# **SEQUENCE & SERIES**

#### 1. **DEFINITION**:

## **Sequence:**

A succession of terms  $a_1, a_2, a_3, a_4$ ...... formed according to some rule or law.

Examples are: 1, 4, 9, 16, 25

$$\frac{x}{1!}, \frac{x^2}{2!}, \frac{x^3}{3!}, \frac{x^4}{4!}, \dots$$

A finite sequence has a finite (i.e. limited) number of terms, as in the first example above. An infinite sequence has an unlimited number of terms, i.e. there is no last term, as in the second and third examples.

#### **Series:**

The indicated sum of the terms of a sequence. In the case of a finite sequence  $a_1$ ,  $a_2$ ,  $a_3$ ,.....,

 $a_n$  the corresponding series is  $a_1 + a_2 + a_3 + \dots + a_n = \sum_{k=1}^n a_k$ . This series has a finite or limited number of terms and is called a finite series.

## 2. ARITHMETIC PROGRESSION (A.P.):

A.P. is a sequence whose terms differ by a fixed number. This fixed number is called the common difference. If a is the first term & d the common difference, then A.P. can be written as

$$a, a + d, a + 2d, \dots, a + (n-1)d, \dots$$

- (a)  $n^{th}$  term of AP  $T_n = a + (n-1)d$ , where  $d = t_n t_{n-1}$
- (b) The sum of the first n terms :  $S_n = \frac{n}{2}[a+\ell] = \frac{n}{2}[2a+(n-1)d]$  where  $\ell$  is  $n^{th}$  term.

#### Note:

- (i) n<sup>th</sup> term of an A.P. is of the form An + B i.e. a linear expression in 'n', in such a case the coefficient of n is the common difference of the A.P. i.e. A.
- (ii) Sum of first 'n' terms of an A.P. is of the form An<sup>2</sup>+Bn i.e. a quadratic expression in 'n', in such case the common difference is twice the coefficient of n<sup>2</sup>. i.e. 2A
- (iii) Also n<sup>th</sup> term  $T_n = S_n S_{n-1}$

#### Illustration 1:

If (x + 1), 3x and (4x + 2) are first three terms of an A.P. then its  $5^{th}$  term is -

(A) 14

(B) 19

(C) 24

(D) 28

$$(x + 1)$$
,  $3x$ ,  $(4x + 2)$  are in AP

$$\Rightarrow 3x - (x+1) = (4x+2) - 3x \Rightarrow x = 3$$

$$\Rightarrow 3x - (x + 1) = (4x + 2) - 3x \qquad \Rightarrow x = 3$$

$$\therefore a = 4, d = 9 - 4 = 5 \qquad \Rightarrow T_5 = 4 + (4)5 = 24 \qquad \text{Ans. (C)}$$

#### Illustration 2:

The sum of first four terms of an A.P. is 56 and the sum of it's last four terms is 112. If its first term is 11 then find the number of terms in the A.P.

#### **Solution:**

$$a + a + d + a + 2d + a + 3d = 56$$

$$4a + 6d = 56$$

$$44 + 6d = 56$$

$$(as a = 11)$$

$$6d = 12$$

hence 
$$d = 2$$

Let total number of terms = n

Now sum of last four terms.

$$a + (n-1)d + a + (n-2)d + a + (n-3)d + a + (n-4)d = 112$$

$$\Rightarrow$$
 4a + (4n - 10)d = 112

$$4a + (4n - 10)d = 112$$
  $\Rightarrow$   $44 + (4n - 10)2 = 112$ 

$$\Rightarrow$$
 4n - 10 = 34

$$\Rightarrow$$
 n = 11

Ans.

#### Illustration 3:

The sum of first n terms of two A.Ps. are in ratio  $\frac{7n+1}{4n+27}$ . Find the ratio of their 11<sup>th</sup> terms.

#### **Solution:**

Let a<sub>1</sub> and a<sub>2</sub> be the first terms and d<sub>1</sub> and d<sub>2</sub> be the common differences of two A.P.s respectively then

$$\frac{\frac{n}{2}[2a_1 + (n-1)d_1]}{\frac{n}{2}[2a_2 + (n-1)d_2]} = \frac{7n+1}{4n+27} \Rightarrow \frac{a_1 + \left(\frac{n-1}{2}\right)d_1}{a_2 + \left(\frac{n-1}{2}\right)d_2} = \frac{7n+1}{4n+27}$$

For ratio of 11<sup>th</sup> terms

$$\frac{n-1}{2} = 10 \implies n = 21$$

so ratio of 11<sup>th</sup> terms is 
$$\frac{7(21)+1}{4(21)+27} = \frac{148}{111} = \frac{4}{3}$$

Ans.

## Do yourself - 1:

- Write down the sequence whose n<sup>th</sup> terms is: (a)  $\frac{2^n}{n}$  (b)  $\frac{3+(-1)^n}{3^n}$ **(i)**
- (ii) For an A.P, show that  $t_m + t_{2n+m} = 2t_{m+n}$
- (iii) If the sum of p terms of an A.P. is q and the sum of its q terms is p, then find the sum of its (p+q) term.

#### 3. PROPERTIES OF A.P.:

- (a) If each term of an A.P. is increased, decreased, multiplied or divided by the some nonzero number, then the resulting sequence is also an A.P.
- (b) Three numbers in A.P. : a d, a, a + d

Four numbers in A.P. : a - 3d, a - d, a + d, a + 3d

Five numbers in A.P. : a-2d, a-d, a, a+d, a+2d

Six numbers in A.P. : a - 5d, a - 3d, a - d, a + d, a + 3d, a + 5d etc.

- (c) The common difference can be zero, positive or negative.
- (d)  $k^{th}$  term from the last =  $(n k + 1)^{th}$  term from the beginning (If total number of terms = n).
- (e) The sum of the two terms of an AP equidistant from the beginning & end is constant and equal to the sum of first & last terms.  $\Rightarrow T_k + T_{n-k+1} = \text{constant} = a + \ell$ .
- (f) Any term of an AP (except the first) is equal to half the sum of terms which are equidistant from it.  $a_n = (1/2)(a_{n-k} + a_{n+k})$ , k < n

For k = 1,  $a_n = (1/2)(a_{n-1} + a_{n+1})$ ; For k = 2,  $a_n = (1/2)(a_{n-2} + a_{n+2})$  and so on.

(g) If a, b, c are in AP, then 2b = a + c.

#### Illustration 4:

Four numbers are in A.P. If their sum is 20 and the sum of their squares is 120, then the middle terms are -

#### Solution:

Let the numbers are a - 3d, a - d, a + d, a + 3d

given, 
$$a - 3d + a - d + a + d + a + 3d = 20$$
  $\Rightarrow$   $4a = 20 \Rightarrow a = 5$   
and  $(a - 3d)^2 + (a - d)^2 + (a + d)^2 + (a + 3d)^2 = 120$   $\Rightarrow$   $4a^2 + 20d^2 = 120$   
 $\Rightarrow$   $4 \times 5^2 + 20d^2 = 120$   $\Rightarrow$   $d^2 = 1 \Rightarrow d = \pm 1$ 

Hence numbers are 2, 4, 6, 8 or 8, 6, 4, 2

Ans. (B)

#### Illustration 5:

If  $a_1$ ,  $a_2$ ,  $a_3$ ,..., $a_n$  are in A.P. where  $a_i > 0$  for all i, show that :

$$\frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}} = \frac{(n-1)}{\sqrt{a_1} + \sqrt{a_n}}$$

#### **Solution:**

L.H.S. 
$$= \frac{1}{\sqrt{a_1} + \sqrt{a_2}} + \frac{1}{\sqrt{a_2} + \sqrt{a_3}} + \dots + \frac{1}{\sqrt{a_{n-1}} + \sqrt{a_n}}$$

$$= \frac{1}{\sqrt{a_2} + \sqrt{a_1}} + \frac{1}{\sqrt{a_3} + \sqrt{a_2}} + \dots + \frac{1}{\sqrt{a_n} + \sqrt{a_{n-1}}}$$

$$= \frac{\sqrt{a_2} - \sqrt{a_1}}{(a_2 - a_1)} + \frac{\sqrt{a_3} - \sqrt{a_2}}{(a_3 - a_2)} + \dots + \frac{\sqrt{a_n} - \sqrt{a_{n-1}}}{a_n - a_{n-1}}$$

Let 'd' is the common difference of this A.P.

then 
$$a_2 - a_1 = a_3 - a_2 = \dots = a_n - a_{n-1} = d$$

Now L.H.S.

$$= \frac{1}{d} \left\{ \sqrt{a_2} - \sqrt{a_1} + \sqrt{a_3} - \sqrt{a_2} + \dots + \sqrt{a_{n-1}} - \sqrt{a_{n-2}} + \sqrt{a_n} - \sqrt{a_{n-1}} \right\} = \frac{1}{d} \left\{ \sqrt{a_n} - \sqrt{a_1} \right\}$$

$$= \frac{a_n - a_1}{a_1 - a_1} - \frac{a_1 + (n-1)d - a_1}{a_1 - a_1} - \frac{1}{a_1 - a_1} - \frac{n-1}{a_1 - a_1} - \frac{n-1}{$$

$$= \frac{a_n - a_1}{d\left(\sqrt{a_n} + \sqrt{a_1}\right)} = \frac{a_1 + (n-1)d - a_1}{d\left(\sqrt{a_n} + \sqrt{a_1}\right)} = \frac{1}{d} \frac{(n-1)d}{(\sqrt{a_n} + \sqrt{a_1})} = \frac{n-1}{\sqrt{a_n} + \sqrt{a_1}} = \text{R.H.S.}$$

## Do yourself - 2:

- (i) Find the sum of first 24 terms of the A.P.  $a_1$ ,  $a_2$ ,  $a_3$ ....., if it is know that  $a_1 + a_5 + a_{10} + a_{15} + a_{20} + a_{24} = 225$ .
- (ii) Find the number of terms common to the two A.P.'s 3, 7, 11, ..... 407 and 2, 9, 16, ....., 709

## 4. GEOMETRIC PROGRESSION (G.P.):

G.P. is a sequence of non zero numbers each of the succeeding term is equal to the preceding term multiplied by a constant. Thus in a GP the ratio of successive terms is constant. This constant factor is called the COMMON RATIO of the sequence & is obtained by dividing any term by the immediately previous term. Therefore a, ar, ar<sup>2</sup>, ar<sup>3</sup>, ar<sup>4</sup>, ...... is a GP with 'a' as the first term & 'r' as common ratio.

- (a)  $n^{th}$  term;  $T_n = a r^{n-1}$
- (b) Sum of the first n terms;  $S_n = \frac{a(r^n 1)}{r 1}$ , if  $r \ne 1$
- (c) Sum of infinite G.P.,  $S_{\infty} = \frac{a}{1-r}$ ; 0 < |r| < 1

#### 5. PROPERTIES OF GP:

- (a) If each term of a G.P. be multiplied or divided by the some non-zero quantity, then the resulting sequence is also a G.P.
- (b) Three consecutive terms of a GP: a/r, a, ar; Four consecutive terms of a GP: a/r³, a/r, ar, ar³ & so on.
- (c) If a, b, c are in G.P. then  $b^2 = ac$ .
- (d) If in a G.P, the product of two terms which are equidistant from the first and the last term, is constant and is equal to the product of first and last term.  $\Rightarrow T_k$ .  $T_{n-k+1} = \text{constant} = \text{a.} \ell$
- (e) If each term of a G.P. be raised to the same power, then resulting sequence is also a G.P.
- (f) In a G.P.,  $T_r^2 = T_{r-k}$ .  $T_{r+k}$ , k < r,  $r \ne 1$

- (g) If the terms of a given G.P. are chosen at regular intervals, then the new sequence is also a G.P.
- (h) If  $a_1$ ,  $a_2$ ,  $a_3$ ..... $a_n$  is a G.P. of positive terms, then  $\log a_1$ ,  $\log a_2$ ,..... $\log a_n$  is an A.P. and vice-versa
- (i) If  $a_1$ ,  $a_2$ ,  $a_3$ .... and  $b_1$ ,  $b_2$ ,  $b_3$ .... are two G.P.'s then  $a_1b_1$ ,  $a_2b_2$ ,  $a_3b_3$ .... &  $\frac{a_1}{b_1}$ ,  $\frac{a_2}{b_2}$ ,  $\frac{a_3}{b_3}$ .... is also in G.P.

## Illustration 6:

If a, b, c, d and p are distinct real numbers such that

$$(a^2 + b^2 + c^2)p^2 - 2p(ab + bc + cd) + (b^2 + c^2 + d^2) \le 0$$
 then a, b, c, d are in

- (A) A.P.
- (B) G.P
- (C) H.P.
- (D) none of these

#### Solution:

Here, the given condition  $\left(a^2+b^2+c^2\right)p^2-2p\left(ab+bc+ca\right)+b^2+c^2+d^2\leq 0$ 

$$\Rightarrow (ap-b)^{2} + (bp-c)^{2} + (cp-d)^{2} \le 0$$

: a square can not be negative

$$\therefore \quad ap - b = 0, bp - c = 0, cp - d = 0 \implies p = \frac{b}{a} = \frac{c}{b} = \frac{d}{c} \implies a, b, c, d \text{ are in G.P.}$$

Ans. (B)

## Illustration 7:

If a, b, c are in G.P., then the equations  $ax^2 + 2bx + c = 0$  and  $dx^2 + 2ex + f = 0$  have a common root

if 
$$\frac{d}{a}$$
,  $\frac{e}{b}$ ,  $\frac{f}{c}$  are in -

- (A) A.P.
- (B) G.P.
- (C) H.P.
- (D) none of these

#### Solution:

a. b. c are in G.P  $\Rightarrow$   $b^2 = ac$ 

Now the equation  $ax^2 + 2bx + c = 0$  can be rewritten as  $ax^2 + 2\sqrt{ac}x + c = 0$ 

$$\Rightarrow \left(\sqrt{a}x + \sqrt{c}\right)^2 = 0 \Rightarrow x = -\sqrt{\frac{c}{a}}, -\sqrt{\frac{c}{a}}$$

If the two given equations have a common root, then this root must be  $-\sqrt{\frac{c}{a}}$ .

Thus 
$$d\frac{c}{a} - 2e\sqrt{\frac{c}{a}} + f = 0 \implies \frac{d}{a} + \frac{f}{c} = \frac{2e}{c}\sqrt{\frac{c}{a}} = \frac{2e}{\sqrt{ac}} = \frac{2e}{b} \implies \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.}$$
 Ans. (A)

#### Illustration 8:

A number consists of three digits which are in G.P. the sum of the right hand and left hand digits exceeds twice the middle digit by 1 and the sum of the left hand and middle digits is two third of the sum of the middle and right hand digits. Find the numbers.

#### Solution:

Let the three digits be a, ar and ar<sup>2</sup> then number is

$$100a + 10ar + ar^2$$
 ....(i)

Given, 
$$a + ar^2 = 2ar + 1$$

or 
$$a(r^2 - 2r + 1) = 1$$

or 
$$a(r-1)^2 = 1$$
 ....(ii

Also given 
$$a + ar = \frac{2}{3} (ar + ar^2)$$

$$\Rightarrow$$
 3 + 3r = 2r + 2r<sup>2</sup>  $\Rightarrow$  2r<sup>2</sup> - r - 3 = 0  $\Rightarrow$  (r + 1)(2r - 3) = 0

$$\therefore$$
 r = -1, 3/2

for 
$$r = -1$$
,  $a = \frac{1}{(r-1)^2} = \frac{1}{4} \notin I$   $\therefore r \neq -1$ 

for 
$$r = 3/2$$
,  $a = \frac{1}{\left(\frac{3}{2} - 1\right)^2} = 4$  {from (ii)}

From (i), number is 
$$400 + 10.4 \cdot \frac{3}{2} + 4 \cdot \frac{9}{4} = 469$$

Ans.

#### Illustration 9:

Find the value of  $0.32\overline{58}$ 

#### Solution:

Let 
$$R = 0.32\overline{58}$$
  $\Rightarrow R = 0.32585858...$  (i)

Here number of figures which are not recurring is 2 and number of figures which are recurring is also 2.

then 
$$100 R = 32.585858...$$
 (ii)

and 
$$10000 R = 3258.5858...$$
 .....(iii)

Subtracting (ii) from (iii), we get

9900 R = 3226 
$$\Rightarrow$$
 R =  $\frac{1613}{4950}$ 

**Aliter Method :** R = .32 + .0058 + .000058 + .000058 + ...

$$= .32 + \frac{58}{10^4} \left( 1 + \frac{1}{10^2} + \frac{1}{10^4} + \dots \infty \right)$$

$$=.32 + \frac{58}{10^4} \left( \frac{1}{1 - \frac{1}{100}} \right)$$

$$= \frac{32}{100} + \frac{58}{9900} = \frac{3168 + 58}{9900} = \frac{3226}{9900} = \frac{1613}{4950}$$

## Do yourself - 3:

- (i) Find a three digit number whose consecutive digits form a G.P. If we subtract 792 from this number, we get a number consisting of the same digits written in the reverse order. Now, if we increase the second digit of the required number by 2, then the resulting digits will form an A.P.
- (ii) If the third term of G.P. is 4, then find the product of first five terms.
- (iii) If a, b, c are respectively the  $p^{th}$ ,  $q^{th}$  and  $r^{th}$  terms of the given G.P., then show that  $(q-r) \log a + (r-p) \log b + (p-q) \log c = 0$ , where a, b, c > 0.
- (iv) Find three numbers in G.P., whose sum is 52 and the sum of whose products in pairs is 624.
- (v) The rational number which equals the number  $2.\overline{357}$  with recurring decimal is -

(A) 
$$\frac{2357}{999}$$

(B) 
$$\frac{2379}{997}$$

(C) 
$$\frac{785}{333}$$

(D) 
$$\frac{2355}{1001}$$

#### 6. HARMONIC PROGRESSION (H.P.):

A sequence is said to be in H.P. if the reciprocal of its terms are in AP.

If the sequence  $a_1$ ,  $a_2$ ,  $a_3$ , ......,  $a_n$  is an HP then  $1/a_1$ ,  $1/a_2$ ,.....,  $1/a_n$  is an AP. Here we do not have the formula for the sum of the n terms of an HP. The general form of a harmonic progression is

$$\frac{1}{a}, \frac{1}{a+d}, \frac{1}{a+2d}, \dots \frac{1}{a+(n-1)d}$$

**Note:** No term of any H.P. can be zero.

(i) If a, b, c are in HP, then 
$$b = \frac{2ac}{a+c}$$
 or  $\frac{a}{c} = \frac{a-b}{b-c}$ 

#### Illustration 10:

The sum of three numbers are in H.P. is 37 and the sum of their reciprocals is 1/4. Find the numbers. *Solution*:

Three numbers are in H.P. can be taken as

then 
$$\frac{1}{a-d}, \frac{1}{a}, \frac{1}{a+d}$$

$$\frac{1}{a-d} + \frac{1}{a} + \frac{1}{a+d} = 37 \qquad ......(i)$$
and 
$$a - d + a + a + d = \frac{1}{4} \implies a = \frac{1}{12}$$
from (i), 
$$\frac{12}{1-12d} + 12 + \frac{12}{1+12d} = 37 \implies \frac{12}{1-12d} + \frac{12}{1+12d} = 25$$

$$\implies \frac{24}{1-144d^2} = 25 \implies 1-144d^2 = \frac{24}{25} \implies d^2 = \frac{1}{25 \times 144}$$

$$\therefore d = \pm \frac{1}{60}$$

$$\therefore$$
 a - d, a, a + d are  $\frac{1}{15}$ ,  $\frac{1}{12}$ ,  $\frac{1}{10}$  or  $\frac{1}{10}$ ,  $\frac{1}{12}$ ,  $\frac{1}{15}$ 

Hence, three numbers in H.P. are 15, 12, 10 or 10, 12, 15

Ans.

#### Illustration 11:

Suppose a is a fixed real number such that  $\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz}$ 

If p, q, r are in A.P., then prove that x, y, z are in H.P.

#### Solution:

$$\therefore$$
 p, q, r are in A.P.

$$\therefore \qquad q - p = r - q \qquad \qquad \dots \dots (i)$$

$$\Rightarrow$$
  $p-q=q-r=k$  (let)

given 
$$\frac{a-x}{px} = \frac{a-y}{qy} = \frac{a-z}{rz}$$
  $\Rightarrow$   $\frac{\frac{a}{x}-1}{p} = \frac{\frac{a}{y}-1}{q} = \frac{\frac{a}{z}-1}{r}$ 

$$\Rightarrow \frac{\left(\frac{a}{x}-1\right)-\left(\frac{a}{y}-1\right)}{p-q} = \frac{\left(\frac{a}{y}-1\right)-\left(\frac{a}{z}-1\right)}{q-r} \text{ (by law of proportion)}$$

$$\Rightarrow \frac{\frac{a}{x} - \frac{a}{y}}{\frac{b}{x}} = \frac{\frac{a}{y} - \frac{a}{z}}{\frac{b}{z}}$$
 {from (i)}

$$\Rightarrow a\left(\frac{1}{x} - \frac{1}{y}\right) = a\left(\frac{1}{y} - \frac{1}{z}\right) \Rightarrow \frac{1}{x} - \frac{1}{y} = \frac{1}{y} - \frac{1}{z}$$

$$\therefore \frac{2}{y} = \frac{1}{x} + \frac{1}{z}$$

$$\therefore \frac{1}{x}, \frac{1}{y}, \frac{1}{z}$$
 are in A.P.

Hence x, y, z are in H.P.

## Do yourself - 4:

- (i) If the 7<sup>th</sup> term of a H.P. is 8 and the 8<sup>th</sup> term is 7. Then find the 28<sup>th</sup> term.
- (ii) In a H.P., if  $5^{th}$  term is 6 and  $3^{rd}$  term is 10. Find the  $2^{nd}$  term.
- (iii) If the pth, qth and rth terms of a H.P. are a,b,c respectively, then prove that  $\frac{q-r}{a} + \frac{r-p}{b} + \frac{p-q}{c} = 0$

#### 7. MEANS

#### (a) ARITHMETIC MEAN:

If three terms are in A.P. then the middle term is called the A.M. between the other two, so if a, b, c are in A.P., b is A.M. of a & c. So A.M. of a and  $c = \frac{a+c}{2} = b$ .

#### n-ARITHMETIC MEANS BETWEEN TWO NUMBERS:

If a,b be any two given numbers & a,  $A_1$ ,  $A_2$ , ......,  $A_n$ , b are in AP, then  $A_1$ ,  $A_2$ ,..........A<sub>n</sub> are the 'n' A.M's between a & b then.  $A_1 = a + d$ ,  $A_2 = a + 2d$ ,.....,  $A_n = a + nd$  or b - d, where

$$d = \frac{b - a}{n + 1}$$

$$\Rightarrow A_1 = a + \frac{b-a}{n+1}, A_2 = a + \frac{2(b-a)}{n+1},....$$

**Note:** Sum of n A.M's inserted between a & b is equal to n times the single A.M. between a & b

i.e.  $\sum_{r=1}^{n} A_r = nA$  where A is the single A.M. between a & b.

#### (b) **GEOMETRIC MEAN:**

If a, b, c are in G.P., then b is the G.M. between a & c,  $b^2 = ac$ . So G.M. of a and  $c = \sqrt{ac} = b$  n-GEOMETRIC MEANS BETWEEN TWO NUMBERS:

If a, b are two given positive numbers & a,  $G_1$ ,  $G_2$ , ....,  $G_n$ , b are in G.P. Then  $G_1$ ,  $G_2$ ,  $G_3$ , ....,  $G_n$  are 'n' G.Ms between a & b. where  $b = ar^{n+1} \Rightarrow r = (b/a)^{1/n+1}$ 

$$G_1 = a(b/a)^{1/n+1},$$
  $G_2 = a(b/a)^{2/n+1}...$   $G_n = a(b/a)^{n/n+1}$   
= ar, = ar<sup>2</sup>, ..... = ar<sup>n</sup> = b/r

**Note:** The product of n G.Ms between a & b is equal to n<sup>th</sup> power of the single G.M. between a & b i.e.  $\prod_{r=1}^{n} G_r = (G)^n$  where G is the single G.M. between a & b

#### (c) HARMONIC MEAN:

If a, b, c are in H.P., then b is H.M. between a & c. So H.M. of a and  $c = \frac{2ac}{a+c} = b$ .

Insertion of 'n' HM's between a and b:

$$a,\ H_{\scriptscriptstyle 1},\ H_{\scriptscriptstyle 2},\ H_{\scriptscriptstyle 3}\ ,......,H_{\scriptscriptstyle n},\ b\to H.P$$

$$\frac{1}{a}, \frac{1}{H_1}, \frac{1}{H_2}, \frac{1}{H_3}, \dots, \frac{1}{H_n}, \frac{1}{b} \to A.P.$$

$$\frac{1}{b} = \frac{1}{a} + (n+1)D \Rightarrow D = \frac{\frac{1}{b} - \frac{1}{a}}{n+1}$$

$$\frac{1}{H_n} = \frac{1}{a} + n \left( \frac{\frac{1}{b} - \frac{1}{a}}{n+1} \right)$$

## **Important note:**

- (i) If A, G, H, are respectively A.M., G.M., H.M. between two positive number a & b then (a)  $G^2 = AH(A, G, H \text{ constitute a GP})$  (b)  $A \ge G \ge H$  (c)  $A = G = H \Leftrightarrow a = b$
- (ii) Let  $a_1, a_2, \dots, a_n$  be n positive real numbers, then we define their arithmetic mean (A), geometric mean (G) and harmonic mean (H) as  $A = \frac{a_1 + a_2 + \dots + a_n}{n}$

G = 
$$(a_1 a_2....a_n)^{1/n}$$
 and H =  $\frac{n}{\left(\frac{1}{a_1} + \frac{1}{a_2} + \frac{1}{a_3} + .... + \frac{1}{a_n}\right)}$ 

It can be shown that  $A \ge G \ge H$ . Moreover equality holds at either place if and only if  $a_1 = a_2 = \dots = a_n$ 

#### Illustration 12:

If  $2x^3 + ax^2 + bx + 4 = 0$  (a and b are positive real numbers) has 3 real roots, then prove that  $a + b \ge 6(2^{1/3} + 4^{1/3})$ .

#### **Solution:**

Let  $\alpha$ ,  $\beta$ ,  $\gamma$  be the roots of  $2x^3 + ax^2 + bx + 4 = 0$ . Given that all the coefficients are positive, so all the roots will be negative.

Let 
$$\alpha_1 = -\alpha$$
,  $\alpha_2 = -\beta$ ,  $\alpha_3 = -\gamma$   $\Rightarrow$   $\alpha_1 + \alpha_2 + \alpha_3 = \frac{a}{2}$ 

$$\alpha_1 \alpha_2 + \alpha_2 \alpha_3 + \alpha_3 \alpha_1 = \frac{b}{2}$$

$$\alpha_1 \alpha_2 \alpha_3 = 2$$

Applying  $AM \ge GM$ , we have

$$\frac{\alpha_1 + \alpha_2 + \alpha_3}{3} \ge (\alpha_1 \alpha_2 \alpha_3)^{1/3} \quad \Rightarrow \quad a \ge 6 \times 2^{1/3}$$

Also 
$$\frac{\alpha_1\alpha_2 + \alpha_2\alpha_3 + \alpha_1\alpha_3}{3} > (\alpha_1\alpha_2\alpha_3)^{2/3}$$
  $\Rightarrow$   $b \ge 6 \times 4^{1/3}$ 

Therefore  $a + b \ge 6(2^{1/3} + 4^{1/3})$ .

#### Illustration 13:

If  $a_i > 0 \ \forall \ i \in N$  such that  $\prod_{i=1}^n a_i = 1$ , then prove that  $(1+a_1)(1+a_2)(1+a_3)....(1+a_n) \ge 2^n$ 

#### **Solution:**

Using A.M.  $\geq$  G.M.

$$1 + a_1 \ge 2\sqrt{a_1}$$
  
 $1 + a_1 \ge 2\sqrt{a_1}$ 

$$1 + a_2 \ge 2\sqrt{a_2}$$

:

$$1 + a_n \ge 2\sqrt{a_n} \ \Rightarrow \ (1 + a_1)(1 + a_2).....(1 + a_n) \ge 2^n(a_1a_2a_3....a_n)^{1/2}$$

As 
$$a_1 a_2 a_3 \dots a_n = 1$$

Hence  $(1 + a_1)(1 + a_2)....(1 + a_n) \ge 2^n$ .



#### Illustration 14:

If a, b, x, y are positive natural numbers such that  $\frac{1}{x} + \frac{1}{y} = 1$  then prove that  $\frac{a^x}{x} + \frac{b^y}{y} \ge ab$ .

#### **Solution:**

Consider the positive numbers  $a^x$ ,  $a^x$ ,.....y times and  $b^y$ ,  $b^y$ ,....x times

For all these numbers,

$$AM = \frac{\{a^{x} + a^{x} + \dots y \text{ time}\} + \{b^{y} + b^{y} + \dots x \text{ times}\}}{x + y} = \frac{ya^{x} + xa^{y}}{(x + y)}$$

$$GM = \left\{ \left( a^{x}.a^{x}.....y \text{ times} \right) \left( b^{y}.b^{y}.....x \text{ times} \right) \right\}^{\frac{1}{(x+y)}} = \left[ \left( a^{xy} \right). \left( b^{xy} \right) \right]^{\frac{1}{(x+y)}} = (ab)^{\frac{xy}{(x+y)}}$$

As 
$$\frac{1}{x} + \frac{1}{y} = 1$$
,  $\frac{x+y}{xy} = 1$ , i.e,  $x + y = xy$ 

So using AM 
$$\geq$$
 GM  $\frac{ya^x + xa^y}{x + y} \geq (ab)^{\frac{xy}{x+y}}$ 

$$\therefore \frac{ya^x + xa^y}{xy} \ge ab \quad \text{or} \quad \frac{a^x}{x} + \frac{a^y}{y} \ge ab.$$

## Do yourself - 5:

- (i) If  $\frac{a^n + b^n}{a^{n-1} + b^{n-1}}$  is the G.M. between a & b then find the value of 'n'.
- (ii) If b is the harmonic mean between a and c, then prove that  $\frac{1}{b-a} + \frac{1}{b-c} = \frac{1}{a} + \frac{1}{c}$ .

#### 8. ARITHMETICO - GEOMETRIC SERIES:

A series, each term of which is formed by multiplying the corresponding term of an A.P. & G.P. is called the Arithmetico-Geometric Series, e.g.  $1 + 3x + 5x^2 + 7x^3 + \dots$ 

Here 1, 3, 5, ...... are in A.P. & 1, x,  $x^2$ ,  $x^3$  ..... are in G.P.

(a) SUM OF N TERMS OF AN ARITHMETICO-GEOMETRIC SERIES:

Let 
$$S_n = a + (a+d)r + (a+2d)r^2 + \dots + [a+(n-1)d]r^{n-1}$$

then 
$$S_n = \frac{a}{1-r} + \frac{dr(1-r^{n-1})}{(1-r)^2} - \frac{[a+(n-1)d]}{1-r}, r \neq 1$$

(b) SUM TO INFINITY:

If 
$$0 < |r| < 1$$
 &  $n \to \infty$ , then  $\lim_{n \to \infty} r^n = 0$ ,  $S_{\infty} = \frac{a}{1-r} + \frac{dr}{(1-r)^2}$ 

Illustration 15:

Find the sum of series  $4 - 9x + 16x^2 - 25x^3 + 36x^4 - 49x^5 + \dots \infty$ .

Solution:

Let 
$$S = 4 - 9x + 16x^2 - 25x^3 + 36x^4 - 49x^5 + \dots \infty$$
  
-  $Sx = -4x + 9x^2 - 16x^3 + 25x^4 - 36x^5 + \dots \infty$ 

ALLEN

On subtraction, we get

$$S(1 + x) = 4 - 5x + 7x^2 - 9x^3 + 11x^4 - 13x^5 + \dots \infty$$

$$-S(1+x)x = -4x + 5x^2 - 7x^3 + 9x^4 - 11x^5 + \dots \infty$$

On subtraction, we get

$$S(1 + x)^2 = 4 - x + 2x^2 - 2x^3 + 2x^4 - 2x^5 + \dots \infty$$

$$=4-x+2x^{2}(1-x+x^{2}-...\infty)=4-x+\frac{2x^{2}}{1+x}=\frac{4+3x+x^{2}}{1+x}$$

$$S = \frac{4 + 3x + x^2}{(1 + x)^3}$$
 Ans.

#### Illustration 16:

Find the sum of series upto n terms  $\left(\frac{2n+1}{2n-1}\right)+3\left(\frac{2n+1}{2n-1}\right)^2+5\left(\frac{2n+1}{2n-1}\right)^3+\dots$ 

#### **Solution:**

For  $x \neq 1$ , let

$$S = x + 3x^{2} + 5x^{3} + \dots + (2n - 3)x^{n-1} + (2n - 1)x^{n} \qquad \dots \dots (i)$$

$$\Rightarrow$$
  $xS = x^2 + 3x^3 + \dots + (2n-5)x^{n-1} + (2n-3)x^n + (2n-1)x^{n+1} \dots$  (ii)

Subtracting (ii) from (i), we get

$$(1-x)S = x + 2x^2 + 2x^3 + \dots + 2x^{n-1} + 2x^n - (2n-1)x^{n+1} = x + \frac{2x^2(1-x^{n-1})}{1-x} - (2n-1)x^{n+1}$$

$$=\frac{x}{1-x}\left[1-x+2x-2x^{n}-(2n-1)x^{n}+(2n-1)x^{n+1}\right]$$

$$\Rightarrow S = \frac{x}{(1-x)^2} [(2n-1)x^{n+1} - (2n+1)x^n + 1 + x]$$

Thus 
$$\left(\frac{2n+1}{2n-1}\right) + 3\left(\frac{2n+1}{2n-1}\right)^2 + \dots + (2n-1)\left(\frac{2n+1}{2n-1}\right)^n$$

$$= \left(\frac{2n+1}{2n-1}\right) \left(\frac{2n-1}{2}\right)^2 \left[ (2n-1) \left(\frac{2n+1}{2n-1}\right)^{n+1} - (2n+1) \left(\frac{2n+1}{2n-1}\right)^n + 1 + \frac{2n+1}{2n-1} \right]$$

$$= \frac{4n^2 - 1}{4} \cdot \frac{4n}{2n - 1} = n(2n + 1)$$

Ans.

## Do yourself - 6:

- (i) Find sum to n terms of the series  $3+5\times\frac{1}{4}+7\times\frac{1}{4^2}+\dots$
- (ii) If the sum to the infinity of the series  $3 + 5r + 7r^2 + \dots$  is  $\frac{44}{9}$ , then find the value of r.
- (iii) If the sum to infinity of the series  $3+(3+d).\frac{1}{4}+(3+2d).\frac{1}{4^2}+\dots$  is  $\frac{44}{9}$  then find d.

## 9. SIGMA NOTATIONS ( $\Sigma$ )

#### **THEOREMS:**

(a) 
$$\sum_{r=1}^{n} (a_r \pm b_r) = \sum_{r=1}^{n} a_r \pm \sum_{r=1}^{n} b_r$$
 (b)  $\sum_{r=1}^{n} k a_r = k \sum_{r=1}^{n} a_r$  (c)  $\sum_{r=1}^{n} k = nk$ ; where k is a constant.

#### 10. RESULTS

(a) 
$$\sum_{r=1}^{n} r = \frac{n(n+1)}{2}$$
 (sum of the first n natural numbers)

**(b)** 
$$\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$$
 (sum of the squares of the first n natural numbers)

(c) 
$$\sum_{r=1}^{n} r^{3} = \frac{n^{2}(n+1)^{2}}{4} = \left[\sum_{r=1}^{n} r\right]^{2} \text{ (sum of the cubes of the first n natural numbers)}$$

(d) 
$$\sum_{r=1}^{n} r^4 = \frac{n}{30} (n+1)(2n+1)(3n^2+3n-1)$$

(e) 
$$\sum_{r=1}^{n} (2r-1) = n^2$$
 (sum of first n odd natural numbers)

(f) 
$$\sum_{r=1}^{n} 2r = n(n+1)$$
 (sum of first n even natural numbers)

#### Note:

If  $n^{th}$  term of a sequence is given by  $T_n = an^3 + bn^2 + cn + d$  where a, b, c, d are constants, then sum of n terms  $S_n = \Sigma T_n = a\Sigma n^3 + b\Sigma n^2 + c\Sigma n + \Sigma d$ 

This can be evaluated using the above results.

#### Illustration 17:

Sum up to 16 terms of the series  $\frac{1^3}{1} + \frac{1^3 + 2^3}{1+3} + \frac{1^3 + 2^3 + 3^3}{1+3+5} + \dots$  is

(D) none of these

#### Solution:

$$t_{n} = \frac{1^{3} + 2^{3} + 3^{3} + \dots + n^{3}}{1 + 3 + 5 + \dots (2n - 1)} = \frac{\left\{\frac{n(n + 1)}{2}\right\}^{2}}{\frac{n}{2}\left\{2 + 2(n - 1)\right\}} = \frac{\frac{n^{2}(n + 1)^{2}}{4}}{n^{2}} = \frac{(n + 1)^{2}}{4} = \frac{n^{2}}{4} + \frac{n}{2} + \frac{1}{4}$$

$$S_{n} = \Sigma t_{n} = \frac{1}{4} \Sigma n^{2} + \frac{1}{2} \Sigma n + \frac{1}{4} \Sigma 1 = \frac{1}{4} \cdot \frac{n(n+1)(2n+1)}{6} + \frac{1}{2} \cdot \frac{n(n+1)}{2} + \frac{1}{4} \cdot n$$

$$S_{16} = \frac{16.17.33}{24} + \frac{16.17}{4} + \frac{16}{4} = 446$$

**Ans.** (C)

## ALLEN

#### 11. METHOD OF DIFFERENCE:

Some times the n<sup>th</sup> term of a sequence or a series can not be determined by the method, we have discussed earlier. So we compute the difference between the successive terms of given sequence for obtained the n<sup>th</sup> terms.

If  $T_1$ ,  $T_2$ ,  $T_3$ ,....., $T_n$  are the terms of a sequence then some times the terms  $T_2 - T_1$ ,  $T_3 - T_2$ ,....... constitute an AP/GP.  $n^{th}$  term of the series is determined & the sum to n terms of the sequence can easily be obtained.

#### Case 1:

- (a) If difference series are in A.P., then Let  $T_n = an^2 + bn + c$ , where a, b, c are constant
- (b) If difference of difference series are in A.P. Let  $T_n = an^3 + bn^2 + cn + d$ , where a, b, c, d are constant

#### Case 2:

- (a) If difference are in G.P., then Let  $T_n = ar^n + b$ , where r is common ratio & a, b are constant
- (b) If difference of difference are in G.P., then

  Let  $T_n = ar^n + bn + c$ , where r is common ratio & a, b, c are constant

  Determine constant by putting  $n = 1, 2, 3 \dots n$  and putting the value of  $T_1, T_2, T_3 \dots n$

and sum of series  $(S_n) = \sum T_n$ 

## Do yourself - 7:

- (i) Find the sum of the series upto n terms  $1 + \frac{1+2}{2} + \frac{1+2+3}{3} + \frac{1+2+3+4}{4} + \dots$
- (ii) Find the sum of 'n' terms of the series whose  $n^{th}$  term is  $t_n = 3n^2 + 2n$ .

#### Miscellaneous Illustration:

#### Illustration 18:

If 
$$\sum_{r=1}^{n} T_r = \frac{n}{8} (n+1)(n+2)(n+3)$$
, then find  $\sum_{r=1}^{n} \frac{1}{T_r}$ .

$$=\sum_{r=1}^{n}T_{r}-\sum_{r=1}^{n-1}T_{r}=\frac{n(n+1)(n+2)(n+3)}{8}-\frac{(n-1)n(n+1)(n+2)}{8}=\frac{n(n+1)(n+2)}{8}[(n+3)-(n-1)]$$

$$T_n = \frac{n(n+1)(n+2)}{8}(4) = \frac{n(n+1)(n+2)}{2}$$

$$\Rightarrow \frac{1}{T_n} = \frac{2}{n(n+1)(n+2)} = \frac{(n+2)-n}{n(n+1)(n+2)} = \frac{1}{n(n+1)} - \frac{1}{(n+1)(n+2)} \qquad .....(i)$$

Ans.

Let 
$$V_n = \frac{1}{n(n+1)}$$

$$\therefore \frac{1}{T_n} = V_n - V_{n+1}$$

Putting n = 1, 2, 3, .... n

$$\Rightarrow \frac{1}{T_1} + \frac{1}{T_2} + \frac{1}{T_3} + \dots + \frac{1}{T_n} = (V_1 - V_{n+1}) \Rightarrow \sum_{r=1}^{n} \frac{1}{T_r} = \frac{n^2 + 3n}{2(n+1)(n+2)}$$

## Illustration 19:

Find the sum of n terms of the series  $1.3.5 + 3.5.7 + 5.7.9 + \dots$ 

#### Solution:

The  $n^{th}$  term is (2n-1)(2n+1)(2n+3)

$$T_n = (2n-1)(2n+1)(2n+3)$$

$$T_{n} = \frac{1}{8} (2n-1) (2n+1) (2n+3) \{ (2n+5) - (2n-3) \}$$

$$= \frac{1}{8} (V_n - V_{n-1})$$
 [Let  $V_n = (2n-1)(2n+1)(2n+3)(2n+5)$ ]

$$S_n = \sum T_n = \frac{1}{8} [V_n - V_0]$$

$$S_n = \frac{(2n-1)(2n+1)(2n+3)(2n+5)}{8} + \frac{15}{8} = n(2n^3 + 8n^2 + 7n - 2)$$

#### Illustration 20:

Find the sum of n terms of the series  $3 + 7 + 14 + 24 + 37 + \dots$ 

#### Solution:

Clearly here the differences between the successive terms are

$$7-3$$
,  $14-7$ ,  $24-14$ , ...... i.e. 4, 7, 10, 13,..., which are in A.P.

Let 
$$S = 3 + 7 + 14 + 24 + \dots + T_n$$

$$S = 3 + 7 + 14 + \dots + T_{n-1} + T_n$$

Subtracting, we get

$$0 = 3 + [4 + 7 + 10 + 13 + \dots (n-1) \text{ terms}] - T_n$$

$$\therefore T_n = 3 + S_{n-1} \text{ of an A.P. whose } a = 4 \text{ and } d = 3.$$

$$T_n = 3 + \left(\frac{n-1}{2}\right)(2.4 + (n-2)3) = \frac{6 + (n-1)(3n+2)}{4} \quad \text{or, } T_n = \frac{1}{2}(3n^2 - n + 4)$$

Now putting  $n = 1, 2, 3, \dots, n$  and adding

$$S_n = \frac{1}{2} \left[ 3 \sum n^2 - \sum n + 4n \right] = \frac{1}{2} \left[ 3 \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 4n \right] = \frac{n}{2} (n^2 + n + 4)$$
 Ans.

## **Aliter Method:**

Let 
$$T_n = an^2 + bn + c$$

Now, 
$$T_1 = 3 = a + b + c$$
 .....(i)

$$T_2 = 7 = 4a + 2b + c$$
 .....(ii)

$$T_2 = 14 = 8a + 3b + c$$
 .....(iii)

Solving (i), (ii) & (iii) we get

$$a = \frac{3}{2}, b = -\frac{1}{2} \& c = 2$$

$$T_n = \frac{1}{2}(3n^2 - n + 4)$$

$$\Rightarrow s_n = \sum T_n = \frac{1}{2} \left[ 3 \sum n^2 - \sum n + 4n \right] = \frac{1}{2} \left[ 3 \frac{n(n+1)(2n+1)}{6} - \frac{n(n+1)}{2} + 4n \right] = \frac{n}{2} (n^2 + n + 4)$$
 Ans.

#### Illustration 21:

Find the sum of n-terms of the series  $1 + 4 + 10 + 22 + \dots$ 

#### Solution:

Let 
$$S = 1 + 4 + 10 + 22 + \dots + T_n$$
 ...... (i)

$$S = 1 + 4 + 10 + \dots + T_{n-1} + T_n \qquad \dots \dots (ii)$$

$$(i) - (ii) \Rightarrow T_n = 1 + (3 + 6 + 12 + \dots + T_n - T_{n-1})$$

$$T_{n} = 1 + 3\left(\frac{2^{n-1} - 1}{2 - 1}\right)$$

$$T_n = 3 \cdot 2^{n-1} - 2$$

So 
$$S_n = \Sigma T_n = 3\Sigma 2^{n-1} - \Sigma 2$$

$$= 3\left(\frac{2^{n}-1}{2-1}\right) - 2n = 3 \cdot 2^{n} - 2n - 3$$

Ans.

#### **Aliter Method:**

Let 
$$T_n = ar^n + b$$
, where  $r = 2$ 

Now 
$$T_1 = 1 = ar + b$$
 ....(i)

$$T_2 = 4 = ar^2 + b$$
 ....(ii)

Solving (i) & (ii), we get

$$a = \frac{3}{2}, b = -2$$

$$T_n = 3.2^{n-1} - 2$$

$$\Rightarrow S_n = \Sigma T_n = 3\Sigma 2^{n-1} - \Sigma 2$$

$$= 3\left(\frac{2^{n}-1}{2-1}\right) - 2n = 3 \cdot 2^{n} - 2n - 3$$

Ans.

#### Illustration 22:

The series of natural numbers is divided into groups (1), (2, 3, 4), (5, 6, 7, 8, 9) ...... and so on. Show that the sum of the numbers in  $n^{th}$  group is  $n^3 + (n-1)^3$ 

## Solution:

The groups are (1), (2, 3, 4), (5, 6, 7, 8, 9) ......

The number of terms in the groups are 1, 3, 5.....

 $\therefore$  The number of terms in the n<sup>th</sup> group = (2n-1)

the last term of the n<sup>th</sup> group is n<sup>2</sup>

If we count from last term common difference should be -1

So the sum of numbers in the n<sup>th</sup> group =  $\left(\frac{2n-1}{2}\right)\left\{2n^2 + (2n-2)(-1)\right\}$ 

$$= (2n-1)(n^2-n+1) = 2n^3-3n^2+3n-1 = n^3+(n-1)^3$$

#### Illustration 23:

Find the natural number 'a' for which  $\sum_{k=1}^{n} f(a+k) = 16(2^{n}-1)$ , where the function f satisfied

f(x+y) = f(x). f(y) for all natural number x,y and further f(1) = 2.

#### Solution:

It is given that

$$f(x+y) = f(x) f(y)$$
 and  $f(1) = 2$ 

$$f(1+1) = f(1) f(1) \Rightarrow f(2) = 2^2, f(1+2) = f(1) f(2) \Rightarrow f(3) = 2^3, f(2+2) = f(2) f(2) \Rightarrow f(4) = 2^4$$

Similarly  $f(k) = 2^k$  and  $f(a) = 2^a$ 

Hence, 
$$\sum_{k=1}^{n} f(a+k) = \sum_{k=1}^{n} f(a)f(k) = f(a)\sum_{k=1}^{n} f(k) = 2^{a}\sum_{k=1}^{n} 2^{k} = 2^{a}\{2^{1} + 2^{2} + \dots + 2^{n}\}$$

$$=2^{a}\left\{\frac{2(2^{n}-1)}{2-1}\right\}=2^{a+1}(2^{n}-1)$$

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But 
$$\sum_{k=1}^{n} f(a+k) = 16(2^{n}-1)$$

$$2^{a+1}(2^n-1) = 16(2^n-1)$$

$$\therefore 2^{a+1} = 2^4$$

$$\therefore \quad a+1=4 \quad \Rightarrow \quad a=3$$

Ans.

## ANSWERS FOR DO YOURSELF

1: (i) (a) 
$$\frac{2}{1}$$
,  $\frac{4}{2}$ ,  $\frac{8}{3}$ ,  $\frac{16}{4}$ ,.....,

**(b)** 
$$\frac{2}{3}, \frac{4}{9}, \frac{2}{27}, \frac{4}{81}, \dots$$
; **(iii)**  $-(p+q)$ 

**(iii)** 
$$-(p+q)$$

- (i) 900 (ii) 14
- **(i)** 931
- **(ii)** 4<sup>5</sup>
- (iv) 4, 12, 36 (v) C

- (i) 2 (ii) 15
- **6:** (i)  $4 + \frac{8}{9} \left( 1 \frac{1}{4^{n-1}} \right) \left( \frac{2n+1}{3 \times 4^{n-1}} \right)$
- **(iii)** 2

7: (i)  $\frac{n(n+3)}{4}$  (ii)  $\frac{n(n+1)(2n+3)}{2}$ 

# **EXERCISE (O-1)**

1.	If $a_1$ , $a_2$ , $a_3$ ,, $a_n$ , are in A.P. such the	hat $a_4 - a_7 + a_{10} = m$ , then the sum of first 13
	terms of this A.P., is:	[JEE-MAINS Online 2013]

- (A) 15 m
- (B) 10 m
- (C) 12 m
- (D) 13 m

**SS0001** 

2. Let 
$$a_1$$
,  $a_2$ ,  $a_3$ , ... be an A.P. such that  $\frac{a_1 + a_2 + ... + a_p}{a_1 + a_2 + a_3 + ... + a_q} = \frac{p^3}{q^3}$ ;  $p \neq q$ . Then  $\frac{a_6}{a_{21}}$  is equal to:

[JEE-MAINS Online 2013]

- (A)  $\frac{121}{1861}$
- (B)  $\frac{11}{41}$  (C)  $\frac{121}{1681}$
- (D)  $\frac{41}{11}$

SS0003

- If a, b, c are in AP, then  $(a c)^2$  equals 3.

  - (A)  $4(b^2 ac)$  (B)  $4(b^2 + ac)$  (C)  $4b^2 ac$
- (D)  $b^2 4ac$

SS0002

- If the sum of the first 11 terms of an arithmetical progression equals that of the first 19 terms, then the 4. sum of its first 30 terms, is
  - (A) equal to 0
- (B) equal to -1
- (C) equal to 1
- (D) non unique

SS0022

- **5.** The first term of an infinite G.P. is 1 and every term is equals to the sum of the successive terms, then its fourth term will be-
  - (A)  $\frac{1}{2}$

- (B)  $\frac{1}{8}$  (C)  $\frac{1}{4}$  (D)  $\frac{1}{16}$

SS0006

- If G be the GM between x and y, then the value of  $\frac{1}{G^2 x^2} + \frac{1}{G^2 y^2}$  is equal to 6.
  - $(A) G^2$
- (B)  $\frac{2}{G^2}$
- (C)  $\frac{1}{G^2}$
- (D)  $3G^2$

SS0011

- If a, b, c are in HP, then  $\frac{a-b}{b-c}$  is equal to

  - (A)  $\frac{a}{b}$  (B)  $\frac{b}{a}$
- (C)  $\frac{a}{a}$
- (D)  $\frac{c}{b}$

SS0012

- If  $a = \sum_{n=0}^{\infty} x^n$ ,  $b = \sum_{n=0}^{\infty} y^n$ ,  $c = \sum_{n=0}^{\infty} (xy)^n$  where |x|, |y| < 1; then-
  - (A) abc = a + b + c

(B) ab + bc = ac + b

(C) ac + bc = ab + c

(D) ab + ac = bc + a

- If r > 1 and  $x = a + \frac{a}{r} + \frac{a}{r^2} + ...$  to  $\infty$ ,  $y = b \frac{b}{r} + \frac{b}{r^2} ...$  to  $\infty$  and  $z = c + \frac{c}{r^2} + \frac{c}{r^4} + ...$  to  $\infty$ , then  $\frac{xy}{z} = \frac{c}{r^2} + \frac{c}{r^4} + ...$ 9.
  - (A)  $\frac{ab}{a}$
- (B)  $\frac{ac}{b}$
- (C)  $\frac{bc}{a}$
- (D) None of these

**SS0008** 

- **10.** In a GP, first term is 1. If  $4T_2 + 5T_3$  is minimum, then its common ratio is
  - (A)  $\frac{2}{5}$
- (B)  $-\frac{2}{5}$
- (C)  $\frac{3}{5}$
- (D)  $-\frac{3}{5}$

SS0009

- If abcd = 1 where a, b, c, d are positive reals then the minimum value of 11.  $a^2 + b^2 + c^2 + d^2 + ab + ac + ad + bc + bd + cd$  is
  - (A)6

- (B) 10
- (C) 12
- (D) 20

SS0033

- If  $a_1, a_2, a_3 \dots a_n \in R^+$  and  $a_1.a_2.a_3...a_n = 1$ , then minimum value of  $(1+a_1+a_1^2)(1+a_2+a_2^2)$  $(1 + a_3 + a_3^2)$  ....  $(1 + a_n + a_n^2)$  is equal to :-
  - (A)  $3^{n+1}$
- (B)  $3^{n}$
- (C)  $3^{n-1}$
- (D) none of these

**SS0014** 

- If a, b, c are positive real numbers such that  $ab^2c^3 = 64$  then minimum value of  $\left(\frac{1}{a} + \frac{2}{b} + \frac{3}{c}\right)$  is equal to:-
  - (A) 6
- (B) 2

- (C) 3
- (D) None of these

SS0015

- $2 + 4 + 7 + 11 + 16 + \dots$  to n terms =
  - (A)  $\frac{1}{6}(n^2 + 3n + 8)$  (B)  $\frac{n}{6}(n^2 + 3n + 8)$  (C)  $\frac{1}{6}(n^2 3n + 8)$  (D)  $\frac{n}{6}(n^2 3n + 8)$

SS0018

- Let  $a_n$ ,  $n \in N$  is an A.P. with common difference 'd' and all whose terms are non-zero. If n approaches **15.** infinity, then the sum  $\frac{1}{a_1 a_2} + \frac{1}{a_2 a_3} + \dots + \frac{1}{a_n a_{n+1}}$  will approach
  - (A)  $\frac{1}{a_1 d}$
- (B)  $\frac{2}{a_1 d}$
- (C)  $\frac{1}{2a_1d}$
- (D)  $a_1d$

SS0027

Let  $\alpha$  and  $\beta$  be the roots of equation  $px^2 + qx + r = 0$ ,  $p \ne 0$ . If p, q, r are in A.P. and  $\frac{1}{\alpha} + \frac{1}{\beta} = 4$ ,

then the value of  $|\alpha - \beta|$  is:

[JEE(Main)-2014]

- $(1) \frac{\sqrt{61}}{9}$
- (2)  $\frac{2\sqrt{17}}{2}$
- (3)  $\frac{\sqrt{34}}{2}$
- $(4) \frac{2\sqrt{13}}{9}$

A				Sequence & S	Series 24	
17.	Three positive numbers form an increasing G.P. If the middle term in numbers are in A.P. Then the common ratio of the G.P. is:				in this G.P. is doubled, the new  [JEE(Main)-2014]	
	(1) $\sqrt{2} + \sqrt{3}$	(2) $3 + \sqrt{2}$	$(3) 2 - \sqrt{3}$	(4) $2 + \sqrt{3}$	SS0110	
18.	If $(10)^9 + 2(11)^1$	$(10)^8 + 3(11)^2 (10)^7 +$	+ $10 (11)^9 = k (10)^9$	, then k is equal to :		
				[JEE(I	Main)-2014]	
	$(1) \frac{121}{10}$	(2) $\frac{441}{100}$	(3) 100	(4) 110	SS0111	
		EXE	RCISE (O-2)			
MU	LTIPLE CORRE	CT:				
1. If $\sin(x - y)$ , $\sin x$ and $\sin(x + y)$ are in H.P., then $\sin x$ . $\sec \frac{y}{2} =$						
	(A) 2	(B) $\sqrt{2}$	$(C) - \sqrt{2}$	(D) - 2	SS0112	
2. If $S = 1^2 + 3^2 + 5^2 + \dots + (99)^2$ then the value of the sum $2^2 + 4^2 + 6^2 + \dots$				$4^2 + 6^2 + \dots + (100)$	$(0)^2$ is	
	(A) $S + 2550$		(B) 172700			
	(C) 171700		(D) $S + 5050$		SS0113	
3. Digits contained by arithmetic mean (N) of the nine distinct number {9, 99, 999, 999999999} is				inct numbers in th	e given set	
	(A) 2	(B) 3	(C) 5	(D) 0	SS0114	
4.	a <sub>1</sub> , a <sub>2</sub> , a <sub>3</sub> ,, a <sub>r</sub>	are in A.P such that a <sub>1</sub>	$+ a_3 + a_5 = -12$ and $a_1$	$a_2 a_3 = 8$ then		
	(A) $a_2 + a_4 + a_6 = -21$		(B) $a_1 a_2 a_3 a_4 a_5 =$	(B) $a_1 a_2 a_3 a_4 a_5 = -20$		
	(C) $a_i < 0 \ \forall \ i \ge 2, i \in N$		(D) $a_i > 0 \ \forall \ i \ge 2$	(D) $a_i > 0 \ \forall \ i \ge 2, i \in N$		
5.	$ \text{If } \frac{1+3+5+\text{upto n terms}}{4+7+10+\text{upto n terms}} = \frac{20}{7\log_{10}x} \text{ and } n = \log_{10}x + \log_{10}x^{\frac{1}{2}} + \log_{10}x^{\frac{1}{4}} + \log_{10}x^{\frac{1}{8}} + $ then				$x^{\frac{1}{8}} + \dots + \infty$	
	(A) $x = 10^5$	(B) $n = 10$	(C) $x = 10^6$	(D) $n = 12$	SS0116	
6.		terms of an AP is Pn +	Qn <sup>2</sup> , where P, Q are c	onstants, then-		
	(A) $T_n = P + (2n + 1)Q$			(B) $T_n = P + (2n - 1)Q$		
	(C) common diff		(D) common diff		SS0117	
7.			then $\frac{a}{b} + \frac{b}{c} + \frac{c}{a}$ is always			

(C) 2

(D) 5

SS0118

(A) 3

(B) 1

node06\B0B0-BA\Kota\LEE[Advanced]\Leader\Wafirs\Shee\Sequence &progression\Eng.p65

(A) 
$$\frac{a}{d} = \frac{1}{2}$$

8.

(B) 
$$\rho = \frac{1}{2}$$
 (C)  $\frac{a}{d} = \frac{1}{3}$ 

$$(C) \frac{a}{d} = \frac{1}{3}$$

In an A.P. with first term 'a' and the common difference d (a,  $d \ne 0$ ), the ratio ' $\rho$ ' of the sum of the first

(D) 
$$\rho = \frac{1}{3}$$

SS0119

## **EXERCISE (S-1)**

1. The sum of n terms of two arithmetic series are in the ratio of (7n + 1): (4n + 27). Find the ratio of their nth term.

SS0058

In an AP of which 'a' is the Ist term, if the sum of the Ist p terms is equal to zero, show that the sum of 2.

the next q terms is 
$$-\left(\frac{aq(p+q)}{p-1}\right)$$

SS0039

- There are nAM's between 1 & 31 such that 7th mean :  $(n-1)^{th}$  mean = 5 : 9, then find the value of n. **3.** SS0043
- 4. The first term of an arithmetic progression is 1 and the sum of the first nine terms equal to 369. The first and the ninth term of a geometric progression coincide with the first and the ninth term of the arithmetic progression. Find the seventh term of the geometric progression.

**SS0044** 

- The sequence  $a_1, a_2, a_3, \dots a_{98}$  satisfies the relation  $a_{n+1} = a_n + 1$  for  $n = 1, 2, 3, \dots 97$  and has **5.** the sum equal to 4949. Evaluate  $\sum_{k=1}^{19} a_{2k}$ . SS0042
- For an increasing G.P.  $a_1, a_2, a_3, \dots, a_n$ , if  $a_6 = 4a_4$ ,  $a_9 a_7 = 192$ , then the value of  $\sum_{i=1}^{\infty} \frac{1}{a_i}$  is 6.

SS0045

7. In a set of four numbers, the first three are in GP & the last three are in A.P. with common difference 6. If the first number is the same as the fourth, find the four numbers.

SS0064

8. If the 10<sup>th</sup> term of an HP is 21 and 21<sup>st</sup> term of the same HP is 10, then find the 210<sup>th</sup> term.

SS0047

The  $p^{th}$  term  $T_p$  of H.P. is q(p+q) and  $q^{th}$  term  $T_q$  is p(p+q) when p>2, q>2. Prove that 9.

(a) 
$$T_{p+q} = pq$$
;

(b) 
$$T_{pq} = p + q;$$
 (c)  $T_{p+q} > T_{pq}$ 

$$(c) T_{p+q} > T_{pq}$$

SS0048

- The harmonic mean of two numbers is 4. The arithmetic mean A & the geometric mean G **10.** satisfy the relation  $2A + G^2 = 27$ . Find the two numbers. SS0049
  - **(b)** The AM of two numbers exceeds their GM by 15 & HM by 27. Find the numbers.

11. If  $A_1, A_2, A_3, \dots, A_{51}$  are arithmetic means inserted between the numbers a and b, then find the value

of 
$$\left(\frac{b+A_{51}}{b-A_{51}}\right) - \left(\frac{A_1+a}{A_1-a}\right)$$
. SS0061

12. If a > 0, then minimum value of  $a + 2a^2 + a^3 + 15 + a^{-1} + a^{-3} + a^{-4}$  is

SS0041

**13.** If a, b, c, d > 0 such that a + 2b + 3c + 4d = 50, then find the maximum value of  $\left(\frac{a^2b^4c^3d}{16}\right)^{1/10}$ 

SS0051

- **14.** Find the n<sup>th</sup> term and the sum to n terms of the sequence :
  - (i) 1+5+13+29+61+...

SS0053

(ii)  $6 + 13 + 22 + 33 + \dots$ 

SS0054

- **15.** Sum the following series to n terms and to infinity:
  - (i)  $\frac{1}{1.4.7} + \frac{1}{4.7.10} + \frac{1}{7.10.13} + \dots$  SS0055
  - (ii)  $\sum_{r=1}^{n} r(r+1)(r+2)(r+3)$  SS0056

## **EXERCISE (S-2)**

Given that the cubic  $ax^3 - ax^2 + 9bx - b = 0$  ( $a \ne 0$ ) has all three positive roots. Find the harmonic mean of the roots independent of a and b, hence deduce that the root are all equal. Find also the minimum value of (a + b) if a and  $b \in N$ .

SS0065

2. If the roots of  $10x^3 - cx^2 - 54x - 27 = 0$  are in harmonic progression, then find c and all the roots.

SS0066

- 3. If a,b,c,d,e be 5 numbers such that a,b,c are in AP; b,c,d are in GP & c,d,e are in HP then:
  - (i) Prove that a,c,e are in GP.
  - (ii) Prove that  $e = (2b a)^2/a$
  - (iii) If a = 2 & e = 18, find all possible values of b,c,d.

SS0063

4. If one AM 'a' and two GM's p and q be inserted between any two given numbers then show that  $p^3 + q^3 = 2apq$ .

SS0062

5. Find the sum of the n terms of the sequence  $\frac{1}{1+1^2+1^4} + \frac{2}{1+2^2+2^4} + \frac{3}{1+3^2+3^4} + \dots$ 

# EXERCISE (JM)

1.	If m is the A.M. of two distinct real numbers $l$ and $n(l, n > 1)$ and $G_1$ , $G_2$ and $G_3$ are three geometr				ee geometric
	means between $l$ and $n$ , then $G_1^4 + 2G_2^4 + G_3^4$ equals -		[JEE(Main)-2015]		
	$(1) 4 lmn^2$	(2) $4 l^2 m^2 n^2$	(3) $4 l^2 mn$	$(4) \ 4 \ lm^2n$	
					SS0070
2.	If the 2 <sup>nd</sup> , 5 <sup>th</sup> and 9 <sup>th</sup>	terms of a non-consta	ant A.P. are in G.P., then the		this G.P. is:- Main)-2016]
	$(1) \frac{7}{4}$	(2) $\frac{8}{5}$	(3) $\frac{4}{3}$	(4) 1	SS0071
3.	If the sum of the first ten terms of the series $\left(1\frac{3}{5}\right)^2 + \left(2\frac{2}{5}\right)^2 + \left(3\frac{1}{5}\right)^2 + 4^2 + \left(4\frac{4}{5}\right)^2 + \dots$ , is $\frac{16}{5}$ m, then m				
	equal to :-				Main)-2016]
	(1) 99	(2) 102	(3) 101	(4) 100	SS0072
4.	If, for a positive into	eger n, the quadratic e	equation,		
	x(x + 1) + (x + 1)	$(x + 2) + \dots + (x + n)$	(x - 1)(x + n) = 10n		
	has two consecutive	e integral solutions, th	en n is equal to:	[JEE(N	Main)-2017]
	(1) 11	(2) 12	(3) 9	(4) 10	SS0073
<b>5.</b>	For any three positi	ve real numbers a, b	and c, $9(25a^2 + b^2) + 25(c^2 + b^2)$	$(2^2 - 3ac) = 15b(3a)$	+ c).
	Then: [JEE(Main)-201				Main)-2017]
	(1) a, b and c are in	G.P.	(2) b, c and a are in	G.P.	
6.					SS0074
			hen $\sum_{n=1}^{10} f(n)$ is equal to :		Main)-2017]
	(1) 255	(2) 330	(3) 165	(4) 190	SS0120
7.	Let $a_1$ , $a_2$ , $a_3$ ,, $a_{49}$ be in A.P. such that $\sum_{k=0}^{12} a_{4k+1} = 416$ and $a_9 + a_{43} = 66$ .				
	If $a_1^2 + a_2^2 + \dots + a_{17}^2$	= 140m, then m is ed	qual to-	[JEE(N	Main)-2018]
	(1) 68	(2) 34	(3) 33	(4) 66	SS0075
8.			d B be the sum of the first		eries
	$1^2 + 2 \cdot 2^2 + 3^2 + 2 \cdot 4^2 + 5^2 + 2 \cdot 6^2 + \dots$ If $B - 2A = 100\lambda$ , then $\lambda$ is equal to:				
	(1) 248	(2) 464	(3) 496		Main)-2018] SS0076
9.	(1) 248 If a, b and c be thre	<b>`</b>	` '	(4) 232 stb, then x canno	
	If a, b and c be three distinct real numbers in G. P. and $a + b + c = xb$ , then x cannot be : [JEE(Main)-Jan 2019]				
				\	, •
	(1) 4	(2) -3	(3) -2	(4) 2	SS0077

11. The sum of the follwing series $1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2++5^2)}{11}+ \text{ up to } 15 \text{ terms, is }: \\ IJEE(Main)-Jan 2 \\ (1) 7820                                   $					ocquence a	25
11. The sum of the follwing series $1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2++5^2)}{11}+ \text{ up to 15 terms, is :} $ $1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2++5^2)}{11}+ \text{ up to 15 terms, is :} $ $1 \text{ [JEE(Main)-Jan 2]} $ $(1) 7820 \qquad (2) 7830 \qquad (3) 7520 \qquad (4) 7510 \qquad \text{SS} $ 12. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expre $\frac{x^my^n}{(1+x^{2m})(1+y^{2n})} \text{ is :-} \qquad \text{[JEE(Main)-Jan 2]} $ $(1) \frac{1}{2} \qquad (2) \frac{1}{4} \qquad (3) \frac{m+n}{6mn} \qquad (4) 1 \qquad \text{SS} $ 13. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$ ; $\alpha$ , $\beta \in [0,\pi]$ , then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to $\frac{1}{2}$ [JEE(Main)-Jan 2] $(1) 0 \qquad (2) -\sqrt{2} \qquad (3) -1 \qquad (4) \sqrt{2} \qquad \text{SS} $ 14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is : $\frac{1}{2}$ [JEE(Main)-Apr 2] $(1) 3221 \qquad (2) 3121 \qquad (3) 3203 \qquad (4) 3303 \qquad \text{SS} $ 15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave a common root, then which one of the following statements is correct? [JEE(Main)-Apr 2]  (1) d,e,f \text{ are in A.P.} \qquad (2) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in G.P.}  SS$				30 15		
11. The sum of the follwing series $1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2++5^2)}{11}+ \text{ up to 15 terms, is :} $ $1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2++5^2)}{11}+ \text{ up to 15 terms, is :} $ $1 \text{ [JEE(Main)-Jan 2]} $ $(1) 7820 \qquad (2) 7830 \qquad (3) 7520 \qquad (4) 7510 \qquad \text{SS} $ 12. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expre $\frac{x^my^n}{(1+x^{2m})(1+y^{2n})} \text{ is :-} \qquad \text{[JEE(Main)-Jan 2]} $ $(1) \frac{1}{2} \qquad (2) \frac{1}{4} \qquad (3) \frac{m+n}{6mn} \qquad (4) 1 \qquad \text{SS} $ 13. If $\sin^4\alpha + 4\cos^4\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$ ; $\alpha$ , $\beta \in [0,\pi]$ , then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to $\frac{1}{2}$ [JEE(Main)-Jan 2] $(1) 0 \qquad (2) -\sqrt{2} \qquad (3) -1 \qquad (4) \sqrt{2} \qquad \text{SS} $ 14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is : $\frac{1}{2}$ [JEE(Main)-Apr 2] $(1) 3221 \qquad (2) 3121 \qquad (3) 3203 \qquad (4) 3303 \qquad \text{SS} $ 15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave a common root, then which one of the following statements is correct? [JEE(Main)-Apr 2]  (1) d,e,f \text{ are in A.P.} \qquad (2) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in G.P.}  SS$	10.	Let a <sub>1</sub> , a <sub>2</sub> ,,	$a_{30}$ be an A. P., $S = \frac{1}{2}$	$\sum_{i=1}^{n} a_i$ and $T = \sum_{i=1}^{n} a_{(2i-1)}$ . I	If $a_5 = 27$ and $S - 2$	2T = 75, then
11. The sum of the follwing series $1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2++5^2)}{11}+ \text{ up to 15 terms, is:} \\ \text{[JEE(Main)-Jan 2]} \\ \text{(1) 7820} \qquad \text{(2) 7830} \qquad \text{(3) 7520} \qquad \text{(4) 7510} \qquad \text{SS} \\ \text{12. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expre} \\ \frac{x^my^n}{(1+x^{2m})(1+y^{2n})} \text{ is:} \\ \text{[JEE(Main)-Jan 2]} \\ \text{(1) } \frac{1}{2} \qquad \text{(2) } \frac{1}{4} \qquad \text{(3) } \frac{m+n}{6mn} \qquad \text{(4) 1} \qquad \text{SS} \\ \text{13. If } \sin^4\alpha+4\cos^4\beta+2=4\sqrt{2}\sin\alpha\cos\beta; \ \alpha, \ \beta\in[0,\pi] \ , \text{ then } \cos(\alpha+\beta)-\cos(\alpha-\beta) \text{ is equal in } \\ \text{[JEE(Main)-Jan 2]} \\ \text{(1) } 0 \qquad \text{(2) } -\sqrt{2} \qquad \text{(3) } -1 \qquad \text{(4) } \sqrt{2} \qquad \text{SS} \\ \text{14. The sum of all natural numbers 'n' such that } 100 < n < 200 \text{ and H.C.F. } (91, n) > 1 \text{ is:} \\ \text{[JEE(Main)-Apr 2]} \\ \text{(1) } 3221 \qquad \text{(2) } 3121 \qquad \text{(3) } 3203 \qquad \text{(4) } 3303 \qquad \text{SS} \\ \text{15. If three distinct numbers a,b,c are in G.P. and the equations } \text{ax}^2 + 2\text{bx} + \text{c} = 0 \text{ and } \text{dx}^2 + 2\text{ex} + \text{have a common root, then which one of the following statements is correct?} \\ \text{[JEE(Main)-Apr 2]} \\ \text{(1) } \text{d,e,} f \text{ are in A.P.} \qquad \text{(2) } \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in G.P.} \qquad \text{SS} \\ \text{(3) } \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.} \qquad \text{(4) d,e,} f \text{ are in G.P.} \qquad \text{SS} \\ \text{(4) d,e,} f \text{ are in G.P.} \qquad \text{(5)} \\ \text{(4) d,e,} f \text{ are in G.P.} \qquad \text{(5)} \\ \text{(4) d,e,} f \text{ are in G.P.} \qquad \text{(5)} \\ \text{(4) d,e,} f \text{ are in G.P.} \qquad \text{(5)} \\ \text{(6)} \\ \text{(6)} \\ \text{(7)} \\ \text{(7)} \\ \text{(7)} \\ \text{(7)} \\ \text{(7)} \\ \text{(8)} \\ \text{(7)} \\ \text{(8)} \\ \text{(8)} \\ \text{(9)} \\ \text{(9)} \\ \text{(1)} \\ \text{(1)} \\ \text{(1)} \\ \text{(2)} \\ \text{(2)} \\ \text{(3)} \\ \text{(4)} \\ \text{(5)} \\ \text{(5)} \\ \text{(6)} \\ \text{(7)} \\ \text{(7)} \\ \text{(8)} \\ \text{(9)} \\ \text{(9)} \\ \text{(1)} \\ \text{(1)} \\ \text{(1)} \\ \text{(1)} \\ \text{(2)} \\ \text{(1)} \\ \text{(2)} \\ \text{(2)} \\ \text{(3)} \\ \text{(4)} \\ \text{(4)} \\ \text{(4)} \\ \text{(4)} \\ \text{(4)} \\ \text{(4)} \\ \text{(5)} \\ (6$						
$1+6+\frac{9(1^2+2^2+3^2)}{7}+\frac{12(1^2+2^2+3^2+4^2)}{9}+\frac{15(1^2+2^2++5^2)}{11}+ \text{ up to 15 terms, is:}$ $[JEE(Main)-Jan 2]$ (1) 7820 (2) 7830 (3) 7520 (4) 7510 SS  12. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expre $\frac{x^my^n}{(1+x^{2m})(1+y^{2n})} \text{ is:} - \qquad \qquad [JEE(Main)-Jan 2]$ (1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1 SS  13. If $\sin^4\alpha+4\cos^4\beta+2=4\sqrt{2}\sin\alpha\cos\beta$ ; $\alpha$ , $\beta\in[0,\pi]$ , then $\cos(\alpha+\beta)-\cos(\alpha-\beta)$ is equal to $(1)$ 0 (2) $-\sqrt{2}$ (3) $-1$ (4) $\sqrt{2}$ SS  14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) $> 1$ is: $[JEE(Main)-Jan 2]$ (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2+2bx+c=0$ and $dx^2+2ex+bave$ a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2] (1) d,e,f are in A.P. (2) $\frac{d}{a},\frac{e}{b},\frac{f}{c} \text{ are in G.P.}$ SS		(1) 57	(2) 47	(3) 42	(4) 52	SS0078
[JEE(Main)-Jan 2  (1) 7820 (2) 7830 (3) 7520 (4) 7510 SS  12. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expre $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is: -  [JEE(Main)-Jan 2  (1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1 SS  13. If $\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cos \beta$ ; $\alpha$ , $\beta \in [0,\pi]$ , then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to $\frac{1}{4}$ (1) 0 (2) $-\sqrt{2}$ (3) $-1$ (4) $\sqrt{2}$ SS  14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) > 1 is:  [JEE(Main)-Apr 2  (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + have a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2  (1) d,e,f are in A.P. (2) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} are in G.P. (3) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} are in A.P. (4) d,e,f are in G.P. SS$	11.	The sum of the fo	ollwing series			
[JEE(Main)-Jan 2  (1) 7820 (2) 7830 (3) 7520 (4) 7510 SS  12. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expre $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is: -  [JEE(Main)-Jan 2  (1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1 SS  13. If $\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2} \sin \alpha \cos \beta$ ; $\alpha$ , $\beta \in [0,\pi]$ , then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to $\frac{1}{4}$ (1) 0 (2) $-\sqrt{2}$ (3) $-1$ (4) $\sqrt{2}$ SS  14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) > 1 is:  [JEE(Main)-Apr 2  (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + have a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2  (1) d,e,f are in A.P. (2) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} are in G.P. (3) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} are in A.P. (4) d,e,f are in G.P. SS$		$9(1^2+2^2)$	$(2+3^2)$ $12(1^2+2^2+3^2+$	$-4^2$ ) $15(1^2+2^2++5^2)$	)	•
(1) 7820 (2) 7830 (3) 7520 (4) 7510 SS  12. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expre $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})}$ is :-  [JEE(Main)-Jan 2  (1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1 SS  13. If $\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2}\sin \alpha \cos \beta$ ; $\alpha$ , $\beta \in [0,\pi]$ , then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal [JEE(Main)-Jan 2  (1) 0 (2) $-\sqrt{2}$ (3) -1 (4) $\sqrt{2}$ SS  14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) > 1 is :  [JEE(Main)-Apr 2  (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $ax^2 + 2cx + bave$ a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2  (1) d,e,f are in A.P. (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P. SS  (3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (4) d,e,f are in G.P. SS		$1+6+\frac{1}{7}$	<del></del>	+ 11		
12. Let x, y be positive real numbers and m, n positive integers. The maximum value of the expre $\frac{x^m y^n}{(1+x^{2m})(1+y^{2n})} \text{ is :-} \qquad \qquad \text{[JEE(Main)-Jan 2]}$ $(1) \frac{1}{2} \qquad (2) \frac{1}{4} \qquad (3) \frac{m+n}{6mn} \qquad (4) \qquad 1 \qquad \text{SS}$ 13. If $\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$ ; $\alpha$ , $\beta \in [0,\pi]$ , then $\cos(\alpha+\beta) - \cos(\alpha-\beta)$ is equal to $\text{[JEE(Main)-Jan 2]}$ $(1) \qquad (2) -\sqrt{2} \qquad (3) -1 \qquad (4) \sqrt{2} \qquad \text{SS}$ 14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is: $\text{[JEE(Main)-Apr 2]}$ $(1) \qquad 3221 \qquad (2) \qquad 3121 \qquad (3) \qquad 3203 \qquad (4) \qquad 3303 \qquad \text{SS}$ 15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave$ a common root, then which one of the following statements is correct? $\text{[JEE(Main)-Apr 2]}$ $(1) \qquad d,e,f \text{ are in A.P.} \qquad (2) \qquad \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in G.P.}$ $(3) \qquad \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.} \qquad (4) \qquad d,e,f \text{ are in G.P.}$						
$\frac{x^{m}y^{n}}{(1+x^{2m})(1+y^{2n})} \text{ is :-} \qquad \qquad \textbf{[JEE(Main)-Jan 2]}$ $(1) \frac{1}{2} \qquad (2) \frac{1}{4} \qquad (3) \frac{m+n}{6mn} \qquad (4) \ 1 \qquad \textbf{SS}$ $\textbf{13.}  \text{If } \sin^{4}\alpha + 4\cos^{4}\beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta; \ \alpha, \ \beta \in [0,\pi] \ , \text{ then } \cos(\alpha+\beta) - \cos(\alpha-\beta) \text{ is equal } \text{!}$ $\textbf{[JEE(Main)-Jan 2]}$ $(1) \ 0 \qquad (2) \ -\sqrt{2} \qquad (3) \ -1 \qquad (4) \ \sqrt{2} \qquad \textbf{SS}$ $\textbf{14.}  \text{The sum of all natural numbers 'n' such that } 100 < n < 200 \text{ and H.C.F. } (91, n) > 1 \text{ is :}$ $\textbf{[JEE(Main)-Apr 2]}$ $(1) \ 3221 \qquad (2) \ 3121 \qquad (3) \ 3203 \qquad (4) \ 3303 \qquad \textbf{SS}$ $\textbf{15.}  \text{If three distinct numbers a,b,c are in G.P. and the equations } ax^{2} + 2bx + c = 0 \text{ and } dx^{2} + 2ex + \text{ have a common root, then which one of the following statements is correct?}$ $\textbf{[JEE(Main)-Apr 2]}$ $(1) \ d,e,f \text{ are in A.P.} \qquad (2) \ \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in G.P.}$ $(3) \ \frac{d}{a}, \frac{e}{b}, \frac{f}{c} \text{ are in A.P.} \qquad (4) \ d,e,f \text{ are in G.P.} \qquad \textbf{SS}$		` '				SS0079
(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1 SSP  13. If $\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$ ; $\alpha$ , $\beta \in [0,\pi]$ , then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to [JEE(Main)-Jan 2]  (1) 0 (2) $-\sqrt{2}$ (3) $-1$ (4) $\sqrt{2}$ SSP  14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) $> 1$ is:  [JEE(Main)-Apr 2]  (1) 3221 (2) 3121 (3) 3203 (4) 3303 SSP  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave$ a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2]  (1) d,e,f are in A.P. (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P. SSP	12.	Let x, y be positive	ve real numbers and m,	n positive integers. The	maximum value of t	the expression
(1) $\frac{1}{2}$ (2) $\frac{1}{4}$ (3) $\frac{m+n}{6mn}$ (4) 1 SSP  13. If $\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$ ; $\alpha$ , $\beta \in [0,\pi]$ , then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to [JEE(Main)-Jan 2]  (1) 0 (2) $-\sqrt{2}$ (3) $-1$ (4) $\sqrt{2}$ SSP  14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) $> 1$ is:  [JEE(Main)-Apr 2]  (1) 3221 (2) 3121 (3) 3203 (4) 3303 SSP  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave$ a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2]  (1) d,e,f are in A.P. (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P. SSP		$\frac{x^m y^n}{}$	is :-		HFF(Mai	in)_Ian 2010]
13. If $\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$ ; $\alpha$ , $\beta \in [0, \pi]$ , then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to [JEE(Main)-Jan 2]  (1) 0 (2) $-\sqrt{2}$ (3) -1 (4) $\sqrt{2}$ SS  14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is:  [JEE(Main)-Apr 2]  (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave$ a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2]  (1) d,e,f are in A.P. (2) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in G.P. (3) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in A.P. (4) d,e,f are in G.P. SS		$(1+x^{2m})(1+y^{2n})$	15			m)-Jan 2017]
13. If $\sin^4 \alpha + 4\cos^4 \beta + 2 = 4\sqrt{2}\sin\alpha\cos\beta$ ; $\alpha$ , $\beta \in [0, \pi]$ , then $\cos(\alpha + \beta) - \cos(\alpha - \beta)$ is equal to [JEE(Main)-Jan 2]  (1) 0 (2) $-\sqrt{2}$ (3) -1 (4) $\sqrt{2}$ SS  14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is:  [JEE(Main)-Apr 2]  (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave$ a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2]  (1) d,e,f are in A.P. (2) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in G.P. (3) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in A.P. (4) d,e,f are in G.P. SS		$(1) \frac{1}{-}$	$(2) \frac{1}{}$	$(3) \frac{m+n}{m+n}$	(4) 1	SS0080
[JEE(Main)-Jan 2 (1) 0 (2) $-\sqrt{2}$ (3) $-1$ (4) $\sqrt{2}$ SS 14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) > 1 is : [JEE(Main)-Apr 2 (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS 15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave a$ common root, then which one of the following statements is correct? [JEE(Main)-Apr 2 (1) d,e,f are in A.P. (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P. SS (3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (4) d,e,f are in G.P. SS		2	4	6mn	(1) 1	550000
[JEE(Main)-Jan 2 (1) 0 (2) $-\sqrt{2}$ (3) $-1$ (4) $\sqrt{2}$ SS 14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. (91, n) > 1 is : [JEE(Main)-Apr 2 (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS 15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave a$ common root, then which one of the following statements is correct? [JEE(Main)-Apr 2 (1) d,e,f are in A.P. (2) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in G.P. SS (3) $\frac{d}{a}, \frac{e}{b}, \frac{f}{c}$ are in A.P. (4) d,e,f are in G.P. SS	13.	If $\sin^4 \alpha + 4\cos^4 \beta$	$3 + 2 = 4\sqrt{2}\sin\alpha\cos\beta; \ \alpha.$	$\beta \in [0, \pi]$ , then $\cos(\alpha)$	$(\alpha + \beta) - \cos(\alpha - \beta)$	is equal to:
14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is:  [JEE(Main)-Apr 2  (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave$ a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2  (1) d,e,f are in A.P. (2) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in G.P.  (3) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in A.P. (4) d,e,f are in G.P. SS						
14. The sum of all natural numbers 'n' such that $100 < n < 200$ and H.C.F. $(91, n) > 1$ is:  [JEE(Main)-Apr 2  (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave$ a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2  (1) d,e,f are in A.P. (2) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in G.P.  (3) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in A.P. (4) d,e,f are in G.P. SS		(1) 0	$(2) -\sqrt{2}$	(3) –1	$(4) \sqrt{2}$	SS0121
[JEE(Main)-Apr 2] (1) 3221 (2) 3121 (3) 3203 (4) 3303 SS  15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2] (1) d,e,f are in A.P. (2) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} are in G.P. (3) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} are in A.P. (4) d,e,f are in G.P. SS$	14.	· /	` '		• =	
(1) 3221 (2) 3121 (3) 3203 (4) 3303 SS 15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave$ a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2]  (1) d,e,f are in A.P.  (2) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in G.P.  (3) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in A.P.  (4) d,e,f are in G.P.  SS						
15. If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + bave a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2]  (1) d,e,f are in A.P.  (2) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} are in G.P.  (3) \frac{d}{a}, \frac{e}{b}, \frac{f}{c} are in A.P.  (4) d,e,f are in G.P.  SS$		(1) 3221	(2) 3121	(3) 3203		SS0082
have a common root, then which one of the following statements is correct?  [JEE(Main)-Apr 2]  (1) d,e,f are in A.P.  (2) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in G.P.  (3) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in A.P.  (4) d,e,f are in G.P.  SSO	15.	If three distinct numbers a,b,c are in G.P. and the equations $ax^2 + 2bx + c = 0$ and $dx^2 + 2ex + f = 0$				
(1) d,e,f are in A.P. (2) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in G.P. (3) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in A.P. (4) d,e,f are in G.P. SS						
(3) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in A.P. (4) d,e,f are in G.P.					[JEE(Mai	n)-Apr 2019]
(3) $\frac{d}{a}$ , $\frac{e}{b}$ , $\frac{f}{c}$ are in A.P. (4) d,e,f are in G.P.		(1) 1 0	T.	d e f		
		(1) $d,e,f$ are in A.P.		(2) -, -, - are	ın G.P.	
		d e f	in A.D.	(1) do Comin (	T D	CC0002
n(n-1)		$(3) = -\frac{1}{a}, \frac{1}{b}, \frac{1}{c}$ are 1	n A.P.	(4) d,e,f are in C	J.P.	SS0083
	1.6	I at the own of	the first is towned of a	man constant A.D. c	a a ha 50	n(n-7)
16. Let the sum of the first n terms of a non-constant A.P., $a_1$ , $a_2$ , $a_3$ , be $50n + \frac{n(n-1)}{2}$	10.	Let the sum of	the first in terms of a	non-constant A.P., a <sub>1</sub>	, a <sub>2</sub> , a <sub>3</sub> , be 501	$n + \frac{1}{2} A$
where A is a constant. If d is the common difference of this A.P., then the ordered pair $(d, a_{50})$ is						
to [JEE(Main)-Apr 2				,		-

 $(2) (A, 50+45A) \qquad (3) (50, 50+46A)$ (4) (50, 50+45A)

(1) (A, 50+46A)

SS0084

The sum  $\frac{3\times1^3}{1^2} + \frac{5\times(1^3+2^3)}{1^2+2^2} + \frac{7\times(1^3+2^3+3^3)}{1^2+2^2+3^2} + \dots$  upto 10th term, is: [JEE(Main)-Apr 2019] **17.** 

(1)660

(2) 620

(3)680

(4)600

If  $a_1$ ,  $a_2$ ,  $a_3$ , .....,  $a_n$  are in A.P. and  $a_1 + a_4 + a_7 + \dots + a_{16} = 114$ , then  $a_1 + a_6 + a_{11} + a_{16}$  is **18.** equal to: [JEE(Main)-Apr 2019]

(1)38

(2)98

(3)76

(4)64

SS0086

- 19. Let a, b and c be in G. P. with common ratio r, where  $a \ne 0$  and  $0 < r \le \frac{1}{2}$ . If 3a, 7b and 15c are the first three terms of an A. P., then the 4<sup>th</sup> term of this A. P. is : [JEE(Main)-Apr 2019]
  - (1)  $\frac{7}{3}$  a
- (2) a

- (3)  $\frac{2}{3}$  a
- (4) 5a

SS0087

- **20.** If  $\alpha$  and  $\beta$  are the roots of the equation  $375x^2 25x 2 = 0$ , then  $\lim_{n \to \infty} \sum_{r=1}^{n} \alpha^r + \lim_{n \to \infty} \sum_{r=1}^{n} \beta^r$  is equal to: [JEE(Main)-Apr 2019]
  - (1)  $\frac{21}{346}$
- $(2) \frac{29}{358}$
- $(3) \frac{1}{12}$
- $(4) \frac{7}{116}$

SS0088

21. If  $\alpha, \beta$  and  $\gamma$  are three consecutive terms of a non-constant G.P. such that the equations  $\alpha x^2 + 2\beta x + \gamma = 0$  and  $x^2 + x - 1 = 0$  have a common root, then  $\alpha(\beta + \gamma)$  is equal to:

[JEE(Main)-Apr 2019]

- (1) βγ
- (2) 0

- $(3) \alpha \gamma$
- (4)  $\alpha\beta$

SS0081

## EXERCISE (JA)

1. Let  $a_1$ ,  $a_2$ ,  $a_3$ ...... $a_{11}$  be real numbers satisfying  $a_1 = 15$ ,  $27 - 2a_2 > 0$  and  $a_k = 2a_{k-1} - a_{k-2}$  for k = 3,4......11.

If  $\frac{a_1^2 + a_2^2 + \dots + a_{11}^2}{11} = 90$ , then the value of  $\frac{a_1 + a_2 + \dots + a_{11}}{11}$  is equal to

[JEE 2010, 3+3]

SS0098

2. The minimum value of the sum of real numbers  $a^{-5}$ ,  $a^{-4}$ ,  $3a^{-3}$ , 1,  $a^{8}$  and  $a^{10}$  with a > 0 is [JEE 2011, 4]

SS0099

3. Let  $a_1, a_2, a_3, \ldots, a_{100}$  be an arithmetic progression with  $a_1 = 3$  and  $S_p = \sum_{i=1}^p a_i, 1 \le p \le 100$ . For any

integer n with  $1 \le n \le 20$ , let m = 5n. If  $\frac{S_m}{S_n}$  does not depend on n, then  $a_2$  is

[JEE 2011, 4]

**SS0100** 

- 4. Let  $a_1$ ,  $a_2$ ,  $a_3$ , .... be in harmonic progression with  $a_1 = 5$  and  $a_{20} = 25$ . The least positive integer n for which  $a_n < 0$  is [JEE 2012, 3 (-1)]
  - (A) 22
- (B) 23
- (C) 24
- (D) 25

- 5. Let  $S_n = \sum_{k=1}^{4n} (-1)^{\frac{k(k+1)}{2}} k^2$ . Then  $S_n$  can take value(s)
- [JEE-Advanced 2013, 4, (-1)]

- (A) 1056
- (B) 1088
- (C) 1120
- (D) 1332
- SS0102

6. Let a,b,c be positive integers such that  $\frac{b}{a}$  is an integer. If a,b,c are in geometric progression and

the arithmetic mean of a,b,c is b + 2, then the value of  $\frac{a^2 + a - 14}{a + 1}$  is [JEE(Advanced)-2014, 3]

SS0104

7. Suppose that all the terms of an arithmetic progression (A.P.) are natural numbers. If the ratio of the sum of the first seven terms to the sum of the first eleven terms is 6:11 and the seventh term lies in between 130 and 140, then the common difference of this A.P. is [JEE 2015, 4M, –0M]

SS0105

8. The sides of the right angled triangle are in arithmetic progression. If the triangle has area 24, then what is the length of its smallest side? [JEE(Advanced)-2017, 3]

SS0107

9. Let X be the set consisting of the first 2018 terms of the arithmetic progression 1, 6, 11, ....., and Y be the set consisting of the first 2018 terms of the arithmetic progression 9, 16, 23, ..... Then, the number of elements in the set  $X \cup Y$  is \_\_\_\_\_ [JEE(Advanced)-2018, 3(0)]

SS0108

10. Let  $\alpha$  and  $\beta$  be the roots of  $x^2 - x - 1 = 0$ , with  $\alpha > \beta$ . For all positive integers n, define

$$a_n = \frac{\alpha^n - \beta^n}{\alpha - \beta}, n \ge 1$$

$$b_1 = 1$$
 and  $b_n = a_{n-1} + a_{n+1}$ ,  $n \ge 2$ .

Then which of the following options is/are correct?

[JEE(Advanced)-2019, 4(-1)]

(1) 
$$a_1 + a_2 + a_3 + ... + a_n = a_{n+2} - 1$$
 for all  $n \ge 1$  (2)  $\sum_{n=1}^{\infty} \frac{a_n}{10^n} = \frac{10}{89}$ 

(3) 
$$\sum_{n=1}^{\infty} \frac{b_n}{10^n} = \frac{8}{89}$$
 (4)  $b_n = \alpha^n + \beta^n \text{ for all } n \ge 1$  SS0122

11. Let AP (a; d) denote the set of all the terms of an infinite arithmetic progression with first term a and common difference d > 0. If AP(1; 3)  $\cap$  AP(2; 5)  $\cap$  AP(3; 7) = AP(a; d) then a + d equals \_\_\_\_ [JEE(Advanced)-2019, 3(0)]

# **ANSWER KEY**

# **EXERCISE (O-1)**

# EXERCISE (O-2)

# **EXERCISE (S-1)**

**1.** 
$$(14n-6)/(8n+23)$$
 **3.**  $n = 14$  **4.** 27 **5.** 2499

3. 
$$n = 14$$

**14.** (i) 
$$2^{n+1} - 3$$
;  $2^{n+2} - 4 - 3n$ 

**14.** (i) 
$$2^{n+1} - 3$$
;  $2^{n+2} - 4 - 3n$  (ii)  $n^2 + 4n + 1$ ;  $(1/6) n(n+1)(2n+13) + n$ 

**15.** (i) 
$$s_n = (1/24) - [1/\{6(3n+1)(3n+4)\}]; \ s_\infty = 1/24$$
 (ii)  $(1/5) n (n+1)(n+2)(n+3)(n+4)$ 

(ii) 
$$(1/5)$$
 n  $(n+1)$   $(n+2)$   $(n+3)$   $(n+4)$ 

## **EXERCISE (S-2)**

**2.** 
$$C = 9: (3, -3/2, -3/5)$$

**2.** 
$$C = 9$$
;  $(3, -3/2, -3/5)$  **3.** (iii)  $b = 4$ ,  $c = 6$ ,  $d = 9$  or  $b = -2$ ,  $c = -6$ ,  $d = -18$ 

9 **or** 
$$b = -2$$
,  $c = -6$ ,  $d = -18$ 

5. 
$$\frac{n(n+1)}{2(n^2+n+1)}$$

# **EXERCISE (JM)**

# EXERCISE (JA)