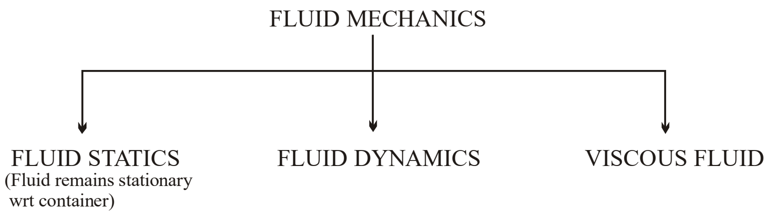
FLUID MECHANICS

KEY CONCEPTS

What Is a Fluid?

A **fluid,** in contrast to a solid, is a substance that can flow. Fluids conform to the boundaries of any container in which we put them. They do so because a fluid cannot sustain a force that is tangential to its surface. a fluid is a substance that flows because it cannot withstand a shearing stress. It can, however, exert a force in the direction perpendicular to its surface.



Fluid includes property → (A) Density (B) Viscosity (C) Bulk modulus of elasticity (D) Pressure (E) Specific gravity.

Assumptions used in fluid mechanics

1. Fluid is incompressible means density remains constant and volume also remains constant.
2. Fluid is non viscous. There is no tangential force between two layers.

DENSITY (ρ)

Mass per unit volume is defined as density. So density at a point of a fluid is represented as

$$\rho = \lim_{\Delta V \to 0} \frac{\Delta m}{\Delta V} = \frac{dm}{dV}$$ Density is a positive scalar quantity.

SI UNIT : kg/m^3 **CGS UNIT :** g/cc **Dimensions :** $[ML^{-3}]$

RELATIVE DENSITY

It is defined as the ratio of the density of the given fluid to the density of pure water at 4°C.

$$\text{Relative density (R.D.)} = \frac{\text{density of given liquid}}{\text{density of pure water at } 4°C}$$

Relative density or specific gravity is a unitless and dimensionless positive scalar physical quantity. Being a dimensionless/unitless quantity R.D. of a substance is same in SI and CGS system.

SPECIFIC GRAVITY

It is defined as the ratio of the specific weight of the given fluid to the specific weight of pure water at 4°C.

$$\text{Specific gravity} = \frac{\text{specific weight of given liquid}}{\text{specific weight of pure water at } 4°C\ (9.81\ kN/m^3)} = \frac{\rho_\ell \times g}{\rho_w \times g} = \frac{\rho_\ell}{\rho_w} = \text{R.D. of the liquid}$$

Thus specific gravity of a liquid is numerically equal to the relative density of that liquid and for calculation purposes they are used interchangeably.

Density of a Mixture of substance in the proportion of mass

Let a number of substances of masses M_1, M_2, M_3 etc., and densities ρ_1, ρ_2, ρ_3 etc. respectively are mixed together. The total mass of the mixture = $M_1 + M_2 + M_3 + \ldots$

The total volume = $\frac{M_1}{\rho_1} + \frac{M_2}{\rho_2} + \frac{M_3}{\rho_3} + \ldots$ therefore, the density of the mixture is $\rho = \frac{M_1 + M_2 + M_3 \ldots}{\frac{M_1}{\rho_1} + \frac{M_2}{\rho_2} + \frac{M_3}{\rho_3} + \ldots}$

For two substances the density of the mixture $\rho = \frac{\rho_1\rho_2(M_1 + M_2)}{\rho_1 M_2 + \rho_2 M_1}$

Density of a mixture of substance in the proportion of volume

Suppose that a number of substances of volume V_1, V_2, V_3 etc. and densities ρ_1, ρ_2, ρ_3 etc. respectively are mixed. The total mass of the mixture is = $\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3 + \ldots$

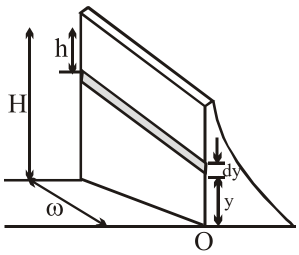
The total volume of the mixture is = $V_1 + V_2 + V_3 + \ldots\ldots\ldots$

Therefore, the density of the mixture is $\rho = \frac{\rho_1 V_1 + \rho_2 V_2 + \rho_3 V_3}{V_1 + V_2 + V_3 + \ldots}$

Therefore, for two substances we can write $\rho = \frac{\rho_1 V_1 + \rho_2 V_2}{V_1 + V_2}$

Example(a) :

Water is filled to a height H behind a dam of width w (fig.). Determine the resultant force exerted by the water on the dam.



Water exerts a force on a dam

Solution :

Let's consider a vertical y axis, starting from the bottom of the dam. Lets consider a thin horizontal strip at a height y above the bottom, such as shown in Fig. We need to consider force due to the pressure of the water only as atmospheric pressure acts on both sides of the dam.

The pressure due to the water at the depth h : $P = \rho gh = \rho g(H - y)$

The force exerted on the shaded strip of area $dA = wdy$:

$dF = P\,dA = \rho g(H - y)\,\omega\,dy$

Integrate to find the total force on the dam :

$$F = \int P\,dA = \int_0^H \rho g(H - y)\omega\,dy = 1/2\ \rho g\omega H^2$$

Example(b) :

In the previuous example find the total torque exerted by the water on dam about a horizontal axis through O. Also find the effective line of action of the total force exerted by the water is at a distance 1/3 H above O.

Solution : .

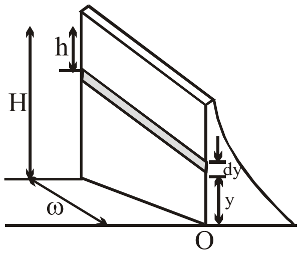
The torque is $\tau = \int d\tau = \int rdF$

From the figure $\tau = \int_0^H y[\rho g(H-y)wdy] = \frac{1}{6}\rho gwH^3$

The total force is given as $\frac{1}{2}\rho gwH^2$

If this were applied at a height y_{eff} such that the torque remains unchanged, we have

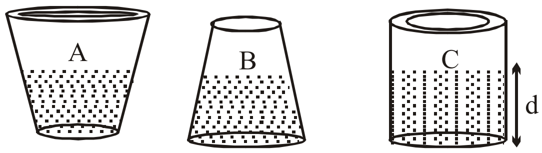
$\frac{1}{6}\rho gwH^3 = y_{eff}\left[\frac{1}{2}\rho gwH^2\right]$ and $y_{eff} = \frac{1}{3}H$



Water exerts a force on a dam

Example(a&b) :

Three vessels having different shapes are as shown in the figure below, they have same base area and the same weight when empty (Fig.). The vessels are filled with mercury to the same level. Neglect the effect of the atmosphere. (a) Which have the largest and which have the smallest pressures at the bottom of the vessel or are they same? (b)Which show the highest weight when weighed on a weighing scale or are they same?



Three differently shaped vessels filled with water to same level.

Solution (a) The mercury at the bottom of each vessel is at the same depth d below the surface. Neglecting the pressure at the surface, the pressures at the bottom must be equal hence:

$P = \rho gd$

(b) The weight of each filled vessel is equal to the weight of the vessel itself plus the weight of the mercury inside. The vessels themselves are of equal weights, but vessel A holds more mercury than C, while vessel B holds less mercury than C. Vessel A weighs the most and vessel B weighs the least.

Example(c) : As the mercury exerts the same downward force on the bottom of each vessel, then why does the vessels weigh differently ?

Soln. In vessel C ,forces due to fluid pressure on the sides of the container are horizontal. Forces on any two diametrically opposite points on the walls of the container are equal and opposite; thus, the net force on the container walls is zero. The force on the bottom is

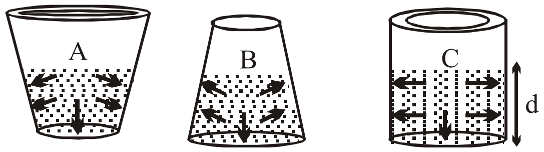
$F = PA = (\rho gd)(\pi r^2)$

The volume of water in the cylinder is $V = \pi r^2 d$, so

$F = \rho gV = (\rho V)g = mg$

The force on the bottom of vessel C is equal to the weight of the water, as expected. The forces due to fluid pressure on the sides of the containers A and B have vertical components also. Hence the force between the fluid and the base of container will not be equal to the weight of the fluid. These containers support the fluid by exerting an upward force equal in magnitude to the weight of the fluid but some force is being applied by the sidewalls and the remaining by the bottom. Fig. shows the forces acting on each container due to the water.

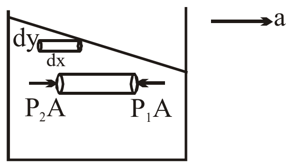
The force on the bottom of vessel A is less than the weight of the mercury in the container, while the force on the bottom of vessel B is greater than the weight of the mercury. In vessel A, the forces on the container walls have downward components as well as horizontal components. The sum of the downward components of the forces on the walls and the downward force on the bottom of the container is equal to the weight of the water. Similarly, the forces on the walls of vessel B have upward components. In each case, the total force on the bottom and sides of the container due to the water is equal to the weight of the water.



Forces exerted on the containers by the water.

Linear Accelerated Motion :

We consider an open container of a liquid that is moving along a straight line with a constant acceleration a as shown in Fig.



Lets consider a small horizontal cylinder of length dx and crossectional area A located y below the free surface of the fluid. This cylinder is accelerating in ground frame with accleration a hence the net horizontal force acting on it should be equal to the product of mass(dm) and acceleration.

$dm = Adx\rho$

$P_2A - P_1A = (Adx\rho)a$

If we say that the right face of the cylinder is y below the free surface of the fluid then the left surface is y + dy below the surface of liquid. Thus

$$P_2 - P_1 = \rho g dy$$

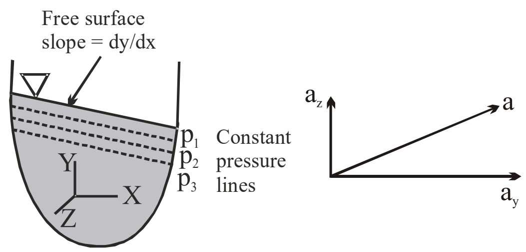
$$\therefore \quad \frac{dy}{dx} = \frac{a}{g}$$

Since the slope of the free surface is coming out to be constant we can say that it must be straight line.

$$\tan\theta = \frac{a}{g}$$

If the container have acceleration along y also than the slope of this line is given by the relationship.

$$\frac{dy}{dx} = -\frac{a_x}{g + a_y}$$



Along a free surface the pressure is constant, so that for the accelerating mass shown in figure the free surface will be inclined if $a_x \neq 0$. In addition, all lines parallel to the free surface will have same presure. For the special circumstance in which $a_x = 0$, $a_y \neq 0$. which corresponds to the mass of fluid accelerating in the vertical direction, Equation indicates that the fluid surface will be horizontal. However, from Equation we see that the pressure variatio is not ρgdy, but is given by the equation.

$$dP = \rho(g + a_y)dy$$

Thus, the pressure on the bottom of a liquid-filled tank which is resting on the floor of an elevator that is accelerating upward will be more than, if the, tank would have been at rest (or moving with a constant velocity). It is to be noted that for a freely falling fluid mass ($a_y = -g$), the pressure variation in all three coordinate directions are zero, which means that the pressure throughout will be same. The pressure throughout a "blob" of a liquid floating in an orbiting space shuttle (a form of free fall) is zero. The only force holding the liquid together is surface tension.

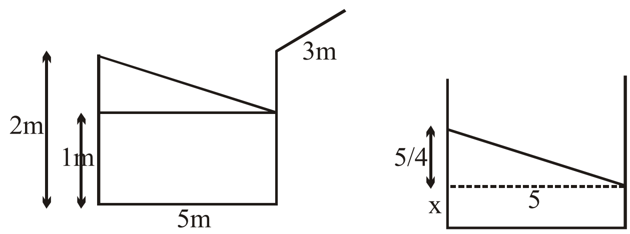
Ques : The cross section of a tank kept on a vehicle is shown in Fig.. The rectangular tank is open to the atmosphere. During motion of the vehicle, the tank is subjected to a constant linear acceleration, $a = 2.5\ m/s^2$. How much fluid will be left inside the tank if initially the tank is half filled. The vessel is 5m wide and 2m high.

Ans.

If the height of the liquid on the left wall is greater than 2m the fluid will be spilled out.

Using equation we can find the angle that the fluid will make with the horizontal.

$$\tan\theta = \frac{2.5}{10} = \frac{1}{4}$$



Lets assume that the dimension of tank in the plane perpendicular to the page is d.

From the geometry its easy to see that free surface on RHS will go down and will rise on LHS. Thus if we assume that fluid on RHS has not touched the floor, we will have fluid taking ashape as described in the diagram. The cuboid part will have volume $x \times 5 \times d$, where x is the height above the bottom.

The wedge part will have the volume $\frac{1}{2} \times h \times 5 \times d$ where h can be found as following

$(h/5) = \tan\theta = (1/4)$

Thus total volume will be $\frac{1}{2} \times (5/4) \times 5 \times d + x \times 5 \times d$ and if we assume there is no spilling than it must beequal to the final volume.

$$\frac{1}{2} \times (5/4) \times 5 \times d + x \times 5 \times d = 1 \times 5 \times d$$

solving we get $x = \frac{3}{8}$

$\therefore$ Total length $\frac{5}{4} + \frac{3}{8} = \frac{10+3}{8} = \frac{13}{8} < 2$

Thus, height is less than 2.

Hence water will not spill.

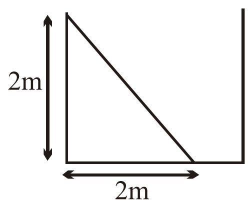
Ex. How much fluid will be left inside the tank if the vehicle accelerates at accleration, a = 10 m/s^2 ?

If the height of the liquid on the left wall is greater than 2m the fluid will be spilled out.

Using equation we can find the angle that the fluid will make with the horizontal.

$$\tan\theta = \frac{10}{10} = 1, \text{ thus } \theta = \frac{\pi}{4}$$

In this case fluid can not remain inside. Fluid having an inclined free surface at 45^0 angle, and covering the bottom of length 5m, will also be 5 m high. This will require the wall to be of 5 m height, which is just 2m for the given vessel. Instead if we think it other way round to keep in contact with the LHS wall, bottom will have to be covered only 2m with the fluid as as shown in the diagram,



Fluid Inside = (1/2) $\times 2 \times 2 \times d$ m^3

Remain inside = 2d m^3

$\therefore$ Thus volume of fluid gone Outside = 3d m^3

Pascal's Principle

When you squeeze one end of a tube to get toothpaste out the other end, you are watching **Pascal's principle** in action. This principle is also the basis for the Heimlich maneuver, in which a sharp pressure increase properly applied to the abdomen is transmitted to the throat, forcefully ejecting food lodged there. The principle was first stated clearly in 1652 by Blaise Pascal (for whom the unit of pressure is named):

A change in the pressure applied to an enclosed incompressible fluid is transmitted undiminished to every portion of the fluid and to the walls of its container.

Demonstrating Pascal's Principle

Consider the case in which the incompressible fluid is a liquid contained in a tall cylinder, as in Fig. The cylinder is fitted with a piston on which a container of lead shot rests. The atmosphere, container, and shot exert pressure p_{ext} on the piston and thus on the liquid. The pressure p at any point P in the liquid is then

$$p = p_{ext} + \rho_{gh}$$

Let us add a little more lead shot to the container to increase pext by an amount p_{ext}. The quantities Δp_{ext}, g and h in Eq. are unchanged, so the pressure change at P is

$$\Delta p = \Delta p_{ext}$$

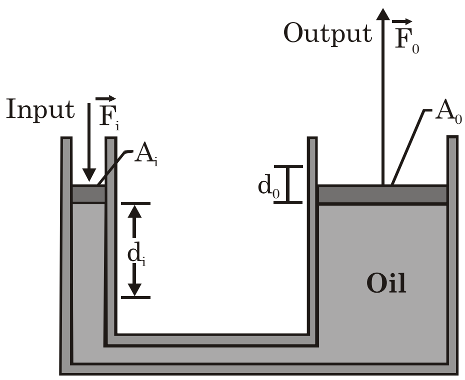
This pressure change is independent of h, so it must hold for all points within the liquid, as Pascal's principle states.

Pascal's Principle and the Hydraulic Lever

Figure shows how Pascal's principle can be made the basis of a hydraulic lever. In operation, let an external force of magnitude F_i be directed downwardon the left-hand (or input) piston, whose surface area is A_i. An incompressible liquid in the device then produces an upward force of magnitude F_0 on the right-hand (or output) piston, whose surface area is A_0. To keep the system in equilibrium, there must be a downward force of magnitude *Fo* on the output piston from an external load (not shown). The force $\vec{F}_l$ applied on the left and the downward force $\vec{F}_0$ from the load on the right produce a change Δp in the pressure of the liquid that is given by

$$\Delta p = \frac{F_i}{A_i} = \frac{F_0}{A_0}$$

$$F_0 = F_l \frac{A_0}{A_i}$$



Equation shows that the output force F_0 on the load must be greater than the input force F_i if $A_0 > A_p$ as is the case in figure.

If we move the input piston downward a distance di, the output piston moves upward a distance do, such that the same volume V of the incompressible liquid is displaced at both pistons.Then

$$V = A_i d_i = A_0 d_0$$

which we can write as

$$d_0 = d_i \frac{A_i}{A_0}$$

This shows that, if Ao > Ai (as in Figure), the output piston moves a smaller distance than the input piston moves.

From Eqs. we can write the output work as

$$W = F_0 d_0 = \left(F_i \frac{A_i}{A_0}\right)\left(d_i \frac{A_i}{A_0}\right) = F_i d_i$$

which shows that the work W done on the input piston by the applied force is equal to the work W done by the output piston in lifting the load placed on it.

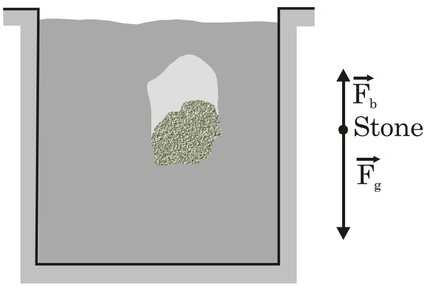
The advantage of a hydraulic lever is this:

With a hydraulic lever, a given force applied over a given distance can be transformed to a greater force applied over a smaller distance.

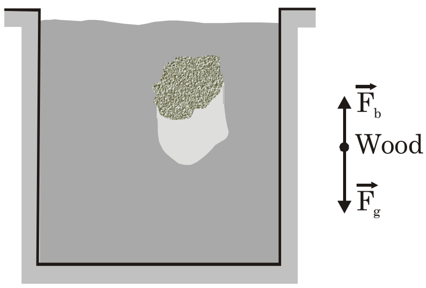
The product of force and distance remains unchanged so that the same work is done. However, there is often tremendous advantage in being able to exert the larger force. Most of us, for example, cannot lift an automobile directly but can with a hydraulic jack, even though we have to pump the handle farther than the automobile rises and in a series of small strokes.

Archimedes' Principle

Figure shows a student in a swimming pool, manipulating a very thin plastic sack (of negligible mass) that is filled with water. She finds that the sack and its contained water are in static equilibrium, tending neither to rise nor to sink.The downward gravitational force $\vec{F}_g$ on the contained water must be balanced by a net upward force from the water surrounding the sack.



(b)



(c)

This net upward force is a **buoyant force** $\vec{F}_b$. It exists because the pressure in the surrounding water increases with depth below the surface.Thus, the pressure near the bottom of the sack is greater than the pressure near the top, which means the forces on the sack due to this pressure are greater in magnitude near the bottom of the sack than near the top. Some of the forces are represented in Fig. (a), where the space occupied by the sack has been left empty. Note that the force vectors drawn near the bottom of that space (with upward components) have longer lengths than those drawn near the top of the sack (with downward components). If we vectorially add all the forces on the sack from the water, the horizontal components cancel and the vertical components add to yield the upward buoyant force $\vec{F}_b$ on the sack. (Force $\vec{F}_b$ is shown to the right of the pool in Fig. (a.)

Because the sack of water is in static equilibrium, the magnitude $\vec{F}_b$ of is equal to the magnitude m_f g of the gravitational force $\vec{F}_g$ on the sack of water: $F_b = m_f g$. (Subscript *f* refers to fluid, here the water.) In words, the magnitude of the buoyant force is equal to the weight of the water in the sack.

In Fig. (b), we have replaced the sack of water with a stone that exactly fills the hole in Fig. (a). The stone is said to displace the water, meaning that the stone occupies space that would otherwise be occupied by water.We have changed nothing about the shape of the hole, so the forces at the hole's surface must be the same as when the water-filled sack was in place. Thus, the same upward buoyant force that acted on the water-filled sack now acts on the stone; that is, the magnitude $\vec{F}_b$ of the buoyant force is equal to $m_f g$, the weight of the water displaced by the stone.

Unlike the water-filled sack, the stone is not in static equilibrium.The downward gravitational force $\vec{F}_g$ on the stone is greater in magnitude than the upward buoyant force, as is shown in the free-body diagram in Fig. *(b)*. The stone thus accelerates downward, sinking to the bottom of the pool. Let us next exactly fill the hole in Fig. (a) with a block of lightweight wood, as in Fig. (c*)*. Again, nothing has changed about the forces at the hole's surface, so the magnitude $\vec{F}_b$ of the buoyant force is still equal to $m_f g$, the weight of the displaced water. Like the stone, the block is not in static equilibrium. However, this time the gravitational force $\vec{F}_g$ is lesser in magnitude than the buoyant force (as shown to the right of the pool), and so the block accelerates upward, rising to the top surface of the water. Our results with the sack, stone, and block apply to all fluids and are summarized in **Archimedes' principle:**

When a body is fully or partially submerged in a fluid, a buoyant force $\vec{F}_b$ from the surrounding fluid acts on the body.The force is directed upward and has a magnitude equal to the weight $m_f g$ of the fluid that has been displaced by the body. The buoyant force on a body in a fluid has the magnitude

$$F_b = m_f g \text{ (buoyant force)}$$

where m_f is the mass of the fluid that is displaced by the body.

Floating

When we release a block of lightweight wood just above the water in a pool, the block moves into the water because the gravitational force on it pulls it downward.

As the block displaces more and more water, the magnitude F_b of the upward buoyant force acting on it increases. Eventually, F_b is large enough to equal the magnitude F_g of the downward gravitational force on the block, and the block comes to rest. The block is then in static equilibrium and is said to be floating in the water. In general, When a body floats in a fluid, the magnitude F_b of the buoyant force on the body is equal to the magnitude F_g of the gravitational force on the body.

We can write this statement as

$$F_b = F_g \text{ (Floating)}$$

From Eq. we know that $F_b = m_f g$. Thus,

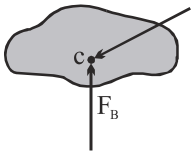
When a body floats in a fluid, the magnitude F_g of the gravitational force on the body is equal to the weight $m_f g$ of the fluid that has been displaced by the body.

We can write this statement as

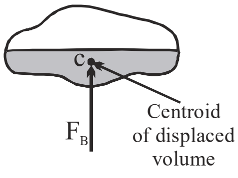
$$F_g = m_f g$$

In other words, a floating body displaces its own weight of fluid.

The location of the line of action of the buoyant force can be determined by adding torques of the forces due to pressure forces, with respect to some convenient axis. The buoyant force must pass through the center of mass of the desplaced volume, as shown in Fig. (c), as it was in translational and rotational equilibirium. The point through which the buoyant force acts is called the center of buoyancy.



(c)



(D)

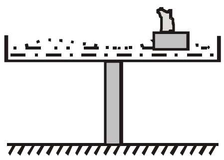
These same results apply to floating bodies which are only partially submerged, as shown in Fig.(d), if the density of the fluid above the liquid surface is very small compared with the liquid in which the body floats. Since the fluid above the surface is usually air, for practial purposes this condition is satisfied.

In the above discussion, the fluid is assumed to have a constant density. If a body is immersed in a fluid in which density varies with depth, such as having multiple layers of fluid, the magnitude of the buoyant force remains equal to the weight of the displaced fluid and the buoyant force passes through the center of mass of the displaced volume.

Q. A wooden block floats vertically in a glass filled with water. How will the level of the water in the glass change if the block is kept in a horizontal position ?

Solution : The level of the water will not change because the quantity of water displaced will remain the same.

Q. A vessel filled with water is placed exactly in middle of a thin wall(fig.). Will the system topple if a small wooden boat carrying some weight is floated in the vessel?



Solution : The system will not topple, since according to Pascal's law the pressure on the bottom of the vessel will be the same everywhere thus the body will still remain in rotational equilibirium

Ex. A homogeneous piece of ice floats in a glass filled with water. How will the level of the water in the glass change when the ice melts ?

Sol. Since the piece of ice floats, the weight of the water displaced by it is equal to the weight of the ice itself or the weight of the water it produces upon melting. For this reason the water formed by the piece of ice will occupy a volume equal to that of the submerged portion, and the level of the water will not change.

Q. A piece of ice is floating in a tub filled with water. How will the level of the water in the tub change when the ice melts ? Consider the following cases :

(1) a stone is frozen in the ice

(2) the ice contains an air bubble

Solution :

(1) The volume of the submerged portion of the piece with the stone is greater than the sum of the volumes of the stone and the water produced by the melting ice. Therefore, the level of the water in the glass will drop.

(2) The weight of the displaced water is equal to that of the ice (the weight of the air in the bubble may be neglected). For this reason, as in conceptual eg., the level of the water will not change.

Example :

A vessel with a body floating in it is kept in elevator accelerating downwards with acceleration a such that $a < g$. Will the body rise or sink further in the vessel?

Solution : The force of bouyancy on the body can be written as $F = \rho V_2(g - a)$, where V_2 is the volume of the submerged portion of the body in the lift. As pressure at a point h below the surface will become $\rho(g - a)h$ instead of ρgh. Applying the Newton's second Law, remembering that the body was accelerating upwards at a.

$Mg - \rho V_2 (g - a) = Ma$

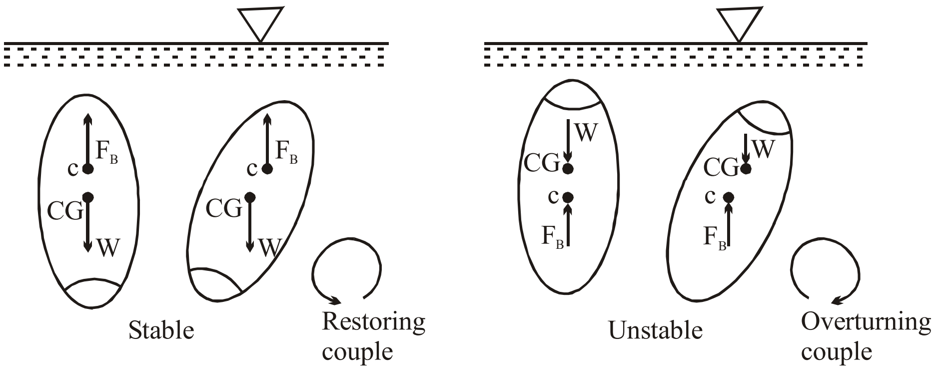
Hence, $V_2 = \frac{M}{\rho}$ thus $V_2 = V$, as in a stationary vessel, $V = \frac{M}{\rho}$. Thus the body does not rise to the surface.

Stability :

The center of buoyancy and center of gravity do not necessarily coincide so the floating or submerged body may not be in stable equilibirium. A small rotation can cause the buoyant force to produce either a restoring or overturning torque. For example, for the completely submerged body shown in Fig., which has a center of gravity below the center of buoyancy, a rotation from its equilibrium position will create a restoring torque by the buoyant force, F_B, which causes the body to rotate back to its original position. Thus, if the center of gravity falls below the center of buoyancy, the body is stable.

However, as shown in Fig., if the center of gravity of the completely submerged body is above the center of buoyancy, the resulting torque formed by the weight and the buoyant force will cause the body to overturn and move to a new equilibrium position. Thus, a completely submerged body with its center of gravity above its center of buoyancy is in an unstable equilibrium position.

For floating bodies the stability problem is more complicated, since as the body rotates the location of the center of buoyancy may change.



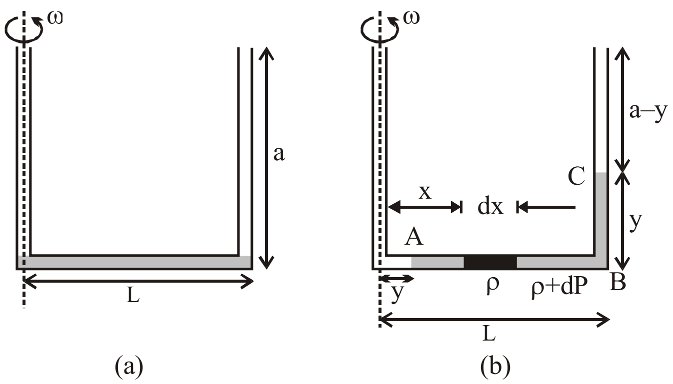
Concept : Length of a horizontal arm of a U-tube is L and ends of both the vertical arms are open to atmospheric pressure P_0. A liquid of density ρ is poured in the tube such that liquid just fills the horizontal part of the tube as shown in figure. Now one end of the opened ends is sealed and the tube is then rotated about a vertical axis passing through the other vertical arm with angular speed ω. If length of each vertical arm is a and in the sealed end liquid rises to a height y, find pressure in the sealed tube during rotation.

The pressure difference across an element of width dx, which is given as

$$dP = dx\rho\omega^2 x$$

Now integrating from A to B, we get

$$P_B - P_A = \int_y^L \rho\omega^2 x dx$$



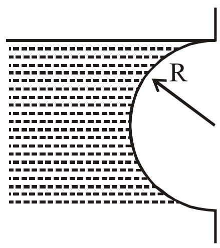
Thus pressure at point C an be given as

$$P_C = P_B - y\rho g$$

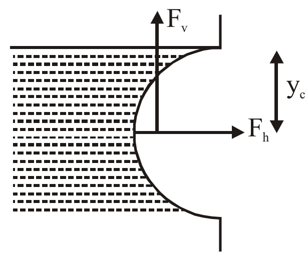
and at point A, pressure is atmospheric, thus we have

$$P_C = \frac{\rho\omega^2}{2}\left(L^2 - y^2\right) + \rho_A - y\rho g$$

Q. A hemisphere of radius R is just submerged is just sinking in water of density ρ. Find the
(a) horizontal thrust.
(b) vertical thrust.
(c) total hydrostatic force.
(d) angle of orientation of total hydrostatic force acting on the hemisphere.
Do not count atmospheric pressure



Sol. (a) Let the horizontal and vertical thrusts on the hemisphere be F_h and F_V respectively



We know that

$$F_h = \rho g y_c A_y$$

where $y_c = R$

and $A_v = \pi R^2$

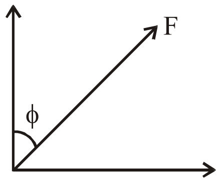
This gives $F_h = \rho g \pi R^3$ (right)

(b) Similarly using the formula $F_v = \rho V g$

Where V = volume of the hemisphere = $\frac{2}{3}\pi R^3$,

we have = $F_V = \frac{2}{3}\rho g \pi R^3$ (up)

(c) Hence the net dydrostatic force on the hemisphere is



$$F = \sqrt{F_h^2 + F_v^2}$$

$$F = \sqrt{\left(\rho g \pi R^3\right)^2 + \left(\frac{2}{3}\rho g \pi R^3\right)^2}$$

$$= \frac{\sqrt{13}}{3}\rho g \pi R^3$$

(d) The angle of orientation of the force F is

$$\phi = \tan^{-1}\frac{F_h}{F_v}$$

$$= \tan^{-1}\frac{\rho g \pi R^3}{\frac{2}{3}\rho g \pi R^3} = \tan^{-1}\frac{3}{2}$$

FLUID DYMANICS & VISCOSITY

Ideal Fluids in Motion

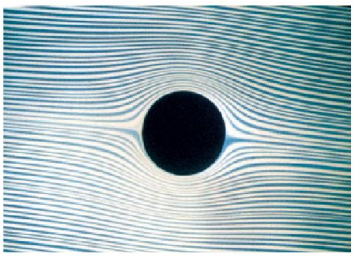
The motion of real fluids is very complicated and not yet fully understood. Instead, we shall discuss the motion of an **ideal fluid,** which is simpler to handle mathematically and yet provides useful results. Here are four assumptions that we make about our ideal fluid; they all are concerned with flow:

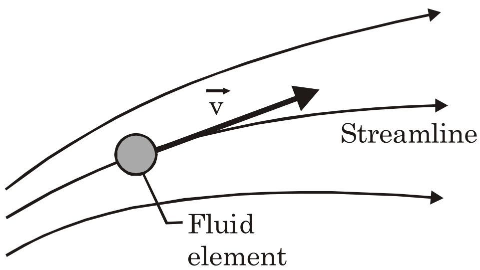
1. **Steady flow** In steady (or laminar) flow, the velocity of the moving fluid at any fixed point does not change with time, either in magnitude or in direction. The gentle flow of water near the center of a quiet stream is steady; the flow in a chain of rapids is not. Figure shows a transition from steady flow to nonsteady (or nonlaminar or turbulent) flow for a rising stream of smoke. The speed of the smoke particles increases as they rise and, at a certain critical speed, the flow changes from steady to nonsteady.
2. **Incompressible flow** We assume, as for fluids at rest, that our ideal fluid is incompressible; that is, its density has a constant, uniform value.
3. **Nonviscous flow** Roughly speaking, the viscosity of a fluid is a measure of how resistive the fluid is to flow. For example, thick honey is more resistive to flow than water, and so honey is said to be more viscous than water. Viscosity is the fluid analog of friction between solids; both are mechanisms by which the kinetic energy of moving objects can be transferred to thermal energy. In the absence of friction, a block could glide at constant speed along a horizontal surface. In the same way, an object moving through a nonviscous fluid would experience no viscous drag force-that is, no resistive force due to viscosity; it could move at constant speed through the fluid.

4. **Irrotational flow** : Although it need not concern us further, we also assume that the flow is irrotational. To test for this property, let a tiny grain of dust move with the fluid. Although this test body may (or may not) move in a circular path, in irrotational flow the test body will not rotate about an axis through its own center of mass. For a loose analogy, the motion of a Ferris wheel is rotational; that of its passengers is irrotational. T8hat the velocity of a particle is always tangent to the path taken by the particle. Here the particle is the fluid element, and its velocity is $\vec{v}$ always tangent to a streamline (Figure). For this reason, two streamlines can never intersect; if they did, then an element arriving at their intersection would have two different velocities simultaneously an impossibility.

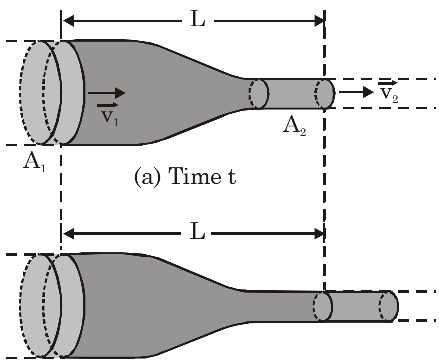
The Equation of Continuity

You may have noticed that you can increase the speed of the water emerging from a garden hose by partially closing the hose opening with your thumb. Apparently the speed v of the water depends on the cross-sectional area A through which the water flows.



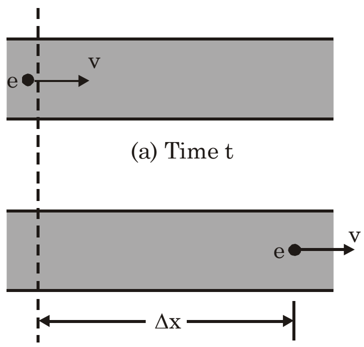


Here we wish to derive an expression that relates v and A for the steady flow of an ideal fluid through a tube with varying cross section, like that in Figure. The flow there is toward the right, and the tube segment shown (part of a longer tube) has length L. The fluid has speeds v_1 at the left end of the segment and v_2 at the right end. The tube has cross-sectional areas A_1 at the left end and A_2 at the right end. Suppose that in a time interval Δt a volume ΔV of fluid enters the tube segment at its left end (that volume is colored purple in Figure. Then, because the fluid is incompressible, an identical volume ΔV must emerge from the right end of the segment (it is colored green in Figure). We can use this common volume ΔV to relate the speeds and areas. To do so, we first consider Fig. , which shows a side view of a tube of uniform crosssectional area A. In Fig.(a), a fluid element e is about to pass through the dashed line drawn across the tube width.The element's speed is v, so during a time interval Δt, the element moves along the tube a distance Δx Δ v Δt.The volume ΔV of fluid that has passed through the dashed line in that time interval Δt is



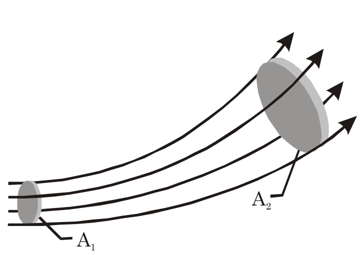
(a) Time t

(b) Time t + Δt



(a) Time t

(b) Time t + Δt



$$\Delta V = A\,\Delta x = Av\,\Delta t.$$

Applying Eq. to both the left and right ends of the tube segment in Fig., we have

$$\Delta V = A_1v_1\,\Delta t = A_2v_2\,\Delta t$$

Or $\qquad A_1v_1 = A_2v_2$ (equation of continuity)

This relation between speed and cross-sectional area is called the **equation of continuity** for the flow of an ideal fluid. It tells us that the flow speed increases when we decrease the cross-sectional area through which the fluid flows (as when we partially close off a garden hose with a thumb). Equation applies not only to an actual tube but also to any so-called tube of flow, or imaginary tube whose boundary consists of streamlines. Such a tube acts like a real tube because no fluid element can cross a streamline; thus, all the fluid within a tube of flow must remain within its boundary. Figure shows a tube of flow in which the cross-sectional area increases from area A_1 to area A_2 along the flow direction. From Eq. we know that, with the increase in area, the speed must decrease, as is indicated by the greater spacing between streamlines at the right in Fig. . Similarly, you can see that in Fig. the speed of the flow is greatest just above and just below the cylinder. We can rewrite Eq. as

$R_V = A_v$ = a constant (volume flow rate, equation of continuity), in which R_V is the **volume flow rate** of the fluid (volume past a given point per unit time). Its SI unit is the cubic meter per second (m^3/s). If the density ρ of the fluid is uniform, we can multiply Eq. by that density to get the **mass flow rate** R_m (mass per unit time):

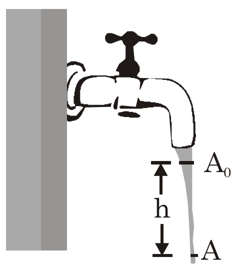
$$R_m = \rho R_V = A_v = \text{a constant (mass flow rate).}$$

The SI unit of mass flow rate is the kilogram per second (kg/s). Equation says that the mass that flows into the tube segment of Fig. each second must be equal to the mass that flows out of that segment each second.

Sample Problem

Figure shows how the stream of water emerging from a faucet "necks down" as it falls. The indicated cross-sectional areas are A_0 = 1.2 cm2 and A =0.35 cm^2. The two levels are separated by a vertical distance h = 45 mm.What is the volume flow rate from the tap?

The volume flow rate through the higher cross section must be the same as that through the lower cross section.



where v_0 and v are the water speeds at the levels corresponding to A_0 and A. From Eq. we can also write, because the water is falling freely with acceleration g,

$$v_2 = v^2_0 - 2gh.$$

Eliminating v between Eqs. and solving for v_0, we obtain

$$v_0 = \sqrt{\frac{2ghA^2}{A_0^2 - A^2}}$$

$$v_0 = \sqrt{\frac{(2)\times(9.8m/s^2)(0.045m)(0.35cm^2)^2}{(1.2cm^2)-(0.35cm^2)^2}}$$

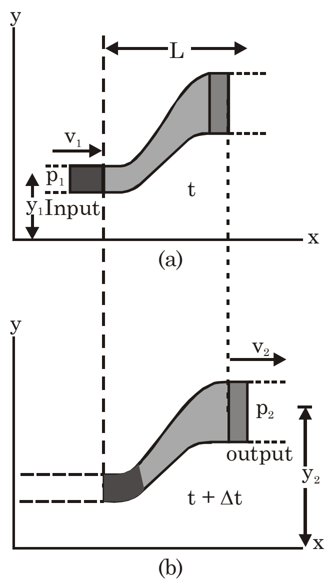
$$v_0 = 0.286 \text{ m/s} = 28.6 \text{ cm/s}.$$

From Eq. , the volume flow rate R_V is then

$$R_V = A_0v_0 = (1.2 \text{ cm}^2)(28.6 \text{ cm/s})$$
$$= 34 \text{ cm}^3/\text{s. Ans.}$$

Bernoulli's Equation

Figure represents a tube through which an ideal fluid is flowing at a steady rate. In a time interval Δt, suppose that a volume of fluid ΔV, colored purple in Fig. , enters the tube at the left (or input) end and an identical volume, colored green in Fig. , emerges at the right (or output) end. The emerging volume must be the same as the entering volume because the fluid is incompressible, with an assumed constant density ρ.



$$p_1 + \frac{1}{2}\rho v_1^2 + \rho g y_1 = p_2 + \frac{1}{2}\rho v_2^2 + \rho g y_2$$

Let y_1, v_1, and p_1 be the elevation, speed, and pressure of the fluid entering at the left, and y_2, v_2, and p_2 be the corresponding quantities for the fluid emerging at the right. By applying the principle of conservation of energy to the fluid, we shall show that these quantities are related by

In general, the term $\frac{1}{2}\rho v^2$ is called the fluid's **kinetic energy density** (kinetic energy per unit volume).We can also write Eq. as

$$p + \frac{1}{2}\rho v^2 + \rho g y \text{ a constant (Bernoulli's equation).}$$

Equations are equivalent forms of **Bernoulli's equation,** after Daniel Bernoulli, who studied fluid flow in the 1700s.* Like the equation of continuity Eq., Bernoulli's equation is not a new principle but simply the reformulation of a familiar principle in a form more suitable to fluid mechanics. As a check, let us apply Bernoulli's equation to fluids at rest, by putting $v_1 = v_2 = 0$ in Eq. The result is

$$p_2 = p_1 + \rho g(y_1 - y_2)$$

Which is equation.

A major prediction of Bernoulli's equation emerges if we take y to be a constant (y = 0, say) so that the fluid does not change elevation as it flows. Equation then becomes

$$p_1 + \frac{1}{2}\rho v_1^2 = p_2 + \frac{1}{2}\rho v_2^2$$

Which tells us that :

If the speed of a fluid element increases as the element travels along a horizontal streamline, the pressure of the fluid must decrease, and conversely.

Put another way, where the streamlines are relatively close together (where the velocity is relatively great), the pressure is relatively low, and conversely. The link between a change in speed and a change in pressure makes sense if you consider a fluid element. When the element nears a narrow region, the higher pressure behind it accelerates it so that it then has a greater speed in the narrow region. When it nears a wide region, the higher pressure ahead of it decelerates it so that it then has a lesser speed in the wide region. Bernoulli's equation is strictly valid only to the extent that the fluid is ideal. If viscous forces are present, thermal energy will be involved. We take no account of this in the derivation that follows.

Proof of Bernoulli's Equation

Let us take as our system the entire volume of the (ideal) fluid shown in Fig. .We shall apply the principle of conservation of energy to this system as it moves from its initial state (Fig. (a)) to its final state (Fig. (b)). The fluid lying between the two vertical planes separated by a distance L in Fig. does not change its properties during this process; we need be concerned only with changes that take place at the input and output ends. First, we apply energy conservation in the form of the work–kinetic energy theorem,

$$W = \Delta K,$$

which tells us that the change in the kinetic energy of our system must equal the net work done on the system. The change in kinetic energy results from the change in speed between the ends of the tube and is

$$\Delta K = \frac{1}{2}\Delta m v_2^2 - \frac{1}{2}\Delta m v_1^2 = \frac{1}{2}\rho \Delta V (v_2^2 - v_1^2)$$

in which Δm (= $\rho\ \Delta V$) is the mass of the fluid that enters at the input end and leaves at the output end during a small time interval Δt.

The work done on the system arises from two sources.The work W_g done by the gravitational force $(\Delta m \vec{g})$ on the fluid of mass Δm during the vertical lift of the mass from the input level to the output level is

$$W_g = \Delta m\, g(y_2 - y_1)$$

$$= -\rho g \Delta V (y_2 - y_1)$$

This work is negative because the upward displacement and the downward gravitational force have opposite directions.

Work must also be done on the system (at the input end) to push the entering fluid into the tube and by the system (at the output end) to push forward the fluid that is located ahead of the emerging fluid. In general, the work done by a force of magnitude F, acting on a fluid sample contained in a tube of area A to move the fluid through a distance Δx, is

$$F\Delta x = (pA)(\Delta x) = p(A\Delta x) = p\Delta v$$

The work done on the system is then $p_1\ \Delta V$, and the work done by the system is $-p_2\ \Delta V$. Their sum Wp is Wp

$$Wp = -p_2\ \Delta V — p_1\ \Delta V$$

$$= -(p_2 - p_1)\ \Delta V.$$

The work–kinetic energy theorem of Eq. now becomes

$$W = Wg + Wp = \Delta K.$$

Substituting from Eqs. yields

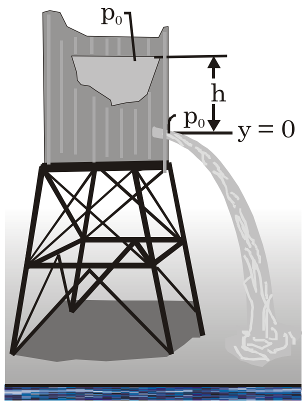
$$-\rho g\Delta V(y_2 - y_1) - \Delta V(p_2 - p_1) = \frac{1}{2}\rho\Delta v(v_2^2 - v_1^2)$$

This, after a slight rearrangement, matches Eq. , which we set out to prove.

Sample Problem : In the old West, a desperado fires a bullet into an open water tank (Fig.), creating a hole a distance h below the water surface.What is the speed v of the water exiting the tank?

Sol. From Eq. $R_V =$ av = Av_0 and thus

$$v_0 = \frac{a}{A}v$$



Because a <<A, we see that v_0 <<v.To apply Bernoulli's equation, we take the level of the hole as our reference level for measuring elevations (and thus gravitational potential energy). Noting that the pressure at the top of the tank and at the bullet hole is the atmospheric pressure p_0 (because both places are exposed to the atmosphere), we write Eq. as

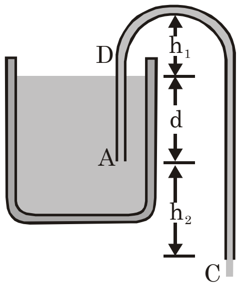
$$p_0 + \frac{1}{2}\rho v_0^2 + \rho gh = p_0 + \frac{1}{2}\rho v^2 + \rho g(0)$$

(Here the top of the tank is represented by the left side of the equation and the hole by the right side. The zero on the right indicates that the hole is at our reference level.) Before we solve Eq. for v, we can use our result that v_0 <<v to simplify it:We assume that v^2_0 and thus the term $\frac{1}{2}\rho v_0^2$ in Eq. , is negligible relative to the other terms, and we drop it. Solving the remaining equation for v then yields

$$v = \sqrt{2gh} \quad \text{Ans.}$$

This is the same speed that an object would have when falling a height h from rest.This is because the work done by atmospheric pressure is cancelling out at open surface and the hole.

Sample Problem : shows a siphon, which is a device for removing liquid from a container. Tube ABC must initially be filled, but once this has been done, liquid will flow through the tube until the liquid surface in the container is level with the tube opening at A. The liquid has density 1000 kg/m^3 and negligible viscosity. The dis-tances shown are h_1 = 25 cm, d =12 cm, and h_2 = 40 cm. (a) With what speed does the liquid emerge from the tube at C? (b) If the atmospheric pressure is 1.0×10^5 Pa, what is the pressure in the liquid at the topmost point B? (c) Theoretically, what is the greatest possible height h_1 that a siphon can lift water?



Solution: You may have used siphon and you may recollect that lower the exit point of the fluid is, faster the fluid flows out.

We consider a point D on the surface of the liquid in the container, in the same tube of flow with points A, B and C. Applying Bernoulli's equation to points D and C, we obtain

$$p_D + \frac{1}{2}\rho v_D^2 + \rho g h_D = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

$$v_C = \sqrt{\frac{2(p_D - p_C)}{\rho} + 2g(h_D - h_C) + v_D^2}$$

$$\approx \sqrt{2g(d + h_2)}$$

where in the last step we set $p_D = p_C = p_{air}$ and $v_D/v_C \approx 0$. Plugging in the values, we obtain

$$v_c = \sqrt{2(9.8 m/s^2)(0.40m + 0.12m)} = 3.2 \text{ m/s}$$

The result confirms our experience.

We now consider points B and C:

$$p_B + \frac{1}{2}\rho v_B^2 + \rho g h_B = p_C + \frac{1}{2}\rho v_C^2 + \rho g h_C$$

Since $v_B = v_C$ by equation of continuity, and $p_C = p_{air}$, Bernoulli's equation becomes

$$p_B = p_C + \rho g(h_C - h_g) = p_{air} - \rho g(h_1 + h_2 + d)$$

$= 1.0 \times 10^5$ Pa $(1.0 \times 10^3$ kg/m $)(9.8$ m/s^2 $)(0.25$ m $+ 0.40$ m $+ 0.12$ m$)$

$= 9.2 \times 10$ Pa.

Since $p_B \geq 0$, we must let $p_{air} - \rho g(h_1 + d + h_2) \geq 0$, which yields

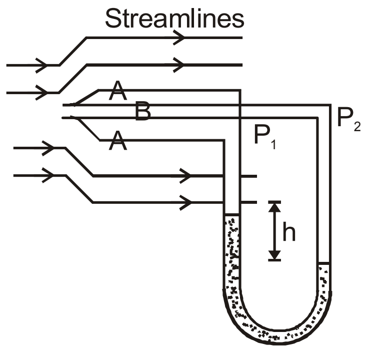
$$h_1 \leq h_{1max} = \frac{p_{air}}{\rho} - d - h_2 \leq \frac{p_{air}}{\rho} = 10.3m$$

Q. In the example just above consider a small hole in the hose at location (1) as indicated. When the siphon is used, will water leak out of the hose, or will air leak into the hose and thus causing the siphon to stop ?

Solution : Whether air will leak out of the hose depends on whether the pressure within the hose at (1) is less than or greater than atmospheric.

Since the hose diameter is constant, it follows from the continuity equation (AV = constant) that the water velocity in the hose is constant throughout. Also the pressure at the end of the hose is atmospheric so the pressure at all the points above it can be shown to be less than the atmospheric pressure. Thus air will leak into the hose pipe stoping it.

Example: Fig. shows a device called pitot's tube. It measures the velocity of moving fluids. Determine the velocity of the fluid in terms of the density ρ, the density of the fluid in manometer (U-tube) σ and the height 'h'.



figure

Solution.

The difference in the two tubes is that liquid will flow into the tube B with full Kinetic Energy while it will just pass over the tube A without directly entering into it.

This problem is based on the use of Bernoulli's principle, on two different situations.

The fluid inside the right tube must be at rest as the fluid exactly at the end is in contact with the fluid in pitot tube, which is at rest.

The velocity υ_1 is the fluid velocity υ_f. The velocity υ_2 of the fluid at point B is zero and the pressure in the right arm is P_2 (called stagnation pressure).

Thus using Bernoulli's principle $\quad P_1 + \frac{1}{2}\rho\upsilon_1^2 = P_2 + \frac{1}{2}\rho\upsilon_2^2$

We get $\quad P_2 = P_1 + \frac{1}{2}\rho\upsilon_f^2$

On the other hand the openings at point A is not along the flow lines, so we dont need to use Bernoulli's eqn. We can simply say that the pressure just outside the opening is same as that within the pitot tube.

Therefore the pressure at the left arm of the manometer is same as the fluid pressre P_f, i.e., $P_1 = P_f$.

Also $\quad P_2 = P_1 + (\rho - \sigma)\, gh \quad$ ....(3)

Generally $\sigma << \rho$, so it is ignored.

Thus $\quad P_2 = P_f + \rho gh \quad$ ....(4)

From eqns. (4) and (3), $\quad \frac{1}{2}\sigma\upsilon_f^2 = \rho gh$

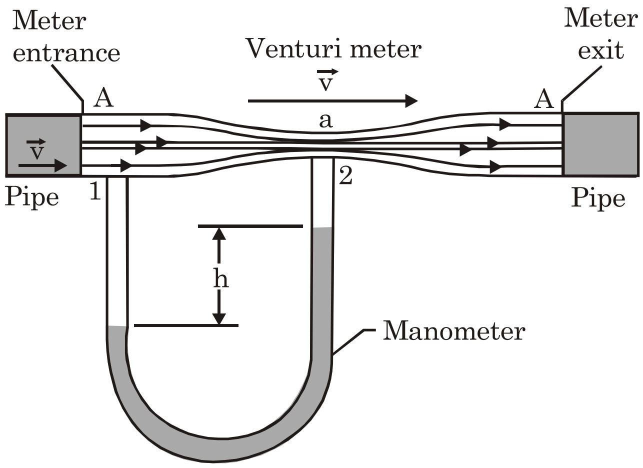
or $\quad \upsilon_f = \sqrt{\frac{2\rho gh}{\sigma}}$

Thus we can see that we have measured the fluid velocity as this was the only difference between the two tubes leading to the pressure difference between the tubes.

Conceptual example:A venturi meter is used to measure the flow speed of a fluid in a pipe. The meter is connected between two sections A venturi meter is used to measure the flow speed of a fluid in a pipe. The meter is connected between two sections of the pipe (Fig.); the cross-sectional area A of the entrance and exit of the meter matches the pipe's crosssectional area. Between the entrance and exit, the fluid flows from the pipe with speed V and then through a narrow "throat" of cross-sectional area a with speed v A manometer connects the wider portion of the meter to the narrower portion. The change in the fluid's speed is accompanied by a change Dp in the fluid's pressure, which causes a height difference h of the liquid in the two arms of the manometer. (Here Dp means pressure in the throat minus pressure in the pipe.) (a) By applying Bernoulli's equation and the equation of continuity to points 1 and 2 in Fig. , show that

$$v = \sqrt{\frac{2a^2\Delta p}{\rho(a^2 - A^2)}}$$

where r is the density of the fluid. (b) Suppose that the fluid is fresh water, that the cross-sectional areas are 64 cm^2 in the pipe and 32 cm^2 in the throat, and that the pressure is 55 kPa in the pipe and 41 kPa in the throat.What is the rate of water flow in cubic meters per second?



(a) The continuity equation yields Av = aV, and Bernoulli's equation yields $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho V^2$ where $\Delta p = p_1 - p_2$. The first equation gives V = (A/a)v. We use this to substitute for V in the second equation, and obtain $\Delta p + \frac{1}{2}\rho v^2 = \frac{1}{2}\rho(A/a)^2 v^2$ We solve for v. The result is

$$v = \sqrt{\frac{2\Delta p}{\rho((A/a)^2 - 1)}} = \sqrt{\frac{2a^2\Delta p}{\rho(A^2 - a^2)}}$$

(b) We substitute values to obtain

$$v = \sqrt{\frac{2(32\times10^{-4}m^2)^2(55\times10^3 Pa - 41\times10^3 Pa)}{(1000 kg/m^3)((64\times10^{-4}m^2)^2 - (32\times10^{-4}m^2)^2)}} = 30.06 \text{ m/s}$$

Consequently, the flow rate is

$$Av = (64 \times 10^{-4}m^2)(3.06 \text{ m/s}) = 2.0 \times 10^{-2} m^3/s.$$

SURFACE TENSION & VISCOSITY

EXPLANATION OF SOME OBSERVED PHENOMENA

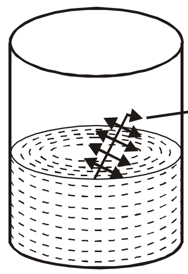
1. Lead balls are spherical in shape.
2. Rain drops and a globule of mercury placed on glass plate are spherical.
3. Hair of a shaving brush/painting brush, when dipped in water spread out, but as soon as it is taken out. Its hair stick together.
4. A greased needle placed gently on the free surface of water in a beaker does not sink.
5. Similarly, insects can walk on the free surface of water without drowning.
6. Bits of Camphor gum move inregularly when placed on water surface.

SURFACE TENSION

Surface Tension is a property of liquid at rest by virtue of which a liquid surface gets contracted to a minimum area and behaves like a stretched membrane.

Surface Tension of a liquid is measured by force per unit length on either side of any imaginary line drawn tangentially over the liquid surface, force being normal to the imaginary line as shown in fig. i.e. Surface tension.

$$(T) = \frac{\text{Total force on either of the imginary line (F)}}{\text{Length of the line } (l)}$$



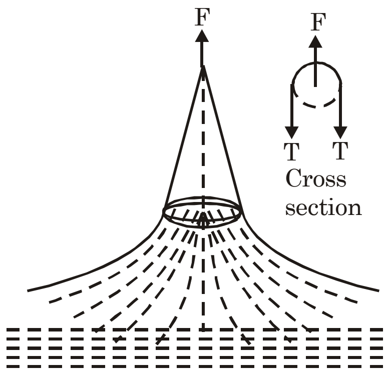
Unit of Surface Tension

In C. G. S. system the unit of surface tension is dyne/cm (dyne cm^{-1}) and SI system its units is Nm^{-1}

Solved Examples

Ex. A ring is cut form a platinum tube of 8.5 cm internal and 8.7 cm external diameter. It is supported horizontally from a pan of a balance so that it comes in contact with the water in a glass vessel. What is the surface tension of water if an extra 3.97 g weight is required to pull it away from water? ($g = 980\ cm/s^2$).

Sol.



The ring is in contact with water along its inner and outer circumference ; so when pulled out the total force on it due to surface tension will be

$$F = T(2\pi\, r_1 + 2\pi t_2)$$

So, $$T = \frac{mg}{2\pi(r_1 + r_2)} \qquad [\because F = mg]$$

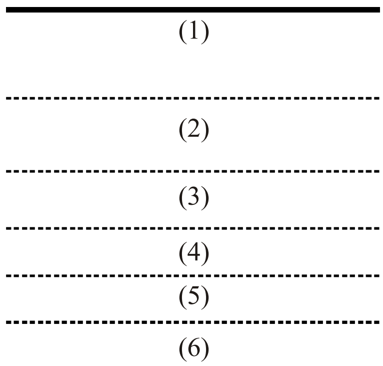
i.e., $$T = \frac{3.97 \times 980}{3.14 \times (8.5 + 8.7)} = 72.13 \text{ dyne/cm}$$

Surface energy

The course of reasoning given below is usually followed to prove that the molecules of the surface layer of a liquid have surplus potential energy. A molecule inside the liquid is acted upon by the forces of attraction from the other molecules which compensate each other on the average. If a molecule is singled out on the surface, the resulting force of attraction from the other molecule is directed into the liquid. For thi reason the molecule tends to move into the liquid, and definite work should be done to bring it to the surface. Therefore, each molecule of the surface layer has excess potential energy equal to this work. The average force that acts on any molecule from the side of all the others, however, is always equal to zero if the liquid is in equilibrium. This is why the work done to move the liquid from a depth to the surface should also be zero. What is the origin, in this case, of the surface energy ?

Sol. The forces of attraction acting on a molecule in the surface layer from all the other molecules produce a resultant directed downward. The closest neighbours, however, exert a force of repulsion on the molecule which is therefore in equilibrium.

Owing to the forces of attraction and repulsion, the density of the liquid is smaller in the surface layer than inside. Indeed, molecule 1 (figure) is acted upon by the force of repulsion from molecule 2 and the forces of attraction from all the other molecules (3, 4, ...). Molecule 2 is acted upon by the forces of repulsion from 3 and 1 and the forces of attraction from the molecules in the deep layers. As a result, distance 1-2 should be greater than 2-3, etc.



This course of reasoning is quite approximate (thermal motion, etc. is disregarded), but nevertheless it gives a qualitatively correct result.

An increase in the surface of the liquid causes new sections of the rarefied surface layer to appear. Here work should be performed against the forces of attraction between the molecules. It is this work that constitutes the surface energy.

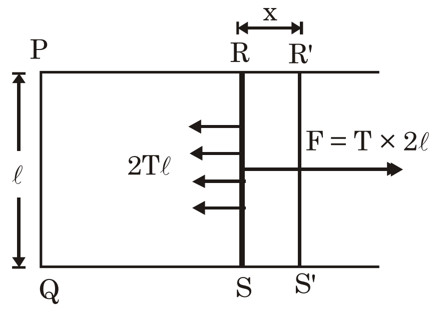
We know that the molecules on the liquid surface experience net downward force. So to bring a molecule from the interior of the liquid to the free surface, some work is required to the done against the intermolecular force of attraction, which will be stored as potential energy of the molecule on the surface. The potential energy of surface molecules per unit area of the surface is called surface is called surface energy. Unit of surface energy is erg cm^{-2} in C.G..S. system and Jm^{-2} in SI system. Dimensional formula of surface energy is $[ML^{\circ}T^{-2}]$ surface energy depends on number of surfaces e.g. a liquid drop is having one liquid air surface while bubble is having two liquid air surface.

Relation between surface tension and surface energy

Consider a rectangular frame PQRS of wire, whose arm RS can slide on the arms PR and QS. If this frame is dipped in a soap solution. then a soap film is produced in the frame PQRS in fig. Due to surface tension (T), the film exerts a force on the frame (towards the interior of the film). Let l be the length of the arm RS, then the force acting on the arm RS towards the film is $F = T \times 2l$ [Since soap film has two surfaces, that is way the length is taken twice.]

$\therefore$ work done, $W = Fx = 2Tlx$

Increase in potential energy of the soap film.



$= EA = 2Elx$ = work done in increasing the area (ΔW)

where E = surface energy of the soap film per unit area.

According the law of conservation of energy, the work done must be equal to the increase in the potential energy.

$\therefore \quad 2Tlx = 2Elx \text{ or } T = E = \dfrac{\Delta W}{A}$

Thus, surface tension is numerically equal to surface energy or work done per unit increase surface area.

Ex. A mercury drop of radius 1 cm is sprayed into 10^6 droplets of equal size. Calculate the energy expanded if surface tension of mercury is 35×10^{-3} N/m.

Sol. If drop of radius R is sprayed into n droplets of equal radius r, then as a drop has only one surface, the initial surface area will be $4\pi R^2$ while final area is $n(4\pi r^2)$. So the increase in area

$$\Delta S = n(4\pi r^2) - 4\pi R^2$$

So energy expended in the process.

$$W = T\Delta S = 4\pi T\,[nr^2 - R^2] \qquad \ldots (1)$$

Now since the total volume of n droplets is the same as that of initial drop, i.e.

$$\frac{4}{3}\pi R^3 = n[(4/3)\,\pi r^3] \quad \text{or} \quad r = R/n^{1/3} \qquad \ldots (2)$$

Putting the value of r from equation (2) in (1)

$$W = 4\pi R^2 T\,(n)^{1/3} - 1$$

Ex. If a number of little droplets of water, each of radius r, coalesce to form a single drop of radius R, show that the rise in temperature will be given by

$$\frac{3T}{J}\left(\frac{1}{r} - \frac{1}{R}\right)$$

where T is the surface tension of water and J is the mechanical equivalent of heat.

Sol. Let n be the number of little droplets.

Since volume will remain constant, hence volume of n little droplets = volume of single drop

$$\therefore \quad n \times \frac{4}{3}\pi r^3 = \frac{4}{3}\pi R^3 \quad \text{or} \quad nr^3 = R^3$$

Decrease in surface area = $n \times 4\pi^3 - 4\pi R^2$

$$\text{or} \quad \Delta A = 4\pi\,[nr^2 - R^2] = 4\pi\left[\frac{nr^3}{r} - R^2\right] = 4\pi\left[\frac{R^3}{r} - R^2\right] = 4\pi R^3\left[\frac{1}{r} - \frac{1}{R}\right]$$

Energy evolved W = T × decrease in surface area $= T \times 4\pi R^3\left[\dfrac{1}{r} - \dfrac{1}{R}\right]$

Heat produced, $Q = \frac{W}{J} = \frac{4\pi TR^3}{J}\left[\frac{1}{r}-\frac{1}{R}\right]$ But $Q = ms\,d\theta$

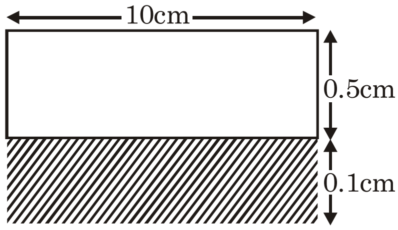
where m is the mass of big drop, s is the specific heat of water and $d\theta$ is the rise in temperature.

$\therefore \quad \frac{4\pi TR^3}{J}\left[\frac{1}{r}-\frac{1}{R}\right]$ = volume of big drop × density of water × sp. heat of water × $d\theta$

or, $\quad \frac{4}{3}\pi R^3 \times 1 \times 1 \times d\theta = \frac{4\pi TR^3}{J}\left(\frac{1}{r}-\frac{1}{R}\right)$ or, $d\theta = \frac{3T}{J}\left[\frac{1}{r}-\frac{1}{R}\right]$

Ex. A film of water is formed between two straight parallel wires each 10cm long and at a separation 0.5 cm. Calculate the work required to increase 1mm distance between them. Surface tension of water 72×10^{-3} N/m.

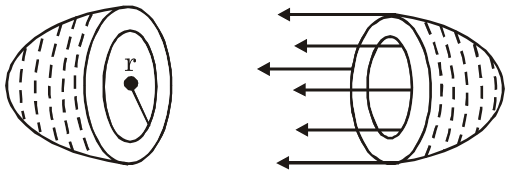
Sol. Here the increase in area is shown by shaded portion in the figure.
Since this a water film, it has two surface, therefore
increase in area, $\Delta S = 2 \times 10 \times 0.1 = 2\text{cm}^2$



$\therefore$ Work required to be done
$W = \Delta S \times T$
$= 2 \times 10^{-4} \times 72 \times 10^{-3}$
$= 144 \times 10^{-7}$ joule
$= 1.44 \times 10^{-5}$ joule

Excess pressure inside A liquid drop and a bubble

1. Inside a bubble : Consider a soap bubble of radius r. Let p be the pressure inside the bubble and p_a outside. The excess pressure = $p - p_a$. Imagine the bubble broken into two halves, and consider one half of it as shown in fig. Since there are two surface, inner and outer, so the force due to surface tension is



F = surface tension × length = T × 2 (circumference of the bubble) = T × 2 (2T πr) ... (1)

The excess pressure $(p - p_a)$ acts on a cross-sectional area πr^2, so the force due to excess pressure is

$\Rightarrow \quad F = (p - p_a)\,\pi r^2$ ... (2)

The surface tension force given by equation (1) must balance the force due to excess pressure given by equation (2) to maintain the equilibrium , i.e. $(p - p_a)\,\pi r^2 = T \times 2(2\pi r)$

or $\quad (p - p_a) = \frac{4T}{r} = p_{excess}$

above expression can also be obtained by equation of excess pressure of curve surface by putting $R_1 = R_2$.

2. Inside the drop : In a drop, there is only one surface and hence excess pressure can be written as

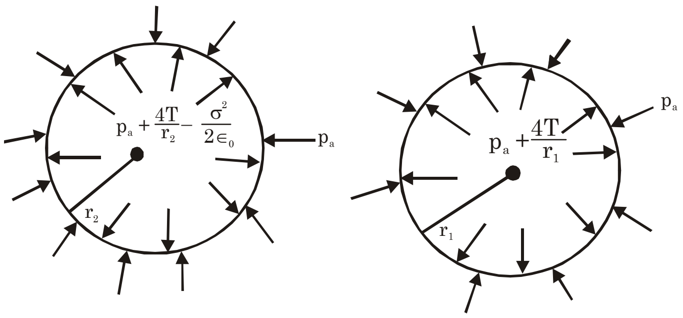
$$(p - p_a) = \frac{2T}{r} = p_{excess}$$

3. Inside air bubble in a liquid :

$$(p - p_a) = \frac{2T}{r} = p_{excess}$$

4. A charged bubble : If bubble is charged, it's radius increases. Bubble has pressure excess due to charge too. Initially pressure inside the bubble

$$= p_a + \frac{4T}{r_1}$$



for charge bubble, pressure inside $= p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\in_0}$, where σ surface is surface charge density.

Taking temperature remains constant, then from Boyle's law

$$\left(p_a + \frac{4T}{r_1}\right)\frac{4}{3}\pi r_1^{\,3} = \left[p_a + \frac{4T}{r_2} - \frac{\sigma^2}{2\in_0}\right]\frac{4}{3}\pi r_2^{\,3}$$

From above expression the radius of charged drop may be calculated. It can conclude that radius of charged bubble increases, i.e. $r_2 > r_1$.

Solved example

Ex. A minute spherical air bubble is rising slowly through a column of mercury contained in a deep jar. If the radius of the bubble at a depth of 100cm is 0.1 mm, calculate its depth where its radius is 0.126 mm, given that the surface tension of mercury is 567 dyne/cm. Assume that the atmospheric pressure is 76cm of mercury.

Sol. The total pressure inside the bubble at depth h_1 is (P atmospheric pressure)

$$= (P + h_1\rho g) + \frac{2T}{r_1} = P_1$$

and the total pressure inside the bubble at depth h_2 is $= (P + h_2\rho g) + \frac{2T}{r_2} = P_2$

Now, according to Boyl's Law

$$P_1V_1 = P_2V_2 \text{ where } V_1 = \frac{4}{3}\pi r_1^{\,3}, \text{ and } V_2 = \frac{4}{3}\pi r_2^{\,3}$$

Hence we get $\left[(P+h_1\rho g)+\frac{2T}{r_1}\right]\frac{4}{3}\pi r_1^{\,3} = \left[(P+h_2\rho g)+\frac{2T}{r_2}\right]\frac{4}{3}\pi r_2^{\,3}$

or, $\left[(P+h_1\rho g)+\frac{2T}{r_1}\right]r_1^{\,3} = \left[(P+h_2\rho g)+\frac{2T}{r_2}\right]r_2^3$

Given that : $h_1 = 100$ cm, $r_1 = 0.1$ mm $= 0.01$ cm, $r_2 = 0.126$ cm, T = 567 dyne / cm, P = 76 cm of mercury. Substituting all the values, we get

$h_2 = 9.48$ cm.

THE FORCE OF COHESION

The force of attraction between the molecules of the same substance is called cohesion.

In case of solids, the force of cohesion is very large and due to this solids have definite shape and size. On the other hand, the force of cohesion in case of liquids is weaker than that of solids. Hence liquids do not have definite shape but have definite volume. The force of cohesion is negligible in case of gases. Because of this fact, gases have neither fixed shape nor volume.

Example

(i) Two drops of a liquid Two drops of a liquid coalesce into one when brought in mutual contact because of the cohesive force.

(ii) It is difficult to separate two sticky plates of glass wetted with water because a large force has to be applied against the cohesive force between the molecules of water.

(iii) It is very difficult to break a drop of mercury into small droplets because of large cohesive force between mercury molecules.

FORCE OF ADHESION

The force of attraction between molecules of different substance is called adhesion.

Example

(i) Adhesive force enables us to write on the black board with a chalk.

(ii) Adhesive force helps us to write on the paper with ink.

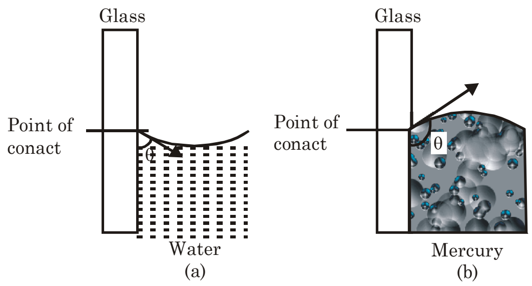
(iii) Large force of adhesion between cement and bricks helps us in construction work.

(iv) Due to force of adhesive, water wets the glass plate.

(v) Fevicol and gum are used in gluing two surfaces together because of adhesive force.

ANGLE OF CONTACT

The angle which the tangent to the liquid surface at the point of contact makes with the solid surface inside the liquid is called angle of contact. Those liquids which we the wall of the container (say in case of water and glass) have meniscus concave upwards and their value of angle of contact is less than 90° (also called acute angle). However, those liquids which don't wet the walls of the container (say in case of mercury and glass) have meniscus convex upwards and their value of angle of contact is greater than 90° (also called obtuse angle). The angle of contact of mercury with glass about 140°, whereas the angle of contact of water with glass is about 8°. But, for pure water, the angle of contact θ with glass is taken as 0°.



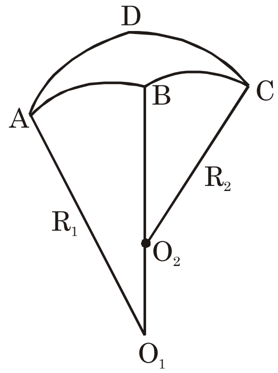
RELATION BETWEEN SURFACE TENSION, RADII OF CURVATURE AND EXCESS PRESSURE ON A CURVED SURFACE.

Let us consider a small element ABCD (fig.) of a curved liquid surface which is convex on the upper side. R_1 and R_2 are the maximum and minimum radii of curvature respectively. They are called the 'principal radii of curvature' of the surface. Let p be the excess pressure on the concave side.

then $p = T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$. If instead of a liquid surface,

we have a liquid film, the above expression will be

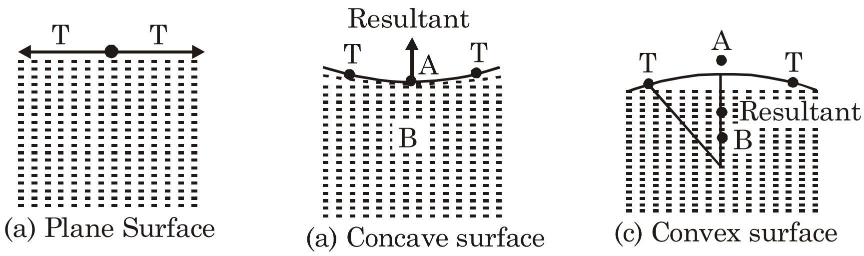
$p = 2T\left(\frac{1}{R_1} + \frac{1}{R_2}\right)$, because a film has two surface.



EXCESS OF PRESSURE INSIDE A CURVED SURFACE

1. Plane surface : If the surface of the liquid is plane [as shown in fig. (a)], the molecule on the liquid surface is attracted equally in all directions. The resultant force due to surface tension is zero. The pressure, therefore on the liquid surface is normal.

2. Concave surface : If the surface is concave upward [as shown in fig. (b)], there will be upward resultant force due to surface tension acting on the molecule. Since the molecule on the surface is in equilibrium, there must be an excess of pressure on the concave side in the downward direction to balance the resultant force of surface tension $p_A - p_B = \frac{2T}{r}$.



(a) Plane Surface (a) Concave surface (c) Convex surface

3. Convex surface : If the surface is convex [as shown fig.(c)], the resultant force due to surface tension acts in the downward direction. Since the molecule on the surface are in equilibrium, there must be an excess of pressure on the concave side of the surface acting in the upward direction to balance the downward resultant force of surface tension. Hence there is always in excess of pressure on concave side of a curved surface over that on the convex side.

$$p_B - p_A = \frac{2T}{r}$$

Solved Examples

Ex. A barometer contains two uniform capillaries of radii 1.44×10^{-3} m and 7.2×10^{-4} m. if the height of the liquid in the tube is 0.2m more than that in the wide tube, calculate the true pressure difference. Density of liquid = 10^3 kg/m^3, surface tension = 72×10^{-3} N/m and g = 9.8 m/s^2.

Sol. Let the pressure in the wide and narrow capillaries of radii r_1 and r_2 respectively be P_1 and P_2. Then pressure just below the mensiscus in the wide and narrow tubes respectively are :

$$\left(P_1 - \frac{2T}{r_1}\right) \text{ and } \left(P_2 - \frac{2T}{r_2}\right) \qquad \left[\text{excess pressure} = \frac{2T}{r}\right]$$

Difference in these pressure = $\left(P_1 - \frac{2T}{r_1}\right) - \left(P_2 - \frac{2T}{r_2}\right) = h\rho g$

$\therefore$ True pressure difference = $P_1 - P_2$

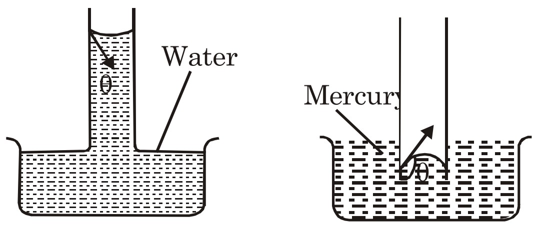
$$= h\rho g + 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

$$= 0.2 \times 10^3 \times 9.8 + 2 \times 72 \times 10^{-3}\left[\frac{1}{1.44 \times 10^{-3}} - \frac{1}{7.2 \times 10^{-4}}\right]$$

$$= 1.86 \times 10^3 = 1860 \text{ N/m}^2$$

Capillarity

A glass tube of very fine bore throughout the length of the tube is called capillary tube. If the capillary tube is dipped in water, the water wets the inner side of the tube and rises in it [shown in figure (a)]. If the same capillary tube is dipped in the mercury, then the mercury is depressed [shown in figure (b)]. The phenomenon of rise or fall of liquids in a capillary tube is called capillarity.

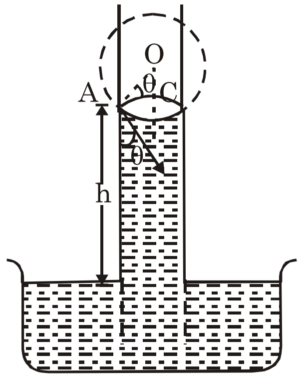


(a) (b)

Particle applications of capillarity

1. The oil in a lamp rises in the wick by capillary action.
2. The tip of nib of a pen is split up, to make a narrow capillary so that the ink rises upto the tin or nib continuously.
3. Sap and water rise upto the top of the leaves of the tree by capillary action.
4. If one end of the towel dips into a bucket of water and the Other end hangs over the bucket the towel soon becomes wet throughout due to capillary action.
5. Ink is absorbed by the blotter due to capillary action.
6. Sandy soil is more dry than clay. It is because the capillaries between sand particles are not so fine as to draw the water up by capillaries.
7. The moisture rises in the capillaries of soil to the surface, where it evaporates. To preserve the moisture m the soil, capillaries must be broken up. This is done by ploughing and leveling the fields.
8. Bricks are porous and behave like capillaries.

Capillary rise (height of a liquid in a capillary tube) ascent formula



consider the liquid which wets the wall of the tube, forms a concave meniscus shown in figure. Consider a capillary tube of radius r dipped in a liquid of surface tension T and density p. Let h be the height through which the liquid rises in the tube. Let p be the pressure on the concave side of the meniscus and p_a be the pressure on the convex side of the meniscus. The excess pressure

$(p - p_a)$ is given by $(p - p_a) = \dfrac{2T}{R}$

Where R is the radius of the meniscus. Due to this excess pressure, the liquid will rise in the capillary tube till it becomes equal to the hydrostatic pressure hpg. Thus in equilibrium state.

Excess pressure = Hydrostatic pressure or $\dfrac{2T}{R} = hpg$

Let θ be the angle of contact and r be the radius of the capillary tube shown in the fig.

From ΔOAC, $\dfrac{OC}{OA} = \cos\theta$ or $R = \dfrac{r}{\cos\theta} \Rightarrow h = \dfrac{2T\cos\theta}{r\rho g}$

The expression is called Ascent formula.

Discussion.

(i) For liquids which wet the glass tube or capillary tube, angle of contact θ < 90º. Hence cos θ = positive ⇒ h = positive. It means that these liquids rise in the capillary tube. Hence, the liquids which wet capillary tube rise in the capillary tube. For example, water milk, kerosene oil, patrol etc.

Solved Examples

Ex. A liquid of specific gravity 1.5 is observed to rise 3.0 cm in a capillary tube of diameter 0.50 mm and the liquid wets the surface of the tube. Calculate th excess pressure inside a spherical bubble of 1.0 cm diameter blown from the same liquid. Angle of contact = 0°.

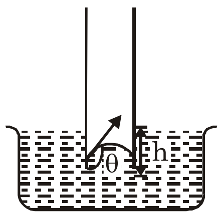
Sol. The surface tension of the liquid is

$$T = \frac{rh\rho g}{2} = \frac{(0.025\text{cm})(3.0\text{cm})(1.5\,\text{gm/cm}^3)(980\,\text{cm/sec}^2)}{2}$$

= 55 dyne/cm.

Hence excess pressure inside a spherical bubble

$$p = \frac{4T}{R} = \frac{4\times 55\text{dyne/cm}}{(0.5\text{cm})} = 440 \text{ dyne/cm}^2.$$



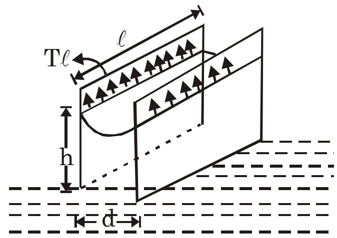
(ii) For liquids which do not wet the glass tube or capillary tube, angle of contact θ > 90º. Hence cos θ = negative ⇒ n = negative. Hence, the liquids which do not wet capillary tube are depressed in the capillary tube. For example, mercury.

(iii) T, θ, ρ and g are constant and hence $h \propto \dfrac{1}{r}$. Thus, the liquid rises more in a narrow tube and less in a wider tube. This is called Jurin's Law.

(iv) If two parallel plates with the spacing 'd' are placed in water reservoir, then height of rise

$\Rightarrow \quad 3Tl = \rho l h d g$

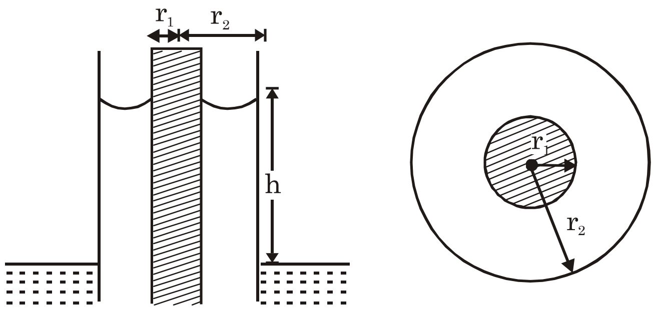
$$h = \frac{2T}{\rho d g}$$



(v) If two concentric tube of radius 'r_1' and 'r_2' (inner one is solid) are placed in water reservoir, then height of rise

$\Rightarrow \quad T\,[2\pi r_1 + 2\pi r_2] = [\pi r_2^2 h - \pi r_1^2 h]\,\rho g$

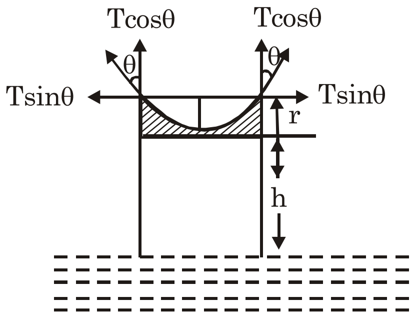
$$h = \frac{2T}{(r_2 - r_1)\rho g}$$



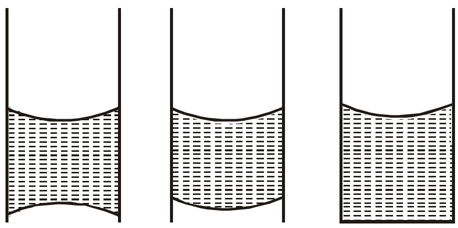
(vi) If weight of the liquid in the menscus is to be consider :

$$T\cos\theta \times 2\pi r = [\pi r^2 h + \frac{1}{3}\,\pi r^2 \times \pi r_1^2 h]\,\rho g$$

$$\left[h + \frac{r}{3}\right] = \frac{2T\cos\theta}{r\rho g}$$



(vii) When capillary tube (radius, 'r') is in vertical position, the upper meniscus is concave and pressure due to surface tension is directed vertically upward and is given by $p_1 = 2T / R_1$ where R_1 = radius of curvature of upper meniscus.



(a) (b) (c)

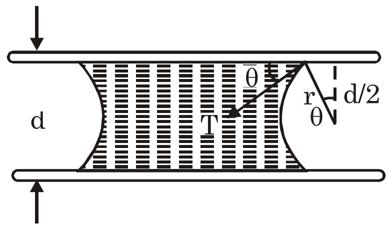
The hydrostatic pressure $p_2 = h\rho g$ is always directed downwards.
If $p_1 > p_2$ i.e. resulting pressure is directed upward. For equilibrium, the pressure due to lower meniscus should be downward. This makes lower meniscus concave downward (fig. (a)). The radius of lower meniscus R_2 can be given by $\frac{2T}{R_2} = (p_1 - p_2)$.

If $p_1 < p_2$ i.e. resulting pressure is directed downward for equilibrium, the pressure due to lower meniscus should be upward. This makes lower meniscus convex upward (fig. b).

The radius of lower meniscus can be given by $\frac{2T}{R_2} = p_2 - p_1$.

If $p_1 = p_2$, then is no resulting pressure, then $p_1 - p_2 = \frac{2T}{R_2} = 0$ or $R_2 = \infty$ i.e. lower surface will be FLAT. (fig.c).

(viii) **Liquid between two plates :** When a small drop of water is placed between two glass plates put face to face, it forms a thin film which is concave outward along its boundary. Let 'R' and 'r' be the radii of curvature of the enclosed film in two perpendicular directions.



Hence the pressure inside the film is less than the atmospheric pressure outside it by an amount p given by $p = T\left(\frac{1}{r} + \frac{1}{r = \infty}\right)$ and we have, $p = \frac{T}{r}$.

If d be the distance between the two plates and θ the angle of contact for water and glass, then, from the figure, $\cos\theta = \frac{\frac{1}{2}d}{r}$ or $\frac{1}{r} = \frac{2\cos\theta}{d}$.

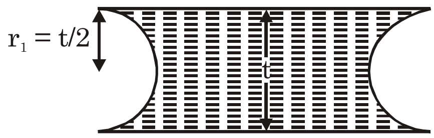
Substituting for $\frac{1}{r}$ in, we get $p = \frac{2T}{d}\cos\theta$.

θ can be taken zero water and glass, i.e. cos θ = 1. Thus the upper plate is pressed downward by the atmospheric pressure minus $\frac{2T}{d}$. Hence the resultant downward pressure acting on the upper plate is $\frac{2T}{d}$. If A be the are of the plate wetted by the film, the resultant force F pressing the upper plate downward is given by F = resultant pressure × area = $\frac{2TA}{d}$. For very nearly plane surface, d will be very small and hence the pressing force F very large. Therefore it will be difficult to separate the two plates normally.

Solved Examples

Ex. A drop of water volume 0.05 cm^3 is pressed between two glass-plates, as a consequence of which, it spreads and occupies an area of 40 cm^2. If the surface tension of water is 70 dyne/cm, find the normal force required to separate out the two glass plates in newton.

Sol. Pressure inside the film is less than outside by an amount, $P = T\left[\frac{1}{r_1} + \frac{1}{r_2}\right]$, where r_1 and r_2 are the radii of curvature of the meniscus. Here $r_1 = 1/2$ and $r_2 = \infty$, then the force required to separate the two glass plates, between which a liquid film is enclosed (figure) is, $F = P \times A = \frac{2AT}{t}$, where t is the thickness of the film, A = area of film.



$$F = \frac{2A^2T}{At} = \frac{2A^2T}{V} = \frac{2\times(40\times10^{-4})^2\times(70\times10^{-3})}{0.05\times10^{-6}} = 45\text{ N}$$

Ex. A glass plate of length 10cm, breadth 1.54 cm and thickness 0.20 cm weigh 8.2 gm in air. It is held vertically with the long side horizontal and the lower half under water. Find the apparent weight of the plate. Surface tension of water 73 dyne per cm, g = 980 cm/sec^2.

Sol. Volume of the portion of the plate immersed in water is

$$10 \times \frac{1}{2}(1.54) \times 0.2 = 1.54\ cm^3.$$

Therefore, if the density of water is taken as 1, then upthrust

= wt. of the water displaced

= 1.54 × 1 × 980 = 1509.2 dynes.

Now, the total length of the plate in contact with the water surface is 2 (10 + 0.2) = 20.4 cm,

∴ downward pull upon the plate due to surface tension

= 20.4 × 73 = 1489.2 dynes

∴ resultant upthrust

= 1509.2 – 1489.2

$= 20.0 \text{ dynes} = \frac{20}{980}$

= 0.0204 gm. wt.

∴ apparent weight of the plate in water

= weight of the plate in air – resultant upthrust

= 8.2 – 0.0204 = 8.1796 gm Ans.

Ex. A glass tube of circular cross-section is closed at one end. This end is weighted and the tube floats vertically in water, heavy end down. How far below the water surface is the end of the tube? Give : Outer radius of the tube 0.14 cm, mass of weighted tube 0.2 gm, surface tension os water 73 dyne/cm and g = 980 cm/sec^2.

Sol. Let l be the length of the tube inside water. The forces acting on the tube are :

(i) Upthrust of water acting upward

$$= \pi r^2 l \times 1 \times 980 = \frac{22}{7} \times (0.14)^2 \, l \times 980 = 60.368 \, l \text{ dyne.}$$

(ii) Weight of the system acting downward

$$= mg = 0.2 \times 980 = 196 \text{ dyne.}$$

(iii) Force of surface tension acting downward

$$= 2\pi rT$$

$$= 2 \times \frac{22}{7} \times 0.14 \times 73 = 64.24 \text{ dyne.}$$

Since the tube is in equilibrium, the upward force is balanced by the downward forces. That is,

$$60.368 \, l = 196 + 64.24 = 260.24.$$

$$\therefore \quad l = \frac{260.24}{60.368} = 4.31 \text{ cm.}$$

Ex. A glass U-tube is such that the diametre of one limb is 3.0 mm and that of the other is 6.00 mm. The tube is inverted vertically with the open ends below the surface of water in a beaker. What is the difference between the heights to which water rises in the two limbs? Surface tension of water is 0.07 nm^{-1}. Assume that the angle of contact between water and glass is 0°.

Sol. Suppose pressure at the points A, B, C and D be P_A, P_B, P_C and P_0 respectively.

The pressure on the concave side of the liquid surface is greater than that on the other side by 2T/R.

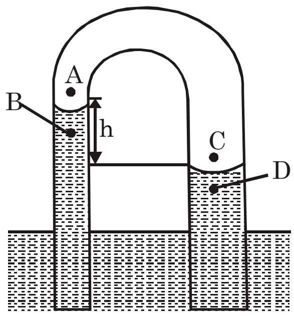
Ang angle of contact θ is given to be 0°, hence R cos 0° = r or R = r

$$\therefore \quad P_A = P_B + 2T / r_1 \quad \text{and} \quad P_C = P_0 + 2T / r_2$$

where r_1 and r_2 are the radii of the two limbs

But $P_A = P_C$

$$\therefore \quad P_B + \frac{2T}{r_1} = P_D + \frac{2T}{r_2}$$



or $$P_D - P_B = 2T\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

where h is the difference in water levels in the two limbs

Now, $$h = \frac{2T}{\rho g}\left(\frac{1}{r_1} - \frac{1}{r_2}\right)$$

Given that $T = 0.07$ Nm^{-1}, $\rho = 1000$ kgm^{-3}

$$r_1 = \frac{3}{2} \text{ mm} = \frac{3}{20} \text{ cm} = \frac{3}{20 \times 100} \text{ m} = 1.5 \times 10^{-3} \text{ m}, \; r_2 = 3 \times 10^{-3} \text{ m}$$

$$\therefore \quad h = \frac{2 \times 0.07}{1000 \times 9.8}\left(\frac{1}{1.5 \times 10^{-3}} - \frac{1}{3 \times 10^{-3}}\right) \text{m} = 4.76 \times 10^{-3} \text{ m} = 4.76 \text{ mm}$$

Ex. Two narrow bores of diameters 3.0 mm and 6.0 mm are joined together to form a U-shaped tube open at both ends. If the U-tube contains water, what is the difference in its levels in the two limbs of the tube? Surface tension of water at the temperature of the experiment is 7.3×10^{-2} Nm^{-1}. Take the angle of contact to be zero.

Sol. Given that $r_1 = \frac{3.0}{2} = 1.5$ mm $= 1.5 \times 10^{-3}$ m, $r_2 = \frac{6.0}{2} = 3.0 \times 10^{-3}$ m,

$T = 7.3 \times 10^{-3}$ Nm^{-1}, $\theta = 0°$ $\rho = 1.0 \times 10^3$ kg m^{-1}, $g = 9.8$ ms^{-2}

When angle of contact is zero degree, the radius of the meniscus equals radius of bore.

Excess pressure in the first bore, $P_1 = \frac{2T}{r_2} = \frac{2 \times 7.3 \times 10^{-3}}{1.5 \times 10^{-3}} = 97.3$ Pascal

Excess pressure in the second before, $P_2 = \frac{2T}{r^2} = \frac{2 \times 7.3 \times 10^{-2}}{3 \times 10^{-3}} = 48.7$ Pascal

Hence, pressure difference in the two limbs of the tube

$$\Delta P = P_1 - P_2 = h\rho g$$

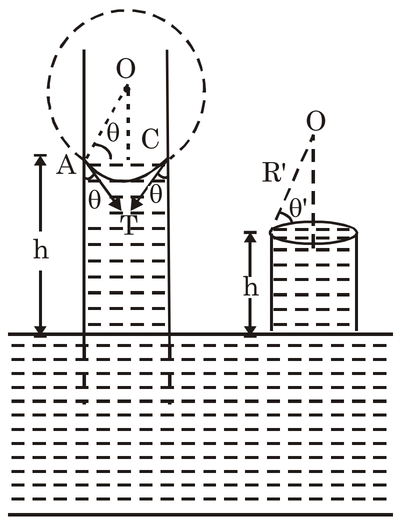
or $$h = \frac{P_1 - P_2}{\rho g} = \frac{97.3 - 48.7}{1.0 \times 10^3 \times 9.8} = 5.0 \text{ mm.}$$

Capillary rise in a tube of insufficient length

We know, the height through which a liquid rises in the capillary tube of radius r is given by

$$\therefore \quad h = \frac{2T}{R\rho g} \text{ or } h\ R = \frac{2T}{\rho g} = \text{constant}$$

When the capillary tube is cut and its length is less then h (i.e. h′). then the liquid rises upto the top of the tube and spreads in such a way that the radius (R′) of the liquid meniscus increases and it becomes more flat so that hR = h′R′ = Constant. Hence the liquid does not overflow.



If $h' < h$ then $R' > R$ or $\frac{r}{\cos\theta'} > \frac{r}{\cos\theta}$

$\Rightarrow \cos\theta' < \cos\theta \quad \Rightarrow \quad \theta' > \theta$

Ex. If a 5cm long capillary tube with 0.1 mm internal diameter open at both ends is slightly dipped in water having surface tension 75 dyne cm^{-1}, state whether (i) water will rise half way in the capillary. (ii) Water will rise up to the upper end of capillary (iii) What will overflow out of the upper end of capillary. Explain your answer.

Sol. Given that surface tension of water, T = 75 dyne/cm

Radius $r = \frac{0.1}{2}$ mm = 0.05 mm = 0.005 cm

density $\rho = 1$ gm/cm^3, angle of contact, $\theta = 0°$

Let h be the height to which water rise in the capillary tube. Then

$$h = \frac{2T\cos\theta}{r\rho g} = \frac{2 \times 75 \times \cos 0°}{0.005 \times 1 \times 981} \text{ cm} = 30.58 \text{ cm}$$

But length of capillary tube, h′ = 5cm

(i) Because $h > \frac{h'}{2}$ therefore the first possibility does not exist.

(ii) Because the tube is of insufficient length therefore the water will rise upto the upper end of the tube.

(iii) The water will not overflow out of the upper end of the capillary. It will rise only upto the upper end of the capillary.

The liquid meniscus will adjust its radius of curvature R′ in such a way that

$$R'h' = Rh \qquad \left[\because\ hR = \frac{2T}{\rho g} = \text{constant}\right]$$

where R is the radius of curvature that the liquid meniscus would possess if the capillary tube were of sufficient length.

$$\therefore \qquad R' = \frac{Rh}{h'} = \frac{rh}{h'} \qquad \left[\because\ R = \frac{r}{\cos\theta} = \frac{r}{\cos 0^\circ} = r\right] = \frac{0.005 \times 30.58}{5} = 0.0306 \text{ cm}$$

Applications of surface tension

(i) The wetting property is made use of in detergents and waterproofing. When the detergent materials are added to liquids, the angle of contact decreases and hence the wettability increases. On the other hand, when water proofing material is added to a fabric, it increases the angle of contact, making the fabric water-repellant.

(ii) The antiseptics have very low value of surface tension. The low value of surface tension prevents the formation of drops that may otherwise block the entrance to skin or a wound. Due to low surface tension the antiseptics spreads properly over the wound. The lubricating oils and paints also have low surface tension. So they can spread properly.

(iii) Surface tension of all lubricating oils and paints is kept low so that they spread over a large area.

(iv) Oil spreads over the surface of water because the surface tension of oil is less than the surface tension of cold water.

(v) A rough sea can be calmed by pouring oil on its surface.

Effect of temperature and impurities on surface tension

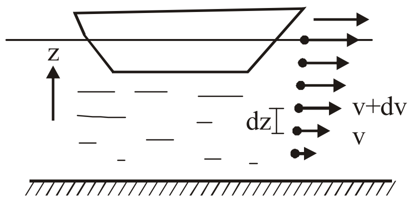
The surface tension of a liquid decreases with the rise in temperature and vice versa. According to Ferguson, $T = T_0\left(1 - \frac{\theta}{\theta_C}\right)^n$ where T_0 is surface tension at 0°C, θ is absolute temperature of the liquid, θ_C is the critical temperature and n is a constant varies slightly from liquid and has means value 1.21. This formula shows that the surface tension becomes zero at the critical temperature, where the while machinery parts get jammed in winter.

The surface tension of a liquid change appreciably with addition of impurities. For example, surface tension of water increases with addition of highly soluble substances like NaCl, $ZnSO_4$ etc. On the other hand surface tension of water gets reduced with addition of sparingly soluble substances like phenol, soap etc.

VISCOSITY

When a layer of a fluid slips or tens to slip on another layer in contact, the two layers exert tangential forces on each other. The directions are such that the relative motion between the layers is opposed. this property of a fluid to oppose relative motion between its layers is called viscosity. The forces between the layers opposing relative motion between them are known as the forces of viscosity. Thus, viscosity may be thought of as the internal friction of a fluid in motion.

If a solid surface is kept in contact with a fluid and is moved, forces of viscosity appear between the solid surface and the fluid layer in contact. the fluid in contact is dragged with the solid. If the viscosity is sufficient, the layer moves with the solid and there is no relative slipping. When a boat moves slowly on the water of a calm river, the water in contact with the boat is dragged with it, whereas the water in contact with the bed of the river remains at rest. Velocities of different layers are different. Let v be the velocity of the layer at a distance z from the bed and v +dv be the velocity at a distance z +dz (figure).



Thus, the velocity differs by dv in going through a distance dz perpendicular to it. The quantity dv/dz is called the velocity gradient.

The force of viscosity between two layers of a fluid is proportional to the velocity gradient in the direction perpendicular to the layers. Also the force is proportional to the area of the layer.

Thus, if F is the force exerted by a layer of area A on a layer in contact,

$$F \propto A \text{ and } F \propto dv/dz$$

or, $$F = -\eta A\, dv/dz$$

The negative sign is included as the force is frictional in nature and opposes relative motion. The constant of proportionality η is called the coefficient of viscosity.

The SI unit of viscosity can be easily worked out from equation. It is N-s/m^2 . However, the corresponding CGS unit dyne-s/cm^2 is in common use and is called a poise in honour of the French scientist Poiseuille. We have

$$1 \text{ poise} = 0.1 \text{ N-s/m}^2$$

TERMINAL VELOCITY

The viscous force on a solid moving through a fluid is proportional to its velocity. When a solid is dropped in a fluid, the forces acting on it are

(a) weight W acting vertically downward,

(b) the viscous force F acting vertically upward and

(c) the buoyancy force B acting vertically upward.

The weight W and the buoyancy B are constant but the force F is proportional to the velocity v, initially, the velocity and hence the viscous force F is zero and the solid is accelerated due to the force W-B. Because of the acceleration, the velocity increases. Accordingly, the viscous force also increases. At a certain instant the viscous force becomes equal to W-B. the net force then becomes zero and the solid falls with constant velocity. This constant velocity is known as the terminal velocity.

Consider a spherical body falling through a liquid. Suppose the density of the body = ρ, density of the liquid = σ, radius of the sphere = r and the terminal velocity = v_0. The viscous force is

$$F = 6\pi\eta r v_0$$

the weight $W = \frac{4}{3}\pi r^3 \rho g$

and the buoyancy force $B = \frac{4}{3}\pi r^3 \sigma g$

We have

$$6\ 6\pi\eta r v_0 = W - B = \frac{4}{3}\pi r^3 \rho g - \frac{4}{3}\pi r^3 \sigma g$$

or $v_0 = \frac{2r^2(\rho - \sigma)g}{9\eta}$

Q. A large wooden plate of area 10 m^2 floating on the surface of a river is made to move horizontally with a speed of 2 m/s by applying a tangential force. If the river is 1 m deep and the water in contact with the bed is stationary, find the tangential force needed to keep the plate moving. Coefficient of viscosity of water at the temperature of the river = 10^{-2} poise.

Sol. The velocity decreases from 2 m/s to zero in 1 m of perpendicular length. Hence, velocity gradient.

$$= dv/dx = 2\ s^{-1}$$

Now, $\eta = \left|\frac{F/A}{dv/dx}\right|$

or, $10^{-3}\ \frac{N-s}{m^2} = \frac{F}{(10m)^2(2s^{-1})}$

or, $F = 0.02$ N.

Q. The velocity of water in a river is 18 km/hr near the surface. If the river is 5 m deep, find the shearing stress between the horizontal layers of water. The coefficient of viscosity of water = 10^{-2} poise.

Sol. The velocity gradient in vertical direction is

$$\frac{dv}{dx} = \frac{18\,km/hr}{5m} = 1.0\,s^{-1}$$

The magnitude of the force of viscosity is

$$F = \eta A \frac{dv}{dx}.$$

The shearing stress is

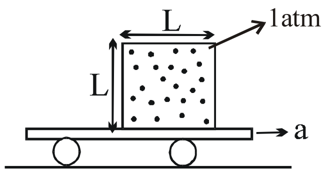
$$F/A = \eta \frac{dv}{dx} = (10^{-2}\text{ poise})\ (1.0\ s^{-1}) = 10^{-3}\ N/m^2$$

EXERCISE (S)

Fluid Statics

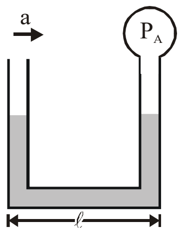
1. A spherical tank of 1.2 m radius is half filled with oil of relative density 0.8. If the tank is given a horizontal acceleration of 10 m/s^2. Calculate the inclination of the oil surface to horizontal and maximum pressure on the tank. **FM0001**

2. A cubical sealed vessel with edge L is placed on a cart, which is moving horizontally with an acceleration 'a' as shown in figure. The cube is filled with an ideal fluid having density ρ. Find the gauge pressure at the centre of the cubical vessel.



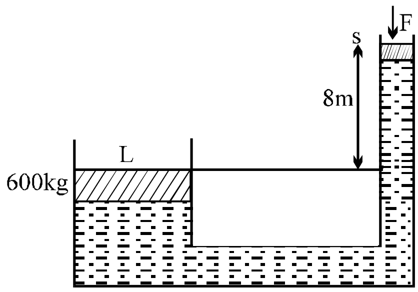
FM0002

3. A liquid of density ρ is filled in a U-tube, whose one end is open & at the other end a bulb is fitted whose pressure is P_A. Now this tube is moved horizontally with acceleration 'a' as shown in the figure. During motion it is found that liquid in both column is at same level at equilibrium. If atmospheric pressure is P_0, then find the value of P_A



FM0003

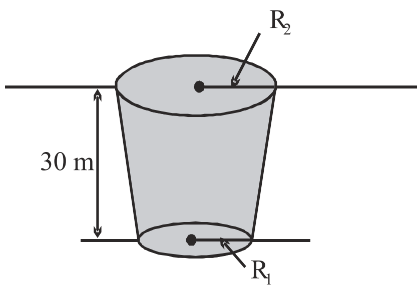
4. For the system shown in the figure, the cylinder on the left at L has a mass of 600 kg and a cross sectional area of 800 cm^2. The piston on the right, at S, has cross sectional area 25 cm^2 and negligible weight. If the apparatus is filled with oil.(ρ = 0.75 gm/cm^3) Find the force F required to hold the system in equilibrium.



FM0004

5. Dams at two different locations are needed to form a lake. When the lake is filled, the water level will be at top of both dams. The second dam is twice as high and twice as wide as the first dam. The force of the water on the second dam is how much greater than the force on the first? (Ignore atmospheric pressure since it is pushing on both sides of the dams.) **FM0005**

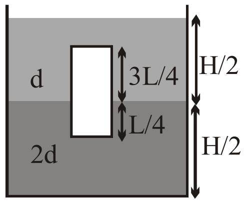
6. As the drawing illustrates, a pond full of water has the shape of an inverted cone with the tip sliced off and has a depth of 30m. The atmospheric pressure above the pond is 1.0×10^5Pa. The circular top surface (radius = R_2) and circular bottom surface (radius = R_1) of the pond are both parallel to the ground. The magnitude of the force acting on the top surface by the liquid is the same as the magnitude of the force acting on the bottom surface by the liquid. Obtain $\frac{R_2}{R_1}$.



FM0006

Buoyancy

7. A container of a large uniform cross-sectional area A resting on a horizontal surface holds two immiscible, non viscous and incompressible liquids of densities d and 2d each of height $\frac{H}{2}$ as shown. The lower density liquid is open to atmosphere. A homogeneous solid cylinder of length $L\left(<\frac{H}{2}\right)$, cross-sectional area $\frac{A}{5}$ is immersed such that it floats with its axis vertical to the liquid -liquid interface with length $\frac{L}{4}$ in denser liquid. Find the density of the solid cylinder.



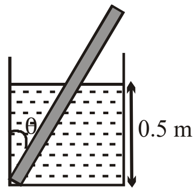
FM0008

8. In air an object weighs 15N, when immersed completely in water the same object weighs 12N. When immersed in another liquid completely, it weighs 13N. Find
(a) the specific gravity of the object and
(b) the specific gravity of the other liquid. **FM0009**

9. A solid sphere of radius R is floating in a liquid of density ρ with half of its volume submerged. If the sphere is slightly pushed and released, it starts performing simple harmonic motion. Find the frequency of these oscillations. **[IIT-JEE 2004]**

FM0167

10. A wooden plank of length 1 m and uniform cross-section is hinged at one end to the bottom of a tank as shown. The tank is filled with water upto a height of 0.5 m. The specific gravity of the plank is 0.5. Find the angle θ made by the plank in equilibrium position

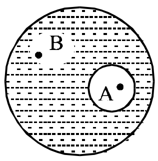


FM0010

11. The volume of an air bubble is doubled as it rises from the bottom of a lake to its surface. If the atmospheric pressure is H m of mercury & the density of mercury is n times that of lake water. Find the depth of the lake. **FM0011**

Surface Tension

12. There is an air bubble of radius R inside a drop of water of radius 3R. Find the ratio of gauge pressure at point A to the gauge pressure at point B.



FM0012

13. Two arms of a U-tube have unequal diameters $d_1 = 1.0$ mm and $d_2 = 1.0$ cm. If water (surface tension 7×10^{-2} N/m) is poured into the tube held in the vertical position, find the difference of level of water in the U-tube. Assume the angle of contact to be zero. **FM0013**

Fluid Dynamics

14. A jet of water having velocity = 10 m/s and stream cross-section = 2 cm^2 hits a flat plate perpendicularly, with the water splashing out parallel to plate. Find the force that the plate experiences.

FM0015

15. A laminar stream is flowing vertically down from a tap of cross-section area 1 cm^2. At a distance 10cm below the tap, the cross-section area of the stream has reduced to 1/2 cm^2. Find the volumetric flow rate of water from the tap. **FM0016**

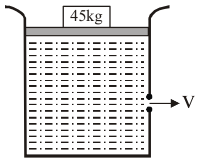
16. Calculate the rate of flow of glycerine of density 1.25×10^3 kg/m^3 through the conical section of a pipe if the radii of its ends are 0.1m & 0.04m and the pressure drop across its length is 10N/m^2.

FM0017

17. A cylindrical vessel open at the top is 20cm high and 10cm in diameter. A circular hole whose cross-sectional area 1 cm^2 is cut at the centre of the bottom of the vessel. Water flows from a tube above it into the vessel at the rate 100 cm^3s^{-1}. Find the height of water in the vessel under steady state.

FM0018

18. A large cylindrical tank of cross-sectional area 1m^2 is filled with water. It has a small hole at a height of 1m from the bottom. A movable piston of mass 5 kg is fitted on the top of the tank such that it can slide in the tank freely. A load of 45 kg is applied on the top of water by piston, as shown in figure. Find the value of v when piston is 7m above the bottom (g = 10 m/s^2)

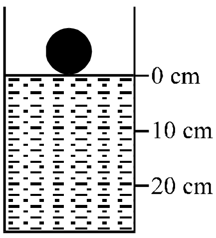


FM0019

Viscosity

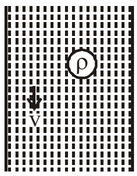
19. A spherical ball of radius 1×10^{-4} m and density 10^4 kg/m^3 falls freely under gravity through a distance h before entering a tank of water. If after entering the water the velocity of the ball does not change, find h. The viscosity of water is 9.8×10^{-6} N-s/m^2. **FM0022**

20. A spherical ball of density ρ and radius 0.003m is dropped into a tube containing a viscous fluid filled up to the 0 cm mark as shown in the figure. Viscosity of the fluid = 1.260 N.m^{-2} and its density $\rho_L = \rho/2$ = 1260 kg.m^{-3}. Assume the ball reaches a terminal speed by the 10 cm mark. Find the time taken by the ball to traverse the distance between the 10 cm and 20 cm mark. (g = acceleration due to gravity = 10 ms^{-2})



FM0023

21. A liquid of density ρ = 1000 kg/m^3 and coefficient of viscosity η = 0.1 Ns/m^2 is flowing down in vertical pipe of large cross section. A small ball of density ρ_0 = 100 kg/m^3 and r = 5 cm will be at rest in flowing liquid, if velocity of flowing liquid is 10 k m/s. Then find the value of k.



FM0024

22. A small sphere falls from rest in a viscous liquid. Due to friction, heat is produced. Find the relation between the rate of production of heat and the radius of the sphere at terminal velocity.

[IIT-JEE 2004]
FM0025

EXERCISE (O)

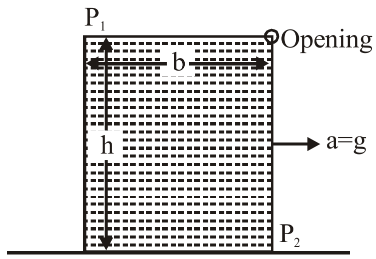
SINGLE CORRECT TYPE QUESTIONS

Fluid Statics

1. A vessel filled with a liquid of density ρ falls vertically downwards with an acceleration a(<g). The guage pressure P at depth h below the free surface of liquid is :

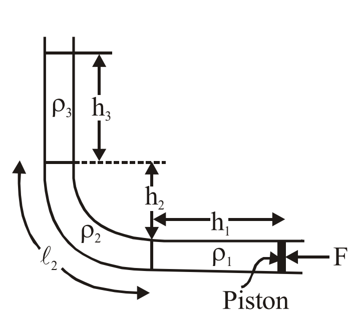
(A) P = h ρ (g + a) (B) P = hρg (C) P = h ρ (g – a) (D) P = hρa **FM0051**

2. A closed rectangular vessel completely filled with a liquid of density ρ moves with an acceleration a = g. The value of the pressure difference $(P_1 - P_2)$ is :



(A) ρgb (B) $\frac{\rho g(b+h)}{2}$ (C) ρ(ab – gh) (D) ρgh **FM0052**

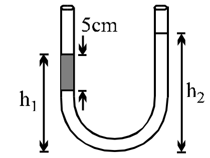
3. A tube is bent into L shape and kept in a vertical plane. If these three liquids are kept in equilibrium by the piston of area A, the value of $\frac{F}{A}$ is :



(A) $(\rho_1h_1 + \rho_2h_2 + \rho_3h_3)g$ (B) $(\rho_1h_1 + \rho_2\ell_2 + \rho_3h_3)g$

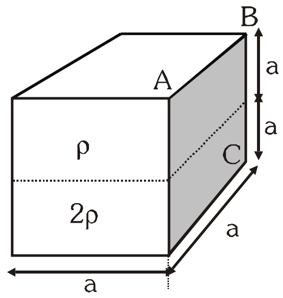
(C) $(\rho_2h_2 + \rho_3h_3)g$ (D) $(\rho_2\ell_2 + \rho_3h_3)g$ **FM0053**

4 An open-ended U-tube of uniform cross-sectional area contains water (density 1.0 gram/centimeter^3) standing initially 20 centimeters from the bottom in each arm. An immiscible liquid of density 4.0grams/ centimeter^3 is added to one arm until a layer 5 centimeters high forms, as shown in the figure above. What is the ratio h_2/h_1 of the heights of the liquid in the two arms?



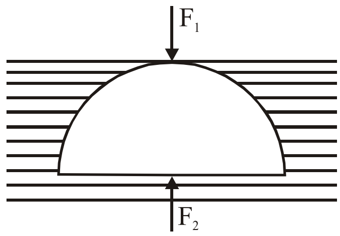
(A) 3/1 (B) 5/2 (C) 2/1 (D) 3/2 **FM0054**

5. A cuboid (a × a × 2a) is filled with two immiscible liquids of density 2ρ & ρ as shown in the figure. Neglecting atmospheric pressure, ratio of force on base & side wall of the cuboid is :-



(A) 2 : 3 (B) 1 : 3 (C) 5 : 6 (D) 6 : 5 **FM0056**

6. A solid hemisphere is just pressed below the liquid, the value of $\frac{F_1}{F_2}$ is (where F_1 and F_2 are the hydrostatic forces acting on the curved and flat surfaces of the hemisphere) (Neglect atmospheric pressure).



(A) $\frac{1}{2}$ (B) $\frac{2}{3}$ (C) $\frac{1}{3}$ (D) none of these

FM0057

7. A thin walled cylindrical metal vessel of linear coefficient of expansion 10^{-3} °C^{-1} contains benzene of volume expansion coefficient 10^{-3}°C^{-1}. If the vessel and its contents are now heated by 10°C, the pressure due to the liquid at the bottom.

(A) increases by 2% (B) decreases by 1%

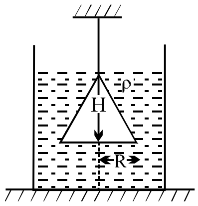
(C) decreases by 2% (D) remains unchanged **FM0058**

8. A cork of density 0.5gcm^{-3} floats on a calm swimming pool. The fraction of the cork's volume which is under water is :-

(A) 0% (B) 25% (C) 10% (D) 50% **FM0059**

Buoyancy

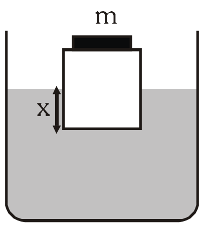
9. A cone of radius R and height H, is hanging inside a liquid of density ρ by means of a string as shown in the figure. The force, due to the liquid acting on the slant surface of the cone is (Neglect atmosphere pressure)



(A) $\rho\pi gHR^2$ (B) $\pi\rho HR^2$ (C) $\frac{4}{3}\pi\rho gHR^2$ (D) $\frac{2}{3}\pi\rho gHR^2$

FM0060

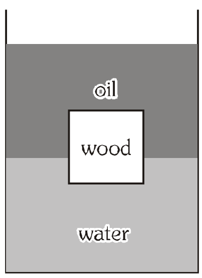
10. A cubical block (a × a × a), with a coin of mass 'm' over it is floating over a liquid of density ρ. In this case x_1 depth of the block is immersed. Now the coin is removed & it is found that x_2 depth is immersed in liquid. Value of $(x_1 - x_2)$ is :-



(A) $\frac{m}{\rho a^2}$ (B) $\frac{\rho a^4}{m}$ (C) $\frac{m}{2\rho a^2}$ (D) data insufficient

FM0061

11. A cubical block of wood of side of 10 cm, floats at the interface between oil and water as shown in figure with its lower face 2 cm below the interface. The density of oil is 0.6 g/cm^3. The mass of the block is



(A) 600 g (B) 680 g (C) 800 g (D) 200 g **FM0063**

12. A solid floats with 2/3 of its volume immersed in a liquid and with 3/4 of its volume immersed in another liquid. What fraction of its volume will be immersed if it floats in a homogenous mixture formed of equal volumes of the liquids?

(A) 6/7 (B) 8/11 (C) 11/16 (D) 12/17 **FM0064**

13. A cuboidal piece of wood has dimensions a, b and c. Its relative density is d. It is floating in a large body of water such that side a is vertical. It is pushed down a bit and released. The time period of SHM executed by it is :

(A) $2\pi\sqrt{\frac{abc}{g}}$ (B) $2\pi\sqrt{\frac{g}{da}}$ (C) $2\pi\sqrt{\frac{bc}{dg}}$ (D) $2\pi\sqrt{\frac{da}{g}}$ **FM0168**

14. A small ball of relative density 0.8 falls into water from a height of 2m. The depth to which the ball will sink is (neglect viscous forces):

(A) 8 m (B) 2 m (C) 6 m (D) 4 m **FM0065**

15. An object of density d_o kept deep inside water of density d_w and released. During the time it moves a vertical distance h with in the water :-

(A) The gravitational potential energy of the water in the vessel increases if $d_0 < d_w$

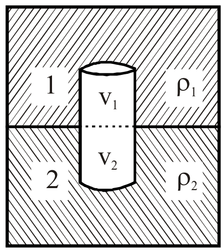
(B) The gravitational potential energy of the water in the vessel decreases if $d_0 < d_w$

(C) The gravitational potential energy of the object increases if $d_0 > d_w$

(D) The gravitational potential energy of the object decreases if $d_0 < d_w$ **FM0066**

16. **Statement-1 :** When a body floats such that it's parts are immersed into two immersible liquids then force exerted by liquid-1 is of magnitude $\rho_1 v_1 g$.

Statement-2 : Total Buogyant force = $\rho_1 v_1 g + \rho_2 v_2 g$



(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true. **FM0067**

17. **Statement-1 :** Submarine sailors are advised that they should not allow it to rest on floor of the ocean.

Statement-2 : The force exerted by a liquid on a submerged body may be downwards.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

(C) Statement-1 is true, statement-2 is false.

(D) Statement-1 is false, statement-2 is true. **FM0068**

Bernoulli

18. Water is flowing steadily through a horizontal tube of non uniform cross-section. If the pressure of water is 4×10^4 N/m^2 at a point where cross-section is 0.02 m^2 and velocity of flow is 2 m/s, what is pressure at a point where cross-section reduces to 0.01 m^2.

(A) 1.4×10^4 N/m^2 (B) 3.4×10^4 N/m^2 (C) 2.4×10^{-4} N/m^2 (D) none of these

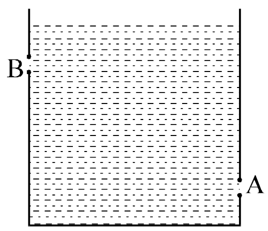
FM0069

19. In the case of a fluid, Bernoulli's theorem expresses the application of the principle of conservation of :-
(A) linear momentum (B) energy (C) mass (D) angular momentum
FM0070

20. Two water pipes P and Q having diameters 2×10^{-2} m and 4×10^{-2}m, respectively, are joined in series with the main supply line of water. The velocity of water flowing in pipe P is :-
(A) 4 times that of Q (B) 2 times that of Q
(C) 1/2 times of that of Q (D) 1/4 times that of Q **FM0071**

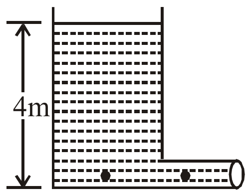
21. A cylinder of height 20 m is completely filled with water. The velocity of efflux of water (in ms^{-1}) through a small hole on the side wall of the cylinder near its bottom, is- **[AIEEE - 2002]**
(A) 10 (B) 20 (C) 25.5 (D) 5 **FM0072**

22. Two identical holes each of cross-sectional area 10^{-3} m^2 are made on the opposite sides of a tank containing water as shown in the figure. As the water comes out of the holes, the tank will experience a net horizontal force of 20 N. The difference in height between the holes A and B is.



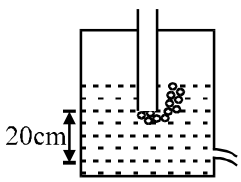
(A) 1 m (B) 0.5 m (C) 2 m (D) 0.25 m **FM0073**

23. A vent tank of large cross-sectional area has a horizontal pipe 0.12 m in diameter at the bottom. This holds a liquid whose density is 1500 kg/m^3 to a height of 4.0 m. Assume the liquid is an ideal fluid in laminar flow. In figure, the velocity with which fluid flows out is :-



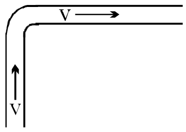
(A) $2\sqrt{5}$ m/s (B) $\sqrt{5}$ m/s (C) $4\sqrt{5}$ m/s (D) $\sqrt{10}$ m/s **FM0074**

24. A tube is attached as shown in closed vessel containing water. The velocity of water coming out from a small hole is :



(A) $\sqrt{2}$ m/s (B) 2 m/s
(C) depends on pressure of air inside vessel (D) None of these **FM0075**

25. A fire hydrant delivers water of density ρ at a volume rate L. The water travels vertically upward through the hydrant and then does 90° turn to emerge horizontally at speed V. The pipe and nozzle have uniform cross-section throughout. The force exerted by the water on the corner of the hydrant is :-



(A) ρVL (B) zero (C) $2\rho VL$ (D) $\sqrt{2}\rho VL$ **FM0076**

26. A large tank is filled with water to a height H. A small hole is made at the base of the tank. It takes T_1 time to decrease the height of water to H/η, $(\eta > 1)$ and it takes T_2 time to take out the rest of water. If $T_1 = T_2$, then the value of η is :

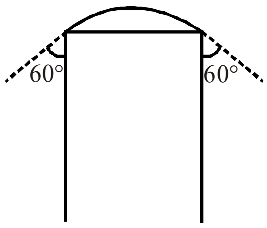
(A) 2 (B) 3 (C) 4 (D) $2\sqrt{2}$ **FM0077**

Surface Tension

27. If two soap bubbles of different radii are connected by a tube,

(A) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(C) air flows from the smaller bubble to the bigger
(D) there is no flow of air. **FM0079**

28. A soap bubble is being blown on a tube of radius 1 cm. The surface tension of the soap solution is 0.05 N/m and the bubble makes an angle of 60° with the tube as shown. The excess of pressure over the atmospheric pressure in the tube is :



(A) 5 Pa (B) 1 Pa (C) 10 Pa (D) 20 Pa **FM0080**

29. When an air bubble rises from the bottom of a deep lake to a point just below the water surface, the pressure of air inside the bubble :-

(A) is greater than the pressure outside it (B) is less than the pressure outside it
(C) increases as the bubble moves up (D) remains same as the bubble moves up

FM0081

30. Two merucry drops (each of radius 'r') merge to form a bigger drop. The surface energy of the bigger drop, if $\frac{1}{\pi}$ is the surface tension (in SI unit), is :

(A) $2^{5/3} r^2$ (B) $4r^2$ (C) $2r^2$ (D) $2^{8/3} r^2$ **FM0082**

31. An open capillary tube is lowered in a vessel with mercury. The difference between the levels of the mercury in the vessel and in the capillary tube $\Delta h = 4.6$mm. What is the radius of curvature of the mercury meniscus in the capillary tube? Surface tension of mercury is 0.46 N/m, density of mercury is 13.6 gm/cc.

(A) $\frac{1}{340}$ m (B) $\frac{1}{680}$ m (C) $\frac{1}{1020}$ m (D) Information insufficient

FM0084

32. A container, whose bottom has round holes with diameter 0.1 mm is filled with water. The maximum height in cm upto which water can be filled without leakage will be what?
Surface tension = 75×10^{-3} N/m and g = 10 m/s^2:

(A) 20 cm (B) 40 cm (C) 30 cm (D) 60 cm **FM0085**

33. If two soap bubbles of different radii are connected by a tube- **[AIEEE - 2004]**

(A) air flows from the bigger bubble to the smaller bubble till the sizes become equal
(B) air flows from bigger bubble to the smaller bubble till the sizes are interchanged
(C) air flows from the smaller bubble to the bigger bubble
(D) there is no flow of air **FM0087**

34. A 20 cm long capillary tube is dipped in water. The water rises upto 8 cm. If the entire arrangement is put in a freely falling elevator, the length of water column in the capillary tube will be- **[AIEEE - 2005]**

(A) 8 cm (B) 10 cm (C) 4 cm (D) 20 cm **FM0088**

Viscosity

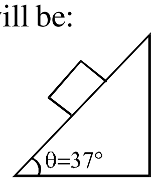
35. Two drops of same radius are falling through air with steady velocity of v cm/s. If the two drops coalesce, what would be the terminal velocity?

(A) 4 v (B) $(4)^{1/3}$ v (C) 2 v (D) 64 v **FM0090**

36. Spherical balls of radius R are falling in a viscous fluid of viscosity η with a velocity v. the retarding viscous force acting on the spherical ball is- **[AIEEE - 2004]**

(A) directly proportional to R but inversely proportional to v
(B) directly proportional to both radius R and velocity v
(C) inversely proportional to both radius R and velocity v
(D) inversely proportional to R but directly proportional to velocity v **FM0091**

37. A cubical block of side 'a' and density 'ρ' slides over a fixed inclined plane with constant velocity 'v'. There is a thin film of viscous fluid of thickness 't' between the plane and the block. Then the coefficient of viscosity of the thin film will be:



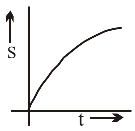
(A) $\frac{3\rho a g t}{5v}$ (B) $\frac{4\rho a g t}{5v}$ (C) $\frac{\rho a g t}{v}$ (D) none of these

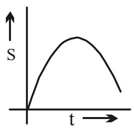
FM0092

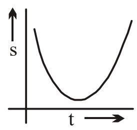
38. There is a 1mm thick layer of glycerine between a flat plate of area 100 cm^2 & a big fixed plate. If the coefficient of viscosity of glycerine is 1.0 kg/m-s then how much force is required to move the plate with a velocity of 7 cm/s?

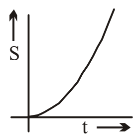
(A) 3.5 N (B) 0.7 N (C) 1.4 N (D) None **FM0093**

39. The displacement of a ball falling from rest in a viscous medium is platted against time. Choose a possible option

(A) 

(B) 

(C) 

(D) 

FM0094

40. An air bubble of radius 1cm is found to rise in a cylindrical vessel of large radius at a steady rate of 0.2cm per second. If the density of the liquid is 1470 kg m^{-3}, then coefficient of viscosity of liquid is approximately equal to

(A) 163 poise (B) 163 centi-poise (C) 140 poise (D) 140 centi-poise

FM0095

41. **Statement-1 :** The free surface of a liquid at rest with respect to stationary container is always normal to the $\vec{g}$.

Statement-2 : Liquids at rest cannot have shear stress.

(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.

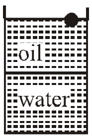
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.

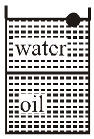
(C) Statement-1 is true, statement-2 is false.

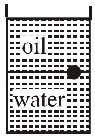
(D) Statement-1 is false, statement-2 is true. **FM0096**

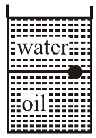
EXERCISE (JM)

1. A ball is made of a material of density ρ where $\rho_{oil} < \rho < \rho_{water}$ with ρ_{oil} and ρ_{water} representing the densities of oil and water, respectively. The oil and water are immiscible. If the above ball is in equilibrium in a mixture of this oil and water, which of the following pictures represents its equilibrium position ? **[AIEEE-2010]**

(1) 

(2) 

(3) 

(4) 

FM0138

2. Water is flowing continuously from a tap having an internal diameter 8×10^{-3} m. The water velocity as it leaves the tap is 0.4 ms^{-1}. The diameter of the water stream at a distance 2×10^{-1} m below the tap is close to :- **[AIEEE-2011]**

(1) 9.6×10^{-3} m (2) 3.6×10^{-3} m (3) 5.0×10^{-3} m (4) 7.5×10^{-3} m

FM0139

3. Work done in increasing the size of a soap bubble from a radius of 3 cm to 5cm is nearly (Surface tension of soap solution = 0.03 Nm^{-1}) :- **[AIEEE-2011]**

(1) 2π mJ (2) $0.4\ \pi$ mJ (3) 4π mJ (4) $0.2\ \pi$ mJ **FM0140**

4. Two mercury drops (each of radius 'r') merge to form a bigger drop. The surface energy of the bigger drop, ifs T is the surface tension, is : **[AIEEE-2011]**

(1) $2^{5/3}\pi r^2 T$ (2) $4\pi r^2 T$ (3) $2\pi r^2 T$ (4) $2^{8/3}\pi r^2 T$ **FM0141**

5. If a ball of steel (density $\rho = 7.8$ g cm^{-3}) attains a terminal velocity of 10 cm s^{-1} when falling in a tank of water (coefficient of viscosity $\eta_{water} = 8.5 \times 10^{-4}$ Pa.s) then its terminal velocity in glycerine ($\rho = 1.2$ g cm^{-3}, $\eta = 13.2$ Pa.s) would be nearly :- **[AIEEE-2011]**

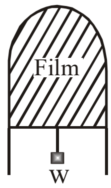
(1) 1.6×10^{-5} cm s^{-1} (2) 6.25×10^{-4} cm s^{-1}
(3) 6.45×10^{-4} cm s^{-1} (4) 1.5×10^{-5} cm s^{-1} **FM0142**

6. A wooden cube (density of wood 'd') of side 'ℓ' floats in a liquid of density 'ρ' with its upper and lower surfaces horizontal. If the cube is pushed slightly down and released, it performs simple harmonic motion of period 'T'. Then, 'T' is equal to :- **[AIEEE-2011]**

(1) $2\pi\sqrt{\frac{\ell\rho}{(\rho-d)g}}$ (2) $2\pi\sqrt{\frac{\ell d}{\rho g}}$ (3) $2\pi\sqrt{\frac{\ell\rho}{dg}}$ (4) $2\pi\sqrt{\frac{\ell d}{(\rho-d)g}}$

FM0169

7. A thin liquid film formed between a U-shaped wire and a light slider supports a weight of 1.5×10^{-2} N (see figure). The length of the slider is 30 cm and its weight negligible. The surface tension of the liquid film is :- **[AIEEE-2012]**



(1) 0.025 Nm^{-1} (2) 0.0125 Nm^{-1} (3) 0.1 Nm^{-1} (4) 0.05 Nm^{-1} **FM0143**

8. A uniform cylinder of length L and mass M having cross- sectional area A is suspended, with its length vertical, form a fixed point by a massless spring, such that it is half submerged in a liquid of density σ at equilibrium position. The extension x_0 of the spring when it is in equilibrium is :

[AIEEE-2013]

(1) $\frac{Mg}{k}$ (2) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{M}\right)$ (3) $\frac{Mg}{k}\left(1-\frac{LA\sigma}{2M}\right)$ (4) $\frac{Mg}{k}\left(1+\frac{LA\sigma}{M}\right)$

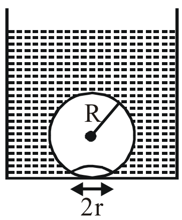
(Here k is spring constant) **FM0144**

9. Assume that a drop of liquid evaporates by decrease in its surface energy, so that its temperature remains unchanged. What should be the minimum radius of the drop for this to be possible? The surface tension is T, density of liquid is ρ and L is its latent heat of vaporization. **[AIEEE-2013]**

(1) $\frac{\rho L}{T}$ (2) $\sqrt{\frac{T}{\rho L}}$ (3) $\frac{T}{\rho L}$ (4) $\frac{2T}{\rho L}$ **FM0145**

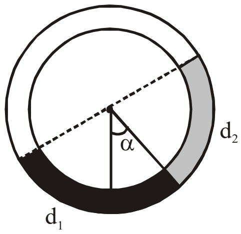
10. On heating water, bubbles being formed at the bottom of the vessel detatch and rise. Take the bubbles to be spheres of radius R and making a circular contact of radius r with the bottom of the vessel. If r << R, and the surface tension of water is T, value of r just before bubbles detatch is:-

(density of water is ρ_w) **[JEE Mains-2014]**



(1) $R^2\sqrt{\frac{\rho_w g}{T}}$ (2) $R^2\sqrt{\frac{3\rho_w g}{T}}$ (3) $R^2\sqrt{\frac{\rho_w g}{3T}}$ (4) $R^2\sqrt{\frac{\rho_w g}{6T}}$ **FM0146**

11. There is a circular tube in a vertical plane. Two liquids which do not mix and of densities d_1 and d_2 are filled in the tube. Each liquid subtends 90° angle at centre. Radius joining their interface makes an angle α with vertical. Ratio $\frac{d_1}{d_2}$ is : **[JEE Mains-2014]**



(1) $\frac{1+\tan\alpha}{1-\tan\alpha}$ (2) $\frac{1+\sin\alpha}{1-\cos\alpha}$ (3) $\frac{1+\sin\alpha}{1-\sin\alpha}$ (4) $\frac{1+\cos\alpha}{1-\cos\alpha}$ **FM0147**

12. The top of a water tank is open to air and its water level is maintained. It is giving out 0.74 m^3 water per minute through a circular opening of 2 cm radius in its wall. The depth of the centre of the opening from the level of water in the tank is close to : **[JEE Main-2019_Jan]**

(1) 9.6 m (2) 4.8 m (3) 2.9 m (4) 6.0 m **FM0170**

13. A cylindrical plastic bottle of negligible mass is filled with 310 ml of water and left floating in a pond with still water. If pressed downward slightly and released, it starts performing simple harmonic motion at angular frequency ω. If the radius of the bottle is 2.5 cm then ω close to : (density of water = 10^3 kg / m^3) **[JEE Main-2019_Jan]**

(1) 5.00 rad s^{-1} (2) 1.25 rad s^{-1} (3) 3.75 rad s^{-1} (4) 2.50 rad s^{-1} **FM0171**

14. Water flows into a large tank with flat bottom at the rate of 10^{-4} m^3s^{-1}. Water is also leaking out of a hole of area 1 cm^2 at its bottom. If the height of the water in the tank remains steady, then this height is: **[JEE Main-2019_Jan]**

(1) 4 cm (2) 2.9 cm (3) 1.7 cm (4) 5.1 cm **FM0172**

15. A liquid of density ρ is coming out of a hose pipe of radius a with horizontal speed v and hits a mesh. 50% of the liquid passes through the mesh unaffected. 25% looses all of its momentum and 25% comes back with the same speed. The resultant pressure on the mesh will be : **[JEE Main-2019_Jan]**

(1) pv^2 (2) $\frac{3}{4}pv^2$ (3) $\frac{1}{2}pv^2$ (4) $\frac{1}{4}pv^2$ **FM0173**

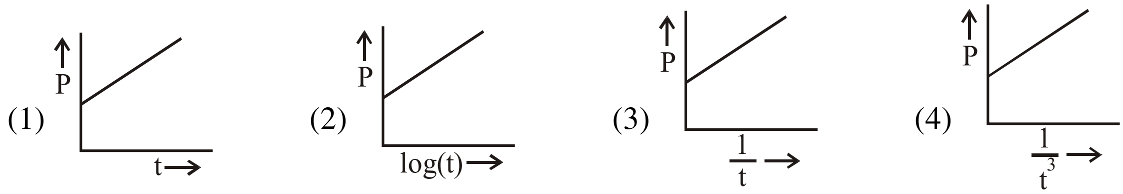
16. A load of mass M kg is suspended from a steel wire of length 2 m and radius 1.0 mm in Searle's apparatus experiment. The increase in length produced in the wire is 4.0 mm. Now the load is fully immersed in a liquid of relative density 2. The relative density of the material of load is 8. The new value of increase in length of the steel wire is : **[JEE Main-2019_Jan]**

(1) 4.0mm (2) 3.0mm (3) 5.0mm (4) zero **FM0174**

17. A long cylindrical vessel is half filled with a liquid. When the vessel is rotated about its own vertical axis, the liquid rises up near the wall. If the radius of vessel is 5 cm and its rotational speed is 2 rotations per second, then the difference in the heights between the centre and the sides, in cm, will be: **[JEE Main-2019_Jan]**

(1) 1.2 (2) 0.1 (3) 2.0 (4) 0.4 **FM0175**

18. A soap bubble, blown by a mechanical pump at the mouth of a tube, increases in volume, with time, at a constant rate. The graph that correctly depicts the time dependence of pressure inside the bubble is given by :- **[JEE Main-2019_Jan]**



FM0176

19. Water from a pipe is coming at a rate of 100 litres per minute. If the radius of the pipe is 5 cm, the Reynolds number for the flow is of the order of : (density of water = 1000 kg/m^3, coefficient of viscosity of water = 1mPas) **[JEE Main-2019_April]**

(1) 10^6 (2) 10^3 (3) 10^4 (4) 10^2 **FM0177**

20. A wooden block floating in a bucket of water has $\frac{4}{5}$ of its volume submerged. When certain amount of an oil is poured into the bucket, it is found that the block is just under the oil surface with half of its volume under water and half in oil. The density of oil relative to that of water is :-

[JEE Main-2019_April]

(1) 0.5 (2) 0.7 (3) 0.6 (4) 0.8 **FM0178**

21. If 'M' is the mass of water that rises in a capillary tube of radius 'r', then mass of water which will rise in a capillary tube of radius '2r' is : **[JEE Main-2019_April]**

(1) 4M (2) M (3) 2M (4) $\frac{M}{2}$ **FM0179**

22. A submarine experiences a pressure of 5.05×10^6 Pa at a depth of d_1 in a sea. When it goes further to a depth of d_2, it experiences a pressure of 8.08×10^6 Pa. ,Then $d_2 - d_1$ is approximately (density of water = 10^3 kg/m^3 and acceleration due to gravity = 10 ms^{-2}) **[JEE Main-2019_April]**

(1) 500 m (2) 400 m (3) 300 m (4) 600 m **FM0180**

23. Water from a tap emerges vertically downwards with an initial speed of 1.0 ms^{-1}. The cross-sectional area of the tap is 10^{-4} m^2. Assume that the pressure is constant throughout the stream of water and that the flow is streamlined. The cross-sectional area of the stream, 0.15 m below the tap would be :

(Take g = 10 ms^{-2}) **[JEE Main-2019_April]**

(1) 1×10^{-5} m^2 (2) 5×10^{-5} m^2 (3) 2×10^{-5} m^2 (4) 5×10^{-4} m^2 **FM0181**

24. A cubical block of side 0.5 m floats on water with 30% of its volume under water. What is the maximum weight that can be put on the block without fully submerging it under water?

(Take density of water = 10^3 kg/m^3) **[JEE Main-2019_April]**

(1) 65.4 kg (2) 87.5 kg (3) 30.1 kg (4) 46.3 kg **FM0182**

25. The radtio of surface tensions of mercury and water is given to be 7.5 while the ratio of thier densities is 13.6. Their contact angles, with glass, are close to 135° and 0°, respectively. It is observed that mercury gets depressed by an amount h in a capillary tube of radius r_1, while water rises by the same amount h in a capillary tube of radius r_2. The ratio, (r_1/r_2), is then close to :

[JEE Main-2019_April]

(1) 2/3 (2) 3/5 (3) 2/5 (4) 4/5 **FM0183**

26. A solid sphere, of radius R acquires a terminal velocity ν_1 when falling (due to gravity) through a viscous fluid having a coefficient of viscosity η. The sphere is broken into 27 identical solid spheres. If each of these spheres acquires a terminal velocity, ν_2, when falling through the same fluid, the ratio (ν_1/ν_2) equals : **[JEE Main-2019_April]**

(1) 1/27 (2) 1/9 (3) 27 (4) 9 **FM0184**

ANSWER KEY

EXERCISE (S)

1. Ans. 45°, $9600\sqrt{2}$ Pa **2. Ans.** $\frac{L}{2}\rho(g + a)$ **3. Ans.** $P_0 - \rho a\ell$ **4. Ans.** 37.5 N

5. Ans. 8 **6. Ans.** 2 **7. Ans.** $\frac{5d}{4}$ **8. Ans.** (a) 5, (b) 2/3

9. Ans. $f = \frac{1}{2\pi}\sqrt{\frac{3g}{2R}}$ **10. Ans.** 45° **11. Ans.** nH **12. Ans.** 4

13. Ans. 2.52 cm **14. Ans.** 20 N **15. Ans.** 4.9 litre/min **16. Ans.** 6.43×10^{-4} m^3/s

17. Ans. 5 cm **18. Ans.** 11 m/s **19. Ans.** 20.4 m **20. Ans.** 5 s

21. Ans. 5 **22. Ans.** $\frac{dQ}{dt} \propto r^5$

EXERCISE (O)

1. Ans. (C)	**2. Ans. (C)**	**3. Ans. (C)**	**4 Ans. (C)**	**5. Ans. (D)**	**6. Ans. (C)**
7. Ans. (C)	**8. Ans. (D)**	**9. Ans. (D)**	**10. Ans. (A)**	**11. Ans. (B)**	**12. Ans. (D)**
13. Ans. (D)	**14. Ans. (A)**	**15. Ans. (B)**	**16. Ans. (D)**	**17. Ans. (A)**	**18. Ans. (B)**
19. Ans. (B)	**20. Ans. (A)**	**21. Ans. (B)**	**22. Ans. (A)**	**23. Ans. (C)**	**24. Ans. (B)**
25. Ans. (D)	**26. Ans. (C)**	**27. Ans. (C)**	**28. Ans. (C)**	**29. Ans. (A)**	**30. Ans. (D)**
31. Ans. (B)	**32. Ans. (C)**	**33. Ans. (C)**	**34. Ans. (D)**	**35. Ans. (B)**	**36. Ans. (B)**
37. Ans. (A)	**38. Ans. (B)**	**39. Ans. (D)**	**40. Ans. (A)**	**41. Ans. (A)**	

EXERCISE (JM)

1. Ans. (3)	**2. Ans. (2)**	**3. Ans. (2)**	**4. Ans. (4)**	**5. Ans. (2)**	**6. Ans. (2)**
7. Ans. (1)	**8. Ans. (3)**	**9. Ans. (4)**	**10. Ans. (3)**	**11. Ans. (1)**	**12. Ans. (2)**
13. Ans. (Bonus)		**14. Ans. (4)**	**15. Ans. (2)**	**16. Ans. (2)**	**17. Ans. (3)**
18. Ans. (Bonus)		**19. Ans. (3)**	**20. Ans. (3)**	**21. Ans. (3)**	**22. Ans. (3)**
23. Ans. (2)	**24. Ans. (2)**	**25. Ans. (3)**	**26. Ans. (4)**		

Important Notes