will decrease

(B) *R* will first increase then decrease while *H* and *T* both will increase

(C) *R* will decrease while *H* and *T* both will increase

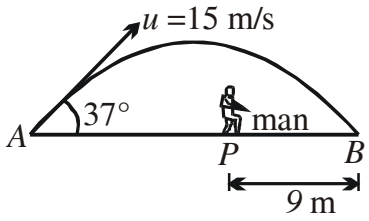
(D) *R* will increase while *H* and *T* both will also increase

KM0156

15. Suppose a player hits several baseballs. Which baseball will be in the air for the longest time?
(A) The one with the farthest range.
(B) The one which reaches maximum height.
(C) The one with the greatest initial velocity.
(D) The one leaving the bat at 45° with respect to the ground.

KM0157

16. A ball is hit by a batsman at an angle of 37° as shown in figure. The man standing at *P* should run at what minimum velocity so that he catches the ball before it strikes the ground. Assume that height of man is negligible in comparison to maximum height of projectile.



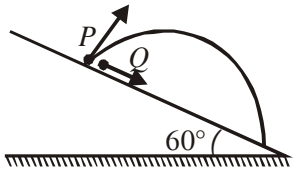
(A) 3 m/s (B) 5 m/s (C) 9 m/s (D) 12 m/s

KM0158

17. A ball is projected horizontally. After 3 s from projection its velocity becomes 1.25 times of the velocity of projection. Its velocity of projection is
(A) 10 m/s (B) 20 m/s (C) 30 m/s (D) 40 m/s

KM0159

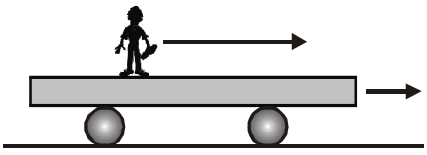
18. A particle *P* is projected from a point on the surface of smooth inclined plane (see figure). Simultaneously another particle *Q* is released on the smooth inclined plane from the same position. *P* and *Q* collide after t = 4 s. The speed of projection of *P* is :-



(A) 5 m/s (B) 10 m/s (C) 15 m/s (D) 20 m/s

KM0160

19. A trolley is moving horizontally with a constant velocity of *v* with respect to earth. A man starts running from one end of the trolley with a velocity 1.5 *v* with respect to trolley. After reaching the opposite end, the man returns back and continues running with a velocity of 1.5 *v* w.r.t. the trolley in the backward direction. If the length of the trolley is *L* then the displacement of the man with respect to earth during the process will be



(A) 2.5 L (B) 1.5 L (C) $\frac{5L}{3}$ (D) $\frac{4L}{3}$

KM0165

20. A particle is thrown from a stationary platform with velocity v at an angle of 60° with the horizontal. The range obtained is R. If the platform moves horizontally in the direction of target with velocity v, the range will increase to :

(A) $\frac{3R}{2}$ (B) $\frac{5R}{2}$ (C) 2 R (D) 3 R

KM0201

21. An elevator car (lift) is moving upward with uniform acceleration of 2 m/s^2. At the instant, when its velocity is 2 m/s upwards a ball is thrown upward from its floor. The ball strikes back the floor 2 s after its projection. Find the velocity of projection of the ball relative to the lift.

(A) 10 m/s ↑ (B) 10 m/s ↓ (C) 12 m/s ↑ (D) 12 m/s ↓

KM0166

22. A flag is mounted on a car moving due North with velocity of 20 km/hr. Strong winds are blowing due East with velocity of 20 km/hr. The flag will point in direction :-

(A) East (B) North-East (C) South-East (D) South-West

KM0167

23. Three ships *A*, *B* & *C* are in motion. Ship *A* moves relative to *B* is with speed *v* towards North-East. Ship B moves relative to *C* with speed *v* towards the North-West. Then relative to *A*, *C* will be moving towards :-

(A) North (B) South (C) East (D) West

KM0168

24. Wind is blowing in the north direction at speed of 2 m/s which causes the rain to fall at some angle with the vertical. With what velocity should a cyclist drive so that the rain appears vertical to him :

(A) 2 m/s south (B) 2 m/s north (C) 4 m/s west (D) 4 m/s south

KM0169

EXERCISE (O-2)

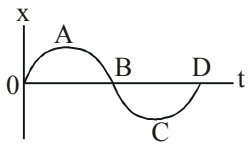
MULTIPLE CORRECT TYPE QUESTIONS

1. A ball is dropped from a building. Somewhere down it crosses a window of length 4 m in 0.5 sec. Speed of ball at top of window is v_1 and at bottom v_2, then choose the **CORRECT** option(s) ($g = 10\ m/s^2$) :-

(A) $v_2 - v_1 = 5$ m/s (B) $v_2 + v_1 = 16$ m/s (C) $\frac{v_2}{v_1} = 9$ (D) $\frac{v_2}{v_1} = \frac{21}{11}$

KM0096

2. A particle has a rectilinear motion and the figure gives its displacement as a function of time. Which of the following statements are true with respect to the motion



(A) in the motion between O and A the velocity is positive and acceleration is negative
(B) between A and B the velocity and acceleration are positive
(C) between B and C the velocity is negative and acceleration is positive
(D) between C and D the acceleration is positive

KM0094

3. A particle moves in the *xy*-plane and at time *t* is at the point (t^2, $t^3 - 2t$). Then
(A) At $t = 2/3$ s, directions of velocity and acceleration are perpendicular
(B) At $t = 0$, directions of velocity and acceleration are perpendicular
(C) At $t = \sqrt{2/3}$ s, particle is moving parallel to *x*-axis
(D) Acceleration of the particle when it is at point (4, 4) is $2\hat{i} + 12\hat{j}$

KM0174

4. A ball is thrown from ground such that it just crosses two poles of equal height kept 80 m apart. The maximum height attained by the ball is 80 m. When the ball passes the first pole, its velocity makes 45° with horizontal. The correct alternatives is/are :- ($g = 10\ m/s^2$)
(A) Time interval between the two poles is 4 s.
(B) Height of the pole is 60 m.
(C) Range of the ball is 160 m.
(D) Angle of projection is $\tan^{-1}(2)$ with horizontal.

KM0214

5. Two particles *A* and B projected along different directions from the same point *P* on the ground with the same velocity of 70 m/s in the same vertical plane. They hit the ground at the same point *Q* such that *PQ* = 480 m. Then :- [g = 9.8 m/s^2]

(A) Ratio of their times of flight is 4 : 5

(B) Ratio of their maximum heights is 9 : 16

(C) Ratio of their minimum speeds during flights is 4 : 3

(D) The bisector of the angle between their directions of projection makes 45° with horizontal

KM0176

6. A particle moves in the *x*-*y* plane with a constant acceleration *g* in the negative *y*-direction. Its equation of motion is $y = ax - bx^2$, where *a* and *b* are constants. Which of the following is/are correct?

(A) The *x*-component of its velocity is constant.

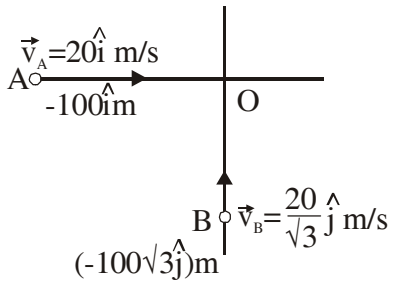
(B) At the origin, the *y*-component of its velocity is a $\sqrt{\frac{g}{2b}}$.

(C) At the origin, its velocity makes an angle $\tan^{-1}(a)$ with the *x*-axis.

(D) The particle moves exactly like a projectile.

KM0175

7. Positions of two vehicles A and B with reference to origin O and their velocities are as shown.



(A) they will collide

(B) distance of closest approach is 100 m.

(C) their relative speed is $\frac{40}{\sqrt{3}}$ m/s

(D) their relative velocity is $\frac{20}{\sqrt{3}}$ m/s

KM0177

COMPREHENSION TYPE QUESTIONS

Paragraph for Question Nos. 8 to 10

A particle leaves the origin with initial velocity $\vec{v}_0 = 11\hat{i} + 14\hat{j}$ m/s. It undergoes a constant acceleration given by $\vec{a} = -\frac{22}{5}\hat{i} + \frac{2}{15}\hat{j}$ m/s².

8. When does the particle cross the y axis ?

(A) 2 sec (B) 4 sec (C) 5 sec (D) 7 sec

KM0179

9. At the instant when particle crosses y-axis, direction in which particle is moving is :-

(A) At angle 37° from +x-axis towards +y-axis

(B) At angle 37° from –x-axis towards +y-axis

(C) At angle 53° from +x-axis towards +y-axis

(D) At angle 53° from –x-axis towards +y-axis

KM0179

10. How far is it from the origin, at that time ?

(A) 70 m (B) 71.67 m (C) 125 m (D) 15 m

KM0179

Paragraph for Question No. 11 to 13

Two projectiles are thrown simultaneously in the same vertical plane from the same point. If their velocities of projection are v_1 and v_2 at angles θ_1 and θ_2 respectively from the horizontal, then answer the following questions

11. The trajectory of particle 1 with respect to particle 2 will be

(A) a parabola (B) a straight line

(C) a vertical straight line (D) a horizontal straight line

KM0178

12. If $v_1\cos\theta_1 = v_2\cos\theta_2$, then choose the ***incorrect*** statement

(A) One particle will remain exactly below or above the other particle

(B) The trajectory of one with respect to other during the flight will be a vertical straight line

(C) Both will have the same range

(D) Both will attain same maximum height

KM0178

13. If $v_1\sin\theta_1 = v_2\sin\theta_2$, then choose the ***correct*** statement

(A) The time of flight of both the particles will be same

(B) The maximum height attained by the particles will be same

(C) The trajectory of one with respect to another during the flight will be a horizontal straight line

(D) None of these

KM0178

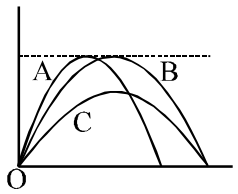
MATRIX MATCH TYPE QUESTION

14. A balloon rises up with constant net acceleration of $10 m/s^2$. After 2 s a particle drops from the balloon. After further 2 s match the following : ($g = 10 m/s^2$)

	Column-I		**Column-II**
(A)	Height of particle from ground	(P)	Zero
(B)	Speed of particle	(Q)	10 SI units
(C)	Displacement of Particle	(R)	40 SI units
(D)	Acceleration of particle	(S)	20 SI units

KM0174

15. Trajectories are shown in figure for three kicked footballs. Initial vertical & horizontal velocity components are u_y and u_x respectively. Ignoring air resistance, choose the correct statement from **Column-II** for the value of variable in **Column-I**.



	Column-I		**Column-II**
(A)	time of flight	(P)	greatest for A only
(B)	u_y/u_x	(Q)	greatest for C only
(C)	u_x	(R)	equal for A and B
(D)	$u_x u_y$	(S)	equal for B and C

KM0182

16. Trajectory of particle in a projectile motion is given as $y = x - \frac{x^2}{80}$. Here, x and y are in meters. For this projectile motion, match the following with $g = 10 m/s^2$.

	Column–I		**Column–II**
(A)	Angle of projection (in degrees)	(P)	20
(B)	Angle of velocity with horizontal after 4s (in degrees)	(Q)	80
(C)	Maximum height (in metres)	(R)	45
(D)	Horizontal range (in metres)	(S)	30
		(T)	60

KM0183

EXERCISE (JM)

1. A particle is moving with velocity $\vec{v} = K(y\hat{i} + x\hat{j})$, where K is a constant. The general equation for its path is : **[AIEEE - 2010]**

(1) $y^2 = x^2$ + constant (2) $y = x^2$ + constant (3) $y^2 = x$ + constant (4) xy = constant

KM0227

2. An object, moving with a speed of 6.25 m/s, is decelerated at a rate given by

$$\frac{dv}{dt} = -2.5\sqrt{v}$$

where v is the instantaneous speed. The time taken by the object, to come to rest, would be :- **[AIEEE-2011]**

(1) 4 s (2) 8 s (3) 1 s (4) 2 s

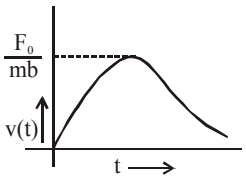
KM0103

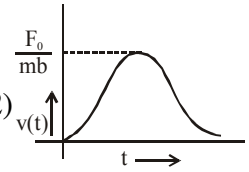
3. A water fountain on the ground sprinkles water all around it. If the speed of water coming out of the fountain is v, the total area around the fountain that gets wet is :- **[AIEEE - 2011]**

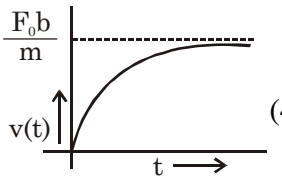
(1) $\frac{\pi}{2}\frac{v^4}{g^2}$ (2) $\pi\frac{v^2}{g^2}$ (3) $\pi\frac{v^2}{g}$ (4) $\pi\frac{v^4}{g^2}$

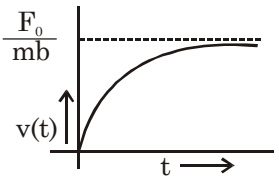
KM0228

4. A particle of mass m is at rest at the origin at time t = 0. It is subjected to a force $F(t) = F_0e^{-bt}$ in the x direction. Its speed v(t) is depicted by which of the following curves ? **[AIEEE - 2012]**

(1) 

(2) 

(3) 

(4) 

KM0229

5. A projectile is given an initial velocity of $(\hat{i} + 2\hat{j})$ m/s, where $\hat{i}$ is along the ground and $\hat{j}$ is along the vertical. If g = 10 m/s^2, the equation of its trajectory is : **[AIEEE - 2013]**

(1) $y = x - 5x^2$ (2) $y = 2x - 5x^2$ (3) $4y = 2x - 5x^2$ (4) $4y = 2x - 25x^2$

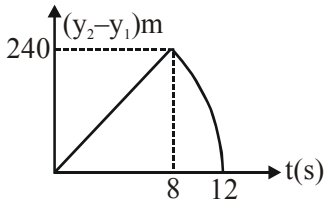
KM0230

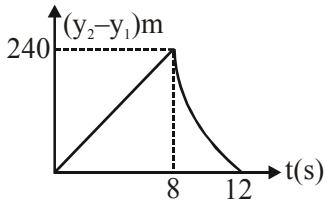
6. From a tower of height H, a particle is thrown vertically upwards with a speed u. The time taken by the particle, to hit the ground, is n times that taken by it to reach the highest point of its path. The relation between H, u and n is : **[JEE-MAIN-2014]**

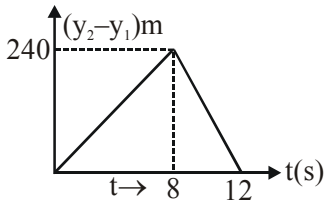
(1) $2gH = nu^2(n-2)$ (2) $gH = (n-2)u^2$ (3) $2gH = n^2u^2$ (4) $gH = (n-2)^2u^2$

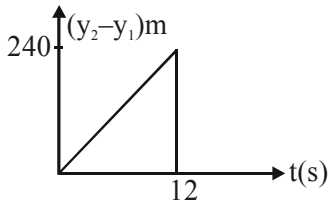
KM0104

7. Two stones are thrown up simultaneously from the edge of a cliff 240 m high with initial speed of 10 m/s and 40 m/s respectively. Which of the following graph best represents the time variation of relative position of the second stone with respect to the first ?
(Assume stones do not rebound after hitting the ground and neglect air resistance, take $g = 10\ m/s^2$) *(The figure are schematic and not drawn to scale)* **[JEE Mains-2015]**

(1) 

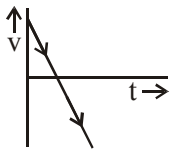
(2) 

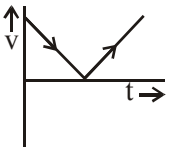
(3) 

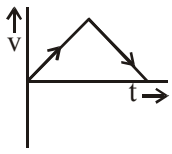
(4) 

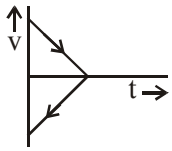
KM0231

8. A body is thrown vertically upwards. Which one of the following graphs correctly represent the velocity vs time? **[JEE Mains-2017]**

(1) 

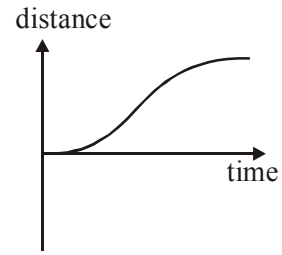
(2) 

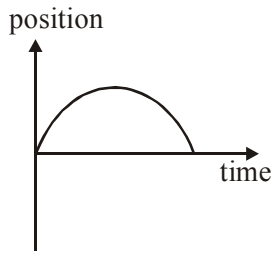
(3) 

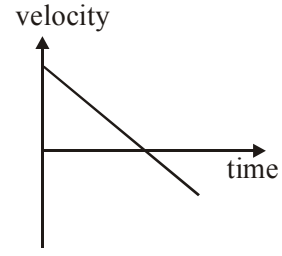
(4) 

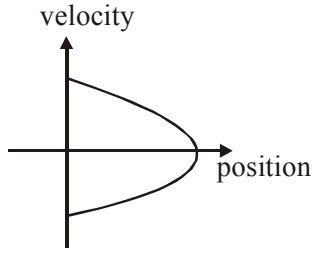
KM0105

9. All the graphs below are intended to represent the same motion. One of them does it incorrectly. Pick it up. **[JEE Mains-2018]**

(1) 

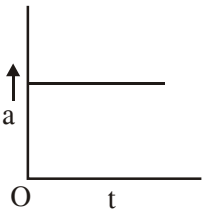
(2) 

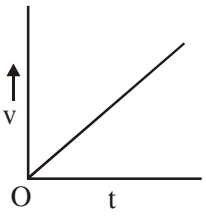
(3) 

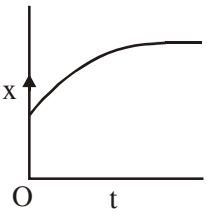
(4) 

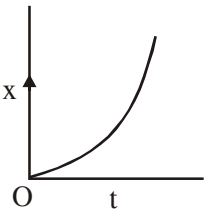
KM0106

10. A particle starts from origin O from rest and moves with a uniform acceleration along the positive x-axis. Identify all figures that correctly represent the motion qualitatively. (a = acceleration, v = velocity, x = displacement, t = time) **[JEE Mains (Online)-2019]**

(A) 

(B) 

(C) 

(D) 

(1) (A), (B), (C) (2) (A) (3) (A), (B), (D) (4) (B), (C)

KM0237

11. Ship A is sailing towards north-east with velocity $\vec{v} = 30\hat{i} + 50\hat{j}$ km/hr where $\hat{i}$ points east and $\hat{j}$, north. Ship B is at a distance of 80 km east and 150 km north of Ship A and is sailing towards west at 10 km/hr. A will be at minimum distance from B in : **[JEE Mains (Online)-2019]**

(1) 4.2 hrs. (2) 2.2 hrs. (3) 3.2 hrs. (4) 2.6 hrs.

KM0238

12. The position of a particle as a function of time t, is given by
$x(t) = at + bt^2 - ct^3$
where a, b and c are constants. When the particle attains zero acceleration, then its velocity will be : **[JEE Mains (Online)-2019]**

(1) $a + \frac{b^2}{4c}$ (2) $a + \frac{b^2}{c}$ (3) $a + \frac{b^2}{2c}$ (4) $a + \frac{b^2}{3c}$

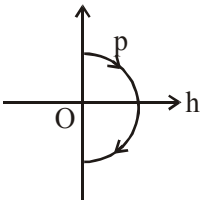
KM0239

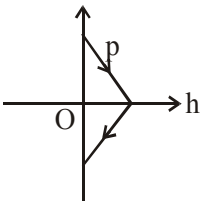
13. The position vector of a particle changes with time according to the relation $\vec{r}(t) = 15t^2\hat{i} + (4 - 20t^2)\hat{j}$. What is the magnitude of the acceleration at t = 1 ? **[JEE Mains (Online)-2019]**

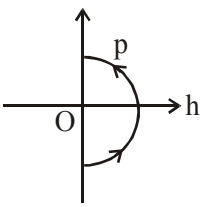
(1) 40 (2) 100 (3) 25 (4) 50

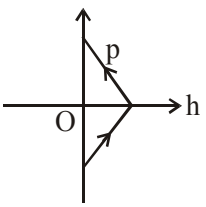
KM0240

14. A ball is thrown vertically up (taken as +z-axis) from the ground. The correct momentum-height (p-h) diagram is : **[JEE Mains (Online)-2019]**

(1) 

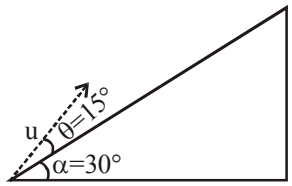
(2) 

(3) 

(4) 

KM0241

15. A plane is inclined at an angle $\alpha = 30°$ with a respect to the horizontal. A particle is projected with a speed $u = 2\ ms^{-1}$ from the base of the plane, making an angle $\theta = 15°$ with respect to the plane as shown in the figure. The distance from the base, at which the particle hits the plane is close to : (Take $g = 10\ ms^{-2}$) **[JEE Mains (Online)-2019]**



(1) 14 cm (2) 20 cm (3) 18 cm (4) 26 cm

KM0242

16. A particle is moving with speed $v = b\sqrt{x}$ along positive x-axis. Calculate the speed of the particle at time $t = \tau$(assume that the particle is at origin at $t = 0$). **[JEE Mains (Online)-2019]**

(1) $\frac{b^2\tau}{4}$ (2) $\frac{b^2\tau}{2}$ (3) $b^2\tau$ (4) $\frac{b^2\tau}{\sqrt{2}}$

KM0243

17. Two particles are projected from the same point with the same speed u such that they have the same range R, but different maximum heights, h_1 and h_2. Which of the following is correct ? **[JEE Mains (Online)-2019]**

(1) $R^2 = 2\ h_1h_2$ (2) $R^2 = 16h_1h_2$ (3) $R^2 = 4\ h_1h_2$ (4) $R^2 = h_1h_2$

KM0244

18. The trajectory of a projectile near the surface of the earth is given as $y = 2x - 9x^2$. If it were launched at an angle θ_0 with speed v_0 then **[JEE Mains (Online)-2019]**
$(g = 10\ ms^{-2})$:

(1) $\theta_0 = \cos^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}ms^{-1}$ (2) $\theta_0 = \sin^{-1}\left(\frac{1}{\sqrt{5}}\right)$ and $v_0 = \frac{5}{3}ms^{-1}$

(3) $\theta_0 = \sin^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}ms^{-1}$ (4) $\theta_0 = \cos^{-1}\left(\frac{2}{\sqrt{5}}\right)$ and $v_0 = \frac{3}{5}ms^{-1}$

KM0245

19. A shell is fired from a fixed artillery gun with an initial speed u such that it hits the target on the ground at a distance R from it. If t_1 and t_2 are the values of the time taken by it to hit the target in two possible ways, the product t_1t_2 is : **[JEE Mains (Online)-2019]**

(1) R/g (2) R/4g (3) 2R/g (4) R/2g

KM0246

20. The position co-ordinates of a particle moving in a 3-D coordinate system is given by

$x = a\cos\omega t$

$y = a\sin\omega t$

and $z = a\omega t$

The speed of the particle is : **[JEE Mains (Online)-2019]**

(1) $a\omega$ (2) $\sqrt{3}\,a\omega$ (3) $\sqrt{2}\,a\omega$ (4) $2a\omega$

KM0247

21. In a car race on straight road, car A takes a time t less than car B at the finish and passes finishing point with a speed 'v' more than that of car B. Both the cars start from rest and travel with constant acceleration a_1 and a_2 respectively. Then 'v' is equal to : **[JEE Mains (Online)-2019]**

(1) $\frac{a_1 + a_2}{2}t$ (2) $\sqrt{2a_1a_2}\,t$ (3) $\frac{2a_1a_2}{a_1 + a_2}t$ (4) $\sqrt{a_1a_2}\,t$

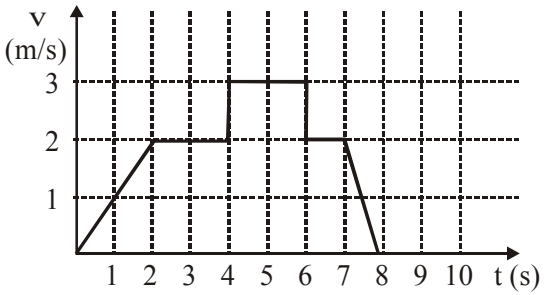
KM0248

22. A particle is moving with a velocity $\vec{v} = K\left(y\hat{i} + x\hat{j}\right)$, where K is a constant. The general equation for its path is: **[JEE Mains (Online)-2019]**

(1) xy = constant (2) $y^2 = x^2$ + constant (3) $y = x^2$ + constant (4) $y^2 = x$ + constant

KM0249

23. A particle starts from the origin at time t = 0 and moves along the positive x-axis. The graph of velocity with respect to time is shown in figure. What is the position of the particle at time t = 5s ? **[JEE Mains (Online)-2019]**



(1) 6 m (2) 9 m (3) 3 m (4) 10 m

KM0250

24. Two guns A and B can fire bullets at speeds 1 km/s and 2 km/s respectively. From a point on a horizontal ground, they are fired in all possible directions. The ratio of maximum areas covered by the bullets fired by the two guns, on the ground is : **[JEE Mains (Online)-2019]**

(1) 1 : 2 (2) 1 : 4 (3) 1 : 8 (4) 1 : 16

KM0251

25. A particle moves from the point $\left(2.0\hat{i}+4.0\hat{j}\right)$ m, at t = 0, with an initial velocity $\left(5.0\hat{i}+4.0\hat{j}\right)$ ms^{-1}. It is acted upon by a constant force which produces a constant acceleration $\left(4.0\hat{i}+4.0\hat{j}\right)ms^{-2}$. What is the distance of the particle from the origin at time 2 s ?

[JEE Mains (Online)-2019]

(1) $20\sqrt{2}$ m (2) $10\sqrt{2}$ m (3) 5 m (4) 15 m

KM0252

26. A passenger train of length 60m travels at a speed of 80 km/hr. Another freight train of length 120 m travels at a speed of 30 km/hr. The ratio of times taken by the passenger train to completely cross the freight train when : (i) they are moving in the same direction, and (ii) in the opposite directions is : **[JEE Mains (Online)-2019]**

(1) $\frac{5}{2}$ (2) $\frac{25}{11}$ (3) $\frac{3}{2}$ (4) $\frac{11}{5}$

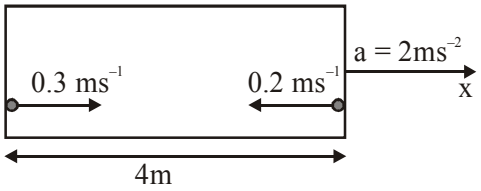
KM0253

EXERCISE (JA)

1. A train is moving along a straight line with a constant acceleration 'a'. A boy standing in the train throws a ball forward with a speed of 10 m/s, at an angle of 60° to the horizontal. The boy has to move forward by 1.15 m inside the train to catch the ball back at the initial height. The acceleration of the train, in m/s^2, is **[IIT-JEE 2011]**

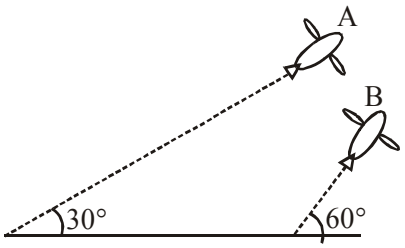
KM0232

2. A rocket is moving in a gravity free space with a constant acceleration of 2 ms^{-2} along + x direction (see figure). The length of a chamber inside the rocket is 4m. A ball is thrown from the left end of the chamber in + x direction with a speed of 0.3 ms^{-1} relative to the rocket. At the same time, another ball is thrown in –x direction with a speed of 0.2 ms^{-1} from its right end relative to the rocket. The time in seconds when the two balls hit each other is **[JEE Advanced 2014]**



KM0233

3. Airplanes A and B are flying with constant velocity in the same vertical plane at angles 30° and 60° with respect to the horizontal respectively as shown in figure. The speed of A is $100\sqrt{3}$ ms^{-1}. At time t = 0 s, an observer in A finds B at a distance of 500 m. This observer sees B moving with a constant velocity perpendicular to the line of motion of A. If at $t = t_0$, A just escapes being hit by B, t_0 in seconds is **[JEE Advanced-2014]**



KM0234

4. Consider an expanding sphere of instantaneous radius R whose total mass remains constant. The expansion is such that the *instantaneous* density ρ remains uniform throughout the volume. The rate of fractional change in density $\left(\frac{1}{\rho}\frac{d\rho}{dt}\right)$ is constant. The velocity v of any point on the surface of the expanding sphere is proportional to : **[JEE Advanced-2017]**

(A) R^3 (B) $\frac{1}{R}$ (C) R (D) $R^{2/3}$

KM0107

5. A ball is projected from the ground at an angle of 45° with the horizontal surface. It reaches a maximum height of 120 m and returns to the ground. Upon hitting the ground for the first time, it loses half of its kinetic energy. Immediately after the bounce, the velocity of the ball makes an angle of 30° with the horizontal surface. The maximum height it reaches after the bounce, in metres, is.....

[JEE Advanced-2018]

KM0235

ANSWER KEY

EXERCISE (S)

1. Ans. 260 m **2. Ans.** $\frac{45}{4}$m **3. Ans.** 30 **4. Ans.** 6

5. Ans. (a) 27 m/s, (b) 108 m, (c) –100 m, (d) 316 m **6. Ans.** 3

7. Ans. (a) 3 m; (b) –3 m/s^2; (c) 1m/s; (d) 3/2 m/s **8. Ans.** 5 **9. Ans.** $20\sqrt{5}$ m/s

10. Ans. 20 s **11. Ans.** 60°, 2 m/s.

12. Ans. (i) 100 m/s (ii) 980 m (iii) 1600 m (iv) $\left(80\hat{i}-140\hat{j}\right)$ **13. Ans.** 4

14. Ans. (i) 7 s, (ii) 175 m **15. Ans.** 6 s, 66 m **16. Ans.** 200 m, 20 m/min, 12 m/min

17. Ans. $\tan^{-1}(1/2)$

EXERCISE (O-1)

1. Ans. (B)	**2. Ans. (C)**	**3. Ans. (A)**	**4. Ans. (B)**	**5. Ans. (B)**	**6. Ans. (A)**
7. Ans. (C)	**8. Ans. (A)**	**9. Ans. (B)**	**10. Ans. (C)**	**11. Ans. (C)**	**12. Ans. (D)**
13. Ans. (B)	**14. Ans. (B)**	**15. Ans. (B)**	**16. Ans. (B)**	**17. Ans. (D)**	**18. Ans. (B)**
19. Ans. (D)	**20. Ans. (D)**	**21. Ans. (C)**	**22. Ans. (C)**	**23. Ans. (B)**	**24.Ans. (B)**

EXERCISE (O-2)

1. Ans. (A, B, D)	**2. Ans. (A,C,D)**	**3. Ans. (A,B,C,D)**	**4. Ans. (A, B,C,D)**
5. Ans. (B, C, D)	**6. Ans. (A, B, C, D)**	**7. Ans. (B, C)**	**8. Ans. (C)**
9. Ans. (D)	**10. Ans. (B)**	**11. Ans. (B)**	**12. Ans. (C, D)**
13. Ans. (A, B, C)	**14. Ans. (A) - (R); (B) - (P) ; (C) - (S) ; (D) - (Q)**		

15. Ans. (A)-R; (B)-P; (C)-Q; (D)-S

16. Ans. (A) → (R) ; (B) → (R) ; (C) → (P) ; (D) → (Q)

EXERCISE (JM)

1. Ans. (1)	**2. Ans. (4)**	**3. Ans. (4)**	**4. Ans. (4)**	**5. Ans. (2)**	**6. Ans. (1)**
7. Ans. (1)	**8. Ans. (1)**	**9. Ans. (1)**	**10. Ans. (3)**	**11. Ans. (4)**	**12. Ans. (4)**
13. Ans. (4)	**14. Ans. (1)**	**15. Ans. (2)**	**16. Ans. (2)**	**17. Ans. (2)**	**18. Ans. (1)**
19. Ans. (3)	**20. Ans. (3)**	**21. Ans. (4)**	**22. Ans. (2)**	**23. Ans. (2)**	**24. Ans. (4)**
25. Ans. (1)	**26. Ans. (4)**				

EXERCISE (JA)

1. Ans. 5	**2. Ans. 8 or 2**	**3. Ans. 5**	**4. Ans. (C)**	**5. Ans. 30 [29.60, 30.40]**