Calculus Formulae

July 28, 2024

1 Limits

1.

$$\lim_{x\to a}\frac{x^n-a^n}{x-a}=na^{n-1}$$

2.

$$\lim_{x\to a}[1+f(x)]^{\frac{1}{g(x)}}=\exp\left[\lim_{x\to a}\frac{f(x)}{g(x)}\right]$$

where $f(x)/g(x)$ is in the form 0/0 or ∞/∞.

3.

$$\lim_{x\to 0}\frac{e^x-1}{x}=1$$

4.

$$\lim_{x\to 0}\frac{e^{kx}-1}{x}=k$$

where k is a constant.

5.

$$\lim_{x\to 0}\frac{\log(1+x)}{x}=1$$

6.

$$\lim_{x\to 0}\frac{\log(1+kx)}{x}=k$$

7.

$$\lim_{x\to 0}x\sin\left(\frac{1}{x}\right)=0$$

8.

$$\lim_{x\to 0}\frac{\sin(x)}{x}=1$$

9.

$$\lim_{x\to 0}\frac{\sin(kx)}{x}=k$$

10.

$$\lim_{x\to 0}\frac{\tan(x)}{x}=1$$

11.

$$\lim_{x\to 0}\frac{\tan(kx)}{x}=k$$

12.

$$\lim_{x\to 0}\frac{\sin^{-1}(x)}{x}=1$$

13.

$$\lim_{x\to 0}\frac{\sin^{-1}(kx)}{x}=k$$

14.

$$\lim_{x\to 0}\frac{\tan^{-1}(x)}{x}=1$$

15.

$$\lim_{x\to 0}\frac{\tan^{-1}(kx)}{x}=k$$

16.

$$\lim_{x\to 0}\frac{1-\cos(kx)}{x^2}=\frac{k^2}{2}$$

17.

$$\lim_{x\to 0}\frac{\log(1+ax)}{x}=a$$

18.

$$\lim_{x\to 0}\frac{a^x-1}{x}=\log(a)$$

This result becomes 1 when $a=e$.

2 Derivatives

1. $y=x^n, y'=nx^{n-1}$

2. $y = \sqrt{x}, y' = \frac{1}{2\sqrt{x}}$

3. $y = \log(x), y' = \frac{1}{x}$

4. $y = 1/x, y' = -1/x^2$

5. $y = e^x, y' = e^x$, exponential function yields itself as the derivative.

6. $y = a^x, y' = a^x \log(a)$

7. $y = \sin(x), y' = \cos(x)$

8. $y = \cos(x), y' = -\sin(x)$

9. $y = \tan(x), y' = -\sec^2(x)$

10. $y = \sec(x), y' = \sec(x)\tan(x)$

11. $y = \csc(x), y' = -\csc(x)\cot(x)$

12. $y = \cot(x), y' = -\csc^2(x)$

13. $y = \sin^{-1}(x), y' = \frac{1}{\sqrt{1-x^2}}$

14. $y = \cos^{-1}(x), y' = \frac{-1}{\sqrt{1-x^2}}$

15. $y = \tan^{-1}(x), y' = \frac{1}{1+x^2}$

16. $y = \cot^{-1}(x), y' = \frac{-1}{1+x^2}$

17. $y = \sec^{-1}(x), y' = \frac{1}{|x|\sqrt{x^2-1}}$

18. $y = \csc^{-1}(x), y' = \frac{-1}{|x|\sqrt{x^2-1}}$

19. **Chain Rule** For a function $y(u)$ where u is also a function in x as $u(x)$,

$$\frac{dy}{dx} = \frac{dy}{du} \times \frac{du}{dx}$$

3 Integration

Integration by substitution This rule is derived from the chain rule for differentiation. Since $[f(u)]' = f'(u)u'$, the reverse is also true when you take the antiderivative. This means that

$$\int f(u) = \int f(u)u' \tag{1}$$

if u is a function in x.

In other words, if $F' = f$,

$$F(u(x)) + C = \int f(u)u'dx \tag{2}$$

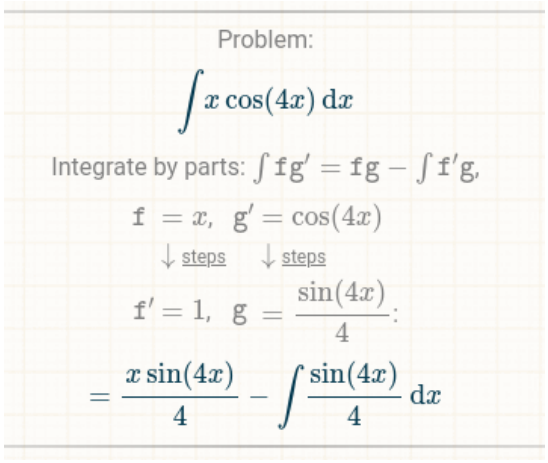
Integration by parts If we need to find $\int u\,dv$, where u can be easily differentiated, and dv can be easily integrated, then

$$\int udv = uv - \int vdu \tag{3}$$

OR

$$\int fg'dx = fg - \int f'gdx$$

Remember, **integrate** dv to get the function v.



https://www.integral-calculator.com/

3.1 Integrals

1. $\int dx = x + C$

2. $\int k\,dx = kx + C$

3. $$\int x^n dx = \frac{x^{n+1}}{n+1} + C$$

4. $$\int \sqrt{x}\,dx = \frac{x^{3/2}}{3/2} + C$$

5.

$$\int x^{-1}dx = \ln(x) + C$$

6.

$$\int e^x dx = e^x + C, \int e^{mx}dx = \frac{e^x}{m} + C$$

7.

$$\int a^x dx = \frac{a^x}{\ln(a)} + C,$$

$$\int a^{mx}dx = \frac{a^{mx}}{m\ln(a)} + C$$

8.

$$\int a^x b^x dx = \frac{(ab)^x}{\ln(ab)} + C$$

9.

$$\int \sin(x)dx = -\cos(x) + C$$

$$\int \sin(mx)dx = \frac{-\cos(mx)}{m} + C$$

10.

$$\int \cos(x)dx = \sin(x) + C$$

$$\int \cos(mx)dx = \frac{\sin(mx)}{m} + C$$

11.

$$\int \tan(x)dx = \ln|\sec(x)| + C$$

$$\int \cot(x)dx = \ln|\sin(x)| + C$$

12.

$$\int \sec(x)dx = \ln|\sec(x) + \tan(x)| + C$$

OR

$$\int \sec(x)dx = \ln\left|\tan\left(\frac{\pi}{4} + \frac{x}{2}\right)\right|$$

13.

$$\int \csc(x)dx = \ln|\csc(x) - \cot(x)| + C$$

OR

$$\int \csc(x)dx = \ln\left|\tan\left(\frac{x}{2}\right)\right| + C$$

14.

$$\int \sec^2(x)dx = \tan(x) + C$$

15.

$$\int \csc^2(x)dx = -\cot(x) + C$$

16.

$$\int \tan^2(x)dx = \int (\sec^2(x) - 1)dx = \tan(x) - x + C$$

17.

$$\int \cot^2(x) = -\cot(x) - x + C$$

18.

$$\int \frac{1}{\sqrt{1-x^2}}dx = \sin^{-1}(x) + C = -\cos^{-1}(x)$$

$$\int \frac{1}{\sqrt{a^2-x^2}}dx = \sin^{-1}\left(\frac{x}{a}\right) + C = -\cos^{-1}\left(\frac{x}{a}\right)$$

19.

$$\int \frac{1}{x^2+1}dx = \tan^{-1}(x) + C = -\cot^{-1}(x)$$

$$\int \frac{1}{x^2+a^2}dx = \frac{1}{a}\tan^{-1}\left(\frac{x}{a}\right) + C$$

20.

$$\int \frac{1}{x\sqrt{x^2-a^2}}dx = \frac{1}{a}\sec^{-1}\left(\frac{x}{a}\right) = -\frac{1}{a}\csc^{-1}\left(\frac{x}{a}\right)$$

21.

$$\int \frac{1}{a^2-x^2}dx = \frac{1}{2a}\ln\left|\frac{a+x}{a-x}\right| + C$$

22.

$$\int \frac{1}{x^2-a^2}dx = \frac{1}{2a}\ln\left|\frac{x-a}{x+a}\right| + C$$

23.

$$\int \sqrt{a^2-x^2}d\ x = \frac{x}{2}\sqrt{a^2-x^2} + \frac{a^2}{2}\sin^2\left(\frac{x}{|a|}\right) + C$$

24.

$$\int \sqrt{x^2-a^2}dx = \frac{x}{2}\sqrt{x^2-a^2} - \frac{a^2}{2}\ln\left|x + \sqrt{x^2-a^2}\right| + C$$

25.

$$\int \sqrt{x^2+a^2}dx = \frac{x}{2}\sqrt{x^2+a^2} + \frac{a^2}{2}\ln\left|x + \sqrt{x^2+a^2}\right| + C$$