SETS

SET : A set is a collection of well defined objects which are distinct from each other

Set are generally denoted by capital letters A, B, C, ... etc. and the elements of the set by a, b, c .... etc.

If a is an element of a set A, then we write $a \in A$ and say a belongs to A.

If a does not belong to A then we write $a \notin A$,

Ex. The collection of first five prime natural numbers is a set containing the elements 2, 3, 5, 7, 11.

SOME IMPORTANT NUMBER SETS :

N = Set of all natural numbers
= {1, 2, 3, 4, ...}

W = Set of all whole numbers
= {0, 1, 2, 3, ...}

Z or I set of all integers
= {.... –3, –2, –1, 0, 1, 2, 3, ...}

Z^+ = Set of all +ve integers
= {1, 2, 3, ...} = N.

Z^- = Set of all –ve integers
= (–1, –2, –3, ...}

Z_0 = The set of all non-zero integers.
= {±1, ±2, ±3, ...}

Q = The set of all rational numbers.

$$= \left\{\frac{p}{q} : p, q \in I, q \neq 0\right\}$$

R = the set of all real numbers.

R–Q = The set of all irrational numbers

e.g. $\sqrt{2}$, $\sqrt{3}$, $\sqrt{5}$, ... π, e, log2 etc. are all irrational numbers.

METHODS TO WRITE A SET :

(i) Roster Method : In this method a set is described by listing elements, separated by commas and enclose then by curly brackets

Ex. The set of vowels of English Alphabet may be described as {a, e, i, o, u}

(ii) Set Builder From : In this case we write down a property or rule p Which gives us all the element of the set

A = {x : P(x)}

Ex. A = {x : x ∈ N and x = 2n for n ∈ N}

i.e. A = {2, 4, 6, ...}

Ex. B = {x^2 : x ∈ z}

i.e. B = {0, 1, 4, 9, ...}

TYPES OF SETS :

Null set or Empty set : A set having no element in it is called an Empty set or a null set or void set it is denoted by φ or { }

Ex. A = {x ∈ N : 5 < x < 6} = φ

A set consisting of at least one element is called a non-empty set or a non-void set.

Singleton : A set consisting of a single element is called a singleton set.

Ex. Then set {0}, is a singleton set

Finite Set : A set which has only finite number of elements is called a finite set.

Ex. A = {a, b, c}

Order of a finite set : The number of elements in a finite set is called the order of the set A and is denoted O(A) or n(A). It is also called cardinal number of the set.

Ex. A = {a, b, c, d} $\Rightarrow$ n(A) = 4

Infinite set : A set which has an infinite number of elements is called an infinite set.

Ex. A = {1, 2, 3, 4, ...} is an infinite set

Equal sets : Two sets A and B are said to be equal if every element of A is a member of B, and every element of B is a member of A.

If sets A and B are equal. We write A = B and A and B are not equal then A $\neq$ B

Ex. A = {1, 2, 6, 7} and B = {6, 1, 2, 7} $\Rightarrow$ A = B

Equivalent sets : Two finite sets A and B are equivalent if their number of elements are same i.e. n(A) = n(B)

Ex. A = {1, 3, 5, 7}, B = {a, b, c, d}

n(A) = 4 and n(B) = 4 $\Rightarrow$ n(A) = n(B)

Note : Equal set always equivalent but equivalent sets may not be equal

Subsets : Let A and B be two sets if every element of A is an element B, then A is called a subset of B if A is a subset of B. we write $A \subseteq B$

Example : A = {1, 2, 3, 4} and B = {1, 2, 3, 4, 5, 6, 7} $\Rightarrow A \subseteq B$

The symbol "$\Rightarrow$" stands for "implies"

Proper subset : If A is a subset of B and A $\neq$ B then A is a proper subset of B. and we write $A \subset B$

Note-1 : Every set is a subset of itself i.e. $A \subseteq A$ for all A

Note-2 : Empty set ϕ is a subset of every set

Note-3 : Clearly $N \subset W \subset Z \subset Q \subset R \subset C$

Note-4 : The total number of subsets of a finite set containing n elements is 2^n

Universal set : A set consisting of all possible elements which occur in the discussion is called a Universal set and is denoted by U

Note : All sets are contained in the universal set

Ex. If A = {1, 2, 3}, B = {2, 4, 5, 6}, C = {1, 3, 5, 7} then U = {1, 2, 3, 4, 5, 6, 7} can be taken as the Universal set.

Power set : Let A be any set. The set of all subsets of A is called power set of A and is denoted by P(A)

Ex.1 Let A = {1, 2} then P(A) = {ϕ, {1}, {2}, {1, 2}}

Ex.2 Let P(ϕ) = {ϕ}

$\because$ P(P(ϕ)) = {ϕ, {ϕ}}

$\because$ P(P(P(ϕ))) = {ϕ, {ϕ}, {{ϕ}}, {ϕ, {ϕ}}}

Note-1 : If A = ϕ then P(A) has one element

Note-2 : Power set of a given set is always non empty

Some Operation on Sets :

(i) **Union of two sets :** $A \cup B = \{x : x \in A \text{ or } x \in B\}$

e.g. A = {1, 2, 3}, B = {2, 3, 4} then $A \cup B$ = {1, 2, 3, 4}

(ii) **Intersection of two sets :** $A \cap B = \{x : x \in A \text{ and } x \in B\}$

e.g. A = {1, 2, 3, }, B = {2, 3, 4} then $A \cap B$ = {2, 3}

(iii) **Difference of two sets :** $A - B = \{x : x \in A \text{ and } x \notin B\}$

e.g. A = {1, 2, 3}, B = {2, 3, 4} ; A − B = {1}

(iv) **Complement of a set :** A' = {x : x ∉ A but x ∈ U} = U − A

e.g. U = {1, 2, ..., 10}, A = {1, 2, 3, 4, 5} then A' = {6, 7, 8, 9, 10}

(v) **De-Morgan Laws :** $(A \cup B)' = A' \cap B'$; $(A \cap B)' = A' \cup B'$

(vi) $A - (B \cup C) = (A - B) \cap (A - C)$; $A - (B \cap C) = (A - B) \cup (A - C)$

(vii) **Distributive Laws :** $A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$; $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(viii) **Commutative Laws :** $A \cup B = B \cup A$; $A \cap B = B \cap A$

(ix) **Associative Laws :** $(A \cup B) \cup C = A \cup (B \cup C)$; $(A \cap B) \cap C = A \cap (B \cap C)$

(x) $A \cap \phi = \phi$; $A \cap U = A$

$A \cup \phi = A$; $A \cup U = U$

(xi) $A \cap B \subseteq A$; $A \cap B \subseteq B$

(xii) $A \subseteq A \cup B$; $B \subseteq A \cup B$

(xiii) $A \subseteq B \Rightarrow A \cap B = A$

(xiv) $A \subseteq B \Rightarrow A \cup B = B$

Disjoint Sets :

IF $A \cap B = \phi$, then A, B are disjoint.

e.g. if A = {1, 2, 3}, B = {7, 8, 9} then $A \cap B = \phi$

Note : $A \cap A' = \phi$ $\therefore$ A, A' are disjoint.

Symmetric Difference of Sets :

$A \Delta B = (A - B) \cup (B - A)$

- $(A')' = A$
- $A \subseteq B \Leftrightarrow B' \subseteq A'$

If A and B are any two sets, then

(i) $A - B = A \cap B'$

(ii) $B - A = B \cap A'$

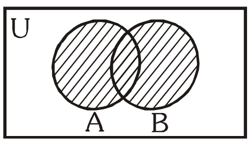
(iii) $A - B = A \Leftrightarrow A \cap B = \phi$

(iv) $(A - B) \cup B = A \cup B$

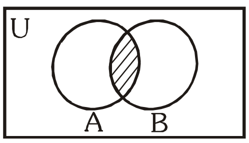
(v) $(A - B) \cap B = \phi$

(vi) $(A - B) \cup (B - A) = (A \cup B) - (A \cap B)$

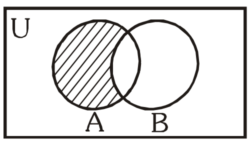
Venn Diagrame :



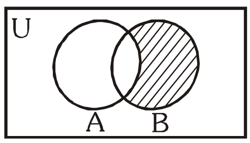
$A \cup B$



$A \cap B$

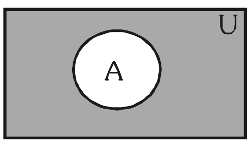


A − B

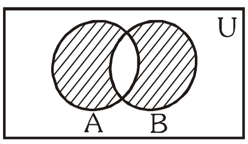


B − A

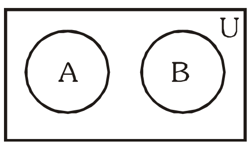
Clearly $(A - B) \cup (B - A) \cup (A \cup B) = A \cup B$



A'



$(A\Delta B)=(A-B) \cup (B-A)$



Disjoint Sets

Note : $A \cap A' = \phi$, $A \cup A' = U$

SOME IMPORTANT RESULTS ON NUMBER OF ELEMENTS IN SETS :

If A, B and C are finite sets, and U be the finite universal set, then

(i) $n(A \cup B) = n(A) + n(B) - n(A \cap B)$

(ii) $n(A \cup B) = n(A) + n(B) \Leftrightarrow$ A, B are disjoint non-void sets.

(iii) $n(A - B) = n(A) - n(A \cap B)$ i.e. $n(A - B) + n(A \cap B) = n(A)$

(iv) $n(A \Delta B)$ = No. of elements which belong to exactly one of A or B

$= n((A - B) \cup (B - A))$

$= n(A - B) + n(B - A)$ [$\because$ (A – B) and (B – A) are disjoint]

$= n(A) - n(A \cap B) + n(B) - n(A \cap B)$

$= n(A) + n(B) - 2n(A \cap B)$

$= n(A) + n(B) - 2n(A \cap B)$

(v) $n(A \cup B \cup C) = n(A) + n(B) + n(C) - n(A \cap B) - n(B \cap C) - n(A \cap C) + n(A \cap B \cap C)$

(vi) Number of elements in exactly two of the sets A, B, C

$= n(A \cap B) + n(B \cap C) + n(C \cap A) - 3n(A \cap B \cap C)$

(vii) number of elements in exactly one of the sets A, B, C

$= n(A) + n(B) + n(C) - 2n(A \cap B) - 2n(B \cap C) - 2n(A \cap C) + 3n(A \cap B \cap C)$

(viii) $n(A' \cup B') = n((A \cap B)') = n(U) - n(A \cap B)$

(ix) $n(A' \cap B') = n((A \cup B)') = n(U) - n(A \cup B)$

Ex. In a group of 1000 people, there are 750 who can speak Hindi and 400 who can speak Bengali. How many can speak Hindi only ?How many can speak Bengali only ? How many can spak both Hindi and Bengali?

Sol. Let A and B be the sets of persons who can speak Hindi and Bengali respectively.

then $n(A \cup B) = 1000$, $n(A) = 750$, $n(B) = 400$.

Number of persons who can speak both Hindi and Bengali

$= n(A \cap B) = n(A) + n(B) - n(A \cup B)$

$= 750 + 400 - 1000 = 150$

Number of persons who can speak Hindi only

$= n(A - B) = n(A) - n(A \cap B) = 750 - 150 = 600$

Number of persons who can speak Bengali only

$= n(B - A) = n(B) - n(A \cap B) = 400 - 150 = 250$

Let us first try and understand what a Venn - Diagram for four sets would look like.

SOLVED EXAMPLES

Ex.1 The set A = [x : x ∈ R, x^2 = 16 and 2x = 6] equal -

(1) φ (2) [14, 3, 4] (3) [3] (4) [4]

Sol.(1) $x^2 = 16 \Rightarrow x = \pm 4$

$2x = 6 \Rightarrow x = 3$

There is no value of x which satisfies both the above equations.

Thus, A = φ

Hence (1) is the correct answer

Ex.2 Let A = {x : x ∈ R, | x | < 1] ; B = [x : x ∈ R, | x – 1 | ≥ 1] and A ∪ B = R – D, then the set D is-

(1) [x : 1 < x ≤ 2] (2) [x : 1 ≤ x < 2] (3) [x : 1 ≤ x ≤ 2] (4) none of these

Sol.(2) A = [x : x ∈ R, –1 < x < 1]

B = [x : x ∈ R : x – 1 ≤ –1 or x – 1 ≥ 1]

= [x : x ∈ R : x ≤ 0 or x ≥ 2]

∴ A ∪ B = R – D

where D = [x : x ∈ R, 1 ≤ x < 2]

Thus (2) is the correct answer.

Ex.3 If aN = {ax : x ∈ N}, then the set 6N ∩ 8N is equal to-

(1) 8N (2) 48N (3) 12N (4) 24N

Sol.(4) 6N = {6, 12, 18, 24, 30, ...}

8N = {8, 16, 24, 32, ...}

∴ 6N ∩ 8N = {24, 48, ...} = 24N

Short cut Method

6N ∩ 8N = 24N [24 is the L.C.M. of 6 and 8]

Ex.4 If P, Q and R subsets of a set A, then R ×(P' ∪ Q')' =

(1) (R ×P) ∩ (R ×Q) (2) (R ×Q) ∩ (R ×P) (3) (R ×P) ∪ (R ×Q) (4) none of these

Sol.(1,2)

R ×(P' ∪ Q')' = R ×[(P')' ∩ (Q')'] = R ×(P ∩ Q) = (R ×P) ∩ (R ×Q)

Hence (1) is the correct answer.

Ex.5 If A = {x, y}, then the power set of A is-

(1) {x^y, y^x} (2) {φ, x, y} (3) {φ, {x} {2y}} (4) {φ, {x}, {y}, {x, y}}

Sol.(4) Clearly P(A) = Power set of A

= set of all subsets of A

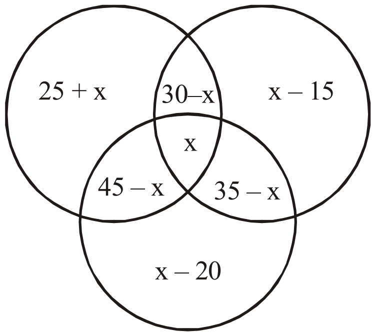
= {φ, {x}, {y}, {x, y}}

∴ (4) holds.

Ex.6 These are 200 students in a school. Out of these, 100 students play cricket, 50 students play hockey and 60 students play basketball. 30 students play both cricket and hockey, 35 students play both hockey and basketball, and 45 students play both basketball and cricket.

(a) What is the maximum number of students who play at least one game ?
(b) What is the maximum number of students who play all the 3 games ?
(c) What is the minimum number of students playing at least one game ?
(d) What is the minimum number of students who play all the 3 games ?

Sol.



Consider the Venn diagram given above :

At first we will convert all the values in terms of x, which can be seen above.

Since the number of students cannot be negative.

$x - 15 \geq 0$

$\therefore \quad x - 20 \geq 0$

So, iv. For the minimum number of students playing all three games, i.e., x = 20.

For the maximum value of x, again none of the categories should have –ve number of students.

$\therefore \quad 30 - x \geq 0$

$x \leq 30$

If x is more than 30, 30 – x would be –ve which is not possible.

Total number of students playing at least one game,

$= 100 + x - 15 + 35 - x + x - 20$

$= 100 + x$

So, the minimum number of students playing at least one game = 100 + 20 = 120

Hence, the maximum number of students playing at least one game = 100 + 30 = 130.

CHECK YOUR GRASP SETS EXERCISE-I

1. If A and B are two sets, then $A \cap (A \cup B)'$ is equal to-

(1) A (2) B
(3) ϕ (4) none of these

ST0001

2. If A is any set, then-

(1) $A \cup A' = \phi$ (2) $A \cup A' = U$
(3) $A \cap A' = U$ (4) none of these

ST0002

3. Let U = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}, A = {1, 2, 5}, B = {6, 7} then $A \cap B'$ is-

(1) B' (2) A (3) A' (4) B.

ST0005

4. If A and B are two sets, then $A \cup B = A \cap B$ iff-

(1) $A \subseteq B$ (2) $B \subseteq A$
(3) A = B (4) none of these

ST0006

5. Let A and B be two sets in the universal set. Then A – B equals-

(1) $A \cap B'$ (2) $A' \cap B$
(3) $A \cap B$ (4) none of these

ST0007

6. Two sets A, B are disjoint iff-

(1) $A \cup B = \phi$ (2) $A \cap B \neq \phi$
(3) $A \cap B = \phi$ (4) None of these

ST0008

7. If $A \subseteq B$, then $A \cap B$ is equal to-

(1) A (2) B (3) A' (4) B'

ST0010

8. If A and B are any two sets, then $A \cup (A \cap B)$ is equal to-

(1) A (2) B (3) A' (4) B'

ST0011

9. If A and B are not disjoint, then $n(A \cup B)$ is equal to-

(1) n(A) + n(B)
(2) $n(A) + n(B) - n(A \cap B)$
(3) $n(A) + n(B) + n(A \cap B)$
(4) n(A).n(B)

ST0012

10. If A = {2, 4, 5}, B = {7, 8, 9} then $n(A \times B)$ is equal to-

(1) 6 (2) 9 (3) 3 (4) 0

ST0013

11. Let A and B be two sets such that n(A) = 70, n(B) = 60 and $n(A \cup B) = 110$. Then $n(A \cap B)$ is equal to-

(1) 240 (2) 20 (3) 100 (4) 120

ST0014

12. Which set is the subset of all given sets ?

(1) {1, 2, 3, 4, ...} (2) {1}
(3) {0} (4) { }

ST0015

13. If $Q = \left\{x : x = \frac{1}{y}, \text{ where } y \in N\right\}$, then-

(1) $0 \in Q$ (2) $1 \in Q$ (3) $2 \in Q$ (4) $\frac{2}{3} \in Q$

ST0016

14. A = {x : x ≠ x} represents-

(1) {0} (2) { } (3) {1} (4) {x}

ST0017

15. Which of the following statements is true ?

(1) $3 \subseteq \{1, 3, 5\}$ (2) $3 \in \{1, 3, 5\}$
(3) $\{3\} \in \{1, 3, 5\}$ (4) $\{3, 5\} \in \{1, 3, 5\}$

ST0018

16. Which of the following is a null set ?

(1) A = {x : x > 1 and x < 1]
(2) B = {x : x + 3 = 3}
(3) C = {ϕ}
(4) D = {x : x ≥ 1 and x ≤ 1}

ST0019

17. $P(A) = P(B) \Rightarrow$

(1) $A \subseteq B$ (2) $B \subseteq A$
(3) A = B (4) none of these

ST0020

18. In a recent survey (conducted by HLL) of 1,000 houses, washing machine, vacuum cleaners and refrigerators were counted. Each house had at least one of these products. 400 had no refrigerators, 380 had no vacuum cleaners, 542 had no washing machines. 294 had both a vacuum cleaner and washing machines, 277 had both a vacuum cleaner and a refrigerator, and 120 had both a refrigerator and a washing machine. How many had only a vacuum cleaner ?

(1) 132 (2) 234
(3) 342 (4) 62

ST0039

19. From 50 students taking examinations in Mathematics, Physics and Chemistry, 37 passed Mathematics, 24 Physics and 43 Chemistry. At most 19 passed Mathematics and Physics, at most 29 passed Mathematics and Chemistry and at most 20 passed Physics and Chemistry. The largest possible number that could have passed all three examinations is -

(1) 11 (2) 12 (3) 13 (4) 14

ST0040

20. Let Z be the set of all integers and $A = \{(a, b) : a^2 + 3b^2 = 28, a, b \in Z\}$ and $B = \{(a, b) : a > b, a, b \in Z\}$. Then, the number of elements in $A \cap B$, is -

(1) 2 (2) 4 (3) 6 (4) 5

ST0041

21. In a class of 25 students, at least one of mathematics or statistics is taken by everybody. 12 have taken mathematics, 8 have taken mathematics but not statistics. Find the difference in the number of students who have taken mathematics and statistics and those who have taken statistics but not maths ?

(1) 9 (2) 10 (3) 18 (4) 8

ST0042

22. In a class of 200 students, 70 played cricket, 60 played hockey and 80 played football. Thirty played cricket and football, 30 played hockey and football, 40 played cricket and hockey.

Find the maximum number of people playing all the three games and also the minimum number of people playing at least one game ?

(1) 200, 100 (2) 30, 110

(3) 30, 120 (4) none of these

ST0043

23. If class with n students is organized into four groups keeping the following conditions :

Each student belongs to exactly two groups and

Each pair of groups has exactly one student in common.

What is the value of n ?

(1) n = 11 (2) n = 7 (3) n = 9 (4) n = 6

ST0044

24. If A = {1, 2, 3, 4}, then the number of subsets of set A containing element 3, is -

(1) 24 (2) 28 (3) 8 (4) 16

ST0045

ANSWER KEY

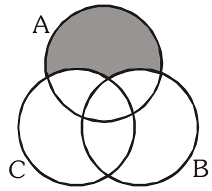
Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	**3**	**2**	**2**	**3**	**1**	**3**	**1**	**1**	**2**	**2**	**2**	**4**	**2**	**2**	**2**
Que.	16	17	18	19	20	21	22	23	24						
Ans.	**1**	**3**	**4**	**4**	**3**	**1**	**3**	**4**	**3**						

PREVIOUS YEAR QUESTIONS SETS EXERCISE-II

1. Sets A and B have 3 and 6 elements respectively. What can be the minimum number of elements in $A \cup B$? **[Roorkee 1991]**
(1) 3 (2) 6 (3) 9 (4) 18
ST0022

2. In a college of 300 students, every student reads 5 new spapers and every newspaper is read by 60 students. The number of newspapers is- **[IIT -1998]**
(1) at least 30 (2) at most 20
(3) exactly 25 (4) none of these
ST0023

3. The shaded region in the given figure is -



(1) $A \cap (B \cup C)$
(2) $A \cup (B \cap C)$
(3) $A \cap (B - C)$
(4) $A - (B \cup C)$
ST0025

4. Let n(U) = 700, n(A) = 200, n(B) = 300 and $n(A \cap B) = 100$, then $n(A' \cap B') =$
(1) 400 (2) 600 (3) 300 (4) 200
ST0026

5. If A = {1, 2, 3, 4, 5}, then the number of proper subsets of A is -
(1) 120 (2) 30 (3) 31 (4) 32
ST0027

6. If $A = \{x : x^2 - 5x + 6 = 0\}$, B = {2, 4}, C = {4, 5}, then $A \times (B \cap C)$ is- **[Kerala P.E.T. 2002]**
(1) {(2, 4), (3, 4)}
(2) {(4, 2), (4, 3)}
(3) {(2, 4), (3, 4), (4, 4)}
(4) {(2, 2), (3, 3), (4, 4), (5, 5)}
ST0029

7. If $A = \{(x, y) : x^2 + y^2 = 25\}$ and $B = \{(x, y) : x^2 + 9y^2 = 144\}$ then $A \cap B$ contains-
(1) one point (2) three points
(3) two points (4) four points
ST0030

8. A class has 175 students. The following data shows the number of students obtaining one or more subjects. Mathematics 100; Physics 70; Chemistry 40; Mathematics and Physics 30; Mathematics and Chemistry 28; Physics and Chemitry 23; Mathematics, Physics and Chemistry 18. How many students have offered Mathematics alone ?
(1) 35 (2) 48 (3) 60 (4) 22
ST0031

9. If A, B and C are three sets such that $A \cap B = A \cap C$ and $A \cup B = A \cup C$, then :- **[AIEEE- 2009]**
(1) B = C (2) $A \cap B = \phi$
(3) A = B (4) A = C
ST0033

10. Two sets A and B are as under
$A = \{(a, b) \in R \times R : |a - 5| < 1$ and $|b - 5| < 1\}$;
$B = \{(a, b) \in R \times R : 4(a - 6)^2 + 9(b - 5)^2 \le 36\}$. Then :- **[AIEEE-2018]**
(1) $A \subset B$
(2) $A \cap B = \phi$ (an empty set)
(3) neither $A \subset B$ nor $B \subset A$
(4) $B \subset A$
ST0034

11. In a class of 60 students, 40 opted for NCC, 30 opted for NSS and 20 opted for both NCC and NSS. If one of these students is selected at random, then the probability that the student selected has opted neither for NCC nor for NSS is : **[JEE(Main) 19]**
(1) $\frac{2}{3}$ (2) $\frac{1}{6}$ (3) $\frac{1}{3}$ (4) $\frac{5}{6}$
ST0046

12. Two newspapers A and B are published in a city. It is known that 25% of the city populations reads A and 20% reads B while 8% reads both A and B. Further, 30% of those who read A but not B look into advertisements and 40% of those who read B but not A also look into advertisements, while 50% of those who read both A and B look into advertisements. Then the percentage of the population who look into advertisement is :- **[JEE(Main) 19]**
(1) 12.8 (2) 13.5 (3) 13.9 (4) 13
ST0036

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12			
Ans.	2	3	4	3	3	1	4	3	1	1	2	3			

RELATIONS

This chapter deals with establishing binary relation between elements of one set and elements of another set according to some particular rule of relationship.

1. CARTESIAN PRODUCT :

The Cartesian product of two sets A, B is a non-void set of all ordered pair (a, b), where $a \in A$ and $b \in B$. This is denoted by $A \times B$

$\therefore$ $A \times B = \{(a, b) \; \forall \; a \in A \text{ and } b \in B\}$

e.g. $A = \{1, 2\}$, $B = \{a, b\}$

$A \times B = \{(1, a), (1, b), (2, a), (2, b)\}$

Note :

(i) $A \times B \neq B \times A$ (Non-commutative)

(ii) $n(A \times B) = n(A)\, n(B)$ and $n(P(A \times B)) = 2^{n(A)n(B)}$

(iii) $A = \phi$ and $B = \phi \iff A \times B = \phi$

(iv) If A and B are two non-empty sets having n elements in common, then $(A \times B)$ and $(B \times A)$ have n^2 elements in common

(v) $A \times (B \cup C) = (A \times B) \cup (A \times C)$

(vi) $A \times (B \cap C) = (A \times B) \cap (A \times C)$

(vii) $A \times (B - C) = (A \times B) - (A \times C)$

Ex. If $n(A) = 7$, $n(B) = 8$ and $n(A \cap B) = 4$, then match the following columns.

(i)	$n(A \cup B)$	(a)	56
(ii)	$n(A \times B)$	(b)	16
(iii)	$n((B \times A) \times A)$	(c)	392
(iv)	$n((A \times B) \cap (B \times A))$	(d)	96
(v)	$n((A \times B) \cup (B \times A)$	(e)	11

Sol. (i) $n(A \cup B) = n(A) + n(B) - n(A \cap B) = 7 + 8 - 4 = 11$

(ii) $n(A \times B) = n(A)\, n(B) = 7 \times 8 = 56 = n(B \times A)$

(iii) $n((B \times A) \times A) = n(B \times A).n(A) = 56 \times 7 = 392$

(iv) $n((A \times B) \cap (B \times A)) = (n((A \cap B))^2 = 4^2 = 16$

(v) $n((A \times B) \cup (B \times A)) = n(A \times B) + n(B \times A) - n(A \times B) \cap (B \times A)$

$= 56 + 56 - 16 = 96$

Ex. If $A = \{2, 4\}$ and $B = \{3, 4, 5\}$, then $(A \cap B) \times (A \cup B)$ is

(1) $\{(2, 2), (3, 4), (4, 2), (5, 4)\}$

(2) $\{(2, 3), (4, 3)(4, 5)\}$

(3) $\{(2, 4), (3, 4), (4, 4), (4, 5)\}$

(4) $\{(4, 2), (4, 3), (4, 4), (4, 5)\}$

Sol. $A \cap B = \{4\}$ and $A \cup B = \{2, 3, 4, 5\}$

$\therefore$ $(A \cap B) \times (A \cup B) = \{(4, 2), (4, 3), (4, 4), (4, 5)\}$

2. RELATION :

Every subset of $A \times B$ defined a relation from set A to set B.

If R is relation from $A \to B$

$R : \{(a, b) \mid (a, b) \in A \times B \text{ and } a\,R\,b\}$

Highlights :

Let A and B be two non empty sets and $R : A \to B$ be a relation such that $R : \{(a, b) \mid (a, b) \in R, a \in A \text{ and } b \in B\}$.

(i) 'b' is called image of 'a' under R.

(ii) 'a' is called pre-image of 'b' under R.

(iii) Domain of R : Collection of all elements of A which has a image in B.

(iv) Range of R : Collection of all elements of B which has a pre-image in A.

Note :

(1) It is not necessary that each and every element of set A has a image in Set B and each and every element of set B has preimage in Set A.

(2) Elements of set A having image in B is not necessarily unique.

(3) Basically relation is the number of subsets of A ×B

Number of non empty relations = no. of ways of selecting a non zero subset of A ×B

$= {}^{mn}C_1 + {}^{mn}C_2 + \ldots\ldots\ldots + {}^{mn}C_{mn} = 2^{mn} - 1$

Total number of relation = 2^{mn} (including void relation)

Examples :

(1) A = {1, 2, 3, 4, 5} and B = {2, 4, 5}

aRb ⇒ a and b are relatively prime or co-prime (i.e. HCF is 1)

[Sol. R= {(1, 2), (1, 4), (1, 5), (2, 5), (3, 2), (3, 4), (3, 5), (4, 5), (5, 2), (5, 4)}]

Domain of R{1, 2, 3, 4, 5}

Range of R{2, 4, 5}

(2) A = {Jaipur, Patna, Kanpur, Lucknow} and B = {Rajasthan, Uttan Pradesh, Bihar}

aRb ⇒ a is capital of b, $a \in A$ and $b \in B$

[Sol. R = {(Jaipur, Rajasthan), (Patna, Bihar), (Lucknow, Uttar Pradesh)}

(3) If A = {1, 3, 5, 7}, B = {2, 4, 6, 8}

Relation is aRb ⇒ $a > b$, $a \in A, b \in B$

Sol. R = {(3, 2), (5, 2), (5, 4), (7, 2), (7, 4), (7, 6)}

Domain = {3, 5, 7}

Range = {2, 4, 6}

3. REPRESENTATION OF A RELATION :

1. Roster form : In this form we represent set of all ordered pairs (a, b) such that $(a, b) \in R$, where $a \in A, b \in B$.

2. Set builder notation : Here we denote the relation by the rule which co relates the two set.

3. Arrow - diagram (Mapping) : This the pictorial notation of any relation

Ex. Let A = {–2, –1, 4}, B = {1, 4, 9}

A relation from A to B i.e. a R b is defined as a is less than b.

This can be represented in the following ways.

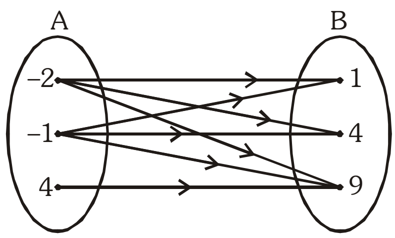
1. Roster form :

R = {(–2, 1), (–2, 4), (–2, 9), (–1, 1), (–1, 4), (–1, 9), (4, 9)}

2. Set builder notation :

R = {(a, b) : $a \in A$ and $b \in B$, a is less than b}

3. Arrow - diagram :



Empty relation (Void relation) : No elements of A is related to any elements of A.

Universal relation : Each elements of A is related to every element of A.

4. INVERSE RELATION :

If relation R is defined from A to B, then the inverse relation would be defined from B to A, i.e.

$R : A \rightarrow B \Rightarrow aRb$ where $a \in A, b \in B$

$R^{-1} : B \rightarrow A \Rightarrow bRa$ where $a \in A, b \in B$

Domain of R = Range of R^{-1}

and Range of R = Domain of R^{-1}

$\therefore$ $R^{-1} = \{(b, a) \mid (a, b) \in R\}$

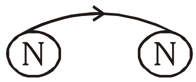
A relation R is defined on the set of 1st ten natural numbers.

e.g. N is a set of first 10 natural nos. $\therefore$ $N = \{1, 2, 3, ..., 10\}$ & $a, b \in N$

$aRb \Rightarrow a + 2b = 10$

$R = \{(2, 4), (4, 3), (6, 2), (8, 1)\}$

$R^{-1} = \{(4, 2), (3, 4), (2, 6), (1, 8)\}$



5. IDENTITY RELATION :

A relation defined on a set A is said to be an identity relation if each & every element of A is related to itself & only to itself.

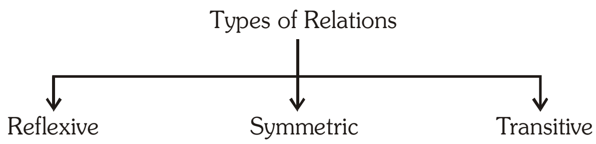
e.g. A relation defined on the set of natural numbers is

$aRb \Rightarrow a = b$ where a & $b \in N$

$R = \{(1, 1), (2, 2), (3, 3), ...\}$

R is an identity relation

6. CLASSIFICATION OF RELATIONS :



(i) **Reflexive :** A relation R on a set A is said to be reflexive if every element of A is related to itself.

i.e. if $(a, b) \in R$, then $(a, a) \in R$. However if there is a single ordered pair of $(a, b) \in R$ such $(a, a) \notin R$, then R is not reflexive.

e.g. A relation defined on (set of natural numbers)

$aRb \Rightarrow$ 'a' divides 'b' $a, b \in N$

R would always contain (a, a) because every natural number divides itself and hence it is a reflexive relation.

Note : Every Identity relation is a reflexive relation but every reflexive relation need not be an Identity.

(ii) **Symmetric :** A relation defined on a set is said to be symmetric if $aRb \Rightarrow bRa$.

If $(a, b) \in R$, then (b, a) must be necessarily there in the same relation.

Examples :

A relation defined on the set of lines.

(1) $aRb \Rightarrow a \parallel b$

It is a symmetric relation because if line 'a' is $\parallel$ to 'b' then the line 'b' is $\parallel$ to 'a'.

where $(a, b) \in L$ {L is set of $\parallel$ lines}

(2) $L_1RL_2 \Rightarrow L_1 \perp L_2$ It is a symmetric relation

$L_1, L_2 \in L$ {L is a set of lines}

(3) $aRb \Rightarrow$ 'a' is brother of 'b' is not a symmetric relation as 'a' may be sister of 'b'.

(4) $aRb \Rightarrow$ 'a' is a cousin of 'b'. This is symmetric relation.

If R is symmetric

(1) $R = R^{-1}$

(2) Range of R = Domain of R

(iii) **Transitive :** A relation on set A is said to be a transitive if aRb and bRc implies aRc

i.e. $(a, b) \in R$ and $(b, c) \in R$, then $(a, c) \in R$

and a, b, c need not be distinct.

Examples :

(1) A relation R defined on a set of natural numbers N with rule aRb $\Rightarrow$ $a \le b$

Let R : {(1, 2), (1, 1)}. on set {1,2}

In this relation a, b, c are not distinct but it is transitive. It is transitive but not symmetric as (2, 1) is missing.

Minimum number of ordered pair that must be added to make it reflexive, symmetric and transitive is 2 i.e. (2, 1) and (2, 2).

(2) Only Transitive $R = \{(x, y) \mid x < y, \ x \in N, y \in N\}$

Only Symmetric $R = \{(x, y) \mid x + y = 10, \ x \in N, y \in N\}$

Only Reflexive $R = \{(x, y) \mid x = y \text{ or } x - y = 1, \ x \in N, y \in N\}$

6. EQUIVALENCE RELATION :

If a relation is Reflexive, Symmetric and Transitive, then it is said to be an equivalence relation.

Exmaples :

(1) A relation defined on N

xRy $\Rightarrow$ $x = y$

R is an equivalence relation.

(2) A relation defined on a set of || lines in a plane

aRb $\Rightarrow$ a || b

It is an equivalence relation.

(3) Relation defined on the set of integer (I)

Prove that : xRy $\Rightarrow$ $(x - y)$ is even is an equivalence relation.

(4) R = {(1, 2), (2, 3)} add minimum number of ordered pairs to make it an equivalence relation.

{(1, 1), (2, 2), (3, 3), (2, 1), (3, 2), (1, 3), (3, 1)} = 7

(5) A = {1, 2, 3, ..., 13, 14}

$R = \{(x, y) \mid 3x - y = 10\}$ $\bar{R}\ \bar{S}\ \bar{T}$

$R = \{(x, y) \mid x \text{ is coefficient of } y\}$ $\bar{R} \cap \bar{S} \cap \bar{T}$

$R = \{(x, y) \mid x \text{ is father of } y\}$ $\bar{R} \cap \bar{S} \cap \bar{T}$

7. PARTIAL ORDER RELATION :

Definition :

A relation R on a set P is called partial order relation if it is reflexive, antisymmetric and transitive. That means that for all x, y and z in P we have:

- x R x;
- if x R y and y R x, then x = y;
- if x R y and y R z, then x R z.

Example :

- The identity relation I on a set P is partial order relation.
- On the set of real numbers $\mathbb{R}$ the relation $\leq$ is partial order relation.
- The relation "is a divisor of" defines partial order on the set of natural numbers N.

CHECK YOUR GRASP — RELATIONS — EXERCISE-I

1. If R is a relation from a finite set A having m elements to a finite set B having n elements, then the number of relations from A to B is-
(1) 2^{mn} (2) $2^{mn}-1$ (3) 2mn (4) m^n
RT0001

2. In the set A = {1, 2, 3, 4, 5}, a relation R is defined by R = {(x, y) | x, y ∈ A and x < y}. Then R is-
(1) Reflexive (2) Symmetric
(3) Transitive (4) None of these
RT0002

3. For real numbers x and y, we write $x\,R\,y \Leftrightarrow x - y + \sqrt{2}$ is an irrational number. Then the relation R is-
(1) Reflexive (2) Symmetric
(3) Transitive (4) none of these
RT0003

4. Let X = {1, 2, 3, 4} and Y = {1, 3, 5, 7, 9}. Which of the following is relations from X to Y-
(1) R_1 = {(x, y) | y = 2 + x, x ∈ X, y ∈ Y}
(2) R_2 = {(1, 1), (2, 1), (3, 3), (4, 3), (5, 5)}
(3) R_3 = {(1, 1), (1, 3), (3, 5), (3, 7), (5, 7)}
(4) R_4 = {(1, 3), (2, 5), (2, 4), (7, 9)} **RT0004**

5. Let L denote the set of all straight lines in a plane. Let a relation R be defined by $\alpha\,R\,\beta \Leftrightarrow \alpha \perp \beta$, α, β ∈ L. Then R is-
(1) Reflexive (2) Symmetric
(3) Transitive (4) none of these **RT0005**

6. Let R be a relation defined in the set of real numbers by a R b ⇔ 1 + ab > 0. Then R is-
(1) Equivalence relation (2) Transitive
(3) Symmetric (4) Anti-symmetric
RT0006

7. Which one of the following relations on R is equivalence relation-
(1) $x\,R_1 y \Leftrightarrow |x| = |y|$ (2) $x\,R_2 y \Leftrightarrow x \geq y$
(3) $x\,R_3 y \Leftrightarrow x \mid y$ (4) $x\,R_4 y \Leftrightarrow x < y$
RT0007

8. Two points P and Q in a plane are related if OP = OQ, where O is a fixed point. This relation is-
(1) Reflexive but not symmetric
(2) Symmetric but not transitive
(3) An equivalence relation
(4) none of these **RT0008**

9. The relation R defined in A = {1, 2, 3} by a R b if $|a^2 - b^2| \leq 5$. Which of the following is false-
(1) R = {(1, 1), (2, 2), (3, 3), (2, 1), (1, 2), (2, 3), (3, 2)
(2) $R^{-1} = R$
(3) Domain of R = {1, 2, 3}
(4) Range of R = {5} **RT0009**

10. Let a relation R is the set N of natural numbers be defined as (x, y) ∈ R if and only if $x^2 - 4xy + 3y^2 = 0$ for all x, y ∈ N. The relation R is-
(1) Reflexive
(2) Symmetric
(3) Transitive
(4) An equivalence relation **RT0010**

11. Let A = {2, 3, 4, 5} and let R = {(2, 2), (3, 3), (4, 4), (5, 5), (2, 3), (3, 2), (3, 5), (5, 3)} be a relation in A. Then R is-
(1) Reflexive and transitive
(2) Reflexive and symmetric
(3) Reflexive and antisymmetric
(4) none of these **RT0011**

12. If A = {2, 3} and B = {1, 2}, then A ×B is equal to-
(1) {(2, 1), (2, 2), (3, 1), (3, 2)}
(2) {(1, 2), (1, 3), (2, 2), (2, 3)}
(3) {(2, 1), (3, 2)}
(4) {(1, 2), (2, 3)} **RT0012**

13. Let R be a relation over the set N × N and it is defined by (a, b) R (c, d) ⇒ a + d = b + c. Then R is-
(1) Reflexive only
(2) Symmetric only
(3) Transitive only
(4) An equivalence relation **RT0013**

14. Let N denote the set of all natural numbers and R be the relation on N × N defined by (a, b) R (c, d) if ad (b + c) = bc(a + d), then R is-
(1) Symmetric only
(2) Reflexive only
(3) Transitive only
(4) An equivalence relation **RT0014**

15. If A = {1, 2, 3}, B = {1, 4, 6, 9} and R is a relation from A to B defined by 'x is greater than y'. Then range of R is-
(1) {1, 4, 6, 9} (2) {4, 6, 9}
(3) {1} (4) none of these
RT0015

16. Let L be the set of all straight lines in the Euclidean plane. Two lines ℓ_1 and ℓ_2 are said to be related by the relation R if ℓ_1 is parallel to ℓ_2. Then the relation R is-
(1) Reflexive (2) Symmetric
(3) Transitive (4) Equivalence **RT0016**

17. A and B are two sets having 3 and 4 elements respectively and having 2 elements in common. The number of relations which can be defined from A to B is-
(1) 2^5 (2) $2^{10} - 1$
(3) $2^{12} - 1$ (4) 2^{12} **RT0017**

18. For n, m $\in$ N, n | m means that n is a factor of m, the relation | is-
(1) reflexive and symmetric
(2) transitive and symmetric
(3) reflexive, transitive and symmetric
(4) reflexive, transitive and not symmetric **RT0018**

19. Let R = {(x, y) : x, y $\in$ A, x + y = 5} where A = {1, 2, 3, 4, 5} then
(1) R is not reflexive, symmetric and not transitive
(2) R is an equivalence relation
(3) R is reflexive, symmetric but not transitive
(4) R is not reflexive, not symmetric but transitive
RT0019

20. Let R be a relation on a set A such that $R = R^{-1}$ then R is-
(1) reflexive
(2) symmetric
(3) transitive
(4) none of these **RT0020**

21. Let x, y $\in$ I and suppose that a relation R on I is defined by x R y if and only if $x \le y$ then
(1) R is partial order ralation
(2) R is an equivalence relation
(3) R is reflexive and symmetric
(4) R is symmetric and transitive **RT0021**

22. Let R be a relation from a set A to a set B, then-
(1) $R = A \cup B$ (2) $R = A \cap B$
(3) $R \subseteq A \times B$ (4) $R \subseteq B \times A$ **RT0022**

23. Given the relation R = = {(1, 2), (2, 3)} on the set A = {1, 2, 3}, the minimum number of ordered pairs which when added to R make it an equivalence relation is- **RT0023**
(1) 5 (2) 6 (3) 7 (4) 8

24. Let P = {(x, y) | $x^2 + y^2 = 1$, x, y $\in$ R} Then P is-
(1) reflexive (2) symmetric
(3) transitive (4) anti-symmetric
RT0024

25. Let X be a family of sets and R be a relation on X defined by 'A is disjoint from B'. Then R is-
(1) reflexive (2) symmetric
(3) anti-symmetric (4) transitive **RT0025**

26. In order that a relation R defined in a non-empty set A is an equivalence relation, it is sufficient that R
(1) is reflexive
(2) is symmetric
(3) is transitive **RT0026**
(4) possesses all the above three properties

27. If R is an equivalence relation in a set A, then R^{-1} is-
(1) reflexive but not symmetric
(2) symmetric but not transitive
(3) an equivalence relation
(4) none of these **RT0027**

28. Let A = {p, q, r}. Which of the following is an equivalence relation in A ?
(1) R_1 = {(p, q), (q, r), (p, r), (p, p)}
(2) R_2 = {(r, q) (r, p), (r, r), (q, q)}
(3) R_3 = {(p, p), (q, q), (r, r), (p, q)}
(4) none of these **RT0028**

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	**1**	**3**	**1**	**1**	**2**	**3**	**1**	**3**	**4**	**1**	**2**	**1**	**4**	**4**	**3**
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28		
Ans.	**4**	**4**	**4**	**1**	**2**	**1**	**3**	**3**	**2**	**2**	**4**	**3**	**4**		

PREVIOUS YEAR QUESTIONS RELATIONS EXERCISE-II

1. Let R = {(1, 3), (4, 2), (2, 4), (2, 3), (3, 1)} be a releation on the set A = {1, 2, 3, 4}. The relation R is- **[AIEEE - 2004]**

(1) transitive (2) not symmetric

(3) reflexive (4) a function **RT0029**

2. Let R = {(3, 3), (6, 6), (9, 9), (12, 12), (6, 12), (3, 9), (3, 12), (3, 6)} be relation on the set A = {3, 6, 9, 12). The relation is- **[AIEEE - 2005]**

(1) reflexive and transitive only

(2) reflexive only

(3) an equilvalence relation

(4) reflexive and symmetric only **RT0030**

3. Let W denote the words in the English dictionary. Define the relation R by : R = {(x, y) ∈ W × W | the words x and y have at least one letter in common}. Then R is- **[AIEEE - 2006]**

(1) reflexive, symmetric and not transitive

(2) reflexive, symmetric and transitive

(3) reflexive, not symmetric and transtive

(4) not reflexive, symmetric and transitive **RT0031**

4. Consider the following relations :-

R = {(x, y) | x, y are real numbers and x = wy for some rational number w} ;

S = $\{(\frac{m}{n}, \frac{p}{q})$ | m, n, p and q are integers such that n, q ≠ 0 and qm = pn}.

Then : **[AIEEE - 2010]**

(1) R is an equivalence relation but S is not an equivalence relation

(2) Neither R nor S is an equivalence relation

(3) S is an equivalence relation but R is not an equivalence relation **RT0032**

(4) R and S both are equivalence relations

5. Let R be the set of real numbers. **[AIEEE - 2011]**

Statement-1:

A = {(x, y) ∈ R × R : y – x is an integer} is an equivalence relation on R.

Statement-2:

B = {(x, y) ∈ R × R : x = αy for some rational number α} is an equivalence relation on R.

(1) Statement-1 is true, Statement-2 is false.

(2) Statement-1 is false, Statement-2 is true

(3) Statement-1 is true, Statement-2 is true; Statement-2 is a correct explanation for Statement-1

(4) Statement-1 is true, Statement-2 is true; Statement-2 is **not** a correct explanation for Statement-1. **RT0033**

6. Let Z be the set of integers.

If $A = \left\{x \in Z : 2^{(x+2)(x^2-5x+6)} = 1\right\}$

and $B = \{x \in Z : -3 < 2x - 1 < 9\}$,

then the number of subsets of the set A × B, is: **[JEE(Main) 19]**

(1) 2^{18} (2) 2^{10} (3) 2^{15} (4) 2^{12}

RT0034

ANSWER KEY

Que.	1	2	3	4	5	6								
Ans.	2	1	1	3	1	3								

HEIGHT AND DISTANCE

1. INTRODUCTION :

One of the important application of trigonometry is in finding the height and distance of the point which are not directly measurable. This is done with the help of trigonometric ratios.

2. ANGLES OF ELEVATION AND DEPRESSION :

Let OP be a horizontal line in the vertical plane in which an object R is given and let OR be joined.

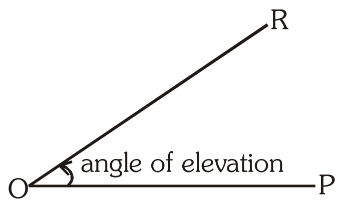


Fig. (a)

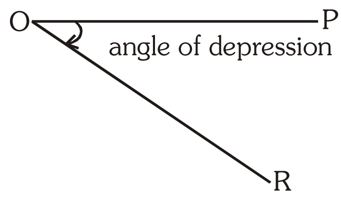


Fig. (b)

In Fig. (a), where the object R is above the horizontal line OP, the angle POR is called the angle of elevation of the object R as seen from the point O. In Fig. (b) where the object R is below the horizontal line OP, the angle POR is called the angle of depression of the object R as seen from the point O.

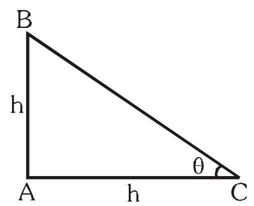
Remark :

Unless stated to the contrary, it is assumed that the height of the observer is neglected, and that the angles of elevation are measured from the ground.

Ex.1 Find the angle of elevation of the sum when the length of shadow of a vertical pole is equal to its height.

Sol. Let height of the pole AB = h and

length of the shadow of the Pole (AC) = h



In ΔABC $\tan\theta = \frac{AB}{AC} = \frac{h}{h} = 1 \Rightarrow \tan\theta = 1$

$\Rightarrow \tan\theta = \tan 45° \Rightarrow \theta = 45°$

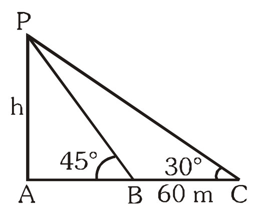
Ex.2 The shadow of the tower standing on a level ground is found to be 60 metres longer when the sun's altitude is 30° than when it is 45°. The height of the tower is-

(1) 60 m (2) $30(\sqrt{3}-1)$m (3) $60\sqrt{3}$ m (4) $30(\sqrt{3}+1)$ m.

Sol.(4) $AC = h\cot 30° = \sqrt{3}\,h$

$AB = h\cot 45° = h$

$\therefore\ BC = AC - AB = h(\sqrt{3}-1) \Rightarrow 60 = h(\sqrt{3}-1)$

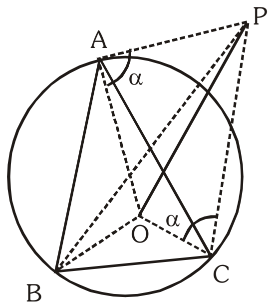


$\therefore\ h = \frac{60}{\sqrt{3}-1} = \frac{60(\sqrt{3}+1)}{3-1} = 30(\sqrt{3}+1)$

Ex.3 The angle of elevation of the tower observed from each of the three point A,B,C on the ground, forming a triangle is the same angle α . If R is the circum - radius of the triangle ABC, then the height of the tower is -

(1) R sin α (2) R cos α (3) R cot α (4) R tan α

Sol.(4) The tower makes equal angles at the vertices of the triangle, therefore foot of the tower is at the circumcentre.



From Δ OCP, OP is perpendicular to OC.

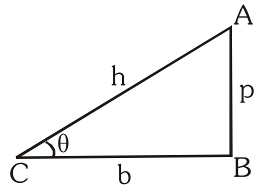
$\angle OCP = \alpha$

so $\tan\alpha = \dfrac{OP}{OA} \Rightarrow OP = OA\tan\alpha$

$OP = R\tan\alpha$

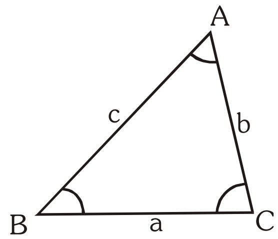
3. SOME USEFUL RESULTS :

- In a triangle ABC,



$\sin\theta = \dfrac{p}{h}$, $\cos\theta = \dfrac{b}{h}$, $\tan\theta = \dfrac{P}{b}$

- In any triangle ABC,

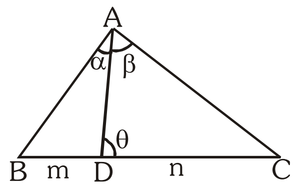


$\dfrac{a}{\sin A} = \dfrac{b}{\sin B} = \dfrac{c}{\sin C}$ [By sine rule]

or cosine formula

i.e. $\cos A = \dfrac{b^2+c^2-a^2}{2bc}$; $\cos B = \dfrac{a^2+c^2-b^2}{2ac}$, $\cos C = \dfrac{a^2+b^2-c^2}{2ab}$

- In any triangle ABC

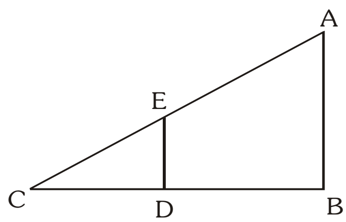


if BD : DC = m : n and $\angle BAD = \alpha$

$\angle CAD = \beta$ and $\angle ADC = \theta$,

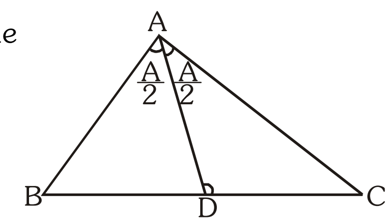
then $(m+n)\cot\theta = m\cot\alpha - n\cot\beta$

- In a triangle ABC, if DE | | AB



then, $\dfrac{AB}{DE} = \dfrac{BC}{DC}$

- In a triangle the internal bisector of an angle divides the opposite side in the ratio of the arms of the angle



$\therefore \dfrac{BD}{DC} = \dfrac{AB}{AC}$

- In an isosceles triangle the median is perpendicular to the base

SOLVED EXAMPLES

Ex.1 A tower subtends an angle of 30° at a point on the same level as its foot, and at a second point h m above the first, the depression of the foot of tower is 60°. The height of the tower is.

(1) h m (2) 3h m (3) $\sqrt{3}$ h m (4) $\frac{h}{3}$ m.

Sol.(4) Let OP be the tower of height x.,A the point on the same level as the foot O of the tower and B be the point h m above A (see Fig.) Then ∠ AOB = 60° and ∠ PAO = 30°. From right-angled triangle AOP, we have

$$OA = x \cot 30°$$

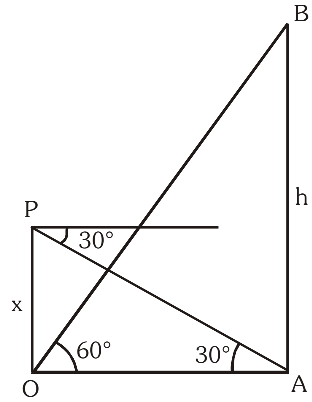
and from right-angled triangle OAB, we have

$$OA = h \cot 60°$$

Therefore, from (1) and (2), we get

$$x \cot 30° = h \cot 60°$$

$$\sqrt{3}\,x = \frac{1}{\sqrt{3}}h \Rightarrow x = \frac{1}{3}h$$



Ex.2 At a point on level ground, the angle of elevation of a vertical tower is found to be such that its tangent is $\frac{5}{12}$. On walking 192 metres towards the tower, the tangent of the angle of elevation is $\frac{3}{4}$. Find the height of the tower.

Sol. Let AB be the tower and let the angle of elevation of its top at C be α. Let D be a point at a distance of 192 metres from C such that the angle of elevation of the top of the tower at D be β.

Let h be the height of the tower and AD = x,

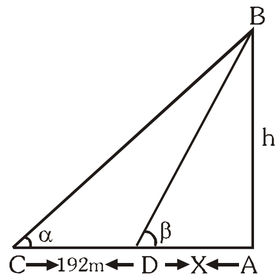
It is given that $\tan\alpha = \frac{5}{12}$ and $\tan\beta = \frac{3}{4}$.

In ΔABC, we have

$$\tan\alpha = \frac{AB}{AC} \Rightarrow \tan\alpha = \frac{h}{192+x} \Rightarrow \frac{5}{12} = \frac{h}{192+x} \quad \text{..... (i)}$$

In ΔABD, we have

$$\tan\beta = \frac{AB}{AD} \Rightarrow \tan\beta = \frac{h}{x} \Rightarrow \frac{3}{4} = \frac{h}{x} \quad \text{..... (ii)}$$



We have to find h. This means that we have to eliminate x from (i) and (ii).

From (ii), we have $3x = 4h \Rightarrow x = \frac{4h}{3}$

Substituting this value of x in (i), we get

$$\frac{5}{12} = \frac{h}{192 + 4h/3} \Rightarrow 5\left(192 + \frac{4h}{3}\right) = 12h$$

$$\Rightarrow 5(576 + 4h) = 36h \Rightarrow 2880 + 20h = 36h$$

$$\Rightarrow 16h = 2880 \Rightarrow h = \frac{2880}{16} = 180$$

Hence, height of tower = 180 metres.

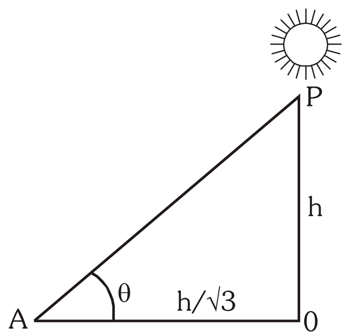
Ex.3 Let α be the solution of $16^{\sin^2\theta}+16^{\cos^2\theta}=10$ in (0, π/4). If the shadow of a vertical pole is $\frac{1}{\sqrt{3}}$ of its height, then then the altitude of the sun is-

(1) α (2) $\frac{\alpha}{2}$ (3) 2α (4) $\frac{\alpha}{3}$

Sol. We have $16^{\sin^2\theta}+16^{\cos^2\theta}=10$

$\Rightarrow 16^{\sin^2\theta}+16^{1-\sin^2\theta}=10 \Rightarrow x+\frac{16}{x}=10$, where $x=16^{\sin^2\theta}$

$\Rightarrow x = 2, 8 \Rightarrow 16^{\sin^2\theta}=2, 8$



$\Rightarrow 2^{4\sin^2\theta}=2, 2^3$

$\Rightarrow 4\sin^2\theta = 2, 3$

$\Rightarrow \sin^2\theta = \frac{1}{2}, \left(\frac{\sqrt{3}}{2}\right)^2 \Rightarrow \theta = \frac{\pi}{6}, \frac{\pi}{3}$

$\therefore \alpha = \frac{\pi}{6}$

Let the altitude of the sun be θ. Then,

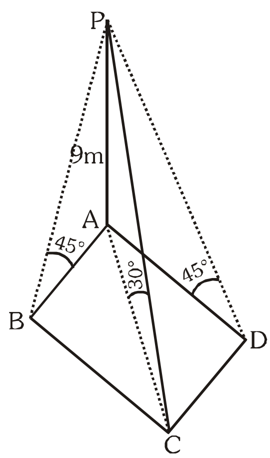
$$\tan\theta = \frac{h}{\frac{h}{\sqrt{3}}} \Rightarrow \tan\theta = \sqrt{3} \Rightarrow \theta = \frac{\pi}{3} \Rightarrow \theta = 2\alpha$$

Ex.4 A vertical lamp-post of height 9 metres stands at the corner of a rectangular field. The angle of elevation of its top from the farthest corner is 30°, while from another corner it is 45°. The area of the field is-

(1) $81\sqrt{2}$ m² (2) $9\sqrt{2}$ m² (3) $81\sqrt{3}$ m² (4) $9\sqrt{3}$ m²

Sol. Let AP be the lamp-post of 9 m standing at corner A of the rectangular field ABCD.

In Δ's BAP and CAP, we have



$$\tan 45° = \frac{PA}{BA} \text{ and } \tan 30° = \frac{PA}{AC}$$

$\Rightarrow$ BA = 9 m and AC = $9\sqrt{3}$ m

$\therefore BC = \sqrt{AC^2 - AB^2} = \sqrt{243-81} = \sqrt{162} = 9\sqrt{2}$ m

Hence, area of the field = AB × BC = $9 \times 9\sqrt{2}$ m² = $81\sqrt{2}$ m²

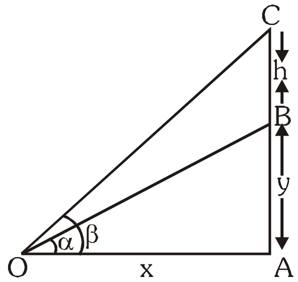
Ex.5 A vertical tower stands on a horizontal plane and is surmounted by a vertical flag staff of height h. At a point on the plane, the angle of elevation of the bottom and the top of the flag staff are α and β respectively. Prove that the height of tower is $\dfrac{h\tan\alpha}{\tan\beta - \tan\alpha}$.

Sol. Let AB be the tower and BC be the flag staff. Let O be a point on the plane containing the foot of the tower such that the angles of elevation of the bottom B and top C of the flagstaff at O are α and β respectively. Let OA = x metres, AB = y metres and BC = h metres.

In ΔOAB, we have

$$\cot\alpha = \frac{OA}{AB} \Rightarrow \cot\alpha = \frac{x}{y}$$

$$\Rightarrow x = y\cot\alpha \qquad \text{...(i)}$$



In ΔOAC, we have

$$\cot\beta = \frac{x}{y+h}$$

$$\Rightarrow x = (y+h)\cot\beta \qquad \text{...(ii)}$$

Equating the values of x from (i) and (ii), we get

$y\cot\alpha = (y+h)\cot\beta$

$\Rightarrow$ $y\cot\alpha - y\cot\beta = h\cot\beta$

$\Rightarrow$ $y(\cot\alpha - \cot\beta) = h\cot\beta$

$$\Rightarrow \quad y = \frac{h\cot\beta}{\cot\alpha - \cot\beta} \Rightarrow y = \frac{h/\tan\beta}{\dfrac{1}{\tan\alpha} - \dfrac{1}{\tan\beta}} = \frac{h\tan\alpha}{\tan\beta - \tan\alpha}$$

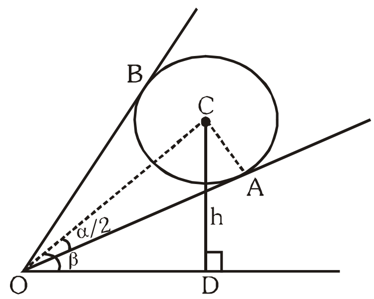
Ex.6 A spherical ball of diameter-δ subtends an angle α at the eye of an observer when the elevation of its centre is β. Prove that the height of the centre of the ball is $\dfrac{1}{2}\delta\sin\beta\operatorname{cosec}\left(\dfrac{\alpha}{2}\right)$.

Sol. O is the position of eye.

As is clear from figure, from ΔODC,

$$OC = \frac{h}{\sin\beta}$$

From ΔOAC,



$$\sin\frac{\alpha}{2} = \frac{CA}{OC} = \frac{\dfrac{\delta}{2}}{h/\sin\beta}$$

$$\Rightarrow \quad h = \frac{1}{2}\delta\sin\beta.\operatorname{cosec}\frac{\alpha}{2}.$$

CHECK YOUR GRASP — HEIGHTS AND DISTANCE — EXERCISE-I

1. An aeroplane flying at a height 300 metre above the ground passes vertically above another plane at an instant when the angles of elevation of the two planes from the same point on the ground are 60° and 45° respectively. Then the height of the lower plane from the ground in metres is-

(1) $100\sqrt{3}$ (2) $100/\sqrt{3}$
(3) 50 (4) $150(\sqrt{3}+1)$.

HD0003

2. From the top of the cliff 300 metres heigh, the top of a tower was observed at an angle of depression 30° and from the foot of the tower the top of the cliff was observed at an angle of elevation 45°. The height of the tower is -

(1) $50(3-\sqrt{3})$m (2) $200(3-\sqrt{3})$m
(3) $100(3-\sqrt{3})$m (4) None of these

HD0005

3. The angles of elevation of the top of a tower at the top and the foot of a pole of height 10 m are 30° and 60° respectively. The height of the tower is-
(1) 10 m (2) 15 m
(3) 20 m (4) None of these

HD0006

4. From the top of a light house 60 m high with its base at sea level the angle of depression of a boat is 15°. The distance of the boat from the light house is-

(1) $60\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$m (2) $60\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$m

(3) $30\left(\frac{\sqrt{3}-1}{\sqrt{3}+1}\right)$m (4) $30\left(\frac{\sqrt{3}+1}{\sqrt{3}-1}\right)$m

HD0008

5. A flag staff on the top of the tower 80 meter high, subtends an angle $\tan^{-1}\left(\frac{1}{9}\right)$ at a point on the ground 100 meters away from the foot of the tower. Find the height of the flag-staff -
(1) 20 m (2) 30 m (3) 25 m (4) 35 m

HD0012

6. A person walking along a straight road observes that a two points 1 km apart, the angles of elevation of a pole in front of him are 30° and 75°. The height of the pole is -

(1) $250\left(\sqrt{3}+1\right)$ m (2) $250\left(\sqrt{3}-1\right)$ m
(3) $225\left(\sqrt{2}-1\right)$ m (4) $225\left(\sqrt{2}+1\right)$ m

HD0013

7. An observer in a boat finds that the angle of elevation of a tower standing on the top of a cliff is 60° and that of the top of cliff is 30°. If the height of the tower be 60 meters, then the height of the cliff is-
(1) 30 m (2) $60\sqrt{3}$ m
(3) $20\sqrt{3}$ m (4) None of these

HD0014

8. ABCD is a square plot. The angle of elevation of the top of a pole standing at D from A and C is 30° and that from B is θ, then tan θ is equal to -
(1) $\sqrt{6}$ (2) $1/\sqrt{6}$
(3) $\sqrt{3}/\sqrt{2}$ (4) $\sqrt{2}/\sqrt{3}$ HD0015

9. The angle of elevation of a ladder against a wall is 58° and the length of foot of the ladder is 9.6 m from the wall. Then the length of the ladder is -
[cos 58° = 0.5299]
(1) 18.11 m (2) 16.11 m
(3) 17.11 m (4) 19.11 m HD0016

10. From the top of a tower, the angle of depression of a point P on the ground is 30°. If the distance of the point P from the tower be 24 meters then height of the tower is.
(1) 12 m (2) $8\sqrt{3}$ m
(3) $24\sqrt{3}$ m (4) $12\sqrt{3}$ m HD0017

11. A tower subtends an angle of 30° at a point on the same level as the foot of the tower. At a second point, h metre above first, point the depression of the foot of the tower is 60°, the horizontal distance of the tower from the points is
(1) h cos 60° (2)(h/3) cot 30°
(3) (h/3) cot 60° (4) h cot 30° HD0018

12. A kite is flying with the string inclined at 75° to the horizon. If the length of the string is 25 m, the height of the kite is-
(1) $(25/2)(\sqrt{3}-1)$ (2) $(25/4)(\sqrt{3}+1)$
(3) $(25/4)(\sqrt{3}+1)^2$ (4) $(25/4)(\sqrt{6}+\sqrt{2})$

HD0020

13. A 6-ft tall man finds that the angle of elevation of the top of a 24-ft-high pillar and the angle of depression of its base are complementary angles. The distance of the man from the pillar is-
(1) $2\sqrt{3}$ ft (2) $8\sqrt{3}$ ft HD0022
(3) $6\sqrt{3}$ ft (4) None of these

14. A flagstaff stands vertically on a pillar, the height of the flagstaff being double the height of the pillar. A man on the ground at a distance finds that both the pillar and the flagstaff subtend equal angles at his eyes. The ratio of the height of the pillar and the distance of the man from the pillar is- HD0023
(1) $\sqrt{3}:1$ (2) 1 : 3 (3) $1:\sqrt{3}$ (4) $\sqrt{3}:2$

15. The angle of elevation of the top of an incomplete vertical pillar at a horizontal distance of 50 m from its base is 45°. If the angle of elevation of the top of the complete pillar the same point is to be 60°, then the height of the incomplete pillar is to be increased by-

(1) $50(\sqrt{3}-1)$ m (2) $50(\sqrt{3}+1)$ m
(3) 50 m (4) $25\sqrt{2}$ m. **HD0025**

16. A vertical tower stands on a declivity which is inclined at 15° to the horizon. From the foot of the tower a man ascends the declivity for 80 feet and then finds that tower subtends an angle of 30°. The height of the tower is-

(1) $20(\sqrt{6}-\sqrt{2})$ (2) $40(\sqrt{6}-\sqrt{2})$
(3) $40(\sqrt{6}+\sqrt{2})$ (4) None of these

HD0026

17. AB is a vertical pole. The point A of pole AB is on the level ground. C is the middle point of AB. P is a point on the level ground. The portion BC substends an angle β at P. If AP = n AB, then tan β is equal to-

(1) $\frac{n}{2n^2+1}$ (2) $\frac{n}{n^2-1}$
(3) $\frac{n}{n^2+1}$ (4) None of these **HD0027**

18. The top of a hill observed from the top and bottom of a building of height h is at angles of elevation p and q respectively. The height of the hill is -

(1) $\frac{h\cot q}{\cot q-\cot p}$ (2) $\frac{h\cot p}{\cot p-\cot q}$
(3) $\frac{h\tan p}{\tan p-\tan q}$ (4) None of these

HD0028

19. A and B are two points 30 m apart in a line on the horizontal plane through the foot of a tower lying on opposite sides of the tower. If the distance of the top of the tower from A and B are 20 m and 15 m respectively, the angle of elevation of the top of the tower at A is-

(1) $\cos^{-1}(43/48)$ (2) $\sin^{-1}(43/48)$
(3) $\cos^{-1}(29/36)$ (4) $\sin^{-1}(29/36)$

HD0029

20. A vertical pole subtends an angle $\tan^{-1}(1/2)$ at a point P on the ground. The angle subtended by the upper half of the pole at the point P is-

(1) $\tan^{-1}(1/4)$ (2) $\tan^{-1}(2/9)$
(3) $\tan^{-1}(1/8)$ (4) $\tan^{-1}(2/3)$ **HD0030**

21. The angle of elevation of the top of a tower standing on a horizontal plane from a point A is α. After walking a distance d towards the foot of the tower, the angle of elevation is found to be β. The height of the tower is-

(1) $\frac{d\sin\alpha\sin\beta}{\sin(\beta-\alpha)}$ (2) $\frac{d\sin\alpha\sin\beta}{\sin(\alpha-\beta)}$
(3) $\frac{d\sin(\beta-\alpha)}{\sin\alpha\sin\beta}$ (4) $\frac{d\sin(\alpha-\beta)}{\sin\alpha\sin\beta}$ **HD0031**

22. The angle of elevation of the top of two vertical towers as seen from the middle point of the line joining the foot of the towers are 60° and 30° respectively. The ratio of the height of the towers is-

(1) 2 : 1 (2) $\sqrt{3}:1$ (3) 3 : 2 (4) 3 : 1

HD0032

23. A person walking along a st. road towards a hill observes at two points, distance $\sqrt{3}$ km, the angles of elevation of the hill to be 30° and 60°. The height of the hill is-

(1) $\frac{3}{2}$ km (2) $\sqrt{\frac{2}{3}}$ km
(3) $\frac{\sqrt{3}+1}{2}$ km (4) $\sqrt{3}$ km **HD0033**

24. The length of the shadow of a vertical pole of height h, thrown by the sun's rays at three different moments are h, 2h and 3h. The sum of the angles of elevation of the rays at these three moments is equal to-

(1) $\frac{\pi}{2}$ (2) $\frac{\pi}{3}$ (3) $\frac{\pi}{4}$ (4) $\frac{\pi}{6}$

HD0034

25. A man standing on a horizontal plane, observes the angle of elevation of the top of a tower to be α. After walking a distance equal to double the height of the tower, the angle of elevation becomes 2α, then α is -

(1) $\frac{\pi}{18}$ (2) $\frac{\pi}{12}$ (3) $\frac{\pi}{6}$ (4) $\frac{\pi}{2}$

HD0039

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	**1**	**3**	**2**	**2**	**1**	**1**	**1**	**2**	**1**	**2**	**2**	**4**	**3**	**3**	**1**
Que.	16	17	18	19	20	21	22	23	24	25					
Ans.	**2**	**1**	**2**	**1**	**2**	**1**	**4**	**1**	**1**	**2**					

PREVIOUS YEAR QUESTIONS HEIGHTS AND DISTANCE EXERCISE-II

1. The upper $\frac{3}{4}$th portion of a vertical pole subtends an an angle $\tan^{-1}\frac{3}{5}$ at a point in the horizontal plane through its foot and at a distance 40 m from the foot. Approximate height of the vertical pole is - **[AIEEE-2002]**

(1) 80 m (2) 20 m (3) 40 m (4) 60 m

HD0035

2. A person standing on the bank of a river observes that the angle of elevation of the top of a tree on the opposite bank of the river is 60° and when he retires 40 meters away from the tree the angle of elevation becomes 30°. The breadth of the river is- **[AIEEE-2004]**

(1) 20 m (2) 30 m (3) 40 m (4) 60 m

HD0036

3. A tower stands at the centre of a circular park. A and B are two points on the boundary of the park such that AB (=a) subtends an angle of 60° at the foot of the tower, and the angle of elevation of the top of the tower from A or B is 30°. The height of the tower is- **[AIEEE-2007]**

(1) $2a/\sqrt{3}$ (2) $2a\sqrt{3}$

(3) $a/\sqrt{3}$ (4) $a\sqrt{3}$ **HD0037**

4. AB is a vertical pole with B at the ground level and A at the top. A man finds that the angle of elevation of the point A from a certain point C on the ground is 60°. He moves away from the pole along the line BC to a point D such that CD = 7 m. From D the angle of elevation of the point A is 45°. Then the height of the pole is- **[AIEEE-2008]**

(1) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}-1}$m (2) $\frac{7\sqrt{3}}{2}(\sqrt{3}+1)$m

(3) $\frac{7\sqrt{3}}{2}(\sqrt{3}-1)$m (4) $\frac{7\sqrt{3}}{2}\frac{1}{\sqrt{3}+1}$m

HD0038

5. ABCD is a trapezium such that AB and CD are parallel and BC ⊥ CD. If ∠ADB = θ, BC = p and CD = q, then AB is equal to **[JEE-MAINS-2013]**

(1) $\frac{(p^2+q^2)\sin\theta}{p\cos\theta+q\sin\theta}$ (2) $\frac{p^2+q^2\cos\theta}{p\cos\theta+q\sin\theta}$

(3) $\frac{p^2+q^2}{p^2\cos\theta+q^2\sin\theta}$ (4) $\frac{(p^2+q^2)\sin\theta}{(p\cos\theta+q\sin\theta)^2}$

HD0040

6. If the angles of elevation of the top of a tower from three collinear points A, B and C, on a line leading to the foot of the tower, are 30°, 45° and 60° respectively, then the ratio, AB : BC, is : **[JEE(Main)-2015]**

(1) $1:\sqrt{3}$ (2) 2 : 3

(3) $\sqrt{3}:1$ (4) $\sqrt{3}:\sqrt{2}$ **HD0041**

7. A man is walking towards a vertical pillar in a straight path, at a uniform speed. At a certain point A on the path, he observes that the angle of elevation of the top of the pillar is 30°. After walking for 10 minutes from A in the same direction, at a point B, he observes that the angle of elevation of the top of the pillar is 60°. Then the time taken (in minutes) by him, form B to reach the pillar, is : **[JEE(Main)-2016]**

(1) 5 (2) 6 (3) 10 (4) 20

HD0042

8. Let a vertical tower AB have its end A on the level ground. Let C be the mid-point of AB and P be a point on the ground such that AP = 2AB. If ∠BPC = β, then tanβ is equal to :- **[JEE(Main)-2017]**

(1) $\frac{4}{9}$ (2) $\frac{6}{7}$ (3) $\frac{1}{4}$ (4) $\frac{2}{9}$

HD0043

9. PQR is a triangular park with PQ = PR = 200 m. A T.V. tower stands at the mid-point of QR. If the angles of elevation of the top of the tower at P,Q and R are respectively 45°, 30° and 30°, then the height of the tower (in m) is - **[JEE(Main)-2018]**

(1) 50 (2) $100\sqrt{3}$

(3) $50\sqrt{2}$ (4) 100 **HD0044**

10. Consider a triangular plot ABC with sides AB=7m, BC=5m and CA=6m. A vertical lamp-post at the mid point D of AC subtends an angle 30° at B. The height (in m) of the lamp-post is: **[JEE(Main)-2019]**

(1) $7\sqrt{3}$ (2) $\frac{2}{3}\sqrt{21}$ (3) $\frac{3}{2}\sqrt{21}$ (4) $2\sqrt{21}$

HD0045

11. Two vertical poles of heights, 20m and 80m stand a part on a horizontal plane. The height (in meters) of the point of intersection of the lines joining the top of each pole to the foot of the other, from this horizontal plane is : **[JEE(Main)-2019]**

(1) 12 (2) 15 (3) 16 (4) 18

HD0047

12. Two poles standing on a horizontal ground are of heights 5m and 10 m respectively. The line joining their tops makes an angle of 15° with ground. Then the distance (in m) between the poles, is :- **[JEE(Main)-2019]**

(1) $\frac{5}{2}(2+\sqrt{3})$ (2) $5(\sqrt{3}+1)$

(3) $5(2+\sqrt{3})$ (4) $10(\sqrt{3}-1)$

HD0048

13. ABC is a triangular park with AB = AC = 100 metres. A vertical tower is situated at the mid-point of BC. If the angles of elevation of the top of the tower at A and B are $\cot^{-1}\left(3\sqrt{2}\right)$ and $\operatorname{cosec}^{-1}\left(2\sqrt{2}\right)$ respectively, then the height of the tower (in metres) is : **[JEE(Main)-2019]**

(1) $10\sqrt{5}$ (2) $\frac{100}{3\sqrt{3}}$ (3) 20 (4) 25

HD0049

14. A 2 m ladder leans against a vertical wall. If the top of the ladder begins to slide down the wall at the rate 25 cm/ sec., then the rate (in cm/sec.) at which the bottom of the ladder slides away from the wall on the horizontal ground when the top of the ladder is 1 m above the ground is : **[JEE(Main)-2019]**

(1) $25\sqrt{3}$ (2) 25

(3) $\frac{25}{\sqrt{3}}$ (4) $\frac{25}{3}$

HD0050

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Ans.	**3**	**1**	**3**	**2**	**1**	**3**	**1**	**4**	**4**	**2**	**3**	**3**	**3**	**3**	

PRINCIPLE OF MATHEMATICAL INDUCTION

Mathematical Induction is a powerful and elegant technique for proving certain types of mathematical statements: general propositions which assert that something is true for all positive integers or for all positive integers greater than some number k.

Let us look at some cases of the type of result that can be proved by induction :

Case-1 : The sum of the first n positive integers {1, 2, 3, . . .} is $\frac{1}{2}n(n + 1)$.

Case-2 : In a convex polygon with n vertices, the greatest number of diagonals that can be drawn is $\frac{1}{2}n(n - 3)$.

As we see, the subject matter of the statements can vary widely. It can include algebra, geometry and many other topics. What is common to all the examples is the number n that appears in the statement. In all cases it is either stated, or implicitly assumed, that n can be any positive integer.

Why do we need proof by induction?

While experimental evidence is insuffficient to gurantee the truthfulness of a statement, it is often not possible to verify the statement for all possisble cases either. For instance, one might assume that $1 + 2 + 3 + ... + n = \frac{1}{2}n(n + 1)$ for all natural numbers n. Of course one easily verifies the statement is true for the first few values of n.

n	1	2	3	4
sum of first n numbers	1	$1+2=3$	$1+2+3=6$	$1+2+3+4=10$
$\frac{1}{2}n(n+1)$	$\frac{1}{2}\times1\times2=1$	$\frac{1}{2}\times2\times3=3$	$\frac{1}{2}\times3\times4=6$	$\frac{1}{2}\times4\times5=10$

Yet we cannot conclude that the statement is true. Maybe it will fail at some unfattempted values, who knows?

To answer this, let's look at another example.

eg **:** If p is any prime number, $2^p - 1$ is also a prime. Let us try some special cases here too.

p	2	3	5	7
2^p-1	3	7	31	127

Since 3, 7, 31, 127 are all primes, we may be satisfied the result is always true. But if we try the next prime, 11, we find that

$$2^{11} - 1 = 2047 = 23 \times 89.$$

So it is not a prime, and our general assertion is therefore FALSE. So how can we verify the statement? A powerful tool is mathematical induction.

What is proof by induction?

One way of thinking about mathematical induction is to regard the statement we are trying to prove as not one proposition, but a whole sequence of propositions, one for each n. The trick used in mathematical induction is to prove the first statement in the sequence, and then prove that if any particular statement is true, then the one after it is also true. This enables us to conclude that all the statements are true.

1. THEOREM-I

If P(n) is statement depending upon n such that

(i) P(1) and is true;

(ii) Assume that P(k) is true for any positive integer $k \in N - \{1\} \Rightarrow$ P(k + 1) is true

then P(n) is true for each $n \in N$

2. THEOREM-II

If P(n) is a statement depending upon n but begining with any positive integer k, then to prove P(n) by Induction, we procced as follows :

(i) Verify the validity of P(n) for n = k.

(ii) Assume that P(m) is true (m > k), $m \in N \Rightarrow P(m + 1)$ is true

Then P(n) is true for each $n \geq k$

3. THEOREM-III

If P(n) is statement depending upon n such that

(i) P(1) and P(2) is true;

(ii) P(k–1) and P(k) is true for some $k \in N - \{1\} \Rightarrow$ P(k + 1) is true

then P(n) is true $\forall\, n \in N$.

SOME USEFUL RESULT BASED ON PRINCIPLE OF MATHEMATICAL INDUCTION :

For any natural number n

(i) $1 + 2 + 3 + ... + n = \Sigma n = \frac{n(n+1)}{2}$

(ii) $1^2 + 2^2 + 3^2 + ... + n^2 = \Sigma n^2 = \frac{n(n+1)(2n+1)}{6}$

(iii) $1^3 + 2^3 + 3^3 + ... + n^3 = \Sigma n^3 = (\Sigma n)^2 = \left\{\frac{n(n+1)}{2}\right\}^2$

(iv) $2 + 4 + 6 + ... + 2n = \Sigma 2n = n(n + 1)$

(v) $1 + 3 + 5 + ... + (2n - 1) = \Sigma(2n - 1) = n^2$

(vi) $x^n - y^n = (x - y)(x^{n-1} + x^{n-2}y + x^{n-3}y^2 + ... + xy^{n-2} + y^{n-1})$

(vii) $x^n + y^n = (x + y)(x^{n-1} - x^{n-2}y + x^{n-3}y^2 + ... - xy^{n-2} + y^{n-1})$

when n is odd positive integer

IMPORTANT TIPS :

(i) Product of r successive integers is divisible by r!

(ii) For $x \neq y$, $x^n - y^n$ is divisible by

(a) x + y, if n is even (b) x – y, if n is even or odd

(iii) $x^n + y^n$ is divisible by

x + y, If n is odd

(iv) For solving objective question related to natural numbers we find out the correct alternative by negative examination of this principle. If the given statement is P(n), then by putting n = 1, 2, 3 .... in P(n) we decide the correct answer. We also use the above formulae established by this principle to find the sum of n terms of a given series. For this we first express T_n as a polynomial in n and then for finding S_n, we put Σ before each term of this polynomial and then use above results of Σn, Σn^2, Σn^3 etc.

SOLVED EXAMPLES

Ex.1 Use the principle of mathematical induction to show that $5^{2n+1} + 3^{n+2}.2^{n-1}$ divisible by 19 for all natural numbers n.

Sol. Let $P(n) = 5^{2n+1} + 3^{n+2}.2^{n-1}$

Step I : For n = 1

$P(1) = 5^{2+1} + 3^{1+2}.2^{1-1}$

= 125 + 27

= 152, which is divisible by 19.

Therefore, the result is true for n = 1.

Step II : Assume that the result is true for n = k, i.e. $P(k) = 5^{2k+1} + 3^{k+2}, 2^{k-1}$ is divisible by 19.

$\Rightarrow$ P(k) = 19r, where r is an integer.

Step III : For n = k + 1

$P(k + 1) = 5^{2(k+1)+1} + 3^{k+1+2}.\ 2^{k+1-1}$

$= 5^{2k+3} + 3^{k+3}.2^{k}$

$= 25.5^{2k+1} + 3.3^{k+2}.2.2^{k-1}$

$= 25.5^{2k} + 6.3^{k+2}.2^{k-1}$

Now $25.5^{2k+1} + 6.3^{k+2}2^{k-1} = 25.(5^{2k+1} + 3^{k-2}.2^{k-1}) - 19.3^{k+2}.2^{k-1}$

i.e. $P(k + 1) = 25\ P(k) - 19.3^{k+2}.2^{k-1}$

But we know that P(k) is divisible by 19. Also $19.3^{k+2}.2^{k-1}$ is clearly divisible by 19.

Hence P(k + 1) is divisible by 19. This shows that the result is true for n = k + 1. Hence by the priciniple of mathematical induction, the result is true for all $n \in N$.

Ex.2. Use the principle of mathematical induction to show that $1.3 + 2.4 + ... + n.(n+2) = \frac{1}{6}n(n+1)(2n+7)$.

Sol. Let P(n) : $1.3+ 2.4 +.....+n.(n+2) = \frac{1}{6}n(n + 1)(2n + 7)$

Step I : For n = 1

LHS of P(1) = $1.3 = 3 = \frac{1}{6}.1.2.9 = \frac{1}{6}.1(1+1)(2.1+7)$ = RHS of P(1)

So P(1) is true

Step II : Now assume P(k) is true, for some natural number k, i.e

$$1.3 + 2.4 +.....+k.(k + 2) = \frac{1}{6}k(k + 1)(2k + 7).$$

Now deduce P(k + 1).

LHS of P(k + 1) = 1.3 + 2.4+......+k.(k+2) + (k + 1). (k + 1 + 2)

= (LHS of P(k)) + (k + 1) (k + 3)

= (RHS of P(k)) + (k + 1) (k + 3), (by inductive assumption)

$$= \frac{1}{6}k(k+1)(2k+7)+(k+1)(k+3)$$

$$= \frac{1}{6}(k+1)(k(2k+7)+6(k+3))$$

$$= \frac{1}{6}(k+1)(2k^2+13k+18)$$

$= \frac{1}{6}(k+1)(k+2)(2k+9)$

$= \frac{1}{6}(k+1)(k+1+1)(2(k+1)+7)$

= RHS of P(k + 1).

So P(k + 1) is true, if P(k) is true.

Hence by induction P(n) is true for all natural numbers n.

Ex.3 Use the principle of mathematical induction to show that for any positive integer number n, $n^3 + 2n$, is divisible by 3.

Sol. Statement P(n) is defined by $n^3 + 2n$ is divisible 3

Step 1 : We first show that P(1) is true. Let n = 1 and calculate $n^3 + 2n$

$1^3 + 2(1) = 3$

Hence P(1) is true.

Step 2 : We now assume that P(k) is true $k^3 + 2k$ is divisible by 3. is equivalent to

$k^3 + 2k = 3M$, where M is a positive integer.

We now consider the algebraic expression $(k + 1)^3 + 2(k + 1)$; expand it and group like terms.

$(k + 1)^3 + 2(k + 1) = k^3 + 3k^2 + 5k + 3$

$= [k^3 + 2k] + [3k^2 + 3k + 3]$

$= 3M + 3[k^2 + k + 1] = 3[M + k^2 + k + 1]$

Hence $(k + 1)^3 + 2(k + 1)$ is also divisible by 3 and therefore statement P(k + 1) is true.

Ex.4 Prove that $3^n > n^2$ for n = 1, n = 2 and use the mathematical induction to prove that $3^n > n^2$ for n, a positive integer greater than 2.

Sol Statement P(n) is defined by

$3^n > n^2$

Step 1 : We first show that P(1) is true. Let n =1 and calculate 3^1 and 1^2 and compare them

$3^1 = 3$

$1^2 = 1$

3 is greater than 1 and hence P(1) is true.

Let us also show that P(2) is true.

$3^2 = 9$

$2^2 = 4$

Hence P(2) is also true.

Step 2 : We now assume that P(k) is true

$3^k > k^2$

Multiply both sides of the above inequality by 3.

$3 * 3^k > 3* k^2$

The left side is equal to 3^{k+1}. For k > 2, we can write

$k^2 > 2k$ and $k^2 > 1$

We now combine the above inequalities by adding the left hand sides and the right hand sides of the two inequalities.

$2k^2 > 2k + 1$

We now add k^2 to both sides of the above inequality to obtain the inequality

$3k^2 > k^2 + 2k + 1$

Factor the right side we can write

$3*k^2 > (k + 1)^2$

If $3 * 3^k > 3*k^2$ and $3*k^2 > (k + 1)^2$ then

$$3 * 3^k > (k + 1)^2$$

Rewrite the left side as 3^{k+1}

$$3^{k+1} > (k + 1)^2$$

* Which proves that P(k + 1) is true.

Ex.5 Let $\{a_n\}$ be a sequence of natrual numbers such that $a_1 = 5$, $a_2 = 13$and $a_{n+2} = 5a_{n+1} - 6a_n$ for all natural numbers n. Prove that $a_n = 2^n + 3^n$ for all natrual numbers n.

Sol. We first check that $a_1 = 5 = 2^1 + 3^1$ and $a_2 = 13 = 2^2 + 3^2$.

Suppose $a_k = 2^k + 3^k$ and $a_{k+1} = 2^{k+1} + 3^{k+1}$ for some natural number k.

Then $a_{k+2} = 5a_{k+1} - 6a_k$

$= 5(2^{k+1} + 3^{k+1}) - 6(2^k + 3^k)$

$= 4.2^k + 9.3^k$

$= 2^{k+2} + 3^{k+2}$

Hence, if the formula holds for n = k and n = k + 1, it also holds for n = k + 2.

By theorem 3, $a_n = 2^n + 3^n$ for all natrual numbers n.

CHECK YOUR GRASP PRINCIPLE OF MATHEMATICAL INDUCTION EXERCISE-I

1. Let P(n) : $n^2 + n$ is an odd integer. It is seen that truth of P(n) $\Rightarrow$ the truth of P(n + 1). Therefore, P(n) is true for all-
(1) $n > 1$ (2) n
(3) $n > 2$ (4) None of these
MI0001

2. If $n \in N$, then $x^{2n-1} + y^{2n-1}$ is divisible by-
(1) $x + y$ (2) $x - y$ (3) $x^2 + y^2$ (4) $x^2 + xy$
MI0002

3. If $n \in N$, then $11^{n+2} + 12^{2n+1}$ is divisible by-
(1) 113 (2) 123
(3) 133 (4) None of these
MI0003

4. If $n \in N$, then $3^{4n+2} + 5^{2n+1}$ is a multiple of-
(1) 14 (2) 16 (3) 18 (4) 20
MI0004

5. For positive integer n, $3^n < n!$ when-
(1) $n \geq 6$ (2) $n > 7$ (3) $n \geq 7$ (4) $n \leq 7$
MI0029

6. If $A = \begin{pmatrix} a & 1 \\ 0 & a \end{pmatrix}$, then for any $n \in N$, A^n equals-
(1) $\begin{pmatrix} na & n \\ 0 & na \end{pmatrix}$ (2) $\begin{pmatrix} a^n & na^{n-1} \\ 0 & a^n \end{pmatrix}$
(3) $\begin{pmatrix} na & 1 \\ 0 & na \end{pmatrix}$ (4) $\begin{pmatrix} a^n & n \\ 0 & a^n \end{pmatrix}$ **MI0030**

7. The sum of n terms of the series
$$\frac{\frac{1}{2}\cdot\frac{2}{2}}{1^3} + \frac{\frac{2}{2}\cdot\frac{3}{2}}{1^3+2^3} + \frac{\frac{3}{2}\cdot\frac{4}{2}}{1^3+2^3+3^3} + \ldots\ldots \text{ is-}$$
(1) $\frac{1}{n(n+1)}$ (2) $\frac{n}{n+1}$ (3) $\frac{n+1}{n}$ (4) $\frac{n+1}{n+2}$
MI0008

8. For all $n \in N$, $7^{2n} - 48n - 1$ is divisible by-
(1) 25 (2) 26 (3) 1234 (4) 2304
MI0009

9. For all positive integral values of n, $3^{2n} - 2n + 1$ is divisible by-
(1) 2 (2) 4 (3) 8 (4) 12
MI0010

10. The smallest positive integer for which the statement $3^{n+1} < 4^n$ holds is-
(1) 1 (2) 2 (3) 3 (4) 4
MI0011

11. For positive integer n, $10^{n-2} > 81n$ when-
(1) $n < 5$ (2) $n > 5$ (3) $n \geq 5$ (4) $n > 6$
MI0012

12. If P is a prime number then $n^p - n$ is divisible by p when n is a
(1) natural number greater than 1
(2) odd number
(3) even number
(4) None of these **MI0013**

13. A student was asked to prove a statement by induction. He proved
(i) P(5) is true and
(ii) Truth of P(n) $\Rightarrow$ truth of p(n + 1), $n \in N$
On the basis of this, he could conclude that P(n) is true for
(1) no $n \in N$ (2) all $n \in N$
(3) all $n \geq 5$ (4) None of these
MI0014

14. **Statement-1 :** $\frac{n^7}{7} + \frac{n^5}{5} + \frac{2n^3}{3} - \frac{n}{105}$ $(\forall\ n \in N)$ is an integer.

Statement-2 : $\frac{n^5}{5} + \frac{n^3}{3} + \frac{7n}{15} + \frac{36}{n}$ $(\forall\ n \in N)$ is an integer.
(1) Statement-I & Statement-II both are correct
(2) Statement-I is correct but Statement-II is incorrect
(3) Statement-II is correct but Statement-I is incorrect
(4) Statement-I & Statement-II both are incorrect.
MI0031

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	
Ans.	**4**	**1**	**3**	**1**	**3**	**2**	**2**	**4**	**1**	**4**	**3**	**1**	**3**	**2**	

PREVIOUS YEAR QUESTIONS PRINCIPLE OF MATHEMATICAL INDUCTION EXERCISE-II

1. The sum of first n terms of the given series $1^2 + 2.2^2 + 3^2 + 2.4^2 + 5^2 + 2.6^2 + ...$ is $\frac{n(n+1)^2}{2}$, when n is even. When n is odd, then sum will be- **[AIEEE-2004]**

(1) $\frac{n(n+1)^2}{2}$ (2) $\frac{1}{2}n^2(n+1)$

(3) $n(n+1)^2$ (4) None **MI0023**

2. Let $S(k) = 1 + 3 + 5 + ... + (2k - 1) = 3 + k^2$, then which of the following is true ? **[AIEEE-2004]**

(1) S(1) is true (2) $S(k) \Rightarrow S(k+1)$

(3) $S(k) \nRightarrow S(k+1)$

(4) Principle of mathematical Induction can be used to prove that formula **MI0024**

3. If $A = \begin{bmatrix} 1 & 0 \\ 1 & 1 \end{bmatrix}$ and $I = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}$, then which one of the following holds for all $n \geq 1$, (by the principal of mathematical induction) **[AIEEE-2005]**

(1) $A^n = nA + (n-1)I$

(2) $A^n = 2^{n-1}A + (n+1)I$

(3) $A^n = nA - (n-1)I$

(4) $A^n = 2^{n-1}A - (n-1)I$ **MI0032**

4. **Statement :1** For every natural number $n \geq 2$

$$\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + ... + \frac{1}{\sqrt{n}} > \sqrt{n}$$

Statement –2 : For every natural number $n \geq 2$,

$\sqrt{n(n+1)} < n+1$. **[AIEEE-2008]**

(1) Statement –1 is false, Statement –2 is true

(2) Statement–1 is true, Statement–2 is false

(3) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1

(4) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for Statement–1

MI0026

5. **Statement - 1:** For each natural number n, $(n + 1)^7 - n^7 - 1$ is divisible by 7.

Statement - 2: For each natural number n, $n^7 - n$ is divisible by 7. **[AIEEE-2011]**

(1) Statement-1 is false, statement-2 is true.

(2) Statement-1 is true, statement-2 is true; Statement-2 is correct explanation for statement-1.

(3) Statement-1 is true, statement-2 is true; Statement-2 is not a correct explanation for statement-1.

(4) Statement-1 is true, statement-2 is false.

MI0027

6. Consider the statement : "P(n): $n^2 - n + 41$ is prime." Then which one of the following is true?

[JEE(Main)-2019]

(1) P(5) is false but P(3) is true

(2) Both P(3) and P(5) are false

(3) P(3) is false but P(5) is true

(4) Both P(3) and P(5) are true **MI0028**

ANSWER KEY

Que.	1	2	3	4	5	6
Ans.	2	2	3	3	2	4

STATISTICS

MEASURES OF CENTRAL TENDENCY :

An average value or a central value of a distribution is the value of variable which is representative of the entire distribution, this representative value are called the measures of central tendency. Generally the following five measures of central tendency.

(a) Mathematical average

(i) Arithmetic mean (ii) Geometric mean (iii) Harmonic mean

(b) Positional average

(i) Median (ii) Mode

1. ARITHMETIC MEAN :

(i) For ungrouped dist. : If $x_1, x_2, ... x_n$ are n values of variate x_i then their A.M. $\overline{x}$ is defined as

$$\overline{x} = \frac{x_1 + x_2 + ... + x_n}{n} = \frac{\sum_{i=1}^{n} x_i}{n}$$

$\Rightarrow$ $\Sigma x_i = n\overline{x}$

(ii) For ungrouped and grouped freq. dist. : If $x_1, x_2, ... x_n$ are values of variate with corresponding frequencies $f_1, f_2, ... f_n$ then their A.M. is given by

$$\overline{x} = \frac{f_1x_1 + f_2x_2 + ... + f_nx_n}{f_1 + f_2 + ... + f_n} = \frac{\sum_{i=1}^{n} f_ix_i}{N}, \quad \text{where } N = \sum_{i=1}^{n} f_i$$

Ex.1 Find the A.M. of the following freq. dist.

x_i	5	8	11	14	17
f_i	4	5	6	10	20

Sol. Here $N = \Sigma f_i = 4 + 5 + 6 + 10 + 20 = 45$

$\Sigma f_ix_i = (5 \times 4) + (8 \times 5) + (11 \times 6) + (14 \times 10) + (17 \times 20) = 606$

$$\therefore \quad \overline{x} = \frac{\Sigma f_ix_i}{N} = \frac{606}{45} = 13.47$$

(iii) By short method : If the value of x_i are large, then the calculation of A.M. by using previous formula is quite tedious and time consuming. In such case we take deviation of variate from an arbitrary point a.

Let $d_i = x_i - a$

$$\therefore \quad \overline{x} = a + \frac{\Sigma f_id_i}{N}, \quad \text{where a is assumed mean}$$

(iv) By step deviation method : Sometime during the application of short method of finding the A.M. If each deviation d_i are divisible by a common number h(let)

Let $u_i = \dfrac{d_i}{h} = \dfrac{x_i - a}{h}$

$$\therefore \quad \overline{x} = a + \left(\frac{\Sigma f_iu_i}{N}\right)h$$

Ex.2 Find the mean of the following freq. dist.

x_i	5	15	25	35	45	55
f_i	12	18	27	20	17	6

Sol. Let assumed mean a = 35, h = 10

here $N = \Sigma f_i = 100$, $u_i = \dfrac{(x_i - 35)}{10}$

$\therefore$ $\Sigma f_i u_i = (12 \times -3) + (18 \times -2) + (27 \times -1) + (20 \times 0) + (17 \times 1) + (6 \times 2) = -70$

$$\therefore \quad \bar{x} = a + \left(\frac{\Sigma f_i u_i}{N}\right) h = 35 + \frac{(-70)}{100} \times 10 = 28$$

(v) Weighted mean : If $w_1, w_2, \ldots\ldots w_n$ are the weights assigned to the values $x_1, x_2, \ldots\ldots x_n$ respectively then their weighted mean is defined as

$$\text{Weighted mean} = \frac{w_1x_1 + w_2x_2 + \ldots.. + w_nx_n}{w_1 + \ldots.. + w_n} = \frac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i}$$

Ex.3 Find the weighted mean of first n natural numbers when their weights are equal to their squares respectively

Sol. $$\text{Weighted Mean} = \frac{1.1^2 + 2.2^2 + \ldots. + n.n^2}{1^2 + 2^2 + \ldots. + n^2} = \frac{1^3 + 2^3 + \ldots.. + n^3}{1^2 + 2^2 + \ldots. + n^2} = \frac{[n(n+1)/2]^2}{[n(n+1)(2n+1)/6]} = \frac{3n(n+1)}{2(2n+1)}$$

(vi) Combined mean : If $\bar{x}_1$ and $\bar{x}_2$ be the means of two groups having n_1 and n_2 terms respectively then the mean (combined mean) of their composite group is given by

$$\text{combined mean} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2}$$

If there are more than two groups then, combined mean $= \dfrac{n_1\bar{x}_1 + n_1\bar{x}_2 + n_3\bar{x}_3 + \ldots.}{n_1 + n_2 + n_3 + \ldots.}$

Ex.4 The mean income of a group of persons is Rs. 400 and another group of persons is Rs. 480. If the mean income of all the persons of these two groups is Rs. 430 then find the ratio of the number of persons in the groups.

Sol. Here $\bar{x}_1 = 400$, $\bar{x}_2 = 480$, $\bar{x} = 430$

$$\because \quad \bar{x} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} \Rightarrow 430 = \frac{400n_1 + 480n_2}{n_1 + n_2}$$

$$\Rightarrow \quad \frac{n_1}{n_2} = \frac{5}{3}$$

(vii) Properties of Arithmetic mean :

- Sum of deviations of variate from their A.M. is always zero i.e. $\Sigma(x_i - \bar{x}) = 0$, $\Sigma f_i(x_i - \bar{x}) = 0$
- Sum of square of deviations of variate from their A.M. is minimum i.e. $\Sigma(x_i - \bar{x})^2$ is minimum
- If $\bar{x}$ is the mean of variate x_i then
  A.M. of $(x_i + \lambda) = \bar{x} + \lambda$
  A.M. of $(\lambda x_i) = \lambda\bar{x}$
  A.M. of $(ax_i + b) = a\bar{x} + b$ (where λ, a, b are constant)
- A.M. is independent of change of assumed mean i.e. it is not effected by any change in assumed mean.

2. MEDIAN :

The median of a series is the value of middle term of the series when the values are written in ascending order. Therefore median, divided an arranged series into two equal parts.

Formulae of median :

(i) For ungrouped distribution : Let n be the number of variate in a series then

$$\text{Median} = \begin{cases} \left(\dfrac{n+1}{2}\right)^{th} \text{term}, & (\text{when n is odd}) \\ \text{Mean of} \left(\dfrac{n}{2}\right)^{th} \text{and} \left(\dfrac{n}{2}+1\right)^{th} \text{terms}, & (\text{when n is even}) \end{cases}$$

(ii) For ungrouped freq. dist. : First we prepare the cumulative frequency (c.f.) column and Find value of N then

$$\text{Median} = \begin{cases} \left(\dfrac{N+1}{2}\right)^{th} \text{term}, & (\text{when N is odd}) \\ \text{Mean of} \left(\dfrac{N}{2}\right)^{th} \text{and} \left(\dfrac{N}{2}+1\right)^{th} \text{terms}, & (\text{when N is even}) \end{cases}$$

(iii) For grouped freq. dist : Prepare c.f. column and find value of $\frac{N}{2}$ then find the class which contain value of c.f. is equal or just greater to N/2, this is median class

$$\therefore \text{ Median} = \ell + \frac{\left(\frac{N}{2}-F\right)}{f} \times h$$

where ℓ — lower limit of median class

f — freq. of median class

F — c.f. of the class preceeding median class

h — Class interval of median class

Ex.5 Find the median of following freq. dist.

class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50
f	8	30	40	12	10

Sol.

class	f_i	c.f.
0 – 10	8	8
10 – 20	30	38
20 – 30	40	78
30 – 40	12	90
40 – 50	10	100

Here $\frac{N}{2} = \frac{100}{2} = 50$ which lies in the value 78 of c.f. hence corresponding class of this c.f. is 20-30 is the median class, so $\ell = 20$, $f = 40$, $F = 38$, $h = 10$

$$\therefore \text{ Median} = \ell + \frac{\left(\frac{N}{2}-F\right)}{f} \times h = 20 + \frac{(50-38)}{40} \times 10 = 23$$

3. MODE :

In a frequency distribution the mode is the value of that variate which have the maximum frequency

Method for determining mode :

(i) For ungrouped dist. : The value of that variate which is repeated maximum number of times

(ii) For ungrouped freq. dist. : The value of that variate which have maximum frequency.

(iii) For grouped freq. dist. : First we find the class which have maximum frequency, this is model calss

$$\therefore \text{ Mode} = \ell + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h$$

where ℓ — lower limit of model class

f_0 — freq. of the model class

f_1 — freq. of the class preceeding model class

f_2 — freq. of the class succeeding model class

h — class interval of model class

Ex. 6 Find the mode of the following frequecy dist

class	0 – 10	10 – 20	20 – 30	30 – 40	40 – 50	50 – 60	60 – 70	70 – 80
f_i	2	18	30	45	35	20	6	3

Sol. Here the class 30–40 has maximum freq. so this is the model class

$\ell = 30$, $f_0 = 45$, $f_1 = 30$, $f_2 = 35$, $h = 10$

$$\therefore \text{ Mode} = \ell + \frac{f_0 - f_1}{2f_0 - f_1 - f_2} \times h = 30 + \frac{45-30}{2\times 45 - 30 - 35} \times 10 = 36$$

4. RELATION BETWEEN MEAN, MEDIAN AND MODE :

In a moderately asymmetric distribution following relation between mean, median and mode of a distribution. It is known as imprical formula.

Mode = 3 Median – 2 Mean

Note (i) Median always lies between mean and mode

(ii) For a symmetric distribution the mean, median and mode are coincide.

5. MEASURES OF DISPERSION :

The dispersion of a statistical distribution is the measure of deviation of its values about the their average (central) value.

It gives an idea of scatteredness of different values from the average value.

Generally the following measures of dispersion are commonly used.

(i) Range (ii) Mean deviation (iii) Variance and standard deviation

(i) Range : The difference between the greatest and least values of variate of a distribution, are called the range of that distribution.

If the distribution is grouped distribution, then its range is the difference between upper limit of the maximum class and lower limit of the minimum class.

$$\text{Also, coefficient of range} = \frac{\text{difference of extreme values}}{\text{sum of extreme values}}$$

Ex.7 Find the range of following numbers 10, 8, 12, 11, 14, 9, 6

Sol. Here greatest value and least value of the distribution are 14 and 6 resp. therefore

Range = 14 – 6 = 8

(ii) Mean deviation (M.D.) : The mean deviation of a distribution is, the mean of absolute value of deviations of variate from their statistical average (Mean, Median, Mode).

If A is any statistical average of a distribution then mean deviation about A is defined as

$$\text{Mean deviation} = \frac{\sum_{i=1}^{n}|x_i - A|}{n} \qquad \text{(for ungrouped dist.)}$$

$$\text{Mean deviation} = \frac{\sum_{i=1}^{n}f_i|x_i - A|}{N} \qquad \text{(for freq. dist.)}$$

Note :- is minimum when it taken about the median

$$\text{Coefficient of Mean deviation} = \frac{\text{Mean deviation}}{A}$$

(where A is the central tendency about which Mean deviation is taken)

Ex.8 Find the mean deviation of number 3, 4, 5, 6, 7

Sol. Here n = 5, $\bar{x} = 5$

$$\therefore \quad \text{Mean deviation} = \frac{\Sigma|x_i - \bar{x}|}{n}$$

$$= \frac{1}{5}[|3-5| + |4-5| + |5-5| + |6-5| + |7-5|]$$

$$= \frac{1}{5}[2+1+0+1+2] = \frac{6}{5} = 1.2$$

Ex.9 Find the mean deviation about mean from the following data

x_i	3	9	17	23	27
f_i	8	10	12	9	5

Sol.

x_i	f_i	f_ix_i	$\|x_i - \bar{x}\|$	$f_i\|x_i - \bar{x}\|$
3	8	24	12	96
9	10	90	6	60
17	12	204	2	24
23	9	207	8	72
27	5	135	12	60
	N = 44	$\Sigma f_ix_i = 660$		$\Sigma f_i\|x_i - \bar{x}\| = 312$

$$\text{Mean } (\bar{x}) = \frac{\Sigma f_ix_i}{N} = \frac{660}{44} = 15$$

$$\text{Mean deviation} = \frac{\Sigma f_i|x_i - \bar{x}|}{N} = \frac{312}{44} = 7.09$$

(iii) Variance and standard deviation : The variance of a distribution is, the mean of squares of deviation of variate from their mean. It is denoted by σ^2 or var(x).

The positive square root of the variance are called the standard deviation. It is denoted by σ or S.D.

Hence standard deviation = $+\sqrt{\text{variance}}$

Formulae for variance :

(i) for ungrouped dist. :

$$\sigma_x^2 = \frac{\Sigma(x_i - \bar{x})^2}{n}$$

$$\sigma_x^2 = \frac{\Sigma x_i^2}{n} - \bar{x}^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2, \quad \text{where } d_i = x_i - a$$

(ii) For freq. dist. :

$$\sigma_x^2 = \frac{\Sigma f_i(x_i - \bar{x})^2}{N}$$

$$\sigma_x^2 = \frac{\Sigma f_i x_i^2}{N} - (\bar{x})^2 = \frac{\Sigma f_i x_i^2}{N} - \left(\frac{\Sigma f_i x_i}{N}\right)^2$$

$$\sigma_d^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$$

$$\sigma_u^2 = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] \quad \text{where } u_i = \frac{d_i}{h}$$

(iii) Coefficient of S.D. = $\frac{\sigma}{\bar{x}}$

Coefficient of variation = $\frac{\sigma}{\bar{x}} \times 100$ (in percentage)

Note :- $\sigma^2 = \sigma_x^2 = \sigma_d^2 = h^2\sigma_u^2$

Ex.10 Find the variance of first n natural numbers

Sol. $\sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2 = \frac{\Sigma n^2}{n} - \left(\frac{\Sigma n}{n}\right)^2 = \frac{n(n+1)(2n+1)}{6n} - \left\{\frac{n(n+1)}{2n}\right\}^2 = \frac{n^2-1}{12}$

Ex.11 If $\sum_{i=1}^{18}(x_i - 8) = 9$ and $\sum_{i=1}^{18}(x_i - 8)^2 = 45$, then find the standard deviation of $x_1, x_2, \ldots x_{18}$

Sol. Let $(x_i - 8) = d_i$

$$\therefore \sigma_x = \sigma_d = \sqrt{\frac{\Sigma d_i^2}{n} - \left(\frac{\Sigma d_i}{n}\right)^2} = \sqrt{\frac{45}{18} - \left(\frac{9}{18}\right)^2} = \sqrt{\frac{5}{2} - \frac{1}{4}} = \frac{3}{2}$$

Ex.12 Find the coefficient of variation of first n natural numbers

Sol. For first n natural numbers.

$$\text{Mean }(\bar{x}) = \frac{n+1}{2}, \text{ S.D.}(\sigma) = \sqrt{\frac{n^2-1}{12}}$$

$$\therefore \text{ coefficient of variance} = \frac{\sigma}{\bar{x}} \times 100 = \sqrt{\frac{n^2-1}{12}} \times \frac{1}{\left(\frac{n+1}{2}\right)} \times 100 = \sqrt{\frac{(n-1)}{3(n+1)}} \times 100$$

6. MEAN SQUARE DEVIATION :

The mean square deviation of a distrubution is the mean of the square of deviations of variate from assumed mean. It is denoted by S^2

Hence $S^2 = \frac{\Sigma(x_i - a)^2}{n} = \frac{\Sigma d_i^2}{n}$ (for ungrouped dist.)

$S^2 = \frac{\Sigma f_i(x_i - a)^2}{N} = \frac{\Sigma f_i d_i^2}{N}$ (for freq. dist.), where $d_i = (x_i - a)$

7. RELATION BETWEEN VARIANCE AND MEAN SQUARE DEVIATION :

$$\because \ \sigma^2 = \frac{\Sigma f_i d_i^2}{N} - \left(\frac{\Sigma f_i d_i}{N}\right)^2$$

$\Rightarrow \sigma^2 = s^2 - d^2$, where $d = \bar{x} - a = \frac{\Sigma f_i d_i}{N}$

$\Rightarrow s^2 = \sigma^2 + d^2 \Rightarrow s^2 \geq \sigma^2$

Hence the variance is the minimum value of mean square deviation of a distribution

Ex.13 Determine the variance of the following frequency dist.

class	0 – 2	2 – 4	4 – 6	6 – 8	8 – 10	10 – 12
f_i	2	7	12	19	9	1

Sol. Let a = 7, h = 2

class	x_i	f_i	$u_i = \frac{x_i - a}{h}$	$f_i u_i$	$f_i u_i^2$
0 – 2	1	2	–3	–6	18
2 – 4	3	7	–2	–14	28
4 – 6	5	12	–1	–12	12
6 – 8	7	19	0	0	0
8 – 10	9	9	1	9	9
10 – 12	11	1	2	2	4
		N = 50		$\Sigma f_i u_i = -21$	$\Sigma f_i u_i^2 = 71$

$$\therefore \ \sigma^2 = h^2\left[\frac{\Sigma f_i u_i^2}{N} - \left(\frac{\Sigma f_i u_i}{N}\right)^2\right] = 4\left[\frac{71}{50} - \left(\frac{-21}{50}\right)^2\right] = 4[1.42 - 0.1764] = 4.97$$

8. MATHEMATICAL PROPERTIES OF VARIANCE :

- $Var.(x_i + \lambda) = Var.(x_i)$

  $Var.(\lambda x_i) = \lambda^2.Var(x_i)$

  $Var(ax_i + b) = a^2.Var(x_i)$

  where λ, a, b, are constant

- If means of two series containing n_1, n_2 terms are $\overline{x}_1, \overline{x}_2$ and their variance's are σ_1^2, σ_2^2 respectively and their combined mean is $\overline{x}$ then the variance σ^2 of their combined series is given by following formula

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{(n_1 + n_2)} \quad \text{where } d_i = \overline{x}_1 - \overline{x},\ d_2 = \overline{x}_2 - \overline{x}$$

i.e. $$\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1 + n_2} + \frac{n_1 n_2}{(n_1 + n_2)^2}(\overline{x}_1 - \overline{x}_2)^2$$

SOLVED EXAMPLES

Ex.1 If in an examination different weights are assigned to different subjects Physics (2), Chemistry (1), English (1), Mathematics (2) A student scores 60 in Physics, 70 in Chemistry, 70 in English and 80 in Mathematics, then weighted mean is-

(1) 60 (2) 70 (3) 80 (4) 85

Sol.(2) Weighted mean $= \dfrac{\sum_{i=1}^{n} w_i x_i}{\sum_{i=1}^{n} w_i} = \dfrac{2\times 60 + 1\times 70 + 1\times 70 + 2\times 80}{6} = 70$

Ex.2 The mean of two groups of sizes 200 and 300 are 25 and 10 respectively.Their standard deviation are 3 and 4 respectively. The variance of combined sample of size 500 is-

(1) 64 (2) 65.2 (3) 67.2 (4) 64.2

Sol.(3) Combined mean $\bar{x} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \dfrac{200\times 25 + 300\times 10}{500} = 16$

Here $d_1 = \bar{x}_1 - \bar{x} = 25 - 16 = 9$ and $d_2 = \bar{x}_2 - \bar{x} = 10 - 16 = -6$

We know that $\sigma^2 = \dfrac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2} = \dfrac{200(9+81) + 300(16+36)}{500} = \dfrac{33600}{500} = 67.2$

Ex.3 If the mean of the series $x_1, x_2, ..., x_n$ is $\bar{x}$, then the mean of the series $x_i + 2i$, i = 1, 2, ..., n will be-

(1) $\bar{x} + n$ (2) $\bar{x} + n + 1$ (3) $\bar{x} + 2$ (4) $\bar{x} + 2n$

Sol.(2) As given $\bar{x} = \dfrac{x_1 + x_2 + ... + x_n}{n}$ ....(1)

If the mean of the series $x_i + 2i$, i = 1, 2, ..., n be $\bar{X}$, then

$$\bar{X} = \frac{(x_1 + 2) + (x_2 + 2.2) + (x_3 + 2.3) + ... + (x_n + 2.n)}{n}$$

$$= \frac{x_1 + x_2 + ... + x_n}{n} + \frac{2(1 + 2 + 3 + ... + n)}{n}$$

$$= \bar{x} + \frac{2n(n+1)}{2n} \quad \text{from (1)}$$

$$= \bar{x} + n + 1$$

Ex.4 The variance of first 20-natural numbers is-

(1) $\dfrac{133}{4}$ (2) $\dfrac{379}{12}$ (3) $\dfrac{133}{2}$ (4) $\dfrac{399}{4}$

Sol.(1) $\because \quad \sigma^2 = \frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2$

$$= \frac{1}{20}[1^2 + 2^2 + ... + 20^2] - \left[\frac{1}{20}(1+2+....+20)\right]^2$$

$$= \frac{1}{20}\frac{20\times 21(2\times 20+1)}{6} - \left[\frac{1}{20}\frac{20\times 21}{2}\right]^2 = \frac{7\times 41}{2} - \frac{441}{4} = \frac{133}{4}.$$

In fact, the variance of first n-natural numbers is $\frac{n^2-1}{12}$

Ex.5 The mean of the following freq. table is 50 and $\Sigma f = 120$

class	0 – 20	20 – 40	40 – 60	60 – 80	80 – 100
f	17	f_1	32	f_2	19

the missing frequencies are-

(1) 28, 24 (2) 24, 36 (3) 36, 28 (4) None of these

Sol.(1) $\Sigma f = 120 = 17 + f_1 + 32 + f_2 + 19$

$\Rightarrow f_1 + f_2 = 52$ ...... (1)

and $\Sigma fx = (10 \times 17) + (30 \times f_1) + (50 \times 32) + (70 \times f_2) + (90 \times 19) = 30f_1 + 70f_2 + 3480$

$$\therefore \quad \bar{x} = \frac{\Sigma fx}{\Sigma f} \Rightarrow 50 = \frac{30f_1 + 70f_2 + 3480}{120}$$

$\Rightarrow 30f_1 + 70f_2 = 2520 \Rightarrow 3f_1 + 7f_2 = 252$ ..... (2)

by (1) and (2) we get $f_1 = 28$, $f_2 = 24$

Ex.6 A student obtained 75%, 80%, 85% marks in three subjects. If the marks of another subject are added then his average marks can not be less than-

(1) 60% (2) 65% (3) 80% (4) 90%

Sol.(1) Total marks obtained from three subjects out of 300 = 75 + 80 + 85 = 240

if the marks of another subject is added then total marks obtained out of 400 is greater than 240

if marks obtained in fourth subject is 0 then

minimum average marks = $\frac{240}{400} \times 100 = 60\%$

Ex.7 The mean and variance of a series containing 5 terms are 8 and 24 respectively. The mean and variance of another series containing 3 terms are also 8 and 24 respectively. The variance of their combined series will be-

(1) 20 (2) 24 (3) 25 (4) 42

Sol.(2) Using $\sigma^2 = \frac{n_1\sigma_1^2 + n_2\sigma_2^2}{n_1+n_2} + \frac{n_1 n_2}{(n_1+n_2)^2}(\bar{x}_1 - \bar{x}_2)^2 \Rightarrow \sigma^2 = \frac{5(24)+3(24)}{5+3} + \frac{5(3)}{(5+3)^2}(8-8)^2 = 24$

Ex.8 The mean deviation about median from the following data 340, 150, 210, 240, 300, 310, 320, is-

(1) 52.4 (2) 52.5 (3) 52.8 (4) none of these

Sol.(3) Arranging the observations in ascending order of magnitude, we have 150, 210, 240, 300, 310, 320, 340. Clearly, the middle observation is 300. So, median = 300

Calculation of Mean deviation

x_i	$\|x_i - 300\|$
340	40
150	150
210	90
240	60
300	0
310	10
320	20
Total	$\sum \|x_i - 300\| = 370$

Mean deviation from median = $\frac{1}{7}\sum|x_i - 300| = \frac{370}{7} = 52.8$

Ex.9 Variance of the data given below is

Size of item	3.5	4.5	5.5	6.5	7.5	8.5	9.5
Frequency	3	7	22	60	85	32	8

(1) 1.29 (2) 2.19 (3) 1.32 (4) none of these

Sol.(3) Let the assumed mean be a = 6.5

Calculation of variance

x_i	f_i	$d_i = x_i - 6.5$	f_id_i	$f_id_i^2$
3.5	3	–3	–9	27
4.5	7	–2	–14	28
5.5	22	–1	–22	22
6.5	60	0	0	0
7.5	85	1	85	85
8.5	32	2	64	128
9.5	8	3	24	72
	$N = \sum f_i = 217$		**$\sum f_id_i = 128$**	**$\sum f_id_i^2 = 362$**

Here N = 217, $\sum f_id_i = 128$ and $\sum f_id_i^2 = 362$

$$\therefore \text{ Var (X)} = \left(\frac{1}{N}\sum f_id_i^2\right) - \left(\frac{1}{N}\sum f_id_i\right)^2 = \frac{362}{217} - \left(\frac{128}{217}\right)^2 = 1.668 - 0.347 = 1.321$$

Ex.10 If a variable takes the value 0, 1, 2......n with frequencies proportional to the bionomial coefficients $^nC_0$, $^nC_1$,.....,^nC_n then the mean of the distribution is-

(1) $\frac{n(n+1)}{4}$ (2) $\frac{n}{2}$ (3) $\frac{n(n-1)}{2}$ (4) $\frac{n(n+1)}{2}$

Sol.(2) $N = \sum f_i = k\,[^nC_0 + {}^nC_1 + ... + {}^nC_n] = k2^n$

$$\sum f_i x_i = k\,[1.{}^nC_1 + 2.\,{}^nC_2 + ... + n\,{}^nC_n] = k\sum_{r=1}^{n} r.{}^nC_r = kn\sum_{r=1}^{n} {}^{n-1}C_{r-1} = kn2^{n-1}$$

Thus $\bar{x} = \frac{1}{2^n}(n\ \ 2^{n-1}) = \frac{n}{2}$.

Ex.11 The mean and variance of 5 observations of an experiment are 4 and 5.2 respectively. If from these observations three are 1, 2 and 6, then the remaining will be-

(1) 2, 9 (2) 5, 6 (3) 4, 7 (4) 3, 8

Sol.(3) As given $\bar{x} = 4$, n = 5 and $\sigma^2 = 5.2$. If the remaining observations are x_1, x_2 then

$$\sigma^2 = \frac{\sum(x_i - \bar{x})^2}{n} = 5.2$$

$$\Rightarrow \frac{(x_1-4)^2 + (x_2-4)^2 + (1-4)^2 + (2-4)^2 + (6-4)^2}{5} = 5.2$$

$\Rightarrow (x_1 - 4)^2 + (x_2 - 4)^2 = 9$ ....(1)

Also $\bar{x} = 4 \Rightarrow \frac{x_1 + x_2 + 1 + 2 + 6}{5} = 4 \Rightarrow x_1 + x_2 = 11$ ....(2)

from eq.(1), (2) $x_1, x_2 = 4, 7$

Ex.12 The mean deviation of the series a, a + d, a + 2d, ..., a + 2nd from its mean is-

(1) $\frac{n+1}{2n+1}|d|$ (2) $\frac{n(n+1)}{2n+1}|d|$ (3) $\frac{n(n-1)}{2n+1}|d|$ (4) none of these

Sol.(2) Number of terms in the series = 2n + 1

$$\therefore \text{ mean } \bar{x} = \frac{a + (a+d) + (a+2d) + ... + (a+2nd)}{2n+1} = \frac{1}{2n+1}\left[\frac{2n+1}{2}(a + a + 2nd)\right] = a + nd$$

Also $\sum|x_i - \bar{x}| = |-nd| + |(1-n)d| + ... + |-d| + 0 + |d| + ... + |nd|$

$$= 2|d|[n + (n-1) + ... + 1] = 2|d|\frac{n(n+1)}{2} = n(n+1)|d|$$

$$\therefore \text{ mean deviation from mean} = \frac{\sum|x_i - \bar{x}|}{N} = \frac{n(n+1)}{2n+1}|d|$$

Ex.13 Let $x_1, x_2, ..., x_n$ be values taken by a variable X and $y_1, y_2, ..., y_n$ be the values taken by a variable Y such that $y_i = ax_i + b$, i = 1, 2, ..., n. Then-

(1) Var (Y) = a^2 Var(X) (2) Var(Y) = a^2 Var(X) + b

(3) Var(Y) = Var(X) + b (4) None of these

Sol.(1) We have,

$$\text{Var}(Y) = \frac{1}{n}\sum_{i=1}^{n}(y_i - \bar{y})^2 \qquad [\because y_i = ax_i + b;\ i = 1, 2, ..., n \Rightarrow \bar{Y} = a\bar{X} + b]$$

$$\Rightarrow \text{Var}(Y) = \frac{1}{n}\sum_{i=1}^{n}a^2(x_i - \bar{X})^2$$

$$\Rightarrow \text{Var}(Y) = a^2\left\{\frac{1}{n}\sum_{i=1}^{n}(x_i - \bar{X})^2\right\} = a^2\,\text{Var}(X)$$

Ex.14 The mean square deviation of a set of n observations $x_1, x_2, ..., x_n$ about a point c is defined as $\frac{1}{n}\sum_{i=1}^{n}(x_i - c)^2$ The mean square deviation about –2 and 2 are 18 and 10 respectively, then standard deviation of this set of observations is-

(1) 3 (2) 2 (3) 1 (4) None of these

Sol.(1) $\because \frac{1}{n}\Sigma(x_i + 2)^2 = 18$ and $\frac{1}{n}\Sigma(x_i - 2)^2 = 10$

$\Rightarrow \Sigma(x_i + 2)^2 = 18n$ and $\Sigma(x_i - 2)^2 = 10n$

$\Rightarrow \Sigma(x_i + 2)^2 + \Sigma(x_i - 2)^2 = 28\,n$ and $\Sigma\,(x_i + 2)^2 - \Sigma(x_i - 2)^2 = 8\,n$

$\Rightarrow 2\Sigma x_i^2 + 8n = 28\,n$ and $8\Sigma x_i = 8n$

$\Rightarrow \Sigma x_i^2 = 10\,n$ and $\Sigma x_i = n$

$$\Rightarrow \frac{\Sigma x_i^2}{n} = 10 \text{ and } \frac{\Sigma x_i}{n} = 1$$

$$\therefore \sigma = \sqrt{\frac{\Sigma x_i^2}{n} - \left(\frac{\Sigma x_i}{n}\right)^2} = \sqrt{10 - (1)^2} = 3$$

CHECK YOUR GRASP STATISTICS EXERCISE-I

Arithmetic mean, weighted mean, Combined mean

1. Mean of the first n terms of the A.P. a, (a + d), (a + 2d), ... is-

(1) $a+\frac{nd}{2}$ (2) $a+\frac{(n-1)d}{2}$

(3) a + (n – 1) d (4) a + nd **SI0001**

2. The A.M. of first n even natural number is -

(1) n(n + 1) (2) $\frac{n+1}{2}$ (3) $\frac{n}{2}$ (4) n + 1

SI0002

3. The A.M. of ${}^nC_0$, ${}^nC_1$, ${}^nC_2$, ...${}^nC_n$ is -

(1) $\frac{2^n}{n}$ (2) $\frac{2^{n+1}}{n}$ (3) $\frac{2^n}{n+1}$ (4) $\frac{2^{n+1}}{n+1}$

SI0003

4. If the mean of numbers 27, 31, 89, 107, 156 is 82, then the mean of numbers 130, 126, 68, 50, 1 will be- **SI0004**

(1) 80 (2) 82 (3) 75 (4) 157

5. If the mean of n observations x_1, x_2, ...x_n is $\overline{x}$, then the sum of deviations of observations from mean is :-

(1) 0 (2) $n\overline{x}$ **SI0005**

(3) $\frac{\overline{x}}{n}$ (4) None of these

6. The mean of 9 terms is 15. if one new term is added and mean become 16, then the value of new term is :- **SI0006**

(1) 23 (2) 25 (3) 27 (4) 30

7. If the mean of first n natural numbers is equal to $\frac{n+7}{3}$, then n is equal to-

(1) 10 (2) 11 **SI0007**

(3) 12 (4) none of these

8. The mean of first three terms is 14 and mean of next two terms is 18. The mean of all the five terms is- **SI0008**

(1) 15.5 (2) 15.0 (3) 15.2 (4) 15.6

9. If the mean of five observations x, x + 2 , x + 4, x+ 6 and x + 8 is 11, then the mean of last three obsevations is- **SI0009**

(1) 11 (2) 13 (3) 15 (4) 17

10. The mean of a set of numbers is $\overline{x}$. If each number is decreased by λ, the mean of the new set is-

(1) $\overline{x}$ (2) $\overline{x}+\lambda$ (3) $\lambda-\overline{x}$ (4) $\overline{x}-\lambda$

SI0010

11. The mean of 50 observations is 36. If its two observations 30 and 42 are deleted, then the mean of the remaining observations is-

(1) 48 (2) 36 **SI0011**

(3) 38 (4) none of these

12. In a frequency dist. , if d_i is deviation of variates from a number ℓ and mean $= \ell+\frac{\Sigma f_i d_i}{\Sigma f_i}$, then ℓ is :-

(1) Lower limit

(2) Assumed mean

(3) Number of observation

(4) Class interval **SI0012**

13. The A.M. of n observation is $\overline{x}$. If the sum of n – 4 observations is K, then the mean of remaining observations is-

(1) $\frac{\overline{x}-K}{4}$ (2) $\frac{n\overline{x}-K}{n-4}$ **SI0013**

(3) $\frac{n\overline{x}-K}{4}$ (4) $\frac{n\overline{x}-(n-4)K}{4}$

14. The mean of values $1,\frac{1}{2},\frac{1}{3},......\frac{1}{n}$ which have frequencies 1, 2, 3, ... n resp., is :-

(1) $\frac{2n+1}{3}$ (2) $\frac{2}{n}$ (3) $\frac{n+1}{2}$ (4) $\frac{2}{n+1}$

SI0014

15. The sum of squares of deviation of variates from their A.M. is always :-

(1) Zero (2) Minimum

(3) Maximum (4) Nothing can be said

SI0015

16. If the mean of following feq. dist. is 2.6, then the value of f is :-

x_i	1	2	3	4	5
f_i	5	4	f	2	3

(1) 1 (2) 3

(3) 8 (4) None of these

SI0016

17. The weighted mean (W.M.) is computed by the formula ?

(1) W.M. = $\frac{\Sigma x_i}{\Sigma w_i}$ (2) W.M. = $\frac{\Sigma w_i}{\Sigma x_i}$

(3) W.M. = $\frac{\Sigma w_i x_i}{\Sigma x_i}$ (4) W.M. = $\frac{\Sigma w_i x_i}{\Sigma w_i}$

SI0017

18. The weighted mean of first n natural numbers when their weights are equal to corresponding natural number, is :-

(1) $\frac{n+1}{2}$ (2) $\frac{2n+1}{3}$ SI0018

(3) $\frac{(n+1)(2n+1)}{6}$ (4) None of these

19. The average income of a group of persons is $\overline{x}$ and that of another group is $\overline{y}$. If the number of persons of both group are in the ratio 4 : 3, then average income of combined group is :-

(1) $\frac{\overline{x}+\overline{y}}{7}$ (2) $\frac{3\overline{x}+4\overline{y}}{7}$ SI0019

(3) $\frac{4\overline{x}+3\overline{y}}{7}$ (4) None of these

20. In a group of students, the mean weight of boys is 65 kg. and mean weight of girls is 55 kg. If the mean weight of all students of group is 61 kg, then the ratio of the number of boys and girls in the group is :- SI0020

(1) 2 : 3 (2) 3 : 1 (3) 3 : 2 (4) 4 : 3

Median, Mode

21. The median of an arranged series of n even observations, will be :-

(1) $\left(\frac{n+1}{2}\right)$ th term

(2) $\left(\frac{n}{2}\right)$ th term

(3) $\left(\frac{n}{2}+1\right)$ th term SI0021

(4) Mean of $\left(\frac{n}{2}\right)$ th and $\left(\frac{n}{2}+1\right)$ th terms

22. The median of the numbers 6, 14, 12, 8, 10, 9, 11, is :- SI0022

(1) 8 (2) 10 (3) 10.5 (4) 11

23. Median of the following freq. dist.

x_i	3	6	10	12	7	15
f_i	3	4	2	8	13	10

(1) 7 (2) 10 SI0023

(3) 8.5 (4) None of these

24. Median is independent of change of :-

(1) only Origin

(2) only Scale

(3) Origin and scale both

(4) Neither origin nor scale SI0024

25. A series which have numbers three 4's, four 5's, five 6's, eight 7's, seven 8's and six 9's then the mode of numbers is :- SI0025

(1) 9 (2) 8 (3) 7 (4) 6

26. Mode of the following freqency distribution

x :	4	5	6	7	8
f :	6	7	10	8	3

SI0026

(1) 5 (2) 6 (3) 8 (4) 10

27. The mode of the following freq. dist is :-

Class	1–10	11–20	21–30	31–40	41–50
f_i	5	7	8	6	4

(1) 24 (2) 23.83 SI0027

(3) 27.16 (4) None of these

Symmetric and asymmetric distribution, Range

28. For a normal dist :-

(1) mean = median

(2) median = mode

(3) mean = mode

(4) mean = median = mode SI0028

29. The relationship between mean, median and mode for a moderately skewed distribution is-

(1) mode = median – 2 mean

(2) mode = 2 median – mean

(3) mode = 2 median – 3 mean

(4) mode = 3 median – 2 mean SI0029

30. The range of observations 2, 3, 5, 9, 8, 7, 6, 5, 7, 4, 3 is :- SI0030

(1) 6 (2) 7 (3) 5.5 (4) 11

Mean Deviation

31. The mean deviation of a frequency dist. is equal to :-

(1) $\frac{\Sigma d_i}{\Sigma f_i}$ (2) $\frac{\Sigma |d_i|}{\Sigma f_i}$ (3) $\frac{\Sigma f_i d_i}{\Sigma f_i}$ (4) $\frac{\Sigma f_i |d_i|}{\Sigma f_i}$

SI0031

32. Mean deviation from the mean for the observation –1, 0, 4 is-

(1) $\sqrt{\frac{14}{3}}$ (2) $\frac{2}{3}$ SI0032

(3) 2 (4) none of these

33. Mean deviation of the observations 70, 42, 63, 34, 44, 54, 55, 46, 38, 48 from median is :-

(1) 7.8 (2) 8.6

(3) 7.6 (4) 8.8 SI0033

34. Mean deviation of 5 observations from their mean 3 is 1.2, then coefficient of mean deviation is :-

(1) 0.24 (2) 0.4 SI0034

(3) 2.5 (4) None of these

35. The mean deviation from median is

(1) greater than the mean deviation from any other central value

(2) less than the mean deviation from any other central value

(3) equal to the mean deviation from any other central value

(4) maximum if all values are positive SI0035

Variance and Standard Deviation

36. The variate x and u are related by $u = \frac{x-a}{h}$ then correct relation between σ_x and σ_u is :-

(1) $\sigma_x = h\sigma_u$ (2) $\sigma_x = h + \sigma_u$

(3) $\sigma_u = h\sigma_x$ (4) $\sigma_u = h + \sigma_x$

SI0036

37. The S.D. of the first n natural numbers is-

(1) $\sqrt{\frac{n^2-1}{2}}$ (2) $\sqrt{\frac{n^2-1}{3}}$

(3) $\sqrt{\frac{n^2-1}{4}}$ (4) $\sqrt{\frac{n^2-1}{12}}$ SI0037

38. The variance of observations 112, 116, 120, 125, 132 is :-

(1) 58.8 (2) 48.8 SI0038

(3) 61.8 (4) None of these

39. If $\sum_{i=1}^{10}(x_i - 15) = 12$ and $\sum_{i=1}^{10}(x_i - 15)^2 = 18$ then the S.D. of observations x_1, x_2 .... x_{10} is :-

(1) $\frac{2}{5}$ (2) $\frac{3}{5}$

(3) $\frac{4}{5}$ (4) None of these

SI0039

40. The S.D. of 7 scored 1, 2, 3, 4, 5, 6, 7 is-

(1) 4 (2) 2

(3) $\sqrt{7}$ (4) none of these

SI0040

41. The variance of series a, a + d, a + 2d, ..., a + 2nd is :-

(1) $\frac{n(n+1)}{2}d^2$ (2) $\frac{n(n+1)}{3}d^2$

(3) $\frac{n(n+1)}{6}d^2$ (4) $\frac{n(n+1)}{12}d^2$ SI0041

42. Variance is independent of change of-

(1) only origin (2) only scale

(3) origin and scale both (4) none of these

SI0042

43. If the coefficient of variation and standard deviation of a distribution are 50% and 20 respectively, then its mean is-

(1) 40 (2) 30

(3) 20 (4) None of these

SI0043

44. If each observation of a dist. whose S.D. is σ, is increased by λ, then the variance of the new observations is -

(1) σ (2) $\sigma+\lambda$

(3) σ^2 (4) $\sigma^2+\lambda$ **SI0044**

45. The variance of 2, 4, 6, 8, 10 is-

(1) 8 (2) $\sqrt{8}$

(3) 6 (4) none of these

SI0045

46. If each observation of a dist., whose variance is σ^2, is multiplied by λ, then the S.D. of the new new observations is-

(1) σ (2) $\lambda\sigma$

(3) $|\lambda|\sigma$ (4) $\lambda^2\sigma$ **SI0046**

47. The standard deviation of variate x_i is σ. Then standard deviation of the variate $\frac{ax_i+b}{c}$, where a , b, c are constants is-

(1) $\left(\frac{a}{c}\right)\sigma$ (2) $\left|\frac{a}{c}\right|\sigma$

(3) $\left(\frac{a^2}{c^2}\right)\sigma$ (4) None of these

SI0047

ANSWER-KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18	19	20
Ans.	2	4	3	3	1	2	2	4	2	4	2	2	3	4	2	1	4	2	3	3
Que.	21	22	23	24	25	26	27	28	29	30	31	32	33	34	35	36	37	38	39	40
Ans.	4	2	3	4	3	2	2	4	4	2	4	3	2	2	2	1	4	2	2	2
Que.	41	42	43	44	45	46	47													
Ans.	2	1	1	3	1	3	2													

PREVIOUS YEAR QUESTIONS STATISTICS EXERCISE-II

1. If the mean deviation of the numbers 1, 1 + d, 1 + 2d, ..., 1 + 100d from their mean is 255, then that d is equal to- **[AIEEE-2009]**

(1) 10.1 (2) 20.2

(3) 10.0 (4) 20.0 **SI0111**

2. **Statement–1 :** The variance of first n even natural numbers is $\frac{n^2-1}{4}$

Statement–2 : The sum of first n natural numbers is $\frac{n(n+1)}{2}$ and the sum of squares of first n natural numbers is $\frac{n(n+1)(2n+1)}{6}$. **[AIEEE-2009]**

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for statement–1. **SI0112**

3. For two data sets each of size is 5, the variances are given to be 4 and 5 and the corresponding measn are given to be 2 and 4 respectively, then the variance of the combined data set is :- **[AIEEE-2010]**

(1) $\frac{5}{2}$ (2) $\frac{11}{2}$

(3) 6 (4) $\frac{13}{2}$ **SI0113**

4. If the mean deviation about the median of the numbers a, 2a, ..., 50a is 50, then |a| equals:- **[AIEEE-2011]**

(1) 4 (2) 5

(3) 2 (4) 3 **SI0114**

5. A scientist is weighing each of 30 fishes. Their mean weight worked out is 30 gm and a standard deviation of 2 gm. Later, it was found that the measuring scale was misaligned and always under reported every fish weight by 2 gm. The correct mean and standard deviation (in gm) of fishes are respectively : **[AIEEE-2011]**

(1) 28, 4 (2) 32, 2

(3) 32, 4 (4) 28, 2 **SI0115**

6. Let $x_1, x_2, \ldots, x_n$ be n observations, and let $\bar{x}$ be their arithmetic mean and σ^2 be their variance. **[AIEEE-2012]**

Statement–1 : Variance of $2x_1, 2x_2, \ldots, 2x_n$ is $4\sigma^2$.

Statement–2 : Arithmetic mean of $2x_1, 2x_2, \ldots, 2x_n$ is $4\bar{x}$.

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true ; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true ; Statement–2 is not a correct explanation for Statement–1. **SI0116**

7. All the students of a class performed poorly in Mathematics. The teacher decided to give grace marks of 10 to each of the students. Which of the following statistical measures will not change even after the grace marks were given? **[JEE-MAINS-2013]**

(1) mean (2) median

(3) mode (4) variance **SI0092**

8. The variance of first 50 even natural numbers is :- **[JEE(Main)-2014]**

(1) $\frac{833}{4}$ (2) 833 (3) 437 (4) $\frac{437}{4}$

SI0093

9. The mean of the data set comprising of 16 observations is 16. If one of the observation valued 16 is deleted and three new observations valued 3, 4 and 5 are added to the data, then the mean of the resultant data, is : **[JEE(Main)-2015]**

(1) 15.8 (2) 14.0 (3) 16.8 (4) 16.0

SI0094

10. If the standard deviation of the numbers 2, 3, a and 11 is 3.5, then which of the following is true ?

(1) $3a^2 - 23a + 44 = 0$ **[JEE(Main)-2016]**
(2) $3a^2 - 26a + 55 = 0$
(3) $3a^2 - 32a + 84 = 0$
(4) $3a^2 - 34a + 91 = 0$

SI0095

11. If $\sum_{i=1}^{9}(x_i - 5) = 9$ and $\sum_{i=1}^{9}(x_i - 5)^2 = 45$, then the standard deviation of the 9 items $x_1, x_2, ..., x_9$ is - **[JEE(Main)-2018]**

(1) 4 (2) 2 (3) 3 (4) 9

SI0096

12. 5 students of a class have an average height 150 cm and variance 18 cm^2. A new student, whose height is 156 cm, joined them. The variance (in cm^2) of the height of these six students is: **[JEE(Main)-2019]**

(1) 22 (2) 20 (3) 16 (4) 18

SI0097

13. A data consists of n observations :

$x_1, x_2, ..., x_n$. If $\sum_{i=1}^{n}(x_i + 1)^2 = 9n$ and $\sum_{i=1}^{n}(x_i - 1)^2 = 5n$, then the standard deviation of this data is : **[JEE(Main)-2019]**

(1) 5 (2) $\sqrt{5}$ (3) $\sqrt{7}$ (4) 2

SI0098

14. The outcome of each of 30 items was observed; 10 items gave an outcome $\frac{1}{2} - d$ each, 10 items gave outcome $\frac{1}{2}$ each and the remaining 10 items gave outcome $\frac{1}{2} + d$ each. If the variance of this outcome data is $\frac{4}{3}$ then |d| equals :- **[JEE(Main)-2019]**

(1) 2 (2) $\frac{\sqrt{5}}{2}$ (3) $\frac{2}{3}$ (4) $\sqrt{2}$

SI0100

15. A student scores the following marks in five tests : 45,54,41,57,43. His score is not known for the sixth test. If the mean score is 48 in the six tests, then the standard deviation of the marks in six tests is **[JEE(Main)-2019]**

(1) $\frac{10}{\sqrt{3}}$ (2) $\frac{100}{\sqrt{3}}$ (3) $\frac{100}{3}$ (4) $\frac{10}{3}$

SI0117

16. If the standard deviation of the numbers –1, 0, 1, k is $\sqrt{5}$ where k > 0, then k is equal to **[JEE(Main)-2019]**

(1) $2\sqrt{\frac{10}{3}}$ (2) $2\sqrt{6}$ (3) $4\sqrt{\frac{5}{3}}$ (4) $\sqrt{6}$

SI0103

17. If for some $x \in R$, the frequency distribution of the marks obtained by 20 students in a test is :

Marks	2	3	5	7
Frequencey	$(x+1)^2$	$2x-5$	x^2-3x	x

then the mean of the marks is : **[JEE(Main)-2019]**

(1) 2.8 (2) 3.2 (3) 3.0 (4) 2.5

SI0104

18. If the data $x_1, x_2, ..., x_{10}$ is such that the mean of first four of these is 11, the mean of the remaining six is 16 and the sum of squares of all of these is 2,000; then the standard deviation of this data is : **[JEE(Main)-2019]**

(1) 4 (2) 2
(3) $\sqrt{2}$ (4) $2\sqrt{2}$

SI0118

ANSWER-KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16	17	18
Ans.	1	2	2	1	2	1	4	2	2	3	2	2	2	4	1	2	1	2

MATHEMATICAL REASONING

1. STATEMENT :

A sentence which is either true or false but cannot be both are called a statement. A sentence which is an exclamatory or a wish or an imperative or an interrogative can not be a statement.

If a statement is true then its truth value is T and if it is false then its truth value is F

For ex.

(i) "New Delhi is the capital of India", a true statement

(ii) "3 + 2 = 6", a false statement

(iii) "Where are you going ?" not a statement beasuse it connot be defined as true or false

Note : A statement cannot be both true and false at a time

2. SIMPLE STATEMENT :

Any statement whose truth value does not depend on other statement are called simple statement

For ex. (i) " $\sqrt{2}$ is an irrational number" (ii) "The set of real number is an infinite set"

3. COMPOUND STATEMENT :

A statement which is a combination of two or more simple statements are called compound statement

Here the simple statements which form a compound statement are known as its sub statements

For ex.

(i) "If x is divisible by 2 then x is even number"

(ii) "ΔABC is equilatral if and only if its three sides are equal"

4. LOGICAL CONNECTIVES :

The words or phrases which combined simple statements to form a compound statement are called logical connectives.

In the following table some possible connectives, their symbols and the nature of the compound statement formed by them

S.N.	Connectives	symbol	use	operation
1.	and	$\wedge$	$p \wedge q$	conjunction
2.	or	$\vee$	$p \vee q$	disjunction
3.	not	$\sim$ or '	$\sim p$ or p'	negation
4.	If .... then	$\Rightarrow$ or $\rightarrow$	$p \Rightarrow q$ or $p \rightarrow q$	Implication or conditional
5.	If and only if (iff)	$\Leftrightarrow$ or $\leftrightarrow$	$p \Leftrightarrow q$ or $p \leftrightarrow q$	Equivalence or Bi-conditional

Explanation :

(i) $p \wedge q \equiv$ statement p and q

($p \wedge q$ is true only when p and q both are true otherwise it is false)

(ii) $p \vee q \equiv$ statement p or q

($p \vee q$ is true if at least one from p and q is true i.e. $p \vee q$ is false only when p and q both are false)

(iii) $\sim p \equiv$ not statement p

($\sim p$ is true when p is false and $\sim p$ is false when p is true)

(iv) $p \Rightarrow q \equiv$ statement p then statement q

($p \Rightarrow q$ is false only when p is true and q is false otherwise it is true for all other cases)

(v) $p \Leftrightarrow q \equiv$ statement p if and only if statement q

($p \Leftrightarrow q$ is true only when p and q both are true or false otherwise it is false)

5. TRUTH TABLE :

A table which shows the relationship between the truth value of compound statement S(p, q, r ....) and the truth values of its sub statements p, q, r, ... is said to be truth table of compound statement S

If p and q are two simple statements then truth table for basic logical connectives are given below

Conjunction

p	q	$p \wedge q$
T	T	T
T	F	F
F	T	F
F	F	F

Disjunction

p	q	$p \vee q$
T	T	T
T	F	T
F	T	T
F	F	F

Negation

p	($\sim p$)
T	F
F	T

Conditional

p	q	$p \rightarrow q$
T	T	T
T	F	F
F	T	T
F	F	T

Biconditional

p	q	$p \rightarrow q$	$q \rightarrow p$	$(p \rightarrow q) \wedge (q \rightarrow p)$ or $p \leftrightarrow q$
T	T	T	T	T
T	F	F	T	F
F	T	T	F	F
F	F	T	T	T

Note : If the compound statement contain n sub statements then its truth table will contain 2^n rows.

6. LOGICAL EQUIVALENCE :

Two compound statements S_1(p, q, r...) and S_2(p, q, r ....) are said to be logically equivalent or simply equivalent if they have same truth values for all logically possibilities

Two statements S_1 and S_2 are equivalent if they have identical truth table i.e. the entries in the last column of their truth table are same. If statements S_1 and S_2 are equivalent then we write $S_1 \equiv S_2$

For ex. The truth table for $(p \rightarrow q)$ and $(\sim p \vee q)$ given as below

p	q	($\sim p$)	$p \rightarrow q$	$\sim p \vee q$
T	T	F	T	T
T	F	F	F	F
F	T	T	T	T
F	F	T	T	T

We observe that last two columns of the above truth table are identical hence compound statements $(p \rightarrow q)$ and $(\sim p \vee q)$ are equivalent

i.e. $\boxed{p \rightarrow q \equiv \sim p \vee q}$

7. TAUTOLOGY AND CONTRADICTION :

(i) Tautology : A statement is said to be a tautology if it is true for all logical possibilities i.e. its truth value always T. it is denoted by t.

For ex. the statement $p \vee \sim(p \wedge q)$ is a tautology

p	q	$p \wedge q$	$\sim(p \wedge q)$	$p \vee \sim(p \wedge q)$
T	T	T	F	T
T	F	F	T	T
F	T	F	T	T
F	F	F	T	T

Clearly, The truth value of $p \vee \sim(p \wedge q)$ is T for all values of p and q. so $p \wedge \sim(p \wedge q)$ is a tautology

(ii) Contradiction : A statement is a contradiction if it is false for all logical possibilities.

i.e. its truth value always F. It is denoted by c.

For ex. The statement $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction

p	q	$\sim p$	$\sim q$	$p \vee q$	$(\sim p \wedge \sim q)$	$(p \vee q) \wedge (\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	F

Clearly, then truth value of $(p \vee q) \wedge (\sim p \wedge \sim q)$ is F for all value of p and q. So $(p \vee q) \wedge (\sim p \wedge \sim q)$ is a contradiction.

Note : The negation of a tautology is a contradiction and negation of a contradiction is a tautology

8. DUALITY :

Two compound statements S_1 and S_2 are said to be duals of each other if one can be obtained from the other by replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$

If a compound statement contains the special variable t (tautology) and c (contradiction) then obtain its dual we replaced t by c and c by t in addition to replacing $\wedge$ by $\vee$ and $\vee$ by $\wedge$.

Note :

(i) the connectives $\wedge$ and $\vee$ are also called dual of each other.

(ii) If $S^*(p, q)$ is the dual of the compound statement $S(p, q)$ then

(a) $S^*(\sim p, \sim q) \equiv \sim S(p, q)$ (ii) $\sim S^*(p, q) \equiv S(\sim p, \sim q)$

For ex. The duals of the following statements

(i) $(p \wedge q) \vee (r \vee s)$ (ii) $(p \vee t) \wedge (p \vee c)$

(iii) $\sim(p \wedge q) \vee [p \wedge \sim(q \vee \sim s)]$

are as given below

(i) $(p \vee q) \wedge (r \wedge s)$

(ii) $(p \wedge c) \vee (p \wedge t)$

(iii) $\sim(p \vee q) \wedge [p \vee \sim(q \wedge \sim s)]$

9. CONVERSE, INVERSE AND CONTRAPOSITIVE OF THE CONDITIONAL STATEMENT ($p \to q$):

(i) Converse : The converse of the conditional statement $p \to q$ is defined as $q \to p$

(ii) Inverse : The inverse of the conditional statement $p \to q$ is defined as $\sim p \to \sim q$

(iii) Contrapositive : The contrapositive of conditional statement $p \to q$ is defined as $\sim q \to \sim p$

10. NEGATION OF COMPOUND STATEMENTS :

If p and q are two statements then

(i) **Negation of conjunction :** $\sim(p \wedge q) \equiv \sim p \vee \sim q$

p	q	~ p	~ q	$(p \wedge q)$	$\sim(p \wedge q)$	$(\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

(ii) Negation of disjunction : $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	~ p	~ q	$(p \vee q)$	$(\sim p \vee q)$	$(\sim p \wedge \sim q)$
T	T	F	F	T	F	F
T	F	F	T	T	F	F
F	T	T	F	T	F	F
F	F	T	T	F	T	T

(iii) Negation of conditional : $\sim(p \to q) \equiv p \wedge \sim q$

p	q	~ q	$(p \to q)$	$\sim(p \to q)$	$(p \wedge \sim q)$
T	T	F	T	F	F
T	F	T	F	T	T
F	T	F	T	F	F
F	F	T	T	F	F

(iv) Negation of biconditional : $\sim(p \leftrightarrow q) \equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

we know that $p \leftrightarrow q \equiv (p \to q) \wedge (q \to p)$

$\therefore \sim(p \leftrightarrow q) \equiv \sim[(p \to q) \wedge (q \to p)]$

$\equiv \sim(p \to q) \vee \sim(q \to p)$

$\equiv (p \wedge \sim q) \vee (q \wedge \sim p)$

Note : The above result also can be proved by preparing truth table for $\sim(p \leftrightarrow q)$ and $(p \wedge \sim q) \vee (q \wedge \sim p)$

11. ALGEBRA OF STATEMENTS :

If p, q, r are any three statements then the some low of algebra of statements are as follow

(i) Idempotent Laws :

(a) $p \wedge p \equiv p$ (b) $p \vee p \equiv p$

i.e. $p \wedge p \equiv p \equiv p \vee p$

p	$(p \wedge p)$	$(p \vee p)$
T	T	T
F	F	F

(ii) Comutative laws :

(a) $p \wedge q \equiv q \wedge p$ (b) $p \vee q \equiv q \vee p$

p	q	$(p \wedge q)$	$(q \wedge p)$	$(p \vee q)$	$(q \vee p)$
T	T	T	T	T	T
T	F	F	F	T	T
F	T	F	F	T	T
F	F	F	F	F	F

(iii) Associative laws :

(a) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(b) $(p \vee q) \vee r \equiv p \vee (q \vee r)$

p	q	r	$(p \wedge q)$	$(q \wedge r)$	$(p \wedge q) \wedge r$	$p \wedge (q \wedge r)$
T	T	T	T	T	T	T
T	T	F	T	F	F	F
T	F	T	F	F	F	F
T	F	F	F	F	F	F
F	T	T	F	T	F	F
F	T	F	F	F	F	F
F	F	T	F	F	F	F
F	F	F	F	F	F	F

Similarly we can proved result (b)

(iv) Distributive laws : (a) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$ (c) $p \wedge (q \wedge r) \equiv (p \wedge q) \wedge (p \wedge r)$

(b) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (d) $p \vee (q \vee r) \equiv (p \vee q) \vee (p \vee r)$

p	q	r	$(q \vee r)$	$(p \wedge q)$	$(p \wedge r)$	$p \wedge (q \vee r)$	$(p \wedge q) \vee (p \wedge r)$
T	T	T	T	T	T	T	T
T	T	F	T	T	F	T	T
T	F	T	T	F	T	T	T
T	F	F	F	F	F	F	F
F	T	T	T	F	F	F	F
F	T	F	T	F	F	F	F
F	F	T	T	F	F	F	F
F	F	F	F	F	F	F	F

Similarly we can prove result (b), (c), (d)

(v) De Morgan Laws : (a) $\sim (p \wedge q) \equiv \sim p \vee \sim q$

(b) $\sim(p \vee q) \equiv \sim p \wedge \sim q$

p	q	$\sim p$	$\sim q$	$(p \wedge q)$	$\sim (p \wedge q)$	$(\sim p \vee \sim q)$
T	T	F	F	T	F	F
T	F	F	T	F	T	T
F	T	T	F	F	T	T
F	F	T	T	F	T	T

Similarly we can proved resulty (b)

(vi) Involution laws (or Double negation laws) : $\sim(\sim p) \equiv p$

p	$\sim p$	$\sim (\sim p)$
T	F	T
F	T	F

(vii) Identity Laws : If p is a statement and t and c are tautology and contradiction respectively then

(a) $p \wedge t \equiv p$ (b) $p \vee t \equiv t$ (c) $p \wedge c \equiv c$ (d) $p \vee c \equiv p$

p	t	c	$(p \wedge t)$	$(p \vee t)$	$(p \wedge c)$	$(p \vee c)$
T	T	F	T	T	F	T
F	T	F	F	T	F	F

(viii) Complement Laws :

(a) $p \wedge (\sim p) \equiv c$ (b) $p \vee (\sim p) \equiv t$ (c) $(\sim t) \equiv c$ (d) $(\sim c) \equiv t$

p	$\sim p$	$(p \wedge \sim p)$	$(p \vee \sim p)$
T	F	F	T
F	T	F	T

(ix) Contrapositive laws : $p \to q \equiv \sim q \to \sim p$

p	q	~ p	~ q	p → q	~ q →~ p
T	T	F	F	T	T
T	F	F	T	F	F
F	T	T	F	T	T
F	F	T	T	T	T

12. QUANTIFIED STATEMENTS AND QUANTIFIERS :

The words or phrases "All", "Some", "None", "There exists a" are examples of quantifiers.

A statement containing one or more of these words (or phrases) is a quantified statement.

E.g. (1) All dogs are poodles

(2) Some books have hard covers

(3) There exists an odd number which is prime.

Note : Phrases "There exists a" and "Atleast one" and the word "some" have the same meaning.

NEGATION OF QUANTIFIED STATEMENTS :

(1) '**None**' is the negation of '**at least one**' or '**some**' or '**few**'

Statement : Some dogs are poodles.

Negation : No dogs are poodles.

Similarly negation of '**some**' is '**none**'

(2) The negation of "**some A are B**" or "**There exist A which is B**" is "**No A are (is) B**" or "**There does not exist any A which is B".**

Statement-1 : Some boys in the class are smart

Statement-2 : There exists a boy in the class who is smart

Statement-3 :Alteast one boy in the class is smart

All the three above statements have same meaning as they all indicate "**existence**" of smart boy in the class.

Negation of these statements is

No boy in the class is smart.

or

There does not exist any boy in the class who is smart.

(3) Negation of "**All A are B**" is "**Some A are not B**".

Statement : All boys in the class are smart.

Negation : Some boys in the class are not smart.

or

There exists a boy in the class who is not smart.

SOLVED EXAMPLES

Ex.1 Which of the following is correct for the statements p and q ?

(1) $p \wedge q$ is true when at least one from p and q is true

(2) $p \rightarrow q$ is true when p is true and q is false

(3) $p \leftrightarrow q$ is true only when both p and q are true

(4) $\sim (p \vee q)$ is true only when both p and q are false

Sol.(4) We know that $p \wedge q$ is true only when both p and q are true so option (1) is not correct

we know that $p \rightarrow q$ is false only when p is true and q is false so option (2) is not correct

we know that $p \leftrightarrow q$ is true only when either p and q both are true or both are flase

so option (3) is not correct

we know that $\sim(p \vee q)$ is true only when $(p \vee q)$ is false

i.e. p and q both are false

So option (4) is correct

Ex.2 $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to-

(1) p (2) ~p (3) q (4) ~q

Sol.(2) $\because \sim(p \vee q) \vee (\sim p \wedge q) \equiv (\sim p \wedge \sim q) \vee (\sim p \wedge q)$ (By Demorgan Law)

$\equiv \sim p \wedge (\sim q \vee q)$ (By distributive laws)

$\equiv \sim p \wedge t$ (By complement laws)

$\equiv \sim p$ (By Identity Laws)

Ex.3 Which of the following is logically equivalent to $(p \wedge q)$?

(1) $p \rightarrow \sim q$ (2) $\sim p \vee \sim q$ (3) $\sim(p \rightarrow \sim q)$ (4) $\sim(\sim p \wedge \sim q)$

Sol.(3) $\because p \rightarrow \sim q \equiv \sim p \vee \sim q \equiv \sim(p \wedge q)$ $(\because p \rightarrow q \equiv \sim p \vee q)$

i.e. $\sim(p \rightarrow \sim q) \equiv p \wedge q$

$\because \sim p \vee \sim q \equiv \sim(p \wedge q)$

and $\sim(\sim p \wedge \sim q) \equiv p \vee q$

Ex.4 If $p \rightarrow (q \vee r)$ is false, then the truth values of p, q, r respectively are-

(1) T, F, F (2) F, F, F (3) F, T, T (4) T, T, F

Sol.(1) We know $p \rightarrow (q \vee r)$ is false only when p is true and $(q \vee r)$ is false. but $(q \vee r)$ is false only when q and r both are false

Hence truth values of p, q, r are respectively T, F, F

Ex.5 Statement $(p \wedge \sim q) \wedge (\sim p \vee q)$ is

(1) a tautology (2) a contradiction

(3) neither a tautology not a contradiction (4) None of these

Sol.(2) $\because (p \wedge \sim q) \wedge (\sim p \vee q)$

$\equiv (p \wedge \sim q) \wedge \sim(p \wedge \sim q)$ (By Demargon Laws)

$\equiv c$, where c is contradiction (By complement laws)

Ex.6 Negation of the statement $p \rightarrow (q \wedge r)$ is-

(1) $\sim p \rightarrow \sim(q \wedge r)$ (2) $\sim p \vee (q \wedge r)$ (3) $(q \wedge r) \rightarrow p$ (4) $p \wedge (\sim q \vee \sim r)$

Sol.(4) $\sim(p \rightarrow (q \wedge r)) \equiv p \wedge \sim(q \wedge r)$ $(\because \sim(p \rightarrow q) \equiv p \wedge \sim q)$

$\equiv p \wedge (\sim q \vee \sim r)$

Ex.7 If $x = 5$ and $y = -2$ then $x - 2y = 9$. The contrapositive of this statement is-

(1) If $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$
(2) If $x - 2y \neq 9$ then $x \neq 5$ and $y \neq -2$
(3) If $x - 2y = 9$ then $x = 5$ and $y = -2$
(4) None of these

Sol.(1) Let p, q, r be the three statements such that

$p : x = 5,\quad q : y = -2$ and $r : x - 2y = 9$

Here given statement is $(p \wedge q) \rightarrow r$ and its contrapositive is $\sim r \rightarrow \sim(p \wedge q)$

i.e. $\sim r \rightarrow (\sim p \vee \sim q)$

i.e. if $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$

Ex.8 Which of the following is wrong ?

(1) $p \rightarrow q$ is logically equivalent to $\sim p \vee q$
(2) If the $(p \vee q) \wedge (q \vee r)$ is true then truth values of p, q, r are T, F, T respectively
(3) $\sim(p \wedge (q \vee r)) \equiv (\sim p \vee \sim q) \wedge (\sim p \vee \sim r)$
(4) The truth value of $p \wedge \sim(p \vee q)$ is always T

Sol.(4) We know that $p \rightarrow q \equiv \sim p \vee q$

If $(p \vee q) \wedge (q \vee r)$ is true then

$(p \vee q)$ and $(q \vee r)$ both are true.

i.e. truth values of p, q, r may be T, F, T respectively

$\because\ \sim(p \wedge (q \vee r)) \equiv \sim((p \wedge q) \vee (p \wedge r) \equiv \sim(p \wedge q) \wedge \sim(p \wedge r) \equiv (\sim p \vee \sim q) \wedge (\sim p \vee \sim r)$

If p is true and q is false then $\sim(p \vee q)$ is false i.e. $p \wedge \sim(p \vee q)$ is false

Ex.9 If $S^*(p, q, r)$ is the dual of the compound statement $S(p, q, r)$ and $S(p, q, r) = \sim p \wedge [\sim(q \vee r)]$ then $S^*(\sim p, \sim q, \sim r)$ is equivalent to-

(1) $S(p, q, r)$
(2) $\sim S(\sim p, \sim q, \sim r)$
(3) $\sim S(p, q, r)$
(4) $S^*(p, q, r)$

Sol.(3) $\because\ S(p, q, r) = \sim p \wedge [\sim(q \vee r)]$

So $S(\sim p, \sim q, \sim r) \equiv \sim(\sim p) \wedge [\sim(\sim q \vee \sim r)] \equiv p \wedge (q \wedge r)$

$S^*(p, q, r) \equiv \sim p \vee [\sim(q \wedge r)]$

$S^*(\sim p, \sim q, \sim r) \equiv p \vee (q \vee r)$

Clearly $S^*(\sim p, \sim q, \sim r) \equiv \sim S(p, q, r)$

Ex.10 The negation of the statement "If a quadrilateral is a square then it is a rhombus"

(1) If a quadrilateral is not a square then is a rhombus it
(2) If a quadrilateral is a square then it is not a rhombus
(3) a quadrilateral is a square and it is not a rhombus
(4) a quadritateral is not a square and it is a rhombus

Sol.(3) Let p and q be the statements as given below

p : a quadrilateral is a square

q : a quadritateral is a rhombus

the given statement is $p \rightarrow q$

$\because\ \sim(p \rightarrow q) \equiv p \wedge \sim q$

Therefore the negation of the given statement is a quadrilateral is a square and it is not a rhombus

CHECK YOUR GRASP MATHEMATICAL REASONING EXERCISE-I

1. The inverse of the statement $(p \wedge \sim q) \rightarrow r$ is-
(1) $\sim(p \vee \sim q) \rightarrow \sim r$ (2) $(\sim p \wedge q) \rightarrow \sim r$
(3) $(\sim p \vee q) \rightarrow \sim r$ (4) None of these
MR0001

2. $(\sim p \vee \sim q)$ is logically equivalent to-
(1) $p \wedge q$ (2) $\sim p \rightarrow q$
(3) $p \rightarrow \sim q$ (4) $\sim p \rightarrow \sim q$ **MR0002**

3. The equivalent statement of $(p \leftrightarrow q)$ is-
(1) $(p \wedge q) \vee (p \vee q)$
(2) $(p \rightarrow q) \vee (q \rightarrow p)$
(3) $(\sim p \vee q) \vee (p \vee \sim q)$
(4) $(\sim p \vee q) \wedge (p \vee \sim q)$ **MR0003**

4. If the compound statement $p \rightarrow (\sim p \vee q)$ is false then the truth value of p and q are respectively-
MR0004
(1) T, T (2) T, F (3) F, T (4) F, F

5. The statement $(p \rightarrow \sim p) \wedge (\sim p \rightarrow p)$ is-
(1) a tautology
(2) a contradiction
(3) neither a tautology nor a contradiction
(4) None of these **MR0005**

6. Negation of the statement $(p \wedge r) \rightarrow (r \vee q)$ is-
(1) $\sim(p \wedge r) \rightarrow \sim(r \vee q)$ (2) $(\sim p \vee \sim r) \vee (r \vee q)$
(3) $(p \wedge r) \wedge (r \wedge q)$ (4) $(p \wedge r) \wedge (\sim r \wedge \sim q)$
MR0006

7. The dual of the statement $\sim p \wedge [\sim q \wedge (p \vee q) \wedge \sim r]$ is-
(1) $\sim p \vee [\sim q \vee (p \vee q) \vee \sim r]$
(2) $p \vee [q \vee (\sim p \wedge \sim q) \vee r]$
(3) $\sim p \vee [\sim q \vee (p \wedge q) \vee \sim r]$
(4) $\sim p \vee [\sim q \wedge (p \wedge q) \wedge \sim r]$ **MR0007**

8. Which of the following is correct-
(1) $(\sim p \vee \sim q) \equiv (p \wedge q)$
(2) $(p \rightarrow q) \equiv (\sim q \rightarrow \sim p)$
(3) $\sim(p \rightarrow \sim q) \equiv (p \wedge \sim q)$
(4) $\sim(p \leftrightarrow q) \equiv (p \rightarrow q) \vee (q \rightarrow p)$ **MR0008**

9. The contrapositive of $p \rightarrow (\sim q \rightarrow \sim r)$ is-
(1) $(\sim q \wedge r) \rightarrow \sim p$ (2) $(q \rightarrow r) \rightarrow \sim p$
(3) $(q \vee \sim r) \rightarrow \sim p$ (4) None of these
MR0009

10. The converse of $p \rightarrow (q \rightarrow r)$ is-
(1) $(q \wedge \sim r) \vee p$ (2) $(\sim q \vee r) \vee p$
(3) $(q \wedge \sim r) \wedge \sim p$ (4) $(q \wedge \sim r) \wedge p$
MR0010

11. If p and q are two statement then $(p \leftrightarrow \sim q)$ is true when-
(1) p and q both are true (2) p and q both are false
(3) p is false and q is true (4) None of these
MR0011

12. Statement $(p \wedge q) \rightarrow p$ is-
(1) a tautology (2) a contradiction
(3) neither (1) nor (2) (4) None of these
MR0012

13 If statements p, q, r have truth values T, F, T respectively then which of the following statement is true-
(1) $(p \rightarrow q) \wedge r$ (2) $(p \rightarrow q) \vee \sim r$
(3) $(p \wedge q) \vee (q \wedge r)$ (4) $(p \rightarrow q) \rightarrow r$
MR0013

14. If statement $p \rightarrow (q \vee r)$ is true then the truth values of statements p, q, r respectively-
(1) T, F, T (2) F, T, F
(3) F, F, F (4) All of these
MR0014

15. Which of the following statement is a contradiction-
(1) $(p \wedge q) \wedge (\sim(p \vee q))$ (2) $p \vee (\sim p \wedge q)$
(3) $(p \rightarrow q) \rightarrow p$ (4) $\sim p \vee \sim q$ **MR0015**

16. The negative of the statement "If a number is divisible by 15 then it is divisible by 5 or 3"
(1) If a number is divisible by 15 then it is not divisible by 5 and 3
(2) A number is divisible by 15 and it is not divisible by 5 or 3
(3) A number is divisible by 15 or it is not divisible by 5 and 3
(4) A number is divisible by 15 and it is not divisible by 5 and 3 **MR0016**

17. If x = 5 and y = –2 then x – 2y = 9. The contrapositive of this statement is-
(1) If $x - 2y \neq 9$ then $x \neq 5$ or $y \neq -2$
(2) If $x - 2y \neq 9$ then $x \neq 5$ and $y \neq -2$
(3) If $x - 2y = 9$ then $x = 5$ and $y = -2$
(4) None of these **MR0017**

18. The negation of the statement "2 + 3 = 5 and 8 < 10" is-
(1) $2 + 3 \neq 5$ and $8 \nless 10$
(2) $2 + 3 \neq 5$ or $8 > 10$
(3) $2 + 3 \neq 5$ or $8 \geq 10$
(4) None of these **MR0018**

19. For any three simple statement p, q, r the statement $(p \wedge q) \vee (q \wedge r)$ is true when-
(1) p and r true and q is false
(2) p and r false and q is true
(3) p, q, r all are false
(4) q and r true and p is false **MR0019**

20. Which of the following statement is a tautology-
(1) $(\sim p \vee \sim q) \vee (p \vee \sim q)$
(2) $(\sim p \vee \sim q) \wedge (p \vee \sim q)$
(3) $\sim p \wedge (\sim p \vee \sim q)$
(4) $\sim q \wedge (\sim p \vee \sim q)$ **MR0020**

21. Which of the following statement is a contradiction-
(1) $(\sim p \vee \sim q) \vee (p \vee \sim q)$
(2) $(p \rightarrow q) \vee (p \wedge \sim q)$
(3) $(\sim p \wedge q) \wedge (\sim q)$
(4) $(\sim p \wedge q) \vee (\sim q)$ **MR0021**

22. The negation of the statement $q \vee (p \wedge \sim r)$ is equivalent to-
(1) $\sim q \wedge (p \rightarrow r)$ (2) $\sim q \wedge \sim(p \rightarrow r)$
(3) $\sim q \wedge (\sim p \wedge r)$ (4) None of these
MR0022

23. Let Q be a non empty subset of N. and q is a statement as given below :-
q : There exists an even number $a \in Q$.
Negation of the statement q will be :-
(1) There is no even number in the set Q.
(2) Every $a \in Q$ is an odd number.
(3) (1) and (2) both
(4) None of these **MR0023**

24. The statement $\sim(p \rightarrow q) \leftrightarrow (\sim p \vee \sim q)$ is-
(1) a tautology
(2) a contradiction
(3) neither a tautology nor a contradiction
(4) None of these **MR0024**

25. Which of the following is equivalent to $(p \wedge q)$
(1) $p \rightarrow \sim q$ (2) $\sim(\sim p \wedge \sim q)$
(3) $\sim(p \rightarrow \sim q)$ (4) None of these
MR0025

26. The dual of the following statement "Reena is healthy and Meena is beautiful" is-
(1) Reena is beaufiful and Meena is healthy
(2) Reena is beautiful or Meena is healthy
(3) Reena is healthy or Meena is beutiful
(4) None of these **MR0026**

27. If p is any statement, t and c are a tautology and a contradiction respectively then which of the following is not correct-
(1) $p \wedge t \equiv p$ (2) $p \wedge c \equiv c$
(3) $p \vee t \equiv c$ (4) $p \vee c \equiv p$ **MR0027**

28. If S*(p, q) is the dual of the compound statement S(p, q) then S*(~p, ~q) is equivalent to-
(1) S(~p, ~q) (2) ~S(p, q)
(3) ~S*(p, q) (4) None of these
MR0028

29. If p is any statement, t is a tautology and c is a contradiction then which fo the following is not correct-
(1) $p \wedge (\sim c) \equiv p$
(2) $p \vee (\sim t) \equiv p$
(3) $t \vee c \equiv p \vee t$
(4) $(p \wedge t) \vee (p \vee c) \equiv (t \wedge c)$ **MR0029**

30. If p, q, r are simple statement with truth values T, F, T respectively then the truth value of $((\sim p \vee q) \wedge \sim r) \rightarrow p$ is-
(1) True (2) False
(3) True if r is false (4) True if q is true
MR0030

31. Which of the following is wrong-
(1) $p \vee \sim p$ is a tautology
(2) $\sim(\sim p) \leftrightarrow p$ is a tautology
(3) $p \wedge \sim p$ is a contradiction
(4) $((p \wedge p) \rightarrow q) \rightarrow p$ is a tautology **MR0031**

32. The statement "If $2^2 = 5$ then I get first class" is logically equivalent to-
(1) $2^2 = 5$ and I do not get first class
(2) $2^2 = 5$ or I do not get first class
(3) $2^2 \neq 5$ or I get first class
(4) None of these **MR0032**

33. If statement $(p \vee \sim r) \rightarrow (q \wedge r)$ is false and statement q is true then statement p is-

(1) true

(2) false

(3) may be true or false

(4) None of these **MR0033**

34. Which of the following statement are not logically equivalent-

(1) $\sim(p \vee \sim q)$ and $(\sim p \wedge q)$

(2) $\sim(p \rightarrow q)$ and $(p \wedge \sim q)$

(3) $(p \rightarrow q)$ and $(\sim q \rightarrow \sim p)$

(4) $(p \rightarrow q)$ and $(\sim p \wedge q)$ **MR0034**

35. Consider the following statements

p : Virat kohli plays cricket.

q : Virat kohli is good at maths

r : Virat kohli is successful.

then negation of the statement "If virat kohli plays cricket and is not good at maths then he is successful" will be :-

(1) $\sim p \wedge (q \wedge r)$ (2) $(\sim p \vee q) \wedge r$

(3) $p \wedge (\sim q \wedge \sim r)$ (4) None of these

MR0035

36. Let p statement "If $2 > 5$ then earth will not rotate" and q be the statement "$2 \not> 5$ or earth will not rotate".

Statement–1 : p and q are equivalent.

Statement–2 : $m \rightarrow n$ and $\sim m \vee n$ are equivalent.

(1) Statement–1 is true, Statement–2 is true; Statement–2 is not the correct explanation of Statement–1.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is false.

(4) Statement–1 is true, Statement–2 is true; Statement–2 is the correct explanation of Statement–1. **MR0036**

37. Which of the following is a tautology :-

(1) $[(\sim p \wedge p) \rightarrow q] \longrightarrow (p \wedge p)$

(2) $[(\sim p \wedge p) \rightarrow q] \longrightarrow (\sim p \rightarrow p)$

(3) $[(\sim p \wedge p) \rightarrow q] \longrightarrow (p \rightarrow p)$

(4) None of these **MR0037**

38. Negation of the statement "No one in the class is fond of music" is :-

(1) everyone in the class is fond of music.

(2) Some of the students in the class are fond of music.

(3) There exists a student in the class who is fond of music.

(4) (2) and (3) both **MR0038**

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15
Ans.	**3**	**3**	**4**	**2**	**2**	**4**	**3**	**2**	**1**	**1**	**3**	**1**	**4**	**4**	**1**
Que.	16	17	18	19	20	21	22	23	24	25	26	27	28	29	30
Ans.	**2**	**1**	**3**	**4**	**1**	**3**	**1**	**3**	**3**	**3**	**3**	**3**	**2**	**4**	**1**
Que.	31	32	33	34	35	36	37	38							
Ans.	**4**	**3**	**3**	**4**	**3**	**4**	**3**	**4**							

PREVIOUS YEAR QUESTIONS MATHEMATICAL REASONING EXERCISE-II

1. **Statement–1 :** $\sim(p \leftrightarrow \sim q)$ is equivalent to $p \leftrightarrow q$.

Statement–2 : $\sim(p \leftrightarrow \sim q)$ is a tautology.

[AIEEE-2009]

(1) Statement–1 is true, Statement–2 is false.

(2) Statement–1 is false, Statement–2 is true.

(3) Statement–1 is true, Statement–2 is true; Statement–2 is a correct explanation for Statement–1.

(4) Statement–1 is true, Statement–2 is true; Statement–2 is not a correct explanation for statement–1.

MR0061

2. Let S be a non-empty subset of R.

Consider the following statement :

p : There is a rational number $x \in S$ such that $x > 0$

which of the following statements is the negation of the statement p ? **[AIEEE-2010]**

(1) There is a rational number $x \in S$ such that $x \le 0$

(2) There is no rational number $x \in S$ such that $x \le 0$

(3) Every rational number $x \in S$ satisfies $x \le 0$

(4) $x \in S$ and $x \le 0 \Rightarrow$ x is not rational

MR0062

3. Consider the following statements

p : Suman is brilliant

q : Suman is rich

r : Suman is honest

The negation of the statement "Suman is brilliant and dishonest if and only if Suman is rich" can be expressed as :- **[AIEEE-2011]**

(1) $\sim q \leftrightarrow \sim p \wedge r$ (2) $\sim(p \wedge \sim r) \leftrightarrow q$

(3) $\sim p \wedge (q \leftrightarrow \sim r)$ (4) $\sim(q \leftrightarrow (p \wedge \sim r))$

MR0063

4. The only statement among the followings that is a tautology is : **[AIEEE-2011]**

(1) $q \rightarrow [p \wedge (p \rightarrow q)]$

(2) $p \wedge (p \vee q)$

(3) $p \vee (p \wedge q)$

(4) $[p \wedge (p \rightarrow q)] \rightarrow q$ **MR0064**

5. The negation of the statement **[AIEEE-2012]** "If I become a teacher, then I will open a school", is :

(1) I will not become a teacher or I will open a school.

(2) I will become a teacher and I will not open a school.

(3) Either I will not become a teacher or I will not open a school.

(4) Neither I will become a teacher nor I will open a school. **MR0039**

6. Consider :

Statement-I : $(p \wedge \sim q) \wedge (\sim p \wedge q)$ is a fallacy.

Statement-II : $(p \rightarrow q) \leftrightarrow (\sim q \rightarrow \sim p)$ is a tuatology. **[JEE(Main)-2013]**

(1) Statement-I is true, Statement-II is true; statement-II is a **correct** explanation for Statement-I.

(2)Statement-I is true, Statement-II is true; statement-II is **not** a correct explanation for Statement-I.

(3) Statement-I is true, Statement-II is false.

(4) Statement-I is false, Statement-II is true.

MR0040

7. The statement $\sim(p \leftrightarrow \sim q)$ is :

(1) equivalent to $p \leftrightarrow q$ **[JEE(Main)-2014]**

(2) equivalent to $\sim p \leftrightarrow q$

(3) a tautology

(4) a fallacy **MR0041**

8. The negation of $\sim s \vee (\sim r \wedge s)$ is equivalent to : **[JEE(Main)-2015]**

(1) $s \vee (r \vee \sim s)$ (2) $s \wedge r$

(3) $s \wedge \sim r$ (4) $s \wedge (r \wedge \sim s)$

MR0042

9. The Boolean Expression $(p \wedge \sim q) \vee q \vee (\sim p \wedge q)$ is equivalent to :- **[JEE(Main)-2016]**

(1) $p \vee \sim q$ (2) $\sim p \wedge q$

(3) $p \wedge q$ (4) $p \vee q$ **MR0043**

10. The following statement

$(p \to q) \to [(\sim p \to q) \to q]$ is : **[JEE(Main)-2017]**

(1) a fallacy

(2) a tautology

(3) equivalent to $\sim p \to q$

(4) equivalent to $p \to \sim q$ **MR0044**

11. The Boolean expression $\sim(p \vee q) \vee (\sim p \wedge q)$ is equivalent to : **[JEE(Main)-2018]**

(1) p (2) q (3) ~q (4) ~p **MR0045**

12. If the Boolean expression $(p \oplus q) \wedge (\sim p \odot q)$ is equivalent to $p \wedge q$, where $\oplus, \odot \in \{\wedge, \vee\}$, then the ordered pair $(\oplus, \odot)$ is : **[JEE(Main)-2019]**

(1) $(\wedge, \vee)$ (2) $(\vee, \vee)$

(3) $(\wedge, \wedge)$ (4) $(\vee, \wedge)$ **MR0046**

13. If q is false and $p \wedge q \leftrightarrow r$ is true, then which one of the following statements is a tautology? **[JEE(Main)-2019]**

(1) $(p \vee r) \to (p \wedge r)$ (2) $p \vee r$

(3) $p \wedge r$ (4) $(p \wedge r) \to (p \vee r)$ **MR0049**

14. Contrapositive of the statement "If two numbers are not equal, then their squares are not equal." is :- **[JEE(Main)-2019]**

(1) If the squares of two numbers are equal, then the numbers are equal.

(2) If the squares of two numbers are equal, then the numbers are not equal.

(3) If the squares of two numbers are not equal, then the numbers are equal.

(4) If the squares of two numbers are not equal, then the numbers are not equal. **MR0050**

15. The contrapositive of the statement "If you are born in India, then you are a citizen of India", is : **[JEE(Main)-2019]**

(1) If you are born in India, then you are not a citizen of India.

(2) If you are not a citizen of India, then you are not born in India.

(3) If you are a citizen of India, then you are born in India.

(4) If you are not born in India, then you are not a citizen of India. **MR0051**

16. If the truth value of the statement $p \to (\sim q \vee r)$ is false(F), then the truth values of the statements p, q, r are respectively : **[JEE(Main)-2019]**

(1) F, T, T (2) T, F, F

(3) T, T, F (4) T, F, T **MR0053**

ANSWER KEY

Que.	1	2	3	4	5	6	7	8	9	10	11	12	13	14	15	16
Ans.	1	3	2,4	4	2	2	1	2	4	2	4	1	4	1	2	3