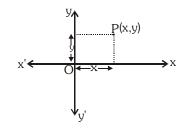
1. INTRODUCTION OF COORDINATE GEOMETRY:

Coordinate geometry is the combination of algebra and geometry. A systematic study of geometry by the use of algebra was first carried out by celebrated French philosopher and mathematician René Descartes. The resulting combination of analysis and geometry is referred as *analytical geometry*.

2. CARTESIAN CO-ORDINATES SYSTEM:

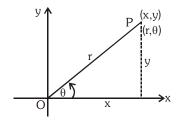
In two dimensional coordinate system, two lines are used; the lines are at right angles, forming a rectangular coordinate system. The horizontal axis is the x-axis and the vertical axis is y-axis. The point of intersection O is the origin of the coordinate system. Distances along the x-axis to the right of the origin are taken as positive, distances to the left as negative.



Distances along the y-axisabove the origin are positive; distances below are negative. The position of a point anywhere in the plane can be specified by two numbers, the coordinates of the point, written as (x, y). The x-coordinate (or abscissa) is the distance of the point from the y-axis in a direction parallel to the x-axis (i.e. horizontally). The y-coordinate (or ordinate) is the distance from the x-axis in a direction parallel to the y-axis (vertically). The origin O is the point (0, 0).

3. POLAR CO-ORDINATES SYSTEM:

A coordinate system in which the position of a point is determined by the length of a line segment from a fixed origin together with the angle that the line segment makes with a fixed line. The origin is called the pole and the line segment is the radius vector (r).



The angle θ between the polar axis and the radius vector is called the vectorial angle. By convention, positive values of θ are measured in an anticlockwise sense, negative values in clockwise sense. The coordinates of the point are then specified as (r, θ) .

If (x,y) are cartesian co-ordinates of a point P, then : $x = r \cos \theta$, $y = r \sin \theta$

$$\text{and} \quad r = \sqrt{x^2 + y^2} \qquad \quad , \quad \theta = tan^{-1} \bigg(\frac{y}{x}\bigg)$$

4. DISTANCE FORMULA AND ITS APPLICATIONS:

If $A(x_1, y_1)$ and $B(x_2, y_2)$ are two points, then $AB = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2}$

Note:

- (i) Three given points A,B and C are collinear, when sum of any two distances out of AB,BC, CA is equal to the remaining third otherwise the points will be the vertices of a triangle.
- (ii) Let A,B,C & D be the four given points in a plane. Then the quadrilateral will be:

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- Square if AB = BC = CD = DA & AC = BD $AC \perp BD$ (a)
- (b) Rhombus if AB = BC = CD = DA and $AC \neq BD$; $AC \perp BD$
- Parallelogram if AB = DC, BC = AD; $AC \neq BD$; AC ∠ BD (c)
- Rectangle if AB = CD, BC = DA, AC = BDAC ∡ BD (d)

Illustration 1: The number of points on x-axis which are at a distance c(c < 3) from the point (2, 3) is

- (A) 2
- (B) 1
- (C) infinite
- (D) no point

Let a point on x-axis is $(x_1, 0)$ then its distance from the point (2, 3)**Solution:**

$$=\sqrt{(x_1-2)^2+9}=c$$
 or $(x_1-2)^2=c^2-9$

$$\therefore x_1 - 2 = \pm \sqrt{c^2 - 9} \text{ since } c < 3 \Rightarrow c^2 - 9 < 0$$

 \therefore x_1 will be imaginary.

Ans. (D)

Illustration 2: The distance between the point $P(a\cos\alpha, a\sin\alpha)$ and $Q(a\cos\beta, a\sin\beta)$, where a>0&

 $\alpha > \beta$, is -

- (A) $4a \sin \frac{\alpha \beta}{2}$ (B) $2a \sin \frac{\alpha + \beta}{2}$ (C) $2a \sin \frac{\alpha \beta}{2}$ (D) $2a \cos \frac{\alpha \beta}{2}$

 $d^2 = \left(a\cos\alpha - a\cos\beta\right)^2 + \left(a\sin\alpha - a\sin\beta\right)^2 = a^2\left(\cos\alpha - \cos\beta\right)^2 + a^2\left(\sin\alpha - \sin\beta\right)^2$ Solution:

$$=a^{2}\left\{2\sin\frac{\alpha+\beta}{2}\sin\frac{\beta-\alpha}{2}\right\}^{2}+a^{2}\left\{2\cos\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}\right\}^{2}$$

$$=4a^{2}\sin^{2}\frac{\alpha-\beta}{2}\left\{\sin^{2}\frac{\alpha+\beta}{2}+\cos^{2}\frac{\alpha+\beta}{2}\right\}=4a^{2}\sin^{2}\frac{\alpha-\beta}{2}\Rightarrow d=2a\sin\frac{\alpha-\beta}{2}$$

Ans. (C)

Do yourself - 1:

- Find the distance between the points P(-3, 2) and Q(2, -1). **(i)**
- If the distance between the points P(-3, 5) and Q(-x, -2) is $\sqrt{58}$, then find the value(s) of x. (ii)
- A line segment is of the length 15 units and one end is at the point (3, 2), if the abscissa of the (iii) other end is 15, then find possible ordinates.

5. **SECTION FORMULA:**

The co-ordinates of a point dividing a line joining the points $P(x_1,y_1)$ and $Q(x_2,y_2)$ in the ratio m:n is given by:

For internal division : $P - R - Q \implies$ R divides line segment PQ, internally.

$$(x, y) \equiv \left(\frac{mx_2 + nx_1}{m + n}, \frac{my_2 + ny_1}{m + n}\right)$$

$$\begin{array}{ccc}
 & m & R(x,y) & n \\
P(x_1,y_1) & & Q(x_2,y_2)
\end{array}$$

(b) For external division: R - P - Q or $P - Q - R \Rightarrow R$ divides line segment PQ, externally.

$$(x, y) \equiv \left(\frac{mx_2 - nx_1}{m - n}, \frac{my_2 - ny_1}{m - n}\right)$$

$$\frac{(PR)}{(QR)} < 1 \quad \Rightarrow \quad \text{R lies on the left of P & } \frac{(PR)}{(QR)} > 1 \qquad \Rightarrow \quad \text{R lies on the right of Q}$$

Illustration 3: Determine the ratio in which y - x + 2 = 0 divides the line joining (3, -1) and (8, 9).

Suppose the line y - x + 2 = 0 divides the line segment joining A(3, -1) and B(8, 9) in the ratio $\lambda: 1$ at a point P, then the co-ordinates of the point P are $\left(\frac{8\lambda + 3}{\lambda + 1}, \frac{9\lambda - 1}{\lambda + 1}\right)$

But P lies on y - x + 2 = 0 therefore $\left(\frac{9\lambda - 1}{\lambda + 1}\right) - \left(\frac{8\lambda + 3}{\lambda + 1}\right) + 2 = 0$

$$\Rightarrow$$
 $9\lambda - 1 - 8\lambda - 3 + 2\lambda + 2 = 0$

$$\Rightarrow$$
 $3\lambda - 2 = 0$ or $\lambda = \frac{2}{3}$

So, the required ratio is $\frac{2}{3}$: 1, i.e., 2:3 (internally) since here λ is positive.

Do yourself - 2:

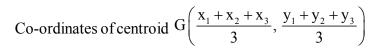
- (i) Find the co-ordinates of the point dividing the join of A(1, -2) and B(4, 7):
 - (a) Internally in the ratio 1:2
- (b) Externally in the ratio of 2:1
- (ii) In what ratio is the line joining A(8, 9) and B(-7, 4) divided by
 - (a) the point (2, 7)
- (b) the x-axis
- (c) the y-axis.

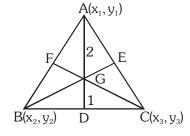
6. CO-ORDINATES OF SOME PARTICULAR POINTS:

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of any triangle ABC, then

(a) Centroid:

The centroid is the point of intersection of the medians (line joining the mid point of sides and opposite vertices). Centroid divides each median in the ratio of 2:1.





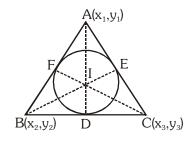
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(b) Incenter:

The incenter is the point of intersection of internal bisectors of the angles of a triangle. Also it is a centre of the circle touching all the sides of a triangle.

Co-ordinates of incenter I $\left(\frac{ax_1 + bx_2 + cx_3}{a + b + c}, \frac{ay_1 + by_2 + cy_3}{a + b + c}\right)$ where a, b, c are the sides of triangle ABC.

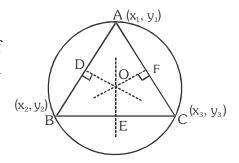


Note:

- (i) Angle bisector divides the opposite sides in the ratio of remaining sides. e.g. $\frac{BD}{DC} = \frac{AB}{AC} = \frac{c}{b}$
- (ii) Incenter divides the angle bisectors in the ratio (b+c): a, (c+a): b, (a+b): c.

(c) Circumcenter:

It is the point of intersection of perpendicular bisectors of the sides of a triangle. If O is the circumcenter of any triangle ABC, then $OA^2 = OB^2 = OC^2$. Also it is a centre of a circle touching all the vertices of a triangle.

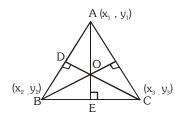


Note:

- (i) If the triangle is right angled, then its circumcenter is the mid point of hypotenuse.
- (ii) Co-ordinates of circumcenter $\left(\frac{x_1 \sin 2A + x_2 \sin 2B + x_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}, \frac{y_1 \sin 2A + y_2 \sin 2B + y_3 \sin 2C}{\sin 2A + \sin 2B + \sin 2C}\right)$

(d) Orthocenter:

It is the point of intersection of perpendiculars drawn from vertices on the opposite sides of a triangle and it can be obtained by solving the equation of any two altitudes.



Note:

- (i) If a triangle is right angled, then orthocenter is the point where right angle is formed.
- (ii) Co-ordinates of circumcenter $\left(\frac{x_1 \tan A + x_2 \tan B + x_3 \tan C}{\tan A + \tan B + \tan C}, \frac{y_1 \tan A + y_2 \tan B + y_3 \tan C}{\tan A + \tan B + \tan C}\right)$

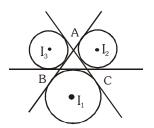
Remarks:

- (i) If the triangle is equilateral, then centroid, incentre, orthocenter, circumcenter, coincide.
- (ii) Orthocentre, centroid and circumcentre are always collinear and centroid divides the line joining orthocentre and circumcentre in the ratio 2:1
- (iii) In an isosceles triangle centroid, orthocentre, incentre & circumcentre lie on the same line.

5

(e) Ex-centers:

The centre of a circle which touches side BC and the extended portions of sides AB and AC is called the ex-centre of \triangle ABC with respect to the vertex A. It is denoted by I₁ and its coordinates



are
$$I_1\left(\frac{-ax_1+bx_2+cx_3}{-a+b+c}, \frac{-ay_1+by_2+cy_3}{-a+b+c}\right)$$

Similarly ex-centers of ΔABC with respect to vertices B and C are denoted by I_2 and I_3 respectively , and

$$I_{2}\left(\frac{ax_{1}-bx_{2}+cx_{3}}{a-b+c},\frac{ay_{1}-by_{2}+cy_{3}}{a-b+c}\right),I_{3}\left(\frac{ax_{1}+bx_{2}-cx_{3}}{a+b-c},\frac{ay_{1}+by_{2}-cy_{3}}{a+b-c}\right)$$

Illustration 4: If $\left(\frac{5}{3}, 3\right)$ is the centroid of a triangle and its two vertices are (0, 1) and (2, 3), then find its third vertex, circumcentre, circumradius & orthocentre.

Solution: Let the third vertex of triangle be (x, y), then

$$\frac{5}{3} = \frac{x+0+2}{3}$$
 \Rightarrow x = 3 and 3 = $\frac{y+1+3}{3}$ \Rightarrow y = 5. So third vertex is (3, 5).

Now three vertices are A(0, 1), B(2, 3) and C(3, 5)

Let circumcentre be P(h, k),

then
$$AP = BP = CP = R$$
 (circumradius) $\Rightarrow AP^2 = BP^2 = CP^2 = R^2$

$$h^{2}+(k-1)^{2}=(h-2)^{2}+(k-3)^{2}=(h-3)^{2}+(k-5)^{2}=R^{2}$$
(i)

from the first two equations, we have

$$h + k = 3$$
 (ii)

from the first and third equation, we obtain

$$6h + 8k = 33$$
 (iii)

On solving, (ii) & (iii), we get

$$h = -\frac{9}{2}, k = \frac{15}{2}$$

Substituting these values in (i), we have

$$R = \frac{5}{2}\sqrt{10} \qquad \qquad \frac{2}{G\left(\frac{5}{3},3\right)} \qquad C\left(-\frac{9}{2},\frac{15}{2}\right)$$

Let $O(x_1, y_1)$ be the orthocentre, then $\frac{x_1 + 2\left(-\frac{9}{2}\right)}{3} = \frac{5}{3} \implies x_1 = 14, \frac{y_1 + 2\left(\frac{15}{2}\right)}{3} = 3$

 \Rightarrow $y_1 = -6$. Hence orthocentre of the triangle is (14, -6).

Illustration 5: The vertices of a triangle are A(0, -6), B(-6, 0) and C(1, 1) respectively, then coordinates of the ex-centre opposite to vertex A is:

(A)
$$\left(\frac{-3}{2}, \frac{-3}{2}\right)$$
 (B) $\left(-4, \frac{3}{2}\right)$ (C) $\left(\frac{-3}{2}, \frac{3}{2}\right)$

(B)
$$\left(-4, \frac{3}{2}\right)$$

$$(C)\left(\frac{-3}{2},\frac{3}{2}\right)$$

Solution:

$$a = BC = \sqrt{(-6-1)^2 + (0-1)^2} = \sqrt{50} = 5\sqrt{2}$$

$$b = CA = \sqrt{(1-0)^2 + (1+6)^2} = \sqrt{50} = 5\sqrt{2}$$

$$c = AB = \sqrt{(0+6)^2 + (-6-0)^2} = \sqrt{72} = 6\sqrt{2}$$

coordinates of ex-centre opposite to vertex A will be:

$$x = \frac{-ax_1 + bx_2 + cx_3}{-a + b + c} = \frac{-5\sqrt{2}.0 + 5\sqrt{2}\left(-6\right) + 6\sqrt{2}\left(1\right)}{-5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} = \frac{-24\sqrt{2}}{6\sqrt{2}} = -4$$

$$y = \frac{-ay_1 + by_2 + cy_3}{-a + b + c} = \frac{-5\sqrt{2}(-6) + 5\sqrt{2} \cdot 0 + 6\sqrt{2}(1)}{-5\sqrt{2} + 5\sqrt{2} + 6\sqrt{2}} = \frac{36\sqrt{2}}{6\sqrt{2}} = 6$$

Hence coordinates of ex-centre is (-4, 6)

Ans. (D)

Do yourself - 3:

- (i) The coordinates of the vertices of a triangle are (0, 1), (2, 3) and (3, 5):
 - Find centroid of the triangle.
 - (b) Find circumcentre & the circumradius.
 - Find orthocentre of the triangle. (c)

7. **AREA OF TRIANGLE:**

Let $A(x_1,y_1)$, $B(x_2,y_2)$ and $C(x_3,y_3)$ are vertices of a triangle, then

Area of
$$\triangle ABC = \begin{vmatrix} 1 \\ 2 \\ x_2 \\ x_3 \\ y_3 \\ 1 \end{vmatrix} \begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = \frac{1}{2} |[x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)]|$$

To remember the above formula, take the help of the following method:

$$= \frac{1}{2} \begin{bmatrix} x_1 \\ y_1 \\ y_2 \end{bmatrix} \underbrace{x_2 \\ y_3} \underbrace{x_3 \\ y_1} \underbrace{x_1} = \begin{bmatrix} \frac{1}{2} \left[(x_1 y_2 - x_2 y_1) + (x_2 y_3 - x_3 y_2) + (x_3 y_1 - x_1 y_3) \right] \end{bmatrix}$$

Remarks:

- If the area of triangle joining three points is zero, then the points are collinear. (i)
- Area of Equilateral triangle: If altitude of any equilateral triangle is P, then its area = $\frac{P^2}{\sqrt{3}}$. If (ii)

'a' be the side of equilateral triangle, then its area = $\left(\frac{a^2\sqrt{3}}{4}\right)$.

Area of quadrilateral with given vertices $A(x_1, y_1)$, $B(x_2, y_2)$, $C(x_3, y_3)$, $D(x_4, y_4)$ (iii)

Area of quad. ABCD =
$$\frac{1}{2} \begin{bmatrix} x_1 & x_2 & x_3 & x_4 & x_1 \\ y_1 & y_2 & y_3 & y_4 & y_1 \end{bmatrix}$$

Note: Area of a polygon can be obtained by dividing the polygon into disjoined triangles and then adding their areas.

Illustration 6: If the vertices of a triangle are (1, 2), (4, -6) and (3, 5) then its area is

(A)
$$\frac{25}{2}$$
 sq. units (B) 12 sq. units (C) 5 sq. units

Solution:

$$\Delta = \frac{1}{2} \left[1(-6-5) + 4(5-2) + 3(2+6) \right] = \frac{1}{2} \left[-11 + 12 + 24 \right] = \frac{25}{2} \text{ square units } \mathbf{Ans.} (\mathbf{A})$$

Illustration 7: The point A divides the join of the points (-5, 1) and (3, 5) in the ratio k:1 and coordinates of points B and C are (1, 5) and (7, -2) respectively. If the area of \triangle ABC be 2 units, then k equals -

(C)
$$7, \frac{31}{9}$$

(C)
$$7, \frac{31}{9}$$
 (D) $9, \frac{31}{9}$

Ans. (C)

Solution:

$$A \equiv \left(\frac{3k-5}{k+1}, \frac{5k+1}{k+1}\right)$$

Area of
$$\triangle ABC = 2$$
 units $\Rightarrow \frac{1}{2} \left[\frac{3k-5}{k+1} (5+2) + 1 \left(-2 - \frac{5k+1}{k+1} \right) + 7 \left(\frac{5k+1}{k+1} - 5 \right) \right] = \pm 2$
 $\Rightarrow 14k - 66 = \pm 4(k+1) \Rightarrow k = 7 \text{ or } \frac{31}{9}$ Ans. (C

Illustration 8: Prove that the co-ordinates of the vertices of an equilateral triangle can not all be rational.

Let $A(x_1, y_1)$, $B(x_2, y_2)$ and $C(x_3, y_3)$ be the vertices of a triangle ABC. If possible let x_1 , **Solution:** y_1 , x_2 , y_2 , x_3 , y_3 be all rational.

Now area of
$$\triangle ABC = \frac{1}{2} |x_1(y_2 - y_3) + x_2(y_3 - y_1) + x_3(y_1 - y_2)| = Rational$$
(i)

Since $\triangle ABC$ is equilateral

:. Area of
$$\triangle ABC = \frac{\sqrt{3}}{4} (\text{side})^2 = \frac{\sqrt{3}}{4} (AB)^2 = \frac{\sqrt{3}}{4} \{ (x_1 - x_2)^2 + (y_1 - y_2)^2 \} = \text{Irrational} \dots$$
 (ii)

From (i) and (ii),

Rational = Irrational

which is contradiction

Hence x_1 , y_1 , x_2 , y_2 , x_3 , y_3 cannot all be rational.

8. CONDITIONS FOR COLLINEARITY OF THREE GIVEN POINTS:

Three given points A (x_1, y_1) , B (x_2, y_2) , C (x_3, y_3) are collinear if any one of the following conditions are satisfied.

(a) Area of triangle ABC is zero i.e.
$$\begin{vmatrix} x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \\ x_3 & y_3 & 1 \end{vmatrix} = 0$$

(b) Slope of AB = slope of BC = slope of AC. i.e.
$$\frac{y_2 - y_1}{x_2 - x_1} = \frac{y_3 - y_2}{x_3 - x_2} = \frac{y_3 - y_1}{x_3 - x_1}$$

(c) Find the equation of line passing through 2 given points, if the third point satisfies the given equation of the line, then three points are collinear.

Do yourself - 4:

- (i) Find the area of the triangle whose vertices are A(1,1), B(7,-3) and C(12,2)
- (ii) Find the area of the quadrilateral whose vertices are A(1,1) B(7,-3), C(12,2) and D(7,21)
- (iii) Prove that the points A(a, b + c), B(b, c + a) and C(c, a + b) are collinear (By determinant method)

9. LOCUS:

The locus of a moving point is the path traced out by that point under one or more geometrical conditions.

(a) Equation of Locus:

The equation to a locus is the relation which exists between the coordinates of any point on the path, and which holds for no other point except those lying on the path.

$\begin{tabular}{ll} \textbf{(b)} & \textbf{Procedure for finding the equation of the locus of a point:} \\ \end{tabular}$

- (i) If we are finding the equation of the locus of a point P, assign coordinates (h, k) to P.
- (ii) Express the given condition as equations in terms of the known quantities to facilitate calculations. We sometimes include some unknown quantities known as parameters.
- (iii) Eliminate the parameters, so that the eliminant contains only h, k and known quantities.
- (iv) Replace h by x, and k by y, in the eliminant. The resulting equation would be the equation of the locus of P.
- **Illustration 9:** The ends of the rod of length ℓ moves on two mutually perpendicular lines, find the locus of the point on the rod which divides it in the ratio $m_1 : m_2$

(A)
$$m_1^2 x^2 + m_2^2 y^2 = \frac{\ell^2}{\left(m_1 + m_2\right)^2}$$

(B)
$$(m_2 x)^2 + (m_1 y)^2 = \left(\frac{m_1 m_2 \ell}{m_1 + m_2}\right)^2$$

(C)
$$(m_1 x)^2 + (m_2 y)^2 = \left(\frac{m_1 m_2 \ell}{m_1 + m_2}\right)^2$$

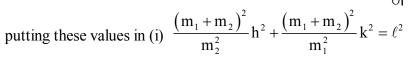
Solution:

Let (h,k) be the point that divide the rod $AB = \ell$, in the ratio $m_1 : m_2$, and OA = a, OB = b say

$$\therefore \quad a^2 + b^2 = \ell^2 \qquad \qquad \dots (i)$$

Now
$$h = \left(\frac{m_2 a}{m_1 + m_2}\right) \Rightarrow a = \left(\frac{m_1 + m_2}{m_2}\right) h$$

$$k = \left(\frac{m_1 b}{m_1 + m_2}\right) \Rightarrow b = \left(\frac{m_1 + m_2}{m_1}\right) k$$



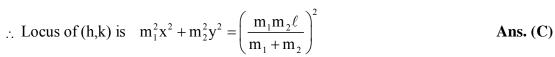


Illustration 10: A(a, 0) and B(-a, 0) are two fixed points of $\triangle ABC$. If its vertex C moves in such a way that $\cot A + \cot B = \lambda$, where λ is a constant, then the locus of the point C is -

(A)
$$y\lambda = 2a$$

(B)
$$y = \lambda a$$

(C)
$$ya = 2\lambda$$

(D) none of these

Solution:

Given that coordinates of two fixed points A and B are (a, 0) and (-a, 0) respectively. Let variable point C is (h, k). From the adjoining figure

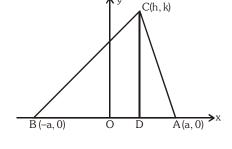
$$\cot A = \frac{DA}{CD} = \frac{a - h}{k}$$

$$\cot B = \frac{BD}{CD} = \frac{a+h}{k}$$

But $\cot A + \cot B = \lambda$, so we have

$$\frac{a-h}{k} + \frac{a+h}{k} = \lambda \implies \frac{2a}{k} = \lambda$$

Hence locus of C is $y\lambda = 2a$



Ans. (A)

Do yourself - 5:

- (i) Find the locus of a variable point which is at a distance of 2 units from the y-axis.
- (ii) Find the locus of a point which is equidistant from both the axes.

10. STRAIGHT LINE:

Introduction : A relation between x and y which is satisfied by co-ordinates of every point lying on a line is called equation of the straight line. Here, remember that every one degree equation in variable x and y always represents a straight line i.e. ax + by + c = 0; $a & b \neq 0$ simultaneously.

- (a) Equation of a line parallel to x-axis at a distance 'a' is y = a or y = -a.
- **(b)** Equation of x-axis is y = 0.
- (c) Equation of a line parallel to y-axis at a distance 'b' is $\mathbf{x} = \mathbf{b}$ or $\mathbf{x} = -\mathbf{b}$.
- (d) Equation of y-axis is x = 0.

Illustration 11: Prove that every first degree equation in x, y represents a straight line.

Solution: Let ax + by + c = 0 be a first degree equation in x, y

where a, b, c are constants.

Let $P(x_1, y_1)$ & $Q(x_2, y_2)$ be any two points on the curve represented by ax + by + c = 0. Thenax₁ + by₁ + c = 0 and $ax_2 + by_2 + c = 0$

Let R be any point on the line segment joining P & Q

Suppose R divides PQ in the ratio λ : 1. Then, the coordinates of R are

$$\left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}\right)$$

We have
$$a\left(\frac{\lambda x_2 + x_1}{\lambda + 1}\right) + b\left(\frac{\lambda y_2 + y_1}{\lambda + 1}\right) + c = \lambda \ 0 + 0 = 0$$

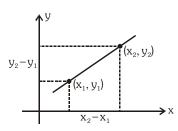
 $\therefore R\left(\frac{\lambda x_2 + x_1}{\lambda + 1}, \frac{\lambda y_2 + y_1}{\lambda + 1}\right) \text{ lies on the curve represented by ax} + by + c = 0. \text{ Thus every}$

point on the line segment joining P & Q lies on ax + by + c = 0.

Hence ax + by + c = 0 represents a straight line.

11. SLOPE OF LINE:

If a given line makes an angle $\theta(0^{\circ} \le \theta < 180^{\circ}, \theta \ne 90^{\circ})$ with the positive direction of x-axis, then slope of this line will be $\tan\theta$ and is usually denoted by the letter **m** i.e. $\mathbf{m} = \tan\theta$. If $A(x_1, y_1)$ and $B(x_2, y_2)$



&
$$x_1 \neq x_2$$
 then slope of line AB = $\frac{y_2 - y_1}{x_2 - x_1}$

Remark:

- (i) If $\theta = 90^{\circ}$, m does not exist and line is parallel to y-axis.
- (ii) If $\theta = 0^{\circ}$, $\mathbf{m} = \mathbf{0}$ and the line is parallel to **x-axis.**
- (iii) Let m₁ and m₂ be slopes of two given lines (none of them is parallel to y-axis)
 - (a) If lines are parallel, $\mathbf{m}_1 = \mathbf{m}_2$ and vice-versa.
 - (b) If lines are perpendicular, $\mathbf{m}_1 \mathbf{m}_2 = -1$ and vice-versa

12. STANDARD FORMS OF EQUATIONS OF A STRAIGHT LINE:

(a) Slope Intercept form: Let m be the slope of a line and c its intercept on y-axis. Then the equation of this straight line is written as: y = mx + c

If the line passes through origin, its equation is written as y = mx

(b) Point Slope form: If m be the slope of a line and it passes through a point (x_1, y_1) , then its equation is written as: $y - y_1 = m(x - x_1)$

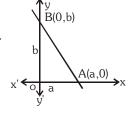
Two point form : Equation of a line passing through two points (x_1, y_1) and (x_2, y_2) is written as : (c)

$$y-y_1 = \frac{y_2-y_1}{x_2-x_1}(x-x_1)$$
 or $\begin{vmatrix} x & y & 1 \\ x_1 & y_1 & 1 \\ x_2 & y_2 & 1 \end{vmatrix} = 0$

Intercept form: If a and b are the intercepts made by a line on the axes of x and y, its equation (d)

is written as: $\frac{x}{a} + \frac{y}{b} = 1$

Length of intercept of line between the coordinate axes = $\sqrt{a^2 + b^2}$ (i)



- Area of triangle AOB = $\frac{1}{2}$ OA.OB = $\frac{1}{2}$ ab
- **Illustration 12:** The equation of the lines which passes through the point (3,4) and the sum of its intercepts on the axes is 14 is -

(A)
$$4x - 3y = 24$$
, $x - y = 7$

(B)
$$4x + 3y = 24$$
, $x + y = 7$

(C)
$$4x + 3y + 24 = 0$$
, $x + y + 7 = 0$

(C)
$$4x + 3y + 24 = 0$$
, $x + y + 7 = 0$ (D) $4x - 3y + 24 = 0$, $x - y + 7 = 0$

Let the equation of the line be $\frac{x}{a} + \frac{y}{b} = 1$ Solution:

....(i)

This passes through (3, 4), therefore $\frac{3}{2} + \frac{4}{5} = 1$(ii)

It is given that $a + b = 14 \implies b = 14 - a$. Putting b = 14 - a in (ii), we get

$$\frac{3}{a} + \frac{4}{14 - a} = 1 \implies a^2 - 13a + 42 = 0 \implies (a - 7)(a - 6) = 0 \implies a = 7, 6$$

For a = 7, b = 14 - 7 = 7 and for a = 6, b = 14 - 6 = 8

Putting the values of a and b in (i), we get the equations of the lines

$$\frac{x}{7} + \frac{y}{7} = 1$$
 and $\frac{x}{6} + \frac{y}{8} = 1$ or $x + y = 7$ and $4x + 3y = 24$ **Ans. (B)**

Illustration 13: Two points A and B move on the positive direction of x-axis and y-axis respectively, such that OA + OB = K. Show that the locus of the foot of the perpendicular from the origin O on the line AB is $(x + y)(x^2 + y^2) = Kxy$.

Solution:

Let the equation of AB be $\frac{x}{a} + \frac{y}{b} = 1$

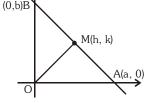
given, a + b = K

from (ii) and (iii),

..... (ii)

now,
$$m_{AB} \times m_{OM} = -1 \implies ah = bk$$

..... (iii)



$$a = \frac{kK}{h+k}$$
 and $b = \frac{hK}{h+k}$

$$\therefore \quad \text{from (i) } \frac{x(h+k)}{k.K} + \frac{y(h+k)}{h.K} = 1$$

as it passes through (h, k)

$$\frac{h(h+k)}{k.K} + \frac{k(h+k)}{h.K} = 1 \Longrightarrow (h+k)(h^2 + k^2) = Khk$$

$$\therefore$$
 locus of (h, k) is $(x + y)(x^2 + y^2) = Kxy$.

(e) Normal form: If p is the length of perpendicular on a line from the origin, and α the angle which this perpendicular makes with positive x-axis, then the equation of this line is written as: $\mathbf{x}\mathbf{cos}\alpha + \mathbf{y}\mathbf{sin}\alpha = \mathbf{p}$ (p is always positive) where $0 \le \alpha < 2\pi$.

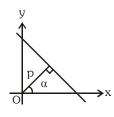


Illustration 14: Find the equation of the straight line on which the perpendicular from origin makes an angle 30° with positive x-axis and which forms a triangle of area $\left(\frac{50}{\sqrt{3}}\right)$ sq. units with the co-ordinates axes.

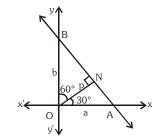
Solution:

$$\angle NOA = 30^{\circ}$$

Let
$$ON = p > 0$$
, $OA = a$, $OB = b$

In
$$\triangle ONA$$
, $\cos 30^\circ = \frac{ON}{OA} = \frac{p}{a} \Rightarrow \frac{\sqrt{3}}{2} = \frac{p}{a}$

or
$$a = \frac{2p}{\sqrt{3}}$$



and in
$$\triangle ONB$$
, $\cos 60^{\circ} = \frac{ON}{OB} = \frac{p}{b} \Rightarrow \frac{1}{2} = \frac{p}{b}$

or
$$b = 2p$$

$$\therefore \text{ Area of } \triangle OAB = \frac{1}{2} \text{ ab} = \frac{1}{2} \left(\frac{2p}{\sqrt{3}} \right) (2p) = \frac{2p^2}{\sqrt{3}}$$

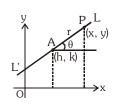
$$\therefore \frac{2p^2}{\sqrt{3}} = \frac{50}{\sqrt{3}} \implies p^2 = 25$$

or
$$p = 5$$

... Using
$$x\cos\alpha + y\sin\alpha = p$$
, the equation of the line AB is $x\cos 30^{\circ} + y\sin 30^{\circ} = 5$

or
$$x\sqrt{3} + y = 10$$

(f) Parametric form: To find the equation of a straight line which passes through a given point A(h, k) and makes a given angle θ with the positive direction of the x-axis. P(x, y) is any point on the line LAL'.



Let AP = r, then $\mathbf{x} - \mathbf{h} = \mathbf{r} \cos \theta$, $\mathbf{y} - \mathbf{k} = \mathbf{r} \sin \theta$ & $\frac{\mathbf{x} - \mathbf{h}}{\cos \theta} = \frac{\mathbf{y} - \mathbf{k}}{\sin \theta} = \mathbf{r}$ is the equation of the straight line LAL'.

Any point P on the line will be of the form $(h + r \cos\theta, k + r \sin\theta)$, where |r| gives the distance of the point P from the fixed point (h, k).

Illustration 15: Equation of a line which passes through point A(2, 3) and makes an angle of 45° with x axis. If this line meet the line x + y + 1 = 0 at point P then distance AP is -

(A)
$$2\sqrt{3}$$

(B)
$$3\sqrt{2}$$

(C)
$$5\sqrt{2}$$

(D)
$$2\sqrt{5}$$

Solution:

Here
$$x_1 = 2$$
, $y_1 = 3$ and $\theta = 45^{\circ}$ hence $\frac{x-2}{\cos 45^{\circ}} = \frac{y-3}{\sin 45^{\circ}} = r$

from first two parts $\Rightarrow x-2=y-3 \Rightarrow x-y+1=0$

Co-ordinate of point P on this line is $\left(2 + \frac{r}{\sqrt{2}}, 3 + \frac{r}{\sqrt{2}}\right)$.

If this point is on line x + y + 1 = 0 then

$$\left(2 + \frac{r}{\sqrt{2}}\right) + \left(3 + \frac{r}{\sqrt{2}}\right) + 1 = 0$$
 $\Rightarrow r = -3\sqrt{2}$; $|r| = 3\sqrt{2}$ Ans. (B)

Illustration 16: A variable line is drawn through O, to cut two fixed straight lines L_1 and L_2 in A_1 and A_2 ,

respectively. A point A is taken on the variable line such that $\frac{m+n}{OA} = \frac{m}{OA_1} + \frac{n}{OA_2}$.

Show that the locus of A is a straight line passing through the point of intersection of L_1 and L_2 where O is being the origin.

Solution:

Let the variable line passing through the origin is
$$\frac{x}{\cos \theta} = \frac{y}{\sin \theta} = r_i$$
 (i)

Let the equation of the line L_1 is $p_1x + q_1y = 1$ (ii)

Equation of the line L_2 is $p_2x + q_2y = 1$ (iii)

the variable line intersects the line (ii) at A_1 and (iii) at A_2 .

Let $OA_1 = r_1$.

Then $A_1 = (r_1 \cos \theta, r_1 \sin \theta) \implies A_1 \text{ lies on } L_1$

$$\Rightarrow r_1 = OA_1 = \frac{1}{p_1 \cos \theta + q_1 \sin \theta}$$

Similarly,
$$r_2 = OA_2 = \frac{1}{p_2 \cos \theta + q_2 \sin \theta}$$

Let OA = r

Let co-ordinate of A are $(h, k) \Rightarrow (h, k) \equiv (r\cos\theta, r\sin\theta)$

Now
$$\frac{m+n}{r} = \frac{m}{OA_1} + \frac{n}{OA_2} \Rightarrow \frac{m+n}{r} = \frac{m}{r_1} + \frac{n}{r_2}$$

$$\Rightarrow$$
 $m + n = m(p_1 r cos\theta + q_1 r sin\theta) + n(p_2 r cos\theta + q_2 r sin\theta)$

$$\Rightarrow$$
 $(p_1h + q_1k - 1) + \frac{n}{m}(p_2h + q_2k - 1) = 0$

Therefore, locus of A is $(p_1x+q_1y-1) + \frac{n}{m}(p_2x+q_2y-1) = 0$

$$\Rightarrow$$
 $L_1 + \lambda L_2 = 0$ where $\lambda = \frac{n}{m}$.

This is the equation of the line passing through the intersection of L₁ and L₂.

Illustration 17: A straight line through P(-2, -3) cuts the pair of straight lines $x^2 + 3y^2 + 4xy - 8x - 6y - 9 = 0$ in Q and R. Find the equation of the line if PQ. PR = 20.

Solution: Let line be $\frac{x+2}{\cos \theta} = \frac{y+3}{\sin \theta} = r$

$$\Rightarrow$$
 $x = r\cos\theta - 2$, $y = r\sin\theta - 3$ (i)

Now,
$$x^2 + 3y^2 + 4xy - 8x - 6y - 9 = 0$$
 (ii)

Taking intersection of (i) with (ii) and considering terms of r² and

constant (as we need PQ . $PR = r_1 . r_2 = product of the roots)$

 $r^{2}(\cos^{2}\theta + 3\sin^{2}\theta + 4\sin\theta\cos\theta) + (\text{some terms})r + 80 = 0$

$$\therefore r_1.r_2 = PQ. PR = \frac{80}{\cos^2 \theta + 4\sin \theta \cos \theta + 3\sin^2 \theta}$$

$$\therefore \cos^2\theta + 4\sin\theta\cos\theta + 3\sin^2\theta = 4 \qquad (\because PQ \cdot PR = 20)$$

$$\therefore \sin^2\theta - 4\sin\theta\cos\theta + 3\cos^2\theta = 0$$

$$\Rightarrow$$
 $(\sin\theta - \cos\theta)(\sin\theta - 3\cos\theta) = 0$

$$\therefore$$
 $\tan\theta = 1$, $\tan\theta = 3$

hence equation of the line is $y + 3 = 1(x + 2) \Rightarrow x - y = 1$

and
$$y + 3 = 3(x + 2) \implies 3x - y + 3 = 0$$
.

Illustration 18: If the line $y - \sqrt{3}x + 3 = 0$ cuts the parabola $y^2 = x + 2$ at A and B, then find the value of PA.PB {where $P = (\sqrt{3}, 0)$ }

Solution: Slope of line $y - \sqrt{3}x + 3 = 0$ is $\sqrt{3}$.

If line makes an angle θ with x-axis, then $\tan \theta = \sqrt{3}$

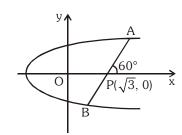
$$\theta = 60^{\circ}$$

$$\frac{x - \sqrt{3}}{\cos 60^{\circ}} = \frac{y - 0}{\sin 60^{\circ}} = r \Longrightarrow \left(\sqrt{3} + \frac{r}{2}, \frac{r\sqrt{3}}{2}\right) \text{ be a point on}$$

the parabola $y^2 = x + 2$

then
$$\frac{3}{4}r^2 = \sqrt{3} + \frac{r}{2} + 2 \implies 3r^2 - 2r - 4(2 + \sqrt{3}) = 0$$

:. PA.PB =
$$r_1 r_2 = \left| \frac{-4(2+\sqrt{3})}{3} \right| = \frac{4(2+\sqrt{3})}{3}$$



Do yourself - 6:

- (i) Reduce the line 2x 3y + 5 = 0,
 - (a) In slope- intercept form and hence find slope & Y-intercept
 - (b) In intercept form and hence find intercepts on the axes.
 - (c) In normal form and hence find perpendicular distance from the origin and angle made by the perpendicular with the positive x-axis.
- (ii) Find distance of point A (2, 3) measured parallel to the line x y = 5 from the line 2x + y + 6 = 0
- (g) General form: We know that a first degree equation in x and y, ax + by + c = 0 always represents a straight line. This form is known as general form of straight line.
 - (i) Slope of this line $=\frac{-a}{b} = -\frac{\text{coeff. of x}}{\text{coeff. of y}}$
 - (ii) Intercept by this line on x-axis = $-\frac{c}{a}$ and intercept by this line on y-axis = $-\frac{c}{b}$
 - (iii) To change the general form of a line to normal form, first take c to right hand side and make it positive, then divide the whole equation by $\sqrt{a^2 + b^2}$.

13. EQUATION OF LINES PARALLEL AND PERPENDICULAR TO A GIVEN LINE:

(a) Equation of line parallel to line ax + by + c = 0

$$ax + by + \lambda = 0$$

(b) Equation of line perpendicular to line ax + by + c = 0

$$bx - ay + k = 0$$

Here λ , k, are parameters and their values are obtained with the help of additional information given in the problem.

14. ANGLE BETWEEN TWO LINES:

(a) If θ be the angle between two lines : $y = m_1 x + c_1$ and $y = m_2 x + c_2$, then $\tan \theta = \pm \left(\frac{m_1 - m_2}{1 + m_1 m_2}\right)$

Note:

- (i) There are two angles formed between two lines but usually the acute angle is taken as the angle between the lines. So we shall find θ from the above formula only by taking positive value of $\tan \theta$.
- (ii) Let m_1 , m_2 , m_3 are the slopes of three lines $L_1 = 0$; $L_2 = 0$; $L_3 = 0$ where $m_1 > m_2 > m_3$ then the interior angles of the Δ ABC found by these formulas are given by,

$$tanA = \frac{m_1 - m_2}{1 + m_1 m_2}$$
; $tanB = \frac{m_2 - m_3}{1 + m_2 m_3}$ & $tanC = \frac{m_3 - m_1}{1 + m_3 m_1}$

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If equation of lines are $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_2=0$, then these lines are -**(b)**

- $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} \neq \frac{c_1}{c_2}$ $\Leftrightarrow a_1 a_2 + b_1 b_2 = 0$ (i) Parallel
- Perpendicular (ii)
- (iii) Coincident $\Leftrightarrow \frac{a_1}{a_2} = \frac{b_1}{b_2} = \frac{c_1}{c_2}$ Illustration 19: If x + 4y 5 = 0 and 4x + ky + 7 = 0 are two perpendicular lines then k is -

- (B)4
- (C) -1
- (D) -4

Solution:

$$m_1 = -\frac{1}{4}$$
 $m_2 = -\frac{4}{k}$

Two lines are perpendicular if $m_1 m_2 = -1$

$$\Rightarrow \left(-\frac{1}{4}\right) \times \left(-\frac{4}{k}\right) = -1 \Rightarrow k = -1$$

Ans. (C)

Illustration 20: A line L passes through the points (1, 1) and (0, 2) and another line M which is perpendicular to L passes through the point (0, -1/2). The area of the triangle formed by these lines with y-axis is -

- (A) 25/8
- (B) 25/16
- (C) 25/4
- (D) 25/32

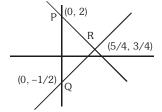
Solution:

Equation of the line L is $y-1 = \frac{-1}{1}(x-1) \implies y = -x+2$

Equation of the line M is y = x - 1/2.

If these lines meet y-axis at P and Q, then PQ = 5/2.

Also x-coordinate of their point of intersection R = 5/4



$$\therefore$$
 area of the $\triangle PQR = \frac{1}{2} \left(\frac{5}{2} \times \frac{5}{4} \right) = 25/16$.

Ans. (B)

Illustration 21: If the straight line 3x + 4y + 5 - k(x + y + 3) = 0 is parallel to y-axis, then the value of k is -

- (A) 1
- (B)2
- (C)3
- (D)4

Solution:

A straight line is parallel to y-axis, if its y - coefficient is zero, i.e. 4 - k = 0 i.e. k = 4Ans. (D)

15. STRAIGHT LINE MAKING A GIVEN ANGLE WITH A LINE:

Equation of line passing through a point (x_1,y_1) and making an angle α , with the line y=mx+c is written as:

$$y - y_1 = \frac{m \pm \tan \alpha}{1 \mp m \tan \alpha} (x - x_1)$$

Illustration 22: Find the equation to the sides of an isosceles right-angled triangle, the equation of whose hypotenuse is 3x + 4y = 4 and the opposite vertex is the point (2, 2).

Solution: The problem can be restated as:

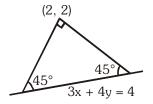
Find the equations of the straight lines passing through the given point (2, 2) and making equal angles of 45° with the given straight line 3x + 4y - 4 = 0. Slope of the line 3x + 4y - 4 = 0 is $m_1 = -3/4$.

$$\Rightarrow \tan 45^\circ = \pm \frac{m - m_1}{1 + m_1 m}, \text{ i.e., } 1 = \pm \frac{m + 3/4}{1 - \frac{3}{4} m}$$

$$m_A = \frac{1}{7}$$
, and $m_B = -7$

Hence the required equations of the two lines are

$$y-2 = m_A(x-2)$$
 and $y-2 = m_B(x-2)$
 $\Rightarrow 7y-x-12 = 0$ and $7x + y = 16$



Ans.

Do yourself - 7:

- (i) Find the angle between the lines 3x + y 7 = 0 and x + 2y 9 = 0.
- (ii) Find the line passing through the point (2, 3) and perpendicular to the straight line 4x 3y = 10.
- (iii) Find the equation of the line which has positive y-intercept 4 units and is parallel to the line 2x 3y 7 = 0. Also find the point where it cuts the x-axis.
- (iv) Classify the following pairs of lines as coincident, parallel or intersecting:

(a)
$$x + 2y - 3 = 0 & -3x - 6y + 9 = 0$$

(b)
$$x + 2y + 1 = 0 & 2x + 4y + 3 = 0$$

(c)
$$3x - 2y + 5 = 0 & 2x + y - 5 = 0$$

16. LENGTH OF PERPENDICULAR FROM A POINT ON A LINE:

Length of perpendicular from a point (x_1, y_1) on the line ax + by + c = 0 is $\left| \frac{ax_1 + by_1 + c}{\sqrt{a^2 + b^2}} \right|$

In particular, the length of the perpendicular from the origin on the line ax + by + c = 0 is $P = \frac{|c|}{\sqrt{a^2 + b^2}}$

Illustration 23: If the algebraic sum of perpendiculars from n given points on a variable straight line is zero then prove that the variable straight line passes through a fixed point.

Solution: Let n given points be (x_i, y_i) where i = 1, 2... n and the variable straight line is ax + by + c = 0.

Given that
$$\sum_{i=1}^{n} \left(\frac{ax_i + by_i + c}{\sqrt{a^2 + b^2}} \right) = 0 \implies a\Sigma x_i + b\Sigma y_i + cn = 0 \implies a\frac{\Sigma x_i}{n} + b\frac{\Sigma y_i}{n} + c = 0.$$

Hence the variable straight line always passes through the fixed point $\left(\frac{\sum x_i}{n}, \frac{\sum y_i}{n}\right)$.

Ans.

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Illustration 24: Prove that no line can be drawn through the point (4, -5) so that its distance from (-2, 3) will be equal to 12.

Solution: Suppose, if possible.

Equation of line through (4, -5) with slope m is y + 5 = m(x - 4)

$$\Rightarrow$$
 mx - y - 4m - 5 = 0

Then
$$\frac{|m(-2)-3-4m-5|}{\sqrt{m^2+1}} = 12$$

$$\Rightarrow |-6m-8| = 12\sqrt{(m^2+1)}$$

On squaring,
$$(6m + 8)^2 = 144(m^2 + 1)$$

$$\Rightarrow$$
 4(3m+4)² = 144(m² + 1) \Rightarrow (3m+4)² = 36(m² + 1)

$$\Rightarrow$$
 27m² - 24m + 20 = 0(i

Since the discriminant of (i) is $(-24)^2 - 4.27.20 = -1584$ which is negative, there is no real value of m. Hence no such line is possible.

17. DISTANCE BETWEEN TWO PARALLEL LINES:

(a) The distance between two parallel lines ax + by + c_1 =0 and ax+by+ c_2 =0 is = $\frac{|c_1 - c_2|}{\sqrt{a^2 + b^2}}$

(**Note**: The coefficients of x & y in both equations should be same)

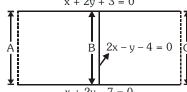
(b) The area of the parallelogram = $\frac{p_1 p_2}{\sin \theta}$, where p_1 & p_2 are distances between two pairs of opposite sides & θ is the angle between any two adjacent sides. Note that area of the parallelogram bounded by the lines $y = m_1 x + c_1$, $y = m_1 x + c_2$ and $y = m_2 x + d_1$,

y =
$$m_2 x + d_2$$
 is given by $\left| \frac{(c_1 - c_2) (d_1 - d_2)}{m_1 - m_2} \right|$.

Illustration 25: Three lines x + 2y + 3 = 0, x + 2y - 7 = 0 and 2x - y - 4 = 0 form 3 sides of two squares. Find the equation of remaining sides of these squares.

Solution: Distance between the two parallel lines is $\frac{|7+3|}{\sqrt{5}} = 2\sqrt{5}$.

 $\frac{7+3}{\sqrt{5}} = 2\sqrt{5}.$



The equations of sides A and C are of the form 2x - y + k = 0.

Since distance between sides A and B

= distance between sides B and C

$$\Rightarrow \frac{|\mathbf{k} - (-4)|}{\sqrt{5}} = 2\sqrt{5} \Rightarrow \frac{\mathbf{k} + 4}{\sqrt{5}} = \pm 2\sqrt{5} \Rightarrow \mathbf{k} = 6, -14.$$

Hence the fourth sides of the two squares are (i) 2x - y + 6 = 0 (ii) 2x - y - 14 = 0.

Ans.

Do yourself - 8:

- (i) Find the distances between the following pair of parallel lines:
 - (a) 3x + 4y = 13, 3x + 4y = 3
 - (b) 3x 4y + 9 = 0, 6x 8y 15 = 0
- (ii) Find the points on the x-axis such that their perpendicular distance from the line $\frac{x}{a} + \frac{y}{b} = 1$ is 'a', a, b > 0.
- (iii) Show that the area of the parallelogram formed by the lines

$$2x - 3y + a = 0$$
, $3x - 2y - a = 0$, $2x - 3y + 3a = 0$ and $3x - 2y - 2a = 0$ is $\frac{2a^2}{5}$ square units.

18. POSITION OF TWO POINTS WITH RESPECT TO A GIVEN LINE:

Let the given line be ax + by + c = 0 and $P(x_1, y_1)$, $Q(x_2, y_2)$ be two points. If the expressions $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have the same signs, then both the points P and Q lie on the same side of the line ax + by + c = 0. If the quantities $ax_1 + by_1 + c$ and $ax_2 + by_2 + c$ have opposite signs, then they lie on the opposite sides of the line.

19. CONCURRENCY OF LINES:

(a) Three lines $a_1x + b_1y + c_1 = 0$; $a_2x + b_2y + c_2 = 0$ and $a_3x + b_3y + c_3 = 0$ are concurrent,

if
$$\begin{vmatrix} a_1 & b_1 & c_1 \\ a_2 & b_2 & c_2 \\ a_3 & b_2 & c_3 \end{vmatrix} = 0$$

(b) To test the concurrency of three lines, first find out the point of intersection of any two of the three lines. If this point lies on the remaining line (i.e. coordinates of the point satisfy the equation of the line) then the three lines are concurrent otherwise not concurrent.

Illustration 27: If the lines ax + by + p = 0, $x\cos\alpha + y\sin\alpha - p = 0$ ($p \ne 0$) and $x\sin\alpha - y\cos\alpha = 0$ are concurrent and the first two lines include an angle $\frac{\pi}{4}$, then $a^2 + b^2$ is equal to -

(D)
$$p^2$$

Solution: Since the given lines are concurrent,

$$\begin{vmatrix} a & b & p \\ \cos \alpha & \sin \alpha & -p \\ \sin \alpha & -\cos \alpha & 0 \end{vmatrix} = 0$$

$$\Rightarrow$$
 a cos α + b sin α + 1 = 0

As ax + by + p = 0 and $x \cos \alpha + y \sin \alpha - p = 0$ include an angle $\frac{\pi}{4}$.

$$\pm \tan \frac{\pi}{4} = \frac{-\frac{a}{b} + \frac{\cos \alpha}{\sin \alpha}}{1 + \frac{a}{b} \frac{\cos \alpha}{\sin \alpha}}$$

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$$\Rightarrow$$
 -a sin α + bcos α = \pm (bsin α + acos α)

$$\Rightarrow$$
 -a sin α + bcos α = ±1 [from (i)](ii)

Squaring and adding (i) & (ii), we get

$$a^2 + b^2 = 2$$
. Ans. (B)

Do yourself - 9:

- (i) Examine the positions of the points (3, 4) and (2, -6) w.r.t. 3x 4y = 8
- (ii) If (2, 9), (-2, 1) and (1, -3) are the vertices of a triangle, then prove that the origin lies inside the triangle.
- (iii) Find the equation of the line joining the point (2, -9) and the point of intersection of lines 2x + 5y 8 = 0 and 3x 4y 35 = 0.
- (iv) Find the value of λ , if the lines 3x 4y 13 = 0, 8x 11 y 33 = 0 and $2x 3y + \lambda = 0$ are concurrent.

20. REFLECTION OF A POINT:

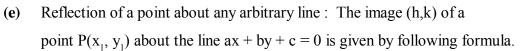
Let P(x, y) be any point, then its image with respect to



(b) y-axis is
$$R(-x, y)$$

(c) origin is
$$S(-x,-y)$$

(d) line
$$y = x$$
 is $T(y, x)$



$$\frac{h - x_1}{a} = \frac{k - y_1}{b} = -2\frac{(ax_1 + by_1 + c)}{a^2 + b^2}$$

and the foot of perpendicular (α,β) from a point (x_1,y_1) on the line ax + by + c = 0 is given by following formula.

$$\frac{\alpha - x_1}{a} = \frac{\beta - y_1}{b} = -\frac{ax_1 + by_1 + c}{a^2 + b^2}$$

$P \xrightarrow{(x_1,y_1)} p \xrightarrow{(x_1,y_1)} x \xrightarrow{(\alpha,\beta)} Q \xrightarrow{(h,k)}$

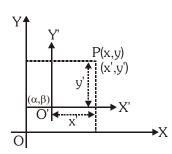
 $R(-x, y) \cdot$

21. TRANSFORMATION OF AXES

(a) Shifting of origin without rotation of axes:

Let P(x, y) with respect to axes OX and OY.

Let $O'(\alpha, \beta)$ is new origin with respect to axes OX and OY and let P(x', y') with respect to axes O'X' and O'Y', where OX and O'X' are parallel and OY and O'Y' are parallel.



Then
$$x = x' + \alpha$$
, $y = y' + \beta$

or
$$x' = x - \alpha$$
, $y' = y - \beta$

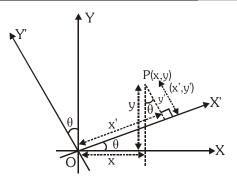
Thus if origin is shifted to point (α, β) without rotation of axes, then new equation of curve can be obtained by putting $x + \alpha$ in place of x and $y + \beta$ in place of y.

Rotation of axes without shifting the origin: **(b)**

Let O be the origin. Let P(x, y) with respect to axes OX and OY and let P (x', y') with respect to axes OX' and OY' where $\angle X'OX = \angle YOY' = \theta$, where θ is measured in anticlockwise direction.

then
$$x = x' \cos \theta - y' \sin \theta$$

 $y = x' \sin \theta + y' \cos \theta$
and $x' = x \cos \theta + y \sin \theta$
 $y' = -x \sin \theta + y \cos \theta$



The above relation between (x, y) and (x', y') can be easily obtained with the help of following table

New Old	x↓	y↓	
x' →	cos θ	sin θ	
y' →	-sin θ	$\cos \theta$	

Illustration 28: Through what angle should the axes be rotated so that the equation

$$9x^2 - 2\sqrt{3}xy + 7y^2 = 10$$
 may be changed to $3x^2 + 5y^2 = 5$?

Let angle be θ then replacing (x, y) by $(x \cos\theta - y \sin\theta, x \sin\theta + y \cos\theta)$ Solution:

then
$$9x^2 - 2\sqrt{3}xy + 7y^2 = 10$$
 becomes

$$9(x\cos\theta - y\sin\theta)^2 - 2\sqrt{3}(x\cos\theta - y\sin\theta)(x\sin\theta + y\cos\theta) + 7(x\sin\theta + y\cos\theta)^2 = 10$$

$$\Rightarrow x^2(9\cos^2\theta - 2\sqrt{3}\sin\theta\cos\theta + 7\sin^2\theta) + 2xy(-9\sin\theta\cos\theta - \sqrt{3}\cos2\theta + 7\sin\theta\cos\theta) + y^2(9\cos^2\theta + 2\sqrt{3}\sin\theta\cos\theta + 7\cos^2\theta) = 10$$

On comparing with $3x^2 + 5y^2 = 5$ (coefficient of xy = 0)

We get
$$-9\sin\theta\cos\theta - \sqrt{3}\cos2\theta + 7\sin\theta\cos\theta = 0$$

or
$$\sin 2\theta = -\sqrt{3}\cos 2\theta$$
 or $\tan 2\theta = -\sqrt{3} = \tan(180^\circ - 60^\circ)$

or
$$2\theta = 120^{\circ}$$
 \therefore $\theta = 60^{\circ}$

Do yourself - 10:

The point (4, 1) undergoes the following transformations, then the match the correct alternatives: (i)

Column-I

(p) (4, -1)

Reflection about x-axis is (A) Reflection about y-axis is

(q) (-4, -1)

(r) $\left(-\frac{12}{25}, -\frac{59}{25}\right)$

Column-II

Reflection about origin is (C)

Reflection about the line y = x is

- (s) (-4, 1)
- Reflection about the line 4x + 3y 5 = 0 is
- (t) (1, 4)
- To what point must the origin be shifted, so that the coordinates of the point (4, 5) become (ii)
- If the axes be turned through an angle tan⁻¹2 (in anticlockwise direction), what does the equation $4xy - 3x^2 = a^2$ become ?

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EQUATION OF BISECTORS OF ANGLES BETWEEN TWO LINES: 22.

If equation of two intersecting lines are $a_1x+b_1y+c_1=0$ and $a_2x+b_2y+c_3=0$, then equation of bisectors of the angles between these lines are written as:

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \pm \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}} \qquad \dots \dots \dots \dots (i)$$

Equation of bisector of angle containing origin: (a)

If the equation of the lines are written with constant terms \mathbf{c}_1 and \mathbf{c}_2 , positive, then the equation of the bisectors of the angle containing the origin is obtained by taking positive sign in (i)

(b) Equation of bisector of acute/obtuse angles:

To find the equation of the bisector of the acute or obtuse angle:

- let ϕ be the angle between one of the two bisectors and one of two given lines. Then if (i) $\phi < 45^{\circ}$ i.e. $2\phi < 90^{\circ}$, the angle bisector will be bisector of acute $tan\phi < 1$ i.e. angle.
- (ii)See whether the constant terms c_1 and c_2 in the two equation are +ve or not. If not then multiply both sides of given equation by -1 to make the constant terms positive. Determine the sign of $a_1a_2 + b_1b_2$

If sign of $a_1a_2 + b_1b_2$	For obtuse angle bisector	For acute angle bisector
+	use + sign in eq. (1)	use-sign in eq. (1)
_	use-sign in eq. (1)	use+sign in eq. (1)

i.e. if $a_1a_2 + b_1b_2 > 0$, then the bisector corresponding to + sign gives obtuse angle bisector

$$\frac{a_1x + b_1y + c_1}{\sqrt{a_1^2 + b_1^2}} = \frac{a_2x + b_2y + c_2}{\sqrt{a_2^2 + b_2^2}}$$

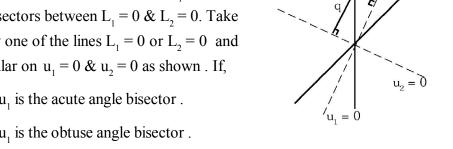
(iii) Another way of identifying an acute and obtuse angle bisector is as follows:

Let $L_1 = 0 \& L_2 = 0$ are the given lines & $u_1 = 0$ and $u_2 = 0$ are the bisectors between $L_1 = 0 \& L_2 = 0$. Take a point P on any one of the lines $L_1 = 0$ or $L_2 = 0$ and drop perpendicular on $u_1 = 0 \& u_2 = 0$ as shown. If,

$$|p| < |q| \implies u_1$$
 is the acute angle bisector.

$$|p| > |q| \implies u_1$$
 is the obtuse angle bisector .

$$|p| = |q| \Rightarrow$$
 the lines $L_1 \& L_2$ are perpendicular.



Note: Equation of straight lines passing through $P(x_1, y_1)$ & equally inclined with the lines $a_1x + b_1y$ $+c_1 = 0 & a_2x + b_2y + c_2 = 0$ are those which are parallel to the bisectors between these two lines & passing through the point P.

Illustration 29: For the straight lines 4x + 3y - 6 = 0 and 5x + 12y + 9 = 0, find the equation of the

(i) bisector of the obtuse angle between them.

(ii) bisector of the acute angle between them.

(iii) bisector of the angle which contains origin.

(iv) bisector of the angle which contains (1, 2).

Solution: Equations of bisectors of the angles between the given lines are

$$\frac{4x+3y-6}{\sqrt{4^2+3^2}} = \pm \frac{5x+12y+9}{\sqrt{5^2+12^2}} \implies 9x-7y-41 = 0 \text{ and } 7x+9y-3=0$$

If θ is the acute angle between the line 4x + 3y - 6 = 0 and the bisector

$$9x - 7y - 41 = 0$$
, then $\tan \theta = \left| \frac{-\frac{4}{3} - \frac{9}{7}}{1 + \left(\frac{-4}{3}\right)\frac{9}{7}} \right| = \frac{11}{3} > 1$

Hence

(i) bisector of the obtuse angle is 9x - 7y - 41 = 0

(ii) bisector of the acute angle is 7x + 9y - 3 = 0

(iii) bisector of the angle which contains origin

$$\frac{-4x - 3y + 6}{\sqrt{(-4)^2 + (-3)^2}} = \frac{5x + 12y + 9}{\sqrt{5^2 + 12^2}} \Rightarrow 7x + 9y - 3 = 0$$

(iv)
$$L_1(1, 2) = 4 \times 1 + 3 \times 2 - 6 = 4 > 0$$

 $L_2(1, 2) = 5 \times 1 + 12 \times 2 + 9 = 38 > 0$

+ve sign will give the required bisector, $\frac{4x+3y-6}{5} = +\frac{5x+12y+9}{13}$ $\Rightarrow 9x-7y-41=0$.

Alternative:

Making c_1 and c_2 positive in the given equation, we get -4x - 3y + 6 = 0 and 5x + 12y + 9 = 0

Since $a_1a_2 + b_1b_2 = -20 - 36 = -56 < 0$, so the origin will lie in the acute angle.

Hence bisector of the acute angle is given by

$$\frac{-4x - 3y + 6}{\sqrt{4^2 + 3^2}} = \frac{5x + 12y + 9}{\sqrt{5^2 + 12^2}} \implies 7x + 9y - 3 = 0$$

Similarly bisector of obtuse angle is 9x - 7y - 41 = 0.

Illustration 30: A ray of light is sent along the line x - 2y - 3 = 0. Upon reaching the line mirror 3x - 2y - 5 = 0, the ray is reflected from it. Find the equation of the line containing the reflected ray.

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Solution:

Let Q be the point of intersection of the incident ray and the line mirror, then

$$x_1 - 2y_1 - 3 = 0$$
 & $3x_1 - 2y_1 - 5 = 0$

on solving these equations, we get

$$x_1 = 1 \& y_1 = -1$$

Since P(-1, -2) be a point lies on the incident ray, so we can find the image of the point P on the reflected ray about the line mirror (by property of reflection).

Let P'(h, k) be the image of point P about line mirror, then

$$\frac{h+1}{3} = \frac{k+2}{-2} = \frac{-2(-3+4-5)}{13} \implies h = \frac{11}{13} \text{ and } k = \frac{-42}{13}.$$

So
$$P'\left(\frac{11}{13}, \frac{-42}{13}\right)$$

Then equation of reflected ray will be

$$(y+1) = \frac{\left(\frac{-42}{13} + 1\right)(x-1)}{\left(\frac{11}{13} - 1\right)}$$

$$\Rightarrow 2y - 29x + 31 = 0.$$

23. FAMILY OF LINES:

If equation of two lines be $P = a_1x + b_1y + c_1 = 0$ and $Q = a_2x + b_2y + c_2 = 0$, then the equation of the lines passing through the point of intersection of these lines is : $P + \lambda Q = 0$ or $a_1x + b_1y + c_1 + \lambda (a_2x + b_2y + c_2) = 0$. The value of λ is obtained with the help of the additional informations given in the problem.

Illustration 31: Prove that each member of the family of straight lines

 $(3\sin\theta + 4\cos\theta)x + (2\sin\theta - 7\cos\theta)y + (\sin\theta + 2\cos\theta) = 0$ (θ is a parameter) passes through a fixed point.

Solution:

The given family of straight lines can be rewritten as

$$(3x + 2y + 1)\sin\theta + (4x - 7y + 2)\cos\theta = 0$$

or,
$$(4x - 7y + 2) + \tan\theta(3x + 2y + 1) = 0$$
 which is of the form $L_1 + \lambda L_2 = 0$

Hence each member of it will pass through a fixed point which is the intersection of

$$4x - 7y + 2 = 0$$
 and $3x + 2y + 1 = 0$ i.e. $\left(\frac{-11}{29}, \frac{2}{29}\right)$.

Do yourself - 11:

- (i) Find the equations of bisectors of the angle between the lines 4x + 3y = 7 and 24x + 7y 31 = 0. Also find which of them is (a) the bisector of the angle containing origin (b) the bisector of the acute angle.
- (ii) Find the equations of the line which pass through the point of intersection of the lines 4x 3y = 1 and 2x 5y + 3 = 0 and is equally inclined to the coordinate axes.
- (iii) Find the equation of the line through the point of intersection of the lines 3x 4y + 1 = 0& 5x + y - 1 = 0 and perpendicular to the line 2x - 3y = 10.

24. PAIR OF STRAIGHT LINES:

- (a) Homogeneous equation of second degree:
 - (i) Let us consider the homogeneous equation of 2nd degree as

$$ax^2 + 2hxy + by^2 = 0$$
(i)

which represents pair of straight lines passing through the origin.

Now, we divide by x^{2} , we get

$$a + 2h\left(\frac{y}{x}\right) + b\left(\frac{y}{x}\right)^2 = 0$$

$$\frac{y}{x} = m$$
 (say)

then
$$a + 2hm + bm^2 = 0$$

.....(ii

if m_1 & m_2 are the roots of equation (ii), then $m_1 + m_2 = -\frac{2h}{b}$, $m_1 m_2 = \frac{a}{b}$

and also,
$$\tan \theta = \left| \frac{m_1 - m_2}{1 + m_1 m_2} \right| = \left| \frac{\sqrt{(m_1 + m_2)^2 - 4m_1 m_2}}{1 + m_1 m_2} \right| = \left| \frac{\sqrt{\frac{4h^2}{b^2} - \frac{4a}{b}}}{1 + \frac{a}{b}} \right| = \pm \frac{2\sqrt{h^2 - ab}}{a + b}$$

These line will be:

- (1) Real and different, if $h^2 ab > 0$ (2) Real and coincident, if $h^2 ab = 0$
- (3) **Imaginary**, if $h^2 ab < 0$
- (ii) The condition that these lines are:
 - (1) At **right angles** to each other is $\mathbf{a} + \mathbf{b} = \mathbf{0}$. i.e. coefficient of $\mathbf{x}^2 + \text{coefficient}$ of $\mathbf{y}^2 = \mathbf{0}$.
 - (2) Coincident is $h^2 = ab$.
 - (3) Equally inclined to the axes of x is h = 0. i.e. coefficient of xy = 0.
- (iii) Homogeneous equation of 2^{nd} degree $ax^2 + 2hxy + by^2 = 0$ always represent a pair of straight lines whose equations are

$$y = \left(\frac{-h \pm \sqrt{h^2 - ab}}{b}\right) x \equiv y = m_1 x \& y = m_2 x \text{ and } m_1 + m_2 = -\frac{2h}{b} ; m_1 m_2 = \frac{a}{b}$$

These straight lines passes through the origin.

- (iv) Pair of straight lines perpendicular to the lines $ax^2 + 2hxy + by^2 = 0$ and through origin are given by $bx^2 2hxy + ay^2 = 0$.
- (v) The product of the perpendiculars drawn from the point (x_1, y_1) on the lines $ax^2 + 2hxy +$

$$by^2 = 0 \text{ is } \left| \frac{ax_1^2 + 2hx_1y_1 + by_1^2}{\sqrt{(a-b)^2 + 4h^2}} \right|$$

Note: A homogeneous equation of degree n represents n straight lines passing through **origin**.



(b) The combined equation of angle bisectors:

The combined equation of angle bisectors between the lines represented by homogeneous equation of 2^{nd} degree is given by $\frac{x^2 - y^2}{a - b} = \frac{xy}{h}$, $a \neq b$, $h \neq 0$.

Note:

(i) If
$$\mathbf{a} = \mathbf{b}$$
, the bisectors are $\mathbf{x}^2 - \mathbf{y}^2 = \mathbf{0}$ i.e. $\mathbf{x} - \mathbf{y} = \mathbf{0}$, $\mathbf{x} + \mathbf{y} = \mathbf{0}$

(ii) If
$$\mathbf{h} = \mathbf{0}$$
, the bisectors are $\mathbf{x}\mathbf{y} = \mathbf{0}$ i.e. $\mathbf{x} = \mathbf{0}$, $\mathbf{y} = \mathbf{0}$.

(iii) The two bisectors are always at **right angles**, since we have coefficient of
$$\mathbf{x}^2$$
 + coefficient of $\mathbf{y}^2 = \mathbf{0}$

(c) General Equation and Homogeneous Equation of Second Degree :

(i) The general equation of second degree
$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$
 represents a pair of straight lines, if $\Delta = abc + 2fgh - af^2 - bg^2 - ch^2 = 0$ i.e.
$$\begin{vmatrix} a & h & g \\ h & b & f \\ g & f & c \end{vmatrix} = 0$$

(ii) If
$$\theta$$
 be the angle between the lines, then $\tan \theta = \pm \frac{2\sqrt{h^2 - ab}}{a + b}$

Obviously these lines are

(1) Parallel, if
$$\Delta = 0$$
, $h^2 = ab$ or if $h^2 = ab$ and $bg^2 = af^2$

(2) Perpendicular, if
$$a + b = 0$$
 i.e. coeff. of $x^2 + \text{coeff.}$ of $y^2 = 0$.

$$ax^{2} + 2hxy + by^{2} + 2gx + 2fy + c = 0$$
 is $\left| \frac{c}{\sqrt{(a-b)^{2} + 4h^{2}}} \right|$

Illustration 32: If
$$\lambda x^2 - 10xy + 12y^2 + 5x - 16y - 3 = 0$$
 represents a pair of straight lines, then λ is equal to -

$$(B)$$
 3

Here
$$a = \lambda$$
, $b = 12$, $c = -3$, $f = -8$, $g = 5/2$, $h = -5$

Using condition $abc + 2fgh - af^2 - bg^2 - ch^2 = 0$, we have

$$\lambda(12)(-3) + 2(-8)(5/2)(-5) - \lambda(64) - 12(25/4) + 3(25) = 0$$

$$\Rightarrow$$
 $-36\lambda + 200 - 64\lambda - 75 + 75 = 0 \Rightarrow $100 \lambda = 200$$

$$\lambda = 2$$

Ans. (C)

Do yourself - 12:

- (i) Prove that the equation $x^2 5xy + 4y^2 = 0$ represents two lines passing through the origin. Also find their equations.
- (ii) If the equation $3x^2 + kxy 10y^2 + 7x + 19y = 6$ represents a pair of lines, find the value of k.
- (iii) If the equation $6x^2 11xy 10y^2 19y 6 = 0$ represents a pair of lines, find their equations. Also find the angle between the two lines.
- (iv) Find the point of intersection and the angle between the lines given by the equation : $2x^2 3xy 2y^2 + 10x + 5y = 0$.

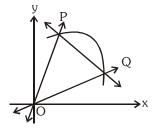
25. EQUATIONS OF LINES JOINING THE POINTS OF INTERSECTION OF A LINE AND A CURVE TO THE ORIGIN :

(a) Let the equation of curve be:

$$ax^2 + 2hxy + by^2 + 2gx + 2fy + c = 0$$
 ...(i)

and straight line be

$$lx + my + n = 0$$
 ...(ii)



Now joint equation of line OP and OQ joining the origin and points of intersection P and Q can be obtained by making the equation (i) homogenous with the help of equation of the line.

Thus required equation is given by

$$ax^2 + 2hxy + by^2 + 2(gx + fy) \left(\frac{\ell x + my}{-n}\right) + c\left(\frac{\ell x + my}{-n}\right)^2 = 0$$

$$\Rightarrow \quad (an^2 + 2gln + cl^2)x^2 + 2(hn^2 + gmn + fln + clm)xy + (bn^2 + 2fmn + cm^2)y^2 = 0 \dots (iii)$$

All points which satisfy (i) and (ii) simultaneously, will satisfy (iii)

- (b) Any second degree curve through the four points of intersection of f(x, y) = 0 & xy = 0 is given by $f(x, y) + \lambda xy = 0$ where f(x, y) = 0 is also a second degree curve.
- **Illustration 35:** The chord $\sqrt{6}y = \sqrt{8}px + \sqrt{2}$ of the curve $py^2 + 1 = 4x$ subtends a right angle at origin then find the value of p.

Solution: $\sqrt{3}y - 2px = 1$ is the given chord. Homogenizing the equation of the curve, we get,

$$py^2 - 4x(\sqrt{3}y - 2px) + (\sqrt{3}y - 2px)^2 = 0$$

$$\Rightarrow$$
 $(4p^2 + 8p)x^2 + (p + 3)y^2 - 4\sqrt{3}xy - 4\sqrt{3}pxy = 0$

Now, angle at origin is 90°

coefficient of x^2 + coefficient of y^2 = 0

$$\therefore 4p^2 + 8p + p + 3 = 0 \implies 4p^2 + 9p + 3 = 0$$

$$p = \frac{-9 \pm \sqrt{81 - 48}}{8} = \frac{-9 \pm \sqrt{33}}{8}.$$

Do yourself - 13:

- Find the angle subtended at the origin by the intercept made on the curve (i) $x^2 - y^2 - xy + 3x - 6y + 18 = 0$ by the line 2x - y = 3.
- Find the equation of the lines joining the origin to the points of intersection of the curve (ii) $2x^2 + 3xy - 4x + 1 = 0$ and the line 3x + y = 1.

ANSWERS FOR DO YOURSELF

- **1:** (i) PQ = $\sqrt{34}$ units; (ii) x = 6 or x = 0
- 2: (i) (a) (2,1); (b) (7,16); (ii) (a) 2:3 (internally); (b) 9:4 (externally); (c) 8:7 (internally)
- 3: (i) (a) $\left(\frac{5}{3}, 3\right)$; (b) $\left(-\frac{9}{2}, \frac{15}{2}\right)$, $\frac{5\sqrt{10}}{2}$, (c) (14, -6)
- **4:** (i) 25 square units; (ii) 132 square units;
- **5**: (i) $x = \pm 2$;
- (ii) $y = \pm x$;
- **6:** (i) (a) $y = \frac{2x}{3} + \frac{5}{3}, \frac{2}{3}, \frac{5}{3}$; (b) $\frac{x}{(-5/2)} + \frac{y}{(5/3)} = 1, -\frac{5}{2}, \frac{5}{3}$;
 - (c) $-\frac{2x}{\sqrt{13}} + \frac{3y}{\sqrt{13}} = \frac{5}{\sqrt{13}}, \frac{5}{\sqrt{13}}, \alpha = \pi \tan^{-1}\left(\frac{3}{2}\right);$ (ii) $13\sqrt{2}/3$ units
- 7: (i) $\theta = 135^{\circ} \text{ or } 45^{\circ}$; (ii) 3x + 4y = 18;
- (iii) 2x 3y + 12 = 0, (-6, 0)
- (iv) (a) Coincident, (b) Parallel, (c) Intersecting
- **8:** (i) (a) 2 (b) 33/10;
- (ii) $\left(\frac{a}{b}\left(b\pm\sqrt{a^2+b^2}\right),0\right)$
- **9:** (i) opposite sides of the line; (iii) -y + x = 11; (iv) $\lambda = -7$
- **10**: (i) (A) \rightarrow (p), (B) \rightarrow (s), (C) \rightarrow (q), (D) \rightarrow (t), (E) \rightarrow (r),; (ii) (7, -4); (iv) $x^2 4y^2 = a^2$
- **11:** (i) x 2y + 1 = 0 & 2x + y 3 = 0; (a) x 2y + 1 = 0; (b) 2x + y 3 = 0
 - (ii) x + y = 2, x = y;
- (iii) 69x + 46y 25 = 0
- **12**: (i) x y = 0 & x 4y = 0;

- (ii) k = -1, or $-\frac{127}{2}$;
- (iii) 2x 5y 2 = 0 & 3x + 2y + 3 = 0; $\pm \tan^{-1} \left(\frac{19}{4} \right)$ (iv) $(-1, 2), 90^{\circ}$
- **13**: (i) $\theta = \pm \tan^{-1} \frac{4}{7}$; (ii) $x^2 y^2 5xy = 0$

1.

EXERCISE (O-1)

Coordinates of the vertices of a triangle ABC are (12,8), (-2,6) and (6,0) then the **correct** statement is-

	(A) triangle is right but not isosceles				
	(B) triangle is isosceles but not right				
	(C) triangle is obtuse				
	(D) the product of the	abscissa of the centroid,	orthocenter and circumce	enter is 160.	SL0003
2.	Find the value of x_1 if the distance between the points $(x_1, 2)$ and $(3, 4)$ be 8.				
	$(A) 3 \pm 2\sqrt{15}$	(B) $3 \pm \sqrt{15}$	(C) $2 \pm 3\sqrt{15}$	(D) $2 \pm \sqrt{15}$	SL0004
3.	IfP(1,2), Q(4,6), R(5,	7) & S(a,b) are the vertice	ces of a parallelogram PC	QRS, then:	
	(A) $a = 2$, $b = 4$	(B) $a = 3$, $b = 4$	(C) $a = 2, b = 3$	(D) $a = 3, b = 5$	SL0009
4.	_		e coordinates of one extre ossible values of the ordi	- · · · · · · · · · · · · · · · · · · ·	
	(A) 3	(B)-4	(C) 12	(D)-6	SL0006
5.	If A and B are the point	ts(-3,4) and $(2,1)$, then t	he co-ordinates of the po	oint C on AB produced	l such that
	AC = 2BC are:				
				(1 5)	
	(A)(2,4)	(B)(3,7)	(C)(7,-2)	$(D)\left(-\frac{1}{2},\frac{5}{2}\right)$	SL0010
6.	The orthocenter of the ordinates of C are:	triangle ABC is 'B' and t	he circumcenter is 'S' (a,	b). If A is the origin th	en the co-
	(A) (2a,2b)	$(B)\left(\frac{a}{2},\frac{b}{2}\right)$	$(C)\left(\sqrt{a^2+b^2},0\right)$	(D) none	SL0012
7.	The medians of a triang is at -	gle meet at $(0,-3)$ and its	two vertices are at (-1,4)	and (5,2). Then the th	nird vertex
	(A)(4,15)	(B) (-4,-15)	(C)(-4,15)	(D)(4,-15)	SL0013
8.	If the two vertices of a	triangle are (7,2) and (1,4	6) and its centroid is (4,6)) then the coordinate of	of the third
	vertex are (a,b) . The value of $(a+b)$, is-				
	(A) 13	(B) 14	(C) 15	(D) 16	SL0011
9.	If in triangle ABC, A	$A \equiv (1,10)$, circumcent	$er = \left(-\frac{1}{3}, \frac{2}{3}\right)$ and ort	hocenter $\equiv \left(\frac{11}{3}, \frac{4}{3}\right)$	then the
	-	oint of side opposite to A			
	(A) (1,-11/3)	(B) (1,5)	(C) (1,-3)	(D)(1,6)	SL0014
10.		(2,-4); Q(4,-2) and R(7,	1). The points P,Q,R -		
	(A) form an equilateral(B) form a right angled	· ·			
	` '	riangle which is not equila	nteral		
	(D) are collinear.		****		SL0007

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11.	A triangle has two of	its vertices at (0,1) and	(2,2) in the cartesian pla	ne. Its third vertex lie	es on the
	x-axis. If the area of the	e triangle is 2 square units	then the sum of the possib	ole abscissae of the thir	d vertex,
	is-				
	(A)-4	(B) 0	(C) 5	(D) 6	SL0016
12.			nce from P to the coordin	=	distance
	• `	,1). The equation of the l	ocus of P in the first quad	rant is -	
	(A) $(x + 1) (y + 1) = 1$		(B) $(x + 1) (y + 1) = 2$		
	(C) $(x-1)(y-1) = 1$		(D) $(x-1)(y-1) = 2$		SL0021
13.			AABC. If the centroid	of ΔABC moves on	the line
	2x + 3y = 1, then the lo				
	(A) 2x + 3y = 9	(B) 2x - 3y = 7	(C) $3x + 2y = 5$	(D) $3x - 2y = 3$	SL0018
14.	A stick of length 10 unit	ts rests against the floor ar	nd a wall of a room. If the	stick begins to slide on	the floor
	then the locus of its mic	ldle point is:			
	(A) $x^2 + y^2 = 2.5$	(B) $x^2 + y^2 = 25$	(C) $x^2 + y^2 = 100$	(D) none	SL0019
15.	A line passes through (2	2,2) and cuts a triangle of	area 9 square units from tl	ne first quadrant. The s	umofall
	possible values for the	slope of such a line, is-			
	(A)-2.5	(B)-2	(C)-1.5	(D)-1	SL0017
16.	The diagonals of a para	allelogram PQRS are alor	ng the lines $x + 3y = 4$ and	16x - 2y = 7. Then PQ	RS must
	be a:				
	(A) rectangle		(B) square		
	(C) cyclic quadrilateral		(D) rhombus		SL0023
17.	A and B are any two po	oints on the positive x and	dy axis respectively satisf	Sying $2(OA) + 3(OB) =$	= 10. If P
	is the middle point of A	B then the locus of P is-			
	(A) 2x + 3y = 5	(B) $2x + 3y = 10$	(C) $3x + 2y = 5$	(D) $3x + 2y = 10$	SL0020
18.	The co-ordinates of the	e orthocentre of the trian	gle bounded by the lines,	4x - 7y + 10 = 0; $x + y$	y = 5 and
	7x + 4y = 15 is-				
	(A)(2,1)	(B) $(-1,2)$	(C)(1,2)	(D)(1,-2)	SL0015

(A)(2,1)

(A)(-1,0)

19. If the x intercept of the line y = mx + 2 is greater than 1/2 then the gradient of the line lies in the interval-(B)(-1/4,0)

 $(C)(-\infty,-4)$

(D)(-4,0)

SL0024

The greatest slope along the graph represented by the equation $4x^2 - y^2 + 2y - 1 = 0$, is-**20.**

(A) - 3

(B) - 2

(C)2

SL0039

21. The extremities of the base of an isosceles triangle ABC are the points A(2,0) and B(0,1). If the equation of the side AC is x = 2 then the slope of the side BC is -

(A) $\frac{3}{4}$

(C) $\frac{3}{2}$

(D) $\sqrt{3}$

SL0025

Number of lines that can be drawn through the point (4,-5) so that its distance from (-2,3) will be equal to 22. 12 is equal to-

(A) 0

(B) 1

(C)2

(D)3

SL0030

23. Two mutually perpendicular straight lines through the origin from an isosceles triangle with the line 2x + y = 5. Then the area of the triangle is:

(A)5

(B)3

(C) 5/2

(D) 1

- The area of the parallelogram formed by the lines 3x + 4y = 7a; 3x + 4y = 7b; 4x + 3y = 7c and 24. 4x + 3y = 7d is-

- (A) $\frac{\left|(a-b)(c-d)\right|}{7}$ (B) $\left|(a-b)(c-d)\right|$ (C) $\frac{\left|(a-b)(c-d)\right|}{49}$ (D) $7\left|(a-b)(c-d)\right|$ **SL0029**
- Consider a parallelogram whose sides are represented by the lines 2x + 3y = 0; 2x + 3y 5 = 0; 25. 3x-4y=0 and 3x-4y=3. The equation of the diagonal not passing through the origin, is-
 - (A) 21x 11y + 15 = 0

(B) 9x - 11y + 15 = 0

(C) 21x - 29y - 15 = 0

(D) 21x - 11y - 15 = 0

SL0037

- If the straight lines, ax + amy + 1 = 0, bx + (m + 1)by + 1 = 0 and cx + (m + 2)cy + 1 = 0, $m \ne 0$ are **26.** concurrent then a,b,c are in:
 - (A) A.P. only for m = 1

(B) A.P. for all m

(C) G.P. for all m

(D) H.P. for all m

SL0035

- 27. A ray of light passing through the point A(1,2) is reflected at a point B on the x-axis and then passes through (5,3). Then the equation of AB is:
 - (A) 5x + 4y = 13

(B) 5x - 4y = -3

(C) 4x + 5y = 14

(D) 4x - 5y = -6

SL0031

- If the line y = mx bisects the angle between the lines $ax^2 + 2hxy + by^2 = 0$ then m is a root of the quadratic 28. equation:
 - (A) $hx^2 + (a b)x h = 0$

- (B) $x^2 + h(a-b)x 1 = 0$
- (C) $(a-b)x^2 + hx (a-b) = 0$
- (D) $(a-b)x^2 hx (a-b) = 0$

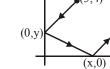
SL0038

- If the equation $ax^2 6xy + y^2 + 2gx + 2fy + c = 0$ represents a pair of lines whose slopes are m and m², **29.** then sum of all possible values of a is-
 - (A) 17
- (B) 19
- (C) 19
- (D)-17

SL0040

- **30.** Suppose that a ray of light leaves the point (3,4), reflects off the y-axis towards the x-axis, reflects off the x-axis, and finally arrives at the point (8,2). The value of x, is-
 - (A) $x = 4\frac{1}{2}$

(B) $x = 4\frac{1}{2}$



(C) $x = 4\frac{2}{3}$

(D) $x = 5\frac{1}{2}$

SL0032

- Let P(-1,0) Q(0,0) and $R(3,3\sqrt{3})$ be three points. The equation of the bisector of the angle PQR is-
- (A) $\sqrt{3} x + y = 0$ (B) $x + \frac{\sqrt{3}}{2} y = 0$ (C) $\frac{\sqrt{3}}{2} x + y = 0$ (D) $x + \sqrt{3} y = 0$

- (A) $-\frac{1}{2}$
- (B) -2
- (C) 1
- (D)2

SL0160

33. The perpendicular bisector of the line segment joining P(1, 4) and Q(k, 3) has y-intercept -4. Then a possible value of k is-

- (A) 1
- (B)2

- (C) -2
- (D)-4

SL0161

[MATRIX LIST TYPE]

Find the equation to the straight line: 34.

Column-I

Column-II

- (P) passing through the point (2, 3) and perpendicular to the straight line 4x - 3y = 10.
- (1) 4y + 11x = 10
- passing through the point (-6, 10) and perpendicular to the straight line 7x + 8y = 5.
- (2) 4y + 3x = 18
- (R) passing through the point (2, -3) and perpendicular to the straight line joining the points (5, 7) and (-6, 3).
- $(3) \quad x + 4y + 16 = 0$ (4) 7y - 8x = 118
- passing through the point (-4, -3) and perpendicular to (S) the straight line joining (1, 3) and (2, 7).

Codes:

P Q R S

- 2 (A) 1 3 4
- (B) 2 4 1 3
- (C) 4 3 2 1
- 3 4 2 (D) 1

SL0042

[MATRIX MATCH]

35. Column-I Column-II

The points (2,-2), (8,4), (5,7) and (-1,1)(A) taken in order constitute the vertices of a

- (P) square
 - SL0043

(B) The points (0,-1), (2,1), (0,3) and (-2,1)taken in order are the vertices of a

rectangle (Q)

(R)

The points (3,-5), (-5,-4), (7,10), (15,9)(C) taken in order are the vertices of a

trapezium

(D) The points (-3,4), (-1,0), (1,0) and (3,4)taken in order are the vertices of a

- (S) parallelogram (T) cyclic quadrilateral

(A)

36. Equation of Straight Line

Column-I

through the point (2,-3).

Which cuts-off an intercept 4 on the x-axis and passes

(P) 2x + y + 1 = 0 **SL0055**

Column-II

(B) Which cuts-off equal intercepts on the co-ordinate axes and passes through (2,5)

(Q) x + y = 7

(C) Which makes an angle of 135° with the axis of x and which cuts the axis of y at a distance -8 from the origin and

(R) 3x - 2y = 12 **SL0057**

(D) Through the point (4,1) and making with the axes in the first quadrant a triangle whose area is 8.

(S) x + 4y = 8 **SL0058**

(T) x + y + 8 = 0

EXERCISE (O-2)

1. If m and b are real numbers and mb > 0, then the line whose equation is y = mx + b cannot contain the point-

(A)(0,2009)

(B)(2009,0)

(C)(0,-2009)

(D)(20,-100)

SL0063

2. The vertex of the right angle of a right angled triangle lies on the straight line 2x - y - 10 = 0 and the two other vertices, at points (2,-3) and (4,1) then the area of triangle in sq. units is-

(A) $\sqrt{10}$

(B) 3

(C) $\frac{33}{5}$

(D) 11

SL0065

3. A triangle ABC is formed by the lines 2x - 3y - 6 = 0; 3x - y + 3 = 0 and 3x + 4y - 12 = 0. If the points $P(\alpha,0)$ and $Q(0,\beta)$ always lie on or inside the \triangle ABC, then:

(A) $\alpha \in [-1,2]$ and $\beta \in [-2,3]$

(B) $\alpha \in [-1,3]$ and $\beta \in [-2,4]$

(C) $\alpha \in [-2,\!4]$ and $\beta \in [-3,\!4]$

(D) $\alpha \in [-1,3]$ and $\beta \in [-2,3]$

SL0066

4. If the straight lines joining the origin and the points of intersection of the curve

$$5x^2 + 12xy - 6y^2 + 4x - 2y + 3 = 0$$
 and $x + ky - 1 = 0$

are equally inclined to the x- axis then the value of k:

(A) is equal to 1

(B) is equal to -1

(C) is equal to 2

(D) does not exist in the set of real numbers

SL0069

Through a point A on the x-axis a straight line is drawn parallel to y-axis so as to meet the pair of straight lines $ax^2 + 2hxy + by^2 = 0$ in B and C. If AB = BC then-

(A) $h^2 = 4ab$

(B) $8h^2 = 9ab$

(C) $9h^2 = 8ab$

(D) $4h^2 = ab$

SL0070

[MULTIPLE CHOICE]

6. The area of triangle ABC is 20 square units. The co-ordinates of vertex A are (-5,0) and B are (3,0). The vertex C lies on the line, x-y=2. The co-ordinates of C are -

(A)(5,3)

(B)(-3,-5)

(C)(-5,-7)

(D)(7,5)

SL0075

- Three vertices of a triangle are A(4,3); B(1,-1) and C(7,k). Value(s) of k for which centroid, orthocentre, 7. incentre and circumcentre of the \triangle ABC lie on the same straight line is/are-
 - (A)7

- (B) 1
- (C) 19/8
- (D) none

SL0072

- 8. A line passes through the origin and makes an angle of $\pi/4$ with the line x-y+1=0. Then:
 - (A) equation of the line is x = 0
 - (B) the equation of the line is y = 0
 - (C) the point of intersection of the line with the given line is (-1,0)
 - (D) the point of intersection of the line with the given line is (0,1)

SL0080

- Equation of a straight line passing through the point (2,3) and inclined at an angle of arc tan $\frac{1}{2}$ with the line 9. y + 2x = 5 is
 - (A) y = 3
- (B) x = 2
- (C) 3x + 4y 18 = 0 (D) 4x + 3y 17 = 0
- If the vertices P,Q,R of a triangle PQR are rational points, which of the following points of the triangle PQR **10.** is/are always rational point(s)?
 - (A) centroid
- (B) incentre
- (C) circumcentre
- (D) orthocentre

SL0074

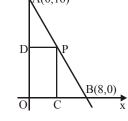
- The lines L_1 and L_2 denoted by $3x^2 + 10xy + 8y^2 + 14x + 22y + 15 = 0$ intersect at the point P and have 11. gradients m_1 and m_2 respectively. The acute angles between them is θ . Which of the following relations hold good?
 - (A) $m_1 + m_2 = 5/4$
 - (B) $m_1 m_2 = 3/8$
 - (C) acute angle between L_1 and L_2 is $\sin^{-1}\left(\frac{2}{5\sqrt{5}}\right)$.
 - (D) sum of the abscissa and ordinate of the point P is −1.

SL0083

Paragraph for Question Nos. 12 to 14

In the diagram, a line is drawn through the points A(0,16) and B(8,0). Point P is chosen in the first quadrant on the line through A and B. Points C and D are chosen on the x and y axis respectively, so that PDOC is a rectangle.

- 12. Perpendicular distance of the line AB from the point (2, 2) is -
 - (A) $\sqrt{4}$
- (B) $\sqrt{10}$
- (C) $\sqrt{20}$
- (D) $\sqrt{50}$



- Sum of the coordinates of the point P if PDOC is a square is -13.
 - (A) $\frac{32}{3}$
- (B) $\frac{16}{3}$
- (C) 16
- (D) 11
- SL0085
- Number of possible ordered pair(s) of all positions of the point P on AB so that the area of the rectangle 14. PDOC is 30 sq. units, is-
 - (A) three
- (B) two
- (C) one
- (D) zero
- **SL0085**

Paragraph for question nos. 15 and 16

An equilateral triangle ABC has its centroid at the origin and the base BC lies along the line x + y = 1.

- Area of the equilateral ΔABC is -**15.**
 - (A) $\frac{3\sqrt{3}}{2}$
- (B) $\frac{3\sqrt{3}}{4}$
- (C) $\frac{3\sqrt{2}}{2}$
- (D) $\frac{2\sqrt{3}}{4}$

SL0087

- Gradient of the other two lines are -**16.**
 - (A) $\sqrt{3}, \sqrt{2}$
- (B) $\sqrt{3}, \frac{1}{\sqrt{3}}$
- (C) $\sqrt{2} + 1, \sqrt{2} 1$ (D) $2 + \sqrt{3}, 2 \sqrt{3}$ **SL0087**

EXERCISE (S-1)

- Line $\frac{x}{6} + \frac{y}{8} = 1$ intersects the x and y axes at M and N respectively. If the coordinates of the point P 1. lying inside the triangle OMN (where 'O' is origin) are (a, b) such that the areas of the triangle POM, PON and PMN are equal. Find the coordinates of the point P. **SL0089**
- 2. Two vertices of a triangle are (4, -3) & (-2, 5). If the orthocentre of the triangle is at (1, 2), find the coordinates of the third vertex. **SL0090**
- **3.** The point A divides the join of P(-5, 1) & Q(3, 5) in the ratio K: 1. Find the two values of K for which the area of triangle ABC, where B is (1, 5) & C is (7, -2), is equal to 2 units in magnitude. SL0091
- 4. A line is such that its segment between the straight lines 5x - y - 4 = 0 and 3x + 4y - 4 = 0 is bisected at the point (1, 5). Obtain the equation. **SL0097**
- **5.** The area of a triangle is 5. Two of its vertices are (2, 1) & (3, -2). The third vertex lies on y = x + 3. Find the third vertex. SL0092
- A variable line, drawn through the point of intersection of the straight lines $\frac{x}{a} + \frac{y}{b} = 1$ & $\frac{x}{b} + \frac{y}{a} = 1$, meets **6.** the coordinate axes in A & B. Find the locus of the mid point of AB. SL0093
- 7. Consider the family of lines $(x-y-6) + \lambda(2x+y+3) = 0$ and $(x+2y-4) + \lambda(3x-2y-4) = 0$. If the lines of these 2 families are at right angle to each other then find the locus of their point of intersection. **SL0096**
- 8. Two consecutive sides of a parallelogram are 4x + 5y = 0 & 7x + 2y = 0. If the equation to one diagonal is 11x + 7y = 9, find the equation to the other diagonal. **SL0109**
- The line 3x + 2y = 24 meets the y-axis at A & the x-axis at B. The perpendicular bisector of AB meets 9. the line through (0, -1) parallel to x-axis at C. Find the area of the triangle ABC. **SL0098**
- If the straight line drawn through the point P($\sqrt{3}$, 2) & inclined at an angle $\frac{\pi}{6}$ with the x-axis, meets **10.** the line $\sqrt{3} x - 4y + 8 = 0$ at Q. Find the length PQ. **SL0099**
- The points (1,3) & (5,1) are two opposite vertices of a rectangle. The other two vertices lie on the line 11. y = 2x + c. Find c & the remaining vertices. **SL0100**
- 12. A straight line L is perpendicular to the line 5x - y = 1. The area of the triangle formed by the line L & the coordinate axes is 5. Find the equation of the line. **SL0101**
- 13. Two equal sides of an isosceles triangle are given by the equations 7x - y + 3 = 0 and x + y - 3 = 0& its third side passes through the point (1, -10). Determine the equation of the third side. **SL0108**

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14. The equations of the perpendicular bisectors of the sides AB & AC of a triangle ABC are x-y+5=0 & x+2y=0, respectively. If the point A is (1,-2) find the equation of the line BC. **SL0106**

- 15. Given vertices A(1, 1), B(4, -2) & C(5, 5) of a triangle, find the equation of the perpendicular dropped from C to the interior bisector of the angle A.
- Consider a triangle ABC with sides AB and AC having the equations $L_1 = 0$ and $L_2 = 0$. Let the centroid, orthocentre and circumcentre of the \triangle ABC are G, H and S respectively. L = 0 denotes the equation of sides BC.
 - (a) If $L_1: 2x y = 0$ and $L_2: x + y = 3$ and G(2, 3) then find the slope of the line L = 0. **SL0103**
 - (b) If $L_1: 2x + y = 0$ and $L_2: x y + 2 = 0$ and H(2, 3) then find the y-intercept of L = 0. **SL0104**
 - (c) If $L_1: x+y-1=0$ and $L_2: 2x-y+4=0$ and S(2, 1) then find the x-intercept of the line L=0.
- 17. Let P be the point (3, 2). Let Q be the reflection of P about the x-axis. Let R be the reflection of Q about the line y=-x and let S be the reflection of R through the origin. PQRS is a convex quadrilateral. Find the area of PQRS.
- 18. A straight line is drawn from the point (1,0) to the curve $x^2 + y^2 + 6x 10y + 1 = 0$, such that the intercept made on it by the curve subtends a right angle at the origin. Find the equations of the line. **SL0110**

EXERCISE (S-2)

- The equations of perpendiculars of the sides AB & AC of triangle ABC are x y 4 = 0 and 2x y 5 = 0 respectively. If the vertex A is (-2, 3) and point of intersection of perpendiculars bisectors is $\left(\frac{3}{2}, \frac{5}{2}\right)$, find the equation of medians to the sides AB & AC respectively.
- 2. The interior angle bisector of angle A for the triangle ABC whose coordinates of the vertices are A(-8, 5); B(-15, -19) and C(1, -7) has the equation ax + 2y + c = 0. Find 'a' and 'c'. **SL0114**
- 3. Find the equation of the straight lines passing through (-2, -7) & having an intercept of length 3 between the straight lines 4x + 3y = 12, 4x + 3y = 3.
- Two sides of a rhombus ABCD are parallel to the lines y = x + 2 & y = 7x + 3. If the diagonals of the rhombus intersect at the point (1, 2) & the vertex A is on the y-axis, find the possible coordinates of A.
- Find the equations of the sides of a triangle having (4, -1) as a vertex, if the lines x 1 = 0 and x y 1 = 0 are the equations of two internal bisectors of its angles.

 $(1) \frac{23}{\sqrt{15}}$

1.

2.

 $(4) \frac{23}{\sqrt{17}}$

[AIEEE-2010]

[AIEEE 2011]

SL0162

EXERCISE (JM)

equation $\frac{x}{c} + \frac{y}{3} = 1$. Then the distance between L and K is:

(2) $\sqrt{17}$

bisector of the acute angle between L_1 and L_2 intersects L_3 at R.

Statement - 1 : The ratio PR : RQ equals $2\sqrt{2}$: $\sqrt{5}$

The line L given by $\frac{x}{5} + \frac{y}{b} = 1$ passes through the point (13, 32). The line K is parallel to L and has the

The lines $L_1: y-x=0$ and $L_2: 2x+y=0$ intersect the line $L_3: y+2=0$ at P and Q respectively. The

(3) $\frac{17}{\sqrt{15}}$

	Statement - 2: In any triangle, bisector of an angle divides the triangle into two similar triangles.				
	(1) Statement-1 is tr	ue, Statement-2 is false.			
	(2) Statement-1 is false, Statement-2 is true				
	(3) Statement-1 is true,	Statement-2 is true; Statem	nent-2 is a correct explanat	ion for Statement-1	SL0163
	(4) Statement-1 is true, S	Statement-2 is true; Stateme	ent-2 is not a correct explan	ation for Statement-	1.
3.		and $ax - y = 1$ intersec	et each other in the first	quadrant. Then the	he set of
	all possible values of			_	E 2011]
	(1)(-1,1]		$(3)[1,\infty)$		SL0164
4.	_	the point $(1, 2)$ to meet the			_
	OPQ, where O is the origin. If the area of the triangle OPQ is least, then the slope of the line PQ				
				[AIEE	E 2012]
	$(1) -\frac{1}{2}$	$(2) -\frac{1}{4}$	(3) –4	(4) -2	SL0124
5.	If the line $2x + y = k$	passes through the poin	nt which divides the line	segment joining th	ne points
	(1, 1) and (2, 4) in t	he ratio 3: 2, then k eq	juals :	[AIEE	E 2012]
	$(1)\frac{11}{5}$	$(2) \frac{29}{5}$	(3) 5	(4) 6	SL0125
	3	3		,	
6.	A ray of light along v	$+\sqrt{3}y = \sqrt{3}$ gets reflected	unon reaching x-axis the	equation of the refl	ected rav
	Titaly of light along X	y y = y y gets reflected	aponitedening A datis, the	equation of the fen	
	is:			[JEE-MA]	N 2013]
	is:	$(2) \sqrt{3}y = x - \sqrt{3}$		[JEE-MA]	N 2013]
7.	is: (1) $y = x + \sqrt{3}$		$(3) \ y = \sqrt{3}x - \sqrt{3}$	[JEE-MA] $(4) \sqrt{3}y = x - 4$	N 2013]
	is: (1) $y = x + \sqrt{3}$	(2) $\sqrt{3}y = x - \sqrt{3}$ the incentre of the triangle to	$(3) \ y = \sqrt{3}x - \sqrt{3}$	[JEE-MA] $(4) \sqrt{3}y = x - 4$	IN 2013] -1 SL0126 des as (0,
7.	is: (1) $y = x + \sqrt{3}$ The x-coordinate of the 1)(1, 1) and (1, 0) is:	(2) $\sqrt{3}y = x - \sqrt{3}$ the incentre of the triangle to	$(3) \ y = \sqrt{3}x - \sqrt{3}$	[JEE-MA] (4) $\sqrt{3}y = x - 1$ f mid points of its sidentification [JEE-MA]	IN 2013] -1 SL0126 des as (0,
7.	is: (1) $y = x + \sqrt{3}$ The x-coordinate of the 1)(1, 1) and (1, 0) is: (1) $2 + \sqrt{2}$	(2) $\sqrt{3}y = x - \sqrt{3}$ the incentre of the triangle to	(3) $y = \sqrt{3}x - \sqrt{3}$ hat has the coordinates of	[JEE-MA] (4) $\sqrt{3}y = x - 4$ f mid points of its side [JEE-MA] (4) $1 - \sqrt{2}$	IN 2013] -1 SL0126 des as (0, N 2013] SL0127
7.	is: (1) $y = x + \sqrt{3}$ The x-coordinate of the 1)(1, 1) and (1, 0) is: (1) $2 + \sqrt{2}$ A light ray emerging from	(2) $\sqrt{3}y = x - \sqrt{3}$ the incentre of the triangle to (2) $2 - \sqrt{2}$	(3) $y = \sqrt{3}x - \sqrt{3}$ hat has the coordinates of (3) $1 + \sqrt{2}$ P(1,3) is reflected at a point ([JEE-MA] (4) $\sqrt{3}y = x - 4$ f mid points of its side [JEE-MA] (4) $1 - \sqrt{2}$	IN 2013] -1 SL0126 des as (0, N 2013] SL0127 e reflected
7. Syst Bug variables from the state of the	is: (1) $y = x + \sqrt{3}$ The x-coordinate of the 1)(1, 1) and (1, 0) is: (1) $2 + \sqrt{2}$ A light ray emerging from ray passes through the passes	(2) $\sqrt{3}y = x - \sqrt{3}$ we incentre of the triangle to the incentre of the in	(3) $y = \sqrt{3}x - \sqrt{3}$ hat has the coordinates of (3) $1 + \sqrt{2}$ P(1,3) is reflected at a point $Q(3)sa of Q(3) is:$	[JEE-MAI] (4) $\sqrt{3}y = x - 4$ f mid points of its side [JEE-MAI] (4) $1 - \sqrt{2}$ Q in the axis of x. If the	IN 2013] -1 SL0126 des as (0, N 2013] SL0127 e reflected
7. Syst Bug variables from the state of the	is: (1) $y = x + \sqrt{3}$ The x-coordinate of the 1)(1, 1) and (1, 0) is: (1) $2 + \sqrt{2}$ A light ray emerging from ray passes through the proof of	(2) $\sqrt{3}y = x - \sqrt{3}$ The incentre of the triangle of the tr	(3) $y = \sqrt{3}x - \sqrt{3}$ hat has the coordinates of (3) $1 + \sqrt{2}$ P(1,3) is reflected at a point 0 sa of Q is: (3) 1	[JEE-MAI] (4) $\sqrt{3}y = x - 4$ f mid points of its side [JEE-MAI] (4) $1 - \sqrt{2}$ Q in the axis of x. If the [JEE-MAIN Online] (4) $\frac{5}{2}$	IN 2013] -1 SL0126 des as (0, N 2013] SL0127 e reflected ne 2013]
7. Syst Bug variables from the state of the	is: (1) $y = x + \sqrt{3}$ The x-coordinate of the 1)(1, 1) and (1, 0) is: (1) $2 + \sqrt{2}$ A light ray emerging from ray passes through the proof of	(2) $\sqrt{3}y = x - \sqrt{3}$ we incentre of the triangle to the incentre of the in	(3) $y = \sqrt{3}x - \sqrt{3}$ that has the coordinates of (3) $1 + \sqrt{2}$ P(1,3) is reflected at a point (3) as a of Q is: (3) 1 • $y = r$ from a right – angle	[JEE-MAI] (4) $\sqrt{3}y = x - \frac{1}{2}$ f mid points of its side [JEE-MAI] (4) $1 - \sqrt{2}$ Q in the axis of x. If the [JEE-MAIN Online] (4) $\frac{5}{2}$ the ded triangle then:	IN 2013] -1 SL0126 des as (0, N 2013] SL0127 e reflected ne 2013] SL0128
7. Syst Bug variables from the state of the	is: (1) $y = x + \sqrt{3}$ The x-coordinate of the 1)(1, 1) and (1, 0) is: (1) $2 + \sqrt{2}$ A light ray emerging from ray passes through the properties of th	(2) $\sqrt{3}y = x - \sqrt{3}$ The incentre of the triangle of the tr	(3) $y = \sqrt{3}x - \sqrt{3}$ that has the coordinates of the coordinates o	[JEE-MA] (4) $\sqrt{3}y = x - \frac{1}{2}$ f mid points of its sident [JEE-MA] (4) $1 - \sqrt{2}$ Q in the axis of x . If the [JEE-MAIN Online] (4) $\frac{5}{2}$ led triangle then: JEE-MAIN Online]	IN 2013] 1 SL0126 des as (0, IN 2013] SL0127 e reflected ne 2013] SL0128 ne 2013]
7. Avianced Naceder Warth Scheen Schaller (Inn Eng pos	is: (1) $y = x + \sqrt{3}$ The x-coordinate of the 1)(1, 1) and (1, 0) is: (1) $2 + \sqrt{2}$ A light ray emerging from ray passes through the properties of th	(2) $\sqrt{3}y = x - \sqrt{3}$ the incentre of the triangle of the triangle of the triangle of the triangle of the point source placed at From R(6,7), then the absciss (2) $\frac{7}{2}$ = p, $ax + 2y = q$ and $ax + q$	(3) $y = \sqrt{3}x - \sqrt{3}$ that has the coordinates of the coordinates o	[JEE-MA] (4) $\sqrt{3}y = x - \frac{1}{2}$ f mid points of its sident [JEE-MA] (4) $1 - \sqrt{2}$ Q in the axis of x . If the [JEE-MAIN Online] (4) $\frac{5}{2}$ led triangle then: JEE-MAIN Online]	IN 2013] 1 SL0126 des as (0, IN 2013] SL0127 e reflected ne 2013] SL0128 ne 2013]

10.	If the x-intercept of some line L is double as that of the line, $3x + 4y = 12$ and the y-intercept of L is as that of the same line, then the slope of L is: [JEE-MAIN Online 20]			•
	as that of the same line, $(1)-3$	1	(3) - 3/8	$(4) -3/16 \qquad \text{SL0130}$
11.	` '	` '		oints $(2a, 0)$ and $(0, a)$
		of the sides is $x = 2a$, then	=	
	was the equation of one		•	JEE-MAIN Online 2013]
	5 .	5 .	_	
	$(1) \frac{5}{2}a^2$	(2) $\frac{5}{4}a^2$	$(3) \frac{23a}{4}$	(4) $5a^2$ SL0131
12.	Let θ_1 be the angle betw	teen two lines $2x + 3y + c$	$c_1 = 0$ and $-x + 5y + c_2 = 0$), and θ_2 be the angle between
	two lines $2x + 3y + c_1 = 0$	and $-x + 5y + c_3 = 0$, who		
		c_3 are proportional, then	$1\boldsymbol{\theta}_1 = \boldsymbol{\theta}_2.$	JEE-MAIN Online 2013]
	Statement-2 : $\theta_1 = \theta_2$ for the statement $\theta_1 = \theta_2$ for $\theta_1 = \theta_2$ for $\theta_2 = \theta_2$ for $\theta_1 = \theta_2$ for $\theta_2 = \theta_2$ for $\theta_1 = \theta_2$ for $\theta_2 = \theta_2$ for		. 2:	
		Statement - 2 is true, Statement	ent-2 is not a correct explana	tion for Statement-1.
	(2) Statement-1 is false a(3) Statement-1 is true a			SL0132
	` '	d Statement - 2 is true, State	ement-2 is a correct explan	
13.			-	fthis triangle lies on the line
		ne vertex C lies on the li		
				(4) 4x + 3y + 3 = 0
				SL0133
14.	If the image of point P(2,	3) in a line L is $Q(4, 5)$ the		(0, 0) in the same line is:
	/10 / 1 - 5	(2) (2 2)	_	JEE-MAIN Online 2013]
1-	(1)(4,5)		(3)(3,4)	(4)(7,7) SL0134
15.				tersection of the lines
	then:	x + 20y + u = 0 lies in the	Tour in quadrant and is eq	[JEE(Main)-2014]
		(2) $2bc + 3ad = 0$	(3) $3bc - 2ad = 0$	- , , -
	(-)	(-)	(6) 555 = 550	SL0135
16.	Let PS be the median of	the triangle with vertices	SP(2,2), Q(6,-1) and R	(7,3). The equation of the
		−1) and parallel to PS is:		[JEE(Main)-2014] (4) $2x - 9y - 11 = 0$
	(1) 4x - 7y - 11 = 0	(2) 2x + 9y + 7 = 0	(3) 4x + 7y + 3 = 0	(4) 2x - 9y - 11 = 0
	T 0.1 1 0.1		(a a) 1 (SL0136
17.		he point $(2, 3)$ in the line		
	(1) circle of radius $\sqrt{2}$		•	[JEE(Main)-2015]
10	(3) straight line parallel t		(4) straight line parall $1 = 0$ and $7x + x = 5 = 0$. If	•
10.				
		_		(4 0)
	$(1)\left(-\frac{10}{3},-\frac{7}{3}\right)$	(2)(-3,-9)	(3)(-3, -8)	(4) $\left(\frac{1}{3}, -\frac{3}{3}\right)$ SL0138
19.	Let k be an integer such t	that triangle with vertices	(k, -3k), (5, k) and $(-k, 2)$) has area 28 sq. units. Then
	the orthocentre of this tr			[JEE(Main)-2017]
18.19.	Two sides of a rhombus a -2), then which one of the (1) $\left(-\frac{10}{3}, -\frac{7}{3}\right)$	are along the lines, $x-y+1$ e following is a vertex of this (2) (-3, -9)	1 = 0 and $7x-y-5=0$. If is rhombus? (3) (-3, -8)	its diagonals intersect at (-1, [JEE(Main)-2016]
	the orthocentre of this tr	angle is at the point:		[JEE(Main)-2017]

 $(3)\left(1,\frac{3}{4}\right)$

 $(2)\left(2,-\frac{1}{2}\right)$

 $(1)\left(2,\frac{1}{2}\right)$

E

 $(4)\left(1,-\frac{3}{4}\right)$

20.	Let the orthocentre and centroid of a triangle be $A(-3, 5)$ and $B(3, 3)$ respectively. If C is the circumcent of this triangle, then the radius of the circle having line segment AC as diameter, is:			
		_		[JEE(Main)-2018]
	(1) $2\sqrt{10}$	(2) $3\sqrt{\frac{5}{2}}$	$(3) \frac{3\sqrt{5}}{2}$	(4) $\sqrt{10}$ SL0140
21.	A straight line through	a fixed point (2, 3) inter	rsects the coordinate axe	s at distinct points P and Q.
	If O is the origin and the r	ectangle OPRQ is compl	eted, then the locus of R is	: [JEE(Main)-2018]
	(1) 2x + 3y = xy	(2) 3x + 2y = xy	(3) $3x + 2y = 6xy$	(4) $3x + 2y = 6$ SL0141
22.		e axes with integral coor		the origin and the other two n S has area 50sq. units, then [JEE(Main)-2019]
	(1) 9	(2) 18	(3) 32	(4) 36 SL0142
23.	If the line $3x + 4y - 24 = 0$ of the triangle OAB, wh		he point A and the y-axis at	the point B, then the incentre [JEE(Main)-2019]
	(1)(3,4)	(2)(2,2)	(3) (4, 4)	(4)(4,3) SL0143
24.	Two vertices of a triangle which quadrant?	e are (0,2) and (4,3). If it	s orthocentre is at the orig	gin, then its third vertex lies in [JEE(Main)-2019]
	(1) Fourth	(2) Second	(3) Third	(4) First SL0144
25.	Two sides of a parallelog (2,4), then one of its vert	-	x + y = 3 and $x - y + 3 = 0$	0. If its diagonals intersect at [JEE(Main)-2019]
	(1)(2,6)	(2)(2,1)	(3)(3,5)	(4)(3,6) SL0145
26.	If a circle of radius R passlocus of the foot of perp			nate axes at A and B, then the [JEE(Main)-2019]
	$(1) (x^2 + y^2)^2 = 4Rx^2y^2$		$(2) (x^2 + y^2)(x + y)$	<u>*</u>
	$(3) (x^2 + y^2)^3 = 4R^2x^2y^2$		$(4) (x^2 + y^2)^2 = 4R^2$	•
27.	Slope of a line passing the P, is	nrough P(2, 3) and inter	secting the line, $x + y = 7$	at a distance of 4 units from [JEE(Main)-2019]
	$(1) \ \frac{\sqrt{5}-1}{\sqrt{5}+1}$	$(2) \; \frac{1 - \sqrt{5}}{1 + \sqrt{5}}$	$(3) \; \frac{1 - \sqrt{7}}{1 + \sqrt{7}}$	(4) $\frac{\sqrt{7}-1}{\sqrt{7}+1}$ SL0147
28.	A rectangle is inscribed in	a circle with a diameter l	ying along the line $3y = x +$	7. If the two adjacent vertices
	of the rectangle are (-8,	5) and (6, 5), then the a	area of the rectangle (in s	• ,
	(1) 70	(2) 0.4	(2) 00	[JEE(Main)-2019]
20	(1) 72	(2) 84	(3) 98	(4) 56 SL0148
29.	<u> </u>	om the origin to th		pts on the coordinate axes and the of 60° with the line [JEE(Main)-2019]
	(1) $(\sqrt{3}+1)x+(\sqrt{3}-1)y$	$y = 8\sqrt{2}$	(2) $(\sqrt{3}-1)x + (\sqrt{3}-1)x + $	$+1)y = 8\sqrt{2}$
	(3) $\sqrt{3}x + y = 8$		(4) $x + \sqrt{3}y = 8$	SL0149

EXERCISE (JA)

1. Consider the lines given by

$$L_1 = x + 3y - 5 = 0$$

$$L_2 = 3x - ky - 1 = 0$$

 $L_3 = 5x + 2y - 12 = 0$

[JEE 2008, 6]

Match the statements / Expression in **Column-I** with the statements / Expressions in **Column-II** and indicate your answer by darkening the appropriate bubbles in the 4×4 matrix given in OMR.

Column-II Column-II

(A) L_1, L_2, L_3 are concurrent, if

- (P) k = -9
- (B) One of L_1 , L_2 , L_3 is parallel to at least one of the other two, if
- $(Q) \qquad k = -\frac{6}{5}$

(C) L_1, L_2, L_3 form a triangle, if

(R) $k = \frac{5}{6}$

(D) L_1, L_2, L_3 do not form a triangle, if

(S) k = 5

SL0154

- 2. Let P, Q, R and S be the points on the plane with position vectors $-2\hat{i} \hat{j}$, $4\hat{i}$, $3\hat{i} + 3\hat{j}$ and $-3\hat{i} + 2\hat{j}$ respectively. The quadrilateral PQRS must be a
 - (A) parallelogram, which is neither a rhombus nor a rectangle

[JEE 2010, 3]

- (B) square
- (C) rectangle, but not a square
- (D) rhombus, but not a square

SL0155

3. A straight line L through the point (3, -2) is inclined at an angle 60° to the line $\sqrt{3}x + y = 1$. If L also intersect the x-axis, then the equation of L is [JEE 2011, 3 (-1)]

(A)
$$y + \sqrt{3}x + 2 - 3\sqrt{3} = 0$$

(B)
$$y - \sqrt{3}x + 2 + 3\sqrt{3} = 0$$

(C)
$$\sqrt{3}y - x + 3 + 2\sqrt{3} = 0$$

(D)
$$\sqrt{3}y + x - 3 + 2\sqrt{3} = 0$$

SL0156

For a > b > c > 0, the distance between (1, 1) and the point of intersection of the lines ax + by + c = 0 and bx + ay + c = 0 is less than $2\sqrt{2}$. Then [JEE-Advanced 2013, 2]

(A)
$$a + b - c > 0$$

(B)
$$a - b + c < 0$$

(C)
$$a - b + c > 0$$

(D)
$$a + b - c < 0$$

SL0157

For a point P in the plane, let $d_1(P)$ and $d_2(P)$ be the distances of the point P from the lines x - y = 0 and x + y = 0 respectively. The area of the region R consisting of all points P lying in the first quadrant of the plane and satisfying $2 \le d_1(P) + d_2(P) \le 4$, is

[JEE(Advanced)-2014, 3]

ANSWER KEY

EXERCISE (0-1)

- **1.** D **2.** A
- **3.** C

- **5.** C
- **6.** A
- **7.** B **8.** B

- **9.** A
- **10.** D
- **11.** A **12.** B
- **13.** A
- **14.** B
- 15. A
- **16.** D

- 17. A
- **18.** C

26. D

- **19.** D
- **20.** C

4. D

- 22. A
- 23. A
- 24. D

- **25.** D
- **27.** A
- **21.** A **28.** A **29.** B
- **30.** B
- **31.** A **32.** C

- **34.** B **33.** D
- **35.** (A) Q,S,T; (B) P,Q,S,T; (C) S; (D) R,T
- **36.** (A) R, (B) Q (C) T (D) S

EXERCISE (O-2)

- **1.** B
- **2.** B
- **3.** D
- **4.** B **5.** B
- **6.** B,D
- **7.** B,C

13. A

- **8.** A,B,C,D
- **9.** B,C
- **10.** A,C,D **11.** B,C,D **12.** C
- **14.** B

- **15.** A
- **16.** D

EXERCISE (S-1)

- **1.** $\left(2, \frac{8}{3}\right)$ **2.** (33, 26) **3.** $K = 7 \text{ or } \frac{31}{9}$ **4.** 83x 35y + 92 = 0

- **5.** $\left(\frac{7}{2}, \frac{13}{2}\right)$ or $\left(-\frac{3}{2}, \frac{3}{2}\right)$ **6.** 2xy(a+b) = ab(x+y) **7.** $x^2 + y^2 3x + 4y 3 = 0$
- 8. x y = 0
- **9.** 91 sq.units
- **10.** 6 units **11.** c = -4; B(2, 0); D(4, 4)
- **12.** $x + 5y + 5\sqrt{2} = 0$ or $x + 5y 5\sqrt{2} = 0$ **13.** x 3y 31 = 0 or 3x + y + 7 = 0
- **14.** 14x + 23y = 40 **15.** x 5 = 0 **16.** (a) 5; (b) 2; (c) 3/2

18. x + y = 1; x + 9y = 1

EXERCISE (S-2)

- 1. x + 4y = 4; 5x + 2y = 8
- 2. a = 11, c = 78
- 3. 7x + 24y + 182 = 0 or x = -2

- **4.** (0,0) or $\left(0,\frac{5}{2}\right)$ **5.** 2x y + 3 = 0, 2x + y 7 = 0, x 2y 6 = 0

EXERCISE (JM)

- **1.** 4 **2.** 1
- **3.** 3
- **5.** 4 **4.** 4
- **6.** 2
 - **7.** 2 **8.** 4
- **9.** 3

21. 2

- **10.** 4 **11.** 1 **22.** 4
 - **12.** 4

24. 2

23. 2

- **13.** 3 **14.** 4 **25.** 4 **26.** 3
- **15.** 3 **27.** 3
- **16.** 2 **28.** 2 **29.** 1 or 2
- **17.** 1 **18.** 4 **19.** 1 **20.** 2
 - **EXERCISE (JA)**
- **1.** (A) S; (B) P,Q; (C) R; (D) P,Q,S
- **2.** A
- **3.** B
- **4.** A or C or A,C **5.** 6