ELECTROSTATICS

KEY CONCEPT

ELECTRIC CHARGE

Charge is the property associated with matter due to which it produces and experiences electrical and magnetic effects. The excess or deficiency of electrons in a body gives the concept of charge.

SI unit of charge : ampere × second i.e. Coulomb Dimension : [A T]

Practical units of charge are ampere × hour (=3600 C) and faraday (= 96500 C)

- Millikan calculated quanta of charge by 'Highest common factor' (H.C.F.) method and it is equal to charge of electron.
- 1 C = 3 × 10^9 state coulomb, 1 absolute - coulomb = 10 C, 1 Faraday = 96500 C.

SPECIFIC PROPERTIES OF CHARGE

- **Charge is a scalar quantity :** It represents excess or deficiency of electrons.
- **Charge is transferable :** If a charged body is put in contact with an another body, then charge can be transferred to another body.
- **Charge is always associated with mass**

  Charge cannot exist without mass though mass can exist without charge.
  - So the presence of charge itself is a convincing proof of existence of mass.
  - In charging, the mass of a body changes.
  - When body is given positive charge, its mass decreases.
  - When body is given negative charge, its mass increases.
- **Charge is quantized**

  The quantization of electric charge is the property by virtue of which all free charges are integral multiple of a basic unit of charge represented by e. Thus charge q of a body is always given by

  q = ne

  n = positive integer or negative integer

  The quantum of charge is the charge that an electron or proton carries.

  Note : Charge on a proton = (–) charge on an electron = 1.6 × 10^{-19} C
- **Charge is conserved**

  In an isolated system, total charge does not change with time, though individual charge may change i.e. charge can neither be created nor destroyed. Conservation of charge is also found to hold good in all types of reactions either chemical (atomic) or nuclear. No exceptions to the rule have ever been found.
- **Charge is invariant**

  Charge is independent of frame of reference. i.e. charge on a body does not change whatever be its speed.
- **Attraction – Repulsion**

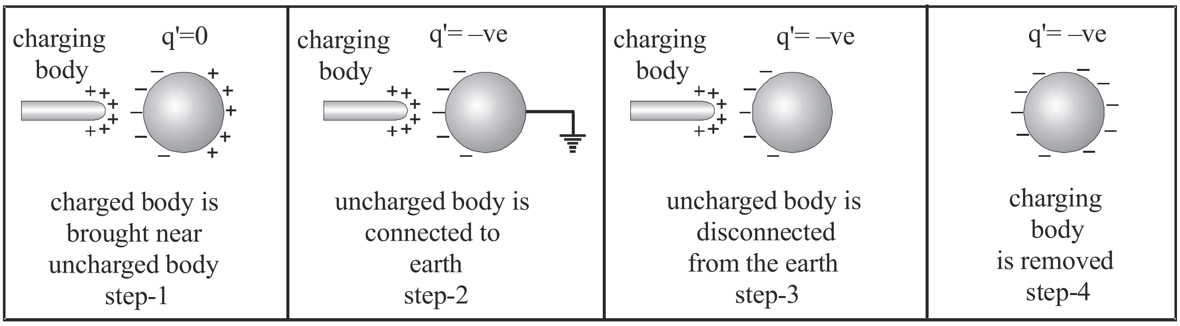
  Similar charges repel each other while dissimilar attract

METHODS OF CHARGING

- **Friction:** If we rub one body with other body, electrons are transferred from one body to the other.
- **Electrostatic induction**

  If a charged body is brought near a metallic neutral body, the charged body will attract opposite charge and repel similar charge present in the neutral body. As a result of this one side of the neutral body becomes negative while the other positive, this process is called 'electrostatic induction'.

  • Charging a body by induction (in four successive steps)



Some important facts associated with induction-

(i) Inducing body neither gains nor loses charge

(ii) The nature of induced charge is always opposite to that of inducing charge

(iii) Induction takes place only in bodies (either conducting or non conducting) and not in particles.

- **Conduction**

  The process of transfer of charge by contact of two bodies is known as conduction. If a charged body is put in contact with uncharged body, the uncharged body becomes charged due to transfer of electrons from one body to the other.

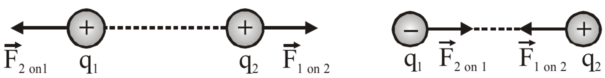
  - The charged body loses some of its charge (which is equal to the charge gained by the uncharged body)
  - The charge gained by the uncharged body is always lesser than initial charge present on the charged body.

COULOMB'S LAW

The electrostatic force of interaction between two static point electric charges is directly proportional to the product of the charges, inversely proportional to the square of the distance between them and acts along the straight line joining the two charges.

Force of electrostatic interaction depends on the nature of medium between the charges. If two points charges q_1 and q_2 separated by a distance r. Let F be the electrostatic force between these two charges. According to Coulomb's law.

$F \propto q_1 q_2$ and $F \propto \frac{1}{r^2}$

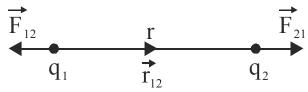


$$F_e = \frac{kq_1q_2}{r^2}$$

where $\left[k = \frac{1}{4\pi \in_0} = 9\times10^9 \frac{Nm^2}{C^2}\right]$ k = coulomb's constant or electrostatic force constant

Coulomb's law in vector form

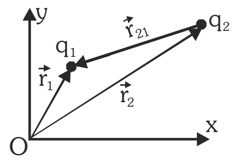
$\vec{F}_{12}$ = force on q_1 due to q_2 = $\dfrac{kq_1q_2}{r^2}\hat{r}_{21}$



$\vec{F}_{21} = \dfrac{kq_1q_2}{r^2}\hat{r}_{12}$ (here $\hat{r}_{12}$ is unit vector from q_1 to q_2)

Coulomb's law in terms of position vector

$$\vec{F}_{12} = \frac{kq_1q_2}{\left|\vec{r}_1 - \vec{r}_2\right|^3}(\vec{r}_1 - \vec{r}_2)$$



Principle of superposition

When a number of charges are interacting, the total force on a given charge is vector sum of the forces exerted on it by all other charges individually

$$F = \frac{kq_0q_1}{r_1^2} + \frac{kq_0q_2}{r_2^2} + \cdots + \frac{kq_0q_i}{r_i^2} + \ldots \frac{kq_0q_n}{r_n^2}$$

in vector form $\vec{F} = kq_0 \sum_{i=1}^{n} \dfrac{q_i}{r_i^2}\hat{r}_i$

Some important points regarding Coulomb's law and electric force

- The law is based on physical observations and is not logically derivable from any other concept. Experiments till today reveal its universal nature.
- The force is a two body interaction, i.e., electrical force between two point charges is independent of presence or absence of other charges and so the principle of superposition is valid, i.e., force on a charged particle due to number of point charges is the resultant of forces due to individual point charges, i.e., $\vec{F}_1 = \vec{F}_{12} + \vec{F}_{13} + \ldots.$
- The net Coulomb's force on two charged particles in free space and in a medium filled upto infinity are

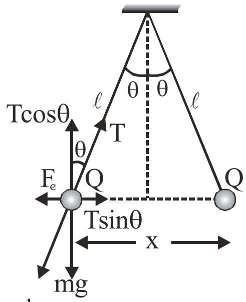
$$F = \frac{1}{4\pi\varepsilon_0}\frac{q_1q_2}{r^2} \text{ and } F' = \frac{1}{4\pi\varepsilon}\frac{q_1q_2}{r^2}. \text{ So } \frac{F}{F'} = \frac{\varepsilon}{\varepsilon_0} = K,$$

- Dielectric constant (K) of a medium is numerically equal to the ratio of the force on two point charges in free space to that in the medium filled upto infinity.
- The law expresses the force between two point charges at rest. In applying it to the case of extended bodies of finite size care should be taken in assuming the whole charge of a body to be concentrated at its 'centre' as this is true only for spherical charged body, that too for external point.

Although net electric force on both particles change in the presence of dielectric but force due to one charge particle on another charge particle does not depend on the medium between them.

❐ Equilibrium of suspended point charge system

For equilibrium position

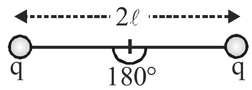


$$T\cos\theta = mg \text{ and } T\sin\theta = F_e = \frac{kQ^2}{x^2} \Rightarrow \tan\theta = \frac{F_e}{mg} = \frac{kQ^2}{x^2 mg}$$

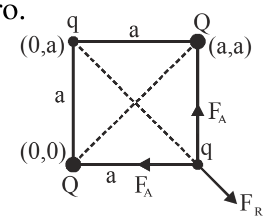
- If θ is small then

$$\tan\theta \approx \sin\theta = \frac{x}{2\ell} \Rightarrow \frac{x}{2\ell} = \frac{kQ^2}{x^2 mg} \Rightarrow x^3 = \frac{2kQ^2\ell}{mg} \Rightarrow x = \left[\frac{Q^2\ell}{2\pi\epsilon_0 mg}\right]^{\frac{1}{3}}$$

- If whole set up is taken into an artificial satellite ($g_{eff} \simeq 0$) $T = F_e = \dfrac{kq^2}{4\ell^2}$



Ex. For the system shown in figure find Q for which resultant force on q is zero.



Sol. For force on q to be zero, charges q and Q must be of opposite of nature.

Net attraction force on q due to charges Q = Repulsion force on q due to q

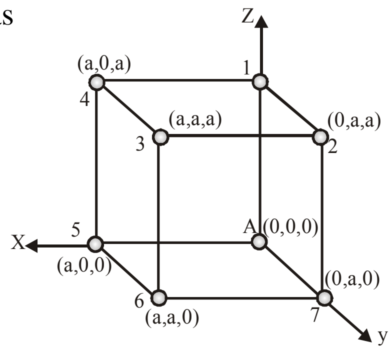
$$\sqrt{2}\, F_A = F_R \Rightarrow \sqrt{2}\,\frac{kQq}{a^2} = \frac{kq^2}{(\sqrt{2}a)^2} \Rightarrow q = 2\sqrt{2}\,Q \text{ Hence } q = -2\sqrt{2}\,Q$$

Ex. Given a cube with point charges q on each of its vertices. Calculate the force exerted on any of the charges due to rest of the 7 charges.

Sol. The net force on particle A can be given by vector sum of force experienced by this particle due to all the other charges on vertices of the cube. For this we use vector form of coulomb's law

$$\vec{F} = \frac{kq_1q_2}{\left|\vec{r}_1 - \vec{r}_2\right|^3}\left(\vec{r}_1 - \vec{r}_2\right)$$

From the figure the different forces acting on A are given as



$$\vec{F}_{A_1} = \frac{kq^2\left(-a\hat{k}\right)}{a^3}$$

$$\vec{F}_{A_2} = \frac{kq^2\left(-a\hat{j} - a\hat{k}\right)}{\left(\sqrt{2}a\right)^3},\ \vec{F}_{A_3} = \frac{kq^2\left(-a\hat{i} - a\hat{j} - a\hat{k}\right)}{\left(\sqrt{3}a\right)^3},$$

$$\vec{F}_{A_4} = \frac{kq^2\left(-a\hat{j} - a\hat{k}\right)}{\left(\sqrt{2}a\right)^3},\ \vec{F}_{A_5} = \frac{kq^2\left(-a\hat{j}\right)}{a^3},\ \vec{F}_{A_6} = \frac{kq^2\left(-a\hat{i} - a\hat{j}\right)}{\left(\sqrt{2}a\right)^3},\ \vec{F}_{A_7} = \frac{kq^2\left(-a\hat{j}\right)}{a^3}$$

The net force experienced by A can be given as

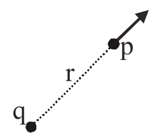
$$\vec{F}_{net} = \vec{F}_{A_1} + \vec{F}_{A_2} + \vec{F}_{A_3} + \vec{F}_{A_4} + \vec{F}_{A_5} + \vec{F}_{A_6} + \vec{F}_{A_7} = \frac{-kq^2}{a^2}\left[\left(\frac{1}{3\sqrt{3}} + \frac{1}{\sqrt{2}} + 1\right)\left(\hat{i} + \hat{j} + \hat{k}\right)\right]$$

ELECTRIC FIELD

In order to explain 'action at a distance', i.e., 'force without contact' between charges it is assumed that a charge or charge distribution produces a field in space surrounding it. So the region surrounding a charge (or charge distribution) in which its electrical effects are perceptible is called the electric field of the given charge. Electric field at a point is characterized either by a vector function of position $\vec{E}$ called 'electric intensity' or by a scalar function of position V called 'electric potential'. The electric field in a certain space is also visualized graphically in terms of 'lines of force.' So electric intensity, potential and lines of force are different ways of describing the same field.

Intensity of electric field due to point charge

Electric field intensity is defined as force on unit test charge.



$$\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{q_0} = \frac{kq}{r^2}\hat{r} = \frac{kq}{r^3}\vec{r}$$

Note : Test charge (q_0) is a fictitious charge that exerts no force on nearby charges but experiences forces due to them.

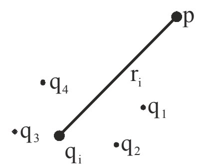
Unit : N/C, V/m **Dimensions** : $[MLT^{-3}A^{-1}]$

Due to discrete distribution of charge

Field produced by a charge distribution for discrete distribution:–

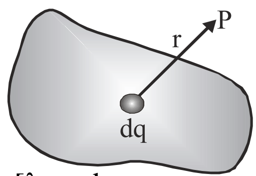
By principle of superposition intensity of electric field due to i[th] charge $\vec{E}_{ip} = \frac{kq}{r_i^3}\vec{r}_i$

$\therefore$ Net electric field due to whole distribution of charge $\vec{E}_p = \sum_{i=1} \vec{E}_i$



Continuous distribution of charge

Treating a small element as particle $\vec{E} = \frac{1}{4\pi\epsilon_0}\int \frac{dq}{r^3}\vec{r}$



Due to linear charge distribution $E = k\int_{\ell} \frac{\lambda d\ell}{r^2}$ [λ = charge per unit length]

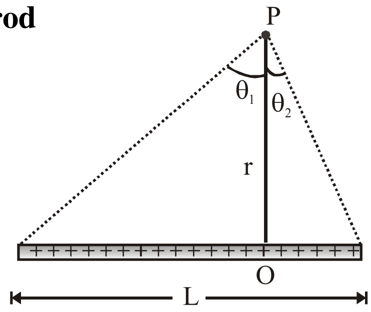
Due to surface charge distribution $E = k\int_{s} \frac{\sigma ds}{r^2}$ [σ = charge per unit area]

Due to volume charge distribution $E = k\int_{v} \frac{\rho dv}{r^2}$ [ρ = charge per unit volume]

Key points

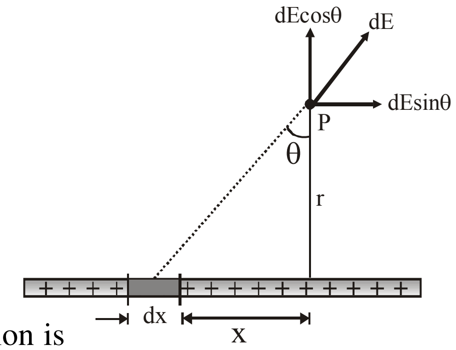
- Charged particle in an electric field always experiences a force either it is at rest or in motion.
- In presence of a dielectric , electric field decreases and becomes $\frac{1}{\epsilon_r}$ times of its value in free space.
- Test charge is always a unit (+ ve) charge. $\vec{E} = \frac{\vec{F}_{test}}{\text{test charge}}$
- If identical charges are placed on each vertices of a regular polygon, then $\vec{E}$ at centre = zero.

Electric field strength at a general point due to a uniformly charged rod



As shown in figure, if P is any general point in the surrounding of rod, to find electric field strength at P, we consider an element on rod of length dx at a distance x from point O as shown in figure. Now if dE be the electric field at P due to the element, then

$$dE = \frac{kdq}{(x^2 + r^2)} \quad \text{Here } dq = \frac{Q}{L}dx$$



Electric field strength in x–direction due to dq at P is

$$dE_x = dE\sin\theta = \left[\frac{kdq}{(x^2+r^2)}\right]\sin\theta = \frac{kQ\sin\theta}{L(x^2+r^2)}dx$$

Here we have $x = r\tan\theta$ and $dx = r\sec^2\theta\, d\theta$

$$\text{Thus } dE_x = \frac{kQ}{L}\,\frac{r\sec^2\theta\, d\theta}{r^2\sec^2\theta}\sin\theta,\ \text{Strength} = \frac{kQ}{Lr}\sin\theta\, d\theta$$

Net electric field strength due to dq at point P in x–direction is

$$E_x = \int dE_x = \frac{kQ}{Lr}\int_{-\theta_2}^{+\theta}\sin\theta\, d\theta = \frac{kQ}{Lr}\left[-\cos\theta\right]_{-\theta_2}^{+\theta_1} = \frac{kQ}{Lr}\left[\cos\theta_2 - \cos\theta_1\right]$$

Similarly, electric field strength at point P due to dq in y–direction is

$$dE_y = dE\cos\theta = \frac{kQdx}{L(r^2+x^2)}\times\cos\theta$$

Again we have $x = r\tan\theta$ and $dx = r\sec^2\theta d\theta$.

$$\text{Thus we have } dE_y = \frac{kQ}{L}\cos\theta \times \frac{r\sec^2\theta}{r^2\sec^2\theta}d\theta = \frac{kQ}{Lr}\cos\theta\, d\theta$$

Net electric field strength at P due to dq in y–direction is

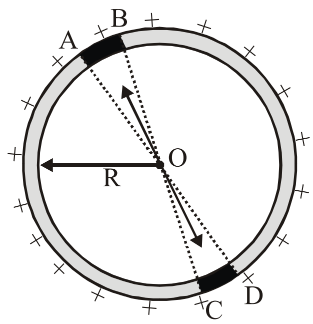
$$E_y = \int dE_y = \frac{kQ}{Lr}\int_{-\theta_2}^{+\theta_1}\cos\theta\, d\theta = \frac{kQ}{Lr}\left[+\sin\theta\right]_{-\theta_2}^{+\theta_1} = \frac{kQ}{Lr}\left[\sin\theta_1 + \sin\theta_2\right]$$

Thus electric field at a general point in the surrounding of a uniformly charged rod which subtend angles θ_1 and θ_2 at the two corners of rod can be given as

in x–direction : $E_x = \frac{kQ}{Lr}(\cos\theta_2 - \cos\theta_1)$ and in y–direction $E_y = \frac{kQ}{Lr}(\sin\theta_1 - \sin\theta_2)$

Electric field due to a uniformly charged ring

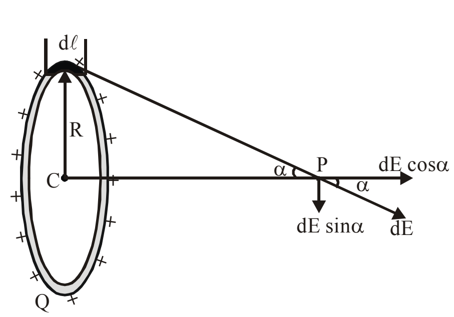
Case – I : At its centre



Here by symmetry we can say that electric field strength at centre due to every small segment on ring is cancelled by the electric field at centre due to the segment exactly opposite to it. The electric field strength at centre due to segment AB is cancelled by that due to segment CD. This net electric field strength at the centre of a uniformly charged ring is $E_0 = 0$

Case II : At a point on the axis of Ring

Here we'll find the electric field strength at point P due to the ring which is situated at a distance x from the ring centre. For this we consider a small section of length $d\ell$ on ring as shown. The charge on this elemental section is $dq = \frac{Q}{2\pi R}d\ell$ [Q= total charge of ring]



Due to the element $d\ell$, electric field strength dE at point P can be given as $dE = \frac{Kdq}{\left(R^2+x^2\right)}$

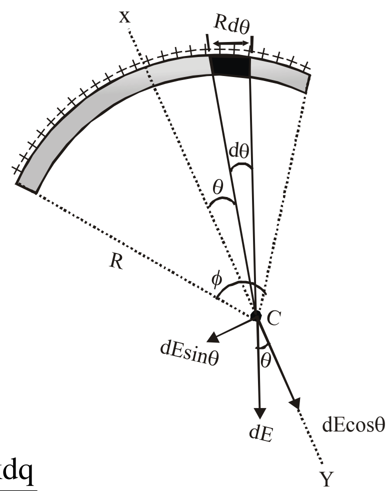
The component of this field strength dE sin α which is normal to the axis of ring will be cancelled out due to the ring section opposite to $d\ell$. The component of electric field strength along the axis of ring $dE\cos\alpha$ due to all the sections will be added up. Hence total electric field strength at point P due to the ring is

$$E_p = \int dE\cos\alpha = \int_0^{2\pi R} \frac{kdq}{\left(R^2+x^2\right)} \times \frac{x}{\sqrt{R^2+x^2}} = \int_0^{2\pi R} \frac{kQx}{2\pi R\left(R^2+x^2\right)}d\ell = \frac{kQx}{2\pi R\left(R^2+x^2\right)^{3/2}}\int_0^{2\pi R} d\ell$$

$$= \frac{kQ_x}{2\pi R\left(R^2+x^2\right)^{3/2}}\left[2\pi R\right] = \frac{kQx}{\left(R^2+x^2\right)^{3/2}}$$

Electric field strength due to a charged circular arc at its centre :

Figure shows a circular arc of radius R which subtend an angle φ at its centre. To find electric field strength at C, we consider a polar segment on arc of angular width $d\theta$ at an angle θ from the angular bisector XY as shown. The length of elemental segment is Rdθ, the charge on this element $d\ell$ is $dq = \left(\frac{Q}{\phi}\right).d\theta$



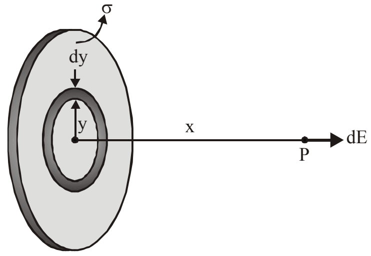
Due to this dq, electric field at centre of arc C is given as $dE = \frac{kdq}{R^2}$

Now electric field component due to this segment dEsinθ which is perpendicular to the angular bisector gets cancelled out in integration and net electric field at centre will be along angular bisector which can be calculated by integrating dEcosθ within limits from $-\frac{\phi}{2}$ to $\frac{\phi}{2}$.

Hence net electric field strength at centre C is $E_C = \int dE\cos\theta$

$$= \int_{-\phi/2}^{+\phi/2} \frac{kQ}{\phi R^2}\cos\theta d\theta = \frac{kQ}{\phi R^2}\int_{-\phi/2}^{+\phi/2}\cos\theta d\theta = \frac{kQ}{\phi R^2}\left[\sin\theta\right]_{-\phi/2}^{+\phi/2} = \frac{kQ}{\phi R^2}\left[\sin\frac{\phi}{2}+\sin\frac{\phi}{2}\right] = \frac{2kQ\sin\left(\frac{\phi}{2}\right)}{\phi R^2}$$

Electric field strength due to a uniformly surface charged disc :



If there is a disc of radius R, charged on its surface with surface charge density σ, we wish to find electric field strength due to this disc at a distance x from the centre of disc on its axis at point P shown in figure.

To find electric field at point P due to this disc, we consider an elemental ring of radius y and width dy in the disc as shown in figure. The charge on this elemental ring $dq = \sigma.2\pi y dy$ [Area of elemental ring $ds = 2\pi y\ dy$]

Now we know that electric field strength due to a ring of radius R, charge Q, at a distance x from its centre on its axis can be given as $E = \dfrac{kQx}{\left(x^2+R^2\right)^{3/2}}$

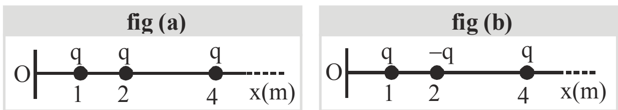
Here due to the elemental ring electric field strength dE at point P can be given as

$$dE = \frac{kdqx}{\left(x^2+y^2\right)^{3/2}} = \frac{k\sigma 2\pi y\ dy\ x}{\left(x^2+y^2\right)^{3/2}}$$

Net electric field at point P due to this disc is given by integrating above expression from 0 to R as

$$E = \int dE = \int_0^R \frac{k\sigma 2\pi xydy}{\left(x^2+y^2\right)^{3/2}} = k\sigma\pi x\int_0^R \frac{2y\ dy}{\left(x^2+y^2\right)^{3/2}} = 2k\sigma\pi x\left[-\frac{1}{\sqrt{x^2+y^2}}\right]_0^R = \frac{\sigma}{2\epsilon_0}\left[1-\frac{x}{\sqrt{x^2+R^2}}\right]$$

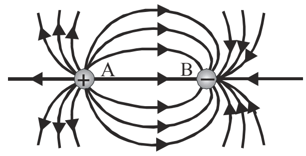
Ex. Calculate the electric field at origin due to infinite number of charges as shown in figures below.



Sol. (a) $E_0 = kq\left[\frac{1}{1}+\frac{1}{4}+\frac{1}{16}+----\right] = \frac{kq.1}{(1-1/4)} = \frac{4kq}{3}$ [$\because$ $S_\infty = \frac{a}{1-r}$, a = 1 and r = $\frac{1}{4}$]

(b) $E_0 = kq\left[\frac{1}{1}-\frac{1}{4}+\frac{1}{16}-----\right] = \frac{kq.1}{\left(1-(-1/4)\right)} = \frac{4kq}{5}$

ELECTRIC LINES OF FORCE



Electric lines of electrostatic field have following properties

(i) Imaginary

(ii) Can never cross each other

(iii) Can never be closed loops

(iv) The number of lines originating or terminating on a charge is proportional to the magnitude of charge. In rationalised MKS system ($1/\varepsilon_o$) electric lines are associated with unit charge, so if a body encloses a charge q , total lines of force associated with it (called flux) will be q/ε_o.

(v) If there is no electric field there will be no lines of force.

(vi) Lines of force per unit area normal to the area at a point represents magnitude of intensity , crowded lines represent strong field while distant lines weak field.

(vii)Tangent to the line of force at a point in an electric field gives the direction of intensity. So a positive charge free to move follow the line of force.

- Lines of force starts from (+ve) charge and ends on (–ve) charge.

Electric flux (ϕ)

The word "flux" comes from a Latin word meaning "to flow" and you can consider the flux of a vector field to be a measure of the flow through an imaginary fixed element of surface in the field.

Electric flux is defined as $\phi_E = \int \vec{E}\cdot d\vec{A}$

This surface integral indicates that the surface in question is to be divided into infinitesimal elements of area $d\vec{A}$ and the scalar quantity $\vec{E}\cdot d\vec{A}$ is to be evaluated for each element and summed over the entire surface.

Electric flux :

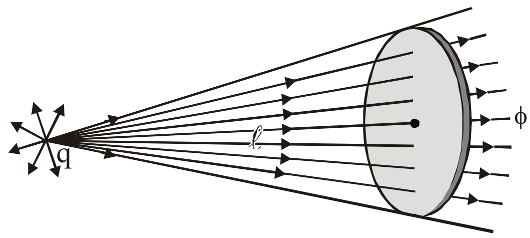
(i) It is a scalar quantity (ii) **Units** (V–m) and N – m^2/C

Dimensions : $[ML^3T^{-3}A^{-1}]$

(iii) The value of ϕ does not depend upon the distribution of charges and the distance between them inside the closed surface.

Electric Flux through a circular Disc :

Figure shows a point charge q placed at a distance ℓ from a disc of radius R. Here we wish to find the electric flux through the disc surface due to the point charge q. We know a point charge q originates electric flux in radially outward direction. The flux is originated in cone shown in figure passes through the disc surface.

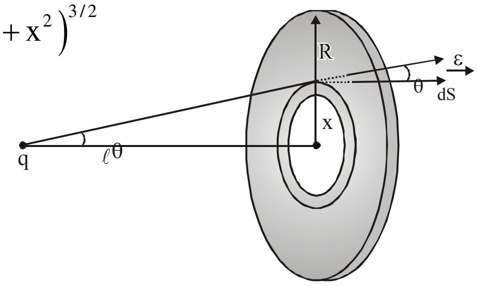


To calculate this flux, we consider on elemental ring an disc surface of radius x and width dx as shown. Area of this ring (strip) is $dS = 2\pi x\ dx$. The electric field due to q at this elemental ring is given as $E = \dfrac{kq}{\left(x^2+\ell^2\right)}$

If $d\phi$ is the flux passing through this elemental ring, then

$$d\phi = EdS\cos\theta = \frac{kq}{\left(x^2+\ell^2\right)} \times 2\pi x\ dx \times \frac{\ell}{\sqrt{x^2+\ell^2}} = \frac{2\pi kq\ell\ x\ dx}{\left(\ell^2+x^2\right)^{3/2}}$$

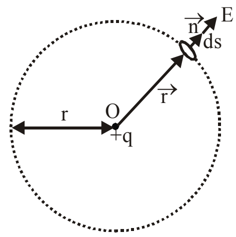
$$\phi = \int d\phi = \int_0^R \frac{q\ell}{2\epsilon_0}\frac{x\ dx}{\left(\ell^2+x^2\right)^{3/2}} = \frac{q\ell}{2\epsilon_0}\int_0^R \frac{x\ dx}{\left(\ell^2+x^2\right)^{3/2}}$$



$$= \frac{q\ell}{2\epsilon_0}\left[-\frac{1}{\sqrt{\ell^2+x^2}}\right]_0^R = \frac{q\ell}{2\epsilon_0}\left[\frac{1}{\ell}-\frac{1}{\sqrt{\ell^2+x^2}}\right]$$

The above result can be obtained in a much simpler way by using the concept of solid angle and Gauss's law.

GAUSS'S LAW



It relates with the total flux of an electric field through a closed surface to the net charge enclosed by that surface and according to it, the total flux linked with a closed surface is $(1/\varepsilon_0)$ times the charge enclosed by the closed surface i.e., $\int_S \vec{E}.\overrightarrow{ds} = \frac{q}{\epsilon_0}$

Regarding Gauss's law it is worth noting that :

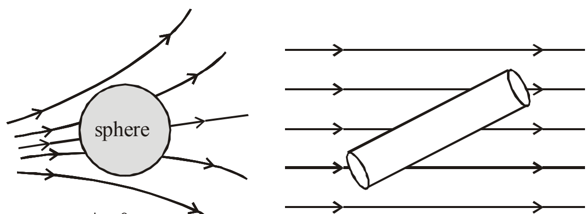
(1) Gauss's law and Coulomb's law are equivalent, i.e., if we assume Coulomb's law we can prove Gauss's law and vice–versa. To prove Gauss's law from Coulomb's law consider a hypothetical spherical surface [called Gaussian–surface] of radius r with point charge q at its centre as shown in figure. By Coulomb's law intensity at a point P on the surface will be, $\vec{E} = \frac{1}{4\pi\varepsilon_0 r^3}\vec{r}$

And hence electric flux linked with area $\overrightarrow{ds} \Rightarrow \vec{E}.\overrightarrow{ds} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^3}\vec{r}.\overrightarrow{ds}$

Here direction of $\vec{r}$ and $\overrightarrow{ds}$ are same, i.e., $\int_S \vec{E}.\overrightarrow{ds} = \frac{1}{4\pi\varepsilon_0}\frac{q}{r^2}\oint_S ds = \frac{1}{4\pi\varepsilon_0}\frac{1}{r^2}\left(4\pi r^2\right)\ \oint_S \vec{E}.\overrightarrow{ds} = \frac{q}{\epsilon_0}$

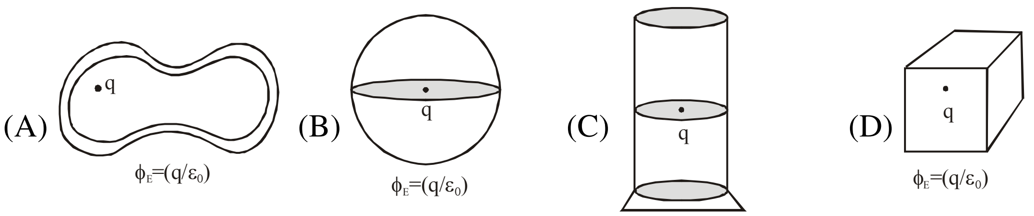
Which is the required result. Though here in proving it we have assumed the surface to be spherical, it is true for any arbitrary surface provided the surface is closed.

(2) (a) If a closed body (not enclosing any charge) is placed in an electric field (either uniform or non–uniform) total flux linked with it will be zero.



$\phi_E=0$ (A) $\phi_E=0$ (B)

(b) If a closed body encloses a charge q, then total flux linked with the body will be $\oint_S \vec{E}.\overrightarrow{ds} = \frac{q}{\epsilon_0}$

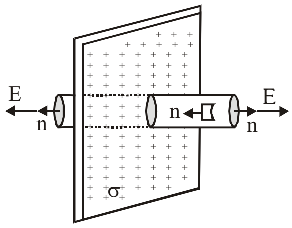


$\phi_E=(q/\varepsilon_0)$ $\phi_E=(q/\varepsilon_0)$ $\phi_E=(q/\varepsilon_0)$ $\phi_E=(q/\varepsilon_0)$

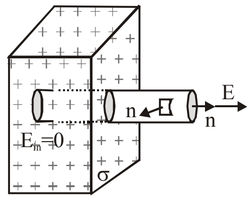
From this expression it is clear that the flux linked with a closed body is independent of the shape and size of the body and position of charge inside it.[figure]

Note : So in case of closed symmetrical body with charge at its centre, flux linked with each half will be $\frac{1}{2}(\phi_E) = \left(\frac{q}{2\varepsilon_0}\right)$ and the symmetrical closed body has n identical faces with point charge at its centre, flux linked with each face will be $\left(\frac{\phi_E}{n}\right) = \left(\frac{q}{n\varepsilon_0}\right)$

(3) Gauss's law is a powerful tool for calculating electric intensity in case of symmetrical charge distribution by choosing a Gaussian– surface in such a way that $\vec{E}$ is either parallel or perpendicular to its various faces. As an example, consider the case of a plane sheet of charge having charge density σ. To calculate E at a point P close to it consider a Gaussian surface in the form of a 'pill box' of cross–section S as shown in figure.



Sheet of charge
(A)



Conductor
(B)

The charge enclosed by the Gaussian–surface = σS and the flux linked with the pill box = ES + 0 + ES = 2ES (as E is parallel to curved surface and perpendicular to plane faces)

So from Gauss's law, $\phi_E = \frac{1}{\varepsilon_0}(q)$, $2ES = \frac{1}{\varepsilon_0}(\sigma S) \Rightarrow E = \frac{\sigma}{2\varepsilon_0}$

(4) If $\vec{E} = 0$, $\phi = \oint \vec{E}.\overrightarrow{ds} = 0$, so q = 0 but if q = 0, $\oint \vec{E}.\overrightarrow{ds} = 0$ So $\vec{E}$ may or may not be zero.

If a dipole is enclosed by a closed surface then, q = 0, so $\oint \vec{E}.\overrightarrow{ds} = 0$, but $\vec{E} \neq 0$

Note : If instead of plane sheet of charge, we have a charged conductor, then as shown in figure (B) $E_{in} = 0$. So $\phi_E = ES$ and hence in this case $E = \frac{\sigma}{\epsilon_0}$. This result can be verified from the fact that intensity at the surface of a charged spherical conductor of radius R is, $E = \frac{1}{4\pi\varepsilon_0}\frac{q}{R^2}$ with $q = 4\pi R^2\sigma$

So for a point close to the surface of conductor, $E = \frac{1}{4\pi\varepsilon_0 R^2} \times \left(4\pi R^2\sigma\right) = \frac{\sigma}{\varepsilon_0}$

Ex. If a point charge q is placed at the centre of a cube.
What is the flux linked (a) with the cube? (b) with each face of the cube?

Sol. (a) According to Gauss's law flux linked with a closed body is $(1/\varepsilon_0)$ times the charge enclosed and here the closed body cube is enclosing a charge q so, $\phi_T = \frac{1}{\varepsilon_0}(q)$

(b) Now as cube is a symmetrical body with 6 –faces and the point charge is at its centre, so electric flux linked with each face will be $\phi_F = \frac{1}{6}(\phi_T) = \frac{q}{6\varepsilon_0}$

Note: (i) Here flux linked with cube or one of its faces is independent of the side of cube.
(ii) If charge is not at the centre of cube (but anywhere inside it), total flux will not change, but the flux linked with different faces will be different.

Ex. Consider $\vec{E} = 3 \times 10^3 \hat{i}$ (N/C) then what is the flux through the square of 10 cm side, if the normal of its plane makes 60° angle with the X axis.

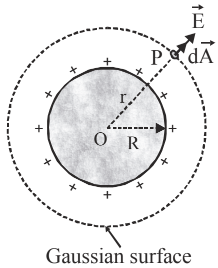
Sol. $\phi = ES\cos\theta = 3 \times 10^3 \times [10 \times 10^{-2}]^2 \times \cos 60° = 3 \times 10^3 \times 10^{-2} \times ½ = 15\ Nm^2/C$

Ex. Find the electric field due to an infinitely long cylindrical charge distribution of radius R and having linear charge density λ at a distance half of the radius from its axis.

Sol. $r = \frac{R}{2}$ point will be inside so $E = \frac{2k\lambda r}{R^2} = \frac{2k\lambda}{R^2}\left(\frac{R}{2}\right) = \frac{\lambda}{4\pi \in_0 R}$

ELECTRIC FIELD DUE TO SOLID CONDUCTING OR HOLLOW SPHERE

- **For outside point (r > R)**



Using Gauss's theorem $\oint \vec{E}.d\vec{s} = \frac{\Sigma q}{\in_0}$

∵ At every point on the Gaussian surface $\vec{E} || d\vec{s}$;

⇒ $\vec{E}.d\vec{s} = E\ ds \cos 0° = E\ ds$

∴ $\oint E.ds = \frac{\Sigma q}{\in_0}$ [E is constant over the gaussian surface] ⇒ $E \times 4\pi r^2 = \frac{q}{\in_0}$ ⇒ $E_p = \frac{q}{4\pi \in_0 r^2}$

- **For surface point r = R** : $E_S = \frac{q}{4\pi \in_0 R^2}$

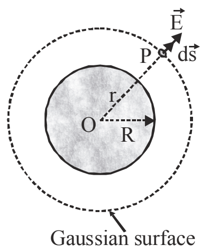
- **For Inside point (r < R)** : Because charge inside the conducting sphere or hollow is zero.

(i.e. Σq = 0) So $\oint \vec{E}.d\vec{s} = \frac{\Sigma q}{\in_0} = 0 \Rightarrow E_{in} = 0$

ELECTRIC FIELD DUE TO SOLID NON CONDUCTING SPHERE

- **Outside (r > R)**

From Gauss's theorem

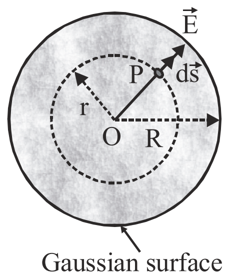


$$\oint_s \vec{E}.d\vec{s} = \frac{\Sigma q}{\in_0} \Rightarrow E \times 4\pi r^2 = \frac{q}{\in_0} \Rightarrow E_P = \frac{q}{4\pi \in_0 r^2}$$

- **At surface (r = R)**

$$E_S = \frac{q}{4\pi \in_0 R^2} \text{ Put } r = R$$

- **Inside (r < R) :**



From Gauss's theorem $\oint_s \vec{E}.d\vec{s} = \frac{\Sigma q}{\in_0}$

Where Σq charge contained within Gaussian surface of radius r

$$E(4\pi r^2) = \frac{\Sigma q}{\in_0} \Rightarrow E = \frac{\Sigma q}{4\pi r^2 \in_0} \quad ...(i)$$

As the sphere is uniformly charged, the volume charge density (charge/volume) ρ is constant throughout the sphere $\rho = \dfrac{q}{\frac{4}{3}\pi R^3}$ $\Rightarrow$ charge enclosed in gaussian surface

$$= \rho\left(\frac{4}{3}\pi r^3\right) = \left(\frac{q}{(4/3)\pi R^3}\right)\left(\frac{4}{3}\pi r^3\right) \Rightarrow \sum q = \frac{qr^3}{R^3}$$

put this value in equation (i) $E_{in} = \dfrac{1}{4\pi\epsilon_0}\dfrac{qr}{R^3}$

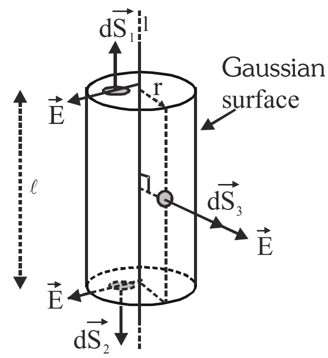
ELECTRIC FIELD DUE TO AN INFINITE LINE DISTRIBUTION OF CHARGE

Let a wire of infinite length is uniformly charged having a constant linear charge density λ.

P is the point where electric field is to be calculated.

Let us draw a coaxial Gaussian cylindrical surfaces of length ℓ .

From Gauss's theorem



$$q\int_{s_1}\vec{E}.d\vec{S}_1 + \int_{s_2}\vec{E}.d\vec{S}_2 + \int_{s_3}\vec{E}.d\vec{S}_3 = \frac{q}{\epsilon_0}$$

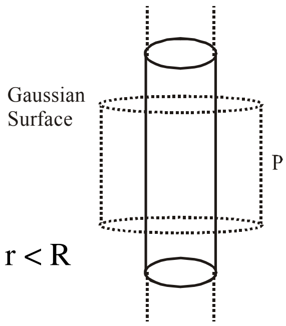
$\Sigma\vec{E} \perp d\vec{S}_1$ so $\vec{E}.d\vec{S}_1 = 0$ and $\vec{E} \perp d\vec{S}_2$ so $\vec{E}.d\vec{S}_2 = 0$

$$E \times 2\pi r\ell = \frac{q}{\epsilon_0} \quad [\because \ \vec{E} \parallel d\vec{S}_3]$$

Charge enclosed in the Gaussian surface $q = \lambda\ell$.

So $E \times 2\pi r\ell = \dfrac{\lambda\ell}{\epsilon_0} \Rightarrow E = \dfrac{\lambda}{2\pi\epsilon_0 r}$ or $E = \dfrac{2k\lambda}{r}$ where $k = \dfrac{1}{4\pi\epsilon_0}$

Charged cylindrical nonconductor of infinite length



Electric field at outside point $\vec{E}_A = \dfrac{2k\lambda}{r}\hat{r}$ $\quad r > R$

Electric field at inside point $\vec{E}_B = \dfrac{\lambda\vec{r}}{2\pi\epsilon_0 R^2}$ $\quad r < R$

KEY POINTS

- Electric field inside a solid conductor is always zero.
- Electric field inside a hollow conductor may or may not be zero ($E \neq 0$ if non zero charge is inside the sphere).
- The electric field due to a circular loop of charge and a point charge are identical provided the distance of the observation point from the circular loop is quite large as compared to its radius i.e. $x >>> R$.

Ex. A charged particle is kept in equilibrium in the electric field between the plates of millikan oil drop experiment. If the direction of the electric field between the plates is reversed, then calculate acceleration of the charged particle.

Sol. Let mass of the particle = m,

charge on particle = q

Intensity of electric field in between plates = E initially mg = qE

After reversing the field ma = mg + qE ⇒ ma = 2mg

∴ Acceleration of particle ⇒ a = 2g

ELECTROSTATIC POTENTIAL ENERGY

Potential energy of a system of particles is defined only in conservative fields. As electric field is also conservative, we define potential energy in it. Potential energy of a system of particles we define as the work done in assembling the system in a given configuration against the interaction forces of particles. Electrostatic potential energy is defined in two ways.

(i) Interaction energy of charged particles of a system

(ii) Self energy of a charged object

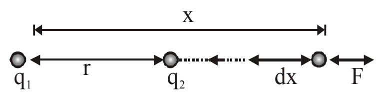
INFINITE SHEET CHARGE

- **Electrostatic Interaction Energy**

Electrostatic interaction energy of a system of charged particles is defined as the external work required to assemble the particles from infinity to the given configuration. When some charged particles are at infinite separation, their potential energy is taken zero as no interaction is there between them. When these charges are brought close to a given configuration, external work is required if the force between these particles is repulsive and energy is supplied to the system, hence final potential energy of system will be positive. If the force between the particle is attractive, work will be done by the system and final potential energy of system will be negative.

- **Interaction Energy of a system of two charged particles**

Figure shows two + ve charges q_1 and q_2 separated by a distance r. The electrostatic interaction energy of this system can be given as work done in bringing q_2 from infinity to the given separation from q_1. It can be calculated as



$$W = \int_{\infty}^{r} \vec{F}.\overrightarrow{dx} = -\int_{\infty}^{r} \frac{kq_1q_2}{x^2}dx$$ [– ve sign shows that x is decreasing]

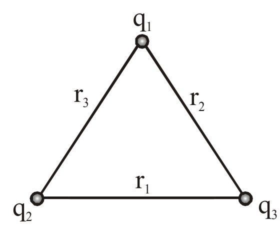
$$W = \frac{kq_1q_2}{r} = U$$ [interaction energy]

If the two charges here are of opposite sign, the potential energy will be negative as

$$U = -\frac{kq_1q_2}{r}$$

- **Interaction Energy for a system of charged particles**

When more than two charged particles are there in a system, the interaction energy can be given by sum of interaction energies of all the pairs of particles. For example if a system of three particles having charges q_1, q_2 and q_3 is given as shown in figure. The total interaction energy of this system can be given as $U = \frac{kq_1q_2}{r_3} + \frac{kq_1q_3}{r_2} + \frac{kq_2q_3}{r_1}$

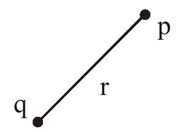


ELECTRIC POTENTIAL

Electric potential is a scalar property of every point in the region of electric field. At a point in electric field potential is defined as the interaction energy of a unit positive charge. If at a point in electric field a charge q_0 has potential energy U, then electric potential at that point can be given as $V = \frac{U}{q_0}$ joule/coulomb

Potential energy of a charge in electric field is defined as work done in bringing the charge from infinity to the given point in electric field. Similarly we can define electric potential as "work done in bringing a unit positive charge from infinity to the given point against the electric forces.

- **Electric Potential due to a point charge in its surrounding :**



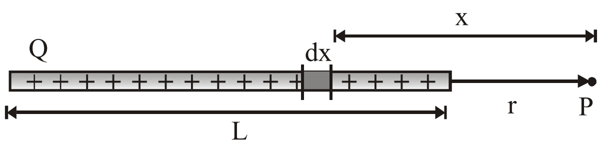
The potential at a point P at a distance r from the charge q , $V_P = \frac{U}{q_0}$

Where U is the potential energy of charge q_0 at point p, $U = \frac{kqq_0}{r}$

Thus potential at point P is $V_P = \frac{kq}{r}$

Electric Potential due to a charge Rod :

Figure shows a rod of length L, uniformly charged with a charge Q. Due to this we'll find electric potential at a point P at a distance r from one end of the rod as shown in figure.



For this we consider an element of width dx at a distance x from the point P.

Charge on this element is $dQ = \frac{Q}{L}dx$

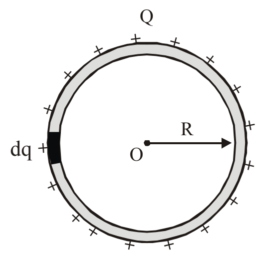
The potential dV due to this element at point P can be given by using the result of a point charge as

$$dV = \frac{kdq}{x} = \frac{kQ}{Lx}dx$$

Net electric potential at point P : $V = \int dV = \int_r^{r+L} \frac{kQ}{Lx} dx = \frac{kQ}{L}\ell n\left(\frac{r+L}{r}\right)$

Electric potential due to a charged ring

Case–I : At its centre



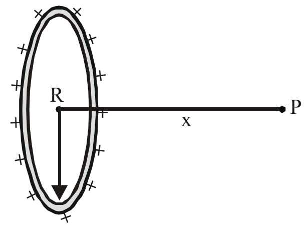
To find potential at the centre C of the ring, we first find potential dV at centre due to an elemental charge dq on ring which is given as $dV = \frac{kdq}{R}$

Total potential at C is $V = \int dV = \int \frac{kdq}{R} = \frac{kQ}{R}$.

As all dq's of the ring are situated at same distance R from the ring centre C, simply the potential due to all dq's is added as being a scalar quantity, we can directly say that the total electric potential at ring centre is $\frac{kQ}{R}$. Here we can also state that even if charge Q is non–uniformly distributed on ring, the electric potential C will remain same.

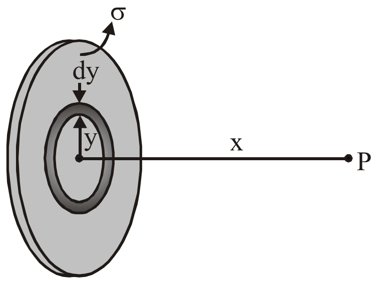
Case II : At a point on axis of ring

We find the electric potential at a point P on the axis of ring as shown, we can directly state the result as here also all points of ring are at same distance $\sqrt{x^2+R^2}$ from the point P, thus the potential at P can be given as $V_P = \frac{kQ}{\sqrt{R^2+x^2}}$



Electric potential due to a uniformly charged disc :

Figure shows a uniformly disc of radius R with surface charge density ρ coul/m². To find electric potential at point P we consider an elemental ring of radius y and width dy, charge on this elemental ring is dq=σ2πy dy. Due to this ring, the electric potential at point P can be given as



$$dV = \frac{kdq}{\sqrt{x^2+y^2}} = \frac{k.\sigma.2\pi y\ dy}{\sqrt{x^2+y^2}}$$

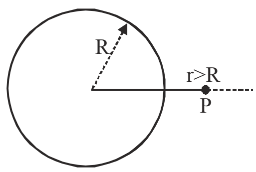
Net electric potential at Point P due to whole disc can be given as

$$V = \int dV = \int_0^R \frac{\sigma}{2\in_0}\cdot\frac{y\ dy}{\sqrt{x^2+y^2}} = \frac{\sigma}{2\in_0}\left[\sqrt{x^2+y^2}\right]_0^R = \frac{\sigma}{2\in_0} = \left[\sqrt{x^2+R^2}-x\right]$$

ELECTRIC POTENTIAL DUE TO HOLLOW OR CONDUCTING SPHERE

- **At outside sphere**

According to definition of electric potential, electric potential at point P



$$V = -\int_\infty^r \vec{E}.d\vec{r} = -\int_\infty^r \frac{q}{4\pi\in_0 r^2}dr \left[\because\ E_{out} = \frac{q}{4\pi\in_0 r^2}\right];\ V = -\frac{q}{4\pi\in_0}\int_\infty^r \frac{1}{r^2}dr = \frac{q}{4\pi\in_0}\left[\frac{1}{r}\right]_\infty^r \frac{q}{4\pi\in_0 r}$$

- **At surface**

$$= V = -\int_\infty^R \vec{E}.d\vec{r} = -\int_\infty^R \frac{q}{4\pi\in_0 r^2}dr \left[\because\ E_{out} = \frac{q}{4\pi\varepsilon_0 r^2}\right];V = -\frac{q}{4\pi\in_0}\int_\infty^R\left(\frac{1}{r^2}\right)dr = \frac{q}{4\pi\in_0}\left[\frac{1}{r}\right]_\infty^R \Rightarrow V = \frac{q}{4\pi\in_0 R}$$

- **Inside the surface** :

$\because$ Inside the surface E = 0 $\left[E = -\frac{dV}{dr}\right]$ $\frac{dV}{dr} = 0$ or V = constant so $V = \frac{q}{4\pi\in_0 R}$

ELECTRIC POTENTIAL DUE TO SOLID NON CONDUCTING SPHERE

- **At outside sphere** Same as conducting sphere.
- **At Surface** Same as conducting sphere.
- **Inside the sphere**

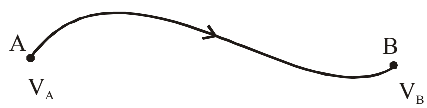
$$V = -\int_{\infty}^{r} \vec{E}.d\vec{r} \qquad \Rightarrow V = -\left[\int_{\infty}^{R} E_1 dr + \int_{R}^{r} E_2 dr\right]$$

$$V = -\left[\int_{\infty}^{R}\left(\frac{kq}{r^2}\right)dr + \int_{R}^{r}\left(\frac{kqr}{R^3}\right)dr\right] \qquad \Rightarrow V = -\left[kq\left(-\frac{1}{r}\right)_{\infty}^{R} + \frac{kq}{R^3}\left(\frac{r^2}{2}\right)_{R}^{r}\right]$$

$$V = -kq\left[-\frac{1}{R} + \frac{r^2}{2R^3} - \frac{R^2}{2R^3}\right] \qquad \Rightarrow V = \frac{kq}{2R^3}\left[3R^2 - r^2\right]$$

Potential Difference Between Two points in electric field

Potential difference between two points in electric field can be defined as work done in displacing a unit positive charge from one point to another against the electric forces.



If a unit +ve charge is displaced from a point A to B as shown work required can be given as

$$V_B - V_A = -\int_A^B \vec{E}.\vec{d}x$$

If a charge q is shifted from point A to B, work done against electric forces can be given as $W = q\,(V_B - V_A)$

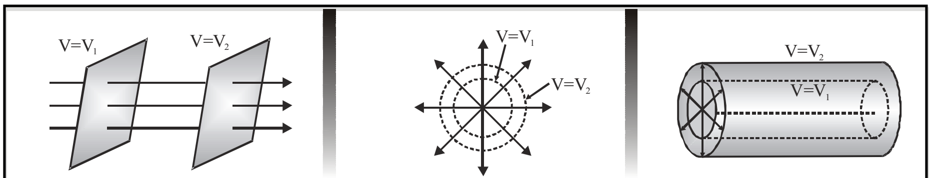
If in a situation work done by electric forces is asked, we use $W = q\,(V_A - V_B)$

If $V_B < V_A$, then charges must have tendency to move toward B (low potential point) it implies that electric forces carry the charge from high potential to low potential points. Hence we can say that in the direction of electric field always electric potential decreases.

Equipotential surfaces

For a given charge distribution, locus of all points having same potential is called 'equipotential surface'.

- Equipotential surfaces can never cross each other (otherwise potential at a point will have two values which is absurd)
- Equipotential surfaces are always perpendicular to direction of electric field.
- If a charge is moved from one point to the other over an equipotential surface then work done $W_{AB} = -U_{AB} = q\,(V_B - V_A) = 0 \quad [\because V_B = V_A]$
- Shapes of equipotential surfaces



for uniform electric field equipotential surfaces are parallel plane | for a point charge, spherical conductor equipotential surfaces are spherical | for a line distribution of charge equipotential surfaces are cylinderical

- The intensity of electric field along an equipotential surface is always zero.

Electric Potential Gradient

The maximum rate of change of potential at right angles to an equipotential surface in an electric field is defined as potential gradient. $\vec{E} = -\vec{\nabla}V$ = – grad V

Note : Potential is a scalar quantity but the gradient of potential is a vector quantity

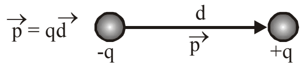
In cartesian co–ordinates $\vec{\nabla}V = \left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$

Ex. If V = –5x + 3y + $\sqrt{15}$ z then E(x, y, z) = ?

Sol. $\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right] = -\left(-5\hat{i} + 3\hat{j} + \sqrt{15}\hat{k}\right) \Rightarrow |\vec{E}| = \sqrt{25+9+15} = \sqrt{49} = 7$ unit

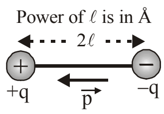
ELECTRIC DIPOLE

A system of two equal and opposite charges separated by a small distance is called electric dipole, shown in figure. Every dipole has a characteristic property called dipole moment. It is defined as the product of magnitude of either charge and the separation between the charges, given as



In some molecules, the centres of positive and negative charges do not coincide. This results in the formation of electric dipole. Atom is non – polar because in it the centres of positive and negative charges coincide. Polarity can be induced in an atom by the application of electric field. Hence it can be called as induced dipole.

- **Dipole Moment** : Dipole moment $\vec{p} = q\vec{d}$



(i) Vector quantity , directed from negative to positive charge

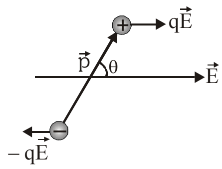
(ii) **Dimension** : [LTA], **Units** : coulomb × metre (or Cm)

(iii) Practical unit is "debye" ≡ Two equal and opposite point charges each having charge 10^{-10} frankline ($\simeq$ e) and separation of 1Å then the value of dipole moment $(\vec{p})$ is 1 debye.

$$1 \text{ Debye} = 10^{-10} \times 10^{-10} \text{ Fr} \times \text{m} = 10^{-20} \times \frac{\text{C} \times \text{m}}{3 \times 10^{9}} \simeq 3.3 \times 10^{-30} \text{ C} \times \text{m}$$

- **Dipole Placed in uniform Electric Field**

Figure shows a dipole of dipole moment $\vec{p}$ placed at an angle θ to the direction of electric field. Here the charges of dipole experience forces qE in opposite direction as shown. $\vec{F}_{net} = \left[q\vec{E} + (-q)\vec{E}\right] = \vec{0}$



Thus we can state that when a dipole is placed in a uniform electric field, net force on the dipole is zero. But as equal and opposite forces act with a separation in their line of action, they produce a couple which tend to align the dipole along the direction of electric field. The torque due to this couple can be given as

τ = Force × separation between lines of actions of forces = qE × d sinθ = pE sin θ

$$\vec{\tau} = \vec{r} \times \vec{F} = \vec{d} \times q\vec{E} = q\vec{d} \times \vec{E} = \vec{p} \times \vec{E}$$

Work done in Rotation of a Dipole in Electric field

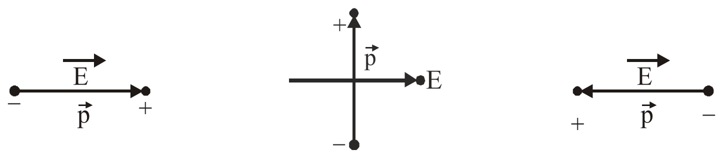
When a dipole is placed in an electric field at an angle θ, the torque on it due to electric field is $\tau = pE \sin\theta$

Work done in rotating an electric dipole from θ_1 to θ_2 [uniform field]

$$dW = \tau\, d\theta \text{ so } W = \int dW = \int \tau d\theta \text{ and } W_{\theta_1 \to \theta_2} = W = \int_{\theta_1}^{\theta_2} pE \sin\theta\, d\theta = pE(\cos\theta_1 - \cos\theta_2)$$

e.g. $W_{0\to180} = pE\,[1-(-1)] = 2\,pE$ $\quad W_{0\to90} = pE\,(1-0) = pE$

If a dipole is rotated from field direction ($\theta = 0°$) to θ then $W = pE\,(1-\cos\theta)$



$\theta = 0$	$\theta = 90°$	$\theta = 180°$
τ = minimum = 0	τ = maximum = pE	τ = minimum = 0
W = minimum = 0	W = pE	W = maximum = 2pE

Electrostatic potential energy :

Electrostatic potential energy of a dipole placed in a uniform field is defined as work done in rotating a dipole from a direction perpendicular to the field to the given direction i.e.,

$$W_{90°\to\theta} = \int_{90°}^{\theta} pE \sin\theta\, d\theta = -pE\cos\theta = -\vec{p}.\vec{E}$$

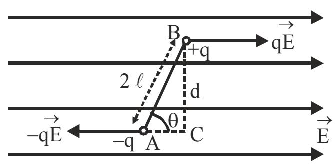
$\vec{E}$ is a conservative field so what ever work is done in rotating a dipole from θ_1 to θ_2 is just equal to change in electrostatic potential energy $W_{\theta_1\to\theta_2} = U_{\theta_2} - U_{\theta_1} = pE(\cos\theta_1 - \cos\theta_2)$

Work done in rotating an electric dipole in an electric field

Suppose at any instant, the dipole makes an angle θ with the electric field.

The torque acting on dipole. $\tau = qEd = (q\, 2\ell \sin\theta)E = pE\sin\theta$

The work done in rotating dipole from θ_1 to θ_2



$$W = \int_{\theta_1}^{\theta_2} \tau d\theta = \int_{\theta_1}^{\theta_2} pE\sin\theta\, d\theta$$

$$W = pE(\cos\theta_1 - \cos\theta_2) = U_2 - U_1 \quad (\because U = -pE\cos\theta)$$

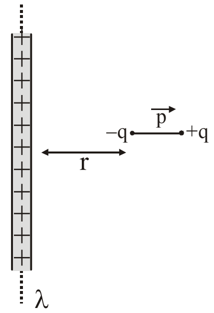
Force on an electric dipole in Non–uniform electric field :

If in a non–uniform electric field dipole is placed at a point where electric field is E, the interaction energy of dipole at this point $U = -\vec{p}.\vec{E}$. Now the force on dipole due to electric field $F = -\frac{\Delta U}{\Delta x}$

For unidirectional variation in electric field, $F = -\frac{d}{dx}\left(\vec{p}.\vec{E}\right)$

If dipole is placed in the direction of electric field then $F = -p\frac{dE}{dx}$

Ex. Calculate force on a dipole in the surrounding of a long charged wire as shown in the figure.



Sol. In the situation shown in figure, the electric field strength due to the wire, at the position of dipole as $E = \frac{2k\lambda}{r}$

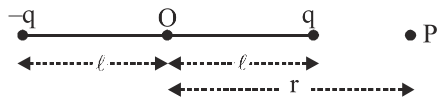
Thus force on dipole is $F = -p.\frac{dE}{dr} = -p\left[-\frac{2k\lambda}{r^2}\right] = \frac{2kp\lambda}{r^2}$

Here –ve charge of dipole is close to wire hence net force an dipole due to wire will be attractive.

ELECTRIC POTENTIAL DUE TO DIPOLE

- **At axial point**

Electric potential due to +q charge $V_1 = \frac{kq}{(r-\ell)}$



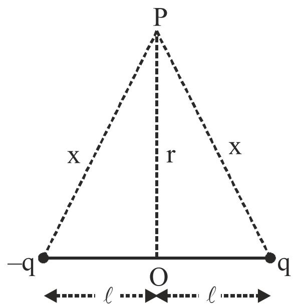
Electric potential due to –q charge $V_2 = \frac{-kq}{(r+\ell)}$

Net electric potential $V = V_1 + V_2 = \frac{kq}{(r-\ell)} + \frac{-kq}{(r+\ell)} = \frac{kq \times 2\ell}{(r^2-\ell^2)} = \frac{kp}{r^2-\ell^2}$

If $r >>> \ell \Rightarrow V = \frac{kp}{r^2}$

- **At equatorial point**

Electric potential of P due to +q charge $V_1 = \frac{kq}{x}$



Electric potential of P due to –q charge $V_2 = -\frac{kq}{x}$

Net potential $V = V_1 + V_2 = \frac{kq}{x} - \frac{kq}{x} = 0 \therefore V = 0$

- **At general point**

$$V = \frac{p\cos\theta}{4\pi\varepsilon_0 r^2} = \frac{\vec{p}\,.\,\vec{r}}{4\pi\varepsilon_0 r^3} \quad \vec{p} = q\vec{a} \text{ electric dipole moment}$$

Electric field due to an electric dipole

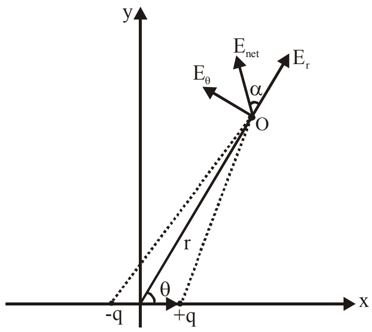


Figure shows an electric dipole placed on x–axis at origin. Here we wish to find the electric field and potential at a point O having coordinates (r, θ). Due to the positive charge of dipole electric field at O is in radially outward direction and due to the negative charge it is radially inward as shown in figure.

$$E_r = -\frac{\partial V}{\partial r} = \frac{2kp\sin\theta}{r^3} \text{ and } E_\theta = -\frac{1}{r}\frac{\partial V}{\partial \theta} = \frac{kp\cos\theta}{r^3}$$

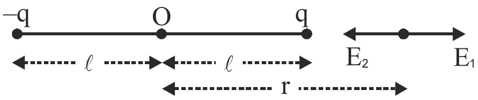
Thus net electric field at point O, $E_{net} = \sqrt{E_r^2 + E_\theta^2} = \frac{kp}{r^3}\sqrt{1+3\sin^2\theta}$

If the direction of E_{net} is at an angle α from radial direction, then $\alpha = \tan^{-1}\frac{E_\theta}{E_r} = \left(\frac{1}{2}\tan\theta\right)$

Thus the inclination of net electric field at point O is (θ + α)

- **At a point on the axis of a dipole :**

Electric field due to +q charge $E_1 = \frac{kq}{(r-\ell)^2}$



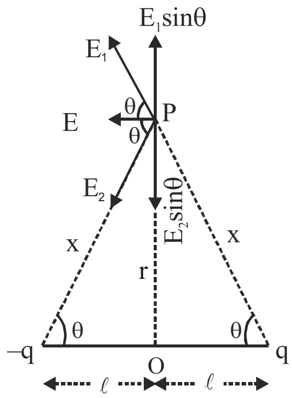
Electric field due to –q charge $E_2 = \frac{kq}{(r+\ell)^2}$

Net electric field $E = E_1 - E_2 = \frac{kq}{(r-\ell)^2} - \frac{kq}{(r+\ell)^2} = \frac{kq \times 4r\ell}{(r^2-\ell^2)^2}$ [∵ $p = q \times 2\ell$ = Dipole moment]

$$E = \frac{2kpr}{(r^2-\ell^2)^2} \text{ If } r >>> \ell \text{ then } E = \frac{2kp}{r^3}$$

- **At a point on equatorial line of dipole :**

Electric field due to +q charge $E_1 = \frac{kq}{x^2}$; Electric field due to –q charge $E_2 = \frac{kq}{x^2}$



Vertical component of E_1 and E_2 will cancel each other and horizontal components will be added So net electric field at P

$E = E_1\cos\theta + E_2\cos\theta$ [∵ $E_1 = E_2$]

$$E = 2E_1\cos\theta = \frac{2kq}{x^2}\cos\theta \because \cos\theta = \frac{\ell}{x} \text{ and } x = \sqrt{r^2+\ell^2}$$

$$E = \frac{2kq\ell}{x^3} = \frac{2kq\ell}{(r^2+\ell^2)^{3/2}} = \frac{kp}{(r^2+\ell^2)^{3/2}} \text{ If } r >>> \ell$$

then $E = \frac{kp}{r^3}$ or $\vec{E} = \frac{-k\vec{p}}{r^3}$

Ex. A short electric dipole is situated at the origin of coordinate axis with its axis along x–axis and equator along y–axis. It is found that the magnitudes of the electric intensity and electric potential due to the dipole are equal at a point distant r = $\sqrt{5}$ m from origin. Find the position vector of the point.

Sol. $\because |E_P| = |V_P| \therefore \frac{kp}{r^3}\sqrt{1+3\cos^2\theta} = \frac{kp\sin\theta}{r^2} \Rightarrow 1 + 3\cos^2\theta = 5\cos^2\theta \Rightarrow \cos\theta = \frac{1}{\sqrt{2}} \Rightarrow \theta = 45^0$

Position vector $\vec{r}$ of point P is $\vec{r} = \frac{\sqrt{5}}{2}(\hat{i}+\hat{j})$

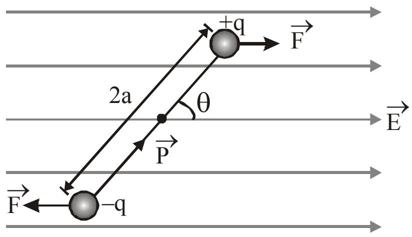
Ex. Prove that the frequency of oscillation of an electric dipole of moment p and rotational inertia I for small amplitudes about its equilibrium position in a uniform electric field strength E is $\frac{1}{2\pi}\sqrt{\left(\frac{pE}{I}\right)}$

Sol. Let an electric dipole (charge q and –q at a distance 2a apart) placed in a uniform external electric field of strength E.

Restoring torque on dipole

$\tau = -pE\sin\theta = -pE\,\theta$ (as θ is small)

Here – ve sign shows the restoring tendency of torque.



$\because \tau = I\alpha \therefore$ angular acceleration = $\alpha = \frac{\tau}{I} = \frac{PE}{I}\theta$

For SHM $\alpha = w^2\theta$ comparing we get $\omega = \sqrt{\frac{pE}{I}}$

Thus frequency of oscillations of dipole n = $\frac{\omega}{2\pi} = \frac{1}{2\pi}\sqrt{\left(\frac{pE}{I}\right)}$

CONDUCTORS

A conductor means infinite no. of free charges which move in random direction so the lattice becomes positively charged. Conductors contain charge carriers, these charge carriers are electrons. In a metal, the outer (valence) electrons part away from their atoms and are free to move. These electrons are free within the metal but not free to leave the metal. The free electrons form a kind of 'gas'; they collide with each other and with the ions, and move randomly in different directions. The positive ions made up of the nuclei and the bound electrons remain held in their fixed positions.

1. Inside a conductor, electrostatic field is zero

Consider a conductor, neutral or charged. There may also be an external electrostatic field. In the static situation, when there is no current inside or on the surface of the conductor, the electric field is zero everywhere inside the conductor. *This fact can be taken as the defining property of a conductor.* A conductor has free electrons. As long as electric field is not zero, the free charge carriers would experience force and drift. In the static situation, the free charges have so distributed themselves that the electric field is zero everywhere inside. ***Electrostatic field is zero inside a conductor.***

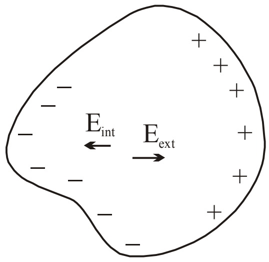
Further Explanation

What happens if conductor is placed in an exterrnal field.

Lets keep a positive charge q charge near a neutral or acharged conductor then the electrons goes close to q.

The redistribution of electrons inside conductor takes place which generates an internal electric field $\vec{E}_{int}$.

$$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{\text{int}}$$



neutral conductor

So an e^- experience $\vec{E}_{net}$

If $E_{int} \neq E_{ext}$, then the e^- move such that they will create a stronger $\vec{E}_{int}$ which will tend to cancel E_{ext}.

This constitutes current and therefore energy conservation is not valid.

$\vec{E}_{net}$ has to be O instantaneously.

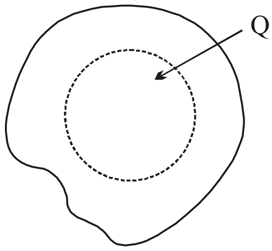
$$\vec{E}_{net} = \vec{E}_{ext} + \vec{E}_{int} = 0$$

2. The interior of a conductor can have no excess charge in the static situation

A neutral conductor has equal amounts of positive and negative charges in every small volume or surface element. When the conductor is charged, the excess charge can reside only on the surface in the static situation.

Explanation

This follows from the Gauss's law. Consider any arbitrary volume element *v* inside a conductor. If we consider any small gaussian surface inside



$$\oint \vec{E} \cdot d\vec{A} = 0 \qquad [\text{as } E = 0]$$

On the closed surface *S* bounding the volume element *v,* electrostatic field is zero. Thus the total electric flux through *S* is zero. Hence, by Gauss's law, there is no net charge enclosed by *S.*

$\Rightarrow$ $q_{enclosed} = 0$

Since the surface S can be made as small as you like, i.e., the volume v can be made vanishingly small. This means *there is no net charge at any point inside the conductor*, and *any excess charge must reside at the surface*.

Note : Thus Solid " conducting" sphere is same as a shell.

Note : You may emphasise again but $q_{in} = 0$ does not imply that $E = 0$ from gauss law.

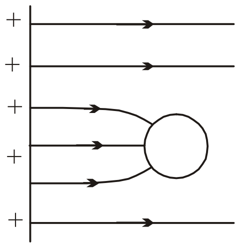
3. At the surface of a charged conductor, electrostatic field must be normal to the surface at every point

If **E** were not normal to the surface, it would have some non-zero component along the surface. Free charges on the surface of the conductor would then experience force and move. In the static situation, therefore, **E** should have no tangential component. Thus *electrostatic field at the surface of a charged conductor must be normal to the surface at every point*. (For a conductor without any surface charge density, field is zero even at the surface.)

Asking question

Consider a neutral conducting sphere placed near a infinite non conducting uniform sheet of charge. Draw the field near the infinite sheet.

Sol.



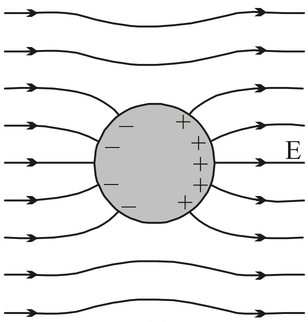
Emphasize following facts from figure

(i) Field is normal to surface conductor

(ii) Field inside conductor is zero

Further discussion

When we place an ideal conductor in an electric field E, the free electrons experience a force in the opposite direction of the field and migrate to one side of the conductor as shown in figure (a).



(a)

(a) The accumulation of electrons leaves one side positively charged and the other negative. This charged distribution creates an electric field in a direction opposite to the applied field. The redistribution of charge takes place till net field inside the conductor is zero. Therefore, in electrostatic equilibrium, the electric field inside an ideal conductor is zero.

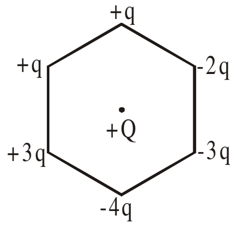
If we place a Gaussian surface, an infinitesimal distance below the surface, the electric field is zero at every point on this Gaussian surface because it is inside the conductor [figure (b)]. Gauss's law then implies that the net charge contained within the Gaussian surface is zero. In electrostatic equilibrium, excess charge on an ideal isolated conductor must reside on the conductor's surface. No free charge can exist anywhere within the electrostatic conductor.

Note: Also if some external field is present then charge distribution on a Solid conducting sphere and a conducting shell will be non uniform.

EXERCISE (S)

Properties of charge and Coulomb's law

1. Six charges are kept at the vertices of a regular hexagon as shown in the figure. If magnitude of force applied by +Q on +q charge is F, then net electric force on the +Q is nF. Find the value of n.



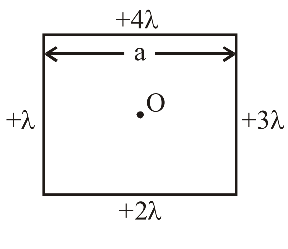
ES0004

Electric field

2. A clock face has negative charges $-q, -2q, -3q, ..., -12q$ fixed at the positions of the corresponding numerals on the dial. Assume that the clock hands do not disturb the net field due to point charges. At what time does the hour hand point in the same direction is electric field at the centre of the dial.

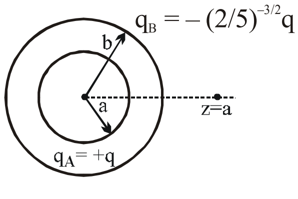
ES0008

3. Four uniformly charged wires of length a are arranged to form a square. Linear charge density of each wire is as shown. Electric field intensity at centre of square is $\frac{nk\lambda}{a}$ then value of n



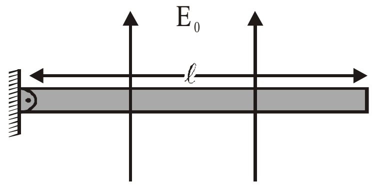
ES0010

4. Two concentric rings, one of radius 'a' and the other of radius 'b' have the charges +q and $-(2/5)^{-3/2}q$ respectively as shown in the figure. Find the ratio b/a if a charge particle placed on the axis at z = a is in equilibrium.



ES0011

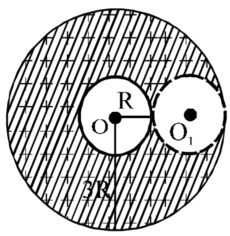
5. A thin insulating uniformly charged (linearly charged density λ) rod is hinged about one of its ends. It can rotate in vertical plane. If rod is in equilibrium by applying vertical electric field E as shown in figure. Find the value of E(in N/C). (Given that mass of rod 2 kg, $\lambda = 10\ C/m$, ℓ=1m, g=10 m/s^2)



ES0013

Gauss' law

6. A thick shell with inner radius R and outer radius 3R has uniform density ρ c/m^3. It has a spherical cavity of radius R as shown in the figure. The electric field at the centre O_1 of the cavity is :-



ES0233

7. A spherical shell has uniform charge density 8.8×10^{-11} C/m^2. If a pin hole is made in the surface of the shell then find the electric field in the hole in N/C. (Take $\epsilon_0 = 8.8 \times 10^{-12}$ S.I. units)

ES0234

Electric potential energy and electric potential

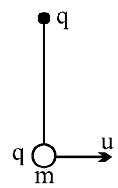
8. A particle of positive charge Q is assumed to have fixed position at P. A second particle of mass m and negative charge –q moves at a constant speed in a circle of radius r_1, centred at P. Derive an expression for the work W that must be done by an external agent on the second particle in order to increase the radius of the circle of motion, centred at P to r_2. Express W in terms of quantities chosen from among m, r_1, r_2, q, Q and ε_0 only.

ES0235

9. The bob of a pendulum has mass m = 1 kg and charge q = 40 μC. Length of pendulum is l = 0.9 m. The point of suspension also has the same charge 40 μC.

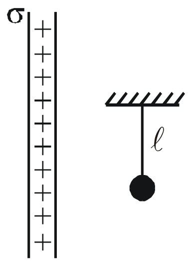
(a) What the minimum speed u should be imparted to the bob so that it can complete vertical circle?

(b) What should be u if q was 20μC each?



ES0236

10. A simple pendulum of length ℓ and bob mass m is hanging in front of a large nonconducting sheet of surface charge density σ. If suddenly a charge $+q$ is given to the bob in the position shown in figure. Find the maximum angle through which the string is deflected from vertical.

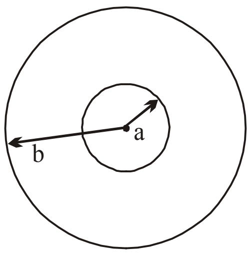


ES0026

11. A particle of mass m carrying charge 'q' is projected with velocity (v) from point P towards an infinite line charge from a distance 'a'. Its speed reduces to zero momentarily at point Q which is at a distance a/2 from the line charge. If another particle with mass m and charge –q is projected with the same velocity v from point P towards the line charge. Its speed is found to be $\frac{Nv}{\sqrt{2}}$ at point 'Q'. Find the value of N.

ES0025

12. Outer cylinder of the coaxial nonconductor of radius 'b' is given a positive potential V relative to the inner cylinder of radius 'a' as shown in the figure (charge distribution is uniform). A charge q (mass = m) is set free with negligible velocity at the surface of the inner cylinder. Find the velocity (in m/s), when it hits the outer cylinder. [consider V = 10, q = –20, m = 1 all in S.I. Units]



ES0027

13. Consider the configuration of a system of four charges each of value +q. Find the work done by external agent in changing the configuration of the system from figure (i) to fig (ii).

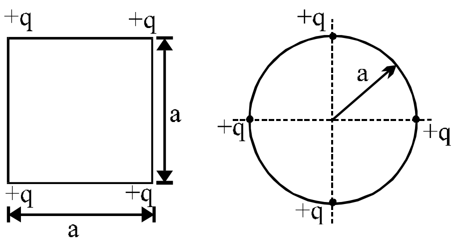


fig (i) fig (ii)

ES0021

14. A positive charge Q is uniformly distributed throughout the volume of a nonconducting sphere of radius R. A point mass having charge $+q$ and mass m is fired towards the centre of the sphere with velocity v from a point at distance $r(r > R)$ from the centre of the sphere. Find the minimum velocity v so that it can penetrate $R/2$ distance of the sphere. Neglect any resistance other than electric interaction. Charge on the small mass remains constant throughout the motion.

ES0028

15. The electric field strength depends only on the x, y and z coordinates according to the law $E = \dfrac{a(x\hat{i} + y\hat{j} + z\hat{k})}{\left(x^2 + y^2 + z^2\right)^{3/2}}$, where a = 122.5 SI unit and is a constant. Find the potential difference (in volt) between (3, 2, 6) and (0, 3, 4).

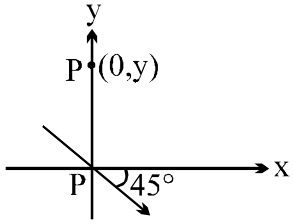
ES0055

Electric dipole

16. A dipole of dipole moment $\vec{p} = 2\hat{i} - 3\hat{j} + 4\hat{k}$ is placed at point A(2,–3,1). The electric potential due to this dipole at the point B(4,–1,0) is (ab) × 10^9 volt here 'a' represents sign (for negative answer select 0 for positive answer select 1. Write the value of $(a+b)^2$. The parameters specified here are in S.I. units.

ES0030

17. A dipole is placed at origin of coordinate system as shown in figure, find the electric field at point P (0, y).



ES0031

18. A charge $+Q$ is fixed at the origin of the coordinate system while a small electric dipole of dipole-moment $\vec{p}$ pointing away from the charge along the x-axis is set free from a point far away from the origin.

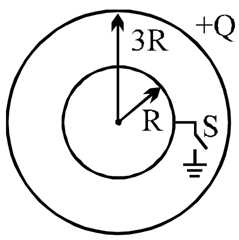
(a) calculate the K.E. of the dipole when it reaches to a point $(d, 0)$ **[IIT-JEE 2003]**

(b) calculate the force on the charge $+Q$ due to the dipole at this moment.

ES0033

Conductor

19. Two thin conducting shells of radii R and $3R$ are shown in figure. The outer shell carries a charge $+Q$ and the inner shell is neutral. The inner shell is earthed with the help of switch S. Find the charge attained by the inner shell.



ES0035

20. A conducting liquid bubble of radius a and thickness t ($t \ll a$) is charged to potential V. If the bubble collapses to a droplet, find the potential on the droplet. **[IIT-JEE 2005]**

ES0037

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

Properties of charges and coulomb's law

1. If an object made of substance A rubs an object made of substance B, then A becomes positively charged and B becomes negatively charged. If, however, an object made of substance A is rubbed against an object made of substance C, then A becomes negatively charged. What will happen if an object made of substance B is rubbed against an object made of substance C?

(A) B becomes positively charged and C becomes positively charged.

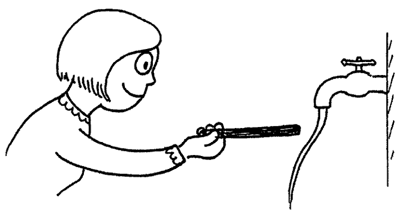
(B) B becomes positively charged and C becomes negatively charged.

(C) B becomes negatively charged and C becomes positively charged.

(D) B becomes negatively charged and C becomes negatively charged.

ES0056

2. In normal cases thin stream of water bends toward a negatively charged rod. When a positively charged rod is placed near the stream, it will bend in the



(A) Opposite direction. (B) Same direction.

(C) It won't bend at all. (D) Can't be predicted.

ES0057

3. Two charged bodies A and B exert repulsive forces on each other. If charge on A is more then that on B, which of the following statement is true.

(A) Body A experiences more Colombian force then B.

(B) Body A experiences less Colombian force then B.

(C) Both of them experience Colombian forces of equal magnitude.

(D) It depends whether the bodies can be treated as point like charges or not.

ES0058

4. Given are four arrangements of three fixed electric charges. In each arrangement, a point labeled P is also identified — test charge, +q, is placed at point P. All of the charges are of the same magnitude, Q, but they can be either positive or negative as indicated. The charges and point P all lie on a straight line. The distances between adjacent items, either between two charges or between a charge and point P, are all the same. Correct order of choices in a decreasing order of magnitude of force on P is

(A) II > I > III > IV (B) I > II > III > IV (C) II > I > IV > III (D) III > IV > I > II

ES0059

5. Two point charge of 100 μC and –4 μC are positioned at points (–2√3, 3√3, –4) and (4√3, –5√3, 6) respectively of a Cartesian coordinate system. Find the force vector on the –4 μC charge? All the coordinates are in meters.

(A) $9\times10^{-4}\left(3\sqrt{3}\hat{i}-4\sqrt{3}\hat{j}+5\hat{k}\right)$ (B) $9\times10^{-4}\left(-3\sqrt{3}\hat{i}+4\sqrt{3}\hat{j}-5\hat{k}\right)$

(C) $2.25\times10^{-4}\left(-3\sqrt{3}\hat{i}+4\sqrt{3}\hat{j}-5\hat{k}\right)$ (D) $2.25\times10^{-4}\left(3\sqrt{3}\hat{i}-4\sqrt{3}\hat{j}+5\hat{k}\right)$

ES0060

Electric field

6. Five positive equal charges are placed at vertices of a regular hexagon and net electric field at the centre is E_1. A negative charge having equal magnitude is placed sixth vertex and then net electric field is E_2. Find $\frac{E_2}{E_1}$.

(A) 2 (B) 1 (C) 3 (D) None of these

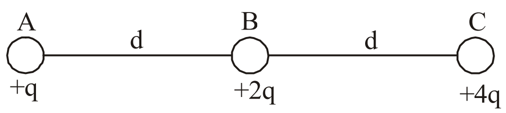
ES0061

7. There are two point charges q_1 and q_2 lying on a circle of unit radius. Electric field intensity at the center of circle due to these charges is $\vec{E}$. Find the position vector of the center with respect to q_2 if the position vector of the center with respect to q_1 is $\vec{r}_1$.

(A) $\frac{\vec{E}+kq_1\vec{r}_1}{kq_2}$ (B) $-\left(\frac{\vec{E}+kq_1\vec{r}_1}{kq_2}\right)$ (C) $\frac{kq_1\vec{r}_1-\vec{E}}{kq_2}$ (D) $\frac{\vec{E}-kq_1\vec{r}_1}{kq_2}$

ES0062

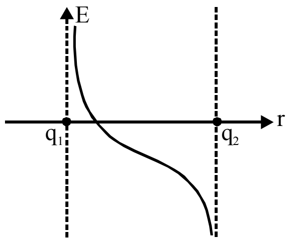
8. Three charges +q, +2q and +4q are connected by strings as shown in the figure. What is ratio of tensions in the strings AB and BC.



(A) 1 : 2 (B) 1 : 3 (C) 2 : 1 (D) 3 : 1

ES0063

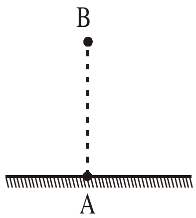
9. The variation of electric field between the two charges q_1 and q_2 along the line joining the charges is plotted against distance from q_1 (taking rightward direction of electric field as positive) as shown in the figure. Then the correct statement is :-



(A) q_1 and q_2 are positive and $q_1 < q_2$ (B) q_1 and q_2 are positive and $q_1 > q_2$
(C) q_1 is positive and q_2 is negative and $q_1 < q_2$ (D) q_1 and q_2 are negative and $q_1 < q_2$

ES0064

10. Particle *B* of charge *Q* and mass m is in equilibrium under weight and electrostatics force applied by a fixed charged *A*, which is directly beneath the particle *B* as shown in figure. When particle B is disturbed along vertical, the equilibrium is



(A) stable (B) unstable
(C) neutral (D) there can not be in equilibrium

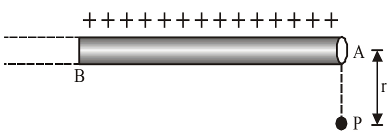
ES0065

11. A charge q is placed at the centroid of an equilateral triangle. Three equal charges Q are placed at the vertices of the triangle. The system of four charges will be in equilibrium if q is equal to :-

(A) $\frac{-Q}{\sqrt{3}}$ (B) $\frac{-Q}{3}$ (C) $-Q\sqrt{3}$ (D) $\frac{Q}{\sqrt{3}}$

ES0066

12. A semi-infinite insulating rod has linear charge density λ. The electric field at the point P shown in figure is :-

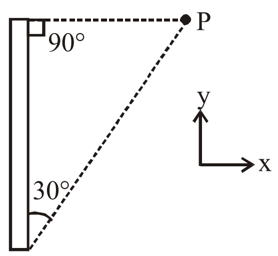


(A) $\frac{2\lambda^2}{(4\pi\varepsilon_0 r)^2}$ at 45° with AB (B) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r^2}$ at 45° with AB

(C) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r}$ at 45° with AB (D) $\frac{\sqrt{2}\lambda}{4\pi\varepsilon_0 r}$ at perpendicular to AB

ES0067

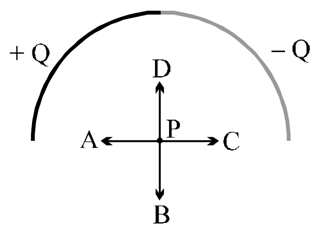
13. The direction (θ) of $\vec{E}$ at point P due to uniformly charged finite rod will be :-



(A) at angle 30^0 from x-axis
(B) 45^0 from x - axis
(C) 60^0 from x-axis
(D) none of these

ES0068

14. As shown in the figure, an insulating rod is set into the shape of a semicircle. The left half of the rod has a charge of + Q uniformly distributed along its length, and the right half of the rod has a charge of –Q uniformly distributed along its length. What vector shows the correct direction of the electric field at point P, the centre of the semicircle ?



(A) A
(B) B
(C) C
(D) D

ES0069

15. A nonconducting ring of radius *R* has uniformly distributed positive charge *Q*. A small part of the ring, of length d, is removed ($d << R$). The electric field at the centre of the ring will now be
(A) directed towards the gap, inversely proportional to R^3.
(B) directed towards the gap, inversely proportional to R^2.
(C) directed away from the gap, inversely proportional to R^3.
(D) directed away from the gap, inversely proportional to R^2.

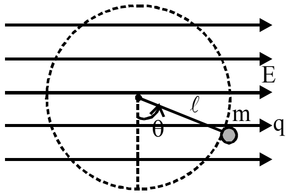
ES0070

16. A particle of mass m, charge –Q is constrained to move along the axis of a ring of radius a. The ring carries a uniform charge density + λ along its circumference. Initially, the particle lies in the plane of the ring at a point where no net force acts on it. The period of oscillation of the particle when it is displaced slightly from its equilibrium position is

(A) $T = 4\pi\sqrt{\frac{\varepsilon_0 ma^2}{\lambda Q}}$
(B) $T = 2\pi\sqrt{\frac{2\varepsilon_0 ma^2}{\lambda Q}}$
(C) $T = 2\pi\sqrt{\frac{4\varepsilon_0 ma^2}{\lambda Q}}$
(D) $T = 2\pi\sqrt{\frac{\varepsilon_0 ma^2}{2\lambda Q}}$

ES0072

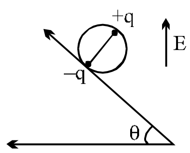
17. A small ball of mass m and charge +q tied with a string of length ℓ, is rotating in a vertical circle under gravity and a uniform horizontal electric field E as shown. The tension in the string will be minimum for:-



(A) $\theta = \tan^{-1}\left(\frac{qE}{mg}\right)$ (B) $\theta = \pi$ (C) $\theta = \pi - \tan^{-1}\left(\frac{qE}{mg}\right)$ (D) $\theta = \pi + \tan^{-1}\left(\frac{qE}{mg}\right)$

ES0074

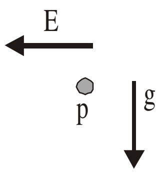
18. A wheel having mass m has charges +q and –q on diametrically opposite points. It remains in equilibrium on a rough inclined plane in the presence of uniform vertical electric field E =

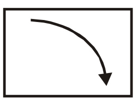
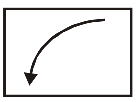
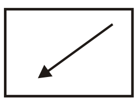
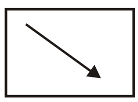


(A) $\frac{mg}{q}$ (B) $\frac{mg}{2q}$ (C) $\frac{mg\tan\theta}{2q}$ (D) none

ES0075

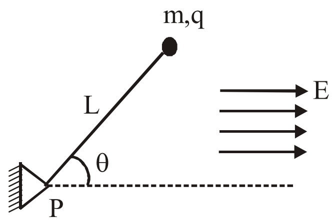
19. A negatively charged particle *p* is placed, initially at rest, in a constant, uniform gravitational field and a constant, uniform electric field as shown in the diagram. What qualitatively, is the shape of the trajectory of the electron ?



(A)  (B)  (C)  (D) 

ES0076

20. A particle of mass m and charge q is attached to a light rod of length L. The rod can rotate freely in the horizontal plane of paper about the other end, which is hinged at P. The entire assembly lies in a uniform electric field E also acting in the plane of paper as shown. The rod is released from rest when it makes an angle θ with the electric field direction. Determine the speed of the particle when the rod is parallel to the electric field.

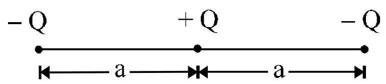


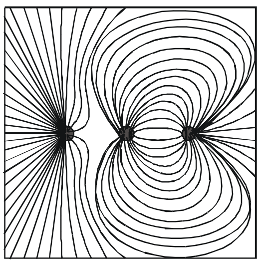
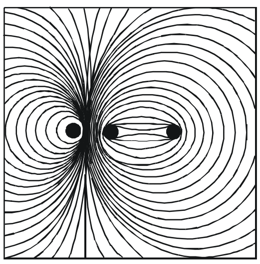
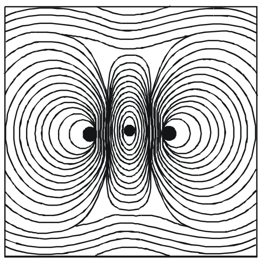
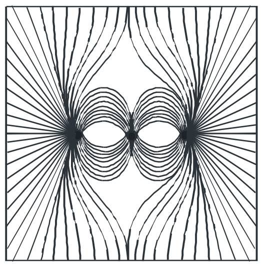
(A) $\left(\frac{2qEL(1-\cos\theta)}{m}\right)^{1/2}$
(B) $\left(\frac{2qEL(1-\sin\theta)}{m}\right)^{1/2}$

(C) $\left(\frac{qEL(1-\cos\theta)}{2m}\right)^{1/2}$
(D) $\left(\frac{2qEL\cos\theta}{m}\right)^{1/2}$

ES0077

21. The fig. shows the distribution of three charges –Q, +Q and –Q on the X-axis. Which of the following figures shows the possible electric field lines for the distribution?

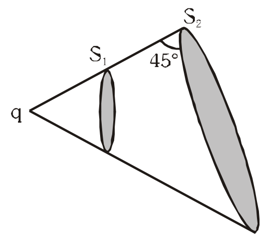


(A)  (B)  (C)  (D) 

ES0078

Gauss' law

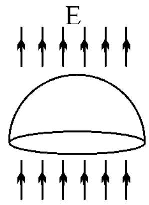
22. In the given figure flux through surface S_1 is ϕ_1 & through S_2 is ϕ_2. Which is correct ?



(A) $\phi_1 = \phi_2$ (B) $\phi_1 > \phi_2$ (C) $\phi_1 < \phi_2$ (D) None of these

ES0079

23. A hemispherical surface (half of a spherical surface) of radius R is located in a uniform electric field E that is parallel to the axis of the hemisphere. What is the magnitude of the electric flux through the hemisphere surface?



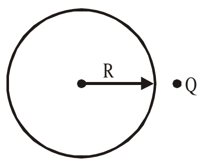
(A) 0 (B) $4\pi R^2E/3$ (C) $2\pi R^2E$ (D) πR^2E

ES0080

24. **Statement 1:** A charge is outside the Gaussian sphere of radius R. Then electric field on the surface of sphere is zero.

and

Statement 2 : As $\oint \vec{E}\cdot d\vec{s} = \frac{q_{in}}{\varepsilon_0}$, for the sphere q_{in} is zero, so $\oint \vec{E}\cdot d\vec{s} = 0$.



(A) Statement-1 is true, statement-2 is true and statement-2 is correct explanation for statement-1.
(B) Statement-1 is true, statement-2 is true and statement-2 is NOT the correct explanation for statement-1.
(C) Statement-1 is true, statement-2 is false.
(D) Statement-1 is false, statement-2 is true.

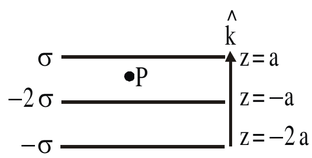
ES0081

25. A sphere of radius R carries charge density proportional to the square of the distance from the center: $\rho = Ar^2$, where A is a positive constant. At a distance of R/2 from the center, the magnitude of the electric field is :-

(A) $A/4\pi\varepsilon_0$ (B) $AR^3/40\varepsilon_0$ (C) $AR^3/24\varepsilon_0$ (D) $AR^3/5\varepsilon_0$

ES0083

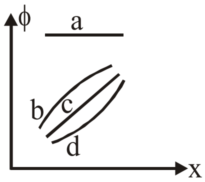
26. Three large parallel plates have uniform surface charge densities as shown in the figure. What is the electric field at *P*? **[IIT-JEE 2005 (Scr)]**



(A) $-\frac{4\sigma}{\epsilon_0}\hat{k}$ (B) $\frac{4\sigma}{\epsilon_0}\hat{k}$ (C) $-\frac{2\sigma}{\epsilon_0}\hat{k}$ (D) $\frac{2\sigma}{\epsilon_0}\hat{k}$

ES0084

27. A line of charge extends along a X-axis whose linear charge density varies directly as x. Imagine a spherical volume with its centre located on X-axis and is moving gradually along it. Which of the graphs shown in figure correspond to the flux ϕ with the x coordinate of the centre of the volume?



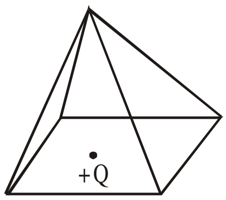
(A) a (B) b (C) c (D) d

ES0085

28. The electric field in a region is given by $\vec{E} = 200\hat{i}$ N/C for x > 0 and $-200\hat{i}$ N/C for x < 0. A closed cylinder of length 2m and cross-section area 10^2 m^2 is kept in such a way that the axis of cylinder is along X-axis and its centre coincides with origin. The total charge inside the cylinder is
[Take : $\varepsilon_0 = 8.85 \times 10^{-12}$ C^2m^2.N)
(A) 0 (B) 1.86×10^{-5}C (C) 1.77×10^{-11}C (D) 35.4×10^{-8} C

ES0086

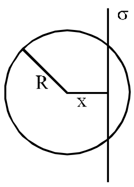
29. A point charge +Q is positioned at the center of the base of a square pyramid as shown. The flux through one of the four identical upper faces of the pyramid is :-



(A) $\frac{Q}{16\varepsilon_0}$ (B) $\frac{Q}{4\varepsilon_0}$ (C) $\frac{Q}{8\varepsilon_0}$ (D) None of these

ES0087

30. An infinite, uniformly charged sheet with surface charge density σ cuts through a spherical Gaussian surface of radius R at a distance x from its center, as shown in the figure. The electric flux Φ through the Gaussian surface is :-



(A) $\frac{\pi R^2 \sigma}{\varepsilon_0}$ (B) $\frac{2\pi\left(R^2 - x^2\right)\sigma}{\varepsilon_0}$ (C) $\frac{\pi(R - x)^2\sigma}{\varepsilon_0}$ (D) $\frac{\pi\left(R^2 - x^2\right)\sigma}{\varepsilon_0}$

ES0088

Electric potential energy and electric potential

31. Two particles X and Y, of equal mass and with unequal positive charges, are free to move and are initially far away from each other. With Y at rest, X begins to move towards it with initial velocity u. After a long time, finally :-
(A) X will stop, Y will move with velocity u.
(B) X and Y will both move with velocities u/2 each.
(C) X will stop, Y will move with velocity < u.
(D) both will move with velocities < u/2.

ES0090

32. Two positively charged particles X and Y are initially far away from each other and at rest. X begins to move towards Y with some initial velocity. The total momentum and energy of the system are p and E.

(A) If Y is fixed, both p and E are conserved.

(B) If Y is fixed, E is conserved, but not p.

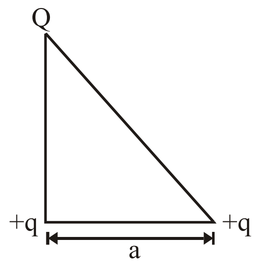
(C) If both are free to move, p is conserved but not E.

(D) If both are free, E is conserved, but not p.

ES0091

33. Three charges Q, +q and +q are placed at the vertices of a right-angled isosceles triangle as shown. The net electrostatic energy of the configuration is zero if Q is equal to :

[JEE 2000(Scr) 1 + 1]



(A) $\frac{-q}{1+\sqrt{2}}$ (B) $\frac{-2q}{2+\sqrt{2}}$ (C) $-2q$ (D) $+q$

ES0093

34. Two fixed charges A and B of 5 μC each are separated by a distance of 6m. C is the mid point of the line joining A and B. A charge 'Q' of –5μC is shot perpendicular to the line joining A and B through C with a kinetic energy of 0.06J. The charge 'Q' comes to rest at a point D. The distance CD is:-

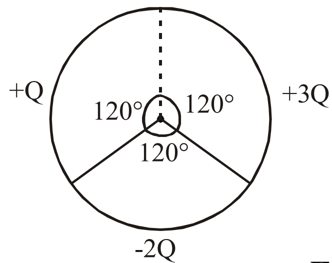
(A) 3 m (B) $\sqrt{3}$ m (C) $3\sqrt{3}$ m (D) 4 m

ES0094

35. Figure shows three circular arcs, each of radius R and total charge as indicated. The net electric potential at the centre of curvature is :-

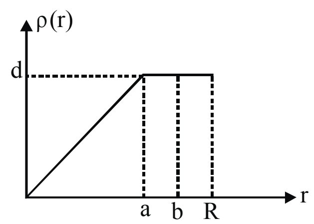
(A) $\frac{Q}{4\pi \in_0 R}$ (B) $\frac{Q}{2\pi \in_0 R}$

(C) $\frac{2Q}{\pi \in_0 R}$ (D) $\frac{Q}{\pi \in_0 R}$



ES0095

36. The nuclear charge (Ze) is non-uniformly distribute within a nucleus of radius R. The charge density ρ(r) [charge per unit volume] is dependent on the radial distance r from the centre of the nucleus as shown in figure. Select correct alternative/s.



(A) Electric field at r = R is independent of b

(B) Electric potential at r = R is proportional to b

(C) Electric field at r = R is proportional to a

(D) Electric potential at r = R is proportional to a

ES0096

37. A solid sphere of radius R is charged uniformly. At what distance from its surface is the electrostatic potential half of the potential at the centre?

(A) R (B) R/2 (C) R/3 (D) 2R

ES0097

38. When a negative charge is released and moves in electric field, it moves toward a position of

(A) lower electric potential and lower potential energy

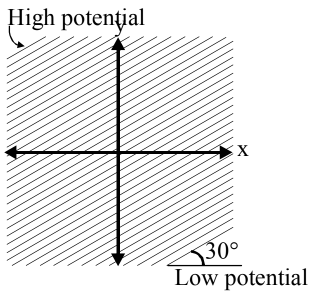
(B) lower electric potential and higher potential energy

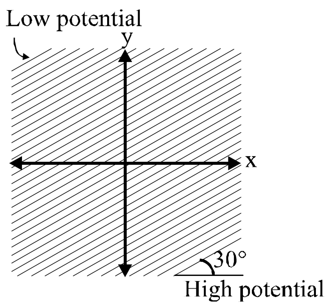
(C) higher electric potential and lower potential energy

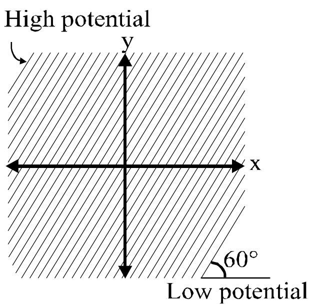
(D) higher electric potential and higher potential energy

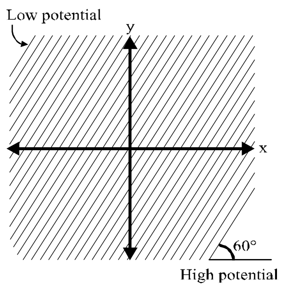
ES0098

39. The electric field intensity at all points in space is given by $\vec{E} = \sqrt{3}\hat{i} - \hat{j}$ volts/metre. The nature of equipotential lines in x-y plane is given by :-

(A) 

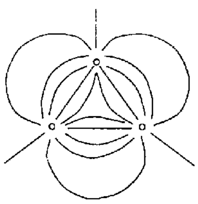
(B) 

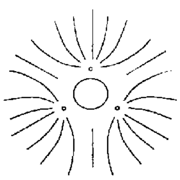
(C) 

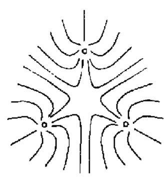
(D) 

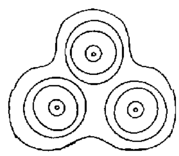
ES0100

40. Three positive charges of equal value q are placed at the vertices of an equilateral triangle. The resulting lines of force should be sketched as in **[JEE 2001 (Scr)]**

(A) 

(B) 

(C) 

(D) 

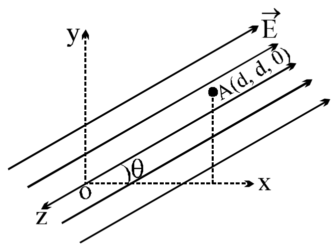
ES0102

41. In an electric field the potential at a point is given by the following relation $V = \frac{343}{r}$. The electric field at $\vec{r} = 3\hat{i} + 2\hat{j} + 6\hat{k}$ is :

(A) $21\hat{i} + 14\hat{j} + 42\hat{k}$ (B) $3\hat{i} + 2\hat{j} + 6\hat{k}$ (C) $\frac{1}{7}(3\hat{i} + 2\hat{j} + 6\hat{k})$ (D) $-(3\hat{i} + 2\hat{j} + 6\hat{k})$

ES0103

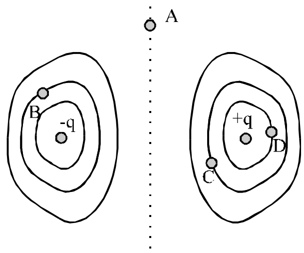
42. A uniform electric field having strength $\vec{E}$ is existing in x-y plane as shown in figure. Find the p.d. between origin O & A(d, d, 0)



(A) Ed (cosθ + sinθ) (B) –Ed (sinθ – cosθ) (C) $\sqrt{2}$ Ed (D) none of these

ES0105

43. Figure shows equi-potential surfaces for a two charges system. At which of the labeled points point will an electron have the highest potential energy?



(A) Point A (B) Point B (C) Point C (D) Point D

ES0106

44. Figure shows some equipotential lines distributed in space. A charged object is moved from point A to point B.

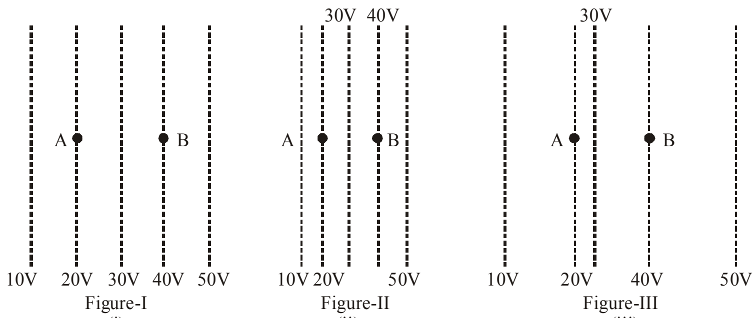


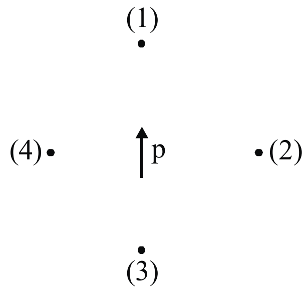
Figure-I (i) Figure-II (ii) Figure-III (iii)

(A) The work done in Fig. (i) is the greatest.
(B) The work done in Fig. (ii) is least.
(C) The work done is the same in Fig. (i), Fig. (ii) and Fig. (iii).
(D) The work done in Fig. (iii) is greater than Fig. (ii)but equal to that in Fig. (i).

ES0107

Electric dipole

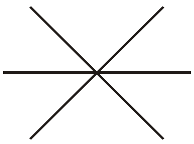
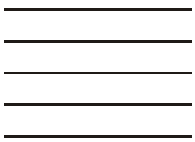
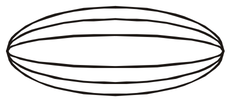
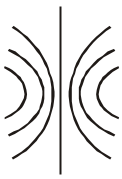
45. The drawing shows four points surrounding an electric dipole. Which one of the following expressions best ranks the electric potential at these four locations?



(A) $1 > 2 = 4 > 3$ (B) $3 > 2 > 4 > 1$ (C) $3 > 2 = 4 > 1$ (D) $2 = 4 > 1 = 3$

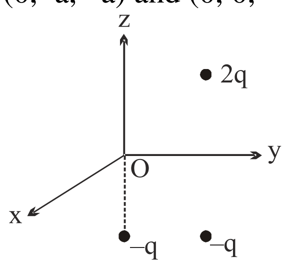
ES0109

46. Which of the following represents the equipotential lines of a dipole (two equal and opposite charges placed at small separation)?

(A)  (B)  (C)  (D) 

ES0110

47. Three point charges 2q, –q and –q are located respectively at (0, a, a), (0, a, –a) and (0, 0, – a) as shown. The dipole moment of this distribution is :-



(A) 2qa in the y–z plane at $\tan^{-1}\left(\frac{1}{4}\right)$ with z-axis

(B) $\sqrt{17}qa$ in the y–z plane at $\tan^{-1}\left(\frac{1}{4}\right)$ with z-axis

(C) $\sqrt{5}qa$ in the x–y plane at $\tan^{-1}$ (4) with y-axis

(D) 4qa in the x–y plane at $\tan^{-1}$ (4) with y-axis

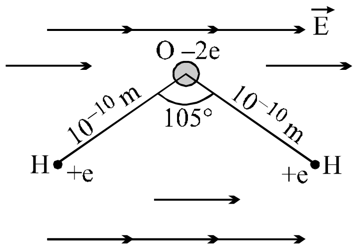
ES0111

48. Point P lies on the axis of a dipole. If the dipole is rotated by 90° anticlock wise, the electric field vector $\vec{E}$ at P will rotate by

(A) 90° clock wise (B) 180° clock wise (C) 90° anti clock wise (D) none

ES0112

49. A water molecule as shown is in a region of uniform electric field $\vec{E} = 1000\,\hat{i}$ V/m. This molecule experiences

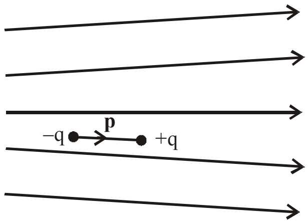


(A) A counterclockwise torque (B) A clockwise torque

(C) A net force to the right (D) A net force to the left

ES0113

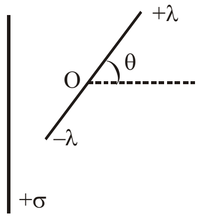
50. Electric field lines in which an electric dipole **p** is placed as shown. Which of the following statements is correct ?



(A) The dipole will not experience any force.
(B) The dipole will experience a force towards right.
(C) The dipole will experience a force towards left.
(D) The dipole will experience a force upwards.

ES0114

51. A large sheet carries uniform surface charge density σ. A rod of length 2ℓ has a linear charge density λ on one half and – λ on the other half. The rod is hinged at mid point O and makes angle θ with the normal to the sheet. The torque experienced by the rod is :-



(A) $\frac{\sigma\lambda\ell^2}{2\varepsilon_0}\cos\theta$ (B) $\frac{\sigma\lambda\ell}{\varepsilon_0}\cos^2\theta$ (C) $\frac{\sigma\lambda\ell^2\sin\theta}{2\varepsilon_0}$ (D) $\frac{\sigma\lambda\ell\sin^2\theta}{\varepsilon_0}$

ES0115

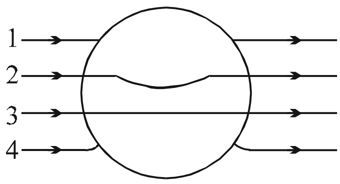
Conductors

52. n small conducting drops of same size are charged to V volts each. If they coalapse to form a single large drop, then its potential will be :-

(A) V/n (B) Vn (C) $Vn^{1/3}$ (D) $Vn^{2/3}$

ES0122

53. A metallic solid sphere is placed in a uniform electric field. The lines of force follow the path (s) shown in figure as :



(A) 1 (B) 2 (C) 3 (D) 4

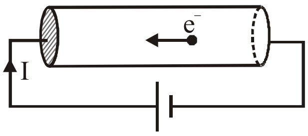
ES0123

CURRENT ELECTRICITY

KEY CONCEPT

In previous chapters we deal largely with electrostatics that is, with charges at rest. With this chapter we begin to focus on electric currents, that is, charges in motion.

ELECTRIC CURRENT



Electric charges in motion constitute an electric current. Any medium having practically free electric charges, free to migrate is a conductor of electricity. The electric charge flows from higher potential energy state to lower potential energy state.

Positive charge flows from higher to lower potential and negative charge flows from lower to higher. Metals such as gold, silver, copper, aluminium etc. are good conductors.

When charge flows in a conductor from one place to the other, then the rate of flow of charge is called electric current (I). When there is a transfer of charge from one point to other point in a conductor, we say that there is an electric current through the area. If the moving charges are positive, the current is in the direction of motion of charge. If they are negative the current is opposite to the direction of motion. If a charge ΔQ crosses an area in time Δt then the average electric current through the area, during this time as

• Average current $I_{av} = \frac{\Delta Q}{\Delta t}$ • Instantaneous current $I = \lim_{\Delta t \to 0} \frac{\Delta Q}{\Delta t} = \frac{dQ}{dt}$

- Current is a fundamental quantity with dimension $[M^0L^0T^0A^1]$
- Current is a scalar quantity with its SI unit **ampere.**

  Ampere : The current through a conductor is said to be one ampere if one coulomb of charge is flowing per second through a cross–section of wire.
- The conventional direction of current is the direction of flow of positive charge or applied field. It is opposite to direction of flow of negatively charged electrons.

- The conductor remains uncharged when current flows through it because the charge entering at one end per second is equal to charge leaving the other end per second.
- For a given conductor current does not change with change in its cross–section because current is simply rate of flow of charge.
- If there are n particles per unit volume each having a charge q and moving with velocity v then current through cross–sectional area A is $I = \frac{\Delta q}{\Delta t} = nqvA$
- If a charge q is moving in a circle of radius r with speed v then its time period is T = 2πr/v. The equivalent current $I = \frac{q}{T} = \frac{qv}{2\pi r}$.

Behavior of conductor in absence of applied potential difference :

In absence of applied potential difference electrons have random motion. The average displacement and average velocity is zero. There is no flow of current due to thermal motion of free electrons in a conductor.

The free electrons present in a conductor gain energy from temperature of surrounding and move randomly in the conductor.

The speed gained by virtue of temperature is called as thermal speed of an electron $\frac{1}{2}mv_{rms}^2 = \frac{3}{2}kt$

So thermal speed $v_{rms} = \sqrt{\frac{3kT}{m}}$ where m is mass of electron

At room temperature T = 300 K, $v_{rms} = 10^5$ m/s

- **Mean free path λ :** $(\lambda \sim 10Å) = \lambda = \frac{\text{total distance travelled}}{\text{number of collisions}}$
- **Relaxation time :** The time taken by an electron between two successive collisions is called as relaxation time τ : $(\tau \sim 10^{-14}s)$, Relaxation time : $\tau = \frac{\text{total time taken}}{\text{number of collisions}}$

Behavior of conductor in presence of applied potential difference :

When two ends of a conductors are joined to a battery then one end is at higher potential and another at lower potential. This produces an electric field inside the conductor from point of higher to lower potential

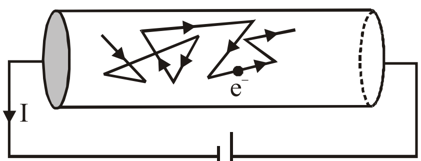
$E = \frac{V}{L}$ where V = emf of the battery, L = length of the conductor.

The field exerts an electric force on free electrons causing acceleration of each electron.

Acceleration of electron $\vec{a} = \frac{\vec{F}}{m} = \frac{-e\vec{E}}{m}$

DRIFT VELOCITY

Drift velocity is defined as the velocity with which the free electrons get drifted towards the positive terminal under the effect of the applied external electric field. In addition to its thermal velocity, due to acceleration given by applied electric field, the electron acquires a velocity component in a direction opposite to the direction of the electric field. The gain in velocity due to the applied field is very small and is lost in the next collision.



Under the action of electric field : Random motion of an electron with superimposed drift

At any given time, an electron has a velocity $\vec{v}_1 = \vec{u}_1 + \vec{a}\tau_1$, where $\vec{u}_1$ = the thermal velocity and $\vec{a}\tau_1$ = the velocity acquired by the electron under the influence of the applied electric field.

τ_1 = the time that has elapsed since the last collision. Similarly, the velocities of the other electrons are $\vec{v}_2 = \vec{u}_2 + \vec{a}\tau_2, \vec{v}_3 = \vec{u}_3 + \vec{a}\tau_3, ... \vec{v}_N = \vec{u}_N + \vec{a}\tau_N$.

The average velocity of all the free electrons in the conductor is equal to the drift velocity $\vec{v}_d$ of the free electrons

$$\vec{v}_d = \frac{\vec{v}_1 + \vec{v}_2 + \vec{v}_3 + ...\vec{v}_N}{N} = \frac{(u_1 + \vec{a}\tau_1) + (\vec{u}_2 + \vec{a}\tau_2) + ... + (\vec{u}_N + \vec{a}\tau_N)}{N} = \frac{(\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N)}{N} + \vec{a}\left(\frac{\tau_1 + \tau_2 + ... + \tau_N}{N}\right)$$

$$\because \frac{\vec{u}_1 + \vec{u}_2 + ... + \vec{u}_N}{N} = 0 \quad \therefore \ \vec{v}_d = \vec{a}\left(\frac{\tau_1 + \tau_2 + ... + \tau_N}{N}\right) \Rightarrow \vec{v}_d = \vec{a}\tau = -\frac{e\vec{E}}{m}\tau$$

Note : Order of drift velocity is 10^{-4} m/s.

Relation between current and drift velocity :

Let n= number density of free electrons and A= area of cross–section of conductor.

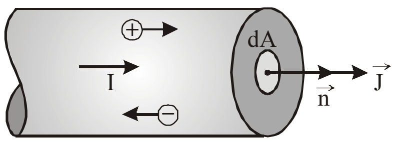
Number of free electrons in conductor of length L = nAL, Total charge on these free electrons $\Delta q = neAL$

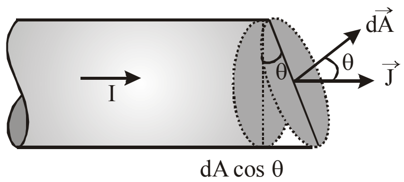
Time taken by drifting electrons to cross conductor $\Delta t = \frac{L}{v_d}$ $\therefore$ current $I = \frac{\Delta q}{\Delta t} = neAL\left(\frac{v_d}{L}\right) = neAv_d$

CURRENT DENSITY (J)

Current is a macroscopic quantity and deals with the overall rate of flow of charge through a section. To specify the current with direction in the microscopic level at a point, the term current density is introduced. Current density at any point inside a conductor is defined as a vector having magnitude equal to current per unit area surrounding that point. Remember area is normal to the direction of charge flow (or current passes) through that point.

- Current density at point P is given by $\vec{J} = \frac{dI}{dA}\vec{n}$





- If the cross–sectional area is not normal to the current, but makes an angle θ with the direction of current then $J = \frac{dI}{dA\cos\theta} \Rightarrow dI = JdA\cos\theta = \vec{J}.d\vec{A} \Rightarrow I = \int \vec{J}.\vec{dA}$
- Current density $\vec{J}$ is a vector quantity. It's direction is same as that of $\vec{E}$. It's S.I. unit is ampere/m^2 and dimension [$L^{-2}A$].

Ex. The current density at a point is $\vec{J} = \left(2\times10^4\hat{j}\right)Jm^{-2}$.

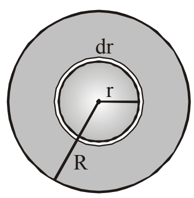
Find the rate of charge flow through a cross sectional area $\vec{S} = \left(2\hat{i} + 3\hat{j}\right)cm^2$

Sol. The rate of flow of charge = current = $I = \int \vec{J}.d\vec{S} \Rightarrow I = \vec{J}.\vec{S} = (2\times10^4)\left[\hat{j}\cdot\left(2\hat{i}+3\hat{j}\right)\right]\times10^{-4}A = 6A$

Ex. A potential difference applied to the ends of a wire made up of an alloy drives a current through it. The current density varies as J = 3 + 2r, where r is the distance of the point from the axis. If R be the radius of the wire, then the total current through any cross section of the wire.

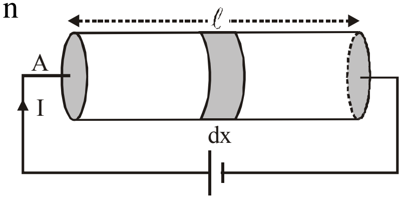
Sol. Consider a circular strip of radius r and thickness dr

$dI = \vec{J}.d\vec{S} = (3+2r)(2\pi r dr)\cos0° = 2\pi(3r+2r^2)dr$



$$I = \int_0^R 2\pi(3r+2r^2)dr = 2\pi\left(\frac{3r^2}{2} + \frac{2}{3}r^3\right)_0^R = 2\pi\left(\frac{3R^2}{2} + \frac{2R^3}{3}\right) \text{ units}$$

RELATION BETWEEN CURRENT DENSITY, CONDUCTIVITY AND ELECTRIC FIELD



Let the number of free electrons per unit volume in a conductor = n

Total number of electrons in dx distance = n (Adx)

Total charge dQ = n (Adx)e

Current $I = \frac{dQ}{dt} = nAe\frac{dx}{dt} = neAv_d$, Current density $J = \frac{I}{A} = nev_d$

$$= ne\left(\frac{eE}{m}\right)\tau \because v_d = \left(\frac{eE}{m}\right)\tau \Rightarrow J = \left(\frac{ne^2\tau}{m}\right)E \Rightarrow J = \sigma E, \text{ where conductivity } \sigma = \frac{ne^2\tau}{m}$$

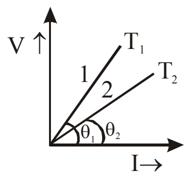
σ depends only on the material of the conductor and its temperature.

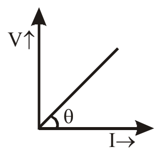
In vector form $\vec{J} = \sigma\vec{E}$ *Ohm's law (at microscopic level)*

RELATION BETWEEN POTENTIAL DIFFERENCE AND CURRENT (Ohm's Law)

If the physical conditions of the conductor (length, temperature, mechanical strain etc.) remains same, then the current flowing through the conductor is directly proportional to the potential difference across it's two ends i.e. $I \propto V \Rightarrow V = IR$ where R is a proportionality constant, known as electric resistance. *Ohm's law (at macroscopic level)*

- Ohm's law is not a universal law. The substances, which obey ohm's law are known as ohmic.
- Graph between V and I for a metallic conductor is a straight line as shown.





Slope of the line $= \tan\theta = \frac{V}{I} = R$

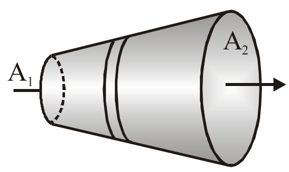
At different temperatures V–I curves are different.

Here $\tan\theta_1 > \tan\theta_2$ So $R_1 > R_2$ i.e. $T_1 > T_2$

- 1 ampere of current means the flow of 6.25×10^{18} electrons per second through any cross section of conductor.
- Current is a scalar quantity but current density is a vector quantity.
- Order of free electron density in conductors = 10^{28} electrons/m^3
-

Terms	Thermal speed v_T	Mean free path λ	Relaxation time τ	Drift speed v_d
Order	10^5 m/s	10 Å	10^{-14} m/s	10^{-4} m/s

- If a steady current flows in a metallic conductor of non uniform cross section.
  Current density and drift velocity depends on area



$$I_1 = I_2, A_1 < A_2 \Rightarrow J_1 > J_2, E_1 > E_2, v_{d_1} > v_{d_2}$$

- If the temperature of the conductor increases, the amplitude of the vibrations of the positive ions in the conductor also increase. Due to this, the free electrons collide more frequently with the vibrating ions and as a result, the average relaxation time decreases.

Ex. What will be the number of electron passing through a heater wire in one minute, if it carries a current of 8 A.

Sol. $I = \frac{Ne}{t} \Rightarrow N = \frac{It}{e} = \frac{8 \times 60}{1.6 \times 10^{-19}} = 3 \times 10^{21}$ electrons

Ex. A current of 1.34 A exists in a copper wire of cross–section 1.0 mm^2. Assuming each copper atom contributes one free electron. Calculate the drift speed of the free electrons in the wire. The density of copper is 8990 kg/m^3 and atomic mass = 63.50.

Sol. Mass of $1m^3$ volume of the copper is = 8990 kg = 8990×10^3 g

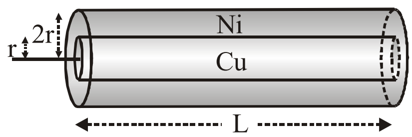
Number of moles in $1m^3$ $= \dfrac{8990\times10^3}{63.5} = 1.4\times10^5$

Since each mole contains 6×10^{23} atoms therefore number of atoms in $1m^3$

$n = (1.4 \times 10^5) \times (6 \times 10^{23}) = 8.4 \times 10^{28}$

$\because I = neAv_d \therefore v_d = \dfrac{I}{neA} = \dfrac{1.34}{8.4\times10^{28}\times1.6\times10^{-19}\times10^{-6}} = 10^{-4}$ m/s = 0.1 mm/s ($\because$ 1 $mm^2 = 10^{-6}$ m^2)

Ex. A copper wire of length 'ℓ' and radius 'r' is nickel plated till its final radius is 2r. If the resistivity of the copper and nickel are ρ_{Cu} and ρ_{Ni}, then find the equivalent resistance of wire?



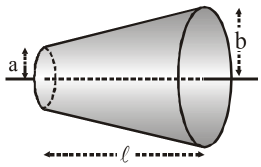
Sol. $R = \rho\dfrac{\ell}{A}$; Resistance of copper wire $R_{Cu} = \rho_{Cu}\dfrac{\ell}{\pi r^2}$

($\because A = \pi r^2$)

$\because A_{Ni} = \pi(2r)^2 - \pi r^2 = 3\pi r^2 \Rightarrow$ Resistance of Nickel wire $R_{Ni} = \rho_{Ni}\dfrac{\ell}{3\pi r^2}$

Both wire are connected in parallel. So equivalent resistance $R = \dfrac{R_{Cu}R_{Ni}}{R_{Cu}+R_{Ni}} = \left(\dfrac{\rho_{Cu}\rho_{Ni}}{3\rho_{Cu}+\rho_{Ni}}\right)\dfrac{\ell}{\pi r^2}$

Ex. Figure shows a conductor of length ℓ carrying current I and having a circular cross – section. The radius of cross section varies linearly from a to b. Assuming that (b – a) << ℓ. Calculate current density at distance x from left end.



Sol. Since radius at left end is a and that of right end is b,

Therefore increase in radius over length ℓ is (b – a).

Hence rate of increase of radius per unit length = $\left(\dfrac{b-a}{\ell}\right)$ Increase in radius over length x = $\left(\dfrac{b-a}{\ell}\right)x$

Since radius at left end is a so radius at distance x, $r = a + \left(\dfrac{b-a}{\ell}\right)x$

Area at this particular section $A = \pi r^2 = \pi\left[a + \left(\dfrac{b-a}{\ell}\right)x\right]^2$

Hence current density $J = \dfrac{I}{A} = \dfrac{I}{\pi r^2} = \dfrac{I}{\pi\left[a + \dfrac{x(b-a)}{\ell}\right]^2}$

RESISTANCE

The resistance of a conductor is the opposition which the conductor offers to the flow of charge. When a potential difference is applied across a conductor, free electrons get accelerated and collide with positive ions and their motion is thus opposed. This opposition offered by the ions is called resistance of the conductor.

Resistance is the property of a conductor by virtue of which it opposes the flow of current in it.

Unit : ohm, volt/ampere,

Dimension = $M L^2 T^{-3} A^{-2}$

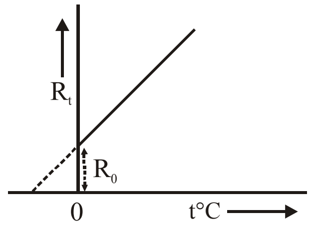
Resistance depends on :

- Length of the conductor ($R \propto \ell$)
- Area of cross-section of the conductor $R \propto \frac{1}{A}$
- Nature of material of the conductor $R = \frac{\rho\ell}{A}$
- Temperature $R_t = R_0 (1 + \alpha\Delta t)$

  Where R_t = Resistance at t° C, R_0 = Resistance at 0° C

  Δt = Change in temperature, α = Temperature coefficient of resistance

  *[*For metals* : α positive *for semiconductors* : α negative]

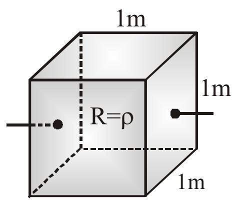


RESISTIVITY

Resistivity : $\rho = RA/\ell$ if ℓ = 1m, A = 1m^2 then ρ = R

The specific resistance of a material is equal to the resistance of the wire of that material with unit cross – section area and unit length.

Resistivity depends on (i) Nature of material (ii) Temperature of material ρ does not depend on the size and shape of the material because it is the characteristic property of the conductor material.



Specific use of conducting materials :

- The **heating element** of devices like heater, geyser, press etc are made of **michrome** because it has high resistivity and high melting point. It does not react with air and acquires steady state when red hot at 800°C.
- **Fuse wire** is made of **tin lead alloy** because it has low melting point and low resistivity. The fuse is used in series, and melts to produce open circuit when current exceeds the safety limit.
- **Resistances** of resistance box are made of **manganin** or **constantan** because they have moderate resistivity and very small temperature coefficient of resistance. The resistivity is nearly independent of temperature.
- The **filament of bulb** is made up of **tungsten** because it has low resistivity, high melting point of 3300 K and gives light at 2400 K. The bulb is filled with inert gas because at high temperature it reacts with air forming oxide.
- The **connection wires** are made of **copper** because it has low resistance and resistivity.

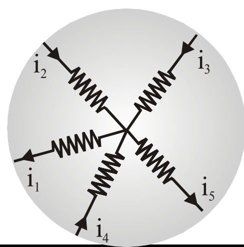
KIRCHHOFF'S LAW

There are two laws given by Kirchhoff for determination of potential difference and current in different branches of any complicated network.

Law of conservation of charge is a consequence of continuity equation

- **First law (Junction Law or Current Law)**

  In an electric circuit, the algebraic sum of the current meeting at any junction in the circuit is zero or Sum of the currents entering the Junction is equal to sum of the current leaving the Junction. $\Sigma i=0$



$i_1 - i_2 - i_3 - i_4 + i_5 = 0 \Rightarrow i_1 + i_5 = i_2 + i_3 + i_4$

This is based on law of conservation of charge.

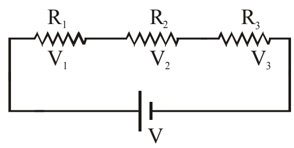
- **Second law (loop rule or potential law)**

  In any closed circuit the algebraic sum of all potential differences and e.m.f. is zero. $\Sigma E - \Sigma IR = 0$ while moving from negative to positive terminal inside the cell, e.m.f. is taken as positive while moving in the direction of current in a circuit the potential drop (i.e. IR) across resistance is taken as positive.

  This law is based on law of conservation of energy.

COMBINATION OF RESISTANCE

Series Combination

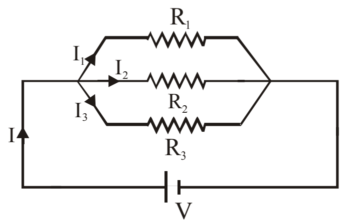


- Same current passes through each resistance
- Voltage across each resistance is directly proportional to it's value

  $V_1 = IR_1,\ V_2 = IR_2\ ,\ V_3 = IR_3$
- Sum of the voltage across resistance is equal to the voltage applied across the circuit.

  $V = V_1 + V_2 + V_3 \Rightarrow IR = IR_1 + IR_2 + IR_3 \Rightarrow\ R = R_1 + R_2 + R_3$ Where R = equivalent resistance

Parallel Combination



- There is same drop of potential across each resistance.
- Current in each resistance is inversely proportional to the value of resistance. $I_1 = \dfrac{V}{R_1}$, $I_2 = \dfrac{V}{R_2}$, $I_3 = \dfrac{V}{R_3}$
- Current flowing in the circuit is sum of the currents in individual resistance.

$$I = I_1 + I_2 + I_3 \ \Rightarrow \frac{V}{R} = \frac{V}{R_1} + \frac{V}{R_2} + \frac{V}{R_3} \Rightarrow \frac{1}{R} = \frac{1}{R_1} + \frac{1}{R_2} + \frac{1}{R_3}$$

- To get maximum resistance, resistance must be connected in series and in series the resultant is greater than largest individual.
- To get minimum resistance, resistance must be connected in parallel and the equivalent resistance of parallel combination is lower than the value of lowest resistance in the combination.
- **In general :**

(i) Resistivity of alloys is greater than their metals.

(ii) Temperature coefficient of alloys is lower than pure metals.

(iv) The resistivity of an insulator (e.g. amber) is greater then the metal by a factor of 10^{22}

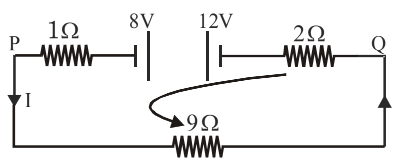
Ex. The resistance 4 R, 16 R, 64 R ... ∞ are connected in series. Find their equivalent resistance.

Sol. Resultant of the given combination $R_{eq} = 4R + 16R + 64R + ...\ \infty = \infty$

Ex. Resistance R, 2R, 4R, 8R...∞ are connected in parallel. What is their resultant resistance ?

Sol. $$\frac{1}{R_{eq}} = \frac{1}{R} + \frac{1}{2R} + \frac{1}{4R} + \frac{1}{8R} ....\infty = \frac{1}{R}\left[1 + \frac{1}{2} + \frac{1}{4} + ...\infty\right] = \frac{1}{R}\left[\frac{1}{1 - \frac{1}{2}}\right] = \frac{2}{R} \Rightarrow R_{eq} = \frac{R}{2}$$

Ex. In the given circuit calculate potential difference between the points P and Q.



Sol. Applying Kirchhoff's voltage law (KVL)

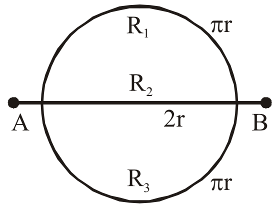
$$12 - 8 = (1)\,I + (9)\,I + (2)\,I \Rightarrow I = \frac{1}{3}\,A$$

Potential difference between the points P and Q, $V_P - V_Q = 9 \times \frac{1}{3} = 3$ volt

Ex. A wire of $\rho_L = 10^{-6}\ \Omega/m$ is turned in the form of a circle of diameter 2 m. A piece of same material is connected in diameter AB. Then find resistance between A and B.

Sol. $\because$ $R = \rho_L \times \text{length}$

$\therefore$ $R_1 = \pi \times 10^{-6}\ \Omega$, $R_2 = 2 \times 10^{-6}\ \Omega$, $R_3 = \pi \times 10^{-6}\ \Omega$



$$\frac{1}{R_{AB}} = \frac{1}{\pi \times 10^{-6}} + \frac{1}{2 \times 10^{-6}} + \frac{1}{\pi \times 10^{-6}}\ ;\ R_{AB} = 0.88 \times 10^{-6}\ \text{ohm.}$$

CELL

Cell convert chemical energy into electrical energy.

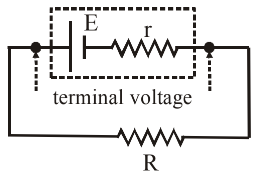
ELECTRO MOTIVE FORCE (E. M. F.)

The potential difference across the terminals of a cell when it is not giving any current is called emf of the cell. The energy given by the cell in the flow of unit charge in the whole circuit (including the cell) is called the emf of the cell.

- emf depends on : (i) nature of electrolyte (ii) metal of electrodes
- emf does not depend on : (i) area of plates (ii) distance between the electrodes (iii) quantity of electrolyte (iv) size of cell

TERMINAL VOLTAGE (V)

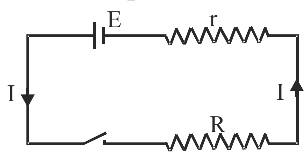
- When current is drawn through the cell or current is supplied to cell then, the potential difference across its terminals called terminal voltage.
- When I current is drawn from cell, then terminal voltage is less than it's e.m.f. $V = E - Ir$



- **Terminal Potential Difference *:*** The potential difference between the two electrodes of a cell in a closed circuit i.e. when current is being drawn from the cell is called terminal potential difference.

(a) When cell is discharging :

Current inside the cell is from cathode to anode.

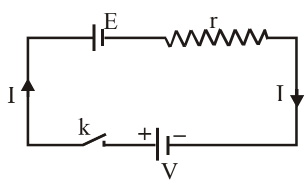


Current $I = \frac{E}{r + R} \Rightarrow E = IR + Ir = V + Ir \Rightarrow V = E - Ir$

When current is drawn from the cell potential difference is less than emf of cell. Greater is the current drawn from the cell smaller is the terminal voltage. When a large current is drawn from a cell its terminal voltage is reduced.

(b) When cell is charging :

Current inside the cell is from anode to cathode.



Current $I = \frac{V - E}{r} \Rightarrow V = E + Ir$

During charging terminal potential difference is greater than emf of cell.

(c) When cell is in open circuit :

In open circuit $R = \infty$ $\therefore$ $I = \frac{E}{R + r} = 0 \Rightarrow V = E$

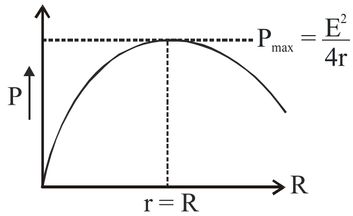
In open circuit terminal potential difference is equal to emf and is the maximum potential difference which a cell can provide.

(d) When cell is short circuited :

In short circuit R = 0 ⇒ $I = \frac{E}{R + r} = \frac{E}{r}$ and V = IR = 0

In short circuit current from cell is maximum and terminal potential difference is zero.

(e) Power transferred to load by cell :



$$P = I^2R = \frac{E^2R}{(r+R)^2} \Rightarrow P = P_{max} \text{ if } \frac{dP}{dR} = 0 \Rightarrow r = R$$

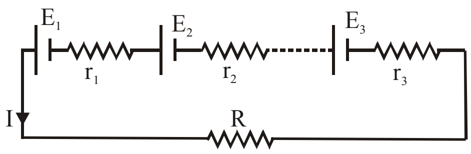
Power transferred by cell to load is maximum when

r = R and $P_{max} = \frac{E^2}{4r} = \frac{E^2}{4R}$

COMBINATION OF CELLS

- **Series combination**

When the cells are connected in series the total e.m.f. of the series combination is equal to the sum of the e.m.f.'s of the individual cells and internal resistance of the cells also come in series.



Equivalent internal resistance $r = r_1 + r_2 + r_3 + ...$

Equivalent emf = $E = E_1 + E_2 + E_3 + ...$

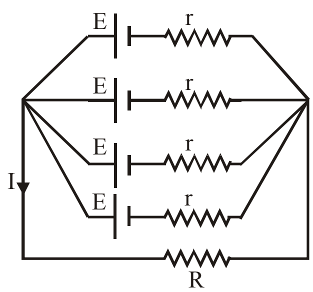
Current $I = \frac{E_{net}}{r_{net} + R}$, If all n cell are identical then $I = \frac{nE}{nr + R}$

• If nr >> R, $I = \frac{E}{r} \simeq$ current from one cell • If nr << R, $I = \frac{nE}{R} \simeq n \times$ current from one cell

- **Parallel combination**

When the cells are connected in parallel, the total e.m.f. of the parallel combination remains equal to the e.m.f. of a single cell and internal resistance of the cell also come in parallel. If m identical cell connected in parallel then total internal resistance of this combination $r_{net} = \frac{r}{m}$. Total e.m.f. of this combination = E

Current in the circuit $I = \frac{E}{R + \frac{r}{m}} = \frac{mE}{mR + r}$



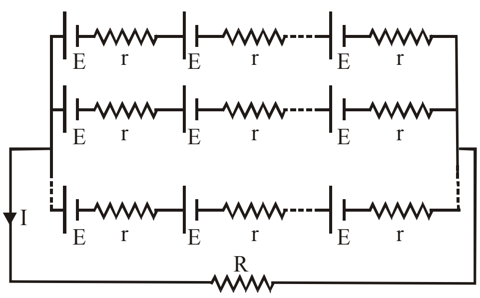
If r << mR I=E/R = Current from one cell

If r>> mR $I = \frac{mE}{r} = m \times$ current from one cell

- **Mixed combination**

If n cells connected in series and their are m such branches in the circuit then total number of identical cell in this circuit is nm. The internal resistance of the cells connected in a row = nr . Since there are such m rows,

Total internal resistance of the circuit $r_{net} = \frac{nr}{m}$



Total e.m.f. of the circuit = total e.m.f. of the cells connected in a row

$E_{net} = nE$

Current in the circuit $I = \frac{E_{net}}{R + r_{net}} = \frac{nE}{R + \frac{nr}{m}}$

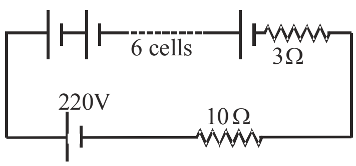
Current in the circuit is maximum when external resistance in the circuit is equal to the total internal resistance of the cells $R = \frac{nr}{m}$

- At the time of charging a cell when current is supplied to the cell, the terminal voltage is greater than the e.m.f. E, V = E + Ir
- Series combination is useful when internal resistance is less than external resistance of the cell.
- Parallel combination is useful when internal resistance is greater than external resistance of the cell.
- Power in R (given resistance) is maximum, if its value is equal to net resistance of remaining circuit.
- Internal resistance of ideal cell = 0
- if external resistance is zero than current given by circuit is maximum.

Ex. A battery of six cells each of e.m.f. 2 V and internal resistance 0.5 Ω is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10 Ω. What will be the charging current.

Sol. Net e.m.f of the battery = 12V and total internal resistance = 3Ω

Total resistance of the circuit = 3 + 10 = 13 Ω



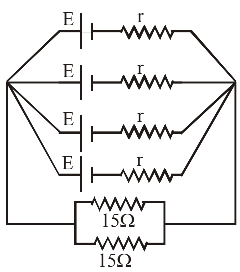
Charging current $I = \frac{\text{Net e.m.f.}}{\text{total resistance}} = \frac{220-12}{13} = 16\text{ A}$

Ex. A battery of six cells each of e.m.f. 2 V and internal resistance 0.5 Ω is being charged by D. C. mains of e.m.f. 220 V by using an external resistance of 10 Ω. What is the potential difference across the battery ?

Sol In case of charging of battery, terminal potential V = E + Ir = 12 + 16 × 3 = 60 volt.

Ex. Four identical cells each of e.m.f. 2V are joined in parallel providing supply of current to external circuit consisting of two 15Ω resistors joined in parallel. The terminal voltage of the equivalent cell as read by an ideal voltmeter is 1.6V calculate the internal resistance of each cell.

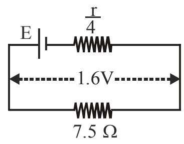
Sol. Total internal resistance of the combination $r_{eq} = \frac{r}{4}$



Total e.m.f. E_{eq} = 2V

Total external resistance $R = \frac{15\times15}{15+15} = \frac{15}{2} = 7.5\Omega$

Current drawn from equivalent cell $I = \dfrac{\text{terminal potential}}{\text{external resistance}} = \dfrac{1.6}{7.5}\text{A}$



$\because\ E - I\left(\dfrac{r}{4}\right) = 1.6 \ \therefore\ E - I\left(\dfrac{r}{4}\right) = 1.6 \Rightarrow r = 7.5\ \Omega$

Ex. The e.m.f. of a primary cell is 2 V, when it is shorted then it gives a current of 4 A. Calculate internal resistance of primary cell.

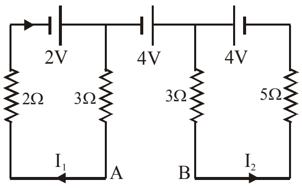
Sol. $I = \dfrac{E}{r+R}$, If cell is shorted then R = 0, $I = \dfrac{E}{r}\ \therefore\ r = \dfrac{E}{I} = \dfrac{2}{4} = 0.5\ \Omega$

Ex. n rows each containing m cells in series, are joined in parallel. Maximum current is taken from this combination in a 3 Ω resistance. If the total number of cells used is 24 and internal resistance of each cell is 0.5 Ω , find the value of m and n.

Sol. Total number of cell mn = 24, For maximum current $\dfrac{mr}{n} = R \Rightarrow 0.5\ m = 3\ n,\ m = \dfrac{3n}{0.5} = 6n$

$\therefore 6n \times n = 24 \Rightarrow n = 2$ and $m \times 2 = 24 \Rightarrow m = 12$

Ex. In the given circuit calculate potential difference between A and B.



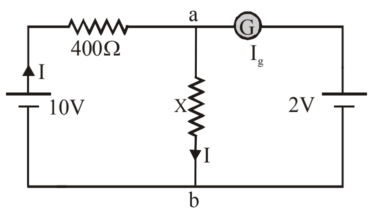
Sol. First applying KVL on left mesh $2 - 3\ I_1 - 2\ I_1 = 0 \Rightarrow I_1 = 0.4$ amp.

Now applying KVL on right mesh. $4 - 5\ I_2 - 3\ I_2 = 0 \Rightarrow I_2 = 0.5$ amp.

Potential difference between points A and B

$V_A - V_B = -3 \times 0.4 - 4 + 3 \times 0.5 = -3.7$ volt.

Ex. In the following circuit diagram, the galvanometer reading is zero. If the internal resistance of cells are negligible then what is the value of X ?

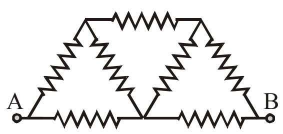


Sol. $\because\ I_g = 0 \qquad \therefore\ I = \dfrac{10}{400+X}$

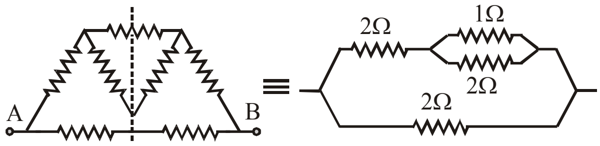
also potential difference across X is 2V ⇒ I X = 2

$$\frac{10X}{400+X} = 2\left(\because\ I = \frac{10}{400+X}\right) \Rightarrow\ X = 100\Omega$$

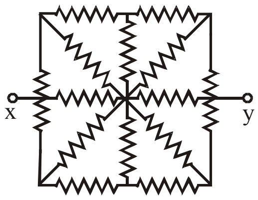
Ex. Each resistance is of 1 Ω in the circuit diagram shown in figure. Find out equivalent resistance between A and B



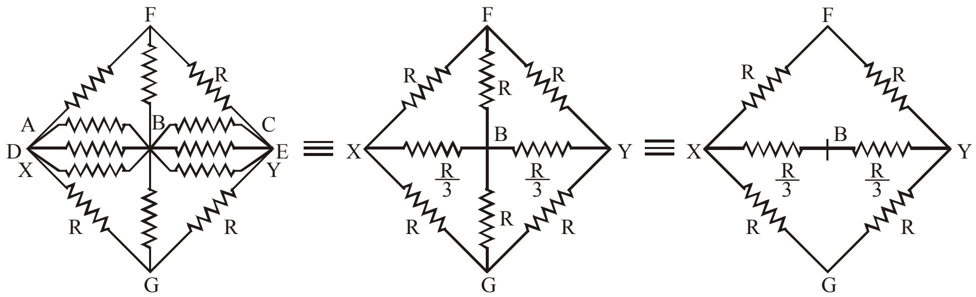
Sol. By symmetric line method $R_{AB} = (2 + 1 \parallel 2) \parallel 2 = \frac{8}{7}\,\Omega$



Ex. Identical resistance of resistance R are connected as in figure then find out net resistance between x and y.



Sol. Given circuit can be modified according to following figures



$$\frac{1}{R_{xy}} = \frac{1}{2R} + \frac{3}{2R} + \frac{1}{2R} = \frac{5}{2R} \Rightarrow R_{xy} = \frac{2R}{5}$$

HEATING EFFECT OF CURRENT

CAUSE OF HEATING

The potential difference applied across the two ends of conductor sets up electric field. Under the effect of electric field, electrons accelerate and as they move, they collide against the ions and atoms in the conductor, the energy of electrons transferred to the atoms and ions appears as heat.

- **Joules's Law of Heating**

When a current I is made to flow through a passive or ohmic resistance R for time t, heat Q is produced such that

$$Q = I^2 R t = P \times t = V I t = \frac{V^2}{R} t$$

Heat produced in conductor does not depend upon the direction of current.

• **SI unit** : joule ;

Practical Units : 1 kilowatt hour (kWh)

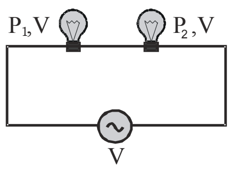
$1\text{kWh} = 3.6 \times 10^6$ joule = 1 unit   1 BTU (British Thermal Unit) = 1055 J

• **Power :** $P = V I = \frac{V^2}{R} = I^2R$ • **SI unit :** Watt

The watt–hour meter placed on the premises of every consumer records the electrical energy consumed.

- **Series combination of resistors (bulbs)**

Total power consumed $P_{total} = \frac{P_1P_2}{P_1 + P_2}$.



If n bulbs are identical $P_{total} = \frac{P}{n}$

In series combination of bulbs : Brightness $\propto$ Power consumed by bulb $\propto$ V $\propto$ R $\propto \frac{1}{P_{rated}}$

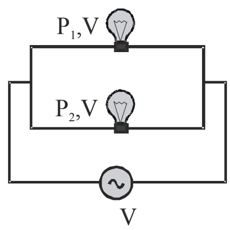
Bulb of lesser wattage will shine more. For same current $P = I^2R$ $\quad P \propto R \quad R\uparrow \Rightarrow P\uparrow$

- **Parallel combination of resistors (bulbs)**

Total power consumed $P_{total} = P_1 + P_2$

If n bulbs are identical $P_{total} = nP$

In parallel combination of bulbs



Brightness $\propto$ Power consumed by bulb $\propto I \propto \frac{1}{R}$

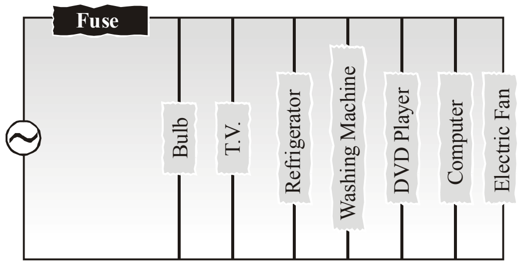
Bulb of greater wattage will shine more.

For same V more power will be consumed in smaller resistance $P \propto \frac{1}{R}$

- Two identical heater coils gives total heat H_S when connected in series and H_p when connected in parallel than $\frac{H_P}{H_S} = 4$ [In this, it is assumed that supply voltage is same]
- If a heater boils m kg water in time T_1 and another heater boils the same water in T_2, then both connected in series will boil the same water in time $T_s = T_1 + T_2$ and in parallel $T_P = \frac{T_1T_2}{T_1 + T_2}$ [Use time taken $\propto$ Resistance]
- Instruments based on heating effect of current, works on both A.C. and D.C. Equal value of A.C. (RMS) and D.C. produces, equal heating effect. That why brightness of bulb is same whether it is operated by A.C. or same value D.C.

FUSE WIRE

The fuse wire for an electric circuit is chosen keeping in view the value of safe current through the circuit.



- The fuse wire should have high resistance per unit length and low melting point.
- However the melting point of the material of fuse wire should be above the temperature that will be reached on the passage of the current through the circuit
- A fuse wire is made of alloys of lead (Pb) and tin (Sn).
- Length of fuse wire is immaterial.
- The material of the filament of a heater should have high resistivity and high melting point.
- The temperature of the wire increases to such a value at which, the heat produced per second equals heat lost per second due to radiation from the surface of wire $I^2\left(\frac{\rho\ell}{\pi r^2}\right) = H \times 2\pi r\ell$ $I^2 \propto r^3$

  H = heat lost per second per unit area due to radiation.

Ex. An electric heater and an electric bulb are rated 500 W, 220 V and 100 W, 220 V respectively. Both are connected in series to a 220 V a.c. mains. Calculate power consumed by (i) heater (ii) bulb.

Sol. $P = \frac{V^2}{R}$ or $R = \frac{V^2}{P}$, For heater. Resistance $R_h = \frac{(220)^2}{500} = 96.8\ \Omega$,

For bulb resistance $R_L = \frac{(220)^2}{100} = 484\ \Omega$

Current in the circuit when both are connected in series $I = \frac{V}{R_L + R_h} = \frac{220}{484 + 96.8} = 0.38\ A$

(i) Power consumed by heater = $I^2R_h = (0.38)^2 \times 96.8 = 13.98$ W

(ii) Power consumed by bulb = $I^2R_L = (0.38)^2 \times 484 = 69.89$ W

Ex. A heater coil is rated 100 W, 200 V. It is cut into two identical parts. Both parts are connected together in parallel, to the same source of 200 V. Calculate the energy liberated per second in the new combination.

Sol. $\because P = \frac{V^2}{R}$ $\therefore R = \frac{V^2}{P} = \frac{(200)^2}{100} = 400\ \Omega$

Resistance of half piece = $\frac{400}{2} = 200\ \Omega$

Resistance of pieces connected in parallel = $\frac{400}{2} = 100\ \Omega$

Energy liberated/second $P = \frac{V^2}{R} = \frac{200 \times 200}{100} = 400$ W

Ex. The power of a heater is 500W at 800°C. What will be its power at 200°C. If $\alpha = 4 \times 10^{-4}$ per °C?

Sol. $P = \dfrac{V^2}{R} \therefore \dfrac{P_{200}}{P_{800}} = \dfrac{R_{800}}{R_{200}} = \dfrac{R_0(1+4\times10^{-4}\times800)}{R_0(1+4\times10^{-4}\times200)} \Rightarrow P_{200} = \dfrac{500\times1.32}{1.08} = 611W$

Ex. When a battery sends current through a resistance R_1 for time t, the heat produced in the resistor is Q. When the same battery sends current through another resistance R_2 for time t, the heat produced in R_2 is again Q. Determine the internal resistance of battery.

Sol. $\left[\dfrac{E}{R_1+r}\right]^2 R_1 = \left[\dfrac{E}{R_2+r}\right]^2 R_2 \Rightarrow r = \sqrt{R_1R_2}$

Ex. How much time heater will take to increase the temperature of 100 g water by 50°C if resistance of heating coil is 484Ω and supply voltage is 220V a.c.

Sol. Heat given by heater = heat taken by water $\Rightarrow \dfrac{V^2}{R} t = \text{ms J}\Delta\theta \Rightarrow \dfrac{220\times220}{484}$

$t = (100 \times 10^{-3})(4.2 \times 10^3)(50) \Rightarrow t = 210s$

GALVANOMETER

The instrument used to measure strength of current, by measuring the deflection of the coil due to torque produced by a magnetic field, is known as galvanometer.

SHUNT

The small resistance connected in parallel to galvanometer coil, in order to control current flowing through the galvanometer, is known as shunt.

- **Merits of shunt**

(i) To protect the galvanometer coil from burning.

(ii) Any galvanometer can be converted into ammeter of desired range with the help of shunt.

(iii) The range an ammeter can be changed by using shunt resistance of different values.

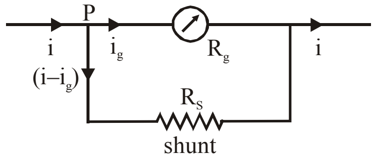
- **Demerits of shunt**

Shunt resistance decreases the sensitivity of galvanometer.

CONVERSION OF GALVANOMETER INTO AMMETER

A galvanometer can be converted into an ammeter by connecting low resistance in parallel to its coil.

- The value of shunt resistance to be connected in parallel to galvanometer coil is given by : $R_S = \dfrac{R_g i_g}{i - i_g}$



Where i = Range of ammeter

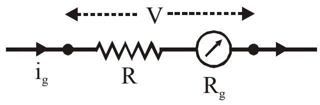
i_g = Current required for full scale deflection of galvanometer.

R_g = Resistance of galvanometer coil.

CONVERSION OF GALVANOMETER INTO VOLTMETER

- The galvanometer can be converted into voltmeter by connecting high resistance in series with its coil.

- The high resistance to be connected in series with galvanometer coil is given by $R = \frac{V}{i_g} - R_g$

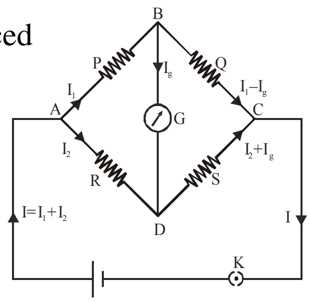


- The rate of variation of deflection depends upon the magnitude of deflection itself and so the accuracy of the instrument.
- A suspended coil galvanometer can measure currents of the order of 10^{-9} ampere.
- I_g is the current for full scale deflection. If the current for a deflection, of one division on the galvanometer scale is k and N is the total number of divisions on one side of the zero of galvanometer scale, then $I_g = k \times N$.
- A ballistic galvanometer is a specially designed moving coil galvanometer, used to measure charge flowing through the circuit for small time intervals.

WHEAT STONE BRIDGE

- The configuration in the adjacent figure is called Wheat Stone Bridge.
- If current in galvanometer is zero ($I_g = 0$) then bridge is said to be balanced

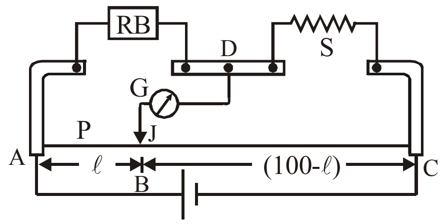
$$V_D = V_B \Rightarrow I_1P = I_2R \ \& \ I_1Q = I_2S \Rightarrow \frac{P}{Q} = \frac{R}{S}$$



- If $\frac{P}{Q} < \frac{R}{S}$ then $V_B > V_D$ and current will flow from B to D.
- If $\frac{P}{Q} > \frac{R}{S}$ then $V_B < V_D$ and current will flow from D to B.

METRE BRIDGE

It is based on principle of whetstone bridge. It is used to find out unknown resistance of wire. AC is 1 m long uniform wire R.B. is known resistance and S is unknown resistance. A cell is connected across 1 m long wire and Galvanometer is connected between Jockey and midpoint D. To find out unknown resistance we touch jockey from A to C and find balance condition. Let balance is at B point on wire.



$AB = \ell$ cm $P = r\ell$

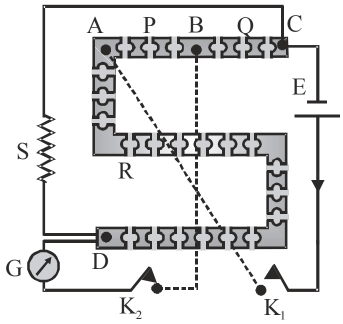
$BC = (100 - \ell)$ cm $Q = r(100 - \ell)$

where r = resistance per unit length on wire.

At balance condition : $\frac{P}{Q} = \frac{R}{S} \Rightarrow \frac{r\ell}{r(100-\ell)} = \frac{R}{S} \Rightarrow S = \frac{(100-\ell)}{\ell}R$

POST OFFICE BOX

It is also based on wheat stone bridge. The resistance of 10Ω, 100Ω, and 1000Ω are often connected between AB and BC. These are known as ratio arms. Resistance from 1Ω to 5000Ω are connected between A and D, this is known arm. Unknown resistance is connected between C and D.

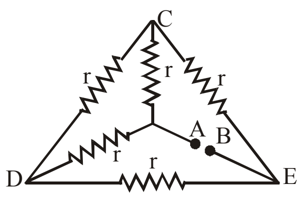


A cell is connected between A and C with key K_1 and Galvanometer is connected between B and D with key K_2.

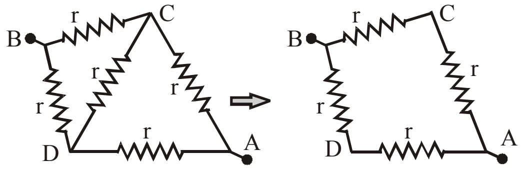
First we select ratio of resistance Q and P. For given value of S we will take value of resistance from known arm in such a way that Galvanometer show null deflection $S = \frac{Q}{P}R$. On decreasing the value of $\frac{Q}{P}$ the sensitivity of the box increases. It is used to find out the breakage in telegraph line in post and telegraph offices.

- To increase the range of an ammeter a shunt is connected in parallel with the galvanometer.
- To convert an ammeter of range I ampere and resistance R_g Ω into an ammeter of range nI ampere, the value of resistance to be connected in parallel will be $R_g(n-1)$
- To increase the range of a voltmeter a high resistance is connected in series with it.
- To convert a voltmeter of resistance R_g Ω and range V volt into a voltmeter of range nV volt, the value of resistance to be connected in series will be $(n-1)R_g$.
- Resistance of ideal ammeter is zero & resistance of ideal voltmeter is infinite.
- The bridge is most sensitive when the resistance in all the four branches of the bridge is of same order.

Ex. In the adjoining network of resistors each is of resistance r Ω. Find the equivalent resistance between point A and B



Sol. Given circuit is balanced Wheat stone Bridge



$$\therefore \frac{1}{R_{AB}} = \frac{1}{2r} + \frac{1}{2r} = \frac{1}{r} \qquad R_{AB} = r$$

Ex. A 100 volt voltmeter whose resistance is 20 kΩ is connected in series to a very high resistance R. When it is joined in a line of 110 volt, it reads 5 volt. What is the magnitude of resistance R ?

Sol. When voltmeter connected in 110 volt line, Current through the voltmeter $I = \frac{110}{(20 \times 10^3 + R)}$

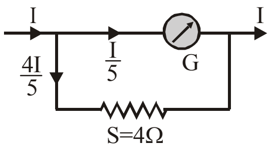
The potential difference across the voltmeter $V = IR_V$

$$\Rightarrow 5 = \frac{110 \times 20 \times 10^3}{(20 \times 10^3 + R)}$$

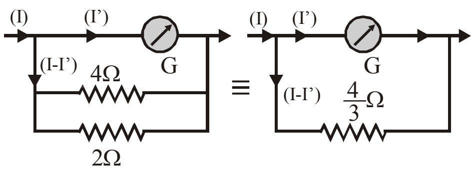
$\Rightarrow 20 \times 10^3 + R = 440 \times 10^3 \Rightarrow R = 420 \times 10^3\ \Omega$

Ex. When a shunt of 4Ω is attached to a galvanometer, the deflection reduces to 1/5th. If an additional shunt of 2Ω is attached what will be the deflection ?

Sol. Initial condition : When shunt of 4Ω used $\frac{I}{5} \times G = \frac{4}{5} I \times 4 \Rightarrow G = 16\Omega$



When additional shunt of 2Ω used $I' \times 16 = (I - I')\frac{4}{3} \Rightarrow I' = \frac{I}{13}$



$\therefore$ it will reduce to $\frac{I}{13}$ of the initial deflection

Ex. A galvanometer having 30 divisions has current sensitivity of 20μA/division. It has a resistance of 25 Ω.

(i) How will you convert it into an ammeter measuring upto 1 ampere.

(ii) How will you convert this ammeter into a voltmeter upto 1 volt.

Sol The current required for full scale deflection $I_g = 20\ \mu A \times 30 = 600\ \mu A = 6 \times 10^{-4} A$

(i) To convert it into ammeter, a shunt is required in parallel with it

shunt resistance $R'_S = \frac{I_g R_g}{(I - I_g)} = \left(\frac{6\times10^{-4}}{1-6\times10^{-4}}\right)25 = 0.015\Omega$

(ii) To convert galvanometer into voltmeter, a high resistance in series with it is required

series resistance $R = \frac{V}{i_g} - R_g = \frac{1}{6\times10^{-4}} - 25 = 1666.67 - 25 = 1641.67\ \Omega$

POTENTIOMETER

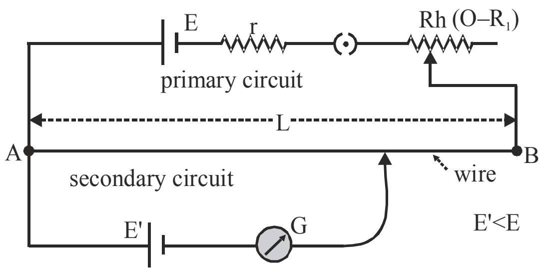
- **Necessity of potentiometer**

  Practically voltameter has a finite resistance. (ideally it should be ∞) in other words it draws some current from the circuit . To overcome this problem potentiometer is used because at the instant of measurement , it draws no current from the circuit. It means its effective resistance is infinite.
- **Working principle of potentiometer**

  Any unknown potential difference is balanced on a known potential difference which is uniformly distributed over entire length of potentiometer wire. This process is named as zero deflection or null deflection method.
- **Potentiometer wire**

  Made up of alloys of magnin, constantan, Eureka. Specific properties of these alloys are high specific resistance, negligible temperature co–efficient of resistance (α). Invariability of resistance of potentiometer wire over a long period.

CIRCUITS OF POTENTIOMETER



- **Primary circuit** contains constant source of voltage rheostat or Resistance Box
- **Secondary, Unknown or galvanometer circuit**

  Let ρ = Resistance per unit length of potentiometer wire
- **Potential gradient (x) (V/m)**

  The fall of potential per unit length of potentiometer wire is called potential gradient.

$$x = \frac{V}{L} = \frac{\text{current} \times \text{resitance of potentiometer wire}}{\text{length of potentiometer wire}} = I\left(\frac{R}{L}\right)$$

  The potential gradient depends only on primary circuit and is independent of secondary circuit.
- **Applications of potentiometer**

• To measure potential difference across a resistance. • To find out emf of a cell • Comparison of two emfs E_1/E_2 • To find out internal resistance of a primary cell • Comparison of two resistance. • To find out an unknown resistance which is connected in series with the given resistance.	• To find out current in a given circuit • Calibration of an ammeter or to have a check on reading of (A) • Calibration of a voltmeter or to have a check on reading of (V) • To find out thermocouple emf (e_t) (mV or mV)

Different between potentiometer and voltmeter	
Potentiomer	**Voltmeter**
• It measures the unknown emf very accurately	It measures the unknown emf approximately.
• While measuring emf it does not draw any current from the driving source of know emf.	While measuring emf it draws some current from the source of emf.
• While measuring unknown potential difference the resistance of potentiometer becomes infinite.	While measuring unknown potential difference the resistance of voltmeter is high but finite.
• It is based on zero deflection method.	It is based on deflection method.
• It has a high sensitivity.	Its sensitivity is low.
• it is used for various applications like measurement of internal restiance of cell, calibration of ammeter and voltmeter, measurement of thermo emf, comparison of emf's etc.	It is only used to measured emf or unknown potential difference.

Ex. There is a definite potential difference between the two ends of a potentiometer. Two cells are connected in such a way that first time help each other, and second time they oppose each other. They are balanced on the potentiometer wire at 120 cm and 60 cm length respectively. Compare the electromotive force of the cells.

Sol. Suppose the potential gradient along the potentiometer wire = x and the emf's of the two cells are E_1 and E_2.

When the cells help each other, the resultant emf = $(E_1 + E_2)$

$$E_1 + E_2 = x \times 120 \text{ cm} \quad ...(i)$$

When the cells oppose each other, the resultant emf = $(E_1 - E_2)$

$$E_1 - E_2 = x \times 60 \text{ cm} \quad ...(ii)$$

From equation (i) and (ii) $\dfrac{E_1+E_2}{E_1-E_2} = \dfrac{120\text{ cm}}{60\text{cm}} = \dfrac{2}{1} \Rightarrow E_1 + E_2 = 2(E_1 - E_2) \Rightarrow 3E_2 = E_1 \Rightarrow \dfrac{E_1}{E_2} = \dfrac{3}{1}$

EXERCISE (S)

Microscophic analysis

1. A copper wire of length L, and cross section area A carries a current I. If the specific resistance of copper is ρ, the electric field in the wire is _ _ _ _ _ _ .

CE0001

2. A copper wire carries a current density j (= current per unit area). Assuming that n = No. of free electrons per unit volume, e = electronic charge, $\langle v \rangle$ = average speed due to thermal agitation. The distance which will be covered by an electron during its displacement ℓ along the wire _ _ _ _ _.

CE0002

3. The total momentum of electrons in a straight wire of length ℓ carrying a current I is _ _ _ _ _ _ (mass of electron = m_e, charge of electron = e)

CE0003

4. Two conductors are made of the same material and have the same length. Conductor A is a solid wire of diameter 1mm. Conductor B is a hollow tube of outer diameter 2mm and inner diameter 1mm. Find the ratio of resistance R_A to R_B.

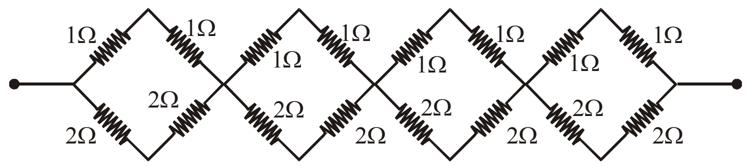
CE0004

Ohm's law and circuit analysis

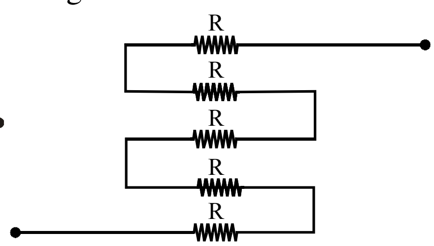
5. (a) Given n resistors each of resistance R, how will you combine them to get the (i) maximum (ii) minimum effective resistance? What Is the ratio of the maximum to minimum resistance?

(b) Given the resistances of 1Ω, 2Ω, 3Ω, how will be combine them to get an equivalent resistance of (i) (11/3) Ω (ii) (11/5) Ω (iii) 6Ω (iv) (6/11) Ω ?

(c) Determine the equivalent resistance of networks shown in figures



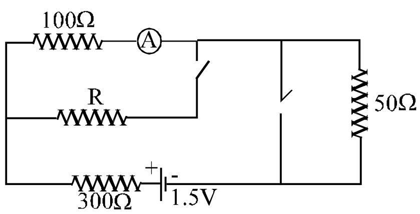
(a)



(b)

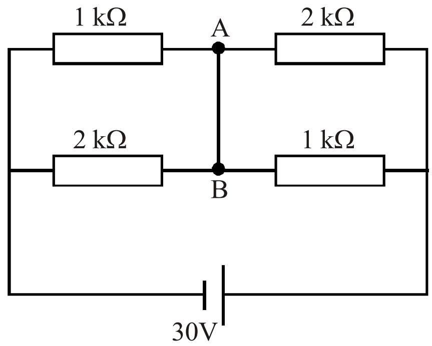
CE0005

6. In the circuit shown in figure the reading of ammeter is the same with both switches open as with both closed. Then find the resistance R. (ammeter is ideal)



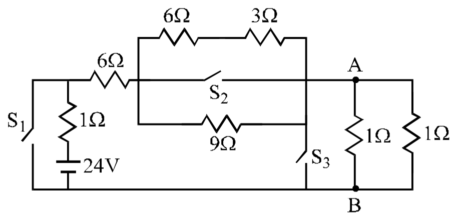
CE0006

7. Find the current (in mA) in the wire between points A and B.



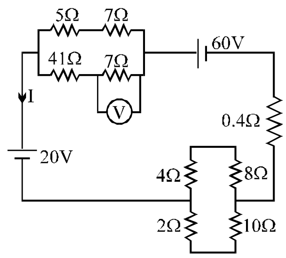
CE0007

8. If the switches S_1, S_2 and S_3 in the figure are arranged such that current through the battery is minimum, find the voltage across points A and B.



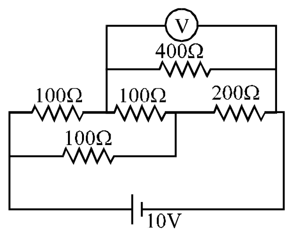
CE0008

9. Find the current I & voltage V in the circuit shown.



CE0009

10. An electrical circuit is shown in the figure. Calculate the potential difference across the resistance of 400 ohm, as will be measured by the voltmeter V of resistance 400 ohm, either by applying Kirchhoff's rules or otherwise.

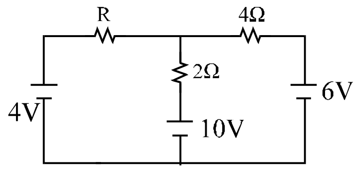


CE0010

11. An enquiring physics student connects a cell to a circuit and measures the current drawn from the cell to I_1. When he joins a second identical cell in series with the first, the current becomes I_2. When the cells are connected are in parallel, the current through the circuit is I_3. Show that relation between the current is $3 I_3 I_2 = 2 I_1 (I_2 + I_3)$

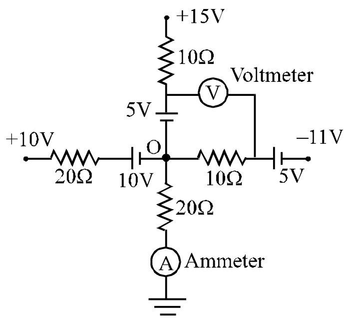
CE0012

12. For what value of R in circuit, current through 4Ω resistance is zero.



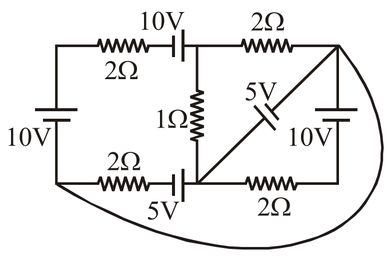
CE0013

13. The potential of certain points in the circuit are maintained at the values indicated. The Voltmeter and Ammeter are ideal. Find the potential of the cross junction point in the circuit (at center O) and the readings of Voltmeter and Ammeter. All cells are ideal.



CE0014

14. In the given circuit diagram, the current through the 1Ω resistor is given by I amp. Fill 2 I in OMR sheet.



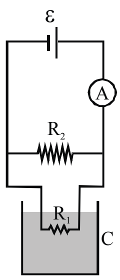
CE0015

Joule heating

15. If a cell of constant E.M.F. produces the same amount of the heat during the same time in two independent resistors R_1 and R_2, when they are separately connected across the terminals of the cell, one after the another, find the internal resistance of the cell.

CE0016

16. The coil of a calorimeter C has a resistance of $R_1 = 60\Omega$. The coil R_1 is connected to the circuit as shown in figure. What is the rise in temperature (°C) of 240 grams of water poured into the calorimeter when it is heated for 7 minutes during which a current flows through the coil and the ammeter shows 3A? The resistance $R_2 = 30\Omega$. [Disregard the resistances of the battery and the ammeter, and the heat losses and heat capacity of the calorimeter and the resistor and specific heat of water = 4200 J/kg°C]

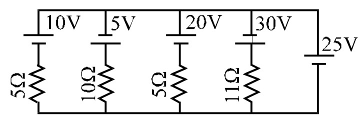


CE0017

17. An electric kettle has two windings. When one of them is switched on, the water in the kettle begins to boil in 15 minutes, and when the other is switched on it takes 30 minutes for water to boil. If the two windings are joined in series and switched on, water in the kettle begin to boil in $\frac{\alpha}{4}$ hr. Assuming no heat loss to the surrounding fill the value of α in OMR sheet.

CE0018

18. Find the current through 25V cell & power supplied by 20V cell in the figure shown.



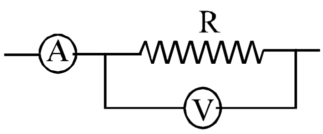
CE0019

19. A person decides to use his bath tub water to generate electric power to run a 40 watt bulb. The bath tub is located at a height of 10 m from the ground & it holds 200 litres of water. If we install a water driven wheel generator on the ground, at what rate should the water drain from the bath tub to light bulb? How long can we keep the bulb on, if the bath tub was full initially. The efficiency of generator is 90%.($g = 10 m/s^2$)

CE0020

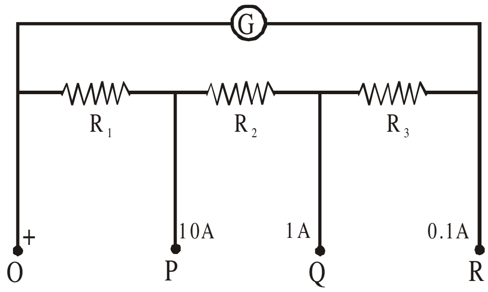
Instruments

20. A part of a circuit is shown in figure. Here reading of ammeter is 5 ampere and voltmeter is 96V & voltmeter resistance is 480 ohm. Then find the resistance R



CE0021

21. The resistance of the galvanometer G in the circuit is 25Ω. The meter deflects full scale for a current of 10 mA. The meter behaves as an ammeter of three different ranges. The range is 0–10A, if the terminals O and P are taken; range is 0–1 A between O and Q; range is 0–0.1 A between O and R. Calculate the resistance R_1, R_2 and R_3.

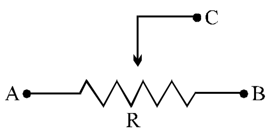


CE0022

22. A galvanometer having 50 divisions provided with a variable shunt is used to measure the current as an ammeter when connected in series with a resistance of 90 Ω and a battery of internal resistance 10Ω. It is observed that when the shunt resistance are 10Ω, 50Ω, respectively the deflection are respectively 9 & 30 divisions. What is the resistance of the galvanometer? Further if the full scale deflection of the galvanometer movement is 300 mA, find the emf of the cell.

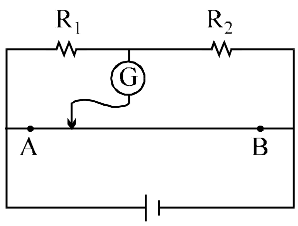
CE0023

23. How a battery is to be connected so that shown rheostat will behave like a potential divider? Also indicate the points about which output can be taken. **[IIT-JEE' 2003]**



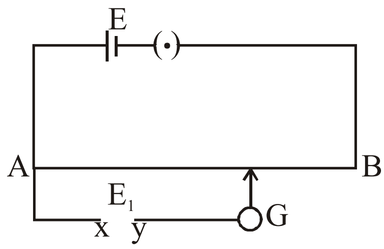
CE0025

24. In the figure shown for which values of R_1 and R_2 the balance point for Jockey is at 40 cm from A. When R_2 is shunted by a resistance of 10 Ω, balance shifts to 50 cm. Find R_1 and R_2. (AB = 1 m):



CE0027

25. While doing an experiment with potentiometer it was found that the deflection is one sided and two casses are possible



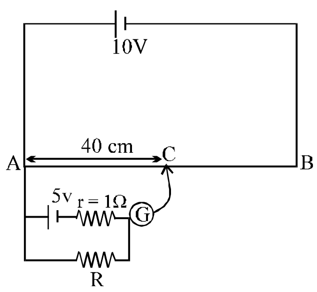
CE0028

(i) the deflection decreased while moving from one end A of the wire to the end B;
(ii) the deflection increased. while the jockey was moved towards the end B. Then
(a) Which terminal +or –ve of the cell E_1, is connected at X in case (i) and how is E_1 related to E?
(b) Which terminal of the cell E_1 is connected at X in case (ii)?

26. In a potentiometer arrangement, a cell of emf 1.25 V gives a balance point at 35.0 cm length of the wire. If the cell is replaced by another cell and the balance point shifts to 63.0 cm, what is the emf of the second cell ?

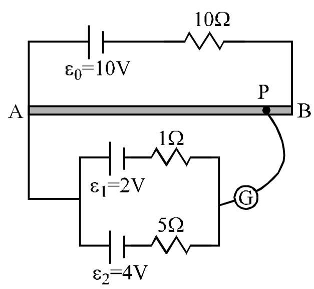
CE0029

27. A potentiometer wire AB is 100 cm long and has a total resistance of 10 ohm. If the galvanometer shows zero deflection at the position C, then find the value of unknown resistance R.



CE0030

28. A battery of emf ε_0 = 10 V is connected across a 1 m long uniform wire having resistance 10Ω/m. Two cells of emf ε_1 = 2V and ε_2 = 4V having internal resistances 1Ω and 5Ω respectively are connected as shown in the figure. If a galvanometer shows no deflection at the point P, find the distance of point P from the point A.



CE0031

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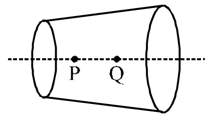
EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

Microscopic analysis

1. A wire has a non-uniform cross-section as shown in figure. A steady current flows through it. The drift speed of electrons at points P and Q is v_P and v_Q.



(A) $v_P = v_Q$ (B) $v_P < v_Q$ (C) $v_P > v_Q$ (D) Data insufficient

CE0042

2. Two wires each of radius of cross section r but of different materials are connected together end to end (in series). If the densities of charge carriers in the two wires are in the ratio 1 : 4, the drift velocity of electrons in the two wires will be in the ratio:

(A) 1 : 2 (B) 2 : 1 (C) 4 : 1 (D) 1 : 4

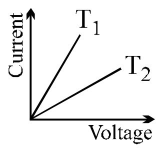
CE0043

3. An insulating pipe of cross-section area 'A' contains an electrolyte which has two types of ions→ their charges being –e and +2e. A potential difference applied between the ends of the pipe result in the drifting of the two types of ions, having drift speed = v (–ve ion) and v/4 (+ve ion). Both ions have the same number per unit volume = n. The current flowing through the pipe is

(A) nev A/2 (B) nev A/4 (C) 5nev A/2 (D) 3nev A/2

CE0044

4. The current in a metallic conductor is plotted against voltage at two different temperatures T_1 and T_2. Which is correct



(A) $T_1 > T_2$ (B) $T_1 < T_2$ (C) $T_1 = T_2$ (D) none

CE0045

5. A metal rod of length 10 cm and a rectangular cross-section of 1 cm $\times \frac{1}{2}$ cm is connected to a battery across opposite faces. The resistance will be

(A) maximum when the battery is connected across 1 cm $\times \frac{1}{2}$ cm faces.

(B) maximum when the battery is connected across 10 cm × 1 cm faces.

(C) maximum when the battery is connected across 10 cm $\times \frac{1}{2}$ cm faces.

(D) same irrespective of the three faces.

CE0046

6. Consider a current carrying wire (current *I*) in the shape of a circle. Note that as the current progresses along the wire, the direction of **j** (current density) changes in an exact manner, while the current *I* remain unaffected. The agent that is *essentially* responsible for is

(A) source of emf.

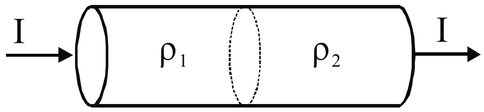
(B) electric field produced by charges accumulated on the surface of wire.

(C) the charges just behind a given segment of wire which push them just the right way by repulsion.

(D) the charges ahead.

CE0047

7. Two long straight cylindrical conductors with resistivities ρ_1 and ρ_2 respectively are joined together as shown in figure. If current I flows through the conductors, the magnitude of the total free charge at the interface of the two conductors is :-



(A) zero (B) $\frac{(\rho_1 - \rho_2) I \varepsilon_0}{2}$ (C) $\varepsilon_0 I |\rho_1 - \rho_2|$ (D) $\varepsilon_0 I |\rho_1 + \rho_2|$

CE0048

8. **Statement 1 :** The drift speed of electrons in metals is small (in the order of a few mm/s) and the charge of an electron is also very small (= 1.6×10^{-19}C), yet we can obtain a large current in a metal.

and

Statement 2: At room temperature, the thermal speed of electron is very high (about 10^7 times the drift speed).

(A) Statement–1 is True, Statement–2 is True ; Statement–2 is a correct explanation for Statement–1.

(B) Statement–1 is True, Statement–2 is True ; Statement–2 is not a correct explanation for Statement–1.

(C) Statement–1 is True, Statement–2 is False.

(D) Statement–1 is False, Statement–2 is True.

CE0049

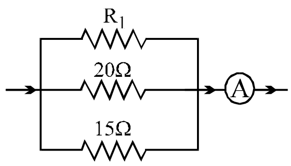
Ohm's law & circuit analysis

9. A storage battery is connected to a charger for charging with a voltage of 12.5Volts. The internal resistance of the storage battery is 1Ω. When the charging current is 0.5 A, the emf of the storage battery is:

(A) 13 Volts (B) 12.5 Volts (C) 12 Volts (D) 11.5 Volts

CE0050

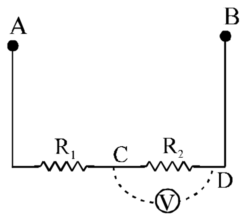
10. In the given circuit the current flowing through the resisitance 20 ohms is 0.3 ampere while the ammetre reads 0.8 ampere. What is the value of R_1?



(A) 30 ohms (B) 40 ohms (C) 50 ohms (D) 60 ohms

CE0051

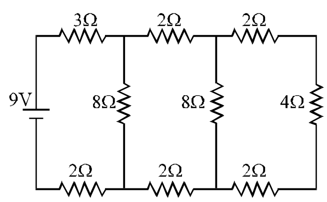
11. Resistances R_1 and R_2 each 60Ω are connected in series as shown in figure. The Potential difference between A and B is kept 120 volt. Then what will be the reading of voltmeter connected between the point C & D if resistance of voltmeter is 120Ω.



(A) 48 V (B) 24 V (C) 40V (D) None

CE0052

12. In the circuit shown in the figure, the current through :



(A) the 3Ω resistor is 0.50 A
(B) the 3Ω resistor is 0.25 A
(C) 4 Ω resistor is 0.50 A
(D) the 4Ω resistor is 0.25 A

CE0053

13. The equivalent resistance of a group of resistances is R. If another resistance is connected in parallel to the group , its new equivalent becomes R_1 & if it is connected in series to the group , its new equivalent becomes R_2 we have :

(A) $R_1 > R$ (B) $R_1 < R$ (C) $R_2 > R$ (D) $R_2 < R$

CE0055

14. An energy source will supply a constant current into the load, if its internal resistance is-

[AIEEE - 2005]

(A) equal to the resistance of the load
(B) very large as compared to the load resistance
(C) zero
(D) non-zero but less than the resistance of the load

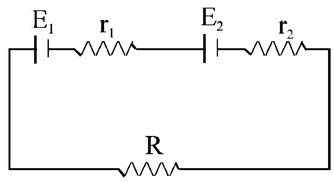
CE0056

15. When a current of 4 A flows within a battery from its positive to negative terminal, the potential difference across the battery is 12 volts. The potential difference across the battery is 9 volts when a current of 2 A flows within it from its negative to its positive terminal. The internal resistance and the e.m.f. of the battery are :-

(A) 0.1 Ω, 4V (B) 0.2 Ω, 5V (C) 0.5 Ω, 10V (D) 0.7 Ω, 10V

CE0057

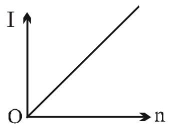
16. Under what condition current passing through the resistance R can be increased by short circuiting the battery of emf E_2. The internal resistances of the two batteries are r_1 and r_2 respectively.

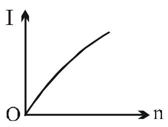


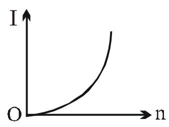
(A) $E_2r_1 > E_1 (R + r_2)$ (B) $E_1r_2 > E_2 (R + r_1)$ (C) $E_2r_2 > E_1 (R + r_2)$ (D) $E_1r_1 > E_2 (R + r_1)$

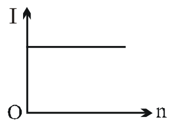
CE0059

17. A battery consists of a variable number n of identical cells having internal resistance connected in series. The terminals of the battery are short circuited and the current I measured. Which one of the graph below shows the relationship between I and n?

(A) 

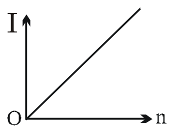
(B) 

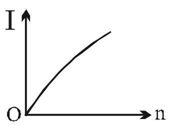
(C) 

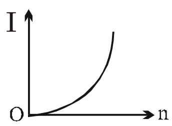
(D) 

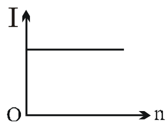
CE0060

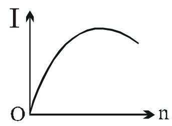
18. In **previous problem**, if the cell had been connected in parallel (instead of in series) which of the above graphs would have shown the relationship between total current I and n?

(A) 

(B) 

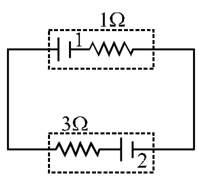
(C) 

(D) 

(E) 

CE0061

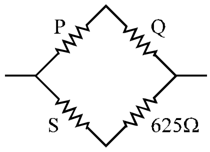
19. In the figure shown, battery 1 has emf = 6 V and internal resistance = 1 Ω. Battery 2 has emf = 2V and internal resistance = 3 Ω. The wires have negligible resistance. What is the potential difference across the terminals of battery 2 ?



(A) 4 V (B) 1.5 V (C) 5 V (D) 0.5 V

CE0062

20. A Wheatstone's bridge is balanced with a resistance of 625 Ω in the third arm, where P, Q and S are in the 1st, 2nd and 4th arm respectively. If P and Q are interchanged, the resistance in the third arm has to be increased by 51Ω to secure balance. The unknown resistance in the fourth arm is :-



(A) 625 Ω (B) 650 Ω (C) 676 Ω (D) 600 Ω

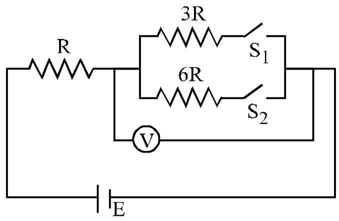
CE0065

21. One end of a Nichrome wire of length 2L and cross-sectional area A is attatched to an end of another Nichrome wire of length L and cross-sectional area 2A. If the free end of the longer wire is at an electric potential of 8.0 volts, and the free end of the shorter wire is at an electric potential of 1.0 volt, the potential at the junction of the two wires is equal to :-

(A) 2.4 V (B) 3.2 V (C) 4.5 V (D) 5.6 V

CE0066

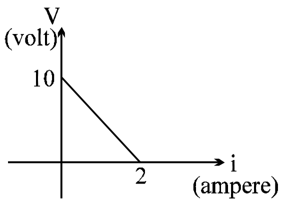
22. In the circuit shown in figure reading of voltmeter is V_1 when only S_1 is closed, reading of voltmeter is V_2 when only S_2 is closed. The reading of voltmeter is V_3 when both S_1 and S_2 are closed then



(A) $V_2 > V_1 > V_3$ (B) $V_3 > V_2 > V_1$ (C) $V_3 > V_1 > V_2$ (D) $V_1 > V_2 > V_3$

CE0067

23. A battery of emf E and internal resistance r is connected across a resistance R. Resistance R can be adjusted to any value greater than or equal to zero. A graph is plotted between the current (i) passing through the resistance and potential difference (V) across it. Select the correct alternative.



(A) internal resistance of battery is 5Ω

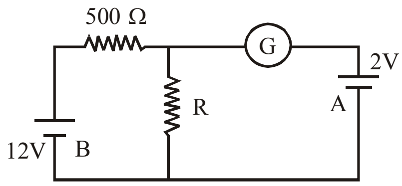
(B) emf of the battery is 20V

(C) maximum current which can be taken from the battery is 4A

(D) V- i graph can never be a straight line as shown in figure.

CE0068

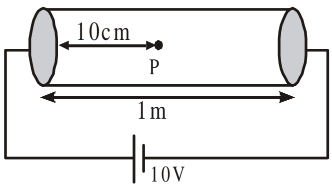
24. In the circuit, the galvanometer G shows zero deflection. If the batteries A and B have negligible internal resistance, the value of the resistor R will be- **[AIEEE - 2005]**



(A) 200 Ω (B) 100 Ω (C) 500 Ω (D) 1000 Ω

CE0069

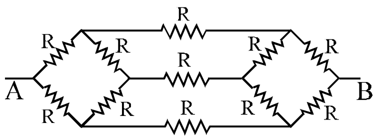
25. A cylindrical solid of length 1m and radius 1 m is connected across a source of emf 10V and negligible internal resistance shown in figure. The resistivity of the rod as a function of x (x measured from left end) is given by $\rho = bx$ [where b is a positive constant]. Find the electric field (in SI unit) at point P at a distance 10 cm from left end.



(A) 1 (B) 2 (C) 3 (D) 4

CE0071

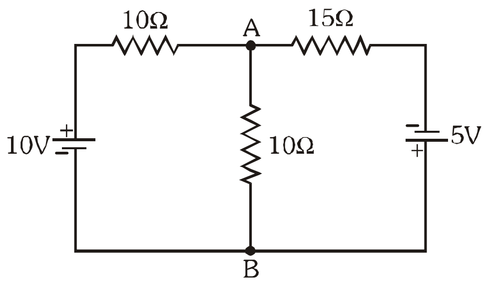
26. The equivalent resistance between the terminal points A and B in the network shown in figure is :-



(A) $\frac{7R}{5}$ (B) $\frac{5R}{6}$ (C) $\frac{7R}{12}$ (D) $\frac{5R}{12}$

CE0073

27. A circuit is arranged as shown. Then, the current from A to B is



(A) +500 mA (B) +250 mA (C) –250 mA (D) –500 mA

CE0075

node06\B0B0-BA\Kota\JEE(Advanced)\Leader\Physics\Sheet\Electrostatics & Electrodynamics\Eng\2. Current electricity.p65

Joule heating

28. Power generated across a uniform wire connected across a supply is H. If the wire is cut into n equal parts and all the parts are connected in parallel across the same supply, the total power generated in the wire is

(A) $\frac{H}{n^2}$ (B) n^2H (C) nH (D) $\frac{H}{n}$

CE0076

29. When electric bulbs of same power, but different marked voltage are connected in series across the power line, their brightness will be :

(A) proportional to their marked voltage
(B) inversely proportional to their marked voltage
(C) proportional to the square of their marked voltage
(D) inversely proportional to the square of their marked voltage

CE0077

30. Rate of dissipation of Joule's heat in resistance per unit volume is (symbols have usual meaning)

(A) σ E (B) σ J (C) J E (D) None

CE0079

31. If the length of the filament of a heater is reduced by 10%, the power of the heater will

(A) increase by about 9% (B) increase by about 11%
(C) increase by about 19% (D) decrease by about 10%

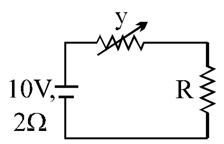
CE0080

32. Two bulbs one of 200 volts, 60 watts & the other of 200 volts, 100 watts are connected in series to a 200 volt supply. The power consumed will be

(A) 37.5 watt (B) 160 watt (C) 62.5 watt (D) 110 watt

CE0081

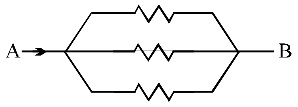
33. In the figure shown the power generated in y is maximum when y = 5Ω. Then R is :-



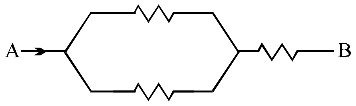
(A) 2 Ω (B) 6 Ω (C) 5 Ω (D) 3 Ω

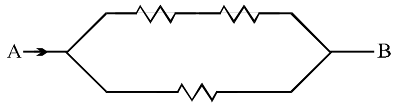
CE0082

34. Arrange the order of power dissipated in the given circuits, if the same current is passing through all circuits and each resistor is 'r' **[IIT-JEE' 2003 (Scr)]**

(1) 

(2) 

(3) 

(4) 

(A) $P_2 > P_3 > P_4 > P_1$ (B) $P_3 > P_2 > P_4 > P_1$ (C) $P_4 > P_3 > P_2 > P_1$ (D) $P_1 > P_2 > P_3 > P_4$

CE0083

35. A variable load R is connected to a voltage source of internal resistance r. Then choose the **INCORRECT** statement out of the following :-

(A) If R = r, maximum power is transferred to the load

(B) If current is maximum, power transfer to load is also maximum

(C) If R << r, the voltage source supplied a fixed current to the load

(D) Power supplied to load is minimum if load is either too low or too high

CE0086

Instruments

36. A galvanometer has a resistance of 20Ω and reads full-scale when 0.2 V is applied across it. To convert it into a 10 A ammeter, the galvanometer coil should have a

(A) 0.01Ω resistor connected across it (B) 0.02Ω resistor connected across it

(C) 200Ω resistor connected in series with it (D) 2000 Ω resistor connected in series with it

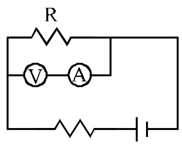
CE0088

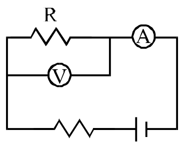
37. A galvanometer coil has a resistance 90 Ω and full scale deflection current 10 mA. A 910Ω resistance is connected in series with the galvanometer to make a voltmeter. If the least count of the voltmeter is 0.1V, the number of divisions on its scale is

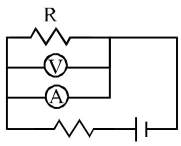
(A) 90 (B) 91 (C) 100 (D) none

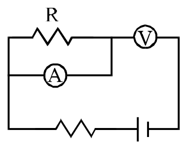
CE0089

38. Which of the following wiring diagrams could be used to experimentally determine R using ohm's law? Assume an ideal voltmeter and an ideal ammeter.

(A)


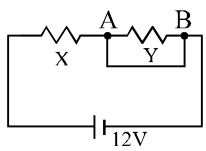
(B)


(C)


(D)


CE0090

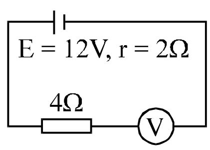
39. When an ammeter of negligible internal resistance is inserted in series with circuit it reads 1A. When the voltmeter of very large resistance is connected across X it reads 1V. When the point A and B are shorted by a conducting wire, the voltmeter measures 10 V across the battery. The internal resistance of the battery is equal to :-



(A) zero (B) 0.5 Ω (C) 0.2 Ω (D) 0.1 Ω

CE0091

40. By error, a student places moving-coil voltmeter V (nearly ideal) in series with the resistance in a circuit in order to read the current, as shown. The voltmeter reading will be



(A) 0 (B) 4V (C) 6V (D) 12V

CE0092

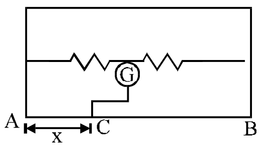
41. In a balanced wheat stone bridge, current in the galvanometer is zero. It remains zero when:
[1] battery emf is increased
[2] all resistances are increased by 10 ohms
[3] all resistances are made five times
[4] the battery and the galvanometer are interchanged
(A) only [1] is correct (B) [1], [2] and [3] are correct
(C) [1], [3] and [4] are correct (D) [1] and [3] are correct

CE0094

42. In a metre bridge experiment, null point is obtained at 20 cm from one end of the wire when resistance X is balanced against another resistance Y. If X < Y, then where will be the new position of the null point from the same end, if one decides to balance a resistance of 4X against Y? **[AIEEE - 2004]**
(A) 50 cm (B) 80 cm (C) 40 cm (D) 70 cm

CE0096

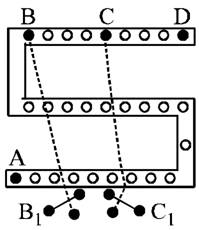
43. In the given circuit, no current is passing through the galvanometer. If the cross-sectional diameter of AB is doubled then for null point of galvanometer the value of AC would **[IIT-JEE' 2003 (Scr)]**



(A) x (B) x/2 (C) 2x (D) None

CE0098

44. For the post office box arrangement to determine the value of unknown resistance, the unknown resistance should be connected between **[IIT-JEE' 2004 (Scr)]**



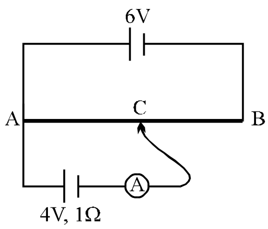
(A) *B* and *C* (B) *C* and *D* (C) *A* and *D* (D) B_1 and C_1

CE0099

45. A potentiometer wire has length 10 m and resistance 10Ω. It is connected to a battery of EMF 11 volt and internal resistance 1 Ω, then the potential gradient in the wire is :-
(A) 10 V/m (B) 1 V/m (C) 0.1 V/m (D) none

CE0100

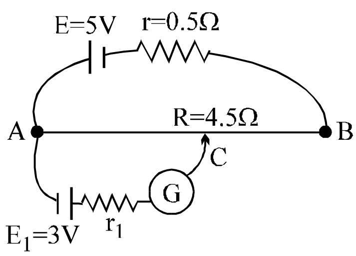
46. A 6 V battery of negligible internal resistance is connected across a uniform wire of length 1 m. The positive terminal of another battery of emf 4V and internal resistance 1 Ω is joined to the point A as shown in figure. The ammeter shows zero deflection when the jockey touches the wire at the point C. The AC is equal to :-



(A) 2/3 m (B) 1/3 m (C) 3/5 m (D) 1/2 m

CE0101

47. In the given potentiometer circuit length of the wire AB is 3 m and resistance is R = 4.5 Ω. The length AC for no deflection in galvanometer is :-



(A) 2 m (B) 1.8 m (C) dependent on r_1 (D) none of these

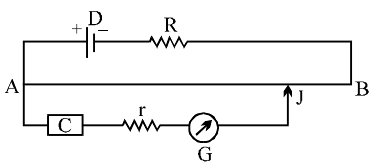
CE0102

48. The length of a potentiometer wire is ℓ. A cell of emf E is balanced at a length $\ell/3$ from the positive end of the wire. If the length of the wire is increased by $\ell/2$. At what distance will the same cell give a balance point.

(A) $\frac{2\ell}{3}$ (B) $\frac{\ell}{2}$ (C) $\frac{\ell}{6}$ (D) $\frac{4\ell}{3}$

CE0103

49. In the given potentiometer circuit, the resistance of the potentiometer wire AB is R_0. C is a cell of internal resistance r. The galvanometer G does not give zero deflection for any position of the jockey J. Which of the following cannot be a reason for this ?



(A) $r > R_0$ (B) $R >> R_0$
(C) emf of C > emf of D (D) The negative terminal of C is connected to A.

CE0105

CAPACITANCE

KEY CONCEPTS

CONCEPT OF CAPACITANCE

When a conductor is charged then its potential rises. The increase in potential is directly proportional to the charge given to the conductor. $Q \propto V \Rightarrow Q = CV$

The constant C is known as the capacity of the conductor.

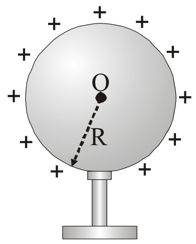
Capacitance is a scalar quantity with dimension $C = \frac{Q}{V} = \frac{Q^2}{W} = \frac{A^2T^2}{M^1L^2T^{-2}} = M^{-1} L^{-2} T^4 A^2$

Unit :- farad, coulomb/volt

The capacity of a conductor is independent of the charge given or its potential raised. It is also independent of nature of material and thickness of the conductor. Theoretically infinite amount of charge can be given to a conductor. But practically the electric field becomes so large that it causes ionisation of medium surrounding it. The charge on conductor leaks reducing its potential.

THE CAPACITANCE OF A SPHERICAL CONDUCTOR

When a charge Q is given to a isolated spherical conductor then its potential rises.



$$V = \frac{1}{4\pi\varepsilon_0}\frac{Q}{R} \Rightarrow C = \frac{Q}{V} = 4\pi\varepsilon_0 R$$

If conductor is placed in a medium then

$$C_{medium} = 4\pi\varepsilon R = 4\pi\varepsilon_0\varepsilon_r R$$

Capacitance depends upon :

(a) Size and Shape of Conductor

(b) Surrounding medium

(c) Presence of other conductors nearby

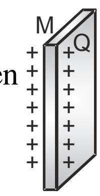
CONDENSER/CAPACITOR

The pair of conductor of opposite charges on which sufficient quantity of charge may be accommodated is defined as condenser.

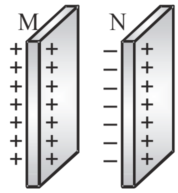
- **Principle of a Condenser**

It is based on the fact that capacitance can be increased by reducing potential keeping the charge constant.

Consider a conducting plate M which is given a charge Q such that its potential rises to V then

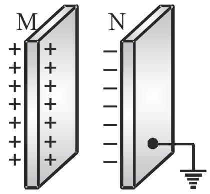


$$C = \frac{Q}{V}$$



Let us place another identical conducting plate N parallel to it such that charge is induced on plate N (as shown in figure). If V_- is the potential at M due to induced negative charge on N and V_+ is the potential at M due to induced positive charge on N, then

$$C' = \frac{Q}{V'} = \frac{Q}{V + V_+ - V_-}$$



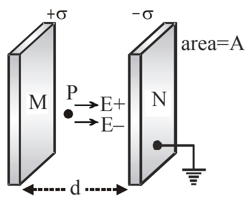
Since V' < V (as the induced negative charge lies closer to the plate M in comparison to induced positive charge). ⇒ C' > C Further, if N is earthed from the outer side (see figure) then $V'' = V_+ - V_-$ (∵ the entire positive charge flows to the earth)

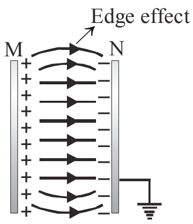
$$C'' = \frac{Q}{V''} = \frac{Q}{V - V_-} \Rightarrow C'' >> C$$

If an identical earthed conductor is placed in the vicinity of a charged conductor then the capacitance of the charged conductor increases appreciable. This is the principle of a parallel plate capacitor.

- **Parallel Plate Capacitor**

(i) Capacitance





It consists of two metallic plates M and N each of area A at separation d. Plate M is positively charged and plate N is earthed. If ε_r is the dielectric constant of the material medium and E is the field at a point P that exists between the two plates, then

I step : Finding electric field $E = E_+ + E_- = \frac{\sigma}{2\varepsilon} + \frac{\sigma}{2\varepsilon} = \frac{\sigma}{\varepsilon} = \frac{\sigma}{\varepsilon_0\varepsilon_r}$ [$\varepsilon = \varepsilon_0\,\varepsilon_r$]

II step : Finding potential difference $V = Ed = \frac{\sigma}{\varepsilon_0\varepsilon_r} d = \frac{qd}{A\varepsilon_0\varepsilon_r}$ $\left(\because E = \frac{V}{d} \text{ and } \sigma = \frac{q}{A}\right)$

III step : Finding capacitance $C = \frac{q}{V} = \frac{\varepsilon_r\varepsilon_0 A}{d}$

(ii) Force between the plates

The two plates of capacitor attract each other because they are oppositely charged.

Electric field due to positive plate $E = \frac{\sigma}{2\varepsilon_0} = \frac{Q}{2\varepsilon_0 A}$

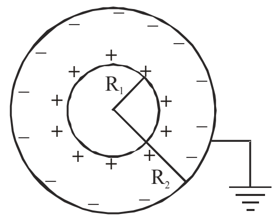
Force on negative charge –Q is $F = -QE = -\frac{Q^2}{2\varepsilon_0 A}$

Magnitude of force $F = \frac{Q^2}{2\varepsilon_0 A} = \frac{1}{2}\varepsilon_0 A E^2$

Force per unit area or energy density or electrostatic pressure $= \frac{F}{A} = u = p = \frac{1}{2}\varepsilon_0 E^2$

- **Spherical Capacitor**

Outer sphere is earthed



When a charge Q is given to inner sphere it is uniformly distributed on its surface A charge –Q is induced on inner surface of outer sphere. The charge +Q induced on outer surface of outer sphere flows to earth as it is grounded.

$E = 0$ for $r < R_1$ and $E = 0$ for $r > R_2$

Potential of inner sphere $V_1 = \frac{Q}{4\pi\varepsilon_0 R_1} + \frac{-Q}{4\pi\varepsilon_0 R_2} \Rightarrow \frac{Q}{4\pi\varepsilon_0}\left(\frac{R_2 - R_1}{R_1 R_2}\right)$

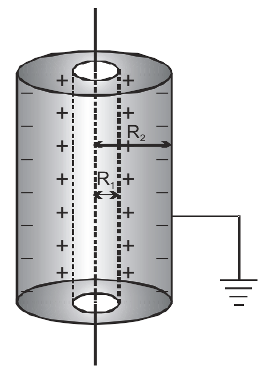
As outer surface is earthed so potential $V_2 = 0$

Potential difference between plates $V = V_1 - V_2 = \frac{Q}{4\pi\varepsilon_0}\frac{(R_2 - R_1)}{R_1 R_2}$

So $C = \frac{Q}{V} = 4\pi\varepsilon_0 \frac{R_1 R_2}{R_2 - R_1}$ (in air or vacuum)

- **Cylindrical Capacitor**

When a charge Q is given to inner cylinder it is uniformly distributed on its surface. A charge –Q is induced on inner surface of outer cylinder. The charge +Q induced on outer surface of outer cylinder flows to earth as it is grounded



Electrical field between cylinders $E = \frac{\lambda}{2\pi\varepsilon_0 r} = \frac{Q/\ell}{2\pi\varepsilon_0 r}$

Potential difference between plates $V = \int_{R_1}^{R_2} \frac{Q}{2\pi\varepsilon_0 r\ell} dr = \frac{Q}{2\pi\varepsilon_0 \ell} \ell n\left(\frac{R_2}{R_1}\right)$

Capacitance $C = \frac{Q}{V} = \frac{2\pi\varepsilon_0 \ell}{\log_e(R_2/R_1)}$

Ex. The stratosphere acts as a conducting layer for the earth. If the stratosphere extends beyond 50 km from the surface of earth, then calculate the capacitance of the spherical capacitor formed between stratosphere and earth's surface. Take radius of earth as 6400 km.

Sol. The capacitance of a spherical capacitor is $C = 4\pi\varepsilon_0\left(\frac{ab}{b-a}\right)$

b = radius of the top of stratosphere layer = 6400 km + 50 km = 6450 km = 6.45×10^6 m

a = radius of earth = 6400 km = 6.4×10^6 m

$\therefore \quad C = \frac{1}{9\times10^9} \times \frac{6.45\times10^6 \times 6.4\times10^6}{6.45\times10^6 - 6.4\times10^6} = 0.092$ F

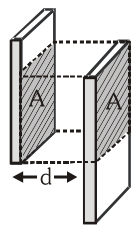
Ex. A cylindrical capacitor has two co-axial cylinders of length 15 cm and radii 1.5 cm and 1.4 cm. The outer cylinder is earthed and the inner cylinder is given a charge of 3.5 μC. Determine the capacitance of the system and the potential of the inner cylinder.

Sol. ℓ = 15 cm = 15 × 10^{-2} m; a = 1.4cm = 1.4 × 10^{-2} m ;
b =1.5 cm = 1.5 × 10^{-2}m; q = 3.5 μC = 3.5 × 10^{-6}C

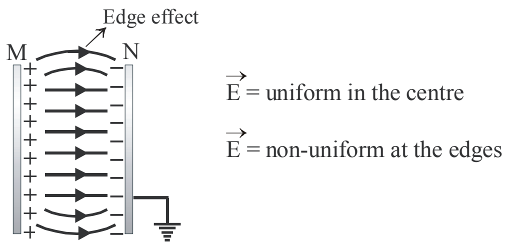
Capacitance $C = \dfrac{2\pi\varepsilon_0\ell}{2.303\log_{10}\left(\dfrac{b}{a}\right)} = \dfrac{2\pi\times 8.854\times10^{-12}\times15\times10^{-2}}{2.303\log_{10}\dfrac{1.5\times10^{-2}}{1.4\times10^{-2}}}$ = 1.21 × 10^{-8} F

Since the outer cylinder is earthed, the potential of the inner cylinder will be equal to the potential difference between them. Potential of inner cylinder, is $V = \dfrac{q}{C} = \dfrac{3.5\times10^{-6}}{1.2\times10^{-10}} = 2.89\times10^4\,V$

- If one of the plates of parallel plate capacitor slides relatively than C decrease (As overlapping area decreases).

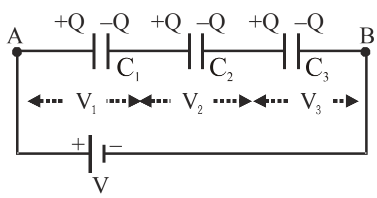


- If both the plates of parallel plate capacitor are touched each other resultant charge and potential became zero.
- Electric field between the plates of a capacitor is shown in figure. Non-uniformity of electric field at the boundaries of the plates is negligible if the distance between the plates is very small as compared to the length of the plates.



COMBINATION OF CAPACITOR

- **Capacitor in series**:
  In this arrangement of capacitors the charge has no alternative path(s) to flow.



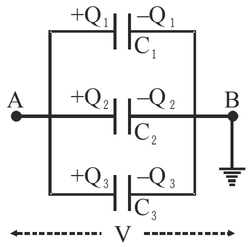
(i) The charges on each capacitor are equal
i.e. $Q = C_1V_1 = C_2V_2 = C_3V_3$

(ii) The total potential difference across AB is shared by the capacitors in the inverse ratio of the capacitances $V = V_1 + V_2 + V_3$
If C_S is the net capacitance of the series combination, then

$$\frac{Q}{C_S} = \frac{Q}{C_1} + \frac{Q}{C_2} + \frac{Q}{C_3} \Rightarrow \frac{1}{C_S} = \frac{1}{C_1} + \frac{1}{C_2} + \frac{1}{C_3}$$

- **Capacitors in parallel**

In such arrangement of capacitors the charge has an alternative path(s) to flow.



(i) The potential difference across each capacitor is same and equal the total potential applied. i.e. $V = V_1 = V_2 = V_3 \Rightarrow V = \frac{Q_1}{C_1} = \frac{Q_2}{C_2} = \frac{Q_3}{C_3}$

(ii) The total charge Q is shared by each capacitor in the direct ratio of the capacitances. $Q = Q_1 + Q_2 + Q_3$

If C_P is the net capacitance for the parallel combination of capacitors :

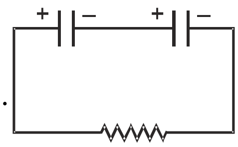
$$C_PV = C_1V + C_2V + C_3V \quad \Rightarrow C_P = C_1 + C_2 + C_3$$

- For a given voltage to store maximum energy capacitors should be connected in parallel.
- If N identical capacitors each having breakdown voltage V are joined in

(i) series then the break down voltage of the combination is equal to NV

(ii) parallel then the breakdown voltage of the combination is equal to V.

- Two capacitors are connected in series with a battery. Now battery is removed and loose wires connected together then final charge on each capacitor is zero.



- If N identical capacitors are connected then $C_{series} = \frac{C}{N}$, $C_{parallel} = NC$
- In DC capacitor's offers infinite resistance in steady state, so there will be no current flows through capacitor branch.

Ex. Capacitor C, 2C, 4C, ... ∞ are connected in parallel, then what will be their effective capacitance ?

Sol. Let the resultant capacitance be $C_{resultant} = C + 2C + 4C + ... \infty = C[1 + 2 + 4 + ... \infty] = C \times \infty = \infty$

Ex. An infinite number of capacitors of capacitance C, 4C, 16C ... ∞ are connected in series then what will be their resultant capacitance ?

Sol. Let the equivalent capacitance of the combination = C_{eq}

$$\frac{1}{C_{eq}} = \frac{1}{C} + \frac{1}{4C} + \frac{1}{16C} + ...\infty = \left[1 + \frac{1}{4} + \frac{1}{16} + ...\infty\right]\frac{1}{C} \text{ (this is G. P.series)}$$

$$\Rightarrow S_\infty = \frac{a}{1-r} \text{ first term a = 1, common ratio r = } \frac{1}{4}$$

$$\Rightarrow \frac{1}{C_{eq}} = \frac{1}{1-\frac{1}{4}} \times \frac{1}{C} \Rightarrow C_{eq} = \frac{3}{4}C$$

ENERGY STORED IN A CHARGED CONDUCTOR/CAPACITOR

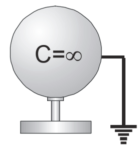
Let C is capacitance of a conductor. On being connected to a battery. It charges to a potential V from zero potential. If q is charge on the conductor at that time then q = CV. Let battery supplies small amount of charge dq to the conductor at constant potential V. Then small amount of work done by the battery against the force exerted by exsiting charge is

$$dW = Vdq = \frac{q}{C}dq \Rightarrow W = \int_0^Q \frac{q}{C}dq = \frac{1}{C}\left[\frac{q^2}{2}\right]_0^Q \Rightarrow W = \frac{Q^2}{2C}$$

where Q is the final charge acquired by the conductor. This work done is stored as potential energy, so

$$U = \frac{Q^2}{2C} = \frac{1}{2}\frac{(CV)^2}{C} = \frac{1}{2}CV^2 = \frac{1}{2}\left(\frac{Q}{V}\right)V^2 = \frac{1}{2}QV \quad \therefore \quad U = \frac{Q^2}{2C} = \frac{1}{2}CV^2 = \frac{1}{2}QV$$

- As the potential of the Earth is assumed to be zero, capacity of earth or a conductor connceted to earth will be infinite $C = \frac{q}{V} = \frac{q}{0} = \infty$



- Actual capacity of the Earth $C = 4\pi\varepsilon_0 R = \frac{1}{9\times10^9}\times 64\times10^5 = 711\,\mu F$
- Work done by battery W_b = (charge given by battery) × (emf) = QV but

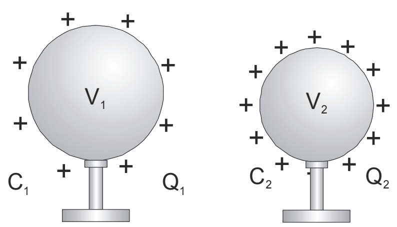
  Energy stored in conductor $= \frac{1}{2}QV$

  so 50% energy supplied by the battery is lost in form of heat.

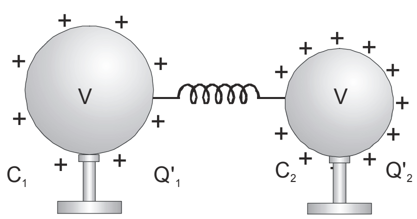
REDISTRIBUTION OF CHARGES AND LOSS OF ENERGY

When two charged conductors are connected by a conducting wire then charge flows from a conductor at higher potential to that at lower potential. This flow of charge stops when the potential of two conductors became equal.

Let the amounts of charges after the conductors are connected are Q_1' and Q_2' respectively and potential is V then



(Before connection)



(After connection)

- **Common potential**

  According to law of Conservation of charge $Q_{\text{before connection}} = Q_{\text{after connection}}$

  $\Rightarrow C_1V_1 + C_2V_2 = C_1V + C_2V$

  Common potential after connection $$\boxed{V = \frac{C_1V_1 + C_2V_2}{C_1 + C_2}}$$

- **Charges after connection**

$$Q_1' = C_1V = C_1\left(\frac{Q_1+Q_2}{C_1+C_2}\right) = \left(\frac{C_1}{C_1+C_2}\right)Q \qquad \text{(Q : Total charge on system)}$$

$$Q_2' = C_2V = C_2\left(\frac{Q_1+Q_2}{C_1+C_2}\right) = \left(\frac{C_2}{C_1+C_2}\right)Q$$

Ratio of the charges after redistribution $\boxed{\frac{Q_1'}{Q_2'} = \frac{C_1V}{C_2V} = \frac{R_1}{R_2} \text{ (in case of spherical conductors)}}$

- **Loss of energy in redistribution**

When charge flows through the conducting wire then **energy is lost mainly on account of Joule effect**, electrical energy is converted into heat energy, so change in energy of this system,

$$\Delta U = U_f - U_i \quad \Rightarrow \left(\frac{1}{2}C_1V^2 + \frac{1}{2}C_2V^2\right) - \left(\frac{1}{2}C_1V_1^2 + \frac{1}{2}C_2V_2^2\right) \Rightarrow \Delta U = -\frac{1}{2}\left(\frac{C_1C_2}{C_1+C_2}\right)(V_1 - V_2)^2$$

Here negative sign indicates that energy of the system decreases in the process.**SOLVED EXAMPLES**

Ex. The plates of a capacitor are charged to a potential difference of 100 V and then connected across a resister. The potential difference across the capacitor decays exponentially with respect to time. After one second the potential difference between the plates of the capacitor is 80 V. What is the fraction of the stored energy which has been dissipated ?

Sol. Energy losses $\Delta U = \frac{1}{2}CV_0^2 - \frac{1}{2}CV^2$

Fractional energy loss $\frac{\Delta U}{U_0} = \frac{\frac{1}{2}CV_0^2 - \frac{1}{2}CV^2}{\frac{1}{2}CV_0^2} = \frac{V_0^2 - V^2}{V_0^2} = \frac{(100)^2 - (80)^2}{(100)^2} = \frac{20 \times 180}{(100)^2} = \frac{9}{25}$

Ex. Two uniformly charged spherical drops at potential V coalesce to form a larger drop. If capacity of each smaller drop is C then find capacity and potential of larger drop.

Sol. When drops coalesce to form a larger drop then total charge and volume remains conserved. If r is radius and q is charge on smaller drop then $C = 4\pi\varepsilon_0 r$ and $q = CV$

Equating volume we get $\frac{4}{3}\pi R^3 = 2 \times \frac{4}{3}\pi r^3 \Rightarrow R = 2^{1/3} r$

Capacitance of larger drop $C' = 4\pi\varepsilon_0 R = 2^{1/3}C$

Charge on larger drop $Q = 2q = 2CV$

Potential of larger drop $V' = \frac{Q}{C'} = \frac{2CV}{2^{1/3}C} = 2^{2/3}V$

EFFECT OF DIELECTRIC

- The insulators in which microscopic local displacement of charges takes place in presence of electric field are known as **dielectrics**.
- Dielectrics are non conductors upto certain value of field depending on its nature. If the field exceeds this limiting value called **dielectric strength** they lose their insulating property and begin to conduct.

- **Dielectric strength** is defined as the maximum value of electric field that a dielectric can tolerate without breakdown. Unit is volt/metre. Dimensions $M^1L^1T^{-3}A^{-1}$

Polar dielectrics

- In absence of external field the centres of positive and negative charge do not coincide-due to asymmetric shape of molecules.
- Each molecule has permanent dipole moment.
- The dipole are randomly oriented so average dipole moment per unit volume of polar dielectric in absence of external field is nearly zero.
- In presence of external field dipoles tends to align in direction of field.

  Ex. Water, Alcohol, CO_2, $HC\ell$, NH_3

Non polar dielectrics

- In absence of external field the centre of positive and negative charge coincides in these atoms or molecules because they are symmetric.
- The dipole moment is zero in normal state.
- In presence of external field they acquire induced dipole moment.

  Ex. Nitrogen, Oxygen, Benzene, Methane

Polarisation :

The alignment of dipole moments of permanent or induced dipoles in the direction applied electric field is called polarisation.

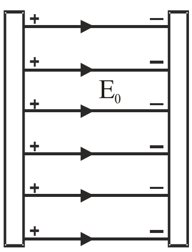
- **Polarisation vector $\vec{P}$**
  This is a vector quantity which describes the extent to which molecules of dielectric become polarized by an electric field or oriented in direction of field.

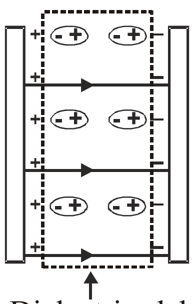
  $\vec{P}$ = the dipole moment per unit volume of dielectric = $n\vec{p}$

  where n is number of atoms per unit volume of dielectric and $\vec{p}$ is dipole moment of an atom or molecule.

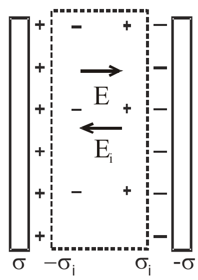
  $$\vec{P} = n\vec{p} = \frac{q_i d}{Ad} = \left(\frac{q_i}{A}\right) = \sigma_i = \text{induced surface charge density.}$$

  Unit of $\vec{P}$ is C/m^2 Dimension is $L^{-2}T^1A^1$





Dielectric slab



Let E_0, V_0, C_0 be electric field, potential difference and capacitance in absence of dielectric. Let E, V, C are electric field, potential difference and capacitance in presence of dielectric respectively.

Electric field in absence of dielectric $E_0 = \frac{V_0}{d} = \frac{\sigma}{\varepsilon_0} = \frac{Q}{\varepsilon_0 A}$

Electric field in presence of dielectric $E = E_0 - E_i = \dfrac{\sigma - \sigma_i}{\varepsilon_0} = \dfrac{Q - Q_i}{\varepsilon_0} = \dfrac{V}{d}$

Capacitance in absence of dielectric $C_0 = \dfrac{Q}{V_0}$

Capacitance in presence of dielectric $C = \dfrac{Q - Q_i}{V}$

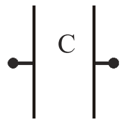
The dielectric constant or relative permittivity K

or $\varepsilon_r = \dfrac{E_0}{E} = \dfrac{V_0}{V} = \dfrac{C}{C_0} = \dfrac{Q}{Q - Q_i} = \dfrac{\sigma}{\sigma - \sigma_i} = \dfrac{\varepsilon}{\varepsilon_0}$

From $K = \dfrac{Q}{Q - Q_i} \Rightarrow Q_i = Q\left(1 - \dfrac{1}{K}\right)$ and $K = \dfrac{\sigma}{\sigma - \sigma_i} \Rightarrow \sigma_i = \sigma\left(1 - \dfrac{1}{K}\right)$

CAPACITY OF DIFFERENT CONFIGURATION

In case of parallel plate capacitor $C = \dfrac{\varepsilon_0 A}{d}$

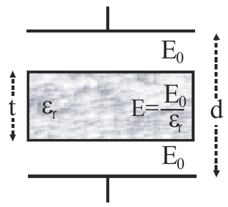


If capacitor is partially filled with dielectric

When the dielectric is filed partially between plates, the thickness of dielatric slab is t(t < d).

If no slab is introduced between the plates of the capacitor, then a field E_0 given by $E_0 = \dfrac{\sigma}{\varepsilon_0}$, exists in a space d.

On inserting the slab of thickness t, a field $E = \dfrac{E_0}{\varepsilon_r}$ exists inside the slab of thickness t and a field E_0 exists in remaining space (d – t). If V is total potential then $V = E_0(d - t) + E\,t$

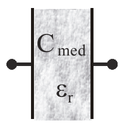


$$\Rightarrow V = E_0\left[d - t + \left(\frac{E}{E_0}\right)t\right] \quad \because \frac{E_0}{E} = \varepsilon_r = \text{Dielectric constant}$$

$$\Rightarrow V = \frac{\sigma}{\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] = \frac{q}{A\varepsilon_0}\left[d - t + \frac{t}{\varepsilon_r}\right] \Rightarrow C = \frac{q}{V} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} = \frac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\varepsilon_r}\right)} \quad ...(i)$$

If medium is fully present between the space.

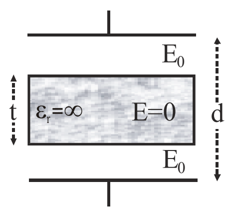
$\because \quad t = d$



Now from equation (i) $C_{medium} = \dfrac{\varepsilon_0 \varepsilon_r A}{d}$

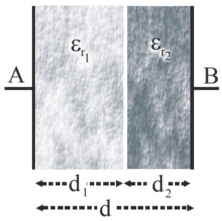
If capacitor is partialy filled by a conducting slab of thickness (t< d).

$\because \quad \varepsilon_r = \infty$ for conductor $C = \dfrac{\varepsilon_0 A}{d - t\left(1 - \frac{1}{\infty}\right)} = \dfrac{\varepsilon_0 A}{(d - t)}$



DISTANCE AND AREA DIVISION BY DIELECTRIC

- **Distance Division**



(i) Distance is divided and area remains same.

(ii) Capacitors are in series.

(iii) Individual capacitances are $C_1 = \dfrac{\varepsilon_0\varepsilon_{r_1}A}{d_1}$, $C_2 = \dfrac{\varepsilon_0\varepsilon_{r_2}A}{d_2}$

These two are in series $\dfrac{1}{C} = \dfrac{1}{C_1} + \dfrac{1}{C_2} \Rightarrow \dfrac{1}{C} = \dfrac{d_1}{\varepsilon_0\varepsilon_{r_1}A} + \dfrac{d_2}{\varepsilon_0\varepsilon_{r_2}A}$

$$\Rightarrow \frac{1}{C} = \frac{1}{\varepsilon_0 A}\left[\frac{d_1\varepsilon_{r_2} + d_2\varepsilon_{r_1}}{\varepsilon_{r_1}\varepsilon_{r_2}}\right] \Rightarrow C = \varepsilon_0 A\left[\frac{\varepsilon_{r_1}\varepsilon_{r_2}}{d_1\varepsilon_{r_2} + d_2\varepsilon_{r_1}}\right]$$

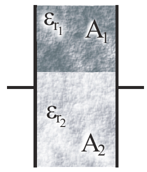
Special case : If $d_1 = d_2 = \dfrac{d}{2} \Rightarrow C = \dfrac{\varepsilon_0 A}{d}\left[\dfrac{2\varepsilon_{r_1}\varepsilon_{r_2}}{\varepsilon_{r_1} + \varepsilon_{r_2}}\right]$

- **Area Division**

(i) Area is divided and distance remains same.

(ii) Capacitors are in parallel.

(iii) Individual capacitances are $C_1 = \dfrac{\varepsilon_0\varepsilon_{r_1}A_1}{d}$ $C_2 = \dfrac{\varepsilon_0\varepsilon_{r_2}A_2}{d}$



These two are in parallel so $C = C_1 + C_2 = \dfrac{\varepsilon_0\varepsilon_{r_1}A_1}{d} + \dfrac{\varepsilon_0\varepsilon_{r_2}A_2}{d} = \dfrac{\varepsilon_0}{d}(\varepsilon_{r_1}A_1 + \varepsilon_{r2}A_2)$

Special case : If $A_1 = A_2 = \dfrac{A}{2}$ Then $C = \dfrac{\varepsilon_0 A}{d}\left(\dfrac{\varepsilon_{r_1} + \varepsilon_{r2}}{2}\right)$

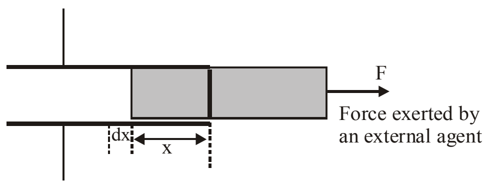
- **Variable Dielectric Constant :**

If the dielectric constant is variable, then equivalent capacitance can be obtained by selecting an element as per the given condition and then integrating.

(i) If different elements are in parallel, then $C = \int dC$, where dC = capacitance of selected differential element.

(ii) If different element are in series, then $\dfrac{1}{C} = \int d\left(\dfrac{1}{C}\right)$ is solved to get equivalent capacitance C.

FORCE ON A DIELECTRIC IN A CAPACITOR



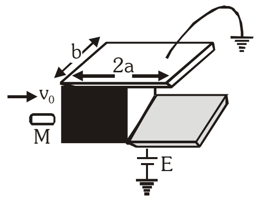
Consider a differential displacement dx of the dielectric as shown in figure always keeping the net force on it zero so that the dielectric moves slowly without acceleration. Then, $W_{Electrostatic} + W_F = 0$, where W_F denotes the work done by external agent in displacement dx

$W_F = -W_{Electrostatic}$ $W_F = \Delta U$

$$\Rightarrow -F.dx = \frac{Q^2}{2}d\left[\frac{1}{C}\right] \left[U = \frac{Q^2}{2C}\right] \Rightarrow -F.dx = \frac{-Q^2}{2C^2}dC \Rightarrow F = \frac{Q^2}{2C^2}\left(\frac{dC}{dx}\right)$$

This is also true for the force between the plates of the capacitor. If the capacitor has battery connected to it, then as the p.d. across the plates is maintained constant. $V = \dfrac{Q}{C} \Rightarrow F = \dfrac{1}{2}V^2\dfrac{dC}{dx}$.

Ex. A parallel plate capacitor is half filled with a dielectric (K) of mass M. Capacitor is attached with a cell of emf E. Plates are held fixed on smooth insulating horizontal surface. A bullet of mass M hits the dielectric elastically and its found that dielectric just leaves out the capacitor. Find speed of bullet and the current as a function of time.



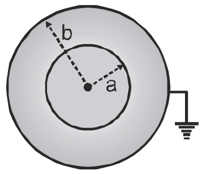
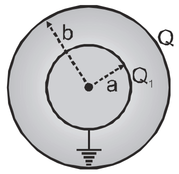
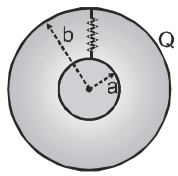
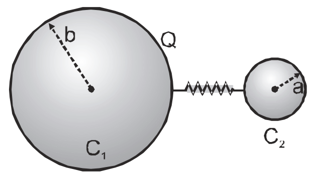
Sol. Since collision is elastic ∴ Velocity of dielectric after collision is v_0.
Dielectric will move and when it is coming out of capacitor a force is applied on it by the capacitor $F = \dfrac{-dU}{dx} = \dfrac{-E^2\varepsilon_0 b(K-1)}{2d}$

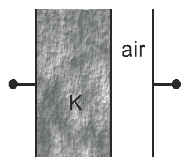
Which decreases its speed to zero, till it comes out it travels a distance a.

$$\frac{1}{2}Mv_0^2 = \frac{E^2\varepsilon_0 b(K-1)a}{2d} \Rightarrow v_0 = E\left[\frac{\varepsilon_0 ab(K-1)}{Md}\right]^{1/2}$$

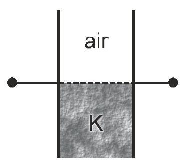
$$i = vE.\frac{dC}{dx}\left[\text{Since}, i = \frac{dq}{dt} = \frac{d}{dt}(EC) = E\frac{dC}{dt} = \frac{EdC}{dx}\frac{dx}{dt}\right]$$

$$i = \left(v_0 - \frac{F}{M}t\right)E\varepsilon_0 b\frac{(K-1)}{d} \quad \text{for } t_0 < t < (t_0 + \frac{v_0 M}{F}) \text{ (where, } t_0 = \frac{a}{v_0})$$

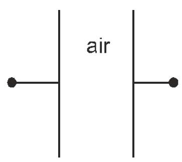
Spherical capacitor outer is earthed	Inner is earthed and outer is given a charge	Connected and outer is given a charge	Connected spheres
			
$C = \dfrac{4\pi\varepsilon_0 ab}{b-a}$ (b > a)	$C = \dfrac{4\pi\varepsilon_0 b^2}{b-a}$ (b > a)	$C = 4\pi\varepsilon_0 b$	$C = C_1 + C_2$ $C = 4\pi\varepsilon_0(a+b)$



$C_1 = \left[\dfrac{2K}{K+1}\right]C$



$C_2 = \left[\dfrac{K+1}{2}\right]C$



$C_3 = C$
when no dielectric is used

$C_2 > C_1 > C_3$

Ex. A capacitor has two circular plates whose radius are 8cm and distance between them is 1mm. When mica (dielectric constant = 6) is placed between the plates, calculate the capacitance of this capacitor and the energy stored when it is given potential of 150 volt.

Sol. Area of plate $\pi r^2 = \pi \times (8 \times 10^{-2})^2 = 0.0201$ m^2 and d = 1mm = 1×10^{-3} m

Capacity of capacitor $C = \dfrac{\varepsilon_0\varepsilon_r A}{d} = \dfrac{8.85\times10^{-12}\times6\times0.0201}{1\times10^{-3}} = 1.068 \times 10^{-9}$ F

Potential difference $V = 150$ volt

Energy stored $U = \frac{1}{2}CV^2 = \frac{1}{2} \times (1.068 \times 10^{-9}) \times (150)^2 = 1.2 \times 10^{-5}$ J

Ex. A parallel-plate capacitor is formed by two plates, each of area 100 cm^2, separated by a distance of 1mm. A dielectric of dielectric constant 5.0 and dielectric strength 1.9×10^7 V/m is filled between the plates. Find the maximum charge that can be stored on the capacitor without causing any dielectric breakdown.

Sol. If the charge on the capacitor = Q

the surface charge density $\sigma = \frac{Q}{A}$ and the electric field = $\frac{Q}{KA\varepsilon_0}$.

This electric field should not exceed the dielectric strength 1.9×10^7 V/m.

$\therefore$ if the maximum charge which can be given is Q

then $\frac{Q}{KA\varepsilon_0} = 1.9 \times 10^7$ V/m, $\because$ A = 100 cm^2 = 10^{-2} m^2

$\Rightarrow Q = (5.0) \times (10^{-2}) \times (8.85 \times 10^{-12}) \times (1.9 \times 10^7) = 8.4 \times 10^{-6}$ C.

Ex. The distance between the plates of a parallel-plate capacitor is 0.05 m. A field of 3×10^4 V/m is established between the plates of capacitor by connecting with battery. Now capacitor is disconnected from the battery and an uncharged metal plate of thickness 0.01 m is inserted between the plates of capacitor. Calculate new potential difference between the plates of capacitor. What would be the potential difference if a plate of same thickness and dielectric constant K = 2 is introduced in place of metal plate ?

Sol. (i) In case of a capacitor as E = (V/d), the potential difference between the plates before the introduction of metal plate

$$V = E \times d = 3 \times 10^4 \times 0.05 = 1.5 \text{ kV}$$

(ii) Now as after charging battery is removed , capacitor is isolated so q = constant. If C' and V' are the capacity and potential after the introduction of plate $q = CV = C'V'$ i.e., $V' = \frac{C}{C'}V$

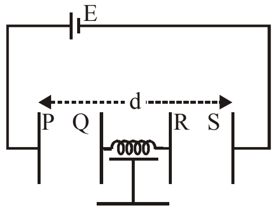
And as $C = \frac{\varepsilon_0 A}{d}$ and $C' = \frac{\varepsilon_0 A}{(d-t)+(t/K)}$, $V' = \frac{(d-t)+(t/K)}{d} \times V$

So in case of metal plate as K = ∞, $V_M = \left[\frac{d-t}{d}\right] \times V = \left[\frac{0.05-0.01}{0.05}\right] \times 1.5 = 1.2 \text{ kV}$

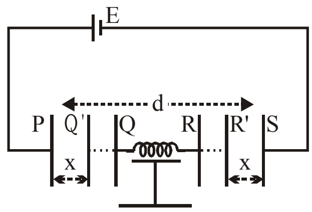
And if instead of metal plate, dielectric with K = 2 is introduced

$$V_D = \left[\frac{(0.05-0.01)+(0.01/2)}{0.05}\right] \times 1.5 = 1.35 \text{ kV}$$

Ex. Two parallel plate capacitors with area A are connected through a conducting spring of natural length ℓ in series as shown. Plates P and S have fixed positions at separation d. Now the plates are connected by a battery of emf E as shown. If the extension in the spring in equilibrium is equal to the separation between the plates, find the spring constant k.



Sol. At any time distance between plates P and Q, R and S is same because force acting on them is same. Let charge on capacitors be q and separation between plates P and Q, R and S be x



Capacitance of capacitor PQ, $C_1 = \dfrac{\varepsilon_0 A}{x}$

Capacitance of capacitor RS, $C_2 = \dfrac{\varepsilon_0 A}{x}$ From KVL $\dfrac{q}{C_1} + \dfrac{q}{C_2} = E \Rightarrow q = \dfrac{\varepsilon_0 AE}{2x}$

At this moment extension in spring, $y = d - 2x - \ell$.

Force on plate Q towards P, $F_1 = \dfrac{q^2}{2A\varepsilon_0} = \dfrac{\varepsilon_0^2 A^2 E^2}{8Ax^2\varepsilon_0} = \dfrac{A\varepsilon_0 E^2}{8x^2}$

Spring force on plate Q due to extension in spring, $F_2 = ky$

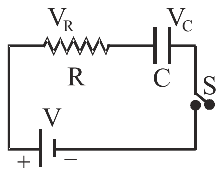
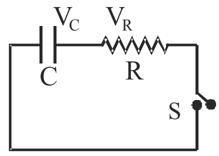
At equilibrium, separation between plates = extension in spring

Thus $x = y = d - 2x - \ell \Rightarrow x = \dfrac{d-\ell}{3}$ ...(i) and $F_1 = F_2$ ...(ii)

From eq. (i) and (ii), $\dfrac{A\varepsilon_0 E^2}{8x^2} = ky = kx \Rightarrow x = \left(\dfrac{A\varepsilon_0 E^2}{8k}\right)^{1/3}$ ...(iii)

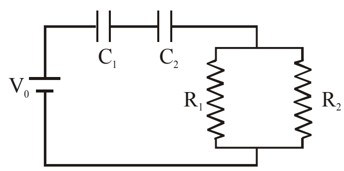
From eq. (i) and (iii), $\left(\dfrac{d-\ell}{3}\right) = \dfrac{A\varepsilon_0 E^2}{8k} \Rightarrow k = \dfrac{A\varepsilon_0 E^2 27}{8(d-\ell)^3}$

CHARGING & DISCHARGING OF A CAPACITOR

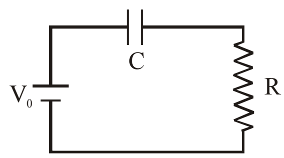
Charging	Discharging
• When a capacitor, resistance, battery, and key is conected in series and key is closed, then	• When a charged capacitor, resistance and keys is conected in series and key is closed. Then energy stored in capacitor is used to circulate current in the circuit.
	
• **Charge at any instant**	• **Charge at any instant**
$V = V_C + V_R = \frac{Q}{C} + IR = \frac{Q}{C} + \frac{dQ}{dt}R$	$V_C + V_R = 0$
$Q = CV\left[1 - e^{-t/RC}\right] = Q_0\left[1 - e^{-t/RC}\right]$	$Q = Q_0 e^{-t/RC}$
At $t = \tau = RC$ = time constant $Q = Q_0[1 - e^{-1}] = 0.632\,Q_0$ So, in charging, charge increases to 63.2% of charge in the time equal to τ.	At $t = \tau = RC$ = time constant $Q = Q_0 e^{-1} = 0.368\,Q_0$ So, in discharging, charge decreases to 36.8% of the initial charge in the time equal to τ.
• **Current at any instant**	• **Current at any instant**
$i = dQ/dt = i_0 e^{-t/RC}$ $\{i_0 = Q_0/RC\}$	$i = dQ/dt = -i_0 e^{-t/RC}$ $\{i_0 = Q_0/RC\}$
• **Potential at any instant**	• **Potential at any instant**
$V = V_0(1 - e^{-t/RC})$	$V = V_0\, e^{-t/RC}$

Ex. Find the time constant for given circuit if

$R_1 = 4\Omega$, $R_2 = 12\Omega$, $C_1 = 3\mu F$ and $C_2 = 6\mu F$.



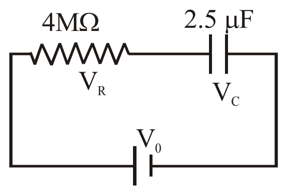
Sol. Given circuit can be reduced to :



$$C = \frac{C_1C_2}{C_1 + C_2} = \frac{3\times6}{3+6} = 2\mu F, \quad R = \frac{R_1R_2}{R_1+R_2} = \frac{4\times12}{4+12} = 3\Omega$$

Time constant = RC = (3) (2×10^{-6}) = 6μs

Ex. A capacitor of 2.5 μF is charged through a series resistor of 4MΩ. In what time the potential drop across the the capacitor will become 3 times that of the resistor. (Given : $\ell n2 = 0.693$)



Sol. $V_C = V_0(1 - e^{-t/RC})$ $\because V_C = 3V_R \therefore V_0 = V_C + \frac{V_C}{3} \Rightarrow V_C = \frac{3}{4}V_0$

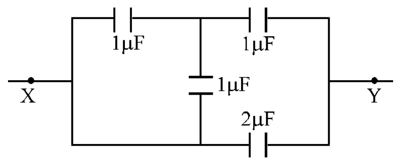
$$\Rightarrow \frac{3}{4}V_0 = V_0(1 - e^{-t/RC}) \Rightarrow \frac{3}{4} = 1 - e^{-t/RC} \Rightarrow \frac{1}{4} = e^{-t/RC} \Rightarrow 4 = e^{t/RC}$$

$$\Rightarrow \frac{t}{RC} = \ell n4 \Rightarrow t = RC\ell n4 = 2RC\,\ell n2 = 2 \times 4 \times 10^6 \times 2.5 \times 10^{-6} \times 0.693 = 13.86 \text{ s}$$

EXERCISE (S)

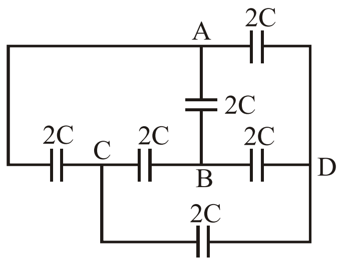
Capacitor Circuits

1. The figure shows a circuit consisting of four capacitors. Find the effective capacitance between X and Y.



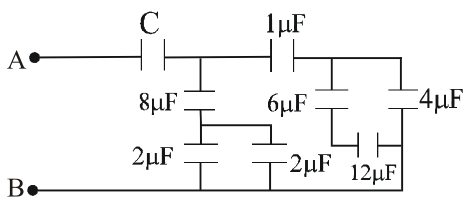
CP0122

2. Equivalent capacitance between A and B is nC. Write n.



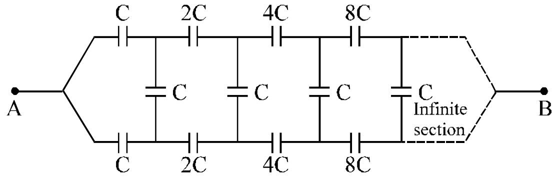
CP0123

3. In the following circuit, the resultant capacitance between A and B is 1 μF. Find the value of C.



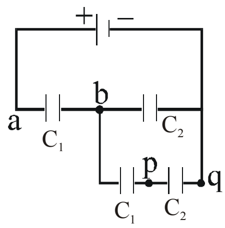
CP0006

4. Find the equivalent capacitance of the circuit between point A and B.



CP0007

5. In the given network if potential difference between p and q is 2V and $C_2 = 3C_1$. Then find the potential difference between a & b.

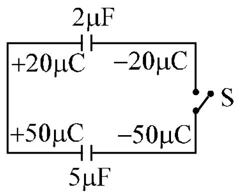


CP0008

6. The plates of a parallel plate capacitor are given charges +4Q and –2Q. The capacitor is then connected across an uncharged capacitor of same capacitance as first one (= C). Find the final potential difference between the plates of the first capacitor.

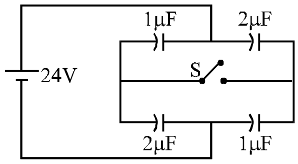
CP0011

7. Find heat produced in the circuit shown in figure on closing the switch S.



CP0009

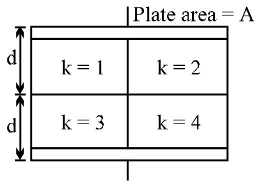
8. The connections shown in figure are established with the switch S open. How much charge will flow through the switch if it is closed ?



CP0010

Dielectrics

9. Find the capacitance of the system shown in figure.

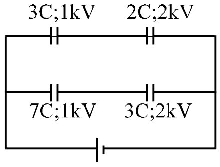


CP0012

10. The plates of a parallel plate capacitor are charged upto 100 volt. A 2 mm thick plate is inserted between the plates, then to maintain the same potential difference, the distance between the capacitor plates is increased by 1.6 mm. Find the dielectric constant of the plate.

CP0013

11. The diagram shows four capacitors with capacitances and break down voltages as mentioned. What should be the maximum value of the external emf source such that no capacitor breaks down?

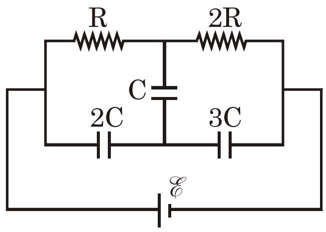


CP0014

12. Two square metallic plates of 1 m side are kept 0.01 m apart, like a parallel plate capacitor, in air in such a way that one of their edges is perpendicular, to an oil surface in a tank filled with an insulating oil. The plates are connected to a battery of e.m.f. 500 volt . The plates are then lowered vertically into the oil at a speed of 0.001 m/s. Calculate the current drawn from the battery during the process. [di-electric constant of oil = 11, $\in_0 = 8.85 \times 10^{-12}$ $C^2/N^2 m^2$]

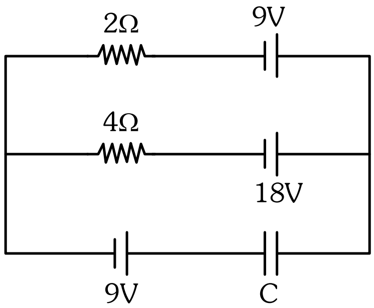
CP0015

13. Find the charge *on the capacitor* C in the figure shown. Internal resistance of a source is to be neglected. (Take : $\mathscr{E}$ = 9 V, C = 2μF)



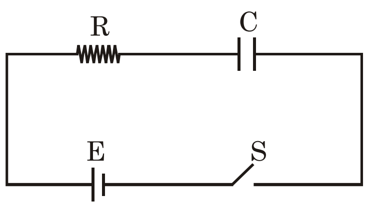
CP0124

14. In the circuit shown, calculate the potential drop across the capacitor.



CP0125

15. At t = 0 switch S is closed. Find ratio of voltage across capacitor and voltage across resistance at time t = 2RCln3 sec. (Capacitor is uncharged before switch S is closed).

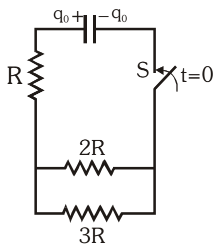


CP0126

node06\B0B0-BA\Kota\JEE(Advanced)\Leader\Physics\Sheet\Electrostatics & Electrodynamics\Eng\3. Capacitance.p65

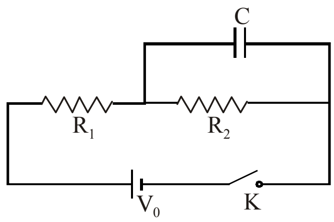
R-C Circuits

16. At $t = 0$ charge on capacitor is q_0. Now switch S is closed. Heat loss in 3R is $x \times 10^{-6}$ J. Then find the value of x. [Given $q_0 = 15\mu C$, $C = 6/55\ \mu F$]



CP0016

17. In the connection shown in the *figure* the switch K is open and the capacitor is uncharged. Then we close the switch and let the capacitor charge up to the maximum and open the switch again. Then (Use the following data : V_0=30 V, R_1=10 kΩ, R_2=5 kΩ.)



(i) the current through R_1 be I_1 immediately after closing the switch

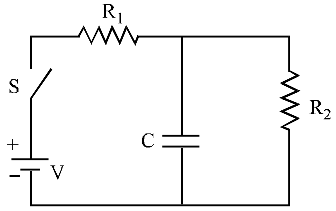
(ii) the current through R_2 be I_2 a long time after the switch was closed

(iii) the current through R_2 be I_3 immediately after reopening the switch

Find the value of $\frac{I_1}{I_2 I_3}$ (in ampere^{-1}).

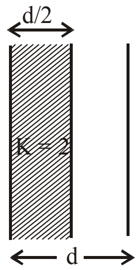
CP0017

18. In the given circuit, the switch S is closed at time $t = 0$. The charge Q on the capacitor at any instant t is given by $Q(t) = Q_0(1 - e^{-\alpha t})$. Find the value of Q_0 and α in terms of given parameters shown in the circuit. **[IIT-JEE 2005]**



CP0018

19. A certain series RC circuit is formed using a resistance R, a capacitor without dielectric having a capacitance C = 2F and a battery of emf E = 3V. The circuit is completed and it is allowed to attain the steady state. After this, at t = 0, half the thickness of the capacitor is filled with a dielectric of constant K = 2 as shown in the figure. The system is again allowed to attain a steady state. What will be the heat generated (in joule) in the circuit between t = 0 and t = ∞ ?

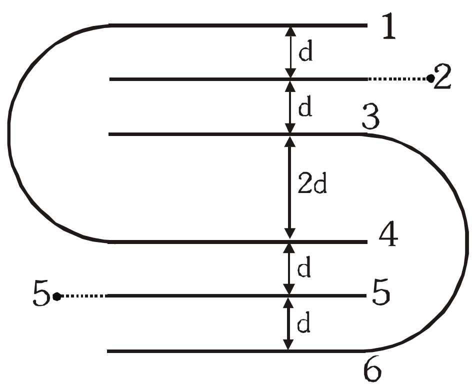


CP0019

20. A capacitor filled with dielectric of permittivity $\varepsilon = 2.1$ loses half the charge acquired during a time interval $\tau = 3.0$ min. Assuming the charge to leak only through the dielectric filler, calculate its resistivity.

CP0020

21. There are six plates of equal area A and separation between the plates is d (d<<A) are arranged as shown in figure. The equivalent capacitance between points 2 and 5, is $\alpha \frac{\epsilon_0 A}{d}$. Then find the value of α.



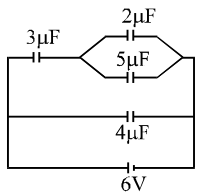
CP0026

EXERCISE (O)

SINGLE CORRECT TYPE QUESTIONS

Capacitor circuits

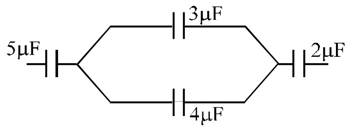
1. In the circuit shown in figure, the ratio of charges on 5μF and 4μF capacitor is :-



(A) 4/5 (B) 3/5 (C) 3/8 (D) 1/2

CP0038

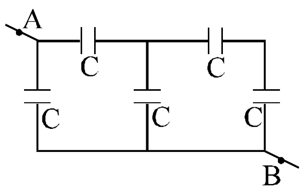
2. If charge on left plate of the 5μF capacitor in the circuit segment shown in the figure is –20μC, the charge on the right plate of 3μF capacitor is :-



(A) +8.57 μC (B) –8.57 μC (C) +11.42 μC (D) –11.42 μC

CP0039

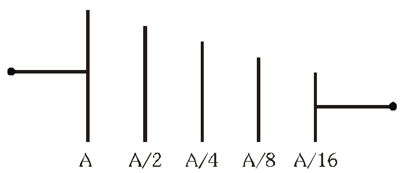
3. What is the equivalent capacitance of the system of capacitors between A & B as shown in the figure.



(A) $\frac{7}{6}C$ (B) 1.6 C (C) C (D) None

CP0040

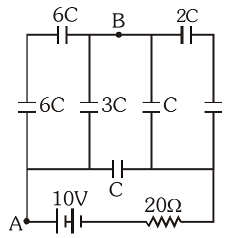
4. 5 Conducting plates each are placed face to face & equi-spaced at distance d. Area of each plate is half the previous plate. If area of first plate is A. Then the equivalent capacitance of the system shown is :-



(A) $\frac{\varepsilon_0 A}{d}$ (B) $\frac{\varepsilon_0 A}{10d}$ (C) $\frac{\varepsilon_0 A}{20d}$ (D) $\frac{\varepsilon_0 A}{30d}$

CP0041

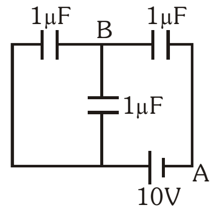
5. For the circuit shown here, the potential difference between points A and B is :-



(A) 2.5 V (B) 7.5 V (C) 10 V (D) Zero

CP0042

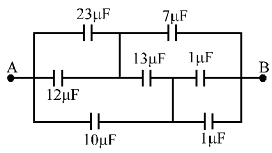
6. If potential of A is 10V, then potential of B is :-



(A) 25/3 V (B) 50/3 V (C) 100/3 V (D) 50 V

CP0043

7. Find the equivalent capacitance across A & B :-



(A) $\frac{28}{3}$ μf (B) $\frac{15}{2}$ μF (C) 15 μF (D) none

CP0044

8. A capacitor of capacitance C is charged to a potential difference V from a cell and then disconnected from it. A charge +Q is now given to its positive plate. The potential difference across the capacitor is now :-

(A) V (B) $V + \frac{Q}{C}$ (C) $V + \frac{Q}{2C}$ (D) $V - \frac{Q}{C}$, if V < CV

CP0045

9. A parallel plate capacitor is made by stacking n equally spaced plates connected alternatively. If the capacitance between any two adjacent plates is C, then the resultant capacitance is :-

[AIEEE-2005]

(A) (n–1)C (B) (n+1)C (C) C (D) nC

CP0046

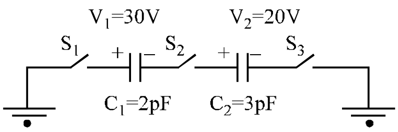
10. For the circuit shown, which of the following statements is true ? **[IIT-JEE 1999]**

(A) with S_1 closed, $V_1 = 15$ V, $V_2 = 20$ V

(B) with S_3 closed, $V_1 = V_2 = 25$ V

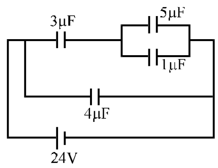
(C) with S_1 & S_2 closed, $V_1 = V_2 = 0$

(D) with S_1 & S_2 closed, $V_1 = 30$ V, $V_2 = 20$ V



CP0047

11. In the circuit shown, the energy stored in 1μF capacitor is :-



(A) 40 μJ (B) 64 μJ (C) 32 μJ (D) none

CP0053

12. A parallel plate capacitor has an electric field of 10^5V/m between the plates. If the charge on the capacitor plate is 1μC, then the force on each capacitor plate is :-

(A) 0.1N (B) 0.05N (C) 0.02N (D) 0.01N

CP0054

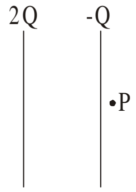
13. In the figure shown the plates of a parallel plate capacitor have unequal charges. Its capacitance is 'C'. P is a point outside the capacitor and close to the plate of charge–Q. The distance between the plates is 'd' then which statement is wrong :-

(A) A point charge at point 'P' will experience electric force due to capacitor

(B) The potential difference between the plates will be $\frac{3Q}{2C}$

(C) The energy stored in the electric field in the region between the plates is $\frac{9Q^2}{8C}$

(D) The force on one plate due to the other plate is $\frac{Q^2}{2\pi \epsilon_0 d^2}$



CP0057

14. Consider a capacitor connected with a battery, capacitor is in steady state. Now plates of capacitor are drawn apart so as to double the separation in two cases.

Case :

(i) Battery remains connected

(ii) Battery is disconnected

Mark the **CORRECT** statement.

(A) In case (i) energy of capacitor increases

(B) In case (i) work done by battery is positive

(C) In case (ii) energy of capacitor increases

(D) In case (ii) potential difference across capacitor decreases

CP0055

15. Two identical capacitors, have the same capacitance C. One of them is charged to potential V_1 and the other to V_2. The negative ends of the capacitors are connected together. When the positive ends are also connected, the decrease in energy of the combined system is :-

[IIT-JEE 2002 (Scr)]

(A) $\frac{1}{4}C\left(V_1^2 - V_2^2\right)$ (B) $\frac{1}{4}C\left(V_1^2 + V_2^2\right)$ (C) $\frac{1}{4}C\left(V_1 - V_2\right)^2$ (D) $\frac{1}{4}C\left(V_1 + V_2\right)^2$

CP0056

Dielectrics

16. A capacitor stores 60μC charge when connected across a battery. When the gap between the plates is filled with a dielectric, a charge of 120μC flows through the battery. The dielectric constant of the material inserted is :

(A) 1 (B) 2 (C) 3 (D) none

CP0062

17. Condenser A has a capacity of 15 μF when it is filled with a medium of dielectric constant 15. Another condenser B has a capacity 1 μF with air between the plates. Both are charged separately by a battery of 100V . After charging, both are connected in parallel without the battery and the dielectric material being removed. The common potential now is :-

(A) 400V (B) 800V (C) 1200V (D) 1600V

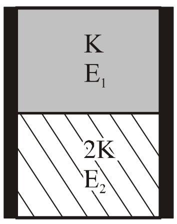
CP0063

18. Three capacitors 2 μF, 3 μF and 5 μF can withstand voltages to 3V, 2V and 1V respectively. Their series combination can withstand a maximum voltage equal to :-

(A) 5 Volts (B) (31/6) Volts (C) (26/5) Volts (D) None

CP0064

19. A parallel plate capacitor is connected from a cell and then isolated from it. Two dielectric slabs of dielectric constant K and 2K are now introduce in the region between upper half and lower half of the plate (as shown in figure). The electric field intensity in upper half of dielectric is E_1 and lower half is E_2 then



(A) $E_1 = 2E_2$

(B) Electrostatic potential energy of upper half is less than that of lower half

(C) Induced charges on both slabs are same

(D) Charge distribution on the plates remains same after insertion of dielectric

CP0065

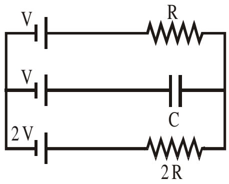
20. Two point charges exert a force F_0 on each other when placed in vacuum. Now the charges are increased to four times, separation between them is doubled and the system is placed is an insulating medium. Now they experience the same force. What should be the dielectric constant of the medium?

(A) 3 (B) 4 (C) 2 (D) 5

CP0066

R-C Circuits

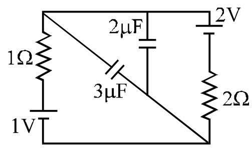
21. In the given circuit, with steady current the potential drop across the capacitor must be :-



(A) V (B) $\frac{V}{2}$ (C) $\frac{V}{3}$ (D) $\frac{2V}{3}$

CP0067

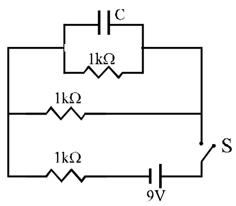
22. In the circuit shown, the charge on the 3μF capacitor at steady state will be



(A) 6 μC (B) 4 μC (C) $\frac{2}{3}$ μC (D) 3 μC

CP0068

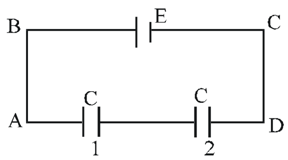
23. A capacitor C = 100 μF is connected to three resistor each of resistance 1 kΩ and a battery of emf 9V. The switch S has been closed for long time so as to charge the capacitor. When switch S is opened, the capacitor discharges with time constant :-



(A) 33 ms (B) 5 ms (C) 3.3 ms (D) 50 ms

CP0069

24. In the adjoining figure, capacitor (1) and (2) have a capacitance 'C' each. When the dielectric of dielectric consatnt K is inserted between the plates of one of the capacitor, the total charge flowing through battery is :-



(A) $\frac{KCE}{K+1}$ from B to C (B) $\frac{KCE}{K+1}$ from C to B

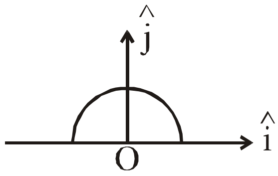
(C) $\frac{(K-1)CE}{2(K+1)}$ from B to C (D) $\frac{(K-1)CE}{2(K+1)}$ from C to B

CP0088

node06\B0B0-BA\Kota\JEE(Advanced)\Leader\Physics\Sheet\Electrostatics & Electrodynamics\Eng\3. Capacitance.p65

EXERCISE-JM

1. A thin semi-circular ring of radius r has a positive charge q distributed uniformly over it. The net field $\vec{E}$ at the centre O is :- **[AIEEE - 2010]**



(1) $\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$ (2) $\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (3) $-\frac{q}{4\pi^2\varepsilon_0 r^2}\hat{j}$ (4) $-\frac{q}{2\pi^2\varepsilon_0 r^2}\hat{j}$

ES0237

2. Let there be a spherically symmetric charge distribution with charge density varying as $\rho(r) = \rho_0\left(\frac{5}{4}-\frac{r}{R}\right)$ upto r = R, and $\rho(r) = 0$ for r > R, where r is the distance from the origin. The electric field at a distance r (r < R) from the origion is given by : **[AIEEE - 2010]**

(1) $\frac{\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$ (2) $\frac{4\pi\rho_0 r}{3\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$

(3) $\frac{\rho_0 r}{4\varepsilon_0}\left(\frac{5}{3}-\frac{r}{R}\right)$ (4) $\frac{4\rho_0 r}{3\varepsilon_0}\left(\frac{5}{4}-\frac{r}{R}\right)$

ES0238

3. Two conductors have the same resistance at 0°C but their temperature coefficients of resistance are α_1 and α_2. The respective temperature coefficients of their series and parallel combinations are nearly : **[AIEEE-2010]**

(1) $\frac{\alpha_1+\alpha_2}{2}, \frac{\alpha_1+\alpha_2}{2}$ (2) $\frac{\alpha_1+\alpha_2}{2}, \alpha_1+\alpha_2$

(3) $\alpha_1+\alpha_2, \frac{\alpha_1+\alpha_2}{2}$ (4) $\alpha_1+\alpha_2, \frac{\alpha_1\alpha_2}{\alpha_1+\alpha_2}$

CE0131

4. Let C be the capacitance of a capacitor discharging through a resistor R. Suppose t_1 is the time taken for the energy stored in the capacitor to reduce to half its initial value and t_2 is the time taken for the charge to reduce to one-fourth its initial value. Then the ratio t_1/t_2 will be : **[AIEEE-2010]**

(1) 2 (2) 1 (3) 1/2 (4) 1/4

CP0099

5. Two identical charged spheres are suspended by strings of equal lengths. The strings make an angle of 30° with each other. When suspended in a liquid of density 0.8 g cm^{-3}, the angle remains the same. If density of the material of the sphere is 1.6 g cm^{-3}, the dielectric constant of the liquid is : **[AIEEE - 2010]**

(1) 1 (2) 4 (3) 3 (4) 2

CP0100

6. Two identical charged spheres suspended from a common point by two massless string of length ℓ are initially a distance d(d << ℓ) apart because of their mutual repulsion. The charge begins to leak from both the spheres at a constant rate. As a result the charges approach each other with a velocity v. Then as a function of distance x between them :- **[AIEEE - 2011]**

(1) $v \propto x^{1/2}$ (2) $v \propto x$ (3) $v \propto x^{-1/2}$ (4) $v \propto x^{-1}$

ES0192

7. The electrostatic potential inside a charged spherical ball is given by $\phi = ar^2 + b$ where r is the distance from the centre; a, b are constant. Then the charge density inside the ball is :- **[AIEEE - 2011]**

(1) $-24\pi a\epsilon_0$ (2) $-6 a\epsilon_0$ (3) $-24\pi a\epsilon_0 r$ (4) $-6 a\epsilon_0 r$

ES0193

8. Two positive charges of magnitude 'q' are placed at the ends of a side (side 1) of a square of side '2a'. Two negative charges of the same magnitude are kept at the other corners. Starting from rest, if a charge Q moves from the middle of side 1 to the centre of square, its kinetic energy at the centre of square is :- **[AIEEE - 2011]**

(1) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1-\frac{1}{\sqrt{5}}\right)$

(2) zero

(3) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1+\frac{1}{\sqrt{5}}\right)$

(4) $\frac{1}{4\pi\epsilon_0}\frac{2qQ}{a}\left(1-\frac{2}{\sqrt{5}}\right)$

ES0194

9. If a wire is stretched to make it 0.1 % longer its resistance will :- **[AIEEE-2011]**

(1) decrease by 0.2% (2) decrease by 0.05%
(3) increase by 0.05% (4) increase by 0.2%

CE0132

10. The current in the primary circuit of a potentiometer is 0.2 A. The specific resistance and cross-section of the potentiometer wire are 4×10^{-7} ohm metre and 8×10^{-7} m^2 respectively. The potential gradient will be equal to :- **[AIEEE - 2011]**

(1) 0.2 V/m (2) 1 V/m (3) 0.5 V/m (4) 0.1 V/m

CE0134

11. A resistor 'R' and 2μF capacitor in series is connected through a switch to 200 V direct supply. Across the capacitor is a neon bulb that lights up at 120 V. Calculate the value of R to make the bulb light up 5s after the switch has been closed. ($\log_{10} 2.5 = 0.4$) **[AIEEE-2011]**

(1) $2.7 \times 10^6\ \Omega$ (2) $3.3 \times 10^7\ \Omega$

(3) $1.3 \times 10^4\ \Omega$ (4) $1.7 \times 10^5\ \Omega$

CP0101

12. Combination of two identical capacitors, a resistor R and a dc voltage source of voltage 6V is used in an experiment on (C–R) circuit. It is found that for a parallel combination of the capacitor the time in which the voltage of the fully charged combination reduces to half its original voltage is 10 second. For series combination the time needed for reducing the voltage of the fully charged series combination by half is :- **[AIEEE-2011]**

(1) 20 second (2) 10 second (3) 5 second (4) 2.5 second

CP0102

13. **This question has Statement-1 and Statement-2. Of the four choices given after the statements, choose the one that best describes the two statements.** **[AIEEE - 2012]**

An insulating solid sphere of radius R has a uniformaly positive charge density ρ. As a result of this uniform charge distribution there is a finite value of electric potential at the centre of the sphere, at the surface of the sphre and also at a point out side the sphere. The electric potential at infinity is zero.

Statement-1: When a charge 'q' is taken from the centre to the surface of the sphere, its potential energy changes by $\frac{q\rho}{3 \epsilon_0}$.

Statement-2 : The electric field at a distance r (r < R) from the centre of the sphere is $\frac{\rho r}{3 \epsilon_0}$

(1) Statement-1 is true, Statement-2 is true and Statement-2 is the correct explanation of Statement-1.

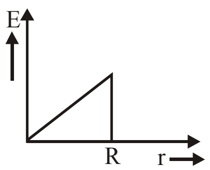
(2) Statement-1 is true, Statement-2 is true and Statement-2 is not the correct explanation of statement-1.

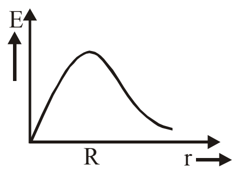
(3) Statement-1 is true, Statement-2 is false

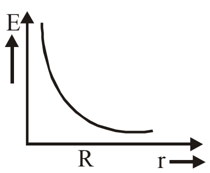
(4) Statement-1 is false, Statement-2 is true

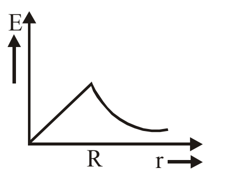
ES0195

14. In a uniformly charged sphere of total charge Q and radius R, the electric field E is plotted as a function of distance from the centre. The graph which would correspond to the above will be :- **[AIEEE - 2012]**

(1) 

(2) 

(3) 

(4) 

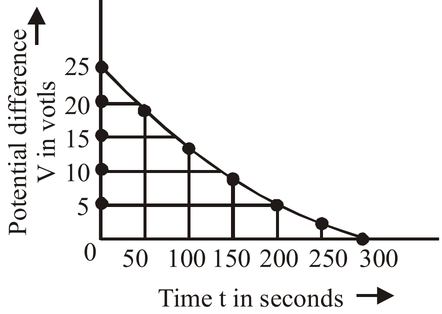
ES0196

15. Two electric bulbs marked 25W-220 V and 100 W-220 V are connected in series to a 440 V supply. Which of the bulbs will fuse ? **[AIEEE - 2012]**

(1) Neither (2) Both (3) 100 W (4) 25 W

CE0135

16. The figure shows an experimental plot for discharging of a capacitor in an R–C circuit. The time constant τ of this circuit lies between:- **[AIEEE 2012]**



(1) 100 sec and 150 sec (2) 150 sec and 200 sec

(3) 0 and 50 sec (4) 50 sec and 100 sec

CP0103

17. Let $[\in_0]$ denote the dimensional formula of the permittivity of vacuum. If M = mass, L = Length, T = Time and A = electric current, then : **[JEE-Main-2013]**

(1) $[\in_0] = [M^{-1} L^{-3} T^2 A]$ (2) $[\in_0] = [M^{-1} L^{-3} T^4 A^2]$

(3) $[\in_0] = [M^{-1} L^2 T^{-1} A^{-2}]$ (4) $[\in_0] = [M^{-1} L^2 T^{-1} A]$

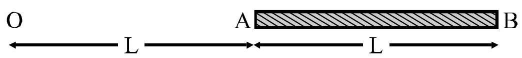
ES0197

18. Two charges, each equal to q, are kept at x = –a and x = a on the x-axis. A particle of mass m and charge $q_0 = \frac{q}{2}$ is placed at the origin. If charge q_0 is given a small displacement (y << a) along the y-axis, the net force acting on the particle is proportional to **[JEE-Main-2013]**

(1) y (2) –y (3) $\frac{1}{y}$ (4) $-\frac{1}{y}$

ES0198

19. A charge Q is uniformly distributed over a long rod AB of length L as shown in the figure. The electric potential at the point O lying at a distance L from the end A is :- **[JEE-Main-2013]**



(1) $\frac{Q}{8\pi \in_0 L}$ (2) $\frac{3Q}{4\pi \in_0 L}$ (3) $\frac{Q}{4\pi \in_0 L \ln 2}$ (4) $\frac{Q \ln 2}{4\pi \in_0 L}$

ES0199

20. The supply voltage to a room is 120V. The resistance of the lead wires is 6Ω. A 60 W bulb is already switched on. What is the decrease of voltage across the bulb, when a 240 W heater is switched on in parallel to the bulb? **[JEE-Main-2013]**

(1) zero Volt (2) 2.9 Volt (3) 13.3 Volt (4) 10.04 Volt

CE0136

21. This question has Statement I and Statement II. Of the four choice given after the Statements, choose the one that best describes the two Statements. **[JEE-Main-2013]**

Statement-I : Higher the range, greater is the resistance of ammeter.

Statement-II : To increase the range of ammeter, additional shunt needs to be used across it.

(1) Statement-I is true, Statement-II is true, Statement-II is the **correct** explanation of Statement-I

(2) Statement-I is true, Statement-II is true, Statement-II is **not** the correct explanation of Statement-I.

(3) Statement-I is **true,** Statement-II is false.

(4) Statement-I is **false,** Statement-II is true.

CE0137

22. Two capacitors C_1 and C_2 are charged to 120V and 200V respectively. It is found that by connecting them together the potential on each one can be made zero. Then : **[JEE-Main-2013]**

(1) $5C_1 = 3C_2$ (2) $3C_1 = 5C_2$ (3) $3C_1 + 5C_2 = 0$ (4) $9C_1 = 4C_2$

CP0104

23. Assume that an electric field $\vec{E} = 30x^2\hat{i}$ exists in space. Then the potential difference $V_A - V_O$, where V_O is the potential at the origin and V_A the potential at x = 2 m is :- **[JEE-Main-2014]**

(1) –80 J (2) 80 J (3) 120 J (4) –120 J

ES0200

24. In a large building, there are 15 bulbs of 40 W, 5 bulbs of 100 W, 5 fans of 80 W and 1 heater of 1kW. The voltage of the electric mains is 220 V. The minimum capacity of the main fuse of the building will be : **[JEE-Main-2014]**

(1) 12 A (2) 14 A (3) 8 A (4) 10 A

CE0138

25. A parallel plate capacitor is made of two circular plates separated by a distance of 5 mm and with a dielectriic of dielectric constant 2.2 between them. When the electric field in the dielectric is 3×10^4 V/m, the charge density of the positive plate will be close to : **[JEE-Main-2014]**

(1) 3×10^4 C/m² (2) 6×10^4 C/m²

(3) 6×10^{-7} C/m² (4) 3×10^{-7} C/m²

CP0105

26. A uniformally charged solid sphere of radius R has potential V_0 (measured with respect to ∞) on its surface. For this sphere the equipotential surfaces with potentials $\frac{3V_0}{2}, \frac{5V_0}{4}, \frac{3V_0}{4}$ and $\frac{V_0}{4}$ have radius R_1, R_2, R_3 and R_4 respectively. Then **[JEE-Main-2015]**

(1) $R_1 = 0$ and $R_2 < (R_4 - R_3)$

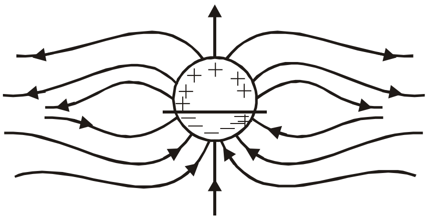
(2) $2R < R_4$

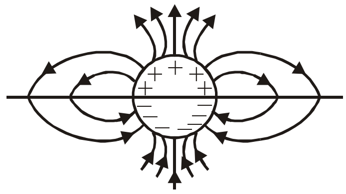
(3) $R_1 = 0$ and $R_2 > (R_4 - R_3)$

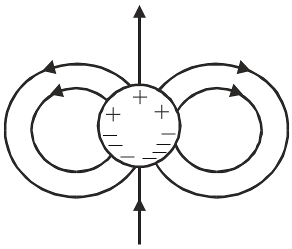
(4) $R_1 \neq 0$ and $(R_2 - R_1) > (R_4 - R_3)$

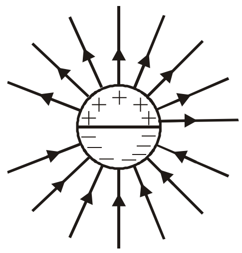
ES0201

27. A long cylindrical shell carries positive surface charge σ in the upper half and negative surface charge –σ in the lower half. The electric field lines around the cylinder will look like figure given in: (figures are schematic and not drawn to scale) **[JEE-Main-2015]**

(1)


(2)


(3)


(4)


ES0202

28. When 5V potential difference is applied across a wire of length 0.1 m, the drift speed of electrons is 2.5×10^{-4} ms^{-1}. If the electron density in the wire is 8×10^{28} m^{-3}, the resistivity of the material is close to :- **[JEE-Main 2015]**

(1) 1.6×10^{-6} Ωm

(2) 1.6×10^{-5} Ωm

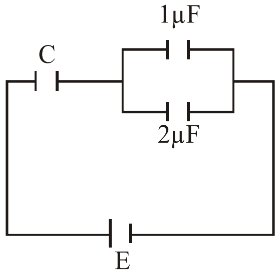
(3) 1.6×10^{-8} Ωm

(4) 1.6×10^{-7} Ωm

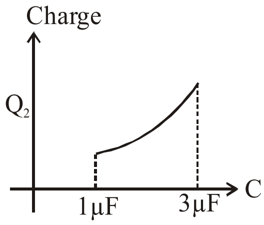
CE0139

29. In the given circuit, charge Q_2 on the 2μF capacitor changes as C is varied from 1μF to 3μF. Q_2 as a function of 'C' is given properly by : (figures are drawn schematically and are not to scale)

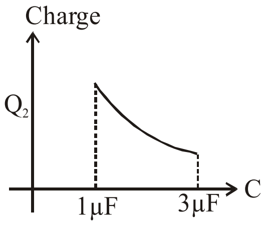
[JEE-Main-2015]



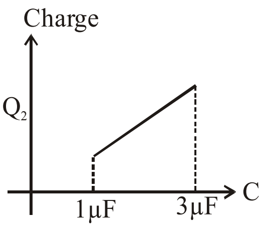
(1)



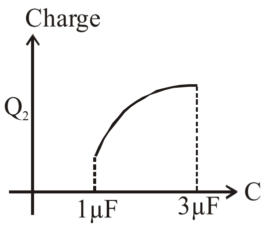
(2)



(3)

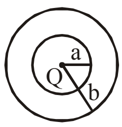


(4)



CP0106

30. The region between two concentric spheres of radii 'a' and 'b', respectively (see figure), has volume charge density $\rho = \frac{A}{r}$, where A is a constant and r is the distance from the centre. At the centre of the spheres is a point charge Q. The value of A such that the electric field in the region between the spheres will be constant, is :- **[JEE-Main-2016]**



(1) $\frac{2Q}{\pi a^2}$ (2) $\frac{Q}{2\pi a^2}$ (3) $\frac{Q}{2\pi(b^2 - a^2)}$ (4) $\frac{2Q}{\pi(a^2 - b^2)}$

ES0203

31. A galvanometer having a coil resistance of 100Ω gives a full scale deflection, when a current of 1 mA is passed through it. The value of the resistance, which can convert this galvanometer into ammeter giving a full scale deflection for a current of 10A, is :- **[JEE-Main 2016]**

(1) 3Ω (2) 0.01Ω (3) 2Ω (4) 0.1Ω

CE0141

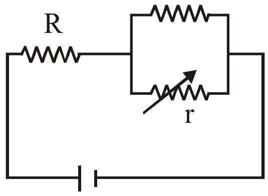
32. A 50Ω resistance is connected to a battery of 5V. A galvanometer of resistance 100 Ω is to be used as an ammeter to measure current through the resistance, for this a resistance r_s is connected to the galvanometer. Which of the following connections should be employed if the measured current is within 1% of the current without the ammeter in the circuit ? **[JEE-Main (Online) 2016]**

(1) r_s = 1Ω in series with galvanometer (2) r_s = 0.5 Ω in parallel with the galvanometer

(3) r_s = 0.5 Ω in series with the galvanometer (4) r_s = 1 Ω in parallel with galvanometer

CE0142

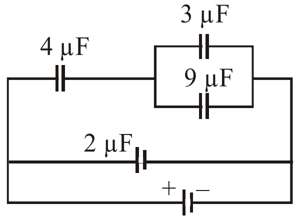
33. In the circuit shown, the resistance r is a variable resistance. If for r = f R, the heat generation in r is maximum then the value of f is : **[JEE-Main (Online) 2016]**



(1) 1 (2) $\frac{1}{4}$ (3) $\frac{1}{2}$ (4) $\frac{3}{4}$

CE0143

34. A combination of capacitors is set up as shown in the figure. The magnitude of the electric field, due to a point charge Q (having a charge equal to the sum of the charges on the 4 μF and 9 μF capacitors), at a point 30 m from it , would equal: **[JEE-Main-2016]**



(1) 480 N/C (2) 240 N/C (3) 360 N/C (4) 420 N/C

CP0107

35. Three capacitors each of 4 μF are to be connected in such a way that the effective capacitance is 6 μF. This can be done by connecting them : **[JEE-Main (Online)-2016]**

(1) two in parallel and one in series (2) all in parallel

(3) two in series and one in parallel (4) all in series

CP0108

36. Which of the following statements is false ? **[JEE-Main-2017]**

(1) A rheostat can be used as a potential divider

(2) Kirchhoff's second law represents energy conservation

(3) Wheatstone bridge is the most sensitive when all the four resistances are of the same order of magnitude.

(4) In a balanced wheatstone bridge if the cell and the galvanometer are exchanged, the null point is disturbed.

CE0146

37. An electric dipole has a fixed dipole moment $\vec{p}$, which makes angle θ with respect to x-axis. When subjected to an electric field $\vec{E}_1 = E\hat{i}$, it experiences a torque $\vec{T}_1 = \tau\hat{k}$. When subjected to another electric field $\vec{E}_2 = \sqrt{3}E_1\hat{j}$ it experiences torque $\vec{T}_2 = -\vec{T}_1$. The angle θ is : **[JEE-Main-2017]**

(1) 60° (2) 90° (3) 30° (4) 45°

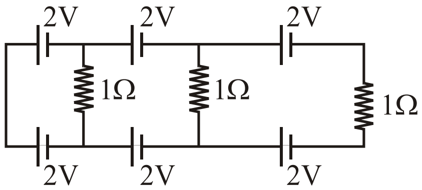
ES0204

38. When a current of 5 mA is passed through a galvanometer having a coil of resistance 15 Ω, it shows full scale deflection. The value of the resistance to be put in series with the galvanometer to convert it into to voltmeter of range 0 – 10 V is :- **[JEE-Main-2017]**

(1) 2.535×10^3 Ω (2) 4.005×10^3 Ω (3) 1.985×10^3 Ω (4) 2.045×10^3 Ω

CE0147

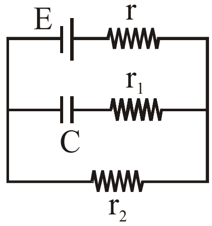
39. In the above circuit the current in each resistance is :- **[JEE-Main-2017]**



(1) 0.5 A (2) 0 A (3) 1 A (4) 0.25 A

CE0148

40. In the given circuit diagram when the current reaches steady state in the circuit, the charge on the capacitor of capacitance C will be : **[JEE-Main-2017]**



(1) $CE\dfrac{r_2}{(r+r_2)}$ (2) $CE\dfrac{r_1}{(r_1+r)}$ (3) CE (4) $CE\dfrac{r_1}{(r_2+r)}$

CP0110

41. Three concentric metal shells A, B and C of respective radii a, b and c (a < b < c) have surface charge densities +σ, –σ and +σ respectively. The potential of shell B is:- **[JEE-Main-2018]**

(1) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{a^2-b^2}{b}+c\right]$ (2) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{b^2-c^2}{b}+a\right]$ (3) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{b^2-c^2}{c}+a\right]$ (4) $\dfrac{\sigma}{\varepsilon_0}\left[\dfrac{a^2-b^2}{a}+c\right]$

ES0205

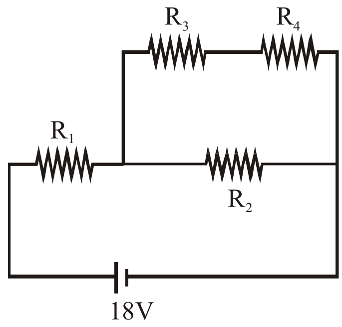
42. In a potentiometer experiment, it is found that no current passes through the galvanometer when the terminals of the cell are connected across 52 cm of the potentiometer wire. If the cell is shunted by a resistance of 5 Ω, a balance is found when the cell is connected across 40 cm of the wire. Find the internal resistance of the cell. **[JEE-Main-2018]**

(1) 1.5 Ω (2) 2 Ω (3) 2.5 Ω (4) 1 Ω

CE0149

43. In the given circuit the internal resistance of the 18 V cell is negligible. If $R_1 = 400\ \Omega$, $R_3 = 100\ \Omega$ and $R_4 = 500\ \Omega$ and the reading of an ideal voltmeter across R_4 is 5V, then the value R_2 will be :

[JEE-Main(online)-2019]

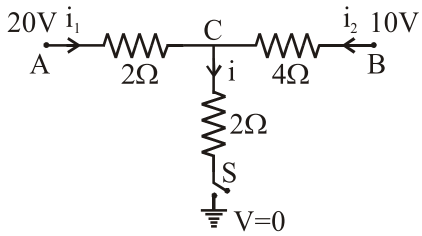


(1) 300 Ω (2) 230 Ω (3) 450 Ω (4) 550 Ω

CE0170

44. When the switch S, in the circuit shown, is closed, then the value of current i will be :

[JEE-Main(online)-2019]

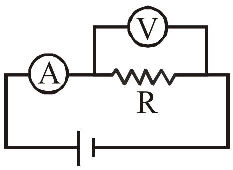


(1) 3 A (2) 5 A (3) 4 A (4) 2 A

CE0171

45. The actual value of resistance R, shown in the figure is 30Ω. This is measured in an experiment as shown using the standard formula $R = \frac{V}{I}$, where V and I are the readings of the voltmeter and ammeter, respectively. If the measured value of R is 5% less, then the internal resistance of the voltmeter is :

[JEE-Main(online)-2019]



(1) 350Ω (2) 570Ω (3) 35 Ω (4) 600 Ω

CE0172

46. Four equal point charges Q each are placed in the xy plane at (0, 2), (4, 2), (4, –2) and (0, –2). The work required to put a fifth charge Q at the origin of the coordinate system will be :

[JEE-Main(online)-2019]

(1) $\frac{Q^2}{2\sqrt{2}\pi\varepsilon_0}$ (2) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{5}}\right)$ (3) $\frac{Q^2}{4\pi\varepsilon_0}\left(1+\frac{1}{\sqrt{3}}\right)$ (4) $\frac{Q^2}{4\pi\varepsilon_0}$

ES0239

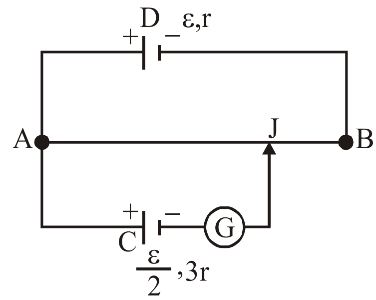
47. A uniform metallic wire has a resistance of 18 Ω and is bent into an equilateral triangle. Then, the resistance between any two vertices of the triangle is : **[JEE-Main(online)-2019]**

(1) 8 Ω (2) 12 Ω (3) 4 Ω (4) 2Ω

CE0173

48. A potentiometer wire AB having length L and resistance 12 r is joined to a cell D of emf ε and internal resistance r. A cell C having emf ε/2 and internal resistance 3r is connected. The length AJ at which the galvanometer as shown in fig. shows no deflection is :

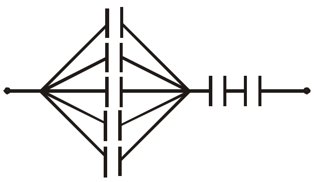
[JEE-Main(online)-2019]

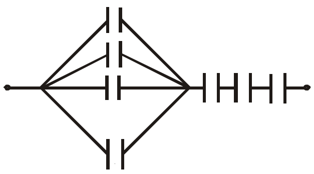


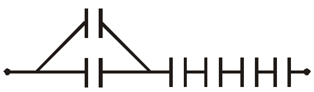
(1) $\frac{5}{12}L$ (2) $\frac{11}{24}L$ (3) $\frac{11}{12}L$ (4) $\frac{13}{24}L$

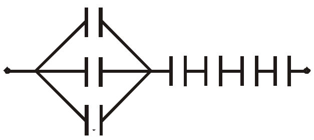
CE0174

49. Seven capacitors, each of capacitance 2 μF, are to be connected in a configuration to obtain an effective capacitance of $\left(\frac{6}{13}\right)\mu F$. Which of the combinations, shown in figures below, will achieve the desired value ? **[JEE-Main(online)-2019]**

(1) 

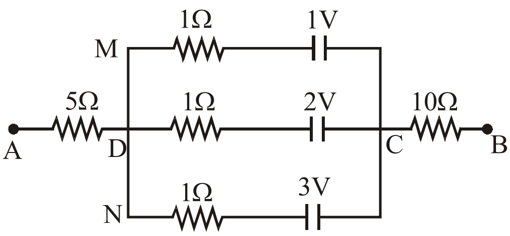
(2) 

(3) 

(4) 

CP0127

50. In the circuit , the potential difference between A and B is :- **[JEE-Main(online)-2019]**



(1) 6 V (2) 1 V (3) 3 V (4) 2 V

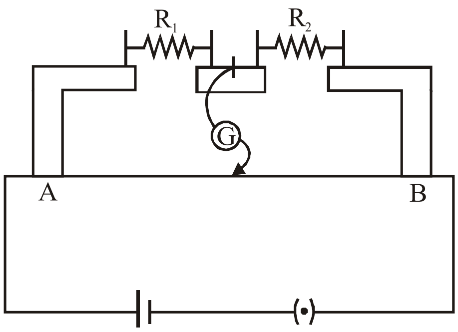
CE0175

51. A galvanometer having a resistance of 20 Ω and 30 divisions on both sides has figure of merit 0.005 ampere/division. The resistance that should be connected in series such that it can be used as a voltmeter upto 15 volt, is :- **[JEE-Main(online)-2019]**

(1) 80 Ω (2) 120 Ω (3) 125 Ω (4) 100 Ω

CE0176

52. In the experimental set up of metre bridge shown in the figure, the null point is obtained at a distance of 40 cm from A. If a 10Ω resistor is connected in series with R_1, the null point shifts by 10 cm. The resistance that should be connected in parallel with $(R_1 + 10)\Omega$ such that the null point shifts back to its initial position is **[JEE-Main(online)-2019]**

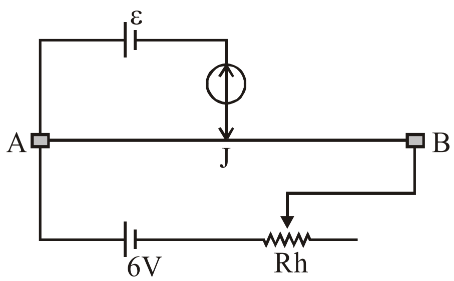


(1) 40 Ω (2) 60 Ω (3) 20 Ω (4) 30Ω

CE0177

53. The resistance of the meter bridge AB is given figure is 4Ω. With a cell of emf ε = 0.5 V and rheostat resistance R_h = 2 Ω the null point is obtained at some point J. When the cell is replaced by another one of emf ε = ε_2 the same null point J is found for R_h = 6 Ω. The emf ε_2 is;

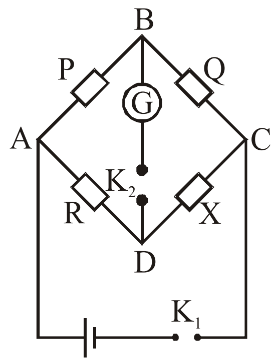
[JEE-Main(online)-2019]



(1) 0.6 V (2) 0.5 V (3) 0.3 V (4) 0.4 V

CE0178

54. In a Wheatstone bridge (see fig.), Resistances P and Q are approximately equal. When R = 400 Ω, the bridge is equal. When R = 400 Ω, the bridge is balanced. On inter-changingP and Q, the value of R, for balance, is 405 Ω. The value of X is close to : **[JEE-Main(online)-2019]**

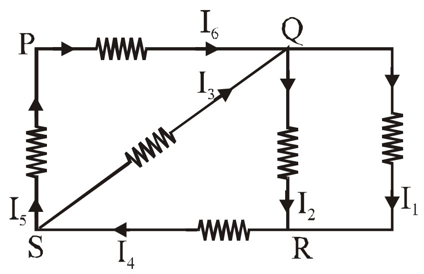


(1) 403.5 ohm (2) 404.5 ohm

(3) 401.5 ohm (4) 402.5 ohm

CE0179

55. In the given circuit diagram, the currents, $I_1 = -0.3A$, $I_4 = 0.8$ A and $I_5 = 0.4$ A, are flowing as shown. The currents I_2,I_3 and I_6,respectively, are : **[JEE-Main(online)-2019]**

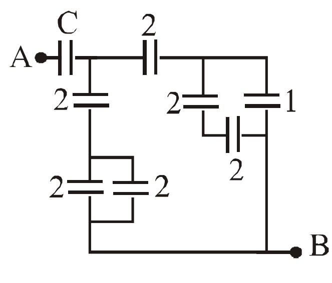


(1) 1.1 A, 0.4 A, 0.4 A (2) –0.4 A, 0.4 A, 1.1 A

(3) 0.4 A, 1.1 A, 0.4 A (4) 1.1 A,–0.4 A, 0.4 A

CE0180

56. In the circuit shown, find C if the effective capacitance of the whole circuit is to be 0.5 μF. All values in the circuit are in μF. **[JEE-Main(online)-2019]**



(1) $\frac{7}{10}\mu F$ (2) $\frac{7}{11}\mu F$ (3) $\frac{6}{5}\mu F$ (4) $4\mu F$

CP0128

57. A galvanometer, whose resistance is 50 ohm, has 25 divisions in it. When a current of 4×10^{-4} A passes through it, its needle (pointer) deflects by one division. To use this galvanometer as a voltmeter of range 2.5 V, it should be connected to a resistance of :

[JEE-Main(online)-2019]

(1) 6250 ohm (2) 250 ohm (3) 200 ohm (4) 6200 ohm

CE0181

58. A parallel plate capacitor with plates of area $1m^2$ each, area t a separation of 0.1 m. If the electric field between the plates is 100 N/C, the magnitude of charge each plate is :-

(Take $\varepsilon_0 = 8.85\times10^{-12}\frac{C^2}{N-m^2}$) **[JEE-Main(online)-2019]**

(1) 7.85×10^{-10} C

(2) 6.85×10^{-10} C

(3) 9.85×10^{-10} C

(4) 8.85×10^{-10} C

ES0240

59. Two electric bulbs, rated at (25 W, 220 V) and (100 W, 220 V), are connected in series across a 220 V voltage source. If the 25 W and 100 W bulbs draw powers P_1 and P_2 respectively, then:

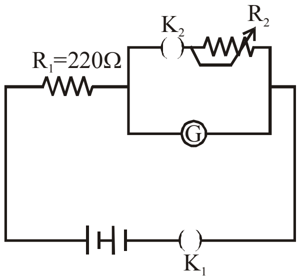
[JEE-Main(online)-2019]

(1) P1 = 9 W, P_2 = 16 W (2) P_1 = 4 W, P_2 = 16W

(3) P_1 = 16 W, P_2 = 4W (4) P_1 16 W, P_2 = 9W

CE0182

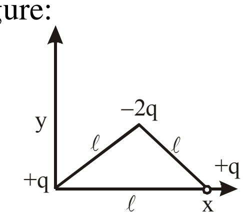
60. The galvanometer deflection, when key K_1 is closed but K_2 is open, equals θ_0 (see figure). On closing K_2 also and adjusting R_2 to 5Ω, the deflection in galvanometer becomes $\frac{\theta_0}{5}$. The resistance of the galvanometer is, then, given by [Neglect the internal resistance of battery]: **[JEE-Main(online)-2019]**



(1) 12Ω (2) 25Ω (3) 5Ω (4) 22Ω

CE0183

61. Determine the electric dipole moment of the system of three charges, placed on the vertices of an equilateral triangle, as shown in the figure: **[JEE-Main(online)-2019]**



(1) $(q\ell)\frac{\hat{i}+\hat{j}}{\sqrt{2}}$ (2) $\sqrt{3}q\ell\frac{\hat{j}-\hat{i}}{\sqrt{2}}$ (3) $-\sqrt{3}q\ell\hat{j}$ (4) $2q\ell\hat{j}$

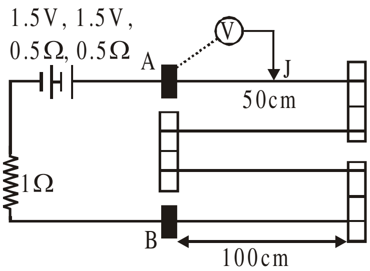
ES0241

62. An ideal battery of 4 V and resistance R are connected in series in the primary circuit of a potentiometer of length 1 m and resistance 5Ω. The value of R, to give a potential difference of 5 mV across 10 cm of potentiometer wire, is : **[JEE-Main(online)-2019]**

(1) 490 Ω (2) 480 Ω (3) 395 Ω (4) 495 Ω

CE0184

63. In the circuit shown, a four-wire potentiometer is made of a 400 cm long wire, which extends between A and B. The resistance per unit length of the potentiometer wire is r = 0.01 Ω/cm. If an ideal voltmeter is connected as shown with jockey J at 50 cm from end A, the expected reading of the voltmeter will be :- **[JEE-Main(online)-2019]**



(1) 0.20 V (2) 0.25 V (3) 0.75 V (4) 0.50V

CE0185

64. A cell of internal resistance r drives current through an external resistance R. The power delivered by the cell to the external resistance will be maximum when :- **[JEE-Main(online)-2019]**

(1) R = 1000 r (2) R = 0.001 r (3) R = 2r (4) R = r

CE0186

65. An electric dipole is formed by two equal and opposite charges q with separation d. The charges have same mass m. It is kept in a uniform electric field E. If it is slightly rotated from its equilibrium orientation, then its angular frequency ω is :- **[JEE-Main(online)-2019]**

(1) $\sqrt{\frac{qE}{2md}}$ (2) $2\sqrt{\frac{qE}{md}}$ (3) $\sqrt{\frac{2qE}{md}}$ (4) $\sqrt{\frac{qE}{md}}$

ES0242

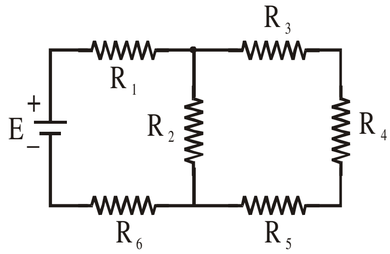
66. The electric field in a region is given by $\vec{E} = (Ax + B)\hat{i}$, where E is in NC^{-1} and x is in metres. The values of constants are A = 20 SI unit and B = 10 SI unit. If the potential at x = 1 is V_1 and that at x = –5 is V_2, then $V_1 - V_2$ is :- **[JEE-Main(online)-2019]**

(1) –48 V (2) –520 V (3) 180 V (4) 320 V

ES0243

67. In the figure shown, what is the current (in Ampere) drawn from the battery ? You are given:
$R_1 = 15\Omega$, $R_2 = 10\ \Omega$, $R_3 = 20\ \Omega$, $R_4 = 5\Omega$, $R_5 = 25\Omega$, $R_6 = 30\ \Omega$, E = 15 V

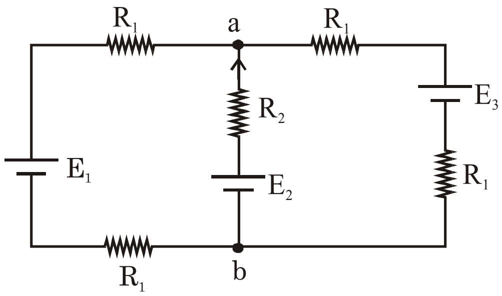
[JEE-Main(online)-2019]



(1) 7/18 (2) 13/24 (3) 9/32 (4) 20/3

CE0187

68. For the circuit shown, with $R_1 = 1.0\Omega$, $R_2 = 2.0\ \Omega$, $E_1 = 2$ V and $E_2 = E_3 = 4$ V, the potential difference between the points 'a' and 'b' is approximately (in V) : **[JEE-Main(online)-2019]**



(1) 2.7 (2) 3.3 (3) 2.3 (4) 3.7

CE0188

69. Voltage rating of a parallel plate capacitor is 500V. Its dielectric can withstand a maximum electric field of 10^6 V/m. The plate area is $10^{-4}\ m^2$. What is the dielectric constant if the capacitance is 15 pF?

(Given $\in_0 = 8.86 \times 10^{-12}\ C^2/Nm^2$) **[JEE-Main(online)-2019]**

(1) 3.8 (2) 4.5 (3) 6.2 (4) 8.5

CP0129

70. The resistance of a galvanometer is 50 ohm and the maximum current which can be passed through it is 0.002 A. What resistance must be connected to it in order to convert it into an ammeter of range 0 – 0.5 A ? **[JEE-Main(online)-2019]**

(1) 0.2 ohm (2) 0.002 ohm (3) 0.02 ohm (4) 0.5 ohm

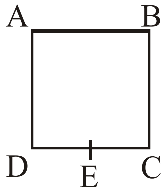
CE0189

71. The parallel combination of two air filled parallel plate capacitors of capacitance C and nC is connected to a battery of voltage, V. When the capacitors are fully charged, the battery is removed and after that a dielectric material of dielectric constant K is placed between the two plates of the first capacitor. The new potential difference of the combined system is :- **[JEE-Main(online)-2019]**

(1) $\frac{V}{K+n}$ (2) V (3) $\frac{(n+1)V}{(K+n)}$ (4) $\frac{nV}{K+n}$

CP0130

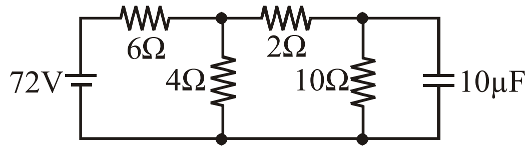
72. A wire of resistance R is bent to form a square ABCD as shown in the figure. The effective resistance between E and C is : (E is mid-point of arm CD) **[JEE-Main(online)-2019]**



(1) R (2) $\frac{1}{16}R$ (3) $\frac{7}{64}R$ (4) $\frac{3}{4}R$

CE0190

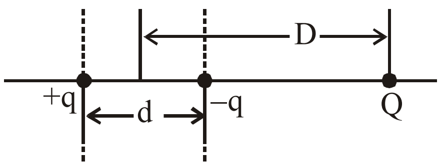
73. Determine the charge on the capacitor in the following circuit : **[JEE-Main(online)-2019]**



(1) 2μC (2) 60μC (3) 200μC (4) 10μC

CE0191

74. A system of three charges are placed as shown in the figure : **[JEE-Main(online)-2019]**



If D >> d, the potential energy of the system is best given by :

(1) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{2D^2}\right]$ (2) $\frac{1}{4\pi\varepsilon_0}\left[+\frac{q^2}{d}+\frac{qQd}{D^2}\right]$

(3) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}+\frac{2qQd}{D^2}\right]$ (4) $\frac{1}{4\pi\varepsilon_0}\left[-\frac{q^2}{d}-\frac{qQd}{D^2}\right]$

ES0244

75. A moving coil galvanometer has resistance 50Ω and it indicates full deflection at 4mA current. A voltmeter is made using this galvanometer and a 5 kΩ resistance. The maximum voltage, that can be measured using this voltmeter, will be close to : **[JEE-Main(online)-2019]**

(1) 10 V (2) 20 V (3) 40 V (4) 15 V

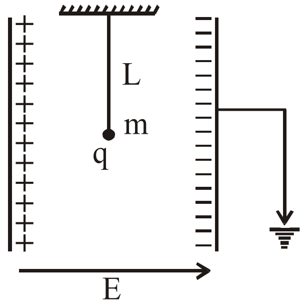
CE0192

76. A capacitor with capacitance 5μF is charged to 5μC. If the plates are pulled apart to reduce the capacitance to 2μF, how much work is done ? **[JEE-Main(online)-2019]**

(1) 3.75×10^{-6} J (2) 2.55×10^{-6} J (3) 2.16×10^{-6} J (4) 6.25×10^{-6} J

CP0131

77. A simple pendulum of length L is placed between the plates of a parallel plate capacitor having electric field E, as shown in figure. Its bob has mass m and charge q. The time period of the pendulum is given by : **[JEE-Main(online)-2019]**



(1) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2+\left(\dfrac{qE}{m}\right)^2}}}$ (2) $2\pi\sqrt{\dfrac{L}{\left(g+\dfrac{qE}{m}\right)}}$ (3) $2\pi\sqrt{\dfrac{L}{\left(g-\dfrac{qE}{m}\right)}}$ (4) $2\pi\sqrt{\dfrac{L}{\sqrt{g^2-\dfrac{q^2E^2}{m^2}}}}$

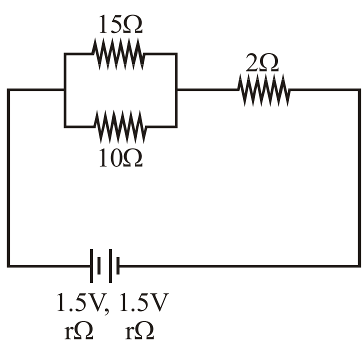
ES0245

78. Space between two concentric conducting spheres of radii a and b (b > a) is filled with a medium of resistivity ρ. The resistance between the two spheres will be : **[JEE-Main(online)-2019]**

(1) $\dfrac{\rho}{4\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$ (2) $\dfrac{\rho}{2\pi}\left(\dfrac{1}{a}-\dfrac{1}{b}\right)$ (3) $\dfrac{\rho}{2\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$ (4) $\dfrac{\rho}{4\pi}\left(\dfrac{1}{a}+\dfrac{1}{b}\right)$

ES0246

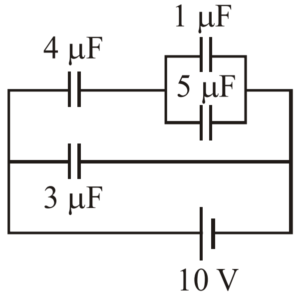
79. In the given circuit, an ideal voltmeter connected across the 10Ω resistance reads 2V. The internal resistance r, of each cell is : **[JEE-Main(online)-2019]**



(1) 1Ω (2) 1.5Ω (3) 0Ω (4) 0.5Ω

CE0193

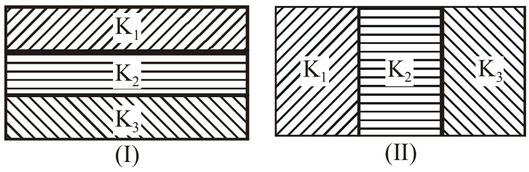
80. In the given circuit, the charge on 4 μF capacitor will be : **[JEE-Main(online)-2019]**



(1) 5.4 μC (2) 24 μC (3) 13.4 μC (4) 9.6 μC

CP0132

81. Two identical parallel plate capacitors, of capacitance C each, have plates of area A, separated by a distance d. The space between the plates of the two capacitors, is filled with three dielectrics, of equal thickness and dielectric constants K_1, K_2 and K_3. The first capacitor is filled as shown in fig. I, and the second one is filled as shown in fig. II. **[JEE-Main(online)-2019]**
If these two modified capacitors are charged by the same potential V, the ratio of the energy stored in the two, would be (E_1 refers to capacitor (I) and E_2 to capacitor (II)) :

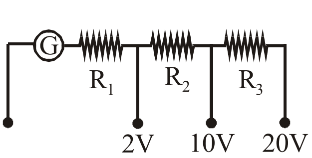


(1) $\frac{E_1}{E_2} = \frac{9K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$ (2) $\frac{E_1}{E_2} = \frac{K_1K_2K_3}{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}$

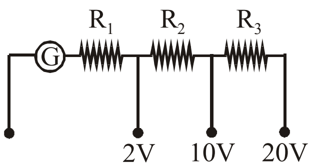
(3) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{K_1K_2K_3}$ (4) $\frac{E_1}{E_2} = \frac{(K_1+K_2+K_3)(K_2K_3+K_3K_1+K_1K_2)}{9K_1K_2K_3}$

CP0133

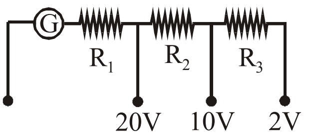
82. A galvanometer of resistance 100Ω has 50 divisions on its scale and has sensitivity of 20 μA/division. It is to be converted to a voltmeter with three ranges, of 0–2 V, 0–10 V and 0–20 V. The appropriate circuit to do so is : **[JEE-Main(online)-2019]**

(1) 

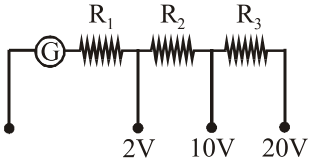
R_1 = 1900 Ω
R_2 = 9900 Ω
R_3 = 19900 Ω

(2) 

R_1 = 2000 Ω
R_2 = 8000 Ω
R_3 = 10000 Ω

(3) 

R_1 = 19900 Ω
R_2 = 9900 Ω
R_3 = 1900 Ω

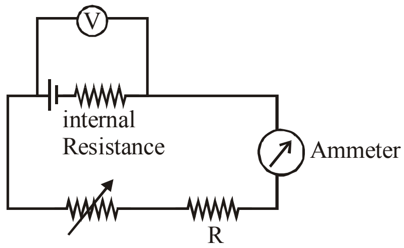
(4) 

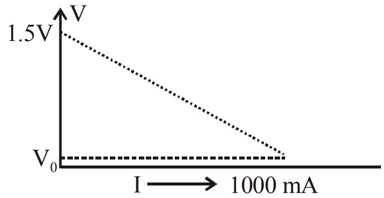
R_1 = 1900 Ω
R_2 = 8000 Ω
R_3 = 10000 Ω

CE0194

83. To verify Ohm's law, a student connects the voltmeter across the battery as, shown in the figure. The measured voltage is plotted as a function of the current, and the following graph is obtained:

[JEE-Main(online)-2019]





If V_0 is almost zero, identify the correct statement:

(1) The value of the resistance R is 1.5 Ω

(2) The emf of the battery is 1.5 V and the value of R is 1.5 Ω

(3) The emf of the battery is 1.5 V and its internal resistance is 1.5 Ω

(4) The potential difference across the battery is 1.5 V when it sends a current of 1000 mA.

CE0195

ANSWER KEY

01_ELECTROSTATICS

EXERCISE (S)

1. Ans. 9 **2. Ans.** 9.30 **3. Ans.** 8 **4. Ans.** 2

5. Ans. 2 **6. Ans.** $E_{O_1} = \frac{7\rho R}{12 \in_o}$ **7. Ans.** 5 **8. Ans.** $W = \frac{Qq}{8\pi\varepsilon_0}\left[\frac{1}{r_1} - \frac{1}{r_2}\right]$

9. Ans. (a) 6.67 ms^{-1} (b) $\sqrt{41}$ m/s **10. Ans.** $2\tan^{-1}\left(\frac{\sigma q}{2\varepsilon_0 mg}\right)$

11. Ans. 2 **12. Ans.** 20 **13. Ans.** $-\frac{kq^2}{a}\left(3-\sqrt{2}\right)$

14. Ans. $\left[\frac{2KQq}{mR}\left(\frac{r-R}{r} + \frac{3}{8}\right)\right]^{1/2}$ **15. Ans.** 7 **16. Ans.** 4

17. Ans. $\frac{kP}{\sqrt{2}\,y^3}(-\hat{i} - 2\hat{j})$ **18. Ans.** (a) $K.E = \frac{P}{4\pi\varepsilon_0}\frac{Q}{d^2}$, (b) $\frac{QP}{2\pi\varepsilon_0 d^3}$ along positive x-axis

19. Ans. $-Q/3$ **20. Ans.** $V' = \left(\frac{a}{3t}\right)^{1/3} V$

EXERCISE (O)

1. Ans. (C)	**2. Ans. (B)**	**3. Ans. (C)**	**4. Ans. (C)**	**5. Ans. (B)**	**6. Ans. (A)**
7. Ans. (D)	**8. Ans. (B)**	**9. Ans. (A)**	**10. Ans. (A)**	**11. Ans. (A)**	**12. Ans. (C)**
13. Ans. (A)	**14. Ans. (C)**	**15. Ans. (A)**	**16. Ans. (B)**	**17. Ans. (D)**	**18. Ans. (B)**
19. Ans. (D)	**20. Ans. (A)**	**21. Ans. (D)**	**22. Ans. (A)**	**23. Ans. (D)**	**24. Ans. (D)**
25. Ans. (B)	**26. Ans. (C)**	**27. Ans. (C)**	**28. Ans. (D)**	**29. Ans. (C)**	**30. Ans. (D)**
31. Ans. (A)	**32. Ans. (B)**	**33. Ans. (B)**	**34. Ans. (D)**	**35. Ans. (B)**	**36. Ans. (A)**
37. Ans. (C)	**38. Ans. (C)**	**39. Ans. (C)**	**40. Ans. (C)**	**41. Ans. (B)**	**42. Ans. (A)**
43. Ans. (B)	**44. Ans. (C)**	**45. Ans. (A)**	**46. Ans. (D)**	**47. Ans. (B)**	**48. Ans. (A)**
49. Ans. (A)	**50. Ans. (C)**	**51. Ans. (C)**	**52. Ans. (D)**	**53. Ans. (D)**	

02_CURRENT ELECTRICITY

EXERCISE (S)

1. Ans. $I\rho/A$ **2. Ans.** $S = e\,n\,l\,\langle v \rangle / j$ **3. Ans.** $p = \frac{I m_e \ell}{e}$ **4. Ans.** 3 : 1

5. Ans. (a) (i) in series, (ii) all in parallel: n^2.

(b) (i) Join 1Ω, 2Ω in parallel and the combination in series with 3Ω,

(ii) parallel combination of 2 Ω and 3 Ω in series with 1 Ω,

(iii) all in series, (iv) all in parallel.

(c) (i) (16/3) Ω, (ii) 5 R.

6. Ans. 600Ω **7. Ans.** 7.5 mA **8. Ans.** 1 V **9. Ans.** I = 2.5 A, V = 3.5 Volts

10. Ans. 20/3 V **12. Ans.** 1 Ω **13. Ans.** $x = \frac{4}{3}$ V, $12\frac{1}{3}$ V, $\frac{1}{15}$ A **14. Ans.** 5

15. Ans. $\sqrt{R_1 R_2}$ **16. Ans.** 25 **17. Ans.** 3 **18. Ans.** 12A, –20W

19. Ans. 4/9 kg/sec., 450 sec **20. Ans.** 20 ohm **21. Ans.** $R_1 = 0.0278\Omega$, $R_2 = 0.25\Omega$, $R_3 = 2.5\Omega$

22. Ans. 233.3Ω, 144V

23. Ans. Battery should be connected across A and B. Out put can be taken across the terminals A and C or B and C

24. Ans. $\frac{10}{3}$ Ω, 5 Ω **25. Ans.** (a) +ve, $E_\ell > E$ (b) –ve **26. Ans.** 2.25 V

27. Ans. 4 ohm **28. Ans.** 46.67 cm

EXERCISE (O)

1. Ans. (C)	**2. Ans. (C)**	**3. Ans. (D)**	**4. Ans. (B)**	**5. Ans. (A)**	**6. Ans. (B)**
7. Ans. (C)	**8. Ans. (B)**	**9. Ans. (C)**	**10. Ans. (D)**	**11. Ans. (A)**	**12. Ans. (D)**
13. Ans. (B,C)	**14. Ans. (B)**	**15. Ans. (C)**	**16. Ans. (B)**	**17. Ans. (D)**	**18. Ans. (A)**
19. Ans. (C)	**20. Ans. (B)**	**21. Ans. (A)**	**22. Ans. (A)**	**23. Ans. (A)**	**24. Ans. (B)**
25. Ans. (B)	**26. Ans. (A)**	**27. Ans. (B)**	**28. Ans. (B)**	**29. Ans. (C)**	**30. Ans. (C)**
31. Ans. (B)	**32. Ans. (A)**	**33. Ans. (D)**	**34. Ans. (A)**	**35. Ans. (B)**	**36. Ans. (B)**
37. Ans. (C)	**38. Ans. (B)**	**39. Ans. (C)**	**40. Ans. (D)**	**41. Ans. (C)**	**42. Ans. (A)**
43. Ans. (A)	**44. Ans. (C)**	**45. Ans. (B)**	**46. Ans. (A)**	**47. Ans. (D)**	**48. Ans. (B)**
49. Ans. (A)					

03_CAPACITANCE

EXERCISE (S)

1. Ans. $\frac{8}{3}\mu F$	**2. Ans. 4**	**3. Ans.** $\frac{32}{23}\ \mu F$	**4. Ans. C**	**5. Ans.** 30 V
6. Ans. 3Q/2C	**7. Ans. 0**	**8. Ans. 12μC**	**9. Ans.** $\frac{25}{24}\frac{\varepsilon_0 A}{d}$	**10. Ans. 5**
11. Ans. 2.5 kV	**12. Ans.** 4.425×10^{-9} Ampere	**13. Ans. 4**		**14. Ans. 3**
15. Ans. 8	**16. Ans. 225**	**17. Ans. 750**	**18. Ans.** $Q_0 = \frac{CVR_2}{R_1+R_2}$ and $\alpha = \frac{R_1+R_2}{CR_1R_2}$	
19. Ans. 3	**20. Ans.** $\rho = \tau/\varepsilon_0\varepsilon \ln 2 = 1.4 \times 10^{13}\ \Omega$.m.		**21. Ans. 1**	

EXERCISE (O)

1. Ans. (C)	**2. Ans. (A)**	**3. Ans. (B)**	**4. Ans. (D)**	**5. Ans. (A)**	**6. Ans. (B)**
7. Ans. (B)	**8. Ans. (C)**	**9. Ans. (A)**	**10. Ans. (D)**	**11. Ans. (C)**	**12. Ans. (B)**
13. Ans. (D)	**14. Ans. (C)**	**15. Ans. (C)**	**16. Ans. (C)**	**17. Ans. (B)**	**18. Ans. (B)**
19. Ans. (B)	**20. Ans. (B)**	**21. Ans. (C)**	**22. Ans. (B)**	**23. Ans. (D)**	**24. Ans. (D)**

EXERCISE (JM)

1. Ans. (4)	**2. Ans. (3)**	**3. Ans. (1)**	**4. Ans. (4)**	**5. Ans. (4)**	**6. Ans. (3)**
7. Ans. (2)	**8. Ans. (1)**	**9. Ans. (4)**	**10. Ans. (4)**	**11. Ans. (1)**	**12. Ans. (4)**
13. Ans. (4)	**14. Ans. (4)**	**15. Ans. (4)**	**16. Ans. (1)**	**17. Ans. (2)**	**18. Ans. (1)**
19. Ans. (4)	**20. Ans. (4)**	**21. Ans. (4)**	**22. Ans. (2)**	**23. Ans. (1)**	**24. Ans. (1)**
25. Ans. (3)	**26. Ans. (1 or 2)**	**27. Ans. (3)**	**28. Ans. (2)**	**29. Ans. (4)**	**30. Ans. (2)**
31. Ans. (2)	**32. Ans. (2)**	**33. Ans. (3)**	**34. Ans. (4)**	**35. Ans. (3)**	**36. Ans. (4)**
37. Ans. (1)	**38. Ans. (3)**	**39. Ans. (2)**	**40. Ans. (1)**	**41. Ans. (1)**	**42. Ans. (1)**
43. Ans.(1)	**44. Ans.(2)**	**45. Ans.(2)**	**46. Ans.(2)**	**47. Ans.(3)**	**48. Ans.(4)**
49. Ans.(4)	**50. Ans.(4)**	**51. Ans.(1)**	**52. Ans.(2)**	**53. Ans.(3)**	**54. Ans.(4)**
55. Ans. (1)	**56. Ans (2)**	**57. Ans.(3)**	**58. Ans.(4)**	**59. Ans.(3)**	**60. Ans.(4)**
61. Ans.(3)	**62. Ans.(3)**	**63. Ans.(2)**	**64. Ans.(4)**	**65. Ans.(3)**	**66. Ans.(3)**
67. Ans.(3)	**68. Ans.(2)**	**69. Ans.(4)**	**70. Ans.(1)**	**71. Ans.(3)**	**72. Ans.(3)**
73. Ans.(3)	**74. Ans.(4)**	**75. Ans.(2)**	**76. Ans.(1)**	**77. Ans.(1)**	**78. Ans.(1)**
79. Ans.(4)	**80. Ans.(2)**	**81. Ans.(1)**	**82. Ans.(4)**	**83. Ans.(3)**	

Important Notes