# SOLUTIONS OF TRIANGLE

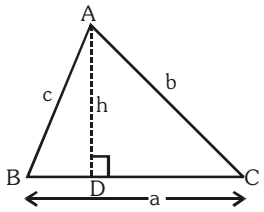
The process of calculating the sides and angles of triangle using given information is called solution of triangle.

In a $\Delta ABC$, the angles are denoted by capital letters A, B and C and the length of the sides opposite these angle are denoted by small letter a, b and c respectively.

## 1. SINE FORMULAE :

In any triangle ABC

$$\frac{a}{\sin A}=\frac{b}{\sin B}=\frac{c}{\sin C}=\lambda=\frac{abc}{2\Delta}=2R$$




where R is circumradius and $\Delta$ is area of triangle.

***Illustration 1 :*** Angles of a triangle are in 4 : 1 : 1 ratio. The ratio between its greatest side and perimeter is

(A) $\frac{3}{2+\sqrt{3}}$ (B) $\frac{\sqrt{3}}{2+\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2-\sqrt{3}}$ (D) $\frac{1}{2+\sqrt{3}}$

***Solution :*** Angles are in ratio 4 : 1 : 1.

$\Rightarrow$ angles are 120°, 30°, 30°.

If sides opposite to these angles are a, b, c respectively, then a will be the greatest side.

Now from sine formula $\frac{a}{\sin 120°}=\frac{b}{\sin 30°}=\frac{c}{\sin 30°}$

$\Rightarrow \quad \frac{a}{\sqrt{3}/2}=\frac{b}{1/2}=\frac{c}{1/2}$

$\Rightarrow \quad \frac{a}{\sqrt{3}}=\frac{b}{1}=\frac{c}{1}=k$ (say)

then $a=\sqrt{3}k$, perimeter $=(2+\sqrt{3})k$

$\therefore$ required ratio $=\frac{\sqrt{3}k}{(2+\sqrt{3})k}=\frac{\sqrt{3}}{2+\sqrt{3}}$ **Ans. (B)**

***Illustration 2 :*** In triangle ABC, if b = 3, c = 4 and $\angle B = \pi/3$, then number of such triangles is -

(A) 1 (B) 2 (C) 0 (D) infinite

***Solution :*** Using sine formulae $\frac{\sin B}{b}=\frac{\sin C}{c}$

$\Rightarrow \quad \frac{\sin \pi/3}{3}=\frac{\sin C}{4} \Rightarrow \frac{\sqrt{3}}{6}=\frac{\sin C}{4} \Rightarrow \sin C=\frac{2}{\sqrt{3}}>1$ which is not possible.

Hence there exist no triangle with given elements. **Ans. (C)**

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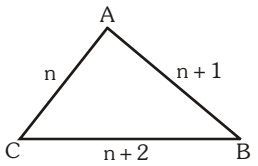
***Illustration 3 :*** The sides of a triangle are three consecutive natural numbers and its largest angle is twice the smallest one. Determine the sides of the triangle.




***Solution :*** Let the sides be n, n + 1, n + 2 cms.

i.e. AC = n, AB = n + 1, BC = n + 2

Smallest angle is B and largest one is A.

Here, $\angle A = 2\angle B$

Also, $\angle A + \angle B + \angle C = 180°$

$\Rightarrow \quad 3\angle B + \angle C = 180° \Rightarrow \angle C = 180° - 3\angle B$

We have, sine law as,

$$\frac{\sin A}{n+2} = \frac{\sin B}{n} = \frac{\sin C}{n+1} \Rightarrow \frac{\sin 2B}{n+2} = \frac{\sin B}{n} = \frac{\sin(180-3B)}{n+1}$$

$$\Rightarrow \frac{\sin 2B}{n+2} = \frac{\sin B}{n} = \frac{\sin 3B}{n+1}$$

(i) (ii) (iii)

from (i) and (ii);

$$\frac{2\sin B\cos B}{n+2} = \frac{\sin B}{n} \Rightarrow \cos B = \frac{n+2}{2n} \quad \text{..... (iv)}$$

and from (ii) and (iii);

$$\frac{\sin B}{n} = \frac{3\sin B - 4\sin^3 B}{n+1} \Rightarrow \frac{\sin B}{n} = \frac{\sin B(3-4\sin^2 B)}{n+1}$$

$$\Rightarrow \frac{n+1}{n} = 3 - 4(1-\cos^2 B) \quad \text{..... (v)}$$

from (iv) and (v), we get

$$\frac{n+1}{n} = -1 + 4\left(\frac{n+2}{2n}\right)^2 \Rightarrow \frac{n+1}{n} + 1 = \left(\frac{n^2+4n+4}{n^2}\right)$$

$$\Rightarrow \frac{2n+1}{n} = \frac{n^2+4n+4}{n^2} \Rightarrow 2n^2 + n = n^2 + 4n + 4$$

$\Rightarrow \quad n^2 - 3n - 4 = 0 \Rightarrow (n-4)(n+1) = 0$

$n = 4$ or $-1$

where $n \neq -1$

$\therefore \quad n = 4$. Hence the sides are 4, 5, 6 **Ans.**

**Do yourself - 1 :**

**(i)** If in a $\Delta ABC$, $\angle A = \frac{\pi}{6}$ and $b : c = 2 : \sqrt{3}$, find $\angle B$.

**(ii)** Show that, in any $\Delta ABC$ : $a\sin(B-C) + b\sin(C-A) + c\sin(A-B) = 0$.

**(iii)** If in a $\Delta ABC$, $\frac{\sin A}{\sin C} = \frac{\sin(A-B)}{\sin(B-C)}$, show that $a^2, b^2, c^2$ are in A.P.

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## 2. COSINE FORMULAE :

**(a)** $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ **(b)** $\cos B = \frac{c^2 + a^2 - b^2}{2ca}$ **(c)** $\cos C = \frac{a^2 + b^2 - c^2}{2ab}$

or $a^2 = b^2 + c^2 - 2bc \cos A$

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***Illustration 4 :*** In a triangle ABC, if $B = 30°$ and $c = \sqrt{3}\, b$, then A can be equal to -

(A) 45° (B) 60° (C) 90° (D) 120°

***Solution :*** We have $\cos B = \frac{c^2 + a^2 - b^2}{2ca} \Rightarrow \frac{\sqrt{3}}{2} = \frac{3b^2 + a^2 - b^2}{2 \times \sqrt{3} b \times a}$

$\Rightarrow \quad a^2 - 3ab + 2b^2 = 0 \Rightarrow (a - 2b)(a - b) = 0$

$\Rightarrow$ Either $a = b \Rightarrow A = 30°$

or $a = 2b \Rightarrow a^2 = 4b^2 = b^2 + c^2 \Rightarrow A = 90°$. **Ans. (C)**

***Illustration 5 :*** In a triangle ABC, $(a^2 - b^2 - c^2) \tan A + (a^2 - b^2 + c^2) \tan B$ is equal to -

(A) $(a^2 + b^2 - c^2) \tan C$ (B) $(a^2 + b^2 + c^2) \tan C$

(C) $(b^2 + c^2 - a^2) \tan C$ (D) none of these

***Solution :*** Using cosine law :

The given expression is equal to $-2bc \cos A \tan A + 2ac \cos B \tan B$

$= 2abc\left(-\frac{\sin A}{a} + \frac{\sin B}{b}\right) = 0$ **Ans. (D)**

**Do yourself - 2 :**

**(i)** If $a : b : c = 4 : 5 : 6$, then show that $\angle C = 2\angle A$.

**(ii)** In any $\Delta ABC$, prove that

(a) $\frac{\cos A}{a} + \frac{\cos B}{b} + \frac{\cos C}{c} = \frac{a^2 + b^2 + c^2}{2abc}$

(b) $\frac{b^2}{a}\cos A + \frac{c^2}{b}\cos B + \frac{a^2}{c}\cos C = \frac{a^4 + b^4 + c^4}{2abc}$

## 3. PROJECTION FORMULAE :

**(a)** $b \cos C + c \cos B = a$ **(b)** $c \cos A + a \cos C = b$ **(c)** $a \cos B + b \cos A = c$

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***Illustration 6 :*** In a $\Delta ABC$, $c\cos^2\frac{A}{2}+a\cos^2\frac{C}{2}=\frac{3b}{2}$, then show a, b, c are in A.P.

***Solution :*** Here, $\frac{c}{2}(1+\cos A)+\frac{a}{2}(1+\cos C)=\frac{3b}{2}$

$\Rightarrow$ a + c + (c cos A + a cos C) = 3b

$\Rightarrow$ a + c + b = 3b {using projection formula}

$\Rightarrow$ a + c = 2b

which shows a, b, c are in A.P.

**Do yourself - 3 :**

**(i)** In a $\Delta ABC$, if $\angle A=\frac{\pi}{4}$, $\angle B=\frac{5\pi}{12}$, show that $a+c\sqrt{2}=2b$.

**(ii)** In a $\Delta ABC$, prove that : (a) $b(a\cos C-c\cos A)=a^2-c^2$ (b) $2\left(b\cos^2\frac{C}{2}+c\cos^2\frac{B}{2}\right)=a+b+c$

## 4. NAPIER'S ANALOGY (TANGENT RULE) :

**(a)** $\tan\left(\frac{B-C}{2}\right)=\frac{b-c}{b+c}\cot\frac{A}{2}$ **(b)** $\tan\left(\frac{C-A}{2}\right)=\frac{c-a}{c+a}\cot\frac{B}{2}$ **(c)** $\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}\cot\frac{C}{2}$

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***Illustration 7 :*** In a $\Delta ABC$, the tangent of half the difference of two angles is one-third the tangent of half the sum of the angles. Determine the ratio of the sides opposite to the angles.

***Solution :*** Here, $\tan\left(\frac{A-B}{2}\right)=\frac{1}{3}\tan\left(\frac{A+B}{2}\right)$ ..... (i)

using Napier's analogy, $\tan\left(\frac{A-B}{2}\right)=\frac{a-b}{a+b}.\cot\left(\frac{C}{2}\right)$ ..... (ii)

from (i) & (ii) ;

$$\frac{1}{3}\tan\left(\frac{A+B}{2}\right)=\frac{a-b}{a+b}.\cot\left(\frac{C}{2}\right)\Rightarrow\frac{1}{3}\cot\left(\frac{C}{2}\right)=\frac{a-b}{a+b}.\cot\left(\frac{C}{2}\right)$$

{as A + B + C = $\pi$ $\therefore\tan\left(\frac{B+C}{2}\right)=\tan\left(\frac{\pi}{2}-\frac{C}{2}\right)=\cot\frac{C}{2}$}

$\Rightarrow$ $\frac{a-b}{a+b}=\frac{1}{3}$ or 3a – 3b = a + b

2a = 4b or $\frac{a}{b}=\frac{2}{1}\Rightarrow\frac{b}{a}=\frac{1}{2}$

Thus the ratio of the sides opposite to the angles is b : a = 1 : 2. **Ans.**

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**Do yourself - 4 :**

**(i)** In any $\Delta ABC$, prove that $\frac{b-c}{b+c}=\frac{\tan\left(\frac{B-C}{2}\right)}{\tan\left(\frac{B+C}{2}\right)}$

**(ii)** If $\Delta ABC$ is right angled at C, prove that : (a) $\tan\frac{A}{2}=\sqrt{\frac{c-b}{c+b}}$ (b) $\sin(A-B)=\frac{a^2-b^2}{a^2+b^2}$

## 5. HALF ANGLE FORMULAE :

$s=\frac{a+b+c}{2}$ = semi-perimeter of triangle.

**(a)** (i) $\sin\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{bc}}$ (ii) $\sin\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{ca}}$ (iii) $\sin\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{ab}}$

**(b)** (i) $\cos\frac{A}{2}=\sqrt{\frac{s(s-a)}{bc}}$ (ii) $\cos\frac{B}{2}=\sqrt{\frac{s(s-b)}{ca}}$ (iii) $\cos\frac{C}{2}=\sqrt{\frac{s(s-c)}{ab}}$

**(c)** (i) $\tan\frac{A}{2}=\sqrt{\frac{(s-b)(s-c)}{s(s-a)}}=\frac{\Delta}{s(s-a)}$ (ii) $\tan\frac{B}{2}=\sqrt{\frac{(s-c)(s-a)}{s(s-b)}}=\frac{\Delta}{s(s-b)}$ (iii) $\tan\frac{C}{2}=\sqrt{\frac{(s-a)(s-b)}{s(s-c)}}=\frac{\Delta}{s(s-c)}$

**(d) Area of Triangle**

$\Delta=\sqrt{s(s-a)(s-b)(s-c)}=\frac{1}{2}bc\sin A=\frac{1}{2}ca\sin B=\frac{1}{2}ab\sin C=\frac{1}{2}ap_1=\frac{1}{2}bp_2=\frac{1}{2}cp_3$,

where $p_1,p_2,p_3$ are altitudes from vertices A,B,C respectively.

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***Illustration 8 :*** If in a triangle ABC, CD is the angle bisector of the angle ACB, then CD is equal to-

(A) $\frac{a+b}{2ab}\cos\frac{C}{2}$ (B) $\frac{2ab}{a+b}\sin\frac{C}{2}$ (C) $\frac{2ab}{a+b}\cos\frac{C}{2}$ (D) $\frac{b\sin\angle DAC}{\sin(B+C/2)}$

***Solution :*** $\Delta CAB = \Delta CAD + \Delta CDB$

$\Rightarrow \frac{1}{2}ab\sin C=\frac{1}{2}b.CD.\sin\left(\frac{C}{2}\right)+\frac{1}{2}a.CD\sin\left(\frac{C}{2}\right)$

$\Rightarrow CD(a+b)\sin\left(\frac{C}{2}\right)=ab\left(2\sin\left(\frac{C}{2}\right)\cos\left(\frac{C}{2}\right)\right)$

So $CD=\frac{2ab\cos(C/2)}{(a+b)}$

and in $\Delta CAD$, $\frac{CD}{\sin\angle DAC}=\frac{b}{\sin\angle CDA}$ (by sine rule)

$\Rightarrow CD=\frac{b\sin\angle DAC}{\sin(B+C/2)}$ **Ans. (C,D)**

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***Illustration 9 :*** If $\Delta$ is the area and 2s the sum of the sides of a triangle, then show $\Delta \le \frac{s^2}{3\sqrt{3}}$.

***Solution :*** We have, $2s = a + b + c$, $\Delta^2 = s(s-a)(s-b)(s-c)$

Now, A.M. $\ge$ G.M.

$$\frac{(s-a)+(s-b)+(s-c)}{3} \ge \{(s-a)(s-b)(s-c)\}^{1/3}$$

or $$\frac{3s-2s}{3} \ge \left(\frac{\Delta^2}{s}\right)^{1/3}$$

or $$\frac{s}{3} \ge \left(\frac{\Delta^2}{s}\right)^{1/3}$$

or $$\frac{\Delta^2}{s} \le \frac{s^3}{27} \quad \Rightarrow \quad \Delta \le \frac{s^2}{3\sqrt{3}}$$ **Ans.**

**Do yourself - 5 :**

**(i)** Given a = 6, b = 8, c = 10. Find

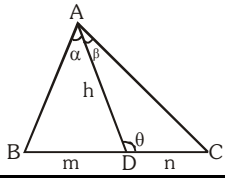
(a) $\sin A$ (b) $\tan A$ (c) $\sin\frac{A}{2}$ (d) $\cos\frac{A}{2}$ (e) $\tan\frac{A}{2}$ (f) $\Delta$

**(ii)** Prove that in any $\Delta ABC$, (abcs) $\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2} = \Delta^2$.

## 6. m-n THEOREM :

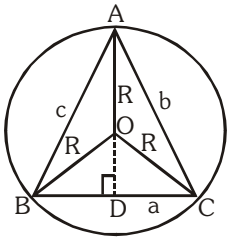
$(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

$(m+n)\cot\theta = n\cot B - m\cot C$.




## 7. RADIUS OF THE CIRCUMCIRCLE 'R' :

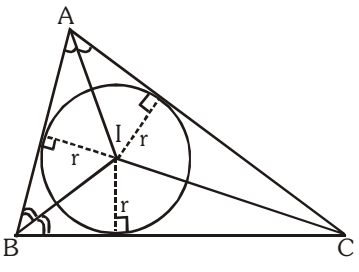
Circumcentre is the point of intersection of perpendicular bisectors of the sides and distance between circumcentre & vertex of triangle is called circumradius 'R'.




$$R = \frac{a}{2\sin A} = \frac{b}{2\sin B} = \frac{c}{2\sin C} = \frac{abc}{4\Delta}.$$

## 8. RADIUS OF THE INCIRCLE 'r' :

Point of intersection of internal angle bisectors is incentre and perpendicular distance of incentre from any side is called inradius 'r'.




$$r = \frac{\Delta}{s} = (s-a)\tan\frac{A}{2} = (s-b)\tan\frac{B}{2} = (s-c)\tan\frac{C}{2} = 4R\sin\frac{A}{2}\sin\frac{B}{2}\sin\frac{C}{2}.$$

$$= a\frac{\sin\frac{B}{2}\sin\frac{C}{2}}{\cos\frac{A}{2}} = b\frac{\sin\frac{A}{2}\sin\frac{C}{2}}{\cos\frac{B}{2}} = c\frac{\sin\frac{B}{2}\sin\frac{A}{2}}{\cos\frac{C}{2}}$$

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***Illustration 10 :*** In a triangle ABC, if a : b : c = 4 : 5 : 6, then ratio between its circumradius and inradius is-

(A) $\frac{16}{7}$ (B) $\frac{16}{9}$ (C) $\frac{7}{16}$ (D) $\frac{11}{7}$

***Solution :*** $\frac{R}{r}=\frac{abc}{4\Delta}\Big/\frac{\Delta}{s}=\frac{(abc)s}{4\Delta^2} \Rightarrow \frac{R}{r}=\frac{abc}{4(s-a)(s-b)(s-c)}$ ....(i)

$\because$ a : b : c = 4 : 5 : 6 $\Rightarrow \frac{a}{4}=\frac{b}{5}=\frac{c}{6}=k$ (say)

$\Rightarrow$ a = 4k, b = 5k, c = 6k

$\therefore s=\frac{a+b+c}{2}=\frac{15k}{2}, s-a=\frac{7k}{2}, s-b=\frac{5k}{2}, s-c=\frac{3k}{2}$

using (i) in these values $\frac{R}{r}=\frac{(4k)(5k)(6k)}{4\left(\frac{7k}{2}\right)\left(\frac{5k}{2}\right)\left(\frac{3k}{2}\right)}=\frac{16}{7}$ **Ans. (A)**

***Illustration 11 :*** If A, B, C are the angles of a triangle, prove that : cosA + cosB + cosC = $1+\frac{r}{R}$.

***Solution :*** $\cos A+\cos B+\cos C=2\cos\left(\frac{A+B}{2}\right).\cos\left(\frac{A-B}{2}\right)+\cos C$

$=2\sin\frac{C}{2}.\cos\left(\frac{A-B}{2}\right)+1-2\sin^2\frac{C}{2}=1+2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\sin\left(\frac{C}{2}\right)\right]$

$=1+2\sin\frac{C}{2}\left[\cos\left(\frac{A-B}{2}\right)-\cos\left(\frac{A+B}{2}\right)\right] \quad \left\{\because \frac{C}{2}=90°-\left(\frac{A+B}{2}\right)\right\}$

$=1+2\sin\frac{C}{2}.2\sin\frac{A}{2}.\sin\frac{B}{2}=1+4\sin\frac{A}{2}.\sin\frac{B}{2}.\sin\frac{C}{2}$

$=1+\frac{r}{R}$ {as, r = 4R sin A/2 . sinB/2 . sinC/2}

$\Rightarrow$ cosA + cosB + cosC = $1+\frac{r}{R}$. Hence proved.

**Do yourself - 6 :**

**(i)** If in $\Delta ABC$, a = 3, b = 4 and c = 5, find

(a) $\Delta$ (b) R (c) r

**(ii)** In a $\Delta ABC$, show that :

(a) $\frac{a^2-b^2}{c}=2R\sin(A-B)$ (b) $r\cos\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}=\frac{\Delta}{4R}$ (c) $a+b+c=\frac{abc}{2Rr}$

**(iii)** Let $\Delta$ & $\Delta'$ denote the areas of a $\Delta$ and that of its incircle. Prove that

$\Delta:\Delta'=\left(\cot\frac{A}{2}.\cot\frac{B}{2}.\cot\frac{C}{2}\right):\pi$

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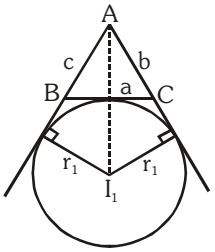
## 9. RADII OF THE EX-CIRCLES :




Point of intersection of two external angles and one internal angle bisectors is excentre and perpendicular distance of excentre from any side is called exradius. If $r_1$ is the radius of escribed circle opposite to $\angle A$ of $\Delta ABC$ and so on, then -

**(a)** $r_1 = \frac{\Delta}{s-a} = s\tan\frac{A}{2} = 4R\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2} = \frac{a\cos\frac{B}{2}\cos\frac{C}{2}}{\cos\frac{A}{2}}$

**(b)** $r_2 = \frac{\Delta}{s-b} = s\tan\frac{B}{2} = 4R\cos\frac{A}{2}\sin\frac{B}{2}\cos\frac{C}{2} = \frac{b\cos\frac{A}{2}\cos\frac{C}{2}}{\cos\frac{B}{2}}$

**(c)** $r_3 = \frac{\Delta}{s-c} = s\tan\frac{C}{2} = 4R\cos\frac{A}{2}\cos\frac{B}{2}\sin\frac{C}{2} = \frac{c\cos\frac{A}{2}\cos\frac{B}{2}}{\cos\frac{C}{2}}$

$I_1$, $I_2$ and $I_3$ are taken as ex-centre opposite to vertex A, B, C repsectively.

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***Illustration 12 :*** Value of the expression $\frac{b-c}{r_1}+\frac{c-a}{r_2}+\frac{a-b}{r_3}$ is equal to -

(A) 1 (B) 2 (C) 3 (D) 0

***Solution :*** $\frac{(b-c)}{r_1}+\frac{(c-a)}{r_2}+\frac{(a-b)}{r_3}$

$\Rightarrow \quad (b-c)\left(\frac{s-a}{\Delta}\right)+(c-a)\left(\frac{s-b}{\Delta}\right)+(a-b).\left(\frac{s-c}{\Delta}\right)$

$\Rightarrow \quad \frac{(s-a)(b-c)+(s-b)(c-a)+(s-c)(a-b)}{\Delta}$

$= \frac{s(b-c+c-a+a-b)-[ab-ac+bc-ba+ac-bc]}{\Delta} = \frac{0}{\Delta} = 0$

Thus, $\frac{b-c}{r_1}+\frac{c-a}{r_2}+\frac{a-b}{r_3} = 0$ **Ans. (D)**

***Illustration 13 :*** If $r_1 = r_2 + r_3 + r$, prove that the triangle is right angled.

***Solution :*** We have, $r_1 - r = r_2 + r_3$

$\Rightarrow \quad \frac{\Delta}{s-a}-\frac{\Delta}{s} = \frac{\Delta}{s-b}+\frac{\Delta}{s-c} \qquad \Rightarrow \quad \frac{s-s+a}{s(s-a)} = \frac{s-c+s-b}{(s-b)(s-c)}$

$\Rightarrow \quad \frac{a}{s(s-a)} = \frac{2s-(b+c)}{(s-b)(s-c)} \qquad \{as,\ 2s = a + b + c\}$

$\Rightarrow \quad \frac{a}{s(s-a)} = \frac{a}{(s-b)(s-c)} \qquad \Rightarrow \quad s^2 - (b + c)\, s + bc = s^2 - as$

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$\Rightarrow$ $s(-a+b+c)=bc$ $\Rightarrow$ $\dfrac{(b+c-a)(a+b+c)}{2}=bc$

$\Rightarrow$ $(b+c)^2-(a)^2=2bc$ $\Rightarrow$ $b^2+c^2+2bc-a^2=2bc$

$\Rightarrow$ $b^2+c^2=a^2$

$\therefore$ $\angle A=90°$. **Ans.**

**Do yourself - 7 :**

**(i)** In an equilateral $\Delta ABC$, R = 2, find

(a) r (b) $r_1$ (c) a

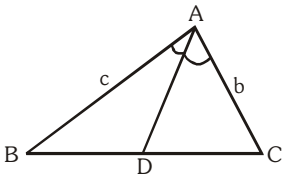
**(ii)** In a $\Delta ABC$, show that

(a) $r_1r_2+r_2r_3+r_3r_1=s^2$ (b) $\dfrac{1}{4}r^2s^2\left(\dfrac{1}{r}-\dfrac{1}{r_1}\right)\left(\dfrac{1}{r}-\dfrac{1}{r_2}\right)\left(\dfrac{1}{r}-\dfrac{1}{r_3}\right)=R$

(c) $\sqrt{rr_1r_2r_3}=\Delta$

## 10. ANGLE BISECTORS & MEDIANS :

An angle bisector divides the base in the ratio of corresponding sides.



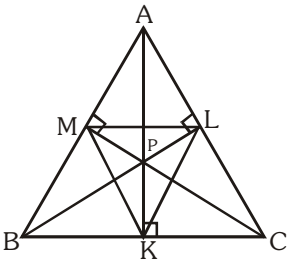

$\dfrac{BD}{CD}=\dfrac{c}{b}$ $\Rightarrow$ $BD=\dfrac{ac}{b+c}$ & $CD=\dfrac{ab}{b+c}$

If $m_a$ and $\beta_a$ are the lengths of a median and an angle bisector from the angle A then,

$m_a=\dfrac{1}{2}\sqrt{2b^2+2c^2-a^2}$ and $\beta_a=\dfrac{2bc\cos\dfrac{A}{2}}{b+c}$

Note that $m_a^2+m_b^2+m_c^2=\dfrac{3}{4}(a^2+b^2+c^2)$

## 11. ORTHOCENTRE :




**(a)** Point of intersection of altitudes is orthocentre & the triangle KLM which is formed by joining the feet of the altitudes is called the pedal triangle.

**(b)** The distances of the orthocentre from the angular points of the $\Delta ABC$ are 2R cosA, 2R cosB, & 2R cosC.

**(c)** The distance of P from sides are 2R cosB cosC, 2R cosC cosA and 2R cosA cosB.

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**Do yourself - 8 :**

**(i)** If x, y, z are the distance of the vertices of $\Delta ABC$ respectively from the orthocentre, then prove that $\frac{a}{x}+\frac{b}{y}+\frac{c}{z}=\frac{abc}{xyz}$

**(ii)** If $p_1, p_2, p_3$ are respectively the perpendiculars from the vertices of a triangle to the opposite sides, prove that

(a) $p_1p_2p_3=\frac{a^2b^2c^2}{8R^3}$ (b) $\Delta=\sqrt{\frac{1}{2}Rp_1p_2p_3}$

**(iii)** In a $\Delta ABC$, AD is altitude and H is the orthocentre prove that AH : DH = (tanB + tanC) : tanA

**(iv)** In a $\Delta ABC$, the lengths of the bisectors of the angle A, B and C are x, y, z respectively.

Show that $\frac{1}{x}\cos\frac{A}{2}+\frac{1}{y}\cos\frac{B}{2}+\frac{1}{z}\cos\frac{C}{2}=\frac{1}{a}+\frac{1}{b}+\frac{1}{c}$.

## 12. THE DISTANCES BETWEEN THE SPECIAL POINTS :

**(a)** The distance between circumcentre and orthocentre is = $R\sqrt{1-8\cos A\cos B\cos C}$

**(b)** The distance between circumcentre and incentre is $=\sqrt{R^2-2Rr}$

**(c)** The distance between incentre and orthocentre is $=\sqrt{2r^2-4R^2\cos A\cos B\cos C}$

**(d)** The distances between circumcentre & excentres are

$OI_1=R\sqrt{1+8\sin\frac{A}{2}\cos\frac{B}{2}\cos\frac{C}{2}}=\sqrt{R^2+2Rr_1}$ & so on.

---

***Illustration 14 :*** Prove that the distance between the circumcentre and the orthocentre of a triangle ABC is $R\sqrt{1-8\cos A\cos B\cos C}$.

***Solution :*** Let O and P be the circumcentre and the orthocentre respectively. If OF is the perpendicular to AB, we have $\angle OAF = 90° - \angle AOF = 90° - C$. Also $\angle PAL = 90° - C$.

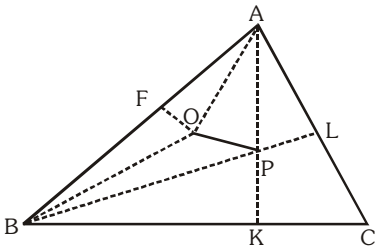
Hence, $\angle OAP = A - \angle OAF - \angle PAL = A - 2(90° - C) = A + 2C - 180°$

$= A + 2C - (A + B + C) = C - B$.

Also OA = R and PA = 2RcosA.

Now in $\Delta AOP$,

$OP^2 = OA^2 + PA^2 - 2OA.\ PA\ \cos OAP$

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$= R^2 + 4R^2\cos^2 A - 4R^2\cos A\cos(C - B)$

$= R^2 + 4R^2\cos A[\cos A - \cos(C - B)]$

$= R^2 - 4R^2\cos A[\cos(B + C) + \cos(C - B)] = R^2 - 8R^2\cos A\cos B\cos C.$

Hence OP = $R\sqrt{1-8\cos A\cos B\cos C}$. **Ans.**

## 13. SOLUTION OF TRIANGLES :

The three sides a,b,c and the three angles A,B,C are called the elements of the triangle ABC. When any three of these six elements (except all the three angles) of a triangle are given, the triangle is known completely; that is the other three elements can be expressed in terms of the given elements and can be evaluated. This process is called the solution of triangles.

* If the three sides a,b,c are given, angle A is obtained from $\tan\frac{A}{2} = \sqrt{\frac{(s-b)(s-c)}{s(s-a)}}$

or $\cos A = \frac{b^2 + c^2 - a^2}{2bc}$ .B and C can be obtained in the similar way.

* If two sides b and c and the included angle A are given, then $\tan\frac{B-C}{2} = \frac{b-c}{b+c}\cot\frac{A}{2}$ gives

$\frac{B-C}{2}$. Also $\frac{B+C}{2} = 90° - \frac{A}{2}$, so that B and C can be evaluated. The third side is given by

$a = b\,\frac{\sin A}{\sin B}$

or $a^2 = b^2 + c^2 - 2bc\cos A$.

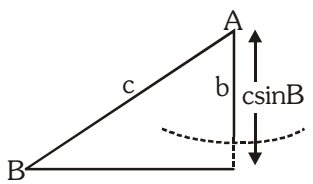
* If two sides b and c and an angle opposite the one of them (say B) are given then

$\sin C = \frac{c}{b}\sin B$, $A = 180° - (B + C)$ and $a = \frac{b\sin A}{\sin B}$ given the remaining elements.

**Case I :**

$b < c \sin B$.

We draw the side c and angle B. Now it is obvious from the figure that there is no triangle possible.

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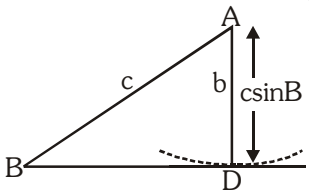
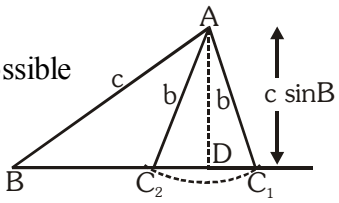
**Case II :**

b = c sin B and B is an acute angle, there is only one triangle possible. and it is right-angled at C.



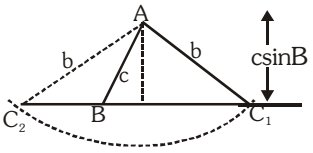

**Case III :**

b > c sin B, b < c and B is an acute angle, then there are two triangles possible for two values of angle C.




**Case IV :**

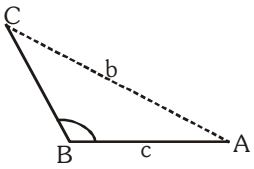
b > c sin B, c < b and B is an acute angle, then there is only one triangle.




**Case V :**

b > c sin B, c > b and B is an obtuse angle.

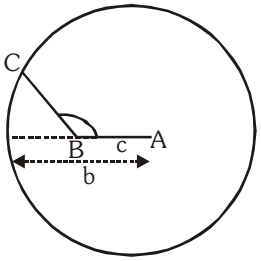
For any choice of point C, b will be greater than c which is a contradication as c > b (given). So there is no triangle possible.




**Case VI :**

b > c sin B, c < b and B is an obtuse angle.

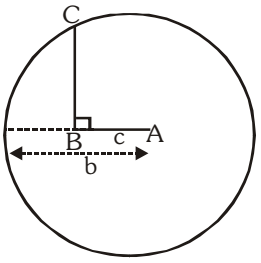
We can see that the circle with A as centre and b as radius will cut the line only in one point. So only one triangle is possible.




**Case VII :**

b > c and B = 90°.

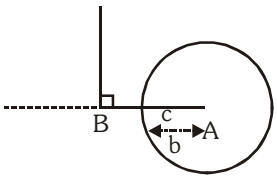
Again the circle with A as centre and b as radius will cut the line only in one point. So only one triangle is possible.




**Case VIII :**

b $\leq$ c and B = 90°.

The circle with A as centre and b as radius will not cut the line in any point. So no triangle is possible.




This is, sometimes, called an ambiguous case.

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**Alternative Method :**

By applying cosine rule, we have $\cos B = \frac{a^2 + c^2 - b^2}{2ac}$

$\Rightarrow \quad a^2 - (2c\cos B)a + (c^2 - b^2) = 0 \Rightarrow a = c\cos B \pm \sqrt{(c\cos B)^2 - (c^2 - b^2)}$

$\Rightarrow \quad a = c\cos B \pm \sqrt{b^2 - (c\sin B)^2}$

This equation leads to following cases :

**Case-I :** If $b < c\sin B$, no such triangle is possible.

**Case-II:** Let $b = c\sin B$. There are further following case :

**(a)** B is an obtuse angle $\Rightarrow$ cosB is negative. There exists no such triangle.

**(b)** B is an acute angle $\Rightarrow$ cosB is positive. There exists only one such triangle.

**Case-III:** Let $b > c\sin B$. There are further following cases :

**(a)** B is an acute angle $\Rightarrow$ cosB is positive. In this case triangle will exist if and only if $c\cos B > \sqrt{b^2 - (c\sin B)^2}$ or $c > b \Rightarrow$ Two such triangle is possible. If $c < b$, only one such triangle is possible.

**(b)** B is an obtuse angle $\Rightarrow$ cosB is negative. In this case triangle will exist if and only if $\sqrt{b^2 - (c\sin B)^2} > |c\cos B| \Rightarrow b > c$. So in this case only one such triangle is possible. If $b < c$ there exists no such triangle.

This is called an ambiguous case.

* If one side a and angles B and C are given, then $A = 180° - (B + C)$, and $b = \frac{a\sin B}{\sin A}, c = \frac{a\sin C}{\sin A}$.

* If the three angles A,B,C are given, we can only find the ratios of the sides a,b,c by using sine rule (since there are infinite similar triangles possible).

***Illustration 15 :*** In the ambiguous case of the solution of triangles, prove that the circumcircles of the two triangles are of same size.

***Solution :*** Let us say b,c and angle B are given in the ambiguous case. Both the triangles will have b and its opposite angle as B. so $\frac{b}{\sin B} = 2R$ will be given for both the triangles. So their circumradii and therefore their sizes will be same.

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***Illustration 16 :*** If a,b and A are given in a triangle and $c_1, c_2$ are the possible values of the third side, prove that $c_1^2 + c_2^2 - 2c_1c_2 \cos 2A = 4a^2\cos^2 A$.

***Solution :*** $\cos A = \dfrac{b^2 + c^2 - a^2}{2bc}$

$\Rightarrow c^2 - 2bc\cos A + b^2 - a^2 = 0.$

$c_1 + c_2 = 2b\cos A$ and $c_1c_2 = b^2 - a^2$.

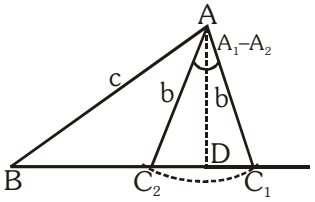
$\Rightarrow c_1^2 + c_2^2 - 2c_1c_2\cos 2A = (c_1 + c_2)^2 - 2c_1c_2(1 + \cos 2A)$

$= 4b^2\cos^2 A - 2(b^2 - a^2)2\cos^2 A = 4a^2\cos^2 A.$

***Illustration 17 :*** If b,c,B are given and b < c, prove that $\cos\left(\dfrac{A_1 - A_2}{2}\right) = \dfrac{c\sin B}{b}$.

***Solution :*** $\angle C_2AC_1$ is bisected by AD.

$\Rightarrow$ In $\Delta AC_2D$, $\cos\left(\dfrac{A_1 - A_2}{2}\right) = \dfrac{AD}{AC_2} = \dfrac{c\sin B}{b}$




Hence proved.

**Do yourself - 9 :**

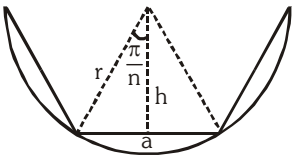
(i) If b,c,B are given and b<c, prove that $\sin\left(\dfrac{A_1 - A_2}{2}\right) = \dfrac{a_1 - a_2}{2b}$

(ii) In a $\Delta ABC$, b,c,B (c > b) are gives. If the third side has two values $a_1$ and $a_2$ such that

$a_1 = 3a_2$, show that $\sin B = \sqrt{\dfrac{4b^2 - c^2}{3c^2}}$.

## 14. REGULAR POLYGON :

A regular polygon has all its sides equal. It may be inscribed or circumscribed.

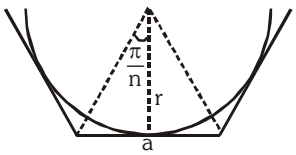



**(a) Inscribed in circle of radius r :**

(i) $a = 2h\tan\dfrac{\pi}{n} = 2r\sin\dfrac{\pi}{n}$

(ii) Perimeter (P) and area (A) of a regular polygon of n sides inscribed in a circle of radius r are given by $P = 2nr\sin\dfrac{\pi}{n}$ and $A = \dfrac{1}{2}nr^2\sin\dfrac{2\pi}{n}$

**(b) Circumscribed about a circle of radius r :**




(i) $a = 2r\tan\dfrac{\pi}{n}$

(ii) Perimeter (P) and area (A) of a regular polygon of n sides circumscribed about a given circle of radius r is given by $P = 2nr\tan\dfrac{\pi}{n}$ and

$A = nr^2\tan\dfrac{\pi}{n}$

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**Do yourself - 10 :**

**(i)** If the perimeter of a circle and a regular polygon of n sides are equal, then

prove that $\frac{\text{area of the circle}}{\text{area of polygon}} = \frac{\tan\frac{\pi}{n}}{\frac{\pi}{n}}$.

**(ii)** The ratio of the area of n-sided regular polygon, circumscribed about a circle, to the area of the regular polygon of equal number of sides inscribed in the circle is 4 : 3. Find the value of n.

## 15. SOME NOTES :

**(a)** (i) If a cos B = b cos A, then the triangle is isosceles.

(ii) If a cos A = b cos B, then the triangle is isosceles or right angled.

**(b)** In right angle triangle

(i) $a^2 + b^2 + c^2 = 8R^2$ (ii) $\cos^2 A + \cos^2 B + \cos^2 C = 1$

**(c)** In equilateral triangle

(i) $R = 2r$ (ii) $r_1 = r_2 = r_3 = \frac{3R}{2}$

(iii) $r : R : r_1 = 1 : 2 : 3$ (iv) $\text{area} = \frac{\sqrt{3}a^2}{4}$ (v) $R = \frac{a}{\sqrt{3}}$

**(d)** (i) The circumcentre lies (1) inside an acute angled triangle (2) outside an obtuse angled triangle & (3) mid point of the hypotenuse of right angled triangle.

(ii) The orthocentre of right angled triangle is the vertex at the right angle.

(iii) The orthocentre, centroid & circumcentre are collinear & centroid divides the line segment joining orthocentre & circumcentre internally in the ratio 2 : 1 except in case of equilateral triangle. In equilateral triangle, all these centres coincide

**(e)** Area of a cyclic quadrilateral $= \sqrt{(s-a)(s-b)(s-c)(s-d)}$

where a, b, c, d are lengths of the sides of quadrilateral and $s = \frac{a+b+c+d}{2}$.

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## ANSWERS FOR DO YOURSELF

**1 :** **(i)** 90°

**5 :** **(i)** **(a)** $\frac{3}{5}$ **(b)** $\frac{3}{4}$ **(c)** $\frac{1}{\sqrt{10}}$ **(d)** $\frac{3}{\sqrt{10}}$ **(e)** $\frac{1}{3}$ **(f)** 24

**6 :** **(i)** **(a)** 6 **(b)** $\frac{5}{2}$ **(c)** 1

**7 :** **(i)** **(a)** 1 **(b)** 3 **(c)** $2\sqrt{3}$

**10 :** **(ii)** 6

# ELEMENTARY EXERCISE

**1.** Angles A, B and C of a triangle ABC are in A.P. If $\frac{b}{c}=\sqrt{\frac{3}{2}}$, then $\angle A$ is equal to

(A) $\frac{\pi}{6}$ (B) $\frac{\pi}{4}$ (C) $\frac{5\pi}{12}$ (D) $\frac{\pi}{2}$ **TS0001**

**2.** If K is a point on the side BC of an equilateral triangle ABC and if $\angle BAK = 15°$, then the ratio of lengths $\frac{AK}{AB}$ is **TS0002**

(A) $\frac{3\sqrt{2}\left(3+\sqrt{3}\right)}{2}$ (B) $\frac{\sqrt{2}\left(3+\sqrt{3}\right)}{2}$ (C) $\frac{\sqrt{2}\left(3-\sqrt{3}\right)}{2}$ (D) $\frac{3\sqrt{2}\left(3-\sqrt{3}\right)}{2}$

**3.** In a triangle ABC, $\angle A = 60°$ and $b : c = \left(\sqrt{3}+1\right) : 2$ then $(\angle B - \angle C)$ has the value equal to

(A) 15° (B) 30° (C) 22.5 ° (D) 45° **TS0014**

**4.** In an acute triangle ABC, $\angle ABC = 45°$, $AB = 3$ and $AC = \sqrt{6}$. The angle $\angle BAC$, is

(A) 60° (B) 65° (C) 75° (D) 15° or 75° **TS0009**

**5.** Let ABC be a right triangle with length of side AB = 3 and hypotenuse AC = 5.

If D is a point on BC such that $\frac{BD}{DC}=\frac{AB}{AC}$, then AD is equal to

(A) $\frac{4\sqrt{3}}{3}$ (B) $\frac{3\sqrt{5}}{2}$ (C) $\frac{4\sqrt{5}}{3}$ (D) $\frac{5\sqrt{3}}{4}$ **TS0003**

**6.** In a triangle ABC, if $a = 6$, $b = 3$ and $\cos(A-B) = \frac{4}{5}$, the area of the triangle is

(A) 8 (B) 9 (C) 12 (D) $\frac{15}{2}$ **TS0018**

**7.** In $\Delta ABC$, if $a = 2b$ and $A = 3B$, then the value of $\frac{c}{b}$ is equal to

(A) 3 (B) $\sqrt{2}$ (C) 1 (D) $\sqrt{3}$ **TS0004**

**8.** If the sides of a triangle are $\sin\alpha$, $\cos\alpha$, $\sqrt{1+\sin\alpha\cos\alpha}$, $0<\alpha<\frac{\pi}{2}$, the largest angle is

(A) 60° (B) 90° (C) 120° (D) 150° **TS0013**

**9.** If the angle A, B and C of a triangle are in an arithmetic progression and if a, b and c denote the lengths of the sides opposite to A, B and C respectively, then the value of expression $E=\left(\frac{a}{c}\sin 2C+\frac{c}{a}\sin 2A\right)$, is

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$ **TS0005**

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**10.** If in a triangle $\sin A : \sin C = \sin (A-B) : \sin (B-C)$, then $a^2, b^2, c^2$ **TS0007**

(A) are in A.P. (B) are in G.P. (C) are in H.P. (D) none of these

**11.** In triangle ABC, if $\cot \frac{A}{2} = \frac{b+c}{a}$, then triangle ABC must be

[Note: All symbols used have usual meaning in $\Delta ABC$.] **TS0016**

(A) isosceles (B) equilateral (C) right angled (D) isoceles right angled

**12.** Consider a triangle ABC and let a, b and c denote the lengths of the sides opposite to vertices A, B and C respectively. If $a = 1$, $b = 3$ and $C = 60°$, then $\sin^2 B$ is equal to

(A) $\frac{27}{28}$ (B) $\frac{3}{28}$ (C) $\frac{81}{28}$ (D) $\frac{1}{3}$ **TS0010**

**13.** The ratio of the sides of a triangle ABC is $1 : \sqrt{3} : 2$. Then ratio of A : B : C is

(A) 3 : 5 : 2 (B) $1 : \sqrt{3} : 2$ (C) 3 : 2 : 1 (D) 1 : 2 : 3 **TS0006**

**14.** In triangle ABC, If $s = 3+\sqrt{3}+\sqrt{2}$, $3B - C = 30°$, $A + 2B = 120°$, then the length of longest side of triangle is

[**Note:** All symbols used have usual meaning in triangle ABC.]

(A) 2 (B) $2\sqrt{2}$ (C) $2(\sqrt{3}+1)$ (D) $\sqrt{3}-1$ **TS0011**

**15.** In a triangle $\tan A : \tan B : \tan C = 1 : 2 : 3$, then $a^2 : b^2 : c^2$ equals

(A) 5 : 8 : 9 (B) 5 : 8 : 12 (C) 3 : 5 : 8 (D) 5 : 8 : 10 **TS0008**

**16.** In $\Delta ABC$, if a,b,c (taken in that order) are in A.P. then $\cot\frac{A}{2}\ \cot\frac{C}{2} =$

[**Note:** All symbols used have usual meaning in triangle ABC. ]

(A) 1 (B) 2 (C) 3 (D) 4 **TS0017**

**17.** In $\Delta ABC$ if $a = 8$, $b = 9$, $c = 10$, then the value of $\frac{\tan C}{\sin B}$ is

(A) $\frac{32}{9}$ (B) $\frac{24}{7}$ (C) $\frac{21}{4}$ (D) $\frac{18}{5}$ **TS0012**

**18.** In triangle ABC, if $\Delta = a^2 - (b-c)^2$, then $\tan A =$

[**Note:** All symbols used have usual meaning in triangle ABC. ]

(A) $\frac{15}{16}$ (B) $\frac{1}{2}$ (C) $\frac{8}{17}$ (D) $\frac{8}{15}$ **TS0015**

**19.** In a triangle ABC, if the sides a, b, c are roots of $x^3 - 11x^2 + 38x - 40 = 0$. If $\sum\left(\frac{\cos A}{a}\right) = \frac{p}{q}$, then find the least value of $(p+q)$ where $p,q \in N$. **TS0020**

**20.** ABC is a triangle such that $\sin(2A+B) = \sin(C-A) = -\sin(B+2C) = \frac{1}{2}$. If A, B, C are in A.P., find A, B, C. **TS0019**

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# EXERCISE (O-1)

**1.** A triangle has vertices A, B and C, and the respective opposite sides have lengths a, b and c. This triangle is inscribed in a circle of radius R. If $b = c = 1$ and the altitude from A to side BC has length $\sqrt{\frac{2}{3}}$, then R equals

(A) $\frac{1}{\sqrt{3}}$ (B) $\frac{2}{\sqrt{3}}$ (C) $\frac{\sqrt{3}}{2}$ (D) $\frac{\sqrt{3}}{2\sqrt{2}}$ **TS0022**

**2.** A circle is inscribed in a right triangle ABC, right angled at C. The circle is tangent to the segment AB at D and length of segments AD and DB are 7 and 13 respectively. Area of triangle ABC is equal to

(A) 91 (B) 96 (C) 100 (D) 104 **TS0028**

**3.** In a triangle ABC, if $a = 13$, $b = 14$ and $c = 15$, then angle A is equal to
(All symbols used have their usual meaning in a triangle.)

(A) $\sin^{-1}\frac{4}{5}$ (B) $\sin^{-1}\frac{3}{5}$ (C) $\sin^{-1}\frac{3}{4}$ (D) $\sin^{-1}\frac{2}{3}$ **TS0024**

**4.** In a triangle ABC, if $b = (\sqrt{3}-1)a$ and $\angle C = 30°$, then the value of $(A - B)$ is equal to
(All symbols used have usual meaning in a triangle.)

(A) 30° (B) 45° (C) 60° (D) 75° **TS0030**

**5.** In triangle ABC, if $AC = 8$, $BC = 7$ and D lies between A and B such that $AD = 2$, $BD = 4$, then the length CD equals

(A) $\sqrt{46}$ (B) $\sqrt{48}$ (C) $\sqrt{51}$ (D) $\sqrt{75}$ **TS0025**

**6.** In a triangle ABC, if $\angle C = 105°$, $\angle B = 45°$ and length of side $AC = 2$ units, then the length of the side AB is equal to

(A) $\sqrt{2}$ (B) $\sqrt{3}$ (C) $\sqrt{2}+1$ (D) $\sqrt{3}+1$ **TS0023**

**7.** In a triangle ABC, if $(a + b + c)(a + b - c)(b + c - a)(c + a - b) = \frac{8a^2b^2c^2}{a^2 + b^2 + c^2}$, then the triangle is

[**Note:** All symbols used have usual meaning in triangle ABC.] **TS0034**

(A) isosceles (B) right angled (C) equilateral (D) obtuse angled

**8.** In triangle ABC, if $2b = a + c$ and $A - C = 90°$, then sin B equals
[**Note:** All symbols used have usual meaning in triangle ABC.] **TS0026**

(A) $\frac{\sqrt{7}}{5}$ (B) $\frac{\sqrt{5}}{8}$ (C) $\frac{\sqrt{7}}{4}$ (D) $\frac{\sqrt{5}}{3}$

**9.** The sides of a triangle are three consecutive integers. The largest angle is twice the smallest one. The area of triangle is equal to **TS0031**

(A) $\frac{5}{4}\sqrt{7}$ (B) $\frac{15}{2}\sqrt{7}$ (C) $\frac{15}{4}\sqrt{7}$ (D) $5\sqrt{7}$

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**10.** The sides a, b, c (taken in that order) of triangle ABC are in A.P.

If $\cos\alpha = \frac{a}{b+c}, \cos\beta = \frac{b}{c+a}, \cos\gamma = \frac{c}{a+b}$ then $\tan^2\left(\frac{\alpha}{2}\right) + \tan^2\left(\frac{\gamma}{2}\right)$ is equal to

[**Note:** All symbols used have usual meaning in triangle ABC. ]

(A) 1 (B) $\frac{1}{2}$ (C) $\frac{1}{3}$ (D) $\frac{2}{3}$ **TS0032**

**11.** AD and BE are the medians of a triangle ABC. If AD = 4, $\angle DAB = \frac{\pi}{6}, \angle ABE = \frac{\pi}{3}$, then area of triangle ABC equals

(A) $\frac{8}{3}$ (B) $\frac{16}{3}$ (C) $\frac{32}{3}$ (D) $\frac{32}{9}\sqrt{3}$ **TS0033**

**12.** In triangle ABC, if $\sin^3 A + \sin^3 B + \sin^3 C = 3\sin A.\sin B.\sin C$, then triangle is **TS0021**

(A) obtuse angled (B) right angled (C) obtuse right angled (D) equilateral

**13.** For right angled isosceles triangle, $\frac{r}{R} =$

[**Note:** All symbols used have usual meaning in triangle ABC. ] **TS0035**

(A) $\tan\frac{\pi}{12}$ (B) $\cot\frac{\pi}{12}$ (C) $\tan\frac{\pi}{8}$ (D) $\cot\frac{\pi}{8}$

**14.** In triangle ABC, If $\frac{1}{a+c} + \frac{1}{b+c} = \frac{3}{a+b+c}$ then angle C is equal to

[**Note:** All symbols used have usual meaning in triangle ABC. ]

(A) 30° (B) 45° (C) 60° (D) 90° **TS0029**

## EXERCISE (O-2)

**Multiple Correct Answer Type :**

**1.** In a triangle ABC, let $2a^2 + 4b^2 + c^2 = 2a(2b + c)$, then which of the following holds good?

[**Note:** All symbols used have usual meaning in a triangle.]

(A) $\cos B = \frac{-7}{8}$ (B) $\sin(A - C) = 0$

(C) $\frac{r}{r_1} = \frac{1}{5}$ (D) $\sin A : \sin B : \sin C = 1 : 2 : 1$ **TS0043**

**2.** In a triangle ABC, if a = 4, b = 8 $\angle C = 60°$, then which of the following relations is (are) correct?

[**Note:** All symbols used have usual meaning in triangle ABC.]

(A) The area of triangle ABC is $8\sqrt{3}$

(B) The value of $\sum \sin^2 A = 2$

(C) Inradius of triangle ABC is $\frac{2\sqrt{3}}{3+\sqrt{3}}$

(D) The length of internal angle bisector of angle C is $\frac{4}{\sqrt{3}}$ **TS0047**

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**3.** In which of the following situations, it is possible to have a triangle ABC?
(All symbols used have usual meaning in a triangle.)

(A) $(a + c - b)(a - c + b) = 4bc$

(B) $b^2 \sin 2C + c^2 \sin 2B = ab$

(C) $a = 3, b = 5, c = 7$ and $C = \frac{2\pi}{3}$

(D) $\cos\left(\frac{A-C}{2}\right) = \cos\left(\frac{A+C}{2}\right)$ **TS0038**

**4.** In a triangle ABC, which of the following quantities denote the area of the triangle?

(A) $\frac{a^2 - b^2}{2}\left(\frac{\sin A \sin B}{\sin(A-B)}\right)$

(B) $\frac{r_1 r_2 r_3}{\sqrt{\sum r_1 r_2}}$

(C) $\frac{a^2 + b^2 + c^2}{\cot A + \cot B + \cot C}$

(D) $r^2 \cot\frac{A}{2} \cdot \cot\frac{B}{2} \cot\frac{C}{2}$ **TS0044**

**5.** In $\Delta ABC$, angle A, B and C are in the ratio 1 : 2 : 3, then which of the following is (are) correct?
(All symbol used have usual meaning in a triangle.)

(A) Circumradius of $\Delta ABC = c$

(B) $a : b : c = 1 : \sqrt{3} : 2$

(C) Perimeter of $\Delta ABC = 3 + \sqrt{3}$

(D) Area of $\Delta ABC = \frac{\sqrt{3}}{8} c^2$ **TS0040**

**6.** In triangle ABC, let $b = 10$, $c = 10\sqrt{2}$ and $R = 5\sqrt{2}$ then which of the following statement(s) is (are) correct?
[**Note:** All symbols used have usual meaning in triangle ABC.]

(A) Area of triangle ABC is 50.

(B) Distance between orthocentre and circumcentre is $5\sqrt{2}$

(C) Sum of circumradius and inradius of triangle ABC is equal to 10

(D) Length of internal angle bisector of $\angle ACB$ of triangle ABC is $\frac{5}{2\sqrt{2}}$ **TS0046**

**7.** In a triangle ABC, let $BC = 1$, $AC = 2$ and measure of angle C is 30°. Which of the following statement(s) is (are) correct?

(A) $2 \sin A = \sin B$

(B) Length of side AB equals $5 - 2\sqrt{3}$

(C) Measure of angle A is less than 30°

(D) Circumradius of triangle ABC is equal to length of side AB **TS0041**

**8.** Given an acute triangle ABC such that $\sin C = \frac{4}{5}$, $\tan A = \frac{24}{7}$ and $AB = 50$. Then-

(A) centroid, orthocentre and incentre of $\Delta ABC$ are collinear

(B) $\sin B = \frac{4}{5}$

(C) $\sin B = \frac{4}{7}$

(D) area of $\Delta ABC = 1200$ **TS0036**

**9.** In $\Delta ABC$, angle A is 120°, $BC + CA = 20$ and $AB + BC = 21$, then

(A) $AB > AC$

(B) $AB < AC$

(C) $\Delta ABC$ is isosceles

(D) area of $\Delta ABC = 14\sqrt{3}$ **TS0037**

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# EXERCISE (S-1)

**1.** Given a triangle ABC with sides $a = 7,\ b = 8$ and $c = 5$. If the value of the expression $\left(\sum \sin A\right)\left(\sum \cot \frac{A}{2}\right)$ can be expressed in the form $\frac{p}{q}$ where $p, q \in N$ and $\frac{p}{q}$ is in its lowest form find the value of $(p + q)$. **TS0052**

**2.** If two times the square of the diameter of the circumcircle of a triangle is equal to the sum of the squares of its sides then prove that the triangle is right angled. **TS0053**

**3.** In acute angled triangle ABC, a semicircle with radius $r_a$ is constructed with its base on BC and tangent to the other two sides. $r_b$ and $r_c$ are defined similarly. If r is the radius of the incircle of triangle ABC then prove that, $\frac{2}{r} = \frac{1}{r_a} + \frac{1}{r_b} + \frac{1}{r_c}$. **TS0058**

**4.** If the length of the perpendiculars from the vertices of a triangle A, B, C on the opposite sides are $p_1, p_2, p_3$ then prove that $\frac{1}{p_1} + \frac{1}{p_2} + \frac{1}{p_3} = \frac{1}{r} = \frac{1}{r_1} + \frac{1}{r_2} + \frac{1}{r_3}$. **TS0059**

**5.** With usual notations, prove that in a triangle ABC
$Rr(\sin A + \sin B + \sin C) = \Delta$ **TS0056**

**6.** With usual notations, prove that in a triangle ABC
$\cot \frac{A}{2} + \cot \frac{B}{2} + \cot \frac{C}{2} = \frac{s^2}{\Delta}$ **TS0054**

**7.** With usual notations, prove that in a triangle ABC
$\cot A + \cot B + \cot C = \frac{a^2 + b^2 + c^2}{4\Delta}$ **TS0057**

**8.** If a,b,c are the sides of triangle ABC satisfying $\log\left(1 + \frac{c}{a}\right) + \log a - \log b = \log 2$.
Also $a(1 - x^2) + 2bx + c(1 + x^2) = 0$ has two equal roots. Find the value of $\sin A + \sin B + \sin C$. **TS0051**

**9.** With usual notations, prove that in a triangle ABC
$\frac{b-c}{r_1} + \frac{c-a}{r_2} + \frac{a-b}{r_3} = 0$ **TS0060**

**10.** With usual notations, prove that in a triangle ABC
$\frac{r_1}{(s-b)(s-c)} + \frac{r_2}{(s-c)(s-a)} + \frac{r_3}{(s-a)(s-b)} = \frac{3}{r}$ **TS0061**

**11.** With usual notations, prove that in a triangle ABC
$\frac{abc}{s} \cos \frac{A}{2} \cos \frac{B}{2} \cos \frac{C}{2} = \Delta$ **TS0062**

**12.** With usual notations, prove that in a triangle ABC
$\frac{1}{r^2} + \frac{1}{r_1^2} + \frac{1}{r_2^2} + \frac{1}{r_3^2} = \frac{a^2 + b^2 + c^2}{\Delta^2}$ **TS0063**

**13.** If $r_1 = r + r_2 + r_3$ then prove that the triangle is a right angled triangle. **TS0064**

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# EXERCISE (S-2)

**1.** With usual notation, if in a $\Delta ABC$, $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$; then prove that, $\frac{\cos A}{7}=\frac{\cos B}{19}=\frac{\cos C}{25}$. **TS0067**

**2.** Given a triangle ABC with AB = 2 and AC = 1. Internal bisector of $\angle BAC$ intersects BC at D. If AD = BD and $\Delta$ is the area of triangle ABC, then find the value of $12\Delta^2$. **TS0073**

**3.** For any triangle ABC, if B = 3C, show that $\cos C=\sqrt{\frac{b+c}{4c}}$ & $\sin\frac{A}{2}=\frac{b-c}{2c}$. **TS0069**

**4.** In a triangle ABC if $a^2+b^2=101c^2$ then find the value of $\frac{\cot C}{\cot A+\cot B}$. **TS0068**

**5.** Two sides of a triangle are of lengths $\sqrt{6}$ and 4 and the angle opposite to smaller side is $30^0$. How many such triangles are possible ? Find the length of their third side and area. **TS0074**

**6.** The triangle ABC (with side lengths a, b, c as usual) satisfies $\log a^2=\log b^2+\log c^2-\log(2bc\cos A)$. What can you say about this triangle? **TS0072**

**7.** The sides of a triangle are consecutive integers $n$, $n+1$ and $n+2$ and the largest angle is twice the smallest angle. Find $n$. **TS0066**

# EXERCISE (JM)

**1.** If 5, 5r, $5r^2$ are the lengths of the sides of a triangle, then r cannot be equal to : **[JEE(Main)-Jan 2019]**

(1) $\frac{3}{2}$ (2) $\frac{3}{4}$ (3) $\frac{5}{4}$ (4) $\frac{7}{4}$ **TS0076**

**2.** With the usual notation, in $\Delta ABC$, if $\angle A+\angle B=120^\circ$, $a=\sqrt{3}+1$ and $b=\sqrt{3}-1$, then the ratio $\angle A:\angle B$, is : **[JEE(Main)-Jan 2019]**

(1) 7 : 1 (2) 5 : 3 (3) 9 : 7 (4) 3 : 1 **TS0077**

**3.** In a triangle, the sum of lengths of two sides is x and the product of the lengths of the same two sides is y. If $x^2-c^2=y$, where c is the length of the third side of the triangle, then the circumradius of the triangle is : **[JEE(Main)-Jan 2019]**

(1) $\frac{y}{\sqrt{3}}$ (2) $\frac{c}{\sqrt{3}}$ (3) $\frac{c}{3}$ (4) $\frac{3}{2}y$ **TS0078**

**4.** Given $\frac{b+c}{11}=\frac{c+a}{12}=\frac{a+b}{13}$ for a $\Delta ABC$ with usual notation. If $\frac{\cos A}{\alpha}=\frac{\cos B}{\beta}=\frac{\cos C}{\gamma}$, then the ordered triad $(\alpha, \beta, \gamma)$ has a value :- **[JEE(Main)-Jan 2019]**

(1) (3, 4, 5) (2) (19, 7, 25) (3) (7, 19, 25) (4) (5, 12, 13) **TS0079**

**5.** If the lengths of the sides of a triangle are in A.P. and the greatest angle is double the smallest, then a ratio of lengths of the sides of this triangle is : **[JEE(Main)-Apr 2019]**

(1) 5 : 9 : 13 (2) 5 : 6 : 7 (3) 4 : 5 : 6 (4) 3 : 4 : 5 **TS0080**

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**6.** The angles A, B and C of a triangle ABC are in A.P. and a : b = 1 : $\sqrt{3}$. If c = 4 cm, then the area (in sq. cm) of this triangle is : **[JEE(Main)-Apr 2019]**

(1) $4\sqrt{3}$ (2) $\frac{2}{\sqrt{3}}$ (3) $2\sqrt{3}$ (4) $\frac{4}{\sqrt{3}}$ **TS0081**

## EXERCISE (JA)

**1.** Let ABC and ABC′ be two non-congruent triangles with sides AB = 4, AC = AC′ = $2\sqrt{2}$ and angle B = 30°. The absolute value of the difference between the areas of these triangles is **[JEE 2009, 5]**

**TS0082**

**2.** (a) If the angle A,B and C of a triangle are in an arithmetic progression and if a,b and c denote the length of the sides opposite to A,B and C respectively, then the value of the expression $\frac{a}{c}\sin 2C + \frac{c}{a}\sin 2A$, is -

(A) $\frac{1}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) 1 (D) $\sqrt{3}$ **TS0083**

(b) Consider a triangle ABC and let a,b and c denote the length of the sides opposite to vertices A,B and C respectively. Suppose a = 6, b = 10 and the area of the triangle is $15\sqrt{3}$. If ∠ACB is obtuse and if r denotes the radius of the incircle of the triangle, then $r^2$ is equal to **TS0084**

(c) Let ABC be a triangle such that $\angle ACB = \frac{\pi}{6}$ and let a,b and c denote the lengths of the sides opposite to A,B and C respectively. The value(s) of x for which $a = x^2 + x + 1$, $b = x^2 - 1$ and $c = 2x + 1$ is/are **[JEE 2010, 3+3+3]**

(A) $-\left(2+\sqrt{3}\right)$ (B) $1+\sqrt{3}$ (C) $2+\sqrt{3}$ (D) $4\sqrt{3}$ **TS0085**

**3.** Let PQR be a triangle of area Δ with a = 2, b = $\frac{7}{2}$ and c = $\frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals **[JEE 2012, 3M, –1M]**

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$ **TS0086**

**4.** In a triangle PQR, P is the largest angle and cos P = $\frac{1}{3}$. Further the incircle of the triangle touches the sides PQ, QR and RP at N, L and M respectively, such that the lengths of PN, QL and RM are consecutive even integers. Then possible length(s) of the side(s) of the triangle is (are)

**[JEE(Advanced) 2013, 3, (–1)]**

(A) 16 (B) 18 (C) 24 (D) 22 **TS0087**

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**5.** In a triangle the sum of two sides is x and the product of the same two sides is y. If $x^2 - c^2 = y$, where c is a third side of the triangle, then the ratio of the in-radius to the circum-radius of the triangle is -

**[JEE(Advanced)-2014, 3(–1)]**

(A) $\frac{3y}{2x(x+c)}$ (B) $\frac{3y}{2c(x+c)}$ (C) $\frac{3y}{4x(x+c)}$ (D) $\frac{3y}{4c(x+c)}$ **TS0088**

**6.** In a triangle XYZ, let x,y,z be the lengths of sides opposite to the angles X,Y,Z, respectively and 2s = x + y + z. If $\frac{s-x}{4} = \frac{s-y}{3} = \frac{s-z}{2}$ and area of incircle of the triangle XYZ is $\frac{8\pi}{3}$, then-

(A) area of the triangle XYZ is $6\sqrt{6}$

(B) the radius of circumcircle of the triangle XYZ is $\frac{35}{6}\sqrt{6}$

(C) $\sin\frac{X}{2}\sin\frac{Y}{2}\sin\frac{Z}{2} = \frac{4}{35}$ **TS0089**

(D) $\sin^2\left(\frac{X+Y}{2}\right) = \frac{3}{5}$ **[JEE(Advanced)-2016, 4(–2)]**

**7.** In a triangle PQR, let $\angle PQR = 30°$ and the sides PQ and QR have lengths $10\sqrt{3}$ and 10, respectively. Then, which of the following statement(s) is (are) TRUE ? **[JEE(Advanced)-2018, 4(–2)]**

(A) $\angle QPR = 45°$

(B) The area of the triangle PQR is $25\sqrt{3}$ and $\angle QRP = 120°$

(C) The radius of the incircle of the triangle PQR is $10\sqrt{3} - 15$

(D) The area of the circumcircle of the triangle PQR is $100\pi$. **TS0090**

**8.** In a non-right-angled triangle $\Delta PQR$, let p, q, r denote the lengths of the sides opposite to the angles at P, Q, R respectively. The median from R meets the side PQ at S, the perpendicular from P meets the side QR at E, and RS and PE intersect at O. If $p = \sqrt{3}$, $q = 1$, and the radius of the circumcircle of the $\Delta PQR$ equals 1, then which of the following options is/are correct ?

**[JEE(Advanced)-2019, 4(–1)]**

(1) Area of $\Delta SOE = \frac{\sqrt{3}}{12}$ (2) Radius of incircle of $\Delta PQR = \frac{\sqrt{3}}{2}(2-\sqrt{3})$

(3) Length of $RS = \frac{\sqrt{7}}{2}$ (4) Length of $OE = \frac{1}{6}$ **TS0091**

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# ANSWERS

## ELEMENTARY EXERCISE

**1.** C **2.** C **3.** B **4.** C **5.** B **6.** B **7.** D **8.** C
**9.** D **10.** A **11.** C **12.** A **13.** D **14.** C **15.** A **16.** C
**17.** A **18.** D **19.** 25 **20.** 45°,60°,75°

## EXERCISE (O-1)

**1.** D **2.** A **3.** A **4.** C **5.** C **6.** D **7.** B **8.** C
**9.** C **10.** D **11.** D **12.** D **13.** C **14.** C

## EXERCISE (O-2)

**1.** B,C **2.** A,B **3.** B,C **4.** A,B,D **5.** B,D **6.** A,B,C **7.** A,C,D
**8.** A,B,D **9.** A,D

## EXERCISE (S-1)

**1.** 107 **8.** $\frac{12}{5}$

## EXERCISE (S-2)

**2.** 9 **4.** 50 **5.** Two tringle $\left(2\sqrt{3}-\sqrt{2}\right)$, $\left(2\sqrt{3}+\sqrt{2}\right)$, $\left(2\sqrt{3}-\sqrt{2}\right)$ & $\left(2\sqrt{3}+\sqrt{2}\right)$ sq. units
**6.** triangle is isosceles **7.** 4

## EXERCISE (JM)

**1.** 4 **2.** 1 **3.** 2 **4.** 3 **5.** 3 **6.** 3

## EXERCISE (JA)

**1.** 4 **2.** (a) D, (b) 3, (c) B **3.** C **4.** B,D **5.** B **6.** A,C,D
**7.** B,C,D **8.** 2,3,4

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# BINOMIAL THEOREM

## 1. BINOMIAL EXPRESSION :

Any algebraic expression which contains two dissimilar terms is called binomial expression.

For example : $x - y$, $xy + \frac{1}{x}, \frac{1}{z} - 1, \frac{1}{(x-y)^{1/3}} + 3$ etc.

## 2. BINOMIAL THEOREM :

The formula by which any positive integral power of a binomial expression can be expanded in the form of a series is known as **BINOMIAL THEOREM.**

If $x, y \in R$ and $n \in N$, then :

$$(x + y)^n = {}^nC_0x^n + {}^nC_1x^{n-1}y + {}^nC_2x^{n-2}y^2 + ..... + {}^nC_rx^{n-r}y^r + ..... + {}^nC_ny^n = \sum_{r=0}^{n} {}^nC_r x^{n-r}y^r$$

This theorem can be proved by induction.

**Observations :**

**(a)** The number of terms in the expansion is ( n+1) i.e. one more than the index.

**(b)** The sum of the indices of x & y in each term is n.

**(c)** The binomial coefficients of the terms **($^nC_0, {}^nC_1$.....)** equidistant from the beginning and the end are equal. i.e. $\mathbf{{}^nC_r = {}^nC_{r-1}}$

**(d)** Symbol $\mathbf{{}^nC_r}$ can also be denoted by $\binom{n}{r}$, **C(n, r)** or $\mathbf{A_r^n}$.

**Some important expansions :**

(i) $(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ..... + {}^nC_nx^n$.

(ii) $(1 - x)^n = {}^nC_0 - {}^nC_1x + {}^nC_2x^2 + ..... + (-1)^n . {}^nC_nx^n$.

**Note :** The coefficient of $x^r$ in $(1 + x)^n = {}^nC_r$ & that in $(1-x)^n = (-1)^r . {}^nC_r$

---

***Illustration 1 :*** Expand : $(y + 2)^6$.

***Solution :*** ${}^6C_0y^6 + {}^6C_1y^5.2 + {}^6C_2y^4.2^2 + {}^6C_3y^3.2^3 + {}^6C_4y^2.2^4 + {}^6C_5y^1 . 2^5 + {}^6C_6 . 2^6$.

$= y^6 + 12y^5 + 60y^4 + 160y^3 + 240y^2 + 192y + 64$.

***Illustration 2 :*** Write first 4 terms of $\left(1 - \frac{2y^2}{5}\right)^7$

***Solution :*** ${}^7C_0, {}^7C_1\left(-\frac{2y^2}{5}\right), {}^7C_2\left(-\frac{2y^2}{5}\right)^2, {}^7C_3\left(-\frac{2y^2}{5}\right)^3$

***Illustration 3 :*** If in the expansion of $(1 + x)^m (1 - x)^n$, the coefficients of x and $x^2$ are 3 and $-6$ respectively then m is - **[JEE 99]**

(A) 6 (B) 9 (C) 12 (D) 24

***Solution :*** $(1 + x)^m (1 - x)^n = \left[1 + mx + \frac{(m)(m-1).x^2}{2} + .....\right]\left[1 - nx + \frac{n(n-1)}{2}x^2 + .....\right]$

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Coefficient of $x = m - n = 3$ .....(i)

Coefficient of $x^2 = -mn + \frac{n(n+1)}{2} + \frac{m(m-1)}{2} = -6$ .....(ii)

Solving (i) and (ii), we get

$m = 12$ and $n = 9$.

**Do yourself - 1 :**

**(i)** Expand $\left(3x^2 - \frac{x}{2}\right)^5$ **(ii)** Expand $(y + x)^n$

**Pascal's triangle :**

| | | |
|---|---|---|
| $(x+y)^0$ | 1 | 1 |
| $(x+y)^1$ | $x + y$ | 1 1 |
| $(x+y)^2$ | $x^2 + 2xy + y^2$ | 1 2 1 |
| $(x+y)^3$ | $x^3 + 3x^2y + 3xy^2 + y^3$ | 1 3 3 1 |
| $(x+y)^4$ | $x^4 + 4x^3y + 6x^2y^2 + 4xy^3 + y^3$ | 1 4 6 4 1 |

Pascal's triangle

(i) **Pascal's triangle -** A triangular arrangement of numbers as shown. The numbers give the binomial coefficients for the expansion of $(x + y)^n$. The first row is for $n = 0$, the second for $n = 1$, etc. Each row has 1 as its first and last number. Other numbers are generated by adding the two numbers immediately to the left and right in the row above.

(ii) Pascal triangle is formed by binomial coefficient.

(iii) The number of terms in the expansion of $(x+y)^n$ is $(n + 1)$ i.e. one more than the index.

(iv) The sum of the indices of x & y in each term is n.

(v) Power of first variable (x) decreases while of second variable (y) increases.

(vi) Binomial coefficients are also called **combinatorial coefficients.**

(vii) Binomial coefficients of the terms equidistant from the begining and end are equal.

(viii) $r^{th}$ term from the beginning in the expansion of $(x + y)^n$ is same as $r^{th}$ term from end in the expansion of $(y + x)^n$.

(ix) $r^{th}$ term from the end in $(x + y)^n$ is **$(n - r + 2)^{th}$ term from the beginning**.

## 3. IMPORTANT TERMS IN THE BINOMIAL EXPANSION :

**(a) General term:** The general term or the $(r+1)^{th}$ term in the expansion of $(x + y)^n$ is given by

$T_{r+1} = {}^nC_r\, x^{n-r}\, y^r$

***Illustration 4 :*** Find : (a) The coefficient of $x^7$ in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$

(b) The coefficient of $x^{-7}$ in the expansion of $\left(ax^2 + \frac{1}{bx}\right)^{11}$

Also, find the relation between a and b, so that these coefficients are equal.

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***Solution :*** (a) In the expansion of $\left(ax^2+\frac{1}{bx}\right)^{11}$, the general term is :

$$T_{r+1} = {}^{11}C_r(ax^2)^{11-r}\left(\frac{1}{bx}\right)^r = {}^{11}C_r.\frac{a^{11-r}}{b^r}.x^{22-3r}$$

putting $22 - 3r = 7$

$\therefore \quad 3r = 15 \quad \Rightarrow \quad r = 5$

$$\therefore \quad T_6 = {}^{11}C_5\frac{a^6}{b^5}.x^7$$

Hence the coefficient of $x^7$ in $\left(ax^2+\frac{1}{bx}\right)^{11}$ is ${}^{11}C_5a^6b^{-5}$. **Ans.**

Note that binomial coefficient of sixth term is ${}^{11}C_5$.

(b) In the expansion of $\left(ax-\frac{1}{bx^2}\right)^{11}$, general term is :

$$T_{r+1} = {}^{11}C_r(ax)^{11-r}\left(\frac{-1}{bx^2}\right)^r = (-1)^r\,{}^{11}C_r\frac{a^{11-r}}{b^r}.x^{11-3r}$$

putting $11 - 3r = -7$

$\therefore \quad 3r = 18 \quad \Rightarrow \quad r = 6$

$$\therefore \quad T_7 = (-1)^6.\ {}^{11}C_6\frac{a^5}{b^6}.x^{-7}$$

Hence the coefficient of $x^{-7}$ in $\left(ax-\frac{1}{bx^2}\right)^{11}$ is ${}^{11}C_6a^5b^{-6}$. **Ans.**

Also given :

Coefficient of $x^7$ in $\left(ax^2+\frac{1}{bx}\right)^{11}$ = coefficient of $x^{-7}$ in $\left(ax-\frac{1}{bx^2}\right)^{11}$

$\Rightarrow \quad {}^{11}C_5a^6b^{-5} = {}^{11}C_6a^5b^{-6}$

$\Rightarrow \quad ab = 1 \qquad (\because \ {}^{11}C_5 = {}^{11}C_6)$

which is the required relation between a and b. **Ans.**

***Illustration 5 :*** Find the number of rational terms in the expansion of $(9^{1/4} + 8^{1/6})^{1000}$.

***Solution :*** The general term in the expansion of $(9^{1/4} + 8^{1/6})^{1000}$ is

$$T_{r+1} = {}^{1000}C_r\left(9^{\frac{1}{4}}\right)^{1000-r}\left(8^{\frac{1}{6}}\right)^r = {}^{1000}C_r\ 3^{\frac{1000-r}{2}}\ 2^{\frac{r}{2}}$$

The above term will be rational if exponents of 3 and 2 are integers

It means $\frac{1000-r}{2}$ and $\frac{r}{2}$ must be integers

The possible set of values of r is {0, 2, 4, ....., 1000}

Hence, number of rational terms is 501 **Ans.**

---

**(b) Middle term :**

The middle term(s) in the expansion of $(x + y)^n$ is (are) :

**(i)** If n is even, there is only one middle term which is given by $T_{(n+2)/2} = {}^nC_{n/2}.\ x^{n/2}.\ y^{n/2}$

**(ii)** If n is odd, there are two middle terms which are $T_{(n+1)/2}$ & $T_{[(n+1)/2]+1}$

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**Important Note :**

Middle term has greatest binomial coefficient and if there are 2 middle terms their coefficients will be equal.

$\Rightarrow$ ${}^nC_r$ will be maximum $\begin{cases} \text{When } r=\frac{n}{2} \text{ if n is even} \\ \text{When } r=\frac{n-1}{2} \text{ or } \frac{n+1}{2} \text{ if n is odd} \end{cases}$

$\Rightarrow$ The term containing greatest binomial coefficient will be middle term in the expansion of $(1+x)^n$

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***Illustration 6 :*** Find the middle term in the expansion of $\left(3x-\frac{x^3}{6}\right)^9$

***Solution :*** The number of terms in the expansion of $\left(3x-\frac{x^3}{6}\right)^9$ is 10 (even). So there are two middle terms.

i.e. $\left(\frac{9+1}{2}\right)^{th}$ and $\left(\frac{9+3}{2}\right)^{th}$ are two middle terms. They are given by $T_5$ and $T_6$

$\therefore \quad T_5 = T_{4+1} = {}^9C_4(3x)^5\left(-\frac{x^3}{6}\right)^4 = {}^9C_4 3^5 x^5 . \frac{x^{12}}{6^4} = \frac{9.8.7.6}{1.2.3.4} . \frac{3^5}{2^4.3^4} x^{17} = \frac{189}{8} x^{17}$

and $T_6 = T_{5+1} = {}^9C_5(3x)^4\left(-\frac{x^3}{6}\right)^5 = -{}^9C_4 3^4 . x^4 . \frac{x^{15}}{6^5} = \frac{-9.8.7.6}{1.2.3.4} . \frac{3^4}{2^5.3^5} x^{19} = -\frac{21}{16} x^{19}$ **Ans.**

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**(c) Term independent of x :**

Term independent of x does not contain x ; Hence find the value of r for which the exponent of x is zero.

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***Illustration 7 :*** The term independent of x in $\left[\sqrt{\frac{x}{3}}+\sqrt{\left(\frac{3}{2x^2}\right)}\right]^{10}$ is -

(A) 1 (B) $\frac{5}{12}$ (C) ${}^{10}C_1$ (D) none of these

***Solution :*** General term in the expansion is

${}^{10}C_r\left(\frac{x}{3}\right)^{\frac{r}{2}}\left(\frac{3}{2x^2}\right)^{\frac{10-r}{2}} = {}^{10}C_r x^{\frac{3r}{2}-10} . \frac{3^{5-r}}{2^{\frac{10-r}{2}}}$ For constant term, $\frac{3r}{2}=10 \Rightarrow r=\frac{20}{3}$

which is not an integer. Therefore, there will be no constant term. **Ans. (D)**

**Do yourself - 2 :**

**(i)** Find the $7^{th}$ term of $\left(3x^2-\frac{1}{3}\right)^{10}$

**(ii)** Find the term independent of x in the expansion : $\left(2x^2-\frac{3}{x^3}\right)^{25}$

**(iii)** Find the middle term in the expansion of : (a) $\left(\frac{2x}{3}-\frac{3}{2x}\right)^6$ (b) $\left(2x^2-\frac{1}{x}\right)^7$

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**(d) Numerically greatest term :**

Let numerically greatest term in the expansion of $(a + b)^n$ be $T_{r+1}$.

$\Rightarrow \quad \begin{cases} |T_{r+1}| \geq |T_r| \\ |T_{r+1}| \geq |T_{r+2}| \end{cases}$ where $T_{r+1} = {}^nC_r a^{n-r} b^r$

Solving above inequalities we get $\frac{n+1}{1+\left|\frac{a}{b}\right|} - 1 \leq r \leq \frac{n+1}{1+\left|\frac{a}{b}\right|}$

**Case I :** When $\frac{n+1}{1+\left|\frac{a}{b}\right|}$ is an integer equal to m, then $T_m$ and $T_{m+1}$ will be numerically greatest term.

**Case II :** When $\frac{n+1}{1+\left|\frac{a}{b}\right|}$ is not an integer and its integral part is m, then $T_{m+1}$ will be the numerically greatest term.

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***Illustration 8 :*** Find numerically greatest term in the expansion of $(3 - 5x)^{11}$ when $x = \frac{1}{5}$

***Solution :*** Using $\frac{n+1}{1+\left|\frac{a}{b}\right|} - 1 \leq r \leq \frac{n+1}{1+\left|\frac{a}{b}\right|}$

$$\frac{11+1}{1+\left|\frac{3}{-5x}\right|} - 1 \leq r \leq \frac{11+1}{1+\left|\frac{3}{-5x}\right|}$$

solving we get $2 \leq r \leq 3$

$\therefore \quad r = 2, 3$

so, the greatest terms are $T_{2+1}$ and $T_{3+1}$.

$\therefore$ Greatest term (when r = 2)

$T_3 = {}^{11}C_2 . 3^9 (-5x)^2 = 55.3^9 = T_4$

From above we say that the value of both greatest terms are equal. **Ans.**

***Illustration 9 :*** Given $T_3$ in the expansion of $(1 - 3x)^6$ has maximum numerical value. Find the range of 'x'.

***Solution :*** Using $\frac{n+1}{1+\left|\frac{a}{b}\right|} - 1 \leq r \leq \frac{n+1}{1+\left|\frac{a}{b}\right|}$

$$\frac{6+1}{1+\left|\frac{1}{-3x}\right|} - 1 \leq 2 \leq \frac{7}{1+\left|\frac{1}{-3x}\right|}$$

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Let $|x| = t$

$$\frac{21t}{3t+1} - 1 \le 2 \le \frac{21t}{3t+1}$$

$$\begin{cases} \frac{21t}{3t+1} \le 3 \\ \frac{21t}{3t+1} \ge 2 \end{cases} \Rightarrow \begin{cases} \frac{4t-1}{3t+1} \le 0 \Rightarrow t \in \left[-\frac{1}{3}, \frac{1}{4}\right] \\ \frac{15t-2}{3t+1} \ge 0 \Rightarrow t \in \left(-\infty, -\frac{1}{3}\right] \cup \left[\frac{2}{15}, \infty\right) \end{cases}$$

Common solution $t \in \left[\frac{2}{15}, \frac{1}{4}\right] \Rightarrow x \in \left[-\frac{1}{4}, -\frac{2}{15}\right] \cup \left[\frac{2}{15}, \frac{1}{4}\right]$

**Do yourself -3 :**

**(i)** Find the numerically greatest term in the expansion of $(3 - 2x)^9$, when $x = 1$.

**(ii)** In the expansion of $\left(\frac{1}{2} + \frac{2x}{3}\right)^n$ when $x = -\frac{1}{2}$, it is known that 3rd term is the greatest term. Find the possible integral values of n.

## 4. PROPERTIES OF BINOMIAL COEFFICIENTS :

$$(1+x)^n = C_0 + C_1x + C_2x^2 + C_3x^3 + \ldots\ldots\ldots + C_nx^n = \sum_{r=0}^{n} {}^nC_r r^r \; ; \; n \in N \qquad \ldots.(i)$$

where $C_0, C_1, C_2, \ldots\ldots\ldots C_n$ are called combinatorial (binomial) coefficients.

**(a)** The sum of all the binomial coefficients is $2^n$.

Put $x = 1$, in (i) we get

$$C_0 + C_1 + C_2 + \ldots\ldots\ldots\ldots + C_n = 2^n \Rightarrow \sum_{r=0}^{n} {}^nC_r = 0 \qquad \ldots.(ii)$$

**(b)** Put $x = -1$ in (i) we get

$$C_0 - C_1 + C_2 - C_3 \ldots\ldots\ldots\ldots + C_n = 0 \Rightarrow \sum_{r=0}^{n} (-1)^r \, {}^nC_r = 0 \qquad \ldots(iii)$$

**(c)** The sum of the binomial coefficients at odd position is equal to the sum of the binomial coefficients at even position and each is equal to $2^{n-1}$.

From (ii) & (iii), $C_0 + C_2 + C_4 + \ldots\ldots\ldots\ldots = C_1 + C_3 + C_5 + \ldots\ldots = 2^{n-1}$

**(d)** ${}^nC_r + {}^nC_{r-1} = {}^{n+1}C_r$

**(e)** $\frac{{}^nC_r}{{}^nC_{r-1}} = \frac{n-r+1}{r}$

**(f)** ${}^nC_r = \frac{n}{r} {}^{n-1}C_{r-1} = \frac{n}{r} \cdot \frac{n-1}{r-1} {}^{n-2}C_{r-2} = \ldots\ldots = \frac{n(n-1)(n-2)\ldots\ldots(n-r+1)}{r(r-1)(r-2)\ldots\ldots\ldots 1}$

**(g)** ${}^nC_r = \frac{r+1}{n+1} \cdot {}^{n+1}C_{r+1}$

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***Illustration 10 :*** Prove that : ${}^{25}C_{10} + {}^{24}C_{10} + \ldots\ldots + {}^{10}C_{10} = {}^{26}C_{11}$

***Solution :*** LHS = ${}^{10}C_{10} + {}^{11}C_{10} + {}^{12}C_{10} + \ldots\ldots\ldots\ldots + {}^{25}C_{10}$

$\Rightarrow$ ${}^{11}C_{11} + {}^{11}C_{10} + {}^{12}C_{10} + \ldots\ldots + {}^{25}C_{10}$

$\Rightarrow$ ${}^{12}C_{11} + {}^{12}C_{10} + \ldots\ldots + {}^{25}C_{10}$

$\Rightarrow$ ${}^{13}C_{11} + {}^{13}C_{10} + \ldots\ldots\ldots {}^{25}C_{10}$

and so on. $\therefore$ LHS = ${}^{26}C_{11}$

**Aliter :**

LHS = coefficient of $x^{10}$ in $\{(1+x)^{10} + (1+x)^{11} + \ldots\ldots\ldots\ldots (1+x)^{25}\}$

$\Rightarrow$ coefficient of $x^{10}$ in $\left[(1+x)^{10}\dfrac{\{1+x\}^{16}-1}{1+x-1}\right]$

$\Rightarrow$ coefficient of $x^{10}$ in $\dfrac{\left[(1+x)^{26}-(1+x)^{10}\right]}{x}$

$\Rightarrow$ coefficient of $x^{11}$ in $\left[(1+x)^{26}-(1+x)^{10}\right] = {}^{26}C_{11} - 0 = {}^{26}C_{11}$

***Illustration 11 :*** A student is allowed to select at most n books from a collection of (2n + 1) books. If the total number of ways in which he can select books is 63, find the value of n.

***Solution :*** Given student selects at most n books from a collection of (2n + 1) books. It means that he selects one book or two books or three books or ..... or n books. Hence, by the given condition-

${}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots\ldots + {}^{2n+1}C_n = 63$ ...(i)

But we know that

${}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots + {}^{2n+1}C_{2n+1} = 2^{2n+1}$ ...(ii)

Since ${}^{2n+1}C_0 = {}^{2n+1}C_{2n+1} = 1$, equation (ii) can also be written as

$2 + ({}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots + {}^{2n+1}C_n) +$

$({}^{2n+1}C_{n+1} + {}^{2n+1}C_{n+2} + {}^{2n+1}C_{n+3} + \ldots\ldots + {}^{2n+1}C_{2n-1} + {}^{2n+1}C_{2n}) = 2^{2n+1}$

$\Rightarrow 2 + ({}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots\ldots + {}^{2n+1}C_n)$

$+ ({}^{2n+1}C_n + {}^{2n+1}C_{n-1} + \ldots\ldots\ldots + {}^{2n+1}C_2 + {}^{2n+1}C_1) = 2^{2n+1}$

$({}^{\because}\ {}^{2n+1}C_r = {}^{2n+1}C_{2n+1-r})$

$\Rightarrow 2 + 2\,({}^{2n+1}C_1 + {}^{2n+1}C_2 + {}^{2n+1}C_3 + \ldots\ldots + {}^{2n+1}C_n) = 2^{2n+1}$ [from (i)]

$\Rightarrow 2 + 2.63 = 2^{2n+1}$ $\Rightarrow 1 + 63 = 2^{2n}$

$\Rightarrow 64 = 2^{2n} \Rightarrow 2^6 = 2^{2n}$ $\therefore 2n = 6$

Hence, n = 3. **Ans.**

***Illustration 12 :*** Prove that :

(i) $C_1 + 2C_2 + 3C_3 + \ldots\ldots\ldots + nC_n = n \cdot 2^{n-1}$

(ii) $C_0 + \dfrac{C_1}{2} + \dfrac{C_2}{3} + \ldots\ldots\ldots + \dfrac{C_n}{n+1} = \dfrac{2^{n+1}-1}{n+1}$

***Solution :*** (i) L.H.S. $= \sum_{r=1}^{n} r.\,{}^{n}C_r = \sum_{r=1}^{n} r.\dfrac{n}{r}.\,{}^{n-1}C_{r-1}$

$= n\sum_{r=1}^{n} {}^{n-1}C_{r-1} = n.\left[{}^{n-1}C_0 + {}^{n-1}C_1 + \ldots.. + {}^{n-1}C_{n-1}\right]$

$= n \cdot 2^{n-1}$

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**Aliter :** (Using method of differentiation)

$(1 + x)^n = {}^nC_0 + {}^nC_1x + {}^nC_2x^2 + ..... + {}^nC_nx^n$ .....(A)

Differentiating (A), we get

$n(1 + x)^{n-1} = C_1 + 2C_2x + 3C_3x^2 + ..... + n.C_nx^{n-1}$.

Put x = 1,

$C_1 + 2C_2 + 3C_3 + ..... + n.C_n = n.2^{n-1}$

(ii) L.H.S. $= \sum_{r=0}^{n} \frac{C_r}{r+1} = \frac{1}{n+1}\sum_{r=0}^{n} \frac{n+1}{r+1} {}^nC_r$

$= \frac{1}{n+1}\sum_{r=0}^{n} {}^{n+1}C_{r+1} = \frac{1}{n+1}\left[{}^{n+1}C_1 + {}^{n+1}C_2 + ..... + {}^{n+1}C_{n+1}\right] = \frac{1}{n+1}\left[2^{n+1} - 1\right]$

**Aliter :** (Using method of integration)

Integrating (A), we get

$\frac{(1+x)^{n+1}}{n+1} + C = C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + ..... + \frac{C_nx^{n+1}}{n+1}$ (where C is a constant)

Put x = 0, we get, $C = -\frac{1}{n+1}$

$\therefore \quad \frac{(1+x)^{n+1} - 1}{n+1} = C_0x + \frac{C_1x^2}{2} + \frac{C_2x^3}{3} + ..... + \frac{C_nx^{n+1}}{n+1}$

Put x = 1, we get $C_0 + \frac{C_1}{2} + \frac{C_2}{3} + ..... \frac{C_n}{n+1} = \frac{2^{n+1} - 1}{n+1}$

Put x = –1, we get $C_0 - \frac{C_1}{2} + \frac{C_2}{3} - ..... = \frac{1}{n+1}$

***Illustration 13 :*** If $(1 + x)^n = \sum_{r=0}^{n} {}^nC_rx^r$, then prove that $C_1^2 + 2.C_2^2 + 3.C_3^2 + ..... + n.C_n^2 = \frac{(2n-1)!}{((n-1)!)^2}$

***Solution :*** $(1 + x)^n = C_0 + C_1x + C_2x^2 + C_2x^3 + ..... + C_n x^n$ .....(i)

Differentiating both the sides, w.r.t. x, we get

$n(1 + x)^{n-1} = C_1 + 2C_2x + 3C_2x^2 + ..... + n.C_nx^{n-1}$ .....(ii)

also, we have

$(x + 1)^n = C_0x^n + C_1x^{n-1} + C_2x^{n-2} + ..... + C_n$ .....(iii)

Multiplying (ii) & (iii), we get

$(C_1 + 2C_2x + 3C_3x^2 + ..... + C_nx^{n-1})(C_0x^n + C_1x^{n-1} + C_2x^{n-2} + .....+ C_n) = n(1 + x)^{2n-1}$

Equating the coefficients of $x^{n-1}$, we get

$C_1^2 + 2C_2^2 + 3C_3^2 + ..... + n.C_n^2 = n.{}^{2n-1}C_{n-1} = \frac{(2n-1)!}{((n-1)!)^2}$ **Ans.**

***Illustration 14 :*** Prove that : $C_0 - 3C_1 + 5C_2 - .....(-1)^n(2n + 1)C_n = 0$

***Solution :*** $T_r = (-1)^r(2r + 1)^nC_r = 2(-1)^r . {}^nC_r + (-1)^r {}^nC_r$

$\Sigma T_r = 2\sum_{r=1}^{n}(-1)^r.r.\frac{n}{r}.{}^{n-1}C_{r-1} + \sum_{r=0}^{n}(-1)^r {}^nC_r = 2\sum_{r=1}^{n}(-1)^r.{}^{n-1}C_{r-1} + \sum_{r=0}^{n}(-1)^r . {}^nC_r$

$= 2\left[{}^{n-1}C_0 - {}^{n-1}C_1 + .....\right] + \left[{}^nC_0 - {}^nC_1 + .....\right] = 0$

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***Illustration 15 :*** Prove that $({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - .... + (-1)^n ({}^{2n}C_{2n})^2 = (-1)^n . {}^{2n}C_n$

***Solution :*** $(1 - x)^{2n} = {}^{2n}C_0 - {}^{2n}C_1x + {}^{2n}C_2x^2 - ....+(-1)^n {}^{2n}C_{2n}x^{2n}$ ....(i)

and $(x + 1)^{2n} = {}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} + {}^{2n}C_2x^{2n-2} +...+{}^{2n}C_{2n}$ ....(ii)

Multiplying (i) and (ii), we get

$(x^2 -1)^{2n} = ({}^{2n}C_0 - {}^{2n}C_1x +....+ (-1)^n {}^{2n}C_{2n}x^{2n}) \times ({}^{2n}C_0x^{2n} + {}^{2n}C_1x^{2n-1} +.... + {}^{2n}C_{2n})$ ....(iii)

Now, coefficient of $x^{2n}$ in R.H.S.

$= ({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - ..... + (-1)^n ({}^{2n}C_{2n})^2$

$\because$ General term in L.H.S., $T_{r+1} = {}^{2n}C_r(x^2)^{2n-r}(-1)^r$

Putting $2(2n - r) = 2n$

$\therefore \quad r = n$

$\therefore \quad T_{n+1} = {}^{2n}C_nx^{2n}(-1)^n$

Hence coeffiecient of $x^{2n}$ in L.H.S. $= (-1)^n.{}^{2n}C_n$

But (iii) is an identity, therefore coefficient of $x^{2n}$ in R.H.S. = coefficient of $x^{2n}$ in L.H.S.

$\Rightarrow \quad ({}^{2n}C_0)^2 - ({}^{2n}C_1)^2 + ({}^{2n}C_2)^2 - .... + (-1)^n ({}^{2n}C_{2n})^2 = (-1)^n . {}^{2n}C_n$

***Illustration 16 :*** Prove that : ${}^nC_0.{}^{2n}C_n - {}^nC_1.{}^{2n-2}Cn_n + {}^nC_2.{}^{2n-4}Cn_n + .... = 2^n$

***Solution :*** L.H.S. = Coefficient of $x^n$ in $[{}^nC_0(1 + x)^{2n} - {}^nC_1(1 + x)^{2n-2} .....]$

$=$ Coefficient of $x^n$ in $[(1 + x)^2 - 1]^n$

$=$ Coefficient of $x^n$ in $x^n(x + 2)^n = 2^n$

***Illustration 17 :*** If $(1 + x)^n = C_0 + C_1 x + C_2x^2 + ..... + C_nx^n$ then show that the sum of the products of the $C_i$'s taken two at a time represented by : $\sum\sum_{0\le i< j\le n} C_iC_j$ is equal to $2^{2n-1} - \frac{2n!}{2.n!n!}$

***Solution :*** Since $(C_0 + C_1 + C_2 +.....+ C_{n-1} + C_n)^2$

$= C_0^2 + C_1^2 + C_2^2 + ..... + C_{n-1}^2 + C_n^2 + 2(C_0C_1 + C_0C_2 + C_0C_3 + ... + C_0C_n + C_1C_2 + C_1C_3 + .....$

$+ C_1C_n + C_2C_3 + C_2C_4 + ... + C_2C_n + .....+C_{n-1}C_n)$

$(2^n)^2 = {}^{2n}C_n + 2 \sum_{0\le i<j\le n} C_iC_j$

Hence $\sum_{0\le i<j\le n} C_iC_j = 2^{2n-1} - \frac{2n!}{2.n!n!}$ **Ans.**

***Illustration 18 :*** If $(1+x)^n = C_0 + C_1x + C_2x^2 +....+ C_nx^n$ then prove that $\sum_{0\le i<j\le n} (C_i + C_j)^2 = (n-1)^{2n}C_n + 2^{2n}$

***Solution :*** L.H.S. $\sum_{0\le i<j\le n} (C_i + C_j)^2$

$= (C_0 + C_1)^2 + (C_0 + C_2)^2 +....+ (C_0 + C_n)^2 + (C_1 + C_2)^2 + (C_1 + C_3)^2 +....$

$+ (C_1 + C_n)^2 + (C_2 + C_3)^2 + (C_2 + C_4)^2 +... + (C_2 + C_n)^2 +....+ (C_{n-1} + C_n)^2$

$= n(C_0^2 + C_1^2 + C_2^2 + .... + C_n^2) + 2 \sum_{0\le i<j\le n} C_iC_j$

$= n.{}^{2n}C_n + 2.\left\{2^{2n-1} - \frac{2n!}{2.n!n!}\right\}$ {from Illustration 17}

$= n .{}^{2n}C_n + 2^{2n} - {}^{2n}C_n = (n - 1) . {}^{2n}C_n + 2^{2n} =$ R.H.S.

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**Do yourself - 4 :**

**(i)** $\binom{n}{0}+\binom{n}{1}+\binom{n}{2}+.....+\binom{n}{n}=$

(A) $2^{n-1}$ (B) ${}^{2n}C_n$ (C) $2^n$ (D) $2^{n+1}$

**(ii)** If $(1+x)^n = C_0 + C_1x + C_2x^2 + ..... + C_nx^n$, $n \in N$. Prove that

(a) $3C_0 - 8C_1 + 13C_2 - 18C_3 + .....$ upto $(n+1)$ terms $= 0$, if $n \geq 2$.

(b) $2C_0 + 2^2\frac{C_1}{2} + 2^3\frac{C_2}{3} + 2^4\frac{C_3}{4} + ..... + 2^{n+1}\frac{C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$

(c) $C_0^2 + \frac{C_1^2}{2} + \frac{C_2^2}{3} + ..... + \frac{C_n^2}{n+1} = \frac{(2n+1)!}{((n+1)!)^2}$

## 5. MULTINOMIAL THEOREM :

Using binomial theorem, we have $(x+a)^n = \sum_{r=0}^{n} {}^nC_r x^{n-r}a^r$, $n \in N$

$= \sum_{r=0}^{n} \frac{n!}{(n-r)!r!}x^{n-r}a^r = \sum_{r+s=n} \frac{n!}{r!s!}x^s a^r$, where $s + r = n$

This result can be generalized in the following form.

$(x_1 + x_2 + ..... + x_k)^n = \sum_{r_1+r_2+....+r_k=n} \frac{n!}{r_1!r_2!....r_k!}x_1^{r_1}x_2^{r_2}....x_k^{r_k}$

The general term in the above expansion $\frac{n!}{r_1!r_2!r_3!.....r_k!}.x_1^{r_1}x_2^{r_2}x_3^{r_3}.....x_k^{r_k}$

The number of terms in the above expansion is equal to the number of non-negative integral solution of the equation $r_1 + r_2 + ..... + r_k = n$ because each solution of this equation gives a term in the above expansion. The number of such solutions is ${}^{n+k-1}C_{k-1}$

**Particular cases :**

(i) $(x+y+z)^n = \sum_{r+s+t=n} \frac{n!}{r!s!t!}x^r y^s z^t$

The above expansion has ${}^{n+3-1}C_{3-1} = {}^{n+2}C_2$ terms

(ii) $(x+y+z+u)^n = \sum_{p+q+r+s=n} \frac{n!}{p!q!r!s!}x^p y^q z^r u^s$

There are ${}^{n+4-1}C_{4-1} = {}^{n+3}C_3$ terms in the above expansion.

---

***Illustration 19 :*** Find the coefficient of $x^2y^3z^4w$ in the expansion of $(x-y-z+w)^{10}$

***Solution :*** $(x-y-z+w)^{10} = \sum_{p+q+r+s=10} \frac{n!}{p!q!r!s!}(x)^p(-y)^q(-z)^r(w)^s$

We want to get $x^2y^3z^4w$ this implies that $p = 2$, $q = 3$, $r = 4$, $s = 1$

$\therefore$ Coefficient of $x^2y^3z^4w$ is $\frac{10!}{2!\,3!\,4!\,1!}(-1)^3(-1)^4 = -12600$ **Ans.**

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***Illustration 20 :*** Find the total number of terms in the expansion of $(1 + x + y)^{10}$ and coefficient of $x^2y^3$.

***Solution :*** Total number of terms $= {}^{10+3-1}C_{3-1} = {}^{12}C_2 = 66$

Coefficient of $x^2y^3 = \frac{10!}{2! \times 3! \times 5!} = 2520$ **Ans.**

***Illustration 21 :*** Find the coefficient of $x^5$ in the expansion of $(2 - x + 3x^2)^6$.

***Solution :*** The general term in the expansion of $(2 - x + 3x^2)^6 = \frac{6!}{r!s!t!} 2^r (-x)^s (3x^2)^t$,

where $r + s + t = 6$.

$= \frac{6!}{r!s!t!} 2^r \times (-1)^s \times (3)^t \times x^{s+2t}$

For the coefficient of $x^5$, we must have $s + 2t = 5$.

But, $r + s + t = 6$,

$\therefore$ $s = 5 - 2t$ and $r = 1 + t$, where $0 \le r, s, t \le 6$.

Now $t = 0 \Rightarrow r = 1, s = 5$.

$t = 1 \Rightarrow r = 2, s = 3$.

$t = 2 \Rightarrow r = 3, s = 1$.

Thus, there are three terms containing $x^5$ and coefficient of $x^5$

$= \frac{6!}{1!\,5!\,0!} \times 2^1 \times (-1)^5 \times 3^0 + \frac{6!}{2!\,3!\,1!} \times 2^2 \times (-1)^3 \times 3^1 + \frac{6!}{3!\,1!\,2!} \times 2^3 \times (-1)^1 \times 3^2$

$= -12 - 720 - 4320 = -5052$. **Ans.**

***Illustration 22 :*** If $(1+x+x^2)^n = \sum_{r=0}^{2n} a_r x^r$, then prove that (a) $a_r = a_{2n-r}$ (b) $\sum_{r=0}^{n-1} a_r = \frac{1}{2}(3^n - a_n)$

***Solution :*** (a) We have

$$\left(1 + x + x^2\right)^n = \sum_{r=0}^{2n} a_r x^r \qquad \text{....(A)}$$

Replace x by $\frac{1}{x}$

$$\therefore \quad \left(1 + \frac{1}{x} + \frac{1}{x^2}\right)^n = \sum_{r=0}^{2n} a_r \left(\frac{1}{x}\right)^r$$

$$\Rightarrow \quad \left(x^2 + x + 1\right)^n = \sum_{r=0}^{2n} a_r x^{2n-r}$$

$$\Rightarrow \quad \sum_{r=0}^{2n} a_r x^r = \sum_{r=0}^{2n} a_r x^{2n-r} \qquad \{\text{Using (A)}\}$$

Equating the coefficient of $x^{2n-r}$ on both sides, we get

$a_{2n-r} = a_r$ for $0 \le r \le 2n$.

Hence $a_r = a_{2n-r}$.

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(b) Putting x=1 in given series, then

$a_0 + a_1 + a_2 + .....+ a_{2n} = (1+1+1)^n$

$a_0 + a_1 + a_2 + .....+ a_{2n} = 3^n$ ....(1)

But $a_r = a_{2n-r}$ for $0 \le r \le 2n$

$\therefore$ series (1) reduces to

$2(a_0 + a_1 + a_2 + .....+ a_{n-1}) + a_n = 3^n.$

$\therefore \quad a_0 + a_1 + a_2 + .....+ a_{n-1} = \frac{1}{2}(3^n - a_n)$

**Do yourself - 5 :**

**(i)** Find the coefficient of $x^2y^5$ in the expansion of $(3 + 2x - y)^{10}$.

## 6. APPLICATION OF BINOMIAL THEOREM :

***Illustration 23 :*** If $\left(6\sqrt{6}+14\right)^{2n+1} = [N] + F$ and $F = N - [N]$; where [.] denotes greatest integer function, then NF is equal to

(A) $20^{2n+1}$ (B) an even integer (C) odd integer (D) $40^{2n+1}$

***Solution :*** Since $\left(6\sqrt{6}+14\right)^{2n+1} = [N] + F$

Let us assume that $f = \left(6\sqrt{6}-14\right)^{2n+1}$; where $0 \le f < 1$.

Now, $[N] + F - f = \left(6\sqrt{6}+14\right)^{2n+1} - \left(6\sqrt{6}-14\right)^{2n+1}$

$$= 2\left[{}^{2n+1}C_1\left(6\sqrt{6}\right)^{2n}(14) + {}^{2n+1}C_3\left(6\sqrt{6}\right)^{2n-2}(14)^3 + ....\right]$$

$\Rightarrow$ [N] + F – f = even integer.

Now $0 < F < 1$ and $0 < f < 1$

so $-1 < F - f < 1$ and F – f is an integer so it can only be zero

Thus $NF = \left(6\sqrt{6}+14\right)^{2n+1}\left(6\sqrt{6}-14\right)^{2n+1} = 20^{2n+1}$. **Ans. (A,B)**

***Illustration 24 :*** Find the last three digits in $11^{50}$.

***Solution :*** Expansion of $(10 + 1)^{50} = {}^{50}C_0 10^{50} + {}^{50}C_1 10^{49} + ..... + {}^{50}C_{48}10^2 + {}^{50}C_{49}10 + {}^{50}C_{50}$

$$= \underbrace{{}^{50}C_0 10^{50} + {}^{50}C_1 10^{49} + .....+ {}^{50}C_{47}10^3}_{1000K} + 49 \times 25 \times 100 + 500 + 1$$

$\Rightarrow$ 1000 K + 123001

$\Rightarrow$ Last 3 digits are 001.

***Illustration 25 :*** Prove that $2222^{5555} + 5555^{2222}$ is divisible by 7.

***Solution :*** When 2222 is divided by 7 it leaves a remainder 3.

So adding & subtracting $3^{5555}$, we get :

$$E = \underbrace{2222^{5555} - 3^{5555}}_{E_1} + \underbrace{3^{5555} + 5555^{2222}}_{E_2}$$

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For $E_1$ : Now since 2222–3 = 2219 is divisible by 7, therefore $E_1$ is divisible by 7
($\because$ $x^n - a^n$ is divisible by x –a)
For $E_2$ : 5555 when devided by 7 leaves remainder 4.
So adding and subtracting $4^{2222}$, we get :

$E_2 = 3^{5555} + 4^{2222} + 5555^{2222} - 4^{2222}$

$= (243)^{1111} + (16)^{1111} + (5555)^{2222} - 4^{2222}$

Again $(243)^{1111} + 16^{1111}$ and $(5555)^{2222} - 4^{2222}$ are divisible by 7
($\because$ $x^n + a^n$ is divisible by x + a when n is odd)
Hence $2222^{5555} + 5555^{2222}$ is divisible by 7.

**Do yourself - 6 :**

**(i)** Prove that $5^{25} - 3^{25}$ is divisible by 2.

**(ii)** Find the remainder when the number $9^{100}$ is divided by 8.

**(iii)** Find last three digits in $19^{100}$.

**(iv)** Let $R = (8+3\sqrt{7})^{20}$ and [.] denotes greatest integer function, then prove that :

(a) [R] is odd (b) $R - [R] = 1 - \frac{1}{(8+3\sqrt{7})^{20}}$

**(v)** Find the digit at unit's place in the number $17^{1995} + 11^{1995} - 7^{1995}$.

## 7. BINOMIAL THEOREM FOR NEGATIVE OR FRACTIONAL INDICES :

If $n \in Q$, then $(1+x)^n = 1 + nx + \frac{n(n-1)}{2!}x^2 + \frac{n(n-1)(n-2)}{3!}x^3 + \ldots\ldots \infty$ provided $|x| < 1$.

**Note :**

(i) When the index n is a positive integer the number of terms in the expansion of $(1+x)^n$ is finite i.e. (n+1) & the coefficient of successive terms are : ${}^nC_0, {}^nC_1, {}^nC_2, \ldots\ldots {}^nC_n$

(ii) When the index is other than a positive integer such as negative integer or fraction, the number of terms in the expansion of $(1+x)^n$ is infinite and the symbol ${}^nC_r$ cannot be used to denote the coefficient of the general term.

(iii) Following expansion should be remembered ($|x| < 1$).

(a) $(1+x)^{-1} = 1 - x + x^2 - x^3 + x^4 - \ldots\ldots \infty$

(b) $(1-x)^{-1} = 1 + x + x^2 + x^3 + x^4 + \ldots\ldots \infty$

(c) $(1+x)^{-2} = 1 - 2x + 3x^2 - 4x^3 + \ldots\ldots \infty$

(d) $(1-x)^{-2} = 1 + 2x + 3x^2 + 4x^3 + \ldots\ldots \infty$

(e) $(1+x)^{-3} = 1 - 3x + 6x^2 - 10x^3 + \ldots\ldots + \frac{(-1)^r (r+1)(r+2)}{2!}x^r + \ldots\ldots$

(f) $(1-x)^{-3} = 1 + 3x + 6x^2 + 10x^3 + \ldots\ldots + \frac{(r+1)(r+2)}{2!}x^r + \ldots\ldots$

(iv) The expansions in ascending powers of x are only valid if x is 'small'. If x is large i.e. $|x| > 1$ then we may find it convenient to expand in powers of 1/x, which then will be small.

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## 8. APPROXIMATIONS :

$(1+x)^n = 1 + nx + \frac{n(n-1)}{1.2}x^2 + \frac{n(n-1)(n-2)}{1.2.3}x^3 .....$

If **x < 1**, the terms of the above expansion go on decreasing and if x be very small, a stage may be reached when we may neglect the terms containing higher powers of x in the expansion. Thus, if x be so small that its square and higher powers may be neglected then $(1+x)^n = 1 + nx$, approximately. This is an approximate value of $(1+x)^n$

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***Illustration 26 :*** If x is so small such that its square and higher powers may be neglected then find the approximate value of $\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{(4+x)^{1/2}}$

***Solution :*** $\frac{(1-3x)^{1/2}+(1-x)^{5/3}}{(4+x)^{1/2}} = \frac{1-\frac{3}{2}x+1-\frac{5x}{3}}{2\left(1+\frac{x}{4}\right)^{1/2}} = \frac{1}{2}\left(2-\frac{19}{6}x\right)\left(1+\frac{x}{4}\right)^{-1/2} = \frac{1}{2}\left(2-\frac{19}{6}x\right)\left(1-\frac{x}{8}\right)$

$= \frac{1}{2}\left(2-\frac{x}{4}-\frac{19}{6}x\right) = 1-\frac{x}{8}-\frac{19}{12}x = 1-\frac{41}{24}x$ **Ans.**

***Illustration 27 :*** The value of cube root of 1001 upto five decimal places is –

(A) 10.03333 (B) 10.00333 (C) 10.00033 (D) none of these

***Solution :*** $(1001)^{1/3} = (1000+1)^{1/3} = 10\left(1+\frac{1}{1000}\right)^{1/3} = 10\left\{1+\frac{1}{3}.\frac{1}{1000}+\frac{1/3(1/3-1)}{2!}\frac{1}{1000^2}+.....\right\}$

$= 10\{1 + 0.0003333 - 0.00000011 + .....\} = 10.00333$ **Ans. (B)**

***Illustration 28 :*** The sum of $1+\frac{1}{4}+\frac{1.3}{4.8}+\frac{1.3.5}{4.8.12}+....\infty$ is -

(A) $\sqrt{2}$ (B) $\frac{1}{\sqrt{2}}$ (C) $\sqrt{3}$ (D) $2^{3/2}$

***Solution :*** Comparing with $1 + nx + \frac{n(n-1)}{2!}x^2 + ....$

$nx = 1/4$ .....(i)

and $\frac{n(n-1)x^2}{2!} = \frac{1.3}{4.8}$

or $\frac{nx(nx-x)}{2!} = \frac{3}{32} \Rightarrow \frac{1}{4}\left(\frac{1}{4}-x\right) = \frac{3}{16}$ (by (i))

$\Rightarrow \left(\frac{1}{4}-x\right) = \frac{3}{4} \Rightarrow x = \frac{1}{4}-\frac{3}{4} = -\frac{1}{2}$ .....(ii)

putting the value of x in (i)

$n(-1/2) = 1/4 \Rightarrow n = -1/2$

$\therefore$ sum of series $= (1+x)^n = (1-1/2)^{-1/2} = (1/2)^{-1/2} = \sqrt{2}$ **Ans. (A)**

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**9. EXPONENTIAL SERIES :**

**(a)** e is an irrational number lying between 2.7 & 2.8. Its value correct upto 10 places of decimal is 2.7182818284.

**(b)** Logarithms to the base 'e' are known as the Napierian system, so named after Napier, their inventor. They are also called **Natural Logarithm.**

**(c)** $e^x = 1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + .....\infty$ ; where x may be any real or complex number & $e = \lim_{n\to\infty}\left(1+\frac{1}{n}\right)^n$

**(d)** $a^x = 1 + \frac{x}{1!}\ell na + \frac{x^2}{2!}\ell n^2 a + \frac{x^3}{3!}\ell n^3 a + .....\infty$, where $a > 0$

**(e)** $e = 1 + \frac{1}{1!} + \frac{1}{2!} + \frac{1}{3!} + .....\infty$

**10. LOGARITHMIC SERIES :**

**(a)** $\ell n\,(1 + x) = x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + .....\infty$, where $-1 < x \le 1$

**(b)** $\ell n\,(1 - x) = -x - \frac{x^2}{2} - \frac{x^3}{3} - \frac{x^4}{4} + .....\infty$, where $-1 \le x < 1$

**Remember :** (i) $1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + .....\infty = \ell n\,2$ (ii) $e^{\ln x} = x$ ; for all $x > 0$

(iii) $\ell n2 = 0.693$ (iv) $\ell n10 = 2.303$

## ANSWERS FOR DO YOURSELF

**1.** **(i)** ${}^5C_0x(3x^2)^5 + {}^5C_1(3x^2)^4\left(-\frac{x}{2}\right) + {}^5C_2(3x^2)^3\left(-\frac{x}{2}\right)^2 + {}^5C_3(3x^2)^2\left(-\frac{x}{2}\right)^3 + {}^5C_4(3x^2)^1\left(-\frac{x}{2}\right)^4 + {}^5C_5\left(-\frac{x}{2}\right)^5$

**(ii)** ${}^nC_0y^n + {}^nC_1y^{n-1}.x + {}^nC_2.y^{n-2}.x^2 + ..... + {}^nC_n.x^n$

**2 :** **(i)** $\frac{70}{3}x^8$ ; **(ii)** $\frac{25!}{10!\,5!}2^{15}3^{10}$ ; **(iii)** (a) –20; (b) $-560x^5, 280x^2$

**3.** **(i)** 4th & 5th i.e. 489888 **(ii)** n = 4, 5, 6

**4.** **(i) C**

**5.** **(i)** –272160 or $-{}^{10}C_5 \times {}^5C_2 \times 108$

**6.** **(ii)** 1 **(iii)** 801 **(v)** 1

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# EXERCISE (O-1)

## [SINGLE CORRECT CHOICE TYPE]

**1.** If the coefficients of $x^7$ & $x^8$ in the expansion of $\left[2+\frac{x}{3}\right]^n$ are equal, then the value of n is :

(A) 15 (B) 45 (C) 55 (D) 56 **BT0001**

**2.** If the constant term of the binomial expansion $\left(2x-\frac{1}{x}\right)^n$ is $-160$, then n is equal to -

(A) 4 (B) 6 (C) 8 (D) 10 **BT0003**

**3.** The coefficient of $x^{49}$ in the expansion of $(x-1)\left(x-\frac{1}{2}\right)\left(x-\frac{1}{2^2}\right)....\left(x-\frac{1}{2^{49}}\right)$ is equal to -

(A) $-2\left(1-\frac{1}{2^{50}}\right)$ (B) +ve coefficient of x (C) –ve coefficient of x (D) $-2\left(1-\frac{1}{2^{49}}\right)$ **BT0004**

**4.** Number of rational terms in the expansion of $\left(\sqrt{2}+\sqrt[4]{3}\right)^{100}$ is :

(A) 25 (B) 26 (C) 27 (D) 28 **BT0007**

**5.** The expression $[x+(x^3-1)^{1/2}]^5+[x-(x^3-1)^{1/2}]^5$ is a polynomial of degree

(A) 5 (B) 6 (C) 7 (D) 8 **BT0006**

**6.** Given $(1-2x+5x^2-10x^3)(1+x)^n = 1+a_1x+a_2x^2+.....$ and that $a_1^2 = 2a_2$ then the value of n is-

(A) 6 (B) 2 (C) 5 (D) 3 **BT0008**

**7.** The sum of the co-efficients of all the even powers of x in the expansion of $(2x^2-3x+1)^{11}$ is -

(A) $2.6^{10}$ (B) $3.6^{10}$ (C) $6^{11}$ (D) none **BT0009**

**8.** Let $\binom{n}{k}$ represents the combination of 'n' things taken 'k' at a time, then the value of the sum

$\binom{99}{97}+\binom{98}{96}+\binom{97}{95}+.....+\binom{3}{1}+\binom{2}{0}$ equals - **BT0011**

(A) $\binom{99}{97}$ (B) $\binom{100}{98}$ (C) $\binom{99}{98}$ (D) $\binom{100}{97}$

## [COMPREHENSION TYPE]

### Paragraph for question nos. 9 to 11

If $n \in N$ and if $(1+4x+4x^2)^n = \sum_{r=0}^{2n} a_r x^r$, where $a_0, a_1, a_2, ....., a_{2n}$ are real numbers.

**9.** The value of $2\sum_{r=0}^{n} a_{2r}$, is

(A) $9^n-1$ (B) $9^n+1$ (C) $9^n-2$ (D) $9^n+2$ **BT0012**

**10.** The value of $2\sum_{r=1}^{n} a_{2r-1}$, is-

(A) $9^n-1$ (B) $9^n+1$ (C) $9^n-2$ (D) $9^n+2$ **BT0012**

**11.** The value of $a_{2n-1}$ is -

(A) $2^{2n}$ (B) $n.2^{2n}$ (C) $(n-1)2^{2n}$ (D) $(n+1)2^{2n}$ **BT0012**

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# EXERCISE (O-2)

## [SINGLE CORRECT CHOICE TYPE]

**1.** Greatest term in the binomial expansion of $(a+2x)^9$ when $a=1$ & $x=\frac{1}{3}$ is :

(A) $3^{rd}$ & $4^{th}$ (B) $4^{th}$ & $5^{th}$ (C) only $4^{th}$ (D) only $5^{th}$ **BT0019**

**2.** If $\sum_{r=1}^{10} r(r-1)\ {}^{10}C_r = k.\ 2^9$, then k is equal to-

(A) 10 (B) 45 (C) 90 (D) 100 **BT0022**

**3.** The sum $\frac{\binom{11}{0}}{1}+\frac{\binom{11}{1}}{2}+\frac{\binom{11}{2}}{3}+.....+\frac{\binom{11}{11}}{12}$ equals $\left(\text{where } \binom{n}{r} \text{denotes } {}^nC_r\right)$

(A) $\frac{2^{11}}{12}$ (B) $\frac{2^{12}}{12}$ (C) $\frac{2^{11}-1}{12}$ (D) $\frac{2^{12}-1}{12}$ **BT0023**

## [MULTIPLE CORRECT CHOICE TYPE]

**4.** In the expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$, the term which does not contain x is-

(A) ${}^{11}C_4-{}^{10}C_3$ (B) ${}^{10}C_7$ (C) ${}^{10}C_4$ (D) ${}^{11}C_5-{}^{10}C_5$ **BT0016**

**5.** Let $(1+x^2)^2(1+x)^n = A_0+A_1x+A_2x^2+.....$ If $A_0, A_1, A_2$ are in A.P. then the value of n is-

(A) 2 (B) 3 (C) 5 (D) 7 **BT0017**

**6.** In the expansion of $\left(x^3+3.2^{-\log_{\sqrt{2}}\sqrt{x^3}}\right)^{11}$ **BT0015**

(A) there appears a term with the power $x^2$ (B) there does not appear a term with the power $x^2$
(C) there appears a term with the power $x^{-3}$ (D) the ratio of the co-efficient of $x^3$ to that of $x^{-3}$ is 1/3

**7.** If it is known that the third term of the binomial expansion $\left(x+x^{\log_{10}x}\right)^5$ is $10^6$ then x is equal to-

(A) 10 (B) $10^{-5/2}$ (C) 100 (D) 5 **BT0014**

# EXERCISE (S-1)

**1.** (a) If the coefficients of $(2r+4)^{th}$, $(r-2)^{th}$ terms in the expansion of $(1+x)^{18}$ are equal, find r. **BT0029**

(b) If the coefficients of the $r^{th}$, $(r+1)^{th}$ & $(r+2)^{th}$ terms in the expansion of $(1+x)^{14}$ are in AP, find r. **BT0030**

(c) If the coefficients of $2^{nd}$, $3^{rd}$ & $4^{th}$ terms in the expansion of $(1+x)^{2n}$ are in AP, show that $2n^2-9n+7=0$. **BT0031**

**2.** Find the term independent of x in the expansion of (i) $\left[\sqrt{\frac{x}{3}}+\frac{\sqrt{3}}{2x^2}\right]^{10}$ (ii) $\left[\frac{1}{2}x^{1/3}+x^{-1/5}\right]^8$ **BT0032**

**3.** Prove that the ratio of the coefficient of $x^{10}$ in $(1-x^2)^{10}$ & the term independent of x in $\left(x-\frac{2}{x}\right)^{10}$ is 1 : 32. **BT0033**

**4.** Find the sum of the series $\sum_{r=0}^{n}(-1)^r.\ {}^nC_r\left[\frac{1}{2^r}+\frac{3^r}{2^{2r}}+\frac{7^r}{2^{3r}}+\frac{15^r}{2^{4r}}+.....\text{up to m terms}\right]$ **BT0058**

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**5.** Find numerically greatest term in the expansion of :

(i) $(2+3x)^9$ when $x=\frac{3}{2}$ **BT0038**

(ii) $(3-5x)^{15}$ when $x=\frac{1}{5}$ **BT0039**

**6.** Find the term independent of x in the expansion of $(1+x+2x^3)\left(\frac{3x^2}{2}-\frac{1}{3x}\right)^9$. **BT0034**

**7.** Let $(1+x^2)^2 \cdot (1+x)^n = \sum_{K=0}^{n+4} a_K \cdot x^K$. If $a_1$, $a_2$ & $a_3$ are in AP, find n. **BT0035**

**8.** Let $f(x) = 1-x+x^2-x^3+.....+x^{16}-x^{17} = a_0+a_1(1+x)+a_2(1+x)^2+.....+a_{17}(1+x)^{17}$, find the value of $a_2$. **BT0036**

**9.** Let $N = {}^{2000}C_1 + 2\cdot{}^{2000}C_2 + 3\cdot{}^{2000}C_3 + ..... + 2000\cdot{}^{2000}C_{2000}$. Prove that N is divisible by $2^{2003}$. **BT0042**

**10.** Find the coefficient of

(a) $x^2y^3z^4$ in the expansion of $(ax-by+cz)^9$. **BT0061**

(b) $a^2b^3c^4d$ in the expansion of $(a-b-c+d)^{10}$. **BT0062**

**11.** Find the coefficient of

(a) $x^4$ in the expansion of $(1+x+x^2+x^3)^{11}$ **BT0059**

(b) $x^4$ in the expansion of $(2-x+3x^2)^6$ **BT0060**

**12.** Find the coefficient of $x^r$ in the expression :

$(x+3)^{n-1}+(x+3)^{n-2}(x+2)+(x+3)^{n-3}(x+2)^2+.....+(x+2)^{n-1}$ **BT0037**

**13.** Given that $(1+x+x^2)^n = a_0+a_1x+a_2x^2+....+a_{2n}x^{2n}$, find the values of : **BT0057**

(i) $a_0+a_1+a_2+.....+a_{2n}$

(ii) $a_0-a_1+a_2-a_3.....+a_{2n}$

**14.** Prove the following identities using the theory of permutation where $C_0, C_1, C_2, ....., C_n$ are the combinatorial coefficients in the expansion of $(1+x)^n$, $n \in N$ :

${}^{100}C_{10}+5.{}^{100}C_{11}+10.{}^{100}C_{12}+10.{}^{100}C_{13}+5.{}^{100}C_{14}+{}^{100}C_{15} = {}^{105}C_{90}$ **BT0047**

**15.** If $C_0, C_1, C_2, ....., C_n$ are the combinatorial coefficients in the expansion of $(1+x)^n$, $n \in N$, then prove the following :

(a) $C_1+2C_2+3C_3+.....+n.C_n = n.2^{n-1}$ **BT0048**

(b) $C_0+2C_1+3C_2+.....+(n+1)C_n = (n+2)2^{n-1}$ **BT0049**

(c) $C_0+3C_1+5C_2+.....+(2n+1)C_n = (n+1)2^n$ **BT0050**

(d) $(C_0+C_1)(C_1+C_2)(C_2+C_3).....(C_{n-1}+C_n) = \frac{C_0.C_1.C_2....C_{n-1}(n+1)^n}{n!}$ **BT0051**

(e) $1.C_o^2+3.C_1^2+5.C_2^2+.....+(2n+1)C_n^2 = \frac{(n+1)(2n)!}{n!\,n!}$ **BT0052**

**16.** Prove that

(a) $\frac{C_1}{C_0}+\frac{2C_2}{C_1}+\frac{3C_3}{C_2}+.....+\frac{n.C_n}{C_{n-1}} = \frac{n(n+1)}{2}$ **BT0053**

(b) $C_0+\frac{C_1}{2}+\frac{C_2}{3}+.....+\frac{C_n}{n+1} = \frac{2^{n+1}-1}{n+1}$ **BT0054**

(c) $2.C_o+\frac{2^2.C_1}{2}+\frac{2^3.C_2}{3}+\frac{2^4.C_3}{4}+.....\frac{2^{n+1}.C_n}{n+1} = \frac{3^{n+1}-1}{n+1}$ **BT0055**

(d) $C_o-\frac{C_1}{2}+\frac{C_2}{3}-.....+(-1)^n\frac{C_n}{n+1} = \frac{1}{n+1}$ **BT0056**

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# EXERCISE (S-2)

**1.** Prove that $\sum_{K=0}^{n} {}^{n}C_K \sin Kx.\cos(n-K)x = 2^{n-1}\sin nx.$ **BT0071**

**2.** Let $a = \left(4^{1/401} - 1\right)$ and let $b_n = {}^{n}C_1 + {}^{n}C_2 \,.\, a + {}^{n}C_3 \,.\, a^2 + \ldots\ldots + {}^{n}C_n \,.\, a^{n-1}$.
Find the value of $(b_{2006} - b_{2005})$ **BT0066**

**3.** Let a and b be the coefficient of $x^3$ in $(1 + x + 2x^2 + 3x^3)^4$ and $(1 + x + 2x^2 + 3x^3 + 4x^4)^4$ respectively. Find the value of $(a - b)$. **BT0063**

**4.** Find the sum of the roots (real or complex) of the equation $x^{2001} + \left(\frac{1}{2} - x\right)^{2001} = 0$. **BT0065**

**5.** Find the coefficient of $x^{49}$ in the polynomial

$$\left(x - \frac{C_1}{C_0}\right)\left(x - 2^2 \cdot \frac{C_2}{C_1}\right)\left(x - 3^2 \cdot \frac{C_3}{C_2}\right) \ldots\ldots\ldots\ldots \left(x - 50^2 \cdot \frac{C_{50}}{C_{49}}\right), \text{ where } C_r = {}^{50}C_r. \quad \textbf{BT0070}$$

**6.** If $\binom{n}{r}$ denotes ${}^{n}C_r$, then

(a) Evaluate : $2^{15}\binom{30}{0}\binom{30}{15} - 2^{14}\binom{30}{1}\binom{29}{14} + 2^{13}\binom{30}{2}\binom{28}{13} \ldots\ldots - \binom{30}{15}\binom{15}{0}$ **BT0072**

(b) Prove that : $\sum_{r=1}^{n}\binom{n-1}{n-r}\binom{n}{r} = \binom{2n-1}{n-1}$ **BT0073**

(c) Prove that : $\binom{n}{r}\binom{r}{k} = \binom{n}{k}\binom{n-k}{r-k}$ **BT0074**

# EXERCISE (JM)

**1.** Let $S_1 = \sum_{j=1}^{10} j(j-1)^{10}C_j$, $S_2 = \sum_{j=1}^{10} j^{10}C_j$ and $S_3 = \sum_{j=1}^{10} j^{2}{}^{10}C_j$. **[AIEEE-2010]**

**Statement–1 :** $S_3 = 55 \times 2^9$.
**Statement–2 :** $S_1 = 90 \times 2^8$ and $S_2 = 10 \times 2^8$.
(1) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.
(2) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1.
(3) Statement–1 is true, Statement–2 is false.
(4) Statement–1 is false, Statement–2 is true. **BT0075**

**2.** The coefficient of $x^7$ in the expansion of $(1 - x - x^2 + x^3)^6$ is :- **[AIEEE 2011]**
(1) – 144 (2) 132 (3) 144 (4) – 132 **BT0076**

**3.** If n is a positive integer, then $\left(\sqrt{3}+1\right)^{2n} - \left(\sqrt{3}-1\right)^{2n}$ is : **[AIEEE 2012]**
(1) a rational number other than positive integers (2) an irrational number
(3) an odd positive integer (4) an even positive integer **BT0077**

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**4.** The term independent of x in expansion of $\left(\frac{x+1}{x^{2/3}-x^{1/3}+1}-\frac{x-1}{x-x^{1/2}}\right)^{10}$ is : **[JEE-Main 2013]**

(1) 4 (2) 120 (3) 210 (4) 310 **BT0078**

**5.** If the coefficients of $x^3$ and $x^4$ in the expansion of $(1+ax+bx^2)(1-2x)^{18}$ in powers of x are both zero, then (a, b) is equal to :- **[JEE(Main)-2014]**

(1) $\left(16,\frac{251}{3}\right)$ (2) $\left(14,\frac{251}{3}\right)$ (3) $\left(14,\frac{272}{3}\right)$ (4) $\left(16,\frac{272}{3}\right)$ **BT0079**

**6.** The sum of coefficients of integral powers of x in the binomial expansion of $\left(1-2\sqrt{x}\right)^{50}$ is : **[JEE(Main)-2015]**

(1) $\frac{1}{2}\left(3^{50}-1\right)$ (2) $\frac{1}{2}\left(2^{50}+1\right)$ (3) $\frac{1}{2}\left(3^{50}+1\right)$ (4) $\frac{1}{2}\left(3^{50}\right)$ **BT0080**

**7.** If the number of terms in the expansion of $\left(1-\frac{2}{x}+\frac{4}{x^2}\right)^n$, $x\neq 0$, is 28, then the sum of the coefficients of all the terms in this expansion, is :- **[JEE(Main)-2016]**

(1) 729 (2) 64 (3) 2187 (4) 243 **BT0081**

**8.** The value of $({}^{21}C_1-{}^{10}C_1)+({}^{21}C_2-{}^{10}C_2)+({}^{21}C_3-{}^{10}C_3)+({}^{21}C_4-{}^{10}C_4)+....+({}^{21}C_{10}-{}^{10}C_{10})$ is :- **[JEE(Main)-2017]**

(1) $2^{20}-2^{10}$ (2) $2^{21}-2^{11}$ (3) $2^{21}-2^{10}$ (4) $2^{20}-2^9$ **BT0082**

**9.** The sum of the co-efficients of all odd degree terms in the expansion of $\left(x+\sqrt{x^3-1}\right)^5+\left(x-\sqrt{x^3-1}\right)^5$, $(x>1)$ is - **[JEE(Main)-2018]**

(1) 0 (2) 1 (3) 2 (4) –1 **BT0083**

**10.** If the fractional part of the number $\frac{2^{403}}{15}$ is $\frac{k}{15}$, then k is equal to : **[JEE(Main)- 2019]**

(1) 14 (2) 6 (3) 4 (4) 8 **BT0084**

**11.** The coefficient of $t^4$ in the expansion of $\left(\frac{1-t^6}{1-t}\right)^3$ is **[JEE(Main)- 2019]**

(1) 12 (2) 15 (3) 10 (4) 14 **BT0085**

**12.** If $\sum_{r=0}^{25}\left\{{}^{50}C_r\cdot{}^{50-r}C_{25-r}\right\}=K\left({}^{50}C_{25}\right)$, then K is equal to : **[JEE(Main)- 2019]**

(1) $2^{25}-1$ (2) $(25)^2$ (3) $2^{25}$ (4) $2^{24}$ **BT0086**

**13.** The sum of the real values of x for which the middle term in the binomial expansion of $\left(\frac{x^3}{3}+\frac{3}{x}\right)^8$ equals 5670 is : **[JEE(Main)- 2019]**

(1) 6 (2) 8 (3) 0 (4) 4 **BT0087**

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**14.** The value of r for which ${}^{20}C_r\ {}^{20}C_0 + {}^{20}C_{r-1}\ {}^{20}C_1 + {}^{20}C_{r-2}\ {}^{20}C_2 + .... {}^{20}C_0\ {}^{20}C_r$ is maximum, is **[JEE(Main)- 2019]**

(1) 20 (2) 15 (3) 11 (4) 10 **BT0088**

**15.** Let $(x + 10)^{50} + (x - 10)^{50} = a_0 + a_1x + a_2x^2 + ..... + a_{50}\ x^{50}$, for all $x \in R$, then $\frac{a_2}{a_0}$ is equal to:- **[JEE(Main)- 2019]**

(1) 12.50 (2) 12.00 (3) 12.75 (4) 12.25 **BT0089**

**16.** Let $S_n = 1 + q + q^2 + ..... + q^n$ and $T_n = 1 + \left(\frac{q+1}{2}\right) + \left(\frac{q+1}{2}\right)^2 + ..... + \left(\frac{q+1}{2}\right)^n$, where q is a real number and $q \neq 1$. If ${}^{101}C_1 + {}^{101}C_2.S_1 + ..... + {}^{101}C_{101}.S_{100} = \alpha T_{100}$, then $\alpha$ is equal to :-**[JEE(Main)- 2019]**

(1) $2^{100}$ (2) 200 (3) $2^{99}$ (4) 202 **BT0090**

**17.** The total number of irrational terms in the binomial expansion of $\left(7^{1/5} - 3^{1/10}\right)^{60}$ is : **[JEE(Main)- 2019]**

(1) 55 (2) 49 (3) 48 (4) 54 **BT0091**

**18.** If some three consecutive in the binomial expansion of $(x + 1)^n$ is powers of x are in the ratio 2 : 15 : 70, then the average of these three coefficient is :- **[JEE(Main)- 2019]**

(1) 964 (2) 625 (3) 227 (4) 232 **BT0092**

**19.** The coefficient of $x^{18}$ in the product $(1+x)(1-x)^{10}(1 + x + x^2)^9$ is : **[JEE(Main)- 2019]**

(1) –84 (2) 84 (3) 126 (4) –126 **BT0093**

**20.** If ${}^{20}C_1 + (2^2)\ {}^{20}C_2 + (3^2)\ {}^{20}C_3 + ..... + (20^2){}^{20}C_{20} = A(2^\beta)$, then the ordered pair $(A, \beta)$ is equal to: **[JEE(Main)- 2019]**

(1) (420, 18) (2) (380, 19) (3) (380, 18) (4) (420, 19) **BT0094**

**21.** The term independent of x in the expansion of $\left(\frac{1}{60} - \frac{x^8}{81}\right) \cdot \left(2x^2 - \frac{3}{x^2}\right)^6$ is equal to : **[JEE(Main)- 2019]**

(1) 36 (2) – 108 (3) – 72 (4) – 36 **BT0095**

## EXERCISE (JA)

**1.** For r = 0, 1,....,10, let $A_r$, $B_r$ and $C_r$ denote, respectively, the coefficient of $x^r$ in the expansions of $(1 + x)^{10}$, $(1 + x)^{20}$ and $(1 + x)^{30}$. Then $\sum_{r=1}^{10} A_r(B_{10}B_r - C_{10}A_r)$ is equal to - **BT0101**

(A) $B_{10} - C_{10}$ (B) $A_{10}\left(B_{10}^2 - C_{10}A_{10}\right)$ (C) 0 (D) $C_{10} - B_{10}$

**[JEE 2010, 5]**

**2.** The coefficients of three consecutive terms of $(1 + x)^{n+5}$ are in the ratio 5 : 10 : 14. Then n = **BT0102**

**[JEE (Advanced) 2013, 4M, –1M]**

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**3.** Coefficient of $x^{11}$ in the expansion of $(1 + x^2)^4(1 + x^3)^7(1 + x^4)^{12}$ is -

(A) 1051 (B) 1106 (C) 1113 (D) 1120 **BT0103**

**[JEE(Advanced)-2014, 3(–1)]**

**4.** The coefficient of $x^9$ in the expansion of $(1+x)(1+x^2)(1+x^3)...(1+x^{100})$ is **[JEE 2015, 4M, –0M]**

**BT0104**

**5.** Let m be the smallest positive integer such that the coefficient of $x^2$ in the expansion of $(1 + x)^2 + (1 + x)^3 + ..... + (1 + x)^{49} + (1 + mx)^{50}$ is $(3n + 1)\,{}^{51}C_3$ for some positive integer n. Then the value of n is **[JEE(Advanced)-2016, 3(0)]**

**BT0105**

**6.** Let $X=\left({}^{10}C_1\right)^2+2\left({}^{10}C_2\right)^2+3\left({}^{10}C_3\right)^2+...+10\left({}^{10}C_{10}\right)^2$, where ${}^{10}C_r, r \in \{1, 2, ..., 10\}$ denote binomial coefficients. Then, the value of $\frac{1}{1430}X$ is _____ . **[JEE(Advanced)-2018, 3(0)]**

**BT0106**

**7.** Suppose $\det\begin{bmatrix} \sum_{k=0}^{n} k & \sum_{k=0}^{n} {}^nC_k\, k^2 \\ \sum_{k=0}^{n} {}^nC_k\, k & \sum_{k=0}^{n} {}^nC_k\, 3^k \end{bmatrix} = 0$, holds for some positive integer n. Then $\sum_{k=0}^{n} \frac{{}^nC_k}{k+1}$ equals

**BT0107**

**[JEE(Advanced)-2019, 3(0)]**

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# ANSWER KEY

## EXERCISE (O-1)

**1.** C **2.** B **3.** A **4.** B **5.** C **6.** A **7.** B

**8.** D **9.** B **10.** A **11.** B

## EXERCISE (O-2)

**1.** B **2.** B **3.** D **4.** A,C,D **5.** A,B **6.** B,C,D **7.** A,B

## EXERCISE (S-1)

**1.** **(a)** r = 6 **(b)** r = 5 or 9 **2.** (i) $\frac{5}{12}$ (ii) $T_6 = 7$ **4.** $\frac{(2^{mn}-1)}{(2^n-1)(2^{mn})}$

**5.** **(i)** $T_7 = \frac{7.3^{13}}{2}$ **(ii)** $455 \times 3^{12}$ **6.** $\frac{17}{54}$ **7.** n = 2 or 3 or 4 **8.** 816

**10.** **(a)** $-1260 \, . \, a^2b^3c^4$ **; (b)** $-12600$ **11.** **(a)** 990 **(b)** 3660 **12.** ${}^nC_r(3^{n-r} - 2^{n-r})$

**13.** **(i)** $3^n$ **(ii)** 1

## EXERCISE (S-2)

**2.** $2^{10}$ **3.** 0 **4.** 500 **5.** – 22100 **6.** (a) $\binom{30}{15}$

## EXERCISE (JM)

**1.** 3 **2.** 1 **3.** 2 **4.** 3 **5.** 4 **6.** 3

**7.** **Bonus**

**Note :** In the problem 'number of terms should be 13 instead of 28', then (1) will be the answer

**8.** 1 **9.** 3 **10.** 4 **11.** 2 **12.** 3 **13.** 3 **14.** 1

**15.** 4 **16.** 1 **17.** 4 **18.** 4 **19.** 2 **20.** 1 **21.** 4

## EXERCISE (JA)

**1.** D **2.** 6 **3.** C **4.** 8 **5.** 5 **6.** 646 **7.** 6.20

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# Important Notes

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