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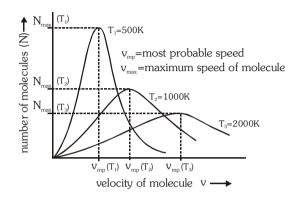
IMPORTANT TOPICS FOR JEE MAIN (1)

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HEAT & THERMODYNAMICS

THEORY

MAXWELL'S LAW OF DISTRIBUTION OF VELOCITIES



GOLDEN KEY POINT

- At any given temperature graph drawn in between molecular velocity and number of molecules is known as velocity distribution curve.
- The velocities of molecules of a gas are in between zero and infinity $(0-\infty)$
- With the increase in the temperature, the most probable velocity and maximum molecule velocity both increases.
- The number of molecules within certain velocity range is constant although the velocity of molecule changes continuously at particular temperature.
- The area enclosed between the (N v) curve and the velocity axis presents the total number of molecules.

On the basis of velocity distribution Maxwell established gives the law of equipartition of energy for gases of any temperature.

Main parts of Carnot's engine are

Source of heat

It is a hot body of very large heat capacity kept at a constant high temperature T₁.

Mechanical arrangements and working substance

It is a cylinder whose walls are perfectly non-conducting and its base is perfectly conducting fitted with non-conducting piston. This piston move without any friction. Ideal gas enclosed in cylinder as a working substance.

Heat sink

It is a cold body at low temperature T₂. It is a body of large heat capacity.

Stand

It is of two types: conducting and nonconducting.

Working

Through a set of reversible processes the working substance is taken back to initial condition to get maximum work from this type of ideal engine.

Processes of Carnot's cycle can be denoted by an indicator diagram.

source

Isothermal expansion $A \rightarrow B$

Initially the cyclinder is taken to be in thermal equilibrium with the high temperature T_1 , this is initial state of working substance denoted by point A (P_1, V_1, T_1) .

After that the piston is allowed to move outward slowly. With the movement of the piston the process is very slow so that it is isothermal.

Heat from reservoir flows through the base of cylinder into the gas so temperature of the gas remains T_1 .

Gas expand and receive heat Q_1 from source and gets state $B(P_2, V_2, T_1)$

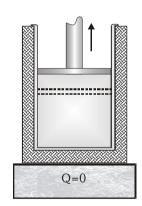
This heat input Q_1 to the gas from path A to B is utilized for doing work W_1 . By path A to B the heat input to the gas = the work done against the external pressure.

$$W_1 = Q_1 = \int_{V_1}^{V_2} P dV = \int_{V_1}^{V_2} \frac{\mu R T_1}{V} \, dV = \mu R T_1 \, \, \ell n \frac{V_2}{V_1}$$



Now cylinder is put in contact with a non-conducting stand and piston is allowed to move outward, because no heat can enter in or leave out so the expansion of gas is adiabatic. The tempeature falls to T_2K and gas describes the adiabatic process from B to **point** $C(P_3, V_3, T_2)$ during this expansion more work is done (W_2) at the expense of the internal energy.

Work done in adiabatic path BC is $W_2 = \frac{\mu R}{\gamma - 1} (T_1 - T_2)$



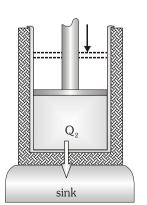
Isothermal compression $C \rightarrow D$

Now the gas cylinder is placed in contact with sink at temperature T_2 . The piston is moved slowly inward so that heat produced during compression passes to the sink. The gas is isothermally compressed from C to **point D**. (P_4, V_4, T_2)

The heat rejected Q_2 to the cold reservoir (sink) at T_2 occurs over this path.

Amount of work done on gas W_3 = amount of heat rejected to the sink

$$\boldsymbol{Q}_2 = \boldsymbol{W}_3 = \mu \boldsymbol{R} \boldsymbol{T}_2 \ell \boldsymbol{n} \left(\frac{\boldsymbol{V}_4}{\boldsymbol{V}_3} \right) \Longrightarrow \boldsymbol{Q}_2 = \mu \boldsymbol{R} \boldsymbol{T}_2 \ell \boldsymbol{n} \left(\frac{\boldsymbol{V}_4}{\boldsymbol{V}_3} \right)$$



Q=0

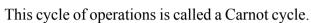


Adiabatic compression $D \rightarrow A$

The cylinder is removed from the sink and is put in contact with insulating stand now piston moves inward. Heat is not allowed to go out and it increases the internal energy of the system. Now work is done on the gas during adiabatic compression from state D to initial point A (P_1, V_1, T_1) .

No heat exchanges occur over the adiabatic path.

Work done on the system $W_4 = \frac{\mu R}{\gamma - 1} (T_2 - T_1)$



In first two steps work is done by engine W₁ and W₂ are positive

In last two steps work is done on gas W_3 and W_4 are negative

The work done in complete cylce W = the area of the closed part of the P–V cycle.

$$W = W_1 + W_2 + W_3 + W_4$$

$$Efficiency of Carnot Engine, \quad \eta = \frac{W}{Q_1} = \frac{\mu R T_1 \ell n \frac{V_2}{V_1} + \mu R T_2 \ell n \frac{V_4}{V_3}}{\mu R T_1 \ell n \frac{V_2}{V_1}}$$

B to C and D to A are adiabatic paths

so
$$T_1 V_2^{(\gamma - 1)} = T_2 V_3^{(\gamma - 1)}$$
 and $T_1 V_1^{(\gamma - 1)} = T_2 V_4^{(\gamma - 1)} \Rightarrow \frac{V_2}{V_1} = \frac{V_3}{V_4}$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1} \\ \Rightarrow \quad \eta = 1 - \frac{Q_2}{Q_1} = 1 - \frac{T_2}{T_1} \\ \frac{Q_1}{T_1} = \frac{Q_2}{T_2}$$

$$\eta = 1 - \frac{1}{Q_1} = 1 - \frac{1}{T_1} \qquad \qquad T_1$$

$$\eta = \frac{T_1 - T_2}{T_1} \times 100\%$$

$$\Rightarrow \quad \eta = \frac{Q_1 - Q_2}{Q_1} \times 100\%$$

The efficiency for the Carnot engine is the best that can be obtained for any heat engine.

The efficiency of a Carnot engine is never 100% because it is 100% only if temperature of sink $T_2 = 0$ k and $T_1 = \infty$ which is impossible.

CARNOT THEOREM

No irreversible engine (I) can have efficiency greater than Carnot reversible engine (R) working

between same hot and cold reservoirs.
$$\eta_R > \eta_1 \implies 1 - \frac{T_2}{T_1} > 1 - \frac{Q_2}{Q_1}$$

4

1. SECOND LAW OF THERMODYNAMICS

The first law of thermodynamics is a generalization of the law of conservation of energy to include heat energy. It tells us that heat and mechanical work are mutually interconvertible.

Second law of thermodynamics tells us in what conditions heat can be converted into useful work.

The following three conditions must be fulfilled to utilize heat for useful work:

- (i) A device called engine with a working substance is essential.
- (ii) The engine must work in a reversible cyclic process.
- (iii) The engine must operate between two temperatures. It will absorb heat from a hot body (called source), and converts a part of it into useful work and reject the rest to a cold body (called sink).

There are two conventional statements of second law:

Kelvin-Planck Statement

It is impossible for an engine working between a cyclic process to extract heat from a reservoir and convert completely into work. In other words, 100% conversion of heat into work is impossible.

Clausius Statement

It is impossible for a self-acting machine, unaided by any external agency to transfer heat from a cold to hot reservoir. In other words heat can not in itself flow from a colder to a hotter body.

2. HEAT ENGINE

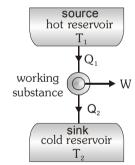
Heat engine is a device which converts heat into work.

Three parts of a heat engine:

- (i) Source of high temperature reservoir at temperature T_1
- (ii) Sink or low temperature reservoir at temperature T₂
- (iii) Working substance.

In a cycle of heat engine the working substance extracts heat \mathbf{Q}_1 from source, does some work W and rejects remaining heat \mathbf{Q}_2 to the sink.





Efficiency of heat engine
$$\eta = \frac{\text{work done (W)}}{\text{heat taken from source (Q_1)}}$$

$$\eta = \frac{T_1 - T_2}{T_1} = \frac{Q_1 - Q_2}{Q_1}$$

Ex. A carnot engine working between 400 K and 800 K has a work output of 1200 J per cycle. What is the amount of heat energy supplied to the engine from source per cycle?

Sol. W = 1200 J,
$$T_1 = 800 \text{ K}$$
, $T_2 = 400 \text{ K}$

$$\therefore \ \eta = 1 - \frac{T_2}{T_1} = \frac{W}{Q_1} \ \Rightarrow \ 1 - \frac{400}{800} = \frac{1200}{Q_1} \ \Rightarrow 0.5 = \frac{1200}{Q_1}$$

Heat energy supplied by source $Q_1 = \frac{1200}{0.5} = 2400$ joule per cycle

- **Ex.** The temperatures T_1 and T_2 of the two heat reservoirs in an ideal carnot engine are 1500°C and 500°C respectively. Which of the following : increasing T_1 by 100°C or decreasing T_2 by 100°C would result in a greater improvement in the efficiency of the engine?
- **Sol.** $T_1 = 1500$ °C = 1500 + 273 = 1773 K and $T_2 = 500$ °C = 500 + 273 = 773 K.

The efficiency of a carnot's engine $\eta = 1 - \frac{T_2}{T_1}$

When the temperature of the source is increased by 100° C, keeping T_2 unchanged, the new temperature of the source is $T'_1 = 1500 + 100 = 1600^{\circ}$ C = 1873 K. The efficiency becomes

$$\eta' = 1 - \frac{T_2}{T_1}' = 1 - \frac{773}{1873} = 0.59$$

On the other hand, if the temperature of the sink is decreased by 100° C, keeping T₁ unchanged, the new temperature of the sink is T'₂ = $500 - 100 = 400^{\circ}$ C = 673 K. The efficiency now becomes

$$\eta^{\prime\prime} = 1 - \frac{T^{\prime}_{2}}{T_{1}} = 1 - \frac{673}{1773} = 0.62$$

Since η'' is greater than η' , decreasing the temperature of the sink by 100°C results in a greater efficiency than increasing the temperatue of the source by 100°C.

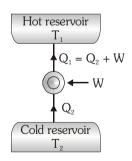
- **Ex.** A heat engine operates between a cold reservoir at temperature $T_2 = 300$ K and a hot reservoir at temperature T_1 . It takes 200 J of heat from the hot reservoir and delivers 120 J of heat to the cold reservoir in a cycle. What could be the minimum temperature of hot reservoir?
- **Sol.** Work done by the engine in a cycle is W = 200 120 = 80 J. $\eta = \frac{W}{Q} = \frac{80}{200} = 0.4$

From carnot's Theorem
$$0.4 \le 1 - \frac{T_2}{T_1} = 1 - \frac{300}{T_1}$$
 or $\frac{300}{T_1} \le 0.6$ or $T_1 \ge \frac{300}{0.6}$ or $T_1 \ge 500$

3. REFRIGERATOR

It is inverse of heat engine. It extracts heat (Q_2) from a cold reservoir, External work W is done on it and rejects heat (Q_1) to hot reservoir. The coefficient of performance of a refrigerator.

$$\beta = \frac{\text{heat extracted from cold reservoir}}{\text{work done on refrigerator}} \ = \frac{Q_2}{W} = \frac{Q_2}{Q_1 - Q_2} = \frac{1}{\frac{Q_1}{Q_2} - 1}$$



For Carnot reversible refrigerator $\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$

$$\therefore \ \beta = \frac{Q_2}{W} = \frac{1}{\left[\frac{Q_1}{Q_2} - 1\right]} = \frac{1}{\left[\frac{T_1}{T_2} - 1\right]} \implies \beta = \frac{T_2}{T_1 - T_2}$$

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A carnot engine works as a refrigrator between 250 K and 300 K. If it receives 750 cal of heat from Ex. the reservoir at the lower temperature. Calculate the amount of heat rejected at the higher temperature.

Sol.
$$T_1 = 300 \text{ K}$$

$$T_2 = 250 \text{ K}$$

$$Q_2 = 750$$

$$Q_1 = ?$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$

$$\frac{Q_1}{Q_2} = \frac{T_1}{T_2}$$
 $Q_1 = \frac{300}{250} \times 750 = 900 \text{ cal}$

The temperature insides & outside of refrigerator are 260 K and 315 K respectively. Assuming that Ex. the refrigerator cycle is reversible, calculate the heat delivered to surroundings for every joule of work done.

$$T_2 = 260 \text{ K}, T_1 = 315 \text{ K}, W = 1 \text{ joule}$$

Coefficient of performance of Carnot refrigerator $\beta = \frac{Q_2}{W} = \frac{T_2}{T_1 - T_2}$

$$\therefore \frac{Q_2}{1} = \frac{260}{315 - 260} = \frac{260}{55}$$

$$\frac{Q_2}{1} = \frac{260}{315 - 260} = \frac{260}{55}$$
 \Rightarrow $Q_2 = \frac{260}{55} = 4.7 \text{ J}$

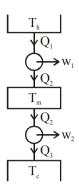
A refrigerator takes heat from water at 0°C and transfer it to room at 27°C. If 100 kg of water is converted in ice at 0°C then calculate the workdone. (Latent heat of ice is 3.4×10^5 J/kg)

Sol. Coefficient of performance (COP) =
$$\frac{T_2}{T_1 - T_2} = \frac{273}{300 - 273} = \frac{273}{27}$$

$$W = \frac{Q_2}{COP} = \frac{mL}{COP} = \frac{100 \times 3.4 \times 10^5}{273 / 27} = \frac{100 \times 3.4 \times 10^5 \times 27}{273} = 3.36 \times 10^6 \, J$$

OBJECTIVE (O)

Suppose that two heat engines are connected in series, such that the heat exhaust of the first engine is used as the heat input of the second engine as shown in figure. The efficiencies of the engines are η_1 and η_2 , respectively. The net efficiency of the combination is given by



(A)
$$\eta_{net} = \eta_2 + (1 - \eta_1)\eta_2$$

(B)
$$\eta_{\text{net}} = \frac{\eta_1}{\left(1 - \eta_1\right)\eta_2}$$

(C)
$$\eta_{\text{net}} = \eta_1 + (1 - \eta_1)\eta_2$$

$$\text{(D)} \ \eta_{\text{net}} = \frac{1 - \eta_1}{\left(1 - \eta_2\right)\eta_2}$$

- 2. "Heat cannot be itself flow from a body at lower temperature to a body at higher temperature" is a statement or consequence of [AIEEE 2003]
 - (A) second law of thermodynamics
- (B) conservation of momentum

(C) conservation of mass

- (D) first law of thermodynamics
- 3. A carnot engine takes 3×10^6 cal of heat from a reservoir at 627 °C and gives it to a sink at 27 °C. The work done by the engine is
 [AIEEE 2003]
 - (A) $4.2 \times 10^6 \,\mathrm{J}$
- (B) $8.4 \times 10^6 \,\text{J}$
- (C) $16.8 \times 10^6 \,\mathrm{J}$
- (D) zero
- 4. The specific heat capacity of a metal at low temperature (T) is given as $C_p(kJk^{-1}kg^{-1}) = 32\left(\frac{T}{400}\right)^3$

A 100 gram vessel of this metal is to be cooled from 20°K to 4°K by a special refrigerator operating at room temperature (27°C). The amount of work required to cool the vessel is:- [AIEEE-2011]

(A) equal to 0.002 kJ

- (B) greater than 0.148 kJ
- (C) between 0.148 kJ and 0.028 kJ
- (D) less than 0.028 kJ
- 5. A Carnot engine, whose efficiency is 40% takes in heat from a source maintained at a temperature of 500 K. It is desired to have an engine of efficiency 60%. Then, the intake temperature for the same exhaust (sink) temperature must be:
 [AIEEE-2012]
 - (A) 600 K
 - (B) efficiency of Carnot engine cannot be made larger than 50%
 - (C) 1200 K
 - (D) 750 K
- 6. A solid body of constant heat capacity 1 J°/C is being heated by keeping it in contact with reservoirs in two ways [JEE-Mains-2015]
 - (i) Sequentially keeping in contact with 2 reservoirs such that each reservoir supplies same amount of heat.
 - (ii) Sequentially keeping in contact with 8 reservoirs such that each reservoir supplies same amount of heat

In both the cases body is brought from initial temperature 100°C to final temperature 200°C. Entropy change of the body in the two cases respectively is -

- (A) ln2, 2ln2
- (B) 2ln2, 8ln2
- (C) ln2, 4ln2
- (D) ln2, ln2

(A)
$$\frac{\gamma M v^2}{2R}$$

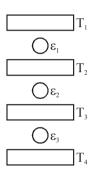
(B)
$$\frac{(\gamma-1)}{2R}$$
 Mv²

(B)
$$\frac{\left(\gamma-1\right)}{2R}$$
 Mv^2 (C) $\frac{\left(\gamma-1\right)}{2\left(\gamma+1\right)R}$ Mv^2 (D) $\frac{\left(\gamma-1\right)}{2\gamma R}$ Mv^2

(D)
$$\frac{(\gamma-1)}{2\gamma R} M v^2$$

8. Two Carrnot engines A and B are operated in series. The first one, A, receives heat at T_1 (= 600 K) and rejects to a reservoir at temperature T₂. The second engine B receives heat rejected by the first engine and, in turn, rejects to a heat reservoir at T₃ (= 400 K). Calculate the temperature T₂ if the work outputs of the two engines are equal: [JEE-Mains-2019_Jan]

Three Carnot engines operate in series between a heat source at a temperature T₁ and a heat sink at 9. temperature T_4 (see figure). There are two other reservoirs at temperature T_2 , and T_3 , as shown, with $T_2 > T_2 > T_3 > T_4$. The three engines are equally efficient if: [JEE-Mains-2019_Jan]



(A)
$$T_2 = (T_1^2 T_4)^{1/3}$$
; $T_3 = (T_1 T_4^2)^{1/3}$

(B)
$$T_2 = (T_1 T_4^2)^{1/3}$$
; $T_3 = (T_1^2 T_4)^{1/3}$

(C)
$$T_2 = (T_1^3 T_4)^{1/4}; T_3 = (T_1 T_4^3)^{1/4}$$

(D)
$$T_2 = (T_1 T_4)^{1/2}; T_3 = (T_1^2 T_4)^{1/3}$$

10. An ideal gas is enclosed in a cylinder at pressure of 2 atm and temperature, 300 K. The mean time between two successive collisions is 6×10^{-8} s. If the pressure is doubled and temperature is increased to 500 K, the mean time between two successive collisions will be close to:

[JEE-Mains-2019_Jan]

(A)
$$4 \times 10^{-8}$$
s

(B)
$$3 \times 10^{-6}$$
s

(C)
$$2 \times 10^{-7}$$
s

(D)
$$0.5 \times 10^{-8}$$
s

A 25 \times 10⁻³ m³ volume cylinder is filled with 1 mol of O₂ gas at room temperature (300K). The 11. molecular diameter of O₂, and its root mean square speed, are found to be 0.3 nm, and 200 m/s, respectively. What is the average collision rate (per second) for an O₂ molecule?

[JEE-Mains-2019_April]

$$(A) \sim 10^{11}$$

(B)
$$\sim 10^{13}$$

$$(C) \sim 10^{10}$$

(D)
$$\sim 10^{12}$$

The number density of molecules of a gas depends on their distance r from the origin as, $n(r) = n_0 e^{-\alpha r^4}$. **12.** Then the total number of molecules is proportional to: [JEE-Mains-2019 April]

(A)
$$n_0 \alpha^{1/4}$$

(B)
$$n_0 \alpha^{-3}$$

(C)
$$n_0 \alpha^{-3/4}$$

(D)
$$\sqrt{n_0} \alpha^{1/2}$$

A Carnot engine has an efficiency of 1/6. When the temperature of the sink is reduced by 62°C, its **13.** efficiency is doubled. The temperatures of the source and the sink are, respectively

[JEE-Mains-2019_April]

CURRENT ELECTRICITY (COLOR CODING)

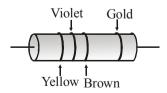
THEORY

COLOUR CODE FOR CARBON RESISTORS

Colour	Strip A	Strip B	Strip C	Strip D (Tolerance)
Black	0	0	10°	
Brown	1	1	10^{1}	
Red	2	2	10^{2}	
Orange	3	3	10^{3}	
Yellow	4	4	10^{4}	
Green	5	5	10 ⁵	
Blue	6	6	10^{6}	
Violet	7	7	10^{7}	
Grey	8	8	10^{8}	
White	9	9	10^{9}	
Gold	-	-	10-1	± 5 %
Silver	-	_	10^{-2}	± 10 %
No colour	-	-	-	± 20 %

Aid to memory BBROY
Great Britain
Very Good watch of gold & silver

- Ex. Draw a colour code for 42 k $\Omega \pm 10\%$ carbon resistance.
- **Sol.** According to colour code colour for digit 4 is yellow, for digit 2 it is red, for 3 colour is orange and 10% tolerance is represented by silver colour. So colour code should be yellow, red, orange and silver.
- **Ex.** What is resistance of following resistor.

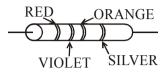


Sol. Number for yellow is 4, Number of violet is 7 Brown colour gives multiplier 10^1 , Gold gives a tolerance of \pm 5% So resistance of resistor is $47 \times 10^1 \Omega \pm 5\% = 470 \pm 5\% \Omega$.

OBJECTIVE (O)

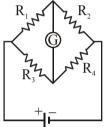
1. A resistance is shown in the figure. Its value and tolerance are given respectively by:

[JEE-Main(online)-2019]



- (A) $27 \text{ K}\Omega$, 20%
- (B) 270 KΩ, 5%
- (C) 270 K Ω , 10%
- (D) 27 K Ω , 10%
- 2. The Wheatstone bridge shown in Fig. here, gets balanced when the carbon resistor used as R_1 has the colour code (Orange, Red, Brown). The resistors R_2 and R_4 are 80Ω and 40Ω , respectively. Assuming that the colour code for the carbon resistors gives their accurate values, the colour code for the carbon resistor, used as R_3 , would be:

 [JEE-Main(online)-2019]



(A) Red, Green, Brown

(B) Brown, Blue, Brown

(C) Grey, Black, Brown

- (D) Brown, Blue, Black
- 3. A 2 W carbon resistor is color coded with green, black, red and brown respectively. The maximum current which can be passed through this resistor is: [JEE-Main(online)-2019]
 - (A) 63 mA
- (B) $0.4 \, \text{mA}$
- (C) 100 mA
- (D) 20 mA
- 4. A 200 Ω resistor has a certain color code. If one replaces the red color by green in the code, the new resistance will be: [JEE-Main(online)-2019]
 - (A) 100Ω
- (B) 400Ω
- (C) 500 Ω
- (D) 300Ω
- **5.** A carbon resistance has a following colour code. What is the value of the resistance?

[JEE-Main(online)-2019]



- (A) 1.64 M $\Omega \pm 5\%$
- (B) 530 k $\Omega \pm 5\%$
- (C) $64 \text{ k}\Omega \pm 10\%$
- (D) 5.3 M $\Omega \pm 5\%$

MAGNETISM

(EARTH MAGNETISM, MAGNETIC PROPERTIES, BAR MAGNET)

THEORY

MISCELLENEOUS

Magnetic field of long Bar magnet



(i) At Axial position :-

Magnetic field at point 'P' due to north pole $B_1 = \frac{\mu_0}{4\pi} \frac{m}{(r-\ell)^2}$ (away from north pole)

Magnetic field at point 'P' due to south pole $B_2 = \frac{\mu_0}{4\pi} \frac{m}{(r+\ell)^2}$ (towards north pole)

Net magnetic field at point 'P'

$$B_{axis} = B_1 - B_2, \ (\because B_1 > B_2) = \frac{\mu_0 m}{4\pi} \left[\frac{1}{(r-\ell)^2} - \frac{1}{(r+\ell)^2} \right] = \frac{\mu_0 m}{4\pi} \left[\frac{4r\ell}{(r^2-\ell^2)^2} \right]$$

$$B_{axis} = \frac{\mu_0}{4\pi} \frac{2Mr}{(r^2 - \ell^2)^2}$$
, where $M = m (2\ell)$

If magnet is short $r \gg \ell$, then $B_{axis} = \frac{\mu_0}{4\pi} \frac{2M}{r^3}$

(ii) At equatorial position :-

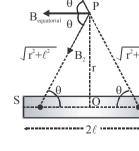
Magnetic field at point 'P' due to north pole :-

$$B_1 = \frac{\mu_0}{4\pi} \frac{m}{\left(\sqrt{r^2 + \ell^2}\right)^2} \dots (1)$$
 (along NP line)

Magnetic field at point 'P' due to south pole :-

$$B_2 = \frac{\mu_0}{4\pi} \frac{m}{\left(\sqrt{r^2 + \ell^2}\right)^2} \dots (2)$$
 (along PS line)

(along PS line)



From equation (1) & (2) $B_1 = B_2 = \frac{\mu_0}{4\pi} \cdot \frac{m}{r^2 + \ell^2} = B$ (Let)

Net magnetic field at point 'P'

$$B_{eq} = 2 B \cos\theta = 2 \cdot \frac{\mu_0}{4\pi} \frac{m}{(r^2 + \ell^2)} \cos\theta , \qquad [\text{where } \cos\theta = \frac{\ell}{\sqrt{(r^2 + \ell^2)}}]$$

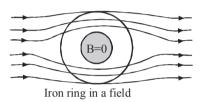
$$= 2 \! \cdot \! \frac{\mu_0}{4\pi} \frac{m}{(r^2 + \ell^2)} \frac{\ell}{\sqrt{(r^2 + \ell^2)}}$$

$$B_{\text{eq.}} = \frac{\mu_0}{4\pi} \cdot \frac{M}{(r^2 + \ell^2)^{3/2}}, \qquad \text{where } M = m(2\ell)$$

If magnet is short $r \gg \ell$, then $B_{eq.} = \frac{\mu_0}{4\pi} \cdot \frac{M}{r^3}$

Magnetic shielding

If a soft iron ring is placed in magnetic field, most of the lines are found to pass through the ring and no lines pass through the space inside the ring. The inside of the ring is thus protected against any external magnetic effect. This phenomenon is called magnetic screening or shielding and is used to protect costly wrist-watches and other instruments from



external magnetic fields by enclosing them in a soft-iron case or box.

- **(i)** Super conductors also provides perfect magnetic screening due to exclusion of lines of force. This effect is called 'Meissner effect'
- Relative magnetic premeability of super conductor is zero. So we (ii) can say that super conductors behaves like perfect dimagnetic.

B=0Super conductor in a field

Dipole - Dipole Interactions:

S.No. Relative position of dipoles

(a)
$$M_1$$
 M_2 M_3 M_4

$$(b) \qquad \stackrel{M_1 \bullet}{\underset{F_m}{\longleftarrow}} \qquad \stackrel{r}{\underset{F_m}{\longrightarrow}} \qquad \stackrel{M}{\underset{F_m}{\longleftarrow}} \qquad \qquad \\$$

(c)
$$M_1$$
 M_2 M_2

Magnetic force (F_m)

$$\frac{\mu_0}{4\pi} \cdot \frac{6M_1M_2}{r^4} \; (along \; r)$$

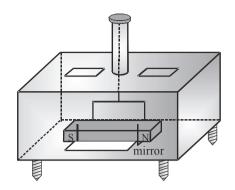
$$\frac{\mu_0}{4\pi} \cdot \frac{3M_1M_2}{r^4} \text{ (along r)}$$

$$\frac{\mu_0}{4\pi} \cdot \frac{3M_1M_2}{r^4}$$
 (perpendicular to r)

VIBRATION MAGNETOMETER:

It is an instrument used to compare the horizontal components of magnetic field of earth of two different places, to compare magnetic fields and magetic moments of two bar magnets. It is also called oscillation magnetometer.

Principle: This device works on the principle, that whenever a freely suspended bar magent horizontal component in earth magnetic field (B_H) is slightly disturbed from its equilibrium position then, it will experience a torque and executes angular S.H.M. *Rotation is possible only in horizontal plane.

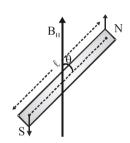


Angular S.H.M of magnetic dipole: When a dipole is suspended in a uniform magnetic field it will align itself parallel to field. Now if it is given a small angular displacement θ about its equilibrium position. The restoring torque acts on it:

$$\tau = -M B_H \sin \theta \implies I \alpha = -M B_H \sin \theta = -M B_H \theta, (\because \sin \theta \simeq \theta)$$

$$\Rightarrow \alpha = \frac{MB_{H}}{I}(-\theta) \Rightarrow \alpha = \omega^{2}(-\theta) \Rightarrow \omega^{2} = \frac{MB_{H}}{I}$$

The time period of angular S.H.M. $\Rightarrow T = 2\pi \sqrt{\frac{I}{MB_{_H}}}$



M = magnetic moment of bar magnet

I = moment of inertia of bar magnet about its geometric axis

Comparision of magnetic moments of magnets of the same size

Let the two magnets of same size have moment of inertia I and magnetic moments M_1 and M_2 . Suspend the two given magnets turn by turn in the metal stirrup of the vibration magnetometer and note the time period in each case.

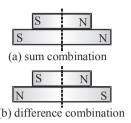
Then
$$T_1 = 2\pi \sqrt{\frac{I}{M_1 B}}$$
 and $T_2 = 2\pi \sqrt{\frac{I}{M_2 B}}$

Dividing,
$$\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}}$$
 or $\frac{M_1}{M_2} = \frac{T_2^2}{T_1^2}$

Since T_1 and T_2 are known therefore the ratio $\frac{M_2}{M_1}$ can be determined.

Comparision of magnetic moments of magnets of different sizes

Let the two magnets have moments of inertia I_1 and I_2 and magnetic moments M_1 and M_2 respectively. Place the two given magnets one upon the other as shown in Fig. (a). This combination is called sum combination'. It has moment of inertia $(I_1 + I_2)$ and magnetic moment $(M_1 + M_2)$. Put this combination in the magnetometer and set it into oscillations. The time period T_1 is determined.



$$T_1 = 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 + M_2)B}}$$
(1)

Now, the two magnets are placed as shown in Fig. (b). This combination is called 'difference combination'. It has moment of inertia $(I_1 + I_2)$ and magnetic moment $(M_1 - M_2)$. This combination is put in the magneto meter and its time period T_2 is determined.

$$T_2 = 2\pi \sqrt{\frac{I_1 + I_2}{(M_1 - M_2)B}}$$
(2)

Dividing,
$$\frac{T_1}{T_2} = \sqrt{\frac{M_1 - M_2}{M_1 + M_2}}$$
 [from equation (1) and (2)]

knowing T_1 and T_2 , we can determine $\frac{M_1}{M_2}$.

© Comparision of earth's magnetic field at two different places

Let the vibrating magnet have moment of inertia I and magnetic moment M. Let it be vibrated in places where earth's magnetic field is $B_{\rm H_1}$ and $B_{\rm H_2}$.

Then,
$$T_1 = 2\pi \sqrt{\frac{I}{MB_{H_1}}}$$
 and $T_2 = 2\pi \sqrt{\frac{I}{MB_{H_2}}}$

 T_1 and T_2 are determined by placing magnetometer at two different places, turn by turn.

Dividing,
$$\frac{T_1}{T_2} = \sqrt{\frac{B_{H_2}}{B_{H_1}}}$$
 or $\frac{T_1^2}{T_2^2} = \frac{B_{H_2}}{B_{H_1}} = \frac{B_2 \cos \theta_2}{B_1 \cos \theta_1} \Rightarrow \frac{B_1}{B_2} = \frac{T_2^2 \cos \theta_2}{T_1^2 \cos \theta_1}$

Knowing T_1 , T_2 and θ_1 , θ_2 the ratio $\frac{B_1}{B_2}$ can be determined.

Magnetic moments of two identical magnets are M and 2M respectively. Both are combined in such Ex. a way that their similar poles are same side. The time period in this is case 'T₁'. If polrity of one of the magnets is reversed its period becomes 'T₂' then find out ratio of their time periods respectively.

$$I = 2M$$

$$M_{\text{system}} = 2M + M = 3M$$

$$I_{\text{system}} = 2I$$

$$T = 2\pi \sqrt{\frac{I}{MB_H}} (I_{system} \rightarrow same, B_H \rightarrow same) \left[T \propto \frac{1}{\sqrt{M}} \right];$$

$$\frac{T_1}{T_2} = \sqrt{\frac{M_2}{M_1}} = \sqrt{\frac{M}{3M}} = \frac{1}{\sqrt{3}}$$



$$M_{\text{system}} = 2M - M = M$$

$$I_{\text{system}} = 2I$$

$$T \propto \frac{1}{\sqrt{M}}$$

A magnet is suspended in such a way when it oscillates in the horizontal plane. It makes 20 oscillations per minute at a place where dip angle is 30° and 15 oscillations per min at a place where dip angle is 60°. Find the ratio of the total earth's magnetic field at the two places.

Sol.
$$f = \frac{1}{2\pi} \sqrt{\frac{MB_H}{I}}$$

$$\Rightarrow f^2 = \frac{1}{4\pi^2} \cdot \frac{MB\cos\theta}{I}$$
 I and M are same in given cases

$$\frac{B_1}{B_2} = \frac{f_1^2}{f_2^2} \times \frac{\cos \theta_2}{\cos \theta_1} = \frac{20 \times 20}{15 \times 15} \times \frac{\cos 60^\circ}{\cos 30^\circ} = \frac{16}{9\sqrt{3}}$$

NEUTRAL POINT

It is a point where net magnetic field is zero.

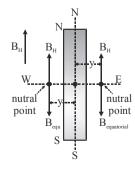
At this point magnetic field of bar magnet or current carrying coil or current carrying wire is just neutralised by magnetic field of earth. (B₁₁)

A compass needle placed at this neutral point can set itself in any direction.

Location of Neutral Points:

When N-pole of magnet directed towards North :- Two neutral (a) points symmetrically located on equatorial line of magnet. Let distance of each neutral point from centre of magnet is 'y' then

$$\begin{split} B_{eq} &= B_{H} \\ B_{H} &= \frac{\mu_{0}}{4\pi} \cdot \frac{M}{(y^{2} + \ell^{2})^{3/2}} \\ \frac{\mu_{0}}{4\pi} \cdot \frac{M}{y^{3}} &= B_{H} \qquad (If y >>> \ell) \end{split}$$

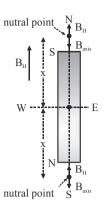


(b) When S-pole of magnet directed towards North: Two neutral points symmetrically located on the axial line of magnet. Let distance of each neutral points from centre of the magnet is x, then

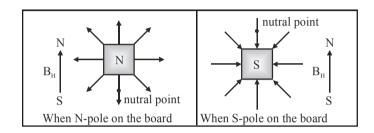
$$B_{axis} = B_{H}$$

$$\Rightarrow B_{\rm H} = \frac{\mu_0}{4\pi} \cdot \frac{2Mx}{(x^2 - \ell^2)^2}$$

$$\frac{\mu_0}{4\pi} \frac{2M}{x^3} = B_H$$
 (If x >>> ℓ)



(c) If magnet is held vertically on the board, then only one neutral point is obtained on the horizontal board.



Ex. The magnetic field at a point x on the axis of a small bar magnet is equal to the field at a point y on the equator of the same magnet. Find the ratio of the distances of x and y from the centre of the magnet.

Sol.
$$B_{axis} = B_{equatorial} \Rightarrow \frac{\mu_0}{4\pi} \frac{2M}{x^3} = \frac{\mu_0}{4\pi} \frac{M}{y^3}$$

$$\Rightarrow \frac{2}{x^3} = \frac{1}{y^3} \Rightarrow \frac{x^3}{y^3} = \frac{2}{1}$$

$$\Rightarrow \frac{x}{y} = 2^{1/3}$$

Ex. A coil of 0.1 m radius and 100 turns placed perpendicular magnetic meridian. When current of 2 ampere is flow through the coil then the neutral point is obtained at the centre. Find out magnetising field of earth.

Sol. Magnetic field at centre of coil
$$B = \frac{\mu_0 NI}{2R} = \mu_0 H_C$$
 $(:H_C = H_e)$

Magnetising field of earth
$$H_e = \frac{NI}{2R} = \frac{100 \times 2}{2 \times 0.1} = 1000 \text{ A/m}$$

Ex. A short magnet of moment $6.75 \text{ A}-\text{m}^2$ produces a neutral point on its axis. If the horizontal component of earth's magnetic field $5 \times 10^{-5} \text{ Wb/m}^2$, Calculate the distance of the neutral point from the centre.

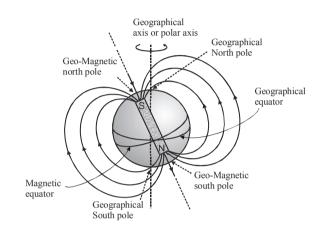
Sol.
$$B_H = \frac{2KM}{d^3} \Rightarrow d = \left(\frac{2KM}{B_H}\right)^{1/3} = \left(\frac{2 \times 10^{-7} \times 6.75}{5 \times 10^{-5}}\right)^{1/3} = 0.3 \text{ m} = 30 \text{ cm}$$

GEO-MAGNETISM

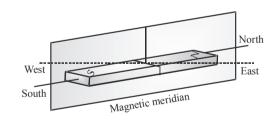
Important definitions:

- (a) Geographic axis: It is a straight line passing through the geographical poles of the earth. It is also called axis of rotation or polar axis of the earth.
- **(b) Geographic Meridian (GM)**: It is a vertical plane at any place which passing through geographical axis of the earth.
- (c) Geographic equator: It is a great circle on the surface of the earth, in a plane perpendicular to the geographic axis. All the points on the geographic equator are at equal distance from the geographic poles.

A great plane which passes through geographic equator and perpendicular to the geographic axis called **geographic equatorial plane**. This plane cuts the earth in two equal parts, a part has geographic north called **northen hemisphere** (NHS) and another part has geographic south called **southern hemi sphere** (SHS).



- (d) Magnetic axis: It is a straight line passing through magnetic poles of the earth. It is inclined to the geographic axis at nearly 17°.
- (e) Magnetic Meridian (MM): (i) It is a vertical plane at any place which passing through magnetic axis of the earth. (ii) It is a vertical plane at any place which passing through axis of free suspended bar magnet or magetic needle.
 - (iii) It is a vertical plane at any place which contains all the magnetic field lines of earth of that place.

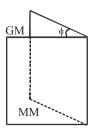


Magnetic equator: It is a great circle on the surface of the earth, in a plane perpendicular to the magnetic axis. All the points on the magnetic equator are at equal distance from the magnetic poles.

MAIN ELEMENTS OF EARTH'S MAGNETIC FIELD

Angle of declination (φ)

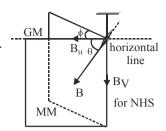
At a given place the acute angle between geographic meridian and the magnetic meridian is called angle of declination, i.e. at a given place it is the angle between the geographical north south direction and the direction indicated by a agnetic compass needle in its equillibrium.



node06\B0B0-BA\Kota\JEE\Advanced\\Leader\Physics\Sheet\Important Topics for JEEMa

Angle of dip (θ)

- (i) It is an angle which the direction of resultant magnetic field of the earth substends with the horizontal line in magnetic meridian at the given place.
- (ii) It is an angle which the axis of freely suspended magnetic needle (up or down) substends with the horizontal line in magnetic meridian at a given place.

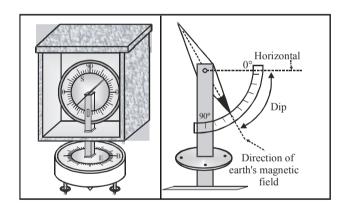


In northen hemi sphere, north pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface. In southern hemi sphere, south pole of freely suspended magnetic needle will dip downwards i.e. towards the earth surface.

Dip circle: Angle of dip at a place is measured by the instrument called 'Dip-circle' in which a magnetic needle is free to rotate in vertical plane. About its horizontal axis. The ends of the needle move over a vertical scale graduated in degree.

Horizontal component of earth magnetic field (B_{μ})

Horizontal component of earth magnetic field at a given place is the component of resultant magnetic field of the earth along the horizontal line in magnetic meridian.



$$B_{H} = B\cos\theta$$
 and $B_{v} = B\sin\theta$ (1

so that
$$\tan \theta = \frac{B_V}{B_H}$$
 and $B = \sqrt{B_H^2 + B_V^2}$ (2)

At magnetic poles $\theta = 90^{\circ}$

 $B_H = 0$ and only B_V exist

• At magnetic equator $\theta = 0^{\circ}$

 $B_v = 0$ and only B_H exist

 ϕ decides the plane in which magnetic field lies at any place, (ϕ) and (θ) decides the direction of magnetic field and (θ) and (θ _H) decides the magnitude of the field.

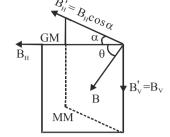
Apparent angle of dip (θ') :

When the plane of vertical scale of dip circle is in the magnetic meridian, the needle rest in the direction of earth's magnetic field. The angle made by the needle with the horizontal is called true dip or actual dip. If the plane of vertical scale of dip circle not kept in magnetic meridian, then the needle will not indicate the correct direction of earth magnetic field.

In this situation the angle made by the needle with the horizontal is called the apparent angle of dip. Suppose the dip circle is set at an angle α to the magnetic meridian. Effective horizontal component in this plane will be $B_H \cos \alpha$ and no effect on vertical component B_V

Apparent angle of dip tan $\theta' = \frac{B'_{V}}{B'_{H}}$

$$\Rightarrow \tan \theta' = \frac{B_V}{B_H \cos \alpha} \Rightarrow \tan \theta' = \frac{\tan \theta}{\cos \alpha}$$



❖ For a vertical plane other than magnetic meridian

$$\alpha \ge 0 \Rightarrow \cos \alpha \le 1 \Rightarrow \tan \theta' \ge \tan \theta \Rightarrow \theta' \ge \theta$$
,

so apparent angle of dip is always more than actual angle of dip at any place.

❖ For a vertical plane perpendicular to magnetic meridian

$$\alpha = 90^{\circ} \Rightarrow \tan \theta' = \frac{\tan \theta}{\cos 90^{\circ}} = \infty$$

 $\theta' = 90^{\circ}$, so in a plane perpendicular to magnetic meridian dip needle becomes just verticle.

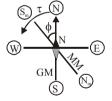
Ex. At a certain place, the horizontal component of earth's magnetic field is $\sqrt{3}$ times of the vertical component. What the angle of dip at that place.

Sol.
$$B_H = \sqrt{3} B_V$$
, $\tan \theta = \frac{B_V}{B_H} = \frac{B_V}{\sqrt{3}B_V} = \frac{1}{\sqrt{3}} = \tan 30^\circ \Rightarrow \theta = 30^\circ$

Ex. A compass needle of magnetic moment is 60 A-m^2 pointing towards geographical north at a certain place where the horizontal component of earth's magnetic field is $40\mu\text{T}$, experiences a torque $1.2 \times 10^{-3} \text{ N-m}$. What is the declination of that place.

Sol.
$$\tau = MB \sin \phi$$

$$\Rightarrow \sin\phi = \frac{\tau}{MB} = \frac{1.2 \times 10^{-3}}{24 \times 10^{-4}} = \frac{1}{2}$$
$$\Rightarrow \phi = 30^{\circ}$$



Ex. If the dip circle is set at 45° to the magnetic meridian, then the apparent dip is 30°. Calculate the true dip.

Sol.
$$\tan \theta' = \frac{\tan \theta}{\cos \alpha}$$

$$\tan\theta = \tan\theta'\cos\alpha = \tan 30^{\circ}\cos 45^{\circ} = \frac{1}{\sqrt{3}} \times \frac{1}{\sqrt{2}}$$

$$\tan\theta = \frac{1}{\sqrt{6}} \implies \theta = \tan^{-1}\left(\frac{1}{\sqrt{6}}\right)$$



Ex. A magnetic needle is free to rotate in a vertical plane and that plane makes an angle of 60° with magnetic meridian. If the needle stays in a direction making an angle of $\tan^{-1}\left(\frac{2}{\sqrt{3}}\right)$ with the horizontal direction, what would be the actual dip at that place?

Sol.
$$\tan\theta = \tan\theta'\cos\alpha$$
 $(\because \theta' = \tan^{-1}\left(\frac{2}{\sqrt{3}}\right), \alpha = 60^{\circ})$

$$\therefore \tan\theta = \tan(\tan^{-1}\frac{2}{\sqrt{3}})\cos 60^{\circ}$$

$$\tan\theta = \frac{2}{\sqrt{3}} \times \frac{1}{2} = \frac{1}{\sqrt{3}} \implies \theta = 30^{\circ}$$

- **Ex.** A 1-meter long narrow solenoid having 1000 turns is placed in magnetic meridian. Find the current in the solenoid which neutralises the earth's horizontal field of 0.36 oersted at the centre of the solenoid.
- **Sol.** The magnetic field intensity at the centre of solenoid is $H = ni = 1000 \text{ i A/m} = 4\pi \text{i}$ ovested (\therefore 1 amp/meter = $4\pi \times 10^{-3}$ oersted)

Since it neutralises the earth's field of 0.36 oersted, it is equal and opposite to the earth's field.

$$\therefore 4\pi i = 0.36$$

$$\Rightarrow$$
 i = $\frac{0.36}{4 \times 3.14}$ = 0.0286 ampere = 28.6 milli-ampere or 28.6 mA

- **Ex.** If θ_1 and θ_2 are angles of dip in two vertical planes at right angle to each other and θ is true dip then prove $\cot^2 \theta = \cot^2 \theta_1 + \cot^2 \theta_2$.
- **Sol.** If the vertical plane in which dip is θ_1 subtends an angle α with meridian than other vertical plane in which dip is θ_2 and is perpendicular to first will make an angle of 90α with magnetic meridian. If θ_1 and θ_2 are apparent dips than

$$\tan\theta_{\rm l} = \frac{B_{\rm V}}{B_{\rm H}\cos\alpha}$$

$$\tan \theta_2 = \frac{B_V}{B_H \cos(90 - \alpha)} = \frac{B_V}{B_H \sin \alpha}$$

$$\cot^{2}\theta_{1} + \cot^{2}\theta_{2} = \frac{1}{(\tan\theta_{1})^{2}} + \frac{1}{(\tan\theta_{2})^{2}} = \frac{B_{H}^{2}\cos^{2}\alpha + B_{H}^{2}\sin^{2}\alpha}{B_{V}^{2}} = \frac{B_{H}^{2}}{B_{V}^{2}} = \left(\frac{B\cos\theta}{B\sin\theta}\right)^{2} = \cot^{2}\theta$$
So $\cot^{2}\theta_{1} + \cot^{2}\theta_{2} = \cot^{2}\theta$

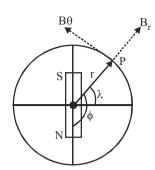
- **Ex.** Considering earth as a short bar magnet show that the angle of dip θ is related to magnetic latitude λ as $\tan\theta = 2\tan\lambda$
- **Sol.** For a magnetic dipole the field components at point P (r, ϕ) are given as

$$B_r = \frac{\mu_0}{4\pi} \ \frac{2M\cos\varphi}{r^3} \ B_\theta = \frac{\mu_0}{4\pi} \ \frac{M\sin\varphi}{r^3}$$

$$\tan \theta = \frac{B_V}{B_H} = -\frac{B_r}{B_{\theta}} = -\frac{\mu_0}{4\pi} \frac{2M\cos\phi}{r^3} \frac{4\pi r^3}{\mu_0 M\sin\phi}$$

or
$$\tan\theta = -2 \cot\phi$$

From figure
$$\phi = \frac{\pi}{2} + \lambda$$
 So $\tan \theta = -2 \cot \left(\frac{\pi}{2} + \lambda \right)$ or $\tan \theta = 2 \tan \lambda$



- Ex. At a certain location in Africa, a compass points 12° west of the geographic north. The north tip of the magnetic needle of a dip circle placed in the plane of the magnetic meridian points 60° above the horizontal. The horizontal component of the earth's field is measured to be 0.16 G. Specify the direction and magnitude of the earth's field at the location.
- **Sol.** From formula, $B_H = B \cos \theta$

$$B = \frac{B_H}{\cos \theta} = B_H \sec \theta = 0.16 \times 2 = 0.32 \text{ G}.$$

The earth's field is 0.32 G, in direction 60° upwards from horizontal, in a plane (magnetic meridian) 12° West of geographical meridian.

- **Ex.** A dip circle shows an apparent dip of 60° at a place where the true dip is 45°. If the dip circle is rotated through 90° what apparent dip will it show?
- **Sol.** Let θ_1 and θ_2 be apparent dip shown by dip circle in two perpendicular positions then true dip θ is given by

$$\cot^2 \theta = \cot^2 \theta_1 + \cot^2 \theta_2 \text{ or } \cot^2 45^\circ = \cot^2 60^\circ + \cot^2 \theta_2$$

or
$$\cot^2 \theta_2 = \frac{2}{3}$$
 or $\cot \theta_2 = 0.816$ giving $\theta_2 = 51^\circ$

IMPORTANT DEFINATIONS AND RELATIONS

Magnetising field or Magnetic intensity $(\vec{\rm H})$

Field in which a material is placed for magnetisation, called as magnetising field.

Magnetising field
$$\vec{H} = \frac{\vec{B}_0}{\mu_0} = \frac{\text{magnetic field}}{\text{permeability of free space}}$$

SI Unit \vec{H} : ampere/meter

Intensity of magnetisation (\vec{l})

When a magnetic material is placed in magnetising field then induced dipole moment per unit volume

of that material is known as intensity of magnetisation $\vec{I} = \frac{\vec{M}}{V}$

SI Unit: ampere/meter
$$\left[\because \frac{\vec{M}}{V} = \frac{\vec{IA}}{V} = \frac{\text{ampere} \times \text{meter}^2}{\text{meter}^3}\right]$$

Magnetic susceptibility (χ_m)

$$\chi_{\rm m} = \frac{I}{H}$$
 [It is a scalar with no units and dimensions]

Physically it represent the ease with which a magnetic material can be magnetised A material with more χ_m , can be change into magnet easily.

Magnetic permeability µ

$$\mu = \frac{B_{_m}}{H} = \frac{total \ magnetic \ field \ inside \ the \ material}{magnetising \ field}$$

It measures the degree to which a magnetic material can be penetrated (or permeated) by the magnetic field lines

SI Unit of
$$\mu$$
: $\mu = \frac{B_m}{H} \equiv \frac{Wb/m^2}{A/m} \equiv \frac{Wb}{A-m} \equiv \frac{H-A}{A-m} = \frac{H}{m}$
[: $\phi = L I$: weber $\equiv henry-ampere$]

Relative permeability
$$\mu_r = \frac{\mu}{\mu_0}$$

It has no units and dimensions.

Relation between permeability and susceptibility

When a magnetic material is placed in magnetic field \vec{B}_0 for magnetisation then total magnetic field in material $\vec{B}_m = \vec{B}_0 + \vec{B}_i$, where $\vec{B}_i =$ induced field.

$$\label{eq:B0} :: \quad \vec{B}_0 = \mu_0 \, \vec{H} \; ; \quad \vec{B}_i = \mu_0 \, \vec{I}$$

$$\therefore \vec{\mathbf{B}}_{m} = \mu_0 \vec{\mathbf{H}} + \mu_0 \vec{\mathbf{I}} \implies \vec{\mathbf{B}}_{m} = \mu_0 (\vec{\mathbf{H}} + \vec{\mathbf{I}}) = \mu_0 \vec{\mathbf{H}} \left(1 + \frac{\mathbf{I}}{\mathbf{H}} \right)$$

$$\Rightarrow \frac{B}{H} = \mu_0 \left(1 + \frac{I}{H} \right) \Rightarrow \mu = \mu_0 (1 + \chi_m) \Rightarrow \mu_r = 1 + \chi_m$$

for vacuum
$$\chi_m = 0$$
, $(\because \mu_r = 1)$
at STP for air $\chi_m = 0.04$
 $(\because \text{ at S.T.P. for air } \mu_r = 1.04)$

CLASSIFICATION OF MAGNETIC MATERIALS

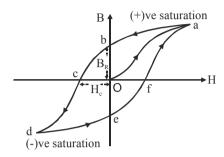
On the basis of magnetic properties of the materials [as magnetisation intensitily (I), Susceptibility (χ) and relative permeability (μ_r)] Faraday devide these materials in three classes –

PROPERTIES	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
Cause of magnetism	Orbital motion of electrons	Spin motion of electrons	Formation of domains
Substance placed in	Poor magnetisation in	Poor magnetisation in	Strong magnetisation in
uniform magnetic	opposite direction.	same direction.	same direction.
field.	Here $B_m < B_0$	Here $B_m > B_0$	Here $B_m >>> B_0$
	M	M M	M
I – H curve	$I \rightarrow Small$, negative,	$I \rightarrow Small$, positive,	$I \rightarrow \text{very large, positive}$
	varies linearly with	varies linearly with	& varies non–linearly
	field	field	with field
	I H	H	I H
$\chi_{\rm m}$ – T curve	$\chi_m \rightarrow \text{small}$, negative &	$\chi_{\rm m} \rightarrow \text{small}$, positive	$\chi_{\rm m} \rightarrow \text{very large, positive}$
	temperature independent	& varies inversely with	& temp. dependent
	$\chi_m \propto T^{\circ}$	temp.	$\chi_{\rm m} \propto \frac{1}{T - T_{\rm C}}$ (Curie Weiss
		$\chi_{\rm m} \propto \frac{1}{T}$ (Curie law)	law) (for $T > T_C$)
			$(T_C = Curie temperature)$
	χ _m Τ	χ _m Τ	$T_{C}(I_{iron})=770^{\circ}C \text{ or } 1043K$
ш	$(\mu < \mu_0)$ $1 > \mu_r > 0$	$2 > \mu_r > 1$ $(\mu > \mu_0)$	$\frac{\text{C' iron'}}{\mu_r >>> 1 \qquad (\mu >>> \mu_0)}$
μ _r Magnetic moment	Atoms donot have any	Atoms have permanent	Atoms have permanent
	•	=	=
of single atom	permanent magnetic	megnetic moment which	megnetic moment which are
	moment	are randomly oriented.	organised in domains.
		(i.e. in absence of	
		external magnetic field	
		the magnetic moment of	
		whole material is zero)	

PROPERTIES	DIAMAGNETIC	PARAMAGNETIC	FERROMAGNETIC
Behaviour of substance in Nonuniform magnetic field	It moves from stronger to weaker magnetic field.	It moves with week force from weaker magnetic field to stronger magnetic field.	Strongly attract from weaker magnetic field to stronger magnetic field.
	N S Weak Field	N S Weak Field	N S Weak Field
	N S Strong Field	N S Strong Field	N S Strong Field
	N S Level depressed in	N S	
	that limb	Level slightly rises	
When rod of material is suspended between poles of magnet.	It becomes perpendicular to the direction of external magnetic field.	If there is strong magnetic field in between the poles then rod becomes parallel to the magnetic field.	Weak magnetic field between magnetic poles can made rod parallel to field direction.
	N S	N S	N S
Magnetic moment of substance in presence of external magnetic field	Value \vec{M} is very less and opposite to \vec{H} .	Value \vec{M} is low but in direction of \vec{H} .	\vec{M} is very high and in direction of \vec{H} .
Examples	Bi, Cu, Ag, Pb, H ₂ O, Hg, H ₂ , He, Ne, Au, Zn, Sb, NaCl, Diamond.(May be found in solid, liquid or gas).	Na, K, Mg, Mn, Sn, Pt, Al, O ₂ (May be found in solid, liquid or gas.)	Fe,Co, Ni all their alloys, Fe ₃ O ₄ Gd, Alnico, etc. (Normally found only in solids) (crystalline solids)

MAGNETIC HYSTERESIS

Only Ferromagnetic materials show magnetic hysteresis, when Ferromagnetic material is placed in external magnetic field for magnetisation then B increases with H non-linearly along Oa. If H is again bring to zero then it decreases along path ab. Due to lagging behind of B with H this curve is known as hysteresis curve. [Lagging of B behind H is called hysteresis]



Cause of hysteresis: By removing external magnetising field (H = 0), the magnetic moment of some domains remains aligned in the applied direction of previous magnetising field which results into a residual magnetism.

- **Residual magnetism** (ob) = B_r = retentivity = remanence Retentivity of a specimen is a measure of the magnetic field remaining in the ferromagnetic specimen when the magnetising field is removed.
- © Coercivity (oc): Coercivity is an measure of magnetising field required to destroy the residual magnetism of the ferromagnetic specimen.

Ferromagnetic materials

Soft magnetic materials

Low retentivity, low coercivity and small hysteresis loss. suitable for making electromagnets, cores of transformers etc. Ex. Soft iron, (used in magnetic shielding)

Hard magnetic materials

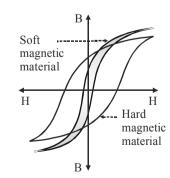
High retentivity, high coercivity and large hysteresis loss suitable for permanent magnet Ex. Steel, Alnico

HYSTERESIS LOSS

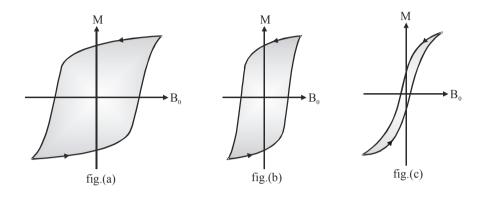
(i) The area of hystersis loop for a ferromagnetic material is equal to the energy loss per cycle of magnetisation and demagnetisation per unit volume.

$$W_{H} = \oint B.dH = \mu_{0} \oint I.dH$$

- (ii) Its value is different for different materials.
- (iii) The work done per cycle per unit volume of material is equal to the area of hysteresis loop.



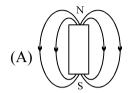
- \therefore Total energy loss in material $W_H = V A n t joule = \frac{V A n t}{J}$ calorie
- i.e W_H = volume of material × area of hysteresis curve × frequency × time.

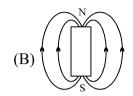


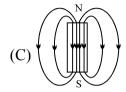
- The materials of both (a) and (b) remain strongly magnetized when B_0 is reduced to zero. The material of (a) is also hard to demagnetize, it would be good for permanent magnets.
- The material of (b) magnetizes and demagnetizes more easily, it could be used as a computer memory material.
- The material of (c) would be useful for transformers and other alternating-current devices where zero hysteresis would be optimal.

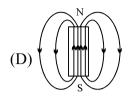
OBJECTIVE (O)

1. The magnetic field lines due to a bar magnet are correctly shown in:- [JEE 2002 (screening)]

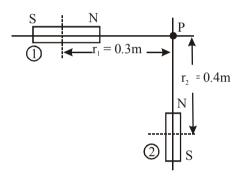




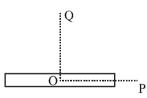




2. Two short magnets of magnetic moment 2Am² and 5Am² respectively are placed along two lines drawn at right angle to each other on the sheet of paper as shown in the figure. What is the magnetic field at the point of intersection of their axis:



- (A) $2.15 \times 10^{-5} \,\mathrm{T}$
- (B) $215 \times 10^{-5} \,\mathrm{T}$
- (C) $2.15 \times 10^{-4} \,\mathrm{T}$
- (D) $21.5 \times 10^{-5} \,\mathrm{T}$
- 3. Two bar magnets of the same mass, same length and breadth but having magnetic moments M and 2M are joined together north pole to north pole and suspended by a string. The time period of assembly in a magnetic field of strength H is 3 seconds. If now the polarity of one of the magnets is reversed and the combination is again made to oscillate in the same field, the time of oscillations is:
 - (A) $\sqrt{3}$ sec
- (B) $3\sqrt{3}$ sec
- (C) 3 sec
- (D) 6 sec
- **4.** When magnetic field at P and Q is same then OP/OQ = ?



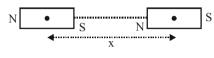
- (A) $\sqrt[3]{2}$
- (B) $\frac{1}{\sqrt[3]{2}}$
- (C) $2\sqrt{2}$
- (D) $\frac{3}{2\sqrt{2}}$
- 5. The magnetic field at a point on the axis of a dipole is 20 Wb/m² Northwards. What will be the magnetic induction at the point if the dipole be rotated anticlockwise by 90°:-
 - (A) 10 Wb/m² Eastwards

(B) 10 Web/m² Southwards

(C) 10 Wb/m² Northwards

(D) 20 Wb/m² Southwards

- 6. A bar magnet of magnetic moment 2A-m² free to rotate about a vertical axis passing through its centre. The magnet is released from rest from east-west position. Then the kinetic energy of the magnet as it takes North-South position is (horizontal component of the earth's field is 25μT and angle of declination is zero:-
 - (A) 25 μJ
- (B) $50 \mu J$
- (C) $100 \mu J$
- (D) $125 \mu J$
- 7. The mid points of two small magnetic dipoles of length d in end-on positions, are separated by a distance x, (x >> d). The force between them is proportional to x^{-n} where n is :-

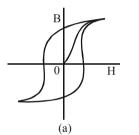


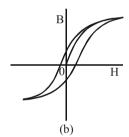
(A)3

(B) 4

(C)2

- (D) 1
- 8. The angle of dip at a place is 30° and the intensity of the vertical component of the earth's magnetic field $V = 6 \times 10^{-5}T$. The total intensity of the earth's magnetic field at this place is:
 - (A) $7 \times 10^{-5} \text{ T}$
- (B) $6 \times 10^{-5} \text{ T}$
- (C) $5 \times 10^{-5} \text{ T}$
- (D) $12 \times 10^{-5} \text{ T}$
- 9. Two types of steel are characterized by the hysteresis loops shown in figure (a) and (b). The loops are obtained in the processes of magnetization and demagnetization of the steels. Which of the two types is best suited for using as the core of a transformer and which, for using as a permanent magnet?





- (A) Both are best suited for transformer
- (B) Both are best suited for permanent magnet
- (C) A is best suited for permanent magnet & B is best suited for core of transformer
- (D) A is best suited for core of transformer & B is best suited for permanent magnet
- 10. A material satisfied the relation $\mu_0(H+M) = 0$, where H and M are magnetic intensity and magnetisation respectively, then material is:-
 - (A) nonmagnetic
- (B) paramagnetic
- (C) ferrormagnetic
- •(D) diamagnetic
- 11. Horizontal magnetic field of earth at Mumbai is 1.5×10^{-4} T in North direction. A small bar magnet of magnetic moment 10 Am^2 is kept on a horizontal table such that its North pole points due North. What is the magnetic field at 10 cm from centre of magnet at a point on its axis due North of it:-
 - (A) 3.5×10^{-4} T
- (B) $5 \times 10^{-5} \text{ T}$
- (C) 2.15×10^{-3} T
- (D) 1.15×10^{-3} T
- 12. In a moving coil galvanometer the number of turns N = 24, area of the coil $A = 2 \times 10^{-3}$ m², and the magnetic field strength B = 0.2 T To increase its current sensitivity by 25% we:-
 - (A) increase B to 0.30 T

(B) decrease A to 1.5×10^{-3} m²

(C) increase N to 30

(D) none of the above

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- 13. A thin rectangular magnet suspended freely has a period of oscillation equal to T. Now it is broken into two equal halves (each having half of the original length) and one piece is made to oscillate freely in the same field. If its period of oscillation is T', the ratio T'/T is
 [AIEEE 2003]
 - (A) $\frac{1}{2\sqrt{2}}$
- (B) $\frac{1}{2}$
- (C) 2
- (D) $\frac{1}{4}$

14. The magnetic lines of force inside a bar magnet:

[AIEEE - 2003]

- (A) are from north-pole to south-pole of the magnet
- (B) do not exist
- (C) depend upon the area of cross-section of the bar magnet
- (D) are from south-pole to north-pole of the magnet
- 15. The length of a magnet is large compared to its width and breadth. The time period of its oscillation in a vibration magnetometer is 2s. The magnet is cut along its length into three equal parts and three parts are then placed on each other with their like poles together. The time period of this combination will be
 [AIEEE 2004]
 - (A) 2s
- (B) 2/3 s
- (C) $2\sqrt{3}$ s
- (D) $2/\sqrt{3}$ s
- 16. The materials suitable for making electromagnets should have-

[AIEEE - 2004]

- (A) high retentivity and high coercivity
- (B) low retentivity and low coercivity
- (C) high retentivity and low coercivity
- (D) low retentivity and high coercivity

ELECTROMAGNETIC INDUCTION & ALTERNATING CURRENT (FORCE & DAMPED OSCILLATION)

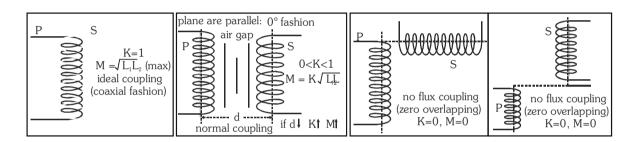
THEORY

DIFFERENT COEFFICIENT OF MUTUAL INDUCTANCE

In terms of L_1 and L_2 : For two magnetically coupled coils:

 $M = K\sqrt{L_1L_2}$ here 'K' is coupling factor between two coils and its range $0 \le K \le 1$

- For ideal coupling $K_{max} = 1 \implies M_{max} = \sqrt{L_1 L_2}$ (where M is geometrical mean of L_1 and L_2)
- For real coupling (0 < K < 1) $M = K_1/L_1$
- Value of coupling factor 'K' decided from fashion of coupling.
- Different fashion of coupling



'K' also defined as $K = \frac{\varphi_s}{\varphi_p} = \frac{\text{mag. flux linked with sec ondary (s)}}{\text{mag. flux linked with primary (p)}}$

RESONANCE

A circuit is said to be resonant when the natural frequency of circuit is equal to frequency of the applied voltage. For resonance both L and C must be present in circuit.

There are two types of resonance:

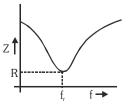
- (i) Series Resonance
 - (ii) Parallel Resonance

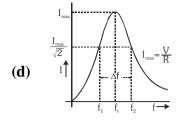
SERIES RESONANCE

- At Resonance (a)
 - (i) $X_1 = X_C$
- (ii) $V_L = V_C$ (iii) $\phi = 0$ (V and I in same phase)
- (iv) $Z_{min} = R$ (impedance minimum) (v) $I_{max} = \frac{V}{R}$ (current maximum)
- **Resonance frequency (b)**

- Variation of Z with f (c)
- If $f \le f_r$ then $X_L \le X_C$ circuit nature capacitive, ϕ (negative) (i)
- At $f = f_r$ then $X_L = X_C$ circuit nature, Resistive, $\phi = zero$ (ii)
- If $f > f_r$ then $X_L > X_C$ circuit nature is inductive, \$\phi\$ (positive) (iii)

Variation of I with f as f increase, Z first decreases then increase





as f increase, I first increase then decreases

• At resonance impedance of the series resonant circuit is minimum so it is called 'acceptor circuit' as it most readily accepts that current out of many currents whose frequency is equal to its natural frequency. In radio or TV tuning we receive the desired station by making the frequency of the circuit equal to that of the desired station.

Half power frequencies

The frequencies at which, power become half of its maximum value called half power frequencies

Band width = $\Delta f = f_2 - f_1$

Quality factor Q: Q-factor of AC circuit basically gives an idea about stored energy & lost energy.

 $Q = 2\pi \frac{\text{maximum energy stored per cycle}}{\text{maximum energy loss per cycle}}$

- (i) It represents the sharpness of resonance.
- (ii) It is unit less and dimension less quantity

(iii)
$$Q = \frac{(X_L)_r}{R} = \frac{(X_C)_r}{R} = \frac{2\pi f_r L}{R} = \frac{1}{R} \sqrt{\frac{L}{C}} = \frac{f_r}{\Delta f} = \frac{f_r}{band\ width}$$

Magnification

At resonance V_L or $V_C = QE$ (where E = supplied voltage)

So at resonance Magnification factor = Q-factor

Sharpness

Sharpness ∞ Quality factor ∞ Magnification factor

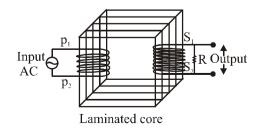
R decrease \Rightarrow Q increases \Rightarrow Sharpness increases

R_1 R_1 R_2 R_3 R_4

Transformers

One of the great advantages of ac over dc for electric-power distribution is that it is much easier to step voltage levels up and down with ac than with dc. The necessary conversion is accomplished by a static device called transformer using the principle of mutual induction.

The figure shows an idealised transformer which consists of two coils or windings, electrically insulated from each other but wound on the same core. The winding to which power is supplied is called primary, the winding from which power is delivered is called the secondary.



The ac source causes an alternating current in the primary which sets up an alternating flux in the core and this induces an emf in each winding of secondary in accordance with Faraday's law. For ideal transformer we assume that primary has negligible resistance and all the flux in core links both primary and secondary.

The primary winding has N_1 turns and secondary has N_2 turns. When the magnetic flux changes because of changing currents in the two coils, the resulting induced emf are

$$E_1 = -N_1 \frac{d\phi_B}{dt}$$
 and $E_2 = -N_2 \frac{d\phi_B}{dt}$

The flux per turn B is same in both primary and the secondary so that the emf per turn is same in each. The ratio of secondary emf E_2 to the primary emf E_1 is therefore equal at any instant to the ratio of secondary to primary turns.

$$\frac{E_2}{E_1} = \frac{N_2}{N_1}$$

If the windings have zero resistance, the induced emf 1 and 2 are equal to the terminal voltage across the primary and the secondary respectively, hence

$$\frac{V_2}{V_1} = \frac{N_2}{N_1}$$

If $N_2 > N_1$ then $V_2 > V_1$ and we have step up transformer, if $N_2 < N_1$ then $V_2 < V_1$ and we have a step down transformer.

If the transformer is assumed to be 100% efficient (no energy losses) the power input is equal to the power output i.e.

$$I_2V_2 = I_1V_1$$
 \therefore $\frac{I_2}{I_1} = \frac{N_1}{N_2}$

All the currents and voltages derived above have same frequency as that of source. The equations obtained above apply to ideal transformers, although some energy is lost but well designed transformers have efficiency more than 95%, this is a good approximation. The causes of energy losses and their rectification is given below

Cause	Rectification
1. Due to poor design and air gaps in the core, all the flux due to primary does not pass through the secondary.	1. By winding the primary and secondary coil one over the other.
2. Resistance of windings causes I ² R loss.	2. In high current, low voltage, these are minimised by using thick wire.
3. The alternating magnetic flux induces eddy currents in the core and causes heating.	3. By using laminated core it can be reduced.
4. Alternating magnetisating of core causes hystersis loss.	4. It is kept minimum by using a magnetic material having low hystersis loss. (e.g. soft iron)

6\B0B0-BA\Kata\JEE(Advanced)\Leader\Physics\Sheet\Important Topics for JEE Mains (1)\En

Damped Harmonic Oscillator:

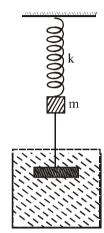
In the previous section we have considered several examples of oscillatory systems. In each case the amplitude of the oscillation does not depend on time and the system, once set to oscillate, will continue to do so forever. Physical systems are never as ideal as that. Even in the most controlled case there will be dissipative elements.

In each of these cases the amplitudes of the oscillation will gradually decrease and the motion will be 'damped'.

Most of the time we are interested in the motion of an object through air or other viscous fluids. The motion of the object is then subject to a resistive force called aerodynamic or viscous drag. Experimentally the resistive force is found to be proportional to the velocity of the body for low relative speed with respect to the medium.

A laboratory model for a damped spring-mass system is shown in figure. Where a vertical spring has at its end a mass m which is connected to massless piston which moves through a liquid. For low speeds it is reasonable to write the frictional force as

$$F_{f_f} = -\alpha v$$
 $(\alpha \text{ in kg/s})$



where α is a positive constant known as damping constant. The equation of motion of a harmonic oscillator becomes

$$m = \frac{d^2x}{dt^2} = -kx - \alpha \frac{dx}{dt}$$

which can be rewritten as

$$\frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = 0$$

where $2\gamma = \alpha/m > 0$ and ω_0^2 , as before is equal to k/m. γ is known as damping coefficient

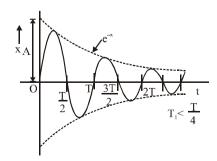
Damped Oscillations

If the dissipative forces are small, i.e. if $\gamma < 0$, the dominant force acting on the mass is the conservation restoring force. The system therefore shows oscillations, even though the amplitudes of oscillation continuously decrease with time.

we get
$$x = Ae^{-\gamma t} \sin(\Omega t + \phi)$$

where the new angular frequency of oscillation Ω is given by $\Omega^2 = \omega_0^2 - \gamma^2$

i.e the frequency shift to a lower value due to damping. The amplitude of oscillation is no longer constant but decreases with time in an exponential fashion. The variation of x with time is shown in figure.



It may be seen that $\sin(\Omega t + \phi)$ is a periodic function with period $2\pi/\Omega$. when this is multiplied with $e^{-\gamma t}$ the zeroes of the product would still be at the same place where $\sin(\Omega t + \phi)$ becomes zero. The maxima however does not fall midway between the two minima because of the exponential $e^{-\gamma t}$ but occurs at a slightly earlier time.

Power Loss in a Damped Oscillation-Q-factor

Because of the work done against the nonconservative forces, the energy stored in an oscillator continuously decreases. The fraction of energy lost in a period with respect to its average value for that period is a measure of the quality of the oscillator. The less the value of this fraction is, the more is the ability of the oscillator to sustain periodic motion. Quantitatively, the quality of an oscillator is measured through a Q-factor defined as

$$Q = 2\pi \times \frac{\text{Average energy stored in one period}}{\text{average loss of energy in that period}}$$

If the damping is light i.e. $\gamma \ll \omega_0$, The instantaneous energy of the oscillator is

$$\begin{split} E\left(t\right) &= \frac{1}{2}mv^{2} + \frac{1}{2}kx^{2} \\ &= \frac{1}{2}mA^{2}\omega_{0}^{2}e^{-2\gamma t} \end{split}$$

where we have replaced k by $m\omega_0^2$. At t = 0 the energy of the oscillator is $E_0 = mA^2 \omega_0^2/2$ so that

$$\mathrm{E}\big(t\big) = \mathrm{E}_0 e^{-2\gamma t}$$

$$P(t) = \frac{dE}{dt} = -2\gamma E_0 e^{-2\gamma t} = 2\gamma E(t)$$

Energy lost in one period can be approximately written as

$$< p(t) > \frac{2\pi}{\Omega} = 2\gamma < E(t) > \frac{2\pi}{\Omega}$$

where the angular bracket < > denotes the time average value during a period at time t. The quality or Q- factor for an oscillator is

$$2\pi \frac{\langle E(t) \rangle}{2\gamma \langle E(t) \rangle \frac{2\pi}{\Omega}} = \frac{\Omega}{2\pi} \cong \frac{\omega_0}{2\gamma}$$

It may be noted that Q is a dimensionless quantity, greater than unity by our assumption that $\gamma << \omega_0$.

Time to reach extreme:

The natural logarithm of two successive maxima is called logarithmic decrement and is denoted by λ . From Equation it follows that

$$\lambda = \frac{2\pi\gamma}{\Omega} = \frac{\pi}{Q}$$

Overdamped Motion

In equation if $\gamma > \omega_0$ we do not get oscillatory solution

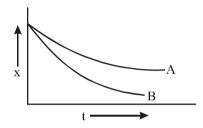
Critical Damping $(\gamma = \omega_0)$

The solution for the displacement is therefore

$$x = (A + Bt)e^{-\gamma t}$$

$$x' = -\frac{mg}{k} (1 + \gamma t) e^{-\gamma t}$$

It may be seen (hat once again there are no oscillation. As $t \to \infty$, $x \to 0$ so that the particle takes an infinite time before reaching the equilibrium position. The difference with the overdamped case is that the approach to the equilibrium is faster. The variation of displacement with time is shown in figure (curve B.)



Forced Oscillations and Resonance

The oscillations in a system can be indefinitely maintained by supplying energy continuously. In mechanical system this can be done by subjecting it to an external force which itself has harmonic time dependence.

An interesting phenomenon occurs if the frequency of the external source is equal or nearly equal to the natural frequency of the system. The amplitude of oscillations is found to increase many folds in such cases. This is called resonance.

Forced Damped Oscillations

In the presence of resistive forces proportional to velocity, equation be comes

$$\frac{d^2x}{dt^2} + 2\gamma \frac{dx}{dt} + \omega_0^2 x = \frac{F_0}{m} \cos \omega t$$

Let us therefore try a solution of the type

$$x = A \cos \omega t + B \sin \omega t + Ce^{-\lambda t}$$

The steady - state displacement is then given by

$$x = C \cos (\omega t + \delta)$$

with

$$C = \frac{F_0}{m} \sqrt{\frac{1}{\left(\omega_0^2 - \omega^2\right)^2 + 4\omega^2 \gamma^2}}$$

and
$$\tan \delta = \frac{2\omega\gamma}{\omega^2 - \omega_0^2}$$

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OBJECTIVE (O)

- 1. In a transformer, number of turns in the primary are 140 and that in the secondary are 280. If current in primary is 4A, then that in the secondary is
 - (A) 4A
- (B) 2A
- (C) 6A
- (D) 10 A
- 2. An ideal efficient transformer has a primary power input of 10kW. The secondary current when the transformer is on load is 25A. If the primary: secondary turns ratio is 8:1, then the potential difference applied to the primary coil is
 - (A) $\frac{10^4 \times 8^2}{25}$ V
 - (B) $\frac{10^4 \times 8}{25}$ V
- (C) $\frac{10^4}{25 \times 8}$ V
- (D) $\frac{10^4}{25 \times 8^2}$ V

- 3. The core of any transformer is laminated so as to -
 - (A) Make it light weight
 - (B) Make it robust and strong
 - (C) Increase the secondary voltage
 - (D) Reduce the energy loss due to eddy current
- **4.** Five particles undergodamped harmonic motion. Values for the spring constant k, the damping constant b, and the mass m are given below. Which leads to the smallest rate of loss of mechanical energy at the initial moment?
 - (A) k = 100 N/m, m = 50 g, b = 8 g/s
 - (B)k = 150N/m, m = 50g, b = 5g/s
 - (C) k = 150N/m, m = I0g, b = 8g/s
 - (D) k = 200N/m, m = 8g, b 6g/s
- 5. An RLC circuit has a capacitance of $12\mu F$, an inductance of 25mH, and a resistance of 60Ω . The current oscillates with an angular frequency of :
 - (A) 1.2×10^3 rad/s
- (B) $1.4 \times 10^3 \text{ rad/s}$
- (C) $1.8 \times 10^3 \text{ rad/s}$
- (D) $2.2 \times 10^3 \text{ rad/s}$

- (E) $2.6 \times 10^3 \text{ rad/s}$
- 6. An RLC circuit has an inductance of 25mH and a capacitance of 5.0μF. The charge on the capacitor does NOT oscillate but rather decays exponentially to zero. The resistance in the circuit must be:
 - (A) greater than or equal to $100\sqrt{2}\Omega$
 - (B) less than $100\sqrt{2}\Omega$ but greater than $50\sqrt{2}\Omega$
 - (C) less than $50\sqrt{2}\Omega$ but greater than $25\sqrt{2}\Omega$
 - (D) less than $25\sqrt{2}\Omega$ but greater than 0
- 7. A simple pendulum has a time period T if there is no air resistance. If a small air resistance is acting on the bob as it oscillates,
 - (A) The time period will be initially more than T and decreases with time.
 - (B) The time period will be less than T initially and increases with time
 - (C) The time period will be less than T and remains constant
 - (D) The time period will be more than T and remains constant.

8. The amplitude of a simple pendulum, oscillating in air with a small spherical bob, decreases from 10cm to 8 cm in 40 seconds. Assuming that Stokes law is valid, and ratio of the coefficient of viscosity of air to that of carbon dioxide is 1.3, the time in which amplitude of this pendulum will reduce from 10 cm to 5 cm in carbondioxide will be close to (ln 5 = 1.601, ln 2 = 0.693).

[JEE Main Online 2014]

(A) 231 s

(B) 208 s

(C) 142 s

(D) 161 s

9. In forced oscillation of a particle the amplitude is maximum for a frequency ω_1 of the force, while the energy is maximum for a frequency ω_2 of the force; then – [AIEEE- 2004]

(A) $\omega_1 = \omega_2$

(B) $\omega_1 > \omega_2$

(C) $\omega_1^{} < \omega_2^{}$ when damping is small and $\omega_1^{} > \omega_2^{}$ when damping is large

(D) $\omega_1 \leq \omega_2$

10. A pendulum with time period of 1s is losing energy due to damping. At certain time its energy is 45 J. If after completing 15 oscillations, its energy has become 15 J, its damping constant (in s⁻¹) is:-

[JEE Mains On line 2015]

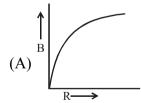
(A) 2

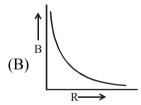
(B) $\frac{1}{15} \ln 3$

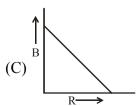
(C) $\frac{1}{2}$

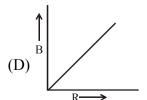
(D) $\frac{1}{30} \ln 3$

11. A charge Q is uniformly distributed over the surface of non-conducting disc of radius R. The disc rotates about an axis perpendicular to its plane and passing through its centre with an angular velocity ω. As a result of this rotation a magnetic field of induction B is obtained at the centre of the disc. If we keep both the amount of charge placed on the disc and its angular velocity to be constant and vary the radius of the disc then the variation of the magnetic induction at the centre of the disc will be represented by the figure:[AIEEE-2012]









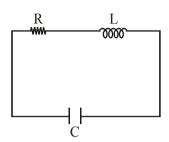
- A coil is suspended in a uniform magnetic field, with the plane of the coil parallel to the magnetic lines **12.** of force. When a current is passed through the coil it starts oscillating; it is very difficult to stop. But if an aluminium plate is placed near to the coil, it stops. This is due to :-[AIEEE - 2012]
 - (A) Electromagnetic induction in the aluminium plate giving rise to electromagnetic damping
 - (B) Development of air current when the plate is placed
 - (C) Induction of electrical charge on the plate
 - (D) Shielding of magnetic lines of force as aluminium is a paramagnetic material
- If a simple pendulum has Significant amplitude (up to a factor of 1/e of original) only in the period 13. between t = 0s to $t = \tau s$, then τ may be called the average life of the pendulum. When the spherical bob of the pendulum suffers a retardation (due to viscous drag) proportional to its velocity, with 'b' as the constant of proportionality, the average lifetime of the pendulum is (assuming damping is small) in [AIEEE-2012] seconds:
 - (A) $\frac{1}{h}$
- (B) $\frac{2}{h}$
- (C) $\frac{0.693}{h}$
- (D) b
- 14. Two short bar magnets of length 1 cm each have magnetic moments 1.20 Am² and 1.00 Am² respectively. They are placed on a horizontal table parallel to each other with their N poles pointing towards the South. They have a common magnetic equator and are separated by a distance of 20.0cm. The value of the resultant horizontal magnetic induction at the midpoint O of the line joining their centres is close to :-[**JEE**(**Mains**) - 2013]

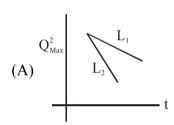
(Horizontal component of earth's magnetic induction is 3.6×10^{-5} Wb/m²)

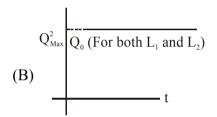
- (A) $3.6 \times 10^{-5} \text{ Wb/m}^2$
- (B) $2.56 \times 10^{-4} \text{ Wb/m}^2$
- (C) $3.50 \times 10^{-4} \text{ Wb/m}^2$
- (D) $5.80 \times 10^{-4} \text{ Wb/m}^2$
- **15.** The amplitude of adamped oscillator decreases to 0.9 times its original magnitude in 5s. In another 10s it will decrease to a times its original magnitude, where a equals: [JEE Main-2013]
 - (A) 0.81
- (B) 0.729
- (C) 0.6
- (D) 0.7
- The coercivity of a small magnet where the ferromagnet gets demagnetized is 3×10^3 A m⁻¹. The 16. current required to be passed in a solenoid of length 10 cm and number of turns 100, so that the magnet gets demagnetized when inside the solenoid, is: [JEE(Mains) - 2014]
 - (A) 3A
- (B) 6 A
- (C) 30 mA
- (D) 60 mA

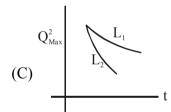
17. An LCR circuit is equivalent to a damped pendulum. In an LCR circuit the capacitor is charged to Q_0 and then connected to the L and R as shown below. If a student plots graphs of the square of maximum charge (Q_{Max}^2) on the capacitor with time (t) for two different values L_1 and L_2 ($L_1 > L_2$) of L then which of the following represents this graph correctly? (plots are schematic and not drawn to scale)

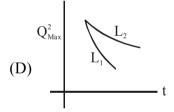
[JEE Main-2015]



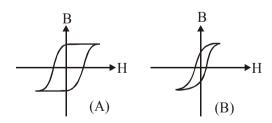








18. Hysteresis loops for two magnetic materials A and B are given below:



These materials are used to make magnets for electric generators, transformer core and electromagnet core. Then it is proper to use ; [JEE Main-2016]

- (A) B for electromagnets and transformers.
- (B) A for electric generators and transformers.
- (C) A for electromagnets and B for electric transformers.
- (D) A for transformers and B for electric generators.

- A magnetic needle of magnetic moment 6.7×10^{-2} Am² and moment of inertia 7.5×10^{-6} kg m² is 19. performing simple harmonic oscillations in a magnetic field of 0.01 T. Time taken for 10 complete oscillations is: [JEE Main-2017]
 - (A) 6.98 s
- (B) 8.76 s
- (C) 6.65 s
- (D) 8.89 s
- For an RLC circuit driven with voltage of amplitude v_m and frequency $w_0 = \frac{1}{\sqrt{LC}}$ the current exhibits 20. resonance. The quality factor, Q is given by :-

[JEE Main-2018]

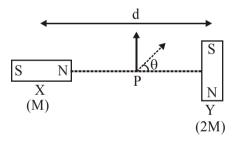
- (A) $\frac{\omega_0 R}{I}$
- (B) $\frac{R}{(\omega_0 C)}$ (C) $\frac{CR}{\omega_0}$
- (D) $\frac{\omega_0 L}{R}$
- The displacement of a damped harmonic oscillator is given by 21. $x(t) = e^{-0.1t} \cos(10\pi t + \phi)$. Here t is in seconds.

The time taken for its amplitude of vibration to drop to half of its initial value is close to:

[JEE Main-2019 April]

- (A) 13 s
- (B)7s
- (C) 27 s
- (D) 4 s
- 22. Two magnetic dipoles X and Y are placed at a separation d, with their axes perpendicular to each other. The dipole moment of Y is twice that of X. A particle of charge q is passing, through their midpoint P, at angle $\theta = 45^{\circ}$ with the horizontal line, as shown in figure. What would be the magnitude of force on the particle at that instant? (d is much larger than the dimensions of the dipole)

[JEE Main-2019_April]



(A) $\sqrt{2} \left(\frac{\mu_0}{4\pi} \right) \frac{M}{(d/2)^3} \times qv$

(B) $\left(\frac{\mu_0}{4\pi}\right) \frac{2M}{\left(d/2\right)^3} \times qv$

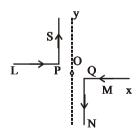
(C) $\left(\frac{\mu_0}{4\pi}\right) \frac{M}{\left(d/2\right)^3} \times qv$

- (D) 0
- A magnetic compass needle oscillates 30 times per minute at a place where the dip is 45°, and 40 23. times per minute where the dip is 30°. If B_1 and B_2 are respectively the total magnetic field due to the earth at the two places, then the ratio B_1/B_2 is best given by : [JEE Main-2019_April]
 - (A) 2.2
- (B) 1.8
- (C) 0.7
- (D) 3.6
- 24. A bar magnet is demagnetized by inserting it inside a solenoid of length 0.2 m, 100 turas, and carrying a current of 5.2 A. The coercivity of the bar magnet is: [JEE Main-2019_Jan]
 - (A) 1200 A/m
- (B) 2600 A/m
- (C) 520 A/jm
- (D) 285 A/m

- 25. At some location on earth the horizontal component of earth's magnetic field is 18×10^{-6} T. At this location, magnetic needle of length 0.12 m and pole strength 1.8 Am is suspended from its mid-point using a thread, it makes 45° angle with horizontal in equilibrium. To keep this needle horizontal, the vertical force that should be applied at one of its ends is : [JEE Main-2019_Jan]
 - (A) $3.6 \times 10^{-5} \text{ N}$
- (B) $6.5 \times 10^{-5} \text{ N}$
- (C) 1.3×10^{-5} N
- (D) $1.8 \times 10^{-5} \text{ N}$
- 26. A magnet of total magnetic moment $10^{-2}\,\hat{i}$ A-m² is placed in a time varying magnetic field, B \hat{i} (cost ω t) where B = 1 Tesla and ω = 0.125 rad/s. The work done for reversing the direction of the magnetic moment at t = 1 second, is : [JEE Main-2019_Jan]
 - (A) 0.007 J
- (B) 0.014 J
- (C) 0.01 J
- (D) 0.028 J
- 27. A paramagnetic substance in the form of a cube with sides 1 cm has a magnetic dipole moment of 20×10^{-6} J/T when a magnetic intensity of 60×10^{3} A/m is applied. Its magnetic susceptibility is:-

[JEE Main-2019_Jan]

- (A) 2.3×10^{-2}
- (B) 3.3×10^{-2}
- (C) 3.3×10^{-4}
- (D) 4.3×10^{-2}
- **28.** A paramagnetic material has 10^{28} atoms/m³. Its magnetic susceptibility at temperature 350 K is 2.8×10^{-4} . Its susceptibility at 300 K is : [**JEE Main-2019_Jan**]
 - (A) 3.672×10^{-4}
- (B) 3.726×10^{-4}
- (C) 3.267×10^{-4}
- (D) 2.672×10^{-4}
- 29. As shown in the figure, two infinitely long, identical wires are bent by 90° and placed in such a way that the segments LP and QM are along the x-axis, while segments PS and QN are parallel to the y-axis. If OP = OQ = 4cm, and the magnitude of the magnetic field at O is 10^{-4} T, and the two wires carry equal currents (see figure), the magnitude of the current in each wire and the direction of the magnetic field at O will be ($\mu_0 = 4\pi \times 10^{-7} \text{ NA}^{-2}$): [JEE Main-2019_Jan]



- (A) 40 A, perpendicular into the page
- (B) 40 A, perpendicular out of the page
- (C) 20 A, perpendicular out of the page
- (D) 20 A, perpendicular into the page

ANSWER KEY

01_HEAT & THERMODYNAMICS

OBJECTIVE (O)

1. Ans. (C) 2. Ans. (A) 3. Ans. (B) 4. Ans. (B) 5. Ans. (D)

6. Ans. (Bonus) 7. Ans. (B) 8. Ans. (C) 9. Ans. (A)

10. Ans. (A) 11. Ans. (D) 12. Ans. (C) 13. Ans. (D)

02_CURRENT ELECTRICITY

(Color coding)

OBJECTIVE (O)

1. Ans. (D) 2. Ans. (B) 3. Ans. (D) 4. Ans. (C) 5. Ans. (B)

03_MAGNETISM

(Earth magnetism, Magnetic properties, Bar magnet)

OBJECTIVE (O)

1. Ans. (D) 2. Ans. (A) 3. Ans. (B) 4. Ans. (A) 5. Ans. (A) 6. Ans. (B)

7. Ans. (B) 8. Ans. (D) 9. Ans. (C) 10. Ans. (D) 11. Ans. (C) 12. Ans. (C)

13. Ans. (B) 14. Ans. (D) 15. Ans. (A) 16. Ans. (B)

04_EMI & AC

(Force & Damped oscillation)

OBJECTIVE (O)

1. Ans. (B) 2. Ans. (B) 3. Ans. (D) 4. Ans. (B) 5. Ans. (B) 6. Ans. (A)

7. Ans. (D) 8. Ans. (D) 9. Ans. (A) 10. Ans. (D) 11. Ans. (B) 12. Ans. (A)

13. Ans. (B) 14. Ans. (B) 15. Ans. (B) 16. Ans. (A) 17. Ans. (C) 18. Ans. (A)

19. Ans. (C) 20. Ans. (D) 21. Ans. (B) 22. Ans. (D) 23. Ans. (C) 24. Ans. (B)

25. Ans. (B) 26. Ans. (B) 27. Ans. (C) 28. Ans. (C) 29. Ans. (D)